

Task 1 – Solve each of the following equations. You must show clear algebraic working. Do not use trial and error.

1) $2x^2 + y^2 - 2xy = 8$
 $y = x + 2$

$$2x^2 + (x + 2)^2 - 2x(x + 2) = 8$$

$$2x^2 + (x + 2)(x + 2) - 2x^2 - 4x = 8$$

$$2x^2 + x^2 + 2x + 2x + 4 - 2x^2 - 4x = 8$$

$$x^2 - 4 = 0$$

$$(x + 2)(x - 2) = 0$$

$$x = -2 \text{ or } x = 2$$

$$y = -2 + 2 = 0$$

$$y = 2 + 2 = 4$$

$x = -2 \text{ or } x = 2$
 $y = 0 \text{ or } y = 4$

2) $x^2 - 4y + 2x = 2y^2 + 2$
 $x = y - 1$

$$(y - 1)^2 - 4y + 2(y - 1) = 2y^2 + 2$$

$$(y - 1)(y - 1) - 4y + 2y - 2 = 2y^2 + 2$$

$$y^2 - y - y + 1 - 4y + 2y - 2 = 2y^2 + 2$$

$$y^2 - 4y - 1 = 2y^2 + 2$$

$$y^2 + 4y + 3 = 0$$

$$(y + 3)(y + 1) = 0$$

$$y = -3 \text{ or } y = -1$$

$$x = -3 - 1 = -4$$

$$x = -1 - 1 = -2$$

$x = -4 \text{ or } x = -2$
 $y = -3 \text{ or } y = -1$

3) $x + y - 2 = 0$
 $x^2 + y^2 + 2xy - 4x = 4 - x^2$

$$y = 2 - x$$

$$x^2 + (2 - x)^2 + 2x(2 - x) - 4x = 4 - x^2$$

$$x^2 + (2 - x)(2 - x) + 2x(2 - x) - 4x = 4 - x^2$$

$$x^2 + 4 - 2x - 2x + x^2 + 4x - 2x^2 - 4x = 4 - x^2$$

$$-4x + 4 = 4 - x^2$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0 \text{ or } x = 4$$

$$y = 2 - 0 = 2$$

$$y = 2 - 4 = -2$$

$x = 0 \text{ or } x = 4$
 $y = 2 \text{ or } y = -2$

Task 2

4) The line **L** with equation $3x + y - 6 = 0$ intersects the curve **C**, with equation $x^2 + y^2 - 4x - 2y = 0$. Work out the coordinates of the points of intersection.

$$3x + y - 6 = 0$$

$$y = -3x + 6$$

$$x^2 + (-3x + 6)^2 - 4x - 2(-3x + 6) = 0$$

$$x^2 + (-3x + 6)(-3x + 6) - 4x - 2(-3x + 6) = 0$$

$$x^2 + 9x^2 - 18x - 18x + 36 - 4x + 6x - 12 = 0$$

$$10x^2 - 34x + 24 = 0$$

$$5x^2 - 17x + 12 = 0$$

$$(5x - 12)(x - 1) = 0$$

$$x = \frac{12}{5} \text{ or } x = 1$$

$$y = -3\left(\frac{12}{5}\right) + 6 = -\frac{6}{5}$$

$$y = -3(1) + 6 = 3$$

$\left(\frac{12}{5}, -\frac{6}{5}\right) \text{ and } (1, 3)$

5) The line **L** with equation $x - 2y + 5 = 0$, intersects the curve **C**, with equation $x^2 + y^2 + 4x - 2y = -3$. Work out the coordinates of the points of intersection.

$$x - 2y + 5 = 0$$

$$x = 2y - 5$$

$$(2y - 5)^2 + y^2 + 4(2y - 5) - 2y = -3$$

$$(2y - 5)(2y - 5) + y^2 + 4(2y - 5) - 2y = -3$$

$$4y^2 - 10y - 10y + 25 + y^2 + 8y - 20 - 2y = -3$$

$$5y^2 - 14y + 5 = -3$$

$$5y^2 - 14y + 8 = 0$$

$$(5y - 4)(y - 2) = 0$$

$$y = \frac{4}{5} \text{ or } y = 2$$

$$x = 2\left(\frac{4}{5}\right) - 5 = -\frac{17}{5}$$

$$x = 2(2) - 5 = -1$$

$\left(-\frac{17}{5}, \frac{4}{5}\right) \text{ and } (-1, 2)$

- 6) The line L with equation $x - 2y = 1$, intersects the curve C, with equation $(x - 3)^2 + (y + 1)^2 = 5$. Work out the coordinates of the points of intersection.

