GENMATHS RESOURCES

RECURRING DECIMAL TO FRACTION PROOF

1) Prove algebraically that the recurring decimal $0.\dot{5}$ can be written as $\frac{5}{9}$.

<u>**Proof**</u>

Let $x = 0.5555 \ldots$

$10x = 5.5555 \ldots$

$10x = 5.5555 \ldots$

$\underline{-(x = 0.5555 \ldots)}$

$9x = 5$

$$x = \frac{5}{9}$$

2) Prove algebraically that the recurring decimal $0.\dot{2}$ can be written as $\frac{2}{9}$.

<u>**Proof**</u>

Let $x = 0.2222 \ldots$

$10x = 2.2222 \ldots$

$10x = 2.2222 \ldots$

$\underline{-(x = 0.2222 \ldots)}$

$9x = 2$

$$x = \frac{2}{9}$$

3) Prove algebraically that the recurring decimal $0.\dot{3}$ can be written as $\frac{1}{3}$.

<u>**Proof**</u>

Let $x = 0.3333 \ldots$

$10x = 3.3333 \ldots$

$10x = 3.3333 \ldots$

$\underline{-(x = 0.3333 \ldots)}$

$9x = 3$

$$x = \frac{3}{9} = \frac{1}{3}$$

4) Prove algebraically that the recurring decimal $0.2\dot{7}$ can be written as $\frac{5}{18}$.

<u>**Proof**</u>

Let $x = 0.27777 \ldots$

$10x = 2.7777 \ldots$

$100x = 27.7777 \ldots$

$100x = 27.7777 \ldots$

$\underline{-(10x = \quad 2.7777 \ldots)}$

$90x = 25$

$$x = \frac{25}{90} = \frac{5}{18}$$

5) Prove algebraically that the recurring decimal $0.\dot{1}\dot{2}$ can be written as $\frac{4}{33}$.

Proof

Let $x = 0.121212\ldots$

$100x = 12.121212\ldots$

$$\begin{aligned} 100x &= 12.1212\ldots \\ -(\quad x &= \ \ 0.1212\ldots) \\ 99x &= 12 \end{aligned}$$

$$x = \frac{12}{99} = \frac{4}{33}$$

6) Prove algebraically that the recurring decimal $0.\dot{4}\dot{5}$ can be written as $\frac{5}{11}$.

Proof

Let $x = 0.454545\ldots$

$100x = 45.454545\ldots$

$$\begin{aligned} 100x &= 45.4545\ldots \\ -(\quad x &= \ \ 0.4545\ldots) \\ 99x &= 45 \end{aligned}$$

$$x = \frac{45}{99} = \frac{5}{11}$$

7) Prove algebraically that the recurring decimal $0.\dot{2}\dot{7}$ can be written as $\frac{3}{11}$.

Proof

Let $x = 0.272727\ldots$

$100x = 27.272727\ldots$

$$\begin{aligned} 100x &= 27.2727\ldots \\ -(\quad x &= \ \ 0.2727\ldots) \\ 99x &= 27 \end{aligned}$$

$$x = \frac{27}{99} = \frac{3}{11}$$

8) Prove algebraically that the recurring decimal $0.\dot{8}\dot{1}$ can be written as $\frac{9}{11}$.

Proof

Let $x = 0.818181\ldots$

$100x = 81.818181\ldots$

$$\begin{aligned} 100x &= 81.8181\ldots \\ -(\quad x &= \ \ 0.8181\ldots) \\ 99x &= 81 \end{aligned}$$

$$x = \frac{81}{99} = \frac{9}{11}$$

9) Prove algebraically that the recurring decimal $0.\dot{3}\dot{6}$ can be written as $\frac{4}{11}$.

Proof

Let $x = 0.363636\ldots$

$100x = 36.363636\ldots$

$$\begin{aligned} 100x &= 36.3636\ldots \\ -(\quad x &= \;\;0.3636\ldots) \\ \hline 99x &= 36 \end{aligned}$$

$$x = \frac{36}{99} = \frac{4}{11}$$

10) Prove algebraically that the recurring decimal $0.\dot{1}2\dot{3}$ can be written as $\frac{41}{333}$.

Proof

Let $x = 0.123123123\ldots$

$1000x = 123.123123\ldots$

$$\begin{aligned} 1000x &= 123.123123\ldots \\ -(\quad x &= \;\;\;0.123123\ldots) \\ \hline 999x &= 123 \end{aligned}$$

$$x = \frac{123}{999} = \frac{41}{333}$$

11) Prove algebraically that the recurring decimal $0.\dot{4}5\dot{6}$ can be written as $\frac{152}{333}$.

Proof

Let $x = 0.456456456\ldots$

$1000x = 456.456456\ldots$

$$\begin{aligned} 1000x &= 456.456456\ldots \\ -(\quad x &= \;\;\;0.456456\ldots) \\ \hline 999x &= 456 \end{aligned}$$

$$x = \frac{456}{999} = \frac{152}{333}$$

12) Prove algebraically that the recurring decimal $0.\dot{1}0\dot{1}$ can be written as $\frac{101}{999}$.

