QUADRATIC FORMULA

Task 1 – Solve each of the following equations using the quadratic formula.

1) $x^2 - 4x - 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{36}}{2}$$

$$\boldsymbol{x = 5 \text{ or } x = -1}$$

2) $x^2 + 2x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{16}}{2}$$

$$\boldsymbol{x = 1 \text{ or } x = -3}$$

3) $x^2 - 6x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{16}}{2}$$

$$\boldsymbol{x = 5 \text{ or } x = 1}$$

4) $4x^2 + 4x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{64}}{8}$$

$$\boldsymbol{x = \frac{1}{2} \text{ or } x = -\frac{3}{2}}$$

5) $x^2 - 4x - 12 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{64}}{2}$$

$$\boldsymbol{x = 6 \text{ or } x = -2}$$

6) $7x^2 + x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(7)(-6)}}{2(7)}$$

$$x = \frac{-1 \pm \sqrt{169}}{14}$$

$$\boldsymbol{x = \frac{6}{7} \text{ or } x = -1}$$

Task 2 – Work out the value of x. Give your answers in the form $x = a + b\sqrt{c}$ where a, b and c are constants.

7) $x^2 + 6x - 11 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-11)}}{2(1)}$$

$$x = \frac{-6 \pm 4\sqrt{5}}{2}$$

$$\boldsymbol{x = -3 \pm 2\sqrt{5}}$$

8) $x^2 - 4x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{4 \pm 2\sqrt{11}}{2}$$

$$\boldsymbol{x = 2 \pm \sqrt{11}}$$

9) $x^2 + 10x + 17 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(17)}}{2(1)}$$

$$x = \frac{-10 \pm 4\sqrt{2}}{2}$$

$$\boldsymbol{x = -5 \pm 2\sqrt{2}}$$

10) $x^2 - 8x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{8 \pm 2\sqrt{13}}{2}$$

$$\boldsymbol{x = 4 \pm \sqrt{13}}$$

11) $x^2 - 2x - 19 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-19)}}{2(1)}$$

$$x = \frac{2 \pm 4\sqrt{5}}{2}$$

$$\boldsymbol{x = 1 \pm 2\sqrt{5}}$$

12) $x^2 - 12x + 31 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(31)}}{2(1)}$$

$$x = \frac{12 \pm 2\sqrt{5}}{2}$$

$$\boldsymbol{x = 6 \pm \sqrt{5}}$$

13) $2x^2 - 4x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{4 \pm 2\sqrt{6}}{4}$$

$$\boldsymbol{x = 1 \pm \frac{1}{2}\sqrt{6}}$$

14) $3x^2 + 6x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-6 \pm 4\sqrt{3}}{6}$$

$$\boldsymbol{x = -1 \pm \frac{2}{3}\sqrt{3}}$$

15) $2x^2 - 4x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{4 \pm 2\sqrt{10}}{4}$$

$$\boldsymbol{x = 1 \pm \frac{1}{2}\sqrt{10}}$$

Task 3 – Solve the following equations. Give your answers to 2 decimal places.

16) $x^2 + 3x - 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)}$$

$$\boldsymbol{x = 1.19 \text{ or } x = -4.19}$$

17) $5x^2 - 2x - 9 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(5)(-9)}}{2(5)}$$

$$\boldsymbol{x = 1.56 \quad \text{or} \quad x = -1.16}$$

18) $4x^2 - 8x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)}$$

$$\boldsymbol{x = 1.87 \quad \text{or} \quad x = 0.13}$$

19) $6x^2 + 5x - 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(6)(-8)}}{2(6)}$$

$$\boldsymbol{x = 0.81 \quad \text{or} \quad x = -1.64}$$

20) $9x^2 - 12x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(9)(2)}}{2(9)}$$

$$\boldsymbol{x = 1.14 \quad \text{or} \quad x = 0.20}$$

21) $-8x^2 + 6x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-8)(5)}}{2(-8)}$$

$$\boldsymbol{x = 1.25 \quad \text{or} \quad x = -0.50}$$

22) $10x^2 + 3x - 5 = -3$

$10x^2 + 3x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(10)(-2)}}{2(10)}$$

