QUADRATIC SIMULTANEOUS EQUATIONS

Task 1 – Solve each of the following equations. You must show clear algebraic working. Do not use trial and error.

1)

$$y = x^2 + 2$$
$$y = 3x$$

$$x^2 + 2 = 3x$$
$$x^2 - 3x + 2 = 0$$
$$(x - 1)(x - 2) = 0$$
$$x = 1 \quad or \quad x = 2$$

$$y = 3(1) = 3$$
$$y = 3(2) = 6$$

$$\boldsymbol{x = 1 \ \text{or} \ x = 2}$$
$$\boldsymbol{y = 3 \ \text{or} \ y = 6}$$

2)

$$y = x^2$$
$$y = 10 - 3x$$

$$x^2 = 10 - 3x$$
$$x^2 + 3x - 10 = 0$$
$$(x + 5)(x - 2) = 0$$
$$x = -5 \quad or \quad x = 2$$

$$y = 10 - 3(-5) = 25$$
$$y = 10 - 3(2) = 4$$

$$\boldsymbol{x = -5 \ \text{or} \ x = 2}$$
$$\boldsymbol{y = 25 \ \text{or} \ y = 4}$$

3)

$$y = 5 + x^2$$
$$y = x + 11$$

$$5 + x^2 = x + 11$$
$$x^2 - x - 6 = 0$$
$$(x + 2)(x - 3) = 0$$
$$x = -2 \quad or \quad x = 3$$

$$y = -2 + 11 = 9$$
$$y = 3 + 11 = 14$$

$$\boldsymbol{x = -2 \ \text{or} \ x = 3}$$
$$\boldsymbol{y = 9 \ \text{or} \ y = 14}$$

4)

$$y = x - 12$$
$$y = x^2 + 9x$$

$$x - 12 = x^2 + 9x$$
$$x^2 + 8x + 12 = 0$$
$$(x + 6)(x + 2) = 0$$
$$x = -6 \quad or \quad x = -2$$

$$y = -6 - 12 = -18$$
$$y = -2 - 12 = -14$$

$$\boldsymbol{x = -6 \ \text{or} \ x = -2}$$
$$\boldsymbol{y = -18 \ \text{or} \ y = -14}$$

5)

$$y = x^2 + 7x + 6$$
$$y = 2x + 2$$

$$x^2 + 7x + 6 = 2x + 2$$
$$x^2 + 5x + 4 = 0$$
$$(x + 4)(x + 1) = 0$$
$$x = -4 \quad or \quad x = -1$$

$$y = 2(-4) + 2 = -6$$
$$y = 2(-1) + 2 = 0$$

$$\boldsymbol{x = -4 \ \text{or} \ x = -1}$$
$$\boldsymbol{y = -6 \ \text{or} \ y = 0}$$

6)

$$x^2 + y^2 = 25$$
$$y = x + 1$$

$$x^2 + (x + 1)^2 = 25$$
$$x^2 + (x + 1)(x + 1) = 25$$
$$x^2 + x^2 + x + x + 1 = 25$$
$$2x^2 + 2x - 24 = 0$$
$$x^2 + x - 12 = 0$$
$$(x + 4)(x - 3) = 0$$
$$x = -4 \quad or \quad x = 3$$

$$y = -4 + 1 = -3$$
$$y = 3 + 1 = 4$$

$$\boldsymbol{x = -4 \ \text{or} \ x = 3}$$
$$\boldsymbol{y = -3 \ \text{or} \ y = 4}$$

7)

$$xy = 12$$
$$y = x - 1$$

$$x(x-1) = 12$$
$$x^2 - x = 12$$
$$x^2 - x - 12 = 0$$
$$(x-4)(x+3) = 0$$
$$x = 4 \quad or \quad x = -3$$

$$y = 4 - 1 = 3$$
$$y = -3 - 1 = -4$$

$$\boldsymbol{x = 4 \quad or \quad x = -3}$$
$$\boldsymbol{y = 3 \quad or \quad y = -4}$$

8)

$$x^2 + y^2 = 29$$
$$y = 2x + 1$$

$$x^2 + (2x+1)^2 = 29$$
$$x^2 + (2x+1)(2x+1) = 29$$
$$x^2 + 4x^2 + 2x + 2x + 1 = 29$$
$$5x^2 + 4x - 28 = 0$$
$$(5x+14)(x-2) = 0$$
$$x = -\frac{14}{5} \quad or \quad x = 2$$

$$y = 2\left(-\frac{14}{5}\right) + 1 = -\frac{23}{5}$$
$$y = 2(2) + 1 = 5$$

$$\boldsymbol{x = -\frac{14}{5} \quad or \quad x = 2}$$
$$\boldsymbol{y = -\frac{23}{5} \quad or \quad y = 5}$$

