Task 1

1) Complete the table of values for $y = x^3 - 2x$

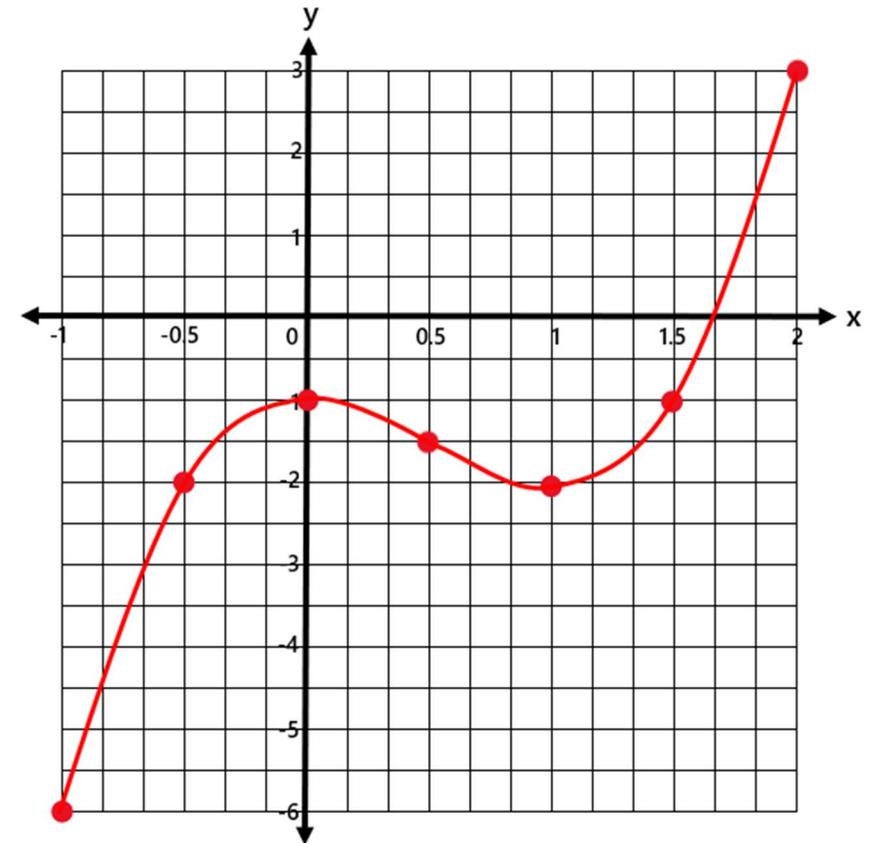| x	-2	-1	0	1	2
y	-4	1	0	-1	4

Draw the graph of $y = x^3 - 2x$ on the grid below.
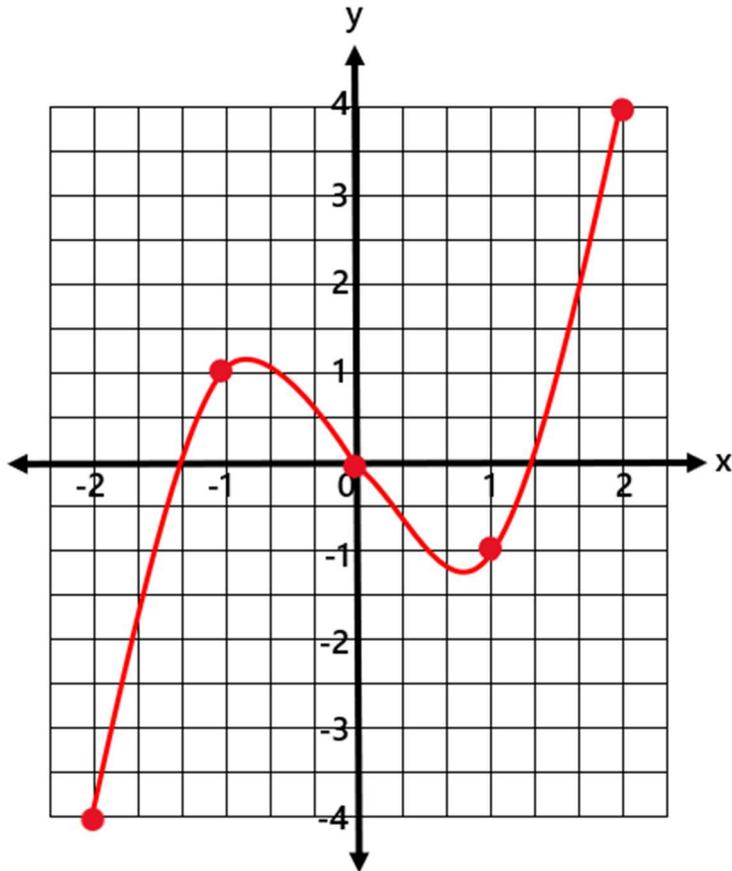
2) Complete the table of values for $y = 2x^3 - 3x^2 - 1$
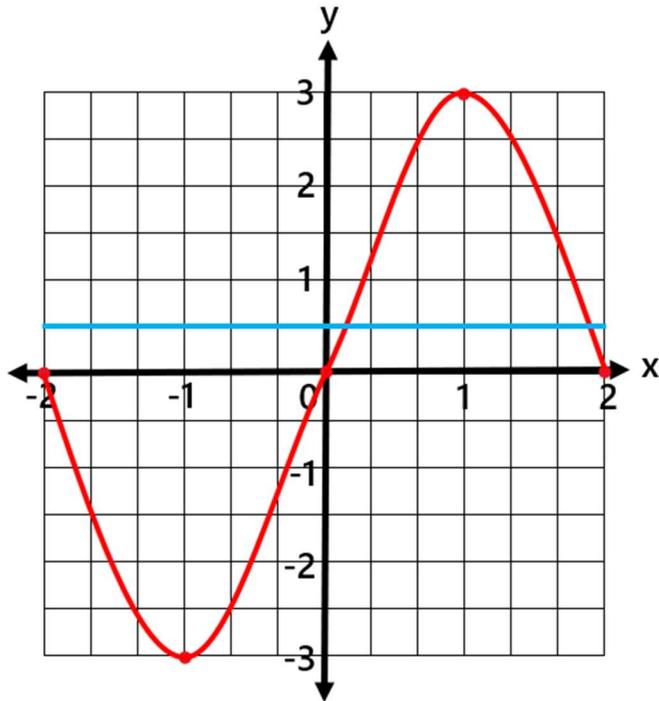
x	-1	-0.5	0	0.5	1	1.5	2
y	-6	-2	-1	-1.5	-2	-1	3

