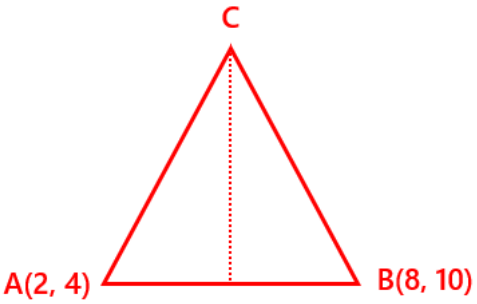
HARDER PARALLEL AND PERPENDICULAR LINES

1) Triangle ABC has vertices $A(2,4)$ and $B(8,10)$.
The line of symmetry of the triangle is the perpendicular bisector of AB.
Find the equation of the perpendicular bisector in the form $ax + by = c$.



Work out the midpoint of AB:

$Midpoint = \left(\frac{2+8}{2}, \frac{4+10}{2}\right) = (5,7)$

Work out the gradient of AB:

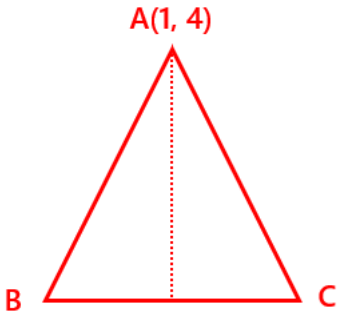
$m_{AB} = \frac{10-4}{8-2} = \frac{6}{6} = 1$

Perpendicular bisector has a gradient of -1:

$y = mx + c$
$7 = -1(5) + c$
$c = 12$
$y = -x + 12$

$\boldsymbol{x + y = 12}$

2) Triangle ABC is isosceles with AB = AC.
A has coordinates $(1,4)$. B and C lie on the line $y = x + 2$.
Find the equation of the line of symmetry. Give your answer in the form $y = mx + c$.

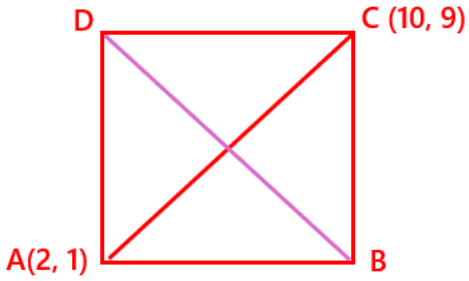


Gradient of BC = 1
Line of symmetry is perpendicular and has gradient -1

Passes through A(1, 4):

$y = mx + c$
$4 = -1(1) + c$
$4 = -1 + c$
$c = 5$
$\boldsymbol{y = -x + 5}$

3) ABCD is a square.
A has coordinates $(2, 1)$ and C has coordinates $(10, 9)$.
Find the equation of the diagonal BD in the form $ax + by = c$.



The diagonals of a square are perpendicular

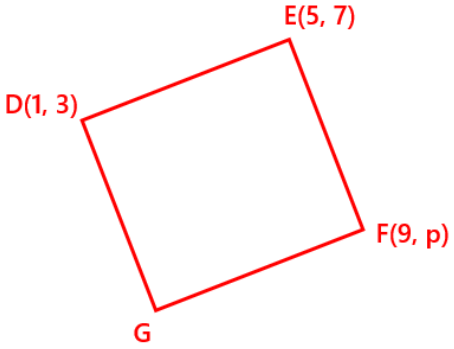
Midpoint of AC = centre of square
$Midpoint = \left(\frac{2+10}{2}, \frac{1+9}{2}\right) = (6, 5)$

Gradient of AC:
$m_{AC} = \frac{9-1}{10-2} = \frac{8}{8} = 1$

BD is perpendicular and has a gradient of -1
Equation through (6,5):
$y = mx + c$
$5 = -1(6) + c$
$c = 11$
$y = -x + 11$

$\boldsymbol{x + y = 11}$

4) DEFG is a square.
The point D has coordinates $(1, 3)$
The point E has coordinates $(5, 7)$.
The point F has coordinates $(9, \mathrm{p})$.
Work out the value of p.
You must show clear, algebraic working.
DE = EF and the line segments are perpendicular



Gradient of DE:
$m_{DE} = \frac{7-3}{5-1} = \frac{4}{4} = 1$

Gradient of EF is -1
Using $E(5,7)$ and $F(9,p)$:
$m = \frac{p-7}{9-5} = \frac{p-7}{4}$

Set equal:
$\frac{p-7}{4} = -1$

$p - 7 = -4$
$\boldsymbol{p = 3}$

5) ABCD is a rectangle.
A has coordinates $(2, 1)$.
B has coordinates $(8, 4)$.
C has coordinates $(12, 0)$.
Work out the equation of the line AD.
Give your answer in the form $px + qy = r$.

