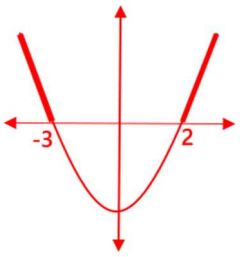
QUADRATIC INEQUALITIES

Task 1 – Solve the following inequalities.

1) $x^2 + x - 6 > 0$

$(x + 3)(x - 2) > 0$

$x = -3 \quad or \quad x = 2$

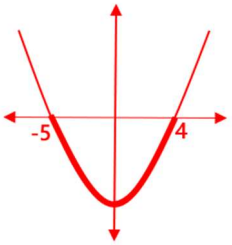


$\boldsymbol{x < -3 \quad or \quad x > 2}$

2) $x^2 + x - 20 \leq 0$

$(x + 5)(x - 4) \leq 0$

$x = -5 \quad or \quad x = 4$

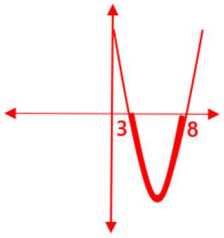


$\boldsymbol{-5 \leq x \leq 4}$

3) $x^2 - 11x + 24 < 0$

$(x - 3)(x - 8) < 0$

$x = 3 \quad or \quad x = 8$

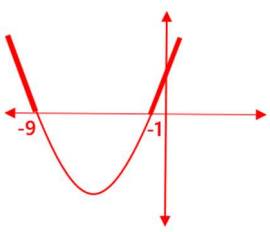


$\boldsymbol{3 < x < 8}$

4) $x^2 + 10x + 9 \geq 0$

$(x + 9)(x + 1) \geq 0$

$x = -9 \quad or \quad x = -1$

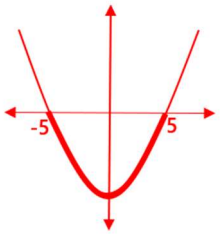


$\boldsymbol{x \leq -9 \quad or \quad x \geq -1}$

5) $x^2 - 25 < 0$

$(x + 5)(x - 5) < 0$

$x = -5 \quad or \quad x = 5$

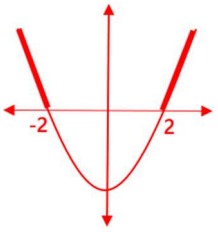


$\boldsymbol{-5 < x < 5}$

6) $x^2 - 4 > 0$

$(x + 2)(x - 2) > 0$

$x = -2 \quad or \quad x = 2$

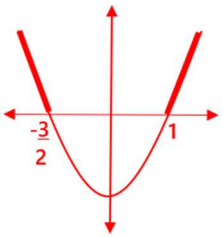


$\boldsymbol{x < -2 \quad or \quad x > 2}$

7) $2x^2 + x - 3 \geq 0$

$(2x + 3)(x - 1) \geq 0$

$x = -\frac{3}{2} \quad or \quad x = 1$

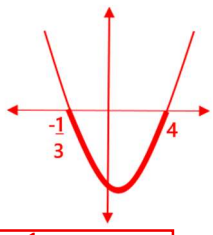


$\boldsymbol{x \leq -\frac{3}{2} \quad or \quad x \geq 1}$

8) $3x^2 - 11x - 4 < 0$

$(3x + 1)(x - 4) < 0$

$x = -\frac{1}{3} \quad or \quad x = 4$

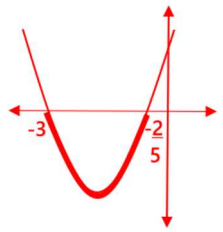


$\boldsymbol{-\frac{1}{3} < x < 4}$

9) $5x^2 + 17x + 6 < 0$

$(5x + 2)(x + 3) < 0$

$x = -\frac{2}{5}$ *or* $x = -3$

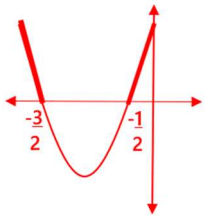


$$\boldsymbol{-3 < x < -\frac{2}{5}}$$

10) $4x^2 + 8x + 3 > 0$

$(2x + 1)(2x + 3) > 0$

$x = -\frac{1}{2}$ *or* $x = -\frac{3}{2}$

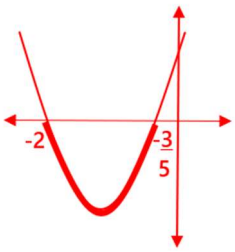


$$\boldsymbol{x < -\frac{3}{2} \quad or \quad x > -\frac{1}{2}}$$

11) $5x^2 + 13x + 6 \leq 0$

$(5x + 3)(x + 2) \leq 0$

$x = -\frac{3}{5}$ *or* $x = -2$



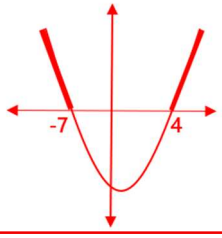
$$\boldsymbol{-2 \leq x \leq -\frac{3}{5}}$$

