

ARITHMETIC SEQUENCES

Task 1 – Work out the next three terms in each of the following sequences.

- 1) 2, 6, 10, 14, ... **18, 22, 26**
- 2) 4, 9, 14, 19, ... **24, 29, 34**
- 3) 12, 15, 18, 21, ... **24, 27, 30**
- 4) 72, 68, 64, 60, ... **56, 52, 48**
- 5) -3, 5, 13, 21, ... **29, 37, 45**
- 6) 0.5, 1, 1.5, 2, ... **2.5, 3, 3.5**
- 7) 100, 93, 86, 79, ... **72, 65, 58**
- 8) -15, -10, -5, 0, ... **5, 10, 15**
- 9) 3.2, 3.5, 3.8, 4.1, ... **4.4, 4.7, 5.0**
- 10) 250, 225, 200, 175, ... **150, 125, 100**
- 11) $a, 2a, 3a, 4a, \dots$ **$5a, 6a, 7a$**
- 12) $x + 2, x + 8, x + 14, \dots$
 $x + 20, x + 26, x + 32$

Task 2 – For each of the following sequences, state the first term and the common difference.

- 13) 18, 24, 30, 36, ... **$a = 18, d = 6$**
- 14) -2, -7, -12, -17, ... **$a = -2, d = -5$**
- 15) 5.5, 5.8, 6.1, 6.4, ... **$a = 5.5, d = 0.3$**
- 16) $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \dots$ **$a = \frac{1}{3}, d = \frac{1}{3}$**
- 17) 90, 75, 60, 45, ... **$a = 90, d = -15$**
- 18) -40, -32, -24, -16, ... **$a = -40, d = 8$**

Task 3 – The n^{th} term formula of arithmetic sequences are given. Work out the first three terms of each sequence.

19) $n + 5$

$$1 + 5 = 6$$

$$2 + 5 = 7$$

$$3 + 5 = 8$$

6, 7, 8

20) $2n + 3$

$$2(1) + 3 = 5$$

$$2(2) + 3 = 7$$

$$2(3) + 3 = 9$$

5, 7, 9

21) $3n + 4$

$$3(1) + 4 = 7$$

$$3(2) + 4 = 10$$

$$3(3) + 4 = 13$$

7, 10, 13

22) $2n + 8$

$$2(1) + 8 = 10$$

$$2(2) + 8 = 12$$

$$2(3) + 8 = 14$$

10, 12, 14

23) $-n + 10$

$$-1 + 10 = 9$$

$$-2 + 10 = 8$$

$$-3 + 10 = 7$$

9, 8, 7

24) $7 - n$

$$7 - 1 = 6$$

$$7 - 2 = 5$$

$$7 - 3 = 4$$

6, 5, 4

25) $-4n + 8$

$$-4(1) + 8 = 4$$

$$-4(2) + 8 = 0$$

$$-4(3) + 8 = -4$$

4, 0, -4

26) $-3n + 9$

$$-3(1) + 9 = 6$$

$$-3(2) + 9 = 3$$

$$-3(3) + 9 = 0$$

6, 3, 0

Task 4 – Work out the n^{th} term formula for each of the following sequences.

27) 6, 10, 14, 18, ...

Common difference = 4

$4n: 4, 8, 12, 16, \dots$

$4n + 2 = 6, 10, 14, 18 \dots$

$4n + 2$

28) -3, 4, 11, 18, ...

Common difference = 7

$7n: 7, 14, 21, 28, \dots$

$7n - 10 = -3, 4, 11, 18 \dots$

$7n - 10$

29) 4, 8, 12, 16, ...

Common difference = 4

$4n$: 4, 8, 12, 16, ...

$$4n$$

30) 4, 9, 14, 19, ...

Common difference = 5

$5n$: 5, 10, 15, 20 ...

$5n - 1$: 4, 9, 14, 19, ...

$$5n - 1$$

31) 25, 20, 15, 10, ...

Common difference = -5

$-5n$: -5, -10, -15, -20 ...

$-5n + 30$: 25, 20, 15, 10 ...

$$-5n + 30$$

32) 0.5, 1.5, 2.5, 3.5, ...

Common difference = 1

n : 1, 2, 3, 4 ...

$n - 0.5$: 0.5, 1.5, 2.5, 3.5 ...

$$n - 0.5$$

33) 100, 97, 94, 91, ...

Common difference = -3

$-3n$: -3, -6, -9, -12 ...

$-3n + 103$: 100, 97, 94, 91 ...

$$-3n + 103$$

34) -12, -7, -2, 3, ...

Common difference = 5

$5n$: 5, 10, 15, 20 ...

