GEOMETRIC SEQUENCES

Task 1 – Find the next term in each of the following sequences.

1) 3, 6, 12, 24, ... **48**
2) 5, 10, 20, 40, ... **80**
3) 2, 20, 200, 2000, ... **20000**
4) 81, 27, 9, 3, ... **1**
5) 5, 40, 320, 2560, ... **20480**
6) ½, 1, 2, 4, ... **8**
7) 100, 50, 25, 12.5, ... **6.25**
8) -2, 6, -18, 54, ... **-162**
9) 7, 21, 63, 189, ... **567**
10) 0.2, 0.8, 3.2, 12.8, ... **51.2**
11) 5, -25, 125, -625, ... **3125**
12) 3a, 6a, 12a, 24a, ... **48a**
13) b + 2, 3b + 6, 9b + 18, ... **27b + 54**

Task 2 – For each of the following sequences, state the first term and the common ratio.

14) 2, 10, 50, 250, ... $\boldsymbol{a = 2, \ r = 5}$
15) 64, 32, 16, 8, ... $\boldsymbol{a = 64, \ r = 0.5}$
16) 3, -6, 12, -24, ... $\boldsymbol{a = 3, \ r = -2}$
17) $\frac{1}{3}$, 1, 3, 9, ... $\boldsymbol{a = \frac{1}{3}, \ r = 3}$
18) 200, 100, 50, 25, ... $\boldsymbol{a = 200, \ r = 0.5}$
19) 5, 15, 45, 135, ... $\boldsymbol{a = 5, \ r = 3}$

Task 3 – The nth term formula of geometric sequences are given. Work out the first three terms of each sequence.

20) $a_n = 2^{n-1}$

$$2^{1-1} = 1$$
$$2^{2-1} = 2$$
$$2^{3-1} = 4$$

1, 2, 4

21) $a_n = 5^{n-1}$

$$5^{1-1} = 1$$
$$5^{2-1} = 5$$
$$5^{3-1} = 25$$

1, 5, 25

22) $a_n = 2 \times 3^{n-1}$

$$2 \times 3^{1-1} = 2$$
$$2 \times 3^{2-1} = 6$$
$$2 \times 3^{3-1} = 18$$

2, 6, 18

23) $a_n = 3 \times 4^{n-1}$

$$3 \times 4^{1-1} = 3$$
$$3 \times 4^{2-1} = 12$$
$$3 \times 4^{3-1} = 48$$

3, 12, 48

24) $a_n = -6 \times 2^{n-1}$

$$-6 \times 2^{1-1} = -6$$
$$-6 \times 2^{2-1} = -12$$
$$-6 \times 2^{3-1} = -24$$

-6, -12, -24

Task 4 – For each of the following sequences, work out the nth term formula.

25) 4, 12, 36, 108, ...

$$a_n = a \times r^{n-1}$$
$$a = 4, r = 3$$

$$\boldsymbol{a_n = 4 \times 3^{n-1}}$$

26) 2, 8, 32, 128, ...

$$a_n = a \times r^{n-1}$$
$$a = 2, r = 4$$

$$\boldsymbol{a_n = 2 \times 4^{n-1}}$$

27) 7, 14, 28, 56, ...

$$a_n = a \times r^{n-1}$$
$$a = 7, r = 2$$

$$\boldsymbol{a_n = 7 \times 2^{n-1}}$$

28) 81, 27, 9, 3, ...

$$a_n = a \times r^{n-1}$$
$$a = 81, r = \frac{1}{3}$$

$$\boldsymbol{a_n = 81 \times \left(\frac{1}{3}\right)^{n-1}}$$

29) ½, 1, 2, 4, ...

$$a_n = a \times r^{n-1}$$
$$a = \frac{1}{2}, r = 2$$

$$\boldsymbol{a_n = \frac{1}{2} \times 2^{n-1}}$$

30) -3, 6, -12, 24, ...
$$a_n = a \times r^{n-1}$$
$$a = -3, r = -2$$

$$\mathbf{a_n = -3 \times (-2)^{n-1}}$$

Task 5

31) Consider the sequence: 5, 15, 45, 135, ... Is 3,645 a term in the sequence? Give a reason for your answer.

n^{th} term:
$$5 \times 3^{n-1}$$

$$5 \times 3^{n-1} = 3645$$
$$3^{n-1} = 729$$
$$3^{n-1} = 3^6$$

$$n - 1 = 6$$
$$n = 7$$

Yes, it is the 7th term in the sequence.

