PARALLEL AND PERPENDICULAR LINES

Task 1

1) Find the equation of the line parallel to $y = 3x + 2$ which passes through $(0, 5)$.

$\boldsymbol{y = 3x + 5}$

2) Find the equation of the line parallel to $y = -2x + 1$ which passes through $(0, -3)$.

$\boldsymbol{y = -2x - 3}$

3) Find the equation of the line parallel to $y = \frac{1}{2}x + 4$ which passes through $(0, 7)$.

$\boldsymbol{y = \frac{1}{2}x + 7}$

4) Find the equation of the line parallel to $y = -\frac{3}{4}x + 6$ which passes through $(0, 1)$.

$\boldsymbol{y = -\frac{3}{4}x + 1}$

5) Find the equation of the line parallel to $y = 4x - 2$ which passes through $(2, 3)$.

$y = mx + c$
$3 = 4(2) + c$
$3 = 8 + c$
$c = -5$

$\boldsymbol{y = 4x - 5}$

6) Find the equation of the line parallel to $y = -x + 6$ which passes through $(3, 2)$.

$y = mx + c$
$2 = -1(3) + c$
$2 = -3 + c$
$c = 5$

$\boldsymbol{y = -x + 5}$

7) Find the equation of the line parallel to $y = \frac{2}{3}x + 1$ which passes through $(6, 5)$.

$y = mx + c$
$5 = \frac{2}{3}(6) + c$
$5 = 4 + c$
$c = 1$

$\boldsymbol{y = \frac{2}{3}x + 1}$

8) Find the equation of the line parallel to $y = -\frac{5}{2}x - 1$ which passes through $(2, -6)$.

$y = mx + c$
$-6 = -\frac{5}{2}(2) + c$
$-6 = -5 + c$
$c = -1$

$\boldsymbol{y = -\frac{5}{2}x - 1}$

9) Find the equation of the line parallel to $2y = 6x + 4$ which passes through $(1, 7)$.

$2y = 6x + 4$
$\div 2 \quad \div 2$
$y = 3x + 2 \qquad m = 3$

$y = mx + c$
$7 = 3(1) + c$
$7 = 3 + c$
$c = 4$

$\boldsymbol{y = 3x + 4}$

10) Find the equation of the line parallel to $3y - 6x = 9$ which passes through $(2, 1)$.

$3y - 6x = 9$
$3y = 6x + 9$
$\div 3 \quad \div 3$
$y = 2x + 3 \qquad m = 2$

$y = mx + c$
$1 = 2(2) + c$
$1 = 4 + c$
$c = -3$

$\boldsymbol{y = 2x - 3}$

Task 2

11) Find the equation of the line perpendicular to $y = 2x + 7$ which passes through $(0, 4)$.

$\boldsymbol{y = -\frac{1}{2}x + 4}$

12) Find the equation of the line perpendicular to $y = -3x + 1$, which passes through $(0, 6)$.

$\boldsymbol{y = \frac{1}{3}x + 6}$

13) Find the equation of the line perpendicular to $y = \frac{1}{2}x - 4$, which passes through $(2, 3)$.

Gradient = -2

$y = mx + c$

$3 = -2(2) + c$

$3 = -4 + c$

$c = 7$

$\boldsymbol{y = -2x + 7}$

14) Find the equation of the line perpendicular to $y = -4x + 2$, which passes through $(1, -1)$.

Gradient = $\frac{1}{4}$

$y = mx + c$

$-1 = \frac{1}{4}(1) + c$

$-1 = \frac{1}{4} + c$

$c = -\frac{5}{4}$

$\boldsymbol{y = \frac{1}{4}x - \frac{5}{4}}$

15) Find the equation of the line perpendicular to $y = 5x - 3$, which passes through $(3, 2)$.

Gradient = $-\frac{1}{5}$

$y = mx + c$

$2 = -\frac{1}{5}(3) + c$

$2 = -\frac{3}{5} + c$

$c = \frac{13}{5}$

$\boldsymbol{y = -\frac{1}{5}x + \frac{13}{5}}$

16) Find the equation of the line perpendicular to $y = -\frac{2}{3}x + 1$, which passes through $(6, 4)$.

Gradient = $\frac{3}{2}$

$y = mx + c$

$4 = \frac{3}{2}(6) + c$

$4 = 9 + c$

$c = -5$

$\boldsymbol{y = \frac{3}{2}x - 5}$

17) Find the equation of the line perpendicular to $y = x + 2$, which passes through $(2, -3)$.