$5n - 17$: -12, -7, -2, 3 ...

$$5n - 17$$

35) $\frac{2}{3}, \frac{5}{3}, \frac{8}{3}, \frac{11}{3}, \dots$

Common difference = 1

n : 1, 2, 3, 4, ...

$n - \frac{1}{3}$: $\frac{2}{3}, \frac{5}{3}, \frac{8}{3}, \frac{11}{3}, \dots$

$$n - \frac{1}{3}$$

Task 5

36) Consider the sequence: 2, 7, 12, 17, ...

Is 58 a term in the sequence? Give a reason for your answer.

No, all terms in the sequence end in 2 or 7.

37) Consider the sequence: 16, 20, 24, 28, ...

Is 80 a term in the sequence? Give a reason for your answer.

$$4n + 12 = 80$$

$$4n = 68$$

$$n = 17$$

Yes, 80 is the 17th term. See working from the n^{th} term formula above.

38) Consider the sequence: 5, 9, 13, 17, ...

Is 45 a term in the sequence? Give a reason for your answer.

$$4n + 1 = 45$$

$$4n = 44$$

$$n = 11$$

Yes, 45 is the 11th term. See working from the n^{th} term formula above.

39) Consider the sequence: 12, 19, 26, 33, ...

Is 200 a term in the sequence? Give a reason for your answer.

$$7n + 5 = 200$$

$$7n = 195$$

$$n = \frac{195}{7}$$

No, 200 is not a term in the sequence. See working from the n^{th} term formula above.

40) Consider the sequence: -8, -3, 2, 7, ...

Is 57 a term in the sequence? Give a reason for your answer.

$$5n - 13 = 57$$

$$5n = 70$$

$$n = 14$$

Yes, 57 is the 14th term in the sequence. See working from the n^{th} term formula above.

- 41) The first four terms of an arithmetic sequence are: 12, 16, 20, 24, ...
Work out the 100th term of the sequence.

Work out the n^{th} term formula:

$$4n: 4, 8, 12, 16, \dots$$

$$4n + 8: 12, 16, 20, 24, \dots$$

$$4n + 8$$

$$4(100) + 8 = \mathbf{408}$$

- 42) The first four terms of an arithmetic sequence are: 4, 10, 16, 22, ...
Work out the 45th term of the sequence.

Work out the n^{th} term formula:

$$6n: 6, 12, 18, 24, \dots$$

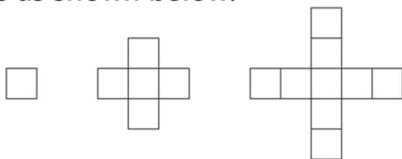
$$6n - 2: 4, 10, 16, 22, \dots$$

$$6n - 2$$

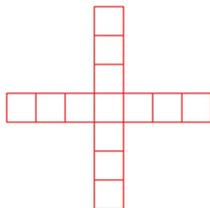
$$6(45) - 2 = \mathbf{268}$$

Task 6

- 43) A sequence of patterns is made from squares as shown below.



- a. Draw the next pattern in the sequence.



- b. How many squares are needed to make pattern number 8?

Rule is to add 4 each time:

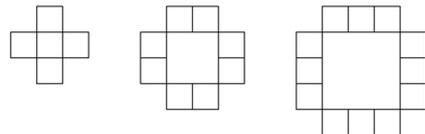
$$4n - 3$$

$$4(8) - 3 = \mathbf{29 \text{ squares}}$$

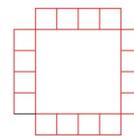
- c. Will there be a pattern with 37 squares? Explain your reasoning.

Yes, the rule is to add four squares each time. Therefore the 10th pattern will have 37 squares.

- 44) A pattern is made from squares as shown below.



- a. Draw the next pattern in the sequence.



- b. How many squares will be in the 6th pattern?

Rule is to add 4 each time:

$$4n + 1$$

$$4(6) + 1 = \mathbf{25 \text{ squares}}$$

- c. Will there be a pattern with 103 squares? Explain your reasoning.

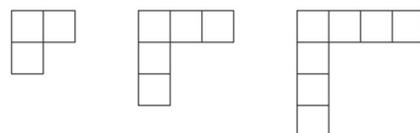
$$4n + 1 = 103$$

$$4n = 102$$

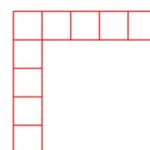
$$n = 25.5$$

No, as demonstrated by the n^{th} term formula.

- 45) An L-shaped pattern made from squares is shown below.



- a. Draw the next pattern in the sequence.



- b. Write an n^{th} term formula for the number of squares in the pattern.

