ITERATION

Task 1

1) Use the iteration formula,

$x_{n+1} = 0.5(x_n + 6)$

with $x_0 = 2$

Find the values of x_1, x_2, and x_3.

$x_1 = 0.5(2 + 6)$
$= 0.5(8)$
$= \mathbf{4}$

$x_2 = 0.5(4 + 6)$
$= 0.5(10)$
$= \mathbf{5}$

$x_3 = 0.5(5 + 6)$
$= 0.5(11)$
$= \mathbf{5.5}$

2) Use the iteration formula,

$x_{n+1} = \frac{{x_n}^2 + 5}{3}$

with $x_0 = 1$

Find the values of x_1, x_2, and x_3.

$x_1 = \frac{1^2 + 5}{3}$

$= \frac{6}{3}$

$= \mathbf{2}$

$x_2 = \frac{2^2 + 5}{3}$

$= \frac{9}{3}$

$= \mathbf{3}$

$x_3 = \frac{3^2 + 5}{3}$

$= \frac{14}{3}$

$= \mathbf{4.67\ (3\ sf)}$

3) The temperature in ℃, of water in a kettle, t seconds after it is turned on is modelled by the equation

$$t_{n+1} = 1.13t_n + 4$$

Starting with $t_0 = 22$, work out the temperature of water in the kettle two seconds after it is turned on.

$t_1 = 1.13(22) + 4 = 28.86$
$t_2 = 1.13(28.86) + 4 = 36.6118$

$\mathbf{36.6\ ℃\ (3\ sf)}$

4) The number of people in a city, in millions, n days after the end of a sports event, is modelled by the formula

$$P_{n+1} = 1.2P_n - 3$$

Given $P_0 = 10$, work out how many people are in the city three days from now.

$P_1 = 1.2(10) - 3 = 9$
$P_2 = 1.2(9) - 3 = 7.8$
$P_3 = 1.2(7.8) - 3 = 6.36$

6,360,000 people

Task 2

5) Show that the equation $x^3 - x - 2 = 0$ has a solution between $x = 1$ and $x = 2$.

$f(1) = 1^3 - 1 - 2 = -2$
$f(2) = 2^3 - 2 - 2 = 4$

There is a change in sign and the graph is continuous, hence there is a solution between 1 and 2.

6) Show that the equation $x^2 - 5x + 4 = 0$ has a solution between $x = 0$ and $x = 2$.

$f(0) = 0^2 - 5(0) + 4 = 4$
$f(2) = 2^2 - 5(2) + 4 = -2$

There is a change in sign and the graph is continuous, hence there is a solution between 0 and 2.

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7) Show that the equation $x^3 - 4x + 1 = 0$ has a solution between $x = 0$ and $x = 1$.

$$f(0) = 0^3 - 4(0) + 1 = 1$$
$$f(1) = 1^3 - 4(1) + 1 = -2$$

There is a change in sign and the graph is continuous, hence there is a solution between 0 and 1.

8) Show that the equation $x^2 + x - 4 = 2$ has a solution between $x = -1$ and $x = 3$.

$$x^2 + x - 4 = 2$$
$$x^2 + x - 6 = 0$$

$$f(-1) = (-1)^2 + (-1) - 6 = -6$$
$$f(3) = 3^2 + 3 - 6 = 6$$

There is a change in sign and the graph is continuous, hence there is a solution between -1 and 3.

9) Show that the equation $x^3 - 2x - 4 = 1$ has a solution between $x = 2$ and $x = 3$.

$$x^3 - 2x - 4 = 1$$
$$x^3 - 2x - 5 = 0$$

$$f(2) = 2^3 - 2(2) - 5 = -1$$
$$f(3) = 3^3 - 2(3) - 5 = 16$$

There is a change in sign and the graph is continuous, hence there is a solution between 2 and 3.