Draw the graph of $y = 2x^3 - 3x^2 - 1$ on the grid below.

3) Complete the table of values for $y = 4x - x^3$

x	-2	-1	0	1	2
y	0	-3	0	3	0

Draw the graph of $y = 4x - x^3$ on the grid below.

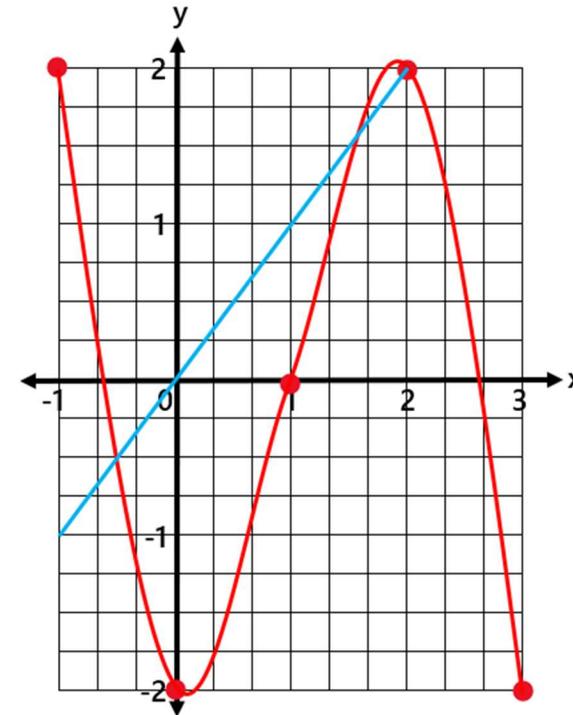Use the same grid to plot the equation of the line $y = 0.5$
See above

How many positive solutions are there to the equation $4x - x^3 = 0.5$?
Give a reason for your answer.
There are two positive solutions, as the graphs intersect at two distinct points.

4) Complete the table of values for $y = -x^3 + 3x^2 - 2$

x	-1	0	1	2	3
y	2	-2	0	2	-2

Draw the graph of $y = -x^3 + 3x^2 - 2$ on the grid below.