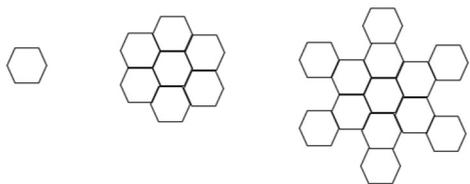
Rule is to add 2 each time:

$$2n: 2, 4, 6, 8, \dots$$

$$2n + 1: 3, 5, 7, 9, \dots$$

$$\mathbf{2n + 1}$$

46) A pattern is made from hexagons as shown below.



Work out the n^{th} term formula for the number of hexagons in the pattern.

Rule is to add 6 each time:

$$6n: 6, 12, 18, \dots$$

$$6n - 5: 1, 7, 13 \dots$$

$$\mathbf{6n - 5}$$

47) A pattern is made from triangles as shown below.



Can the pattern be described using an arithmetic sequence? Why or why not?

1st pattern: 4 triangles

2nd pattern: 9 triangles

3rd pattern: 16 triangles

No, the pattern has a common difference of 5 and then 7. An arithmetic sequence has the same difference from term to term.

Challenge

48) The first four terms of two sequences are shown below.

Sequence A: 3, 7, 11, 15, ...

Sequence B: 1, 6, 11, 16, ...

For what value of n are the n^{th} terms equal?

n^{th} term formula of sequence A:

$$4n: 4, 8, 12, 16, \dots$$

$$4n - 1: 3, 7, 11, 15, \dots$$

$$4n - 1$$

n^{th} term formula of sequence B:

$$5n: 5, 10, 15, 20, \dots$$

$$5n - 4: 1, 6, 11, 16 \dots$$

$$5n - 4$$

$$4n - 1 = 5n - 4$$

$$-1 = n - 4$$

$$n = 3$$

3rd term

49) The 5th term of an arithmetic sequence is 18. The 12th term is 46. Work out:

a. The common difference

$$a + (n - 1) \times d$$

5th term:

$$a + (5 - 1) \times d = 18$$

$$a + 4d = 18$$

12th term:

$$a + (12 - 1) \times d = 46$$

$$a + 11d = 46$$

Subtract the equations:

$$a + 11d = 46$$

$$-(a + 4d) = 18$$

$$7d = 28$$

$$\mathbf{d = 4}$$

b. The first term

Substitute into equation above:

$$a + 4d = 18$$

$$a + 4(4) = 18$$

$$a + 16 = 18$$

$$\mathbf{a = 2}$$

c. The formula for the n^{th} term

$$a_n = a + (n - 1) \times d$$

$$= 2 + (n - 1) \times 4$$

$$= 2 + 4n - 4$$

$$= \mathbf{4n - 2}$$

50) Sequence A has a first term of 4 and a common difference of 6. Sequence B has a first term of 16 and a common difference of 3. One term in each of the sequences is the same. Which term is it?

n^{th} term formula of sequence A:

$$a_n = a + (n - 1) \times d$$

$$= 4 + (n - 1) \times 6$$

$$= 4 + 6n - 6$$

$$= \mathbf{6n - 2}$$

n^{th} term formula of sequence B:

$$a_n = a + (n - 1) \times d$$

$$= 16 + (n - 1) \times 3$$

$$= 16 + 3n - 3$$

$$= \mathbf{3n + 13}$$

$$6n - 2 = 3n + 13$$

$$3n - 2 = 13$$

$$3n = 15$$

$$n = \mathbf{5}$$

The 5th term of each sequence is the same.

51) An arithmetic sequence has an n^{th} term formula

$$a_n = bn + c$$

- a. Work out the 3rd and 4th terms of the sequence in terms of b and c .

$$\begin{aligned} a_3 &= b(3) + c \\ &= 3b + c \end{aligned}$$

$$\begin{aligned} a_4 &= b(4) + c \\ &= 4b + c \end{aligned}$$

A different arithmetic sequence has an n^{th} term formula

$$a_n = 2.5bn - 2c$$

- b. Work out the 3rd and 4th terms of the sequence in terms of b and c .

$$\begin{aligned} a_3 &= 2.5b(3) - 2c \\ &= 7.5b - 2c \end{aligned}$$

$$\begin{aligned} a_4 &= 2.5b(4) - 2c \\ &= 10b - 2c \end{aligned}$$

- c. The 3rd term of the first sequence is 10. The 4th term of the first sequence is equal to the 4th term of the second sequence. Work out the values of b and c .

3rd term of first sequence:

$$3b + c = 10$$

Setting the 4th terms equal:

$$4b + c = 10b - 2c$$

$$4b + 3c = 10b$$

$$3c = 6b$$

$$c = 2b$$

Substitute into 3rd term of first sequence:

$$3b + 2b = 10$$

$$5b = 10$$

$$b = 2$$

$$c = 2 \times 2 = 4$$

$$\mathbf{b = 2, c = 4}$$