32) Consider the sequence: 2, 6, 18, 54, ... Is 486 a term in the sequence? Give a reason for your answer.

n^{th} term:
$$2 \times 3^{n-1}$$

$$2 \times 3^{n-1} = 486$$
$$3^{n-1} = 243$$
$$3^{n-1} = 3^5$$

$$n - 1 = 5$$
$$n = 6$$

Yes, it is the 6th term in the sequence.

33) Consider the sequence: -4, 8, -16, 32, ... Is -512 a term in the sequence? Give a reason for your answer.

n^{th} term:
$$-4 \times (-2)^{n-1}$$

$$-4 \times (-2)^{n-1} = -512$$
$$(-2)^{n-1} = 128$$

No, $(-2)^7 = -128$

Challenge

34) The nth term formula for two geometric sequences are as follows:

Sequence A: $3 \times 2^{n-1}$

Sequence B: $192 \times \left(\frac{1}{2}\right)^{n-1}$

For what value of n are the nth terms equal? You must show clear algebraic working. Do not use trial and error.

Set n^{th} term formulas equal:
$$3 \times 2^{n-1} = 192 \times \left(\frac{1}{2}\right)^{n-1}$$
$$2^{n-1} = 64 \times \left(\frac{1}{2}\right)^{n-1}$$
$$2^{n-1} = 64 \times (2^{-1})^{n-1}$$
$$2^{n-1} = 64 \times 2^{1-n}$$
$$\frac{2^{n-1}}{2^{1-n}} = 64$$
$$2^{n-1-(1-n)} = 64$$
$$2^{2n-2} = 64$$
$$2^{2n-2} = 2^6$$

$$2n - 2 = 6$$
$$2n = 8$$
$$n = 4$$

The 4th terms of the sequences are equal.

35) The 4th term of a geometric sequence is 54. The 7th term is 1458. Work out:

a. The common ratio
$$a_4 \quad a_5 \quad a_6 \quad a_7$$
$$54 \times r^3 = 1458$$
$$r^3 = 27$$
$$\mathbf{r = 3}$$

b. The first term
$$a \quad a_2 \quad a_3 \quad a_4$$
$$a \times r^3 = 54$$
$$a \times 3^3 = 54$$
$$a \times 27 = 54$$
$$\mathbf{a = 2}$$

c. The formula for the nth term
$$a_n = a \times r^{n-1}$$
$$= \mathbf{2 \times 3^{n-1}}$$

36) The first term of Sequence A is 5 and the common ratio is 3. The first term of Sequence B is 405 and the common ratio is $\frac{1}{3}$. For which term number do the sequences have the same value? You must show clear algebraic working. Do not use trial and error.

n$^{\text{th}}$ term formula:
$$a_n = a \times r^{n-1}$$

n$^{\text{th}}$ term of sequence A:
$$a_n = 5 \times 3^{n-1}$$

n$^{\text{th}}$ term of sequence B:
$$a_n = 405 \times \left(\frac{1}{3}\right)^{n-1}$$

Setting equal:
$$5 \times 3^{n-1} = 405 \times \left(\frac{1}{3}\right)^{n-1}$$

$$3^{n-1} = 81 \times \left(\frac{1}{3}\right)^{n-1}$$

$$3^{n-1} = 81 \times (3^{-1})^{n-1}$$

$$3^{n-1} = 81 \times 3^{1-n}$$

$$\frac{3^{n-1}}{3^{1-n}} = 81$$

$$3^{n-1-(1-n)} = 81$$

$$3^{2n-2} = 3^4$$

$$2n - 2 = 4$$
$$2n = 6$$
$$n = 3$$

The third term of the sequences are the same.