12) $x^2 + 4x - 21 \geq x + 7$

$x^2 + 3x - 28 \geq 0$

$(x + 7)(x - 4) \geq 0$

$x = -7$ *or* $x = 4$



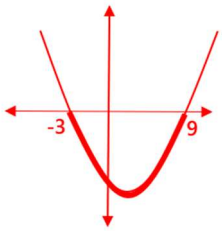
$$\boldsymbol{x \leq -7 \quad or \quad x \geq 4}$$

13) $x^2 - 8x + 10 \leq -2x + 37$

$x^2 - 6x - 27 \leq 0$

$(x + 3)(x - 9) \leq 0$

$x = -3$ *or* $x = 9$



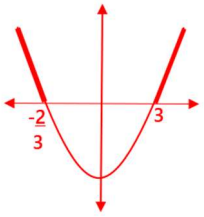
$$\boldsymbol{-3 \leq x \leq 9}$$

14) $2x^2 - 2x - 1 \geq -x^2 - x + 5$

$3x^2 - x - 6 \geq 0$

$(3x + 2)(x - 3) \geq 0$

$x = -\frac{2}{3}$ *or* $x = 3$



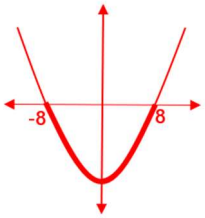
$$\boldsymbol{x \leq -\frac{2}{3} \quad or \quad x \geq 3}$$

15) $x^2 - 3x - 64 < -3x$

$x^2 - 64 < 0$

$(x + 8)(x - 8) < 0$

$x = -8$ *or* $x = 8$



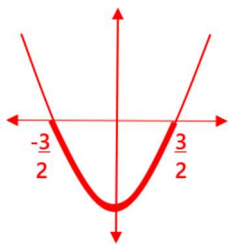
$$\boldsymbol{-8 < x < 8}$$

16) $4x^2 - 17 < -8$

$4x^2 - 9 < 0$

$(2x + 3)(2x - 3) < 0$

$x = -\frac{3}{2}$ *or* $x = \frac{3}{2}$



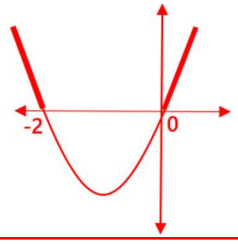
$$\boldsymbol{-\frac{3}{2} < x < \frac{3}{2}}$$

17) $x^2 + 2x + 4 > 4$

$x^2 + 2x > 0$

$x(x + 2) > 0$

$x = 0 \quad or \quad x = -2$



$\boldsymbol{x < -2 \quad or \quad x > 0}$

Challenge

18) A rectangle has a width of x cm and a length of $(x - 3)$ cm. The area of the rectangle must be at least 40 cm. Given that $x > 3$, form an algebraic inequality and solve for the possible values of x.

$x(x - 3) \geq 40$

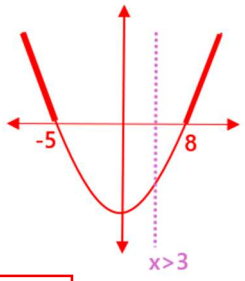
$x^2 - 3x \geq 40$

$x^2 - 3x - 40 \geq 0$

$(x + 5)(x - 8) \geq 0$

$x = -5 \quad or \quad x = 8$

$x \leq -5 \quad or \quad x \geq 8 \quad (x > 3)$



$\boldsymbol{x \geq 8}$

19) A ball is thrown upward, and its height, in metres, after t seconds is given by:

$$h(t) = -t^2 + 6t$$

For which values of t, is the ball above 5 metres? Show clear algebraic working.

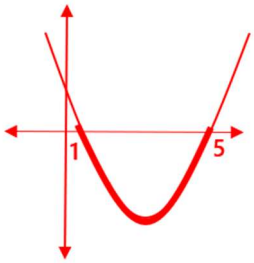
$-t^2 + 6t > 5$

$-t^2 + 6t - 5 > 0$

$t^2 - 6t + 5 < 0$

$(t - 1)(t - 5) < 0$

$t = 1 \quad or \quad t = 5$



$\boldsymbol{1 < t < 5}$

20) A company's profit, in thousands of pounds, is given by:

$$P(x) = -3x^2 + 7x - 1$$

The company wants the profit to be positive.

For what values of x is $P(x) > 0$?

Give your answers as exact values.

$-3x^2 + 7x - 1 > 0$

$3x^2 - 7x + 1 < 0$

Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{7 \pm \sqrt{37}}{6}$$

$$\boldsymbol{\frac{7 - \sqrt{37}}{6} < x < \frac{7 + \sqrt{37}}{6}}$$