AB is perpendicular to AD

Gradient of AB:
$m_{AB} = \frac{4-1}{8-2} = \frac{3}{6} = \frac{1}{2}$

Gradient of AD = -2
Passes through A(2,1):
$y = mx + c$
$1 = -2(2) + c$
$1 = -4 + c$
$c = 5$
$y = -2x + 5$

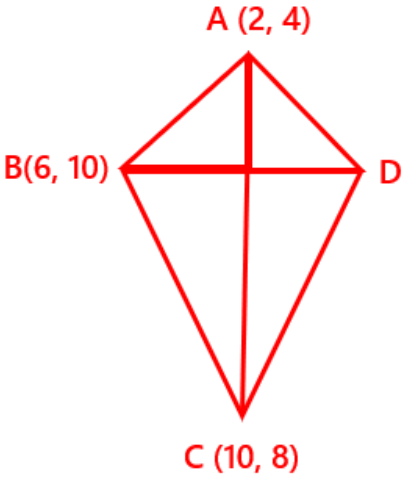
$\boldsymbol{2x + y = 5}$

6) ABCD is a kite with AB = AD and CB = CD.
The coordinates of A are $(2, 4)$.
The coordinates of B are $(6, 10)$.
The coordinates of C are $(10, 8)$.

a. Work out the equation of BD. Give your answer in the form $ax + by + c = 0$.

$m_{AC} = \frac{4-8}{2-10} = \frac{-4}{-8} = \frac{1}{2}$

$m_{BD} = -2$



Equation of BD:
Passes through (6, 10)
$y = mx + c$
$10 = -2(6) + c$
$10 = -12 + c$
$c = 22$

$y = -2x + 22$
$\boldsymbol{2x + y - 22 = 0}$

b. Work out the coordinates of the point D.
The point of intersection of AC and BD is the midpoint of BD

Equation of AC
Passes through (2, 4):
$y = mx + c$
$4 = \frac{1}{2}(2) + c$
$4 = 1 + c$
$c = 3$
$y = \frac{1}{2}x + 3$

Set equations equal to find point of intersection:

$-2x + 22 = \frac{1}{2}x + 3$

$-4x + 44 = x + 6$

$5x = 38$

$x = \frac{38}{5}$

$y = -2\left(\frac{38}{5}\right) + 22 = \frac{34}{5}$

Midpoint of BD is $\left(\frac{38}{5}, \frac{34}{5}\right)$

Solve for the x and y coordinates of D using (6, 10) and the midpoint:

$\frac{6 + x}{2} = \frac{38}{5}$

$6 + x = \frac{76}{5}$

$x = \frac{46}{5}$

$\frac{10 + y}{2} = \frac{34}{5}$

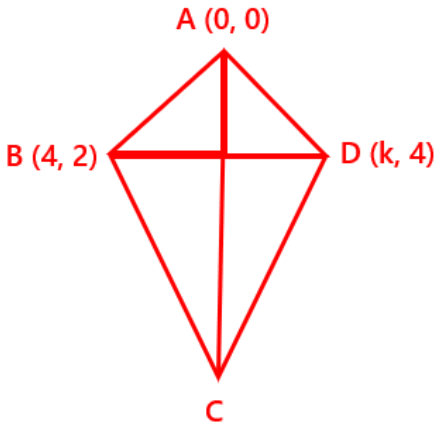
$10 + y = \frac{68}{5}$

$y = \frac{18}{5}$

$\boldsymbol{D\left(\frac{46}{5}, \frac{18}{5}\right)}$

7) ABCD is a kite with AB = AD and CB = CD.
The coordinates of the points are: $A(0, 0)$, $B(4, 2)$, $D(k, 4)$.
Given that $k > 0$, work out the value of k.
You must demonstrate clear, algebraic working.