$$\boldsymbol{x = 0.32 \quad \text{or} \quad x = -0.62}$$

23) $3x^2 - 7x + 10 = 13$

$3x^2 - 7x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(3)(-3)}}{2(3)}$$

$$\boldsymbol{x = 2.70 \quad \text{or} \quad x = -0.37}$$

24) $15x^2 + 2x + 7 = 10$

$15x^2 + 2x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(15)(-3)}}{2(15)}$$

$$\boldsymbol{x = 0.39 \quad \text{or} \quad x = -0.52}$$

25) $16x^2 - 15x + 1 = 3x - 4$

$16x^2 - 18x + 5 = 0$

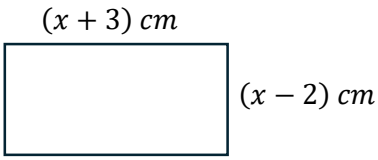
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{18 \pm \sqrt{(-18)^2 - 4(16)(5)}}{2(16)}$$

$$\boldsymbol{x = 0.63 \quad \text{or} \quad x = 0.50}$$

Challenge

26) A rectangle is shown below. The area of the rectangle is 40 cm². Work out the value of x. Give your answer to 2 decimal places.



$Area = b \times h$

$(x+3)(x-2) = 40$

$x^2 - 2x + 3x - 6 = 40$

$x^2 + x - 46 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-46)}}{2(1)}$

$x = 6.30$ or ~~$x = -7.30$~~

$\boldsymbol{x = 6.30\ cm}$

27) A ball is projected upward from a platform 5 m above the ground with velocity 18 m/s. Its height is given by:

$$h = -4.9t^2 + 18t + 5$$

Work out the time when the ball hits the ground. Give your answer to 2 decimal places.

The ball will hit the ground when h = 0:

$-4.9t^2 + 18t + 5 = 0$

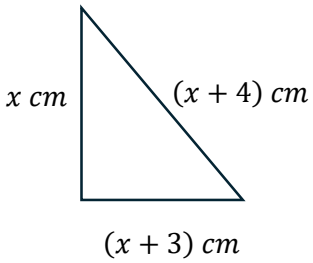
$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \dfrac{-18 \pm \sqrt{18^2 - 4(-4.9)(5)}}{2(-4.9)}$

$t = -0.26$ or $t = 3.93$

$\boldsymbol{t = 3.93\ seconds}$

28) A right-angled triangle is shown below.



Work out the values of x. Give your answers in the form $x = a \pm b\sqrt{c}$.

$a^2 + b^2 = c^2$

$x^2 + (x+3)^2 = (x+4)^2$

$x^2 + (x+3)(x+3) = (x+4)(x+4)$

$x^2 + x^2 + 3x + 3x + 9 = x^2 + 4x + 4x + 16$

$2x^2 + 6x + 9 = x^2 + 8x + 16$

$x^2 - 2x - 7 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2(1)}$

$\boldsymbol{x = 1 \pm 2\sqrt{2}\ cm}$

29) Solve for x. Give your answers to 2 decimal places.

$$\frac{3}{x-2} + \frac{5}{x+1} = 2$$

$\dfrac{3(x+1)}{(x+1)(x-2)} + \dfrac{5(x-2)}{(x+1)(x-2)} = 2$

$\dfrac{3(x+1) + 5(x-2)}{(x+1)(x-2)} = 2$

$3x + 3 + 5x - 10 = 2(x+1)(x-2)$

$8x - 7 = 2(x^2 - 2x + x - 2)$

$8x - 7 = 2(x^2 - x - 2)$

$8x - 7 = 2x^2 - 2x - 4$

$2x^2 - 10x + 3 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{10 \pm \sqrt{(-10)^2 - 4(2)(3)}}{2(2)}$

$\boldsymbol{x = 4.68\ (2\ dp)}$ or $\boldsymbol{x = 0.32\ (2\ dp)}$

30) Find k such that $x^2 + kx + 9 = 0$ has exactly one solution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Has one solution when the discriminant is equal to zero:

$\sqrt{b^2 - 4ac} = 0$

$\sqrt{k^2 - 4(1)(9)} = 0$

$k^2 - 36 = 0$

$k^2 = 36$

$\boldsymbol{k = \pm 6}$