

## Task 1

- 1) Prove that for any positive integer  $n$ ,  $3n + 18 + n + 6$  is always a multiple of 4.
- 2) Prove that for any positive integer  $n$ ,  $5n^2 + 16 + n^2 - 4 + 6n$  is always a multiple of 6.
- 3) Prove that for any positive integer  $n$ ,  $16n + 5 + 4n + 25$  is always a multiple of 10.
- 4) Prove that for any integer  $n$ ,  $9n + 19 + 12n + 3$  is always one more than a multiple of 3.
- 5) Prove that for any positive integer  $n$ ,  $3(6n + 7) + 2(3n - 1)$  is always one more than a multiple of 6.
- 11) Prove that the sum of two consecutive integers is always odd.
- 12) Prove that the sum of three consecutive integers is always a multiple of 3.
- 13) Prove that the difference between two odd integers is always even.
- 14) Prove that the sum of two consecutive even integers is always even.
- 15) Prove that the difference between two consecutive square numbers is always odd.
- 16) Prove that the product of two consecutive integers is always even.

## Task 2

- 6) Prove that the sum of two even integers is always even.
- 7) Prove that the product of two odd integers is always odd.
- 8) Prove that  $4n^2 + 4n + 1$  is always a square number, for all positive integer values of  $n$ .
- 9) Prove that  $(2n + 1)^2 - (2n - 1)^2$ , is always a multiple of 8, for all positive integer values of  $n$ .
- 10) Prove that  $(3n + 4)^2 - (3n - 4)^2$  is always a multiple of 16, for all positive integer values of  $n$ .
- 17) Prove that the square of any multiple of 3 is also a multiple of 9.
- 18) Prove that the square of any odd integer is also odd.
- 19) Prove that the square of any even integer is a multiple of 4.
- 20) Show that the difference between the squares of two consecutive integers equals the sum of those integers.
- 21) Prove that the sum of the squares of two consecutive integers is always odd.
- 22) Prove that  $2(x^2 + y^2 + xy) - (x^2 + y^2)$ , is a square number for all positive integer values of  $n$ .

### Challenge

23) Prove that the difference between the cubes of two consecutive integers can be written as  $(3n(n + 1) + 1)$

24) Prove that  $n^3 - n$  is always a multiple of 6, for any positive integer value of  $n$ .

25) Prove that the product of two consecutive even integers is a multiple of 8.