Task 3

10)

a. Show that the equation $x^2 + x - 31 = 0$ can be rearranged into $x = \sqrt{31 - x}$

$$x^2 + x - 31 = 0$$
$$x^2 + x = 31$$
$$x^2 = 31 - x$$
$$x = \sqrt{31 - x}$$

b. Starting with $x_0 = 6$, use the iteration formula $x_{n+1} = \sqrt{31 - x_n}$ twice to find an estimate to the solution of $x^2 + x - 31 = 0$ to 3 decimal places.

$$x_1 = \sqrt{31 - 6} = \sqrt{25} = 5$$
$$x_2 = \sqrt{31 - 5} = \sqrt{26} = 5.099019514$$

$$\mathbf{5.099\ (3\ dp)}$$

11)

a. Show that the equation $x^3 - 2 = x$ can be rearranged into $x = \sqrt[3]{x + 2}$

$$x^3 - 2 = x$$
$$x^3 = x + 2$$
$$x = \sqrt[3]{x + 2}$$

b. Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \sqrt[3]{x_n + 2}$ three times to find an estimate to the solution of $x^3 - 2 = x$ to 2 decimal places.

$$x_1 = \sqrt[3]{1 + 2} = 1.44224957$$
$$x_2 = \sqrt[3]{ans + 2} = 1.509897449$$
$$x_3 = \sqrt[3]{ans + 2} = 1.519724305$$

$$\boldsymbol{x = 1.52\ (2\ dp)}$$

12)

a. Show that the equation $x^2 - 3x - 4 = 0$ can be rearranged into $x = \sqrt{3x + 4}$

$$x^2 - 3x - 4 = 0$$
$$x^2 = 3x + 4$$
$$x = \sqrt{3x + 4}$$

b. Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt{3x_n + 4}$ three times to find an estimate to the solution of $x^2 - 3x - 4 = 0$ to 2 decimal places.

$$x_1 = \sqrt{3(2) + 4} = 3.16227766$$
$$x_2 = \sqrt{3(ans) + 4} = 3.672442373$$
$$x_3 = \sqrt{3(ans) + 4} = 3.875219622$$

$$\boldsymbol{x = 3.89\ (2\ dp)}$$

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Challenge

13) Find the solution to the equation

$$x^3 + 3x = 7$$

using the iteration formula

$$x_{n+1} = \sqrt[3]{7 - 3x_n}$$

and the initial value $x_0 = 1$

Give your answer to 2 decimal places.

$$x_1 = \sqrt[3]{7 - 3(1)} = 1.587401052$$
$$x_2 = \sqrt[3]{7 - 3(ans)} = 1.307997415$$
$$x_3 = \sqrt[3]{7 - 3(ans)} = 1.454328367$$
$$x_4 = \sqrt[3]{7 - 3(ans)} = 1.381563532$$
$$x_5 = \sqrt[3]{7 - 3(ans)} = 1.418679822$$
$$x_6 = \sqrt[3]{7 - 3(ans)} = 1.399993288$$
$$x_7 = \sqrt[3]{7 - 3(ans)} = 1.409463125$$
$$x_8 = \sqrt[3]{7 - 3(ans)} = 1.404680022$$
$$x_9 = \sqrt[3]{7 - 3(ans)} = 1.407099975$$
$$x_{10} = \sqrt[3]{7 - 3(ans)} = 1.405876671$$

$$x = \mathbf{1.41}\ (\mathbf{2}\ \boldsymbol{dp})$$

14) Find the solution to the equation

$$x^3 + 4x = 10$$

using the iteration formula

$$x_{n+1} = \sqrt[3]{10 - 4x_n}$$

and the initial value $x_1 = 2$

Give your answer to 2 decimal places.

$$x_1 = \sqrt[3]{10 - 4(2)} = 1.25992105$$
$$x_2 = \sqrt[3]{10 - 4(ans)} = 1.70543999$$
$$x_3 = \sqrt[3]{10 - 4(ans)} = 1.470264817$$
$$x_4 = \sqrt[3]{10 - 4(ans)} = 1.602981515$$
$$x_5 = \sqrt[3]{10 - 4(ans)} = 1.530924575$$
$$x_6 = \sqrt[3]{10 - 4(ans)} = 1.570866183$$
$$x_7 = \sqrt[3]{10 - 4(ans)} = 1.548980984$$
$$x_8 = \sqrt[3]{10 - 4(ans)} = 1.561048504$$
$$x_9 = \sqrt[3]{10 - 4(ans)} = 1.554417641$$
$$x_{10} = \sqrt[3]{10 - 4(ans)} = 1.558068154$$
$$x_{11} = \sqrt[3]{10 - 4(ans)} = 1.556060544$$

$$\boldsymbol{x} = \mathbf{1.56}\ (\mathbf{2}\ \boldsymbol{dp})$$

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