Proof

Let $x = 0.101101101\ldots$

$1000x = 101.101101\ldots$

$$\begin{aligned} 1000x &= 101.101101\ldots \\ -(\quad x &= \;\;\;0.101101\ldots) \\ \hline 999x &= 101 \end{aligned}$$

$$x = \frac{101}{999}$$

13) Prove algebraically that the recurring decimal $0.4\dot{3}$ can be written as $\frac{13}{30}$.

Proof

Let $x = 0.4333333\ldots$

$10x = 4.3333333\ldots$

$100x = 43.333333\ldots$

$$\begin{aligned} 100x &= 43.3333\ldots \\ -(10x &= 4.3333\ldots) \\ 90x &= 39 \end{aligned}$$

$$x = \frac{39}{90} = \frac{13}{30}$$

14) Prove algebraically that the recurring decimal $3.3\dot{7}\dot{5}$ can be written as $3\,\frac{62}{165}$.

Proof

Let $x = 3.37575\ldots$

$10x = 33.7575\ldots$

$1000x = 3375.7575\ldots$

$$\begin{aligned} 1000x &= 3375.7575\ldots \\ -(10x &= 33.7575\ldots) \\ 990x &= 3342 \end{aligned}$$

$$x = \frac{3342}{990} = \frac{557}{165} = 3\frac{62}{165}$$

15) Prove algebraically that the recurring decimal $4.0\dot{1}\dot{6}$ can be written as $4\,\frac{8}{495}$.

Proof

Let $x = 4.01616\ldots$

$10x = 40.1616\ldots$

$1000x = 4016.1616\ldots$

$$\begin{aligned} 1000x &= 4016.1616\ldots \\ -(10x &= 40.1616\ldots) \\ 990x &= 3976 \end{aligned}$$

$$x = \frac{3976}{990} = \frac{1988}{495} = 4\frac{8}{495}$$

16) Prove algebraically that the recurring decimal $5.\dot{1}2\dot{3}$ can be written as $5\,\frac{41}{333}$.

Proof

Let $x = 5.123123\ldots$

$1000x = 5123.123123\ldots$

$$\begin{aligned} 1000x &= 5123.123123\ldots \\ -(x &= 5.123123\ldots) \\ 999x &= 5118 \end{aligned}$$

$$x = \frac{5118}{999} = \frac{1706}{333} = 5\frac{41}{333}$$

17) Prove algebraically that the recurring decimal $0.12\dot{3}\dot{4}$ can be written as $\frac{611}{4950}$.

<u>Proof</u>

Let $x = 0.12343434\ldots$

$100x = 12.343434\ldots$

$10000x = 1234.343434\ldots$

$$\begin{aligned} 10000x &= 1234.3434\ldots \\ -(100x &= 12.3434\ldots) \\ \hline 9900x &= 1222 \end{aligned}$$

$$x = \frac{1222}{9900} = \frac{611}{4950}$$

18) Use algebra to work out the value of $0.\dot{5}\dot{2} \div 0.\dot{8}\dot{4}$. Give your answer as a fraction in simplest form.

$x = 0.5252\ldots$

$100x = 52.5252\ldots$

$100x - x = 52$

$99x = 52$

$x = \frac{52}{99}$

$y = 0.8484\ldots$

$100y = 84.8484\ldots$

$100y - y = 84$

$99y = 84$

$y = \frac{84}{99}$

$\frac{52}{99} \div \frac{84}{99}$

$= \frac{52}{99} \times \frac{99}{84} = \frac{52}{84} = \mathbf{\frac{13}{21}}$

19) Use algebra to work out the value of $0.1\dot{2} + 0.4\dot{3}$. Give your answer as a fraction in simplest form.

$x = 0.1222\ldots$

$10x = 1.2222\ldots$

$100x = 12.2222\ldots$

$100x - 10x = 11$

$90x = 11$

$x = \frac{11}{90}$

$y = 0.4333\ldots$

$10y = 4.3333\ldots$

$100y = 43.3333\ldots$

$100y - 10y = 39$

$90y = 39$

$y = \frac{39}{90}$

$\frac{11}{90} + \frac{39}{90} = \frac{50}{90} = \mathbf{\frac{5}{9}}$

20) Use algebra to work out the value of $2.2\dot{7} \times 3.5\dot{6}$. Give your answer as a mixed number in simplest form.

$x = 2.2777\ldots$
$10x = 22.7777\ldots$
$100x = 227.7777\ldots$

$100x - 10x = 205$

$90x = 205$

$x = \frac{205}{90} = \frac{41}{18}$

$y = 3.5666\ldots$
$10y = 35.6666\ldots$
$100y = 356.6666\ldots$

$100y - 10y = 321$

$90y = 321$

$y = \frac{321}{90} = \frac{107}{30}$

$\frac{41}{18} \times \frac{107}{30} = \frac{4387}{540} = \mathbf{8\frac{67}{540}}$

21) Prove algebraically that $0.\dot{9} = 1$

<u>Proof</u>

Let $x = 0.9999\ldots$
$10x = 9.9999\ldots$

$10x = 9.9999\ldots$
$\underline{-\ \ (x = 0.9999\ldots)}$
$9x = 9$

$x = \frac{9}{9} = 1$