Gradient = -1

$y = mx + c$

$-3 = -1(2) + c$

$-3 = -2 + c$

$c = -1$

$\boldsymbol{y = -x - 1}$

18) Find the equation of the line perpendicular to $y = -\frac{1}{4}x + 3$, which passes through $(4, 2)$.

Gradient = 4

$y = mx + c$

$2 = 4(4) + c$

$2 = 16 + c$

$c = -14$

$\boldsymbol{y = 4x - 14}$

19) Find the equation of the line perpendicular to $2y = 8x + 6$, which passes through $(1, 2)$.

$2y = 8x + 6$

$\div 2 \quad \div 2$

$y = 4x + 3$

Gradient = $-\frac{1}{4}$

$y = mx + c$

$2 = -\frac{1}{4}(1) + c$

$2 = -\frac{1}{4} + c$

$c = \frac{9}{4}$

$$\boldsymbol{y = -\frac{1}{4}x + \frac{9}{4}}$$

20) Find the equation of the line perpendicular to $3y - 3x = 6$, which passes through $(3, 1)$.

$3y - 3x = 6$

$3y = 3x + 6$

$\div 3 \quad \div 3$

$y = x + 2$

Gradient = -1

$y = mx + c$

$1 = -1(3) + c$

$1 = -3 + c$

$c = 4$

$$\boldsymbol{y = -x + 4}$$

Task 3

21) Line 1 passes through the points $(1, 2)$ and $(3, 6)$. Work out the equation of Line 2, which is parallel and passes through the point $(0, 1)$.

$m = \frac{6-2}{3-1} = \frac{4}{2} = 2$

$$\boldsymbol{y = 2x + 1}$$

22) Line 1 passes through the points $(1, 1)$ and $(3, 5)$. Work out the equation of Line 2, which is perpendicular and passes through the point $(0, 2)$.

$m_1 = \frac{5-1}{3-1} = \frac{4}{2} = 2$

$m_2 = -\frac{1}{2}$

$$\boldsymbol{y = -\frac{1}{2}x + 2}$$

23) Line 1 passes through the points $(2, 3)$ and $(4, 7)$. Work out the equation of Line 2, which is parallel and passes through the point $(1, 1)$.

$m = \frac{7-3}{4-2} = \frac{4}{2} = 2$

$y = mx + c$

$1 = 2(1) + c$

$1 = 2 + c$

$c = -1$

$$\boldsymbol{y = 2x - 1}$$

24) Line 1 passes through the points $(0, 0)$ and $(-8, -9)$. Work out the equation of Line 2, which is perpendicular and passes through the point $(2, 1)$.

$m_1 = \frac{-9-0}{-8-0} = \frac{-9}{-8} = \frac{9}{8}$

$m_2 = -\frac{8}{9}$

$y = mx + c$

$1 = -\frac{8}{9}(2) + c$

$1 = -\frac{16}{9} + c$

$c = \frac{25}{9}$

$$\boldsymbol{y = -\frac{8}{9}x + \frac{25}{9}}$$

25) Line 1 passes through the points $(1, 3)$ and $(-2, -5)$. Work out the equation of Line 2, which is parallel and passes through the point $(6, -4)$.

$$m = \frac{-5-3}{-2-1} = \frac{-8}{-3} = \frac{8}{3}$$

$$y = mx + c$$
$$-4 = \frac{8}{3}(6) + c$$
$$-4 = 16 + c$$
$$c = -20$$

$$\boldsymbol{y = \frac{8}{3}x - 20}$$

26) Line 1 passes through the points $(10, m)$ and $(8, 2m)$. Line 2 passes through the points $(4, 8)$ and $(-2, m)$. Given that the lines are parallel, work out the value of m.

$$m_1 = \frac{2m-m}{8-10} = \frac{m}{-2}$$
$$m_2 = \frac{m-8}{-2-4} = \frac{m-8}{-6}$$

$$\frac{m}{-2} = \frac{m-8}{-6}$$

$$-2(m-8) = -6m$$
$$-2m + 16 = -6m$$
$$16 = -4m$$
$$\boldsymbol{m = -4}$$

27) Line 1 passes through the points $(2, 2)$ and $(4, p)$. Line 2 passes through the points $(3, 5)$ and $(5, 7)$. Given that the lines are perpendicular, work out the value of p.

$$m_1 = \frac{p-2}{4-2} = \frac{p-2}{2}$$
$$m_2 = \frac{7-5}{5-3} = \frac{2}{2} = 1$$

$$\frac{p-2}{2} = -1$$
$$p - 2 = -2$$
$$\boldsymbol{p = 0}$$

28) Determine whether the following two lines are parallel, perpendicular or neither.

$$y = 3x + 8$$
$$4y = 16 + 12x$$

Second equation can be written as:

$$4y = 12x + 16$$
$$\div 4 \qquad \div 4$$
$$y = 3x + 4$$

The lines are parallel.