9)

$$y = x^2 + 2x + 1$$
$$y = x + 1$$

$$x^2 + 2x + 1 = x + 1$$
$$x^2 + x = 0$$
$$x(x+1) = 0$$
$$x = 0 \quad or \quad x = -1$$

$$y = 0 + 1 = 1$$
$$y = -1 + 1 = 0$$

$$\boldsymbol{x = 0 \quad or \quad x = -1}$$
$$\boldsymbol{y = 1 \quad or \quad y = 0}$$

10)

$$x + y = 5$$
$$x^2 + y^2 = 17$$

$$y = 5 - x$$
$$x^2 + (5-x)^2 = 17$$
$$x^2 + (5-x)(5-x) = 17$$
$$x^2 + 25 - 5x - 5x + x^2 = 17$$
$$2x^2 - 10x + 8 = 0$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$x = 4 \quad or \quad x = 1$$

$$y = 5 - 4 = 1$$
$$y = 5 - 1 = 4$$

$$\boldsymbol{x = 4 \quad or \quad x = 1}$$
$$\boldsymbol{y = 1 \quad or \quad y = 4}$$

11) Give your solutions to 3 significant figures.

$$xy = 8$$
$$y = 2x - 2$$

$$x(2x-2) = 8$$
$$2x^2 - 2x - 8 = 0$$
$$x^2 - x - 4 = 0$$
$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$$
$$x = \frac{1 \pm \sqrt{17}}{2}$$

$$y = 2\left(\frac{1+\sqrt{17}}{2}\right) - 2 = 3.1231\ldots$$
$$y = 2\left(\frac{1-\sqrt{17}}{2}\right) - 2 = -5.1231\ldots$$

$$\boldsymbol{x = 2.56\ (3\ sf) \quad or \quad x = -1.56\ (3\ sf)}$$
$$\boldsymbol{y = 3.12\ (3\ sf) \quad or \quad y = -5.12\ (3\ sf)}$$

12) Give your solutions to 3 significant figures.

$$y = x^2 + x$$
$$y = 3x + 4$$

$$x^2 + x = 3x + 4$$
$$x^2 - 2x - 4 = 0$$
$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$
$$x = 1 \pm \sqrt{5}$$

$$y = 3(1+\sqrt{5}) + 4 = 13.708\ldots$$
$$y = 3(1-\sqrt{5}) + 4 = 0.2917\ldots$$

$$\boldsymbol{x = 3.24\ (3\ sf) \quad or \quad x = -1.24\ (3\ sf)}$$
$$\boldsymbol{y = 13.7\ (3\ sf) \quad or \quad y = 0.292\ (3\ sf)}$$

Challenge

13) A curve **C** has equation $y = x^2 - 2$. A line **L** has equation $x + y = 4$. The line intersects the curve at two points.

Work out the midpoint of the two points of intersection.

$$y = x^2 - 2$$

$$x + y = 4$$
$$y = 4 - x$$

$$x^2 - 2 = 4 - x$$
$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x = -3 \quad or \quad x = 2$$

$$y = 4 - -3 = 7$$
$$y = 4 - 2 = 2$$

Points of intersection:
$(-3, 7) \quad and \quad (2, 2)$

Midpoint:
$$\left(\frac{2 + (-3)}{2}, \frac{2 + 7}{2}\right)$$
$$\boxed{\left(-\frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{9}}{\mathbf{2}}\right)}$$

14) A curve **C** has equation $y = 2x - x^2$. A line **L** has equation $x - y = 2$. The line intersects the curve at two points.

Work out the distance between the two points of intersection. Give your answer as an exact value.

$$y = 2x - x^2$$

$$x - y = 2$$
$$y = x - 2$$

$$2x - x^2 = x - 2$$
$$x^2 - x - 2 = 0$$
$$(x - 2)(x + 1) = 0$$
$$x = 2 \quad or \quad x = -1$$

$$y = 2 - 2 = 0$$
$$y = -1 - 2 = -3$$

Points of intersection:
$(2, 0) \quad and \quad (-1, -3)$

Distance:
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1 - 2)^2 + (-3 - 0)^2}$$
$$= \sqrt{(-3)^2 + (-3)^2}$$
$$= \sqrt{9 + 9}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}$$

$$\boxed{\textbf{Distance} = 3\sqrt{2}}$$