AB = AD



Use distance formula:

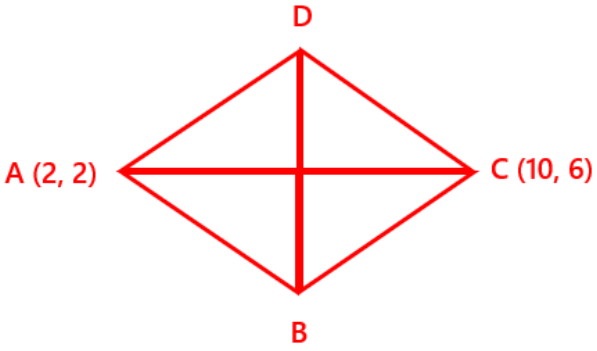
$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(4 - 0)^2 + (2 - 0)^2}$
$= \sqrt{16 + 4}$
$= \sqrt{20}$

$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(k - 0)^2 + (4 - 0)^2}$
$= \sqrt{k^2 + 16}$

Setting equal:

$\sqrt{20} = \sqrt{k^2 + 16}$
$20 = k^2 + 16$
$k^2 = 4$
$k = \pm 2$
$\boldsymbol{k = 2}$

8) ABCD is a rhombus.
A is the point $(2, 2)$ and C is the point $(10, 6)$.
The diagonal BD passes through the midpoint of AC and is perpendicular to AC.
Find the equation of BD. Give your answer in the form $ax + by = c$.



BD passes through the midpoint of AC:

$Midpoint = \left(\frac{2+10}{2}, \frac{2+6}{2}\right) = (6, 4)$

Gradient AC:

$m_{AC} = \frac{6-2}{10-2} = \frac{4}{8} = \frac{1}{2}$

Gradient of BD = -2

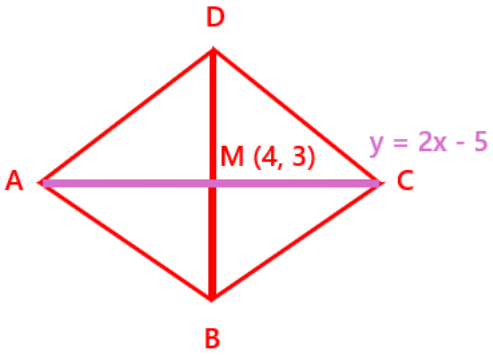
$y = mx + c$
$4 = -2(6) + c$
$4 = -12 + c$
$c = 16$
$y = -2x + 16$

$\mathbf{2x + y = 16}$

9) ABCD is a rhombus.
The diagonals of the rhombus intersect at the point $M(4, 3)$.
A and C lie on the line $5 + y = 2x$.
The diagonal BD is perpendicular to AC and passes through the point M.
Work out the coordinates of the point where BD intersects the x-axis.

Equation of AC:

$5 + y = 2x$
$y = 2x - 5$ Gradient of AC = 2



Gradient of BD = $-\frac{1}{2}$

Passes through M(4, 3):

$y = mx + c$

$3 = -\frac{1}{2}(4) + c$

$3 = -2 + c$

$c = 5$

$y = -\frac{1}{2}x + 5$

Intersects the x-axis when y = 0:

$0 = -\frac{1}{2}x + 5$

$\frac{1}{2}x = -5$

$x = -10$

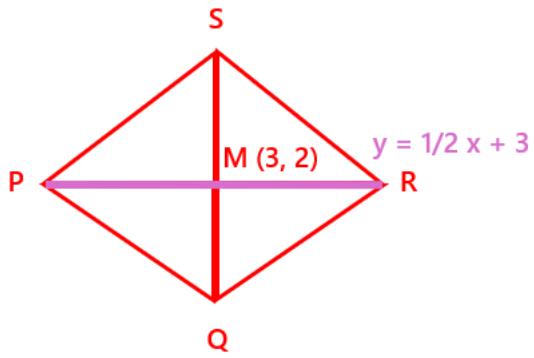
$\mathbf{(-10, 0)}$

10) PQRS is a rhombus.
The diagonals of the rhombus intersect at the point $M(3, 2)$.
The diagonal PR lies on the line with the equation $2y = x + 6$.
The diagonal QS is perpendicular to PR and passes through the point M.
Work out the coordinates of the point where the diagonal QS intersects the x-axis.