29) Determine whether the following two lines are parallel, perpendicular or neither.

$$2y = \frac{1}{2}x + 8$$
$$y = 4x + 10$$

First equation can be written as:

$$2y = \frac{1}{2}x + 8$$
$$\div 2 \qquad \div 2$$
$$y = \frac{1}{4}x + 4$$

The lines are neither.

30) Determine whether the following two lines are parallel, perpendicular or neither.

$$8 - 5y = 25x$$
$$10y = -50x + 12$$

First equation can be written as:

$$8 - 5y = 25x$$
$$-5y = 25x - 8$$
$$\div (-5) \quad \div (-5)$$
$$y = -5x + \frac{8}{5}$$

Second equation can be written as:

$$10y = -50x + 12$$
$$\div 10 \quad \div 10$$
$$y = -5x + \frac{6}{5}$$

The lines are parallel.

31) Determine whether the following two lines are parallel, perpendicular or neither.

$$3y + 2x = 17$$
$$-6x + 4y + 10 = 0$$

First equation can be written as:

$3y + 2x = 17$
$3y = -2x + 17$
$\div 3 \quad \div 3$
$y = -\frac{2}{3}x + \frac{17}{3}$

Second equation can be written as:

$-6x + 4y + 10 = 0$
$4y = 6x - 10$
$\div 4 \quad \div 4$
$y = \frac{3}{2}x - \frac{5}{2}$

The lines are perpendicular.

Challenge

32) A line segment has endpoints $(2, 3)$ and $(6, 7)$. Work out the equation of the perpendicular bisector.

The perpendicular bisector passes through the midpoint of the line segment.

$$Midpoint = \left(\frac{2+6}{2}, \frac{3+7}{2}\right) = (4, 5)$$

Gradient of the line segment:

$$m = \frac{7-3}{6-2} = \frac{4}{4} = 1$$

Gradient of perpendicular bisector = -1

$y = mx + c$
$5 = -1(4) + c$
$5 = -4 + c$
$c = 9$

$\boldsymbol{y = -x + 9}$

33) The line $y = 2x + 5$ intersects the vertical line $x = 2$. Find the equation of the line perpendicular to $y = 2x + 5$ at their point of intersection.

Work out the y-coordinate of the point of intersection:

$y = 2(2) + 5 = 9$
$(2, 9)$

Gradient = $-\frac{1}{2}$

$y = mx + c$
$9 = -\frac{1}{2}(2) + c$
$9 = -1 + c$
$c = 10$

$\boldsymbol{y = -\frac{1}{2}x + 10}$

34) Line 1 has the equation $8y - 64 = 16x$. Point Q has an x-coordinate of 2 and lies on Line 1. Line 2 is perpendicular to Line 1 and passes through the point Q.

a. Write the equation of Line 2 in the form $ax + by + c = 0$, where a, b and c are integers to be found.

Equation of line 1:

$8y - 64 = 16x$

$8y = 16x + 64$

$y = 2x + 8$

Work out y-coordinate of point Q:

$y = 2(2) + 8 = 12$

$Q(2, 12)$

Gradient of Line 2 = $-\frac{1}{2}$

$y = mx + c$

$12 = -\frac{1}{2}(2) + c$

$12 = -1 + c$

$c = 13$

$y = -\frac{1}{2}x + 13$

$\frac{1}{2}x + y - 13 = 0$

$\times 2 \qquad \times 2$

$\boldsymbol{x + 2y - 26 = 0}$

$\boldsymbol{a = 1, \quad b = 2, \quad c = -26}$

b. Line 1 and Line 2 cross the x-axis at the points R and S respectively. Work out the area of the triangle QRS.

Cross x-axis when y = 0

x-coordinate of R:

$0 = 2x + 8$

$2x = -8$

$x = -4$

$(-4, 0)$

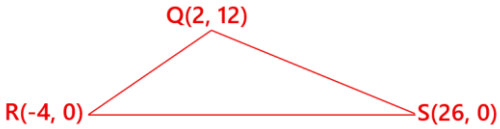
x-coordinate of S:

$0 = -\frac{1}{2}x + 13$

$\frac{1}{2}x = 13$

$x = 26$

$(26, 0)$



Base of triangle = $26 - (-4) = 30$

Height of triangle = $2 - 0 = 2$

Area= $\frac{b \times h}{2} = \frac{30 \times 2}{2} = \boldsymbol{30\ units^2}$