$$\begin{aligned}x - 2y &= 1 \\x &= 2y + 1\end{aligned}$$

$$\begin{aligned}(2y + 1 - 3)^2 + (y + 1)^2 &= 5 \\(2y - 2)^2 + (y + 1)^2 &= 5 \\(2y - 2)(2y - 2) + (y + 1)(y + 1) &= 5 \\4y^2 - 4y - 4y + 4 + y^2 + y + y + 1 &= 5 \\5y^2 - 6y + 5 &= 5 \\5y^2 - 6y &= 0 \\y(5y - 6) &= 0 \\y = 0 \text{ or } y &= \frac{6}{5}\end{aligned}$$

$$x = 2(0) + 1 = 1$$

$$x = 2\left(\frac{6}{5}\right) + 1 = \frac{17}{5}$$

$$\boxed{(1, 0) \text{ and } \left(\frac{17}{5}, \frac{6}{5}\right)}$$

Challenge

- 7) Solve the equations:

$$\begin{aligned}2x - 3y + 6 &= 0 \\2x^2 + y^2 - 4xy + 4x &= 6\end{aligned}$$

Do not use trial and improvement. You must show clear algebraic working.

$$\begin{aligned}2x - 3y + 6 &= 0 \\2x + 6 &= 3y \\y &= \frac{2x + 6}{3}\end{aligned}$$

$$2x^2 + \left(\frac{2x + 6}{3}\right)^2 - 4x\left(\frac{2x + 6}{3}\right) + 4x = 6$$

Multiply all terms by 9:

$$\begin{aligned}18x^2 + (2x + 6)^2 - 12x(2x + 6) + 36x &= 54 \\18x^2 + (2x + 6)(2x + 6) - 12x(2x + 6) + 36x &= 54 \\18x^2 + 4x^2 + 12x + 12x + 36 - 24x^2 - 72x + 36x &= 54 \\-2x^2 - 12x + 36 &= 54 \\-2x^2 - 12x - 18 &= 0 \\x^2 + 6x + 9 &= 0 \\(x + 3)(x + 3) &= 0 \\x &= -3\end{aligned}$$

$$y = \frac{2(-3) + 6}{3} = 0$$

$$\boxed{x = -3, \quad y = 0}$$

- 8) The line L with equation $y = \frac{x}{2} - 1$, intersects the curve C, with equation $x^2 + y^2 = 2xy + 4$ at two points. Work out the length of the line segment joining these points.

$$\begin{aligned}y &= \frac{x}{2} - 1 \\x^2 + \left(\frac{x}{2} - 1\right)^2 &= 2x\left(\frac{x}{2} - 1\right) + 4 \\x^2 + \left(\frac{x}{2} - 1\right)\left(\frac{x}{2} - 1\right) &= 2x\left(\frac{x}{2} - 1\right) + 4 \\x^2 + \frac{x^2}{4} - \frac{x}{2} - \frac{x}{2} + 1 &= x^2 - 2x + 4 \\x^2 + \frac{x^2}{4} - x + 1 &= x^2 - 2x + 4\end{aligned}$$

Multiply all terms by 4:

$$\begin{aligned}4x^2 + x^2 - 4x + 4 &= 4x^2 - 8x + 16 \\5x^2 - 4x + 4 &= 4x^2 - 8x + 16 \\x^2 + 4x - 12 &= 0 \\(x + 6)(x - 2) &= 0 \\x = -6 \text{ or } x &= 2\end{aligned}$$

$$y = -\frac{6}{2} - 1 = -4$$

$$y = \frac{2}{2} - 1 = 0$$

Points of intersection:

$$(-6, -4) \text{ and } (2, 0)$$

$$\begin{aligned}\text{Distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(2 - -6)^2 + (0 - -4)^2} \\&= \sqrt{8^2 + 4^2} \\&= \sqrt{80} \\&= 4\sqrt{5}\end{aligned}$$

$$\boxed{\text{Length} = 4\sqrt{5}}$$