Equation of PR:

$2y = x + 6$

$y = \frac{1}{2}x + 3$



Gradient of PR $= \frac{1}{2}$

Gradient of QS = -2

Passes through M(3, 2):

$y = mx + c$

$2 = -2(3) + c$

$2 = -6 + c$

$c = 8$

$y = -2x + 8$

Intersects the x-axis when y = 0:

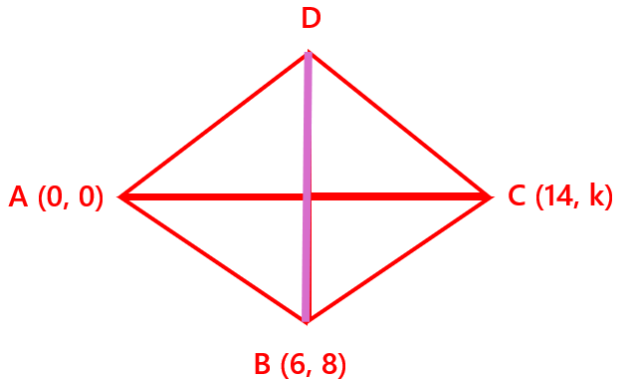
$0 = -2x + 8$

$2x = 8$

$x = 4$

$(\mathbf{4}, \mathbf{0})$

11) ABCD is a rhombus.
A has coordinates $(0, 0)$.
B has coordinates $(6, 8)$.
C has coordinates $(14, k)$.
Given all sides of the rhombus are equal, and $k > 2$, work out the equation of the line BD.
Give your answer in the form $y = mx + c$.



Use distance to solve for k

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(6 - 0)^2 + (8 - 0)^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100}$$

$$CB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(14 - 6)^2 + (k - 8)^2}$$
$$= \sqrt{64 + (k - 8)^2}$$

Set equal:

$$64 + (k - 8)^2 = 100$$
$$(k - 8)^2 = 36$$
$$k - 8 = \pm\sqrt{36}$$

$k - 8 = -6$ or $k - 8 = 6$
~~$k = 2$~~ $k = 14$

BD passes through the midpoint of AC:

$$Midpoint = \left(\frac{0+14}{2}, \frac{0+14}{2}\right) = (7,7)$$

Gradient of BD:

$$m_{BD} = \frac{8-7}{6-7} = \frac{1}{-1} = -1$$

Equation of BD
Passes through (6, 8):

$$y = mx + c$$
$$8 = -1(6) + c$$
$$8 = -6 + c$$
$$c = 14$$

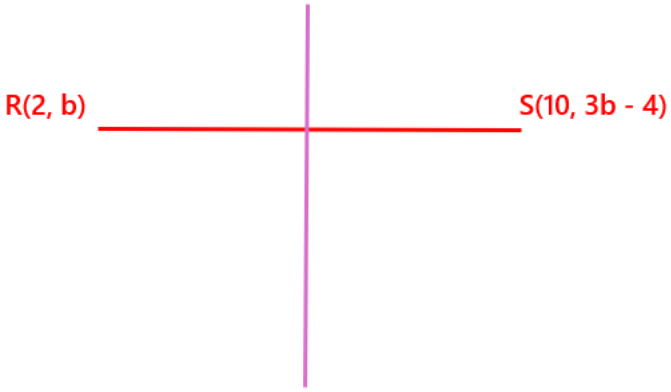
$$\boldsymbol{y = -x + 14}$$

12) RS is a straight line drawn on a square grid.
The grid has a scale of 1 cm for 1 unit on each axis.
R has coordinates $(2, b)$ and S has coordinates $(10, 3b - 4)$, where $b > 0$.
The length of RS is 10 cm.
Find an equation of the perpendicular bisector of RS.
Give your answer in the form $ay = bx + c$, where a, b and c are integers.

Use distance of RS to solve for b:

$RS = \sqrt{(10-2)^2 + ((3b-4)-b)^2}$
$10 = \sqrt{8^2 + (2b-4)^2}$
$10 = \sqrt{64 + (2b-4)^2}$
$100 = 64 + (2b-4)^2$
$36 = (2b-4)^2$
$2b - 4 = \pm 6$

$2b - 4 = 6$ or $2b - 4 = -6$
$2b = 10$ $2b = -2$
$b = 5$ ~~$b = -1$~~



$R = (2, 5)$
$S = (10, 3(5) - 4) = (10, 11)$

Midpoint of RS:

$Midpoint = \left(\frac{2+10}{2}, \frac{5+11}{2}\right) = (6, 8)$

Gradient of RS:

$m_{RS} = \frac{11-5}{10-2} = \frac{6}{8} = \frac{3}{4}$

Perpendicular gradient:

$m = -\frac{4}{3}$

Equation of perpendicular bisector:

$y = mx + c$

$8 = -\frac{4}{3}(6) + c$

$8 = -8 + c$

$c = 16$

$y = -\frac{4}{3}x + 16$

$\mathbf{3y = -4x + 48}$