Use the same grid to plot the equation $y = x$.
See above

State the number of solutions to the equation $-x^3 + 3x^2 - 2 = x$.
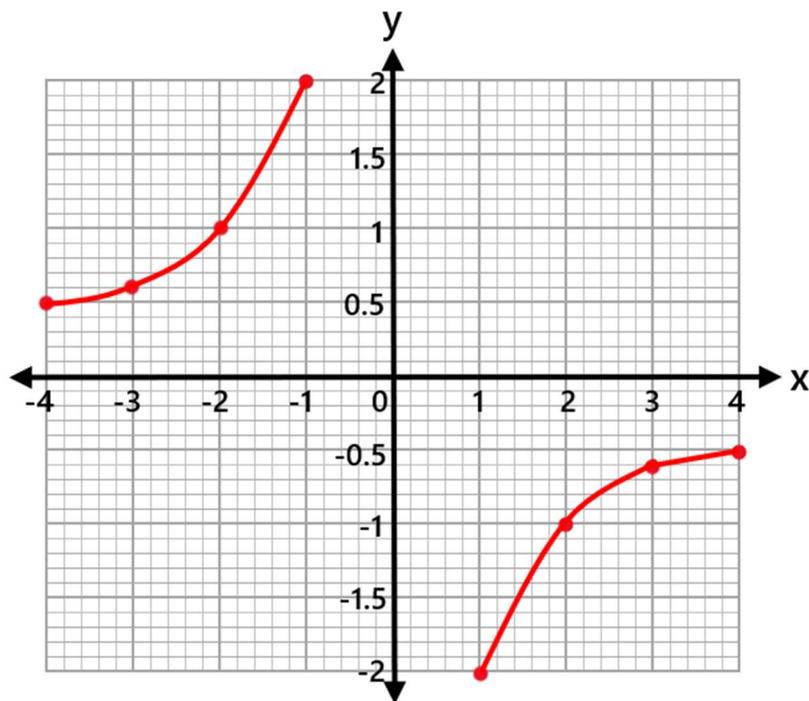Give a reason for your answer.
There are three solutions, as the graphs intersect at three distinct points.

Task 2

5) Complete the table of values for $y = -\frac{2}{x}$

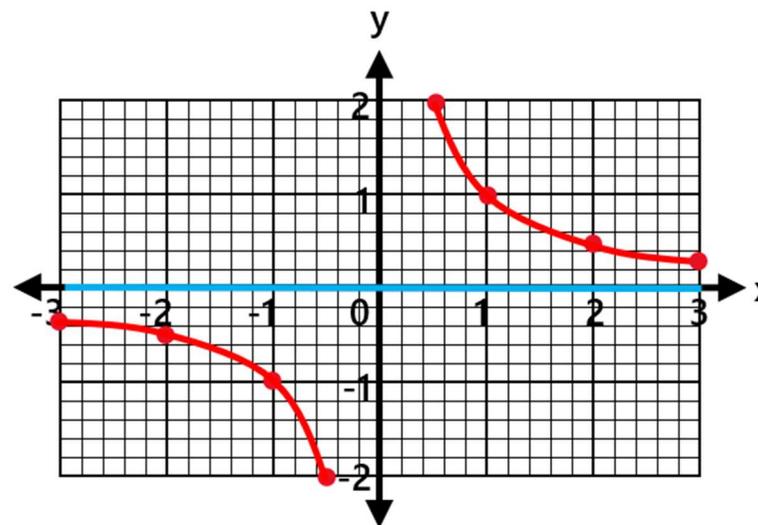| x	-4	-3	-2	-1	1	2	3	4
y	$\frac{1}{2}$	$\frac{2}{3}$	1	2	-2	-1	$-\frac{2}{3}$	$-\frac{1}{2}$

Draw the graph of $y = -\frac{2}{x}$ on the grid below.

6) Complete the table of values for $y = \frac{1}{x}$

x	-3	-2	-1	-0.5	0.5	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$	$\frac{1}{3}$

Draw the graph of $y = \frac{1}{x}$ on the grid below.

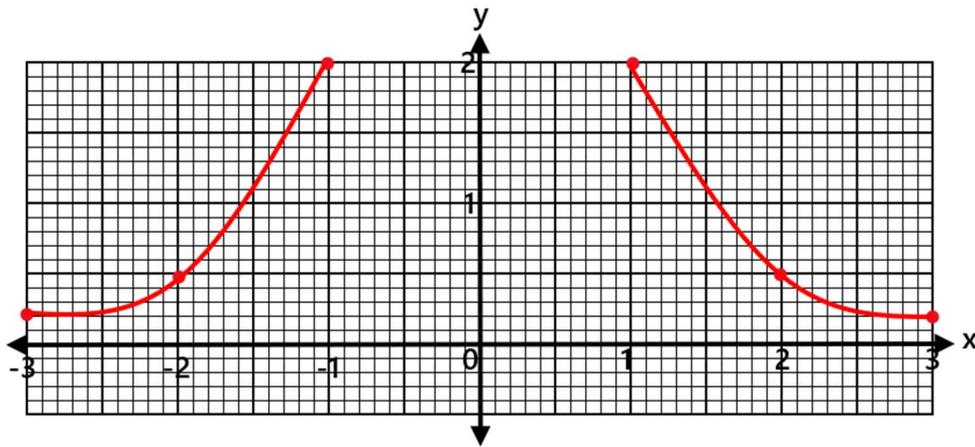Use the same grid to plot the equation of the line $y = 0$
See above

Explain why the equations $y = \frac{1}{x}$ and $y = 0$ have no solutions.
There are no points of intersection, as shown on the graph. The reciprocal graph has a horizontal asymptote at $y = 0$.

7) Complete the table of values for $y = \frac{2}{x^2}$

| x	-3	-2	-1	1	2	3
y	$\frac{2}{9}$	$\frac{1}{2}$	2	2	$\frac{1}{2}$	$\frac{1}{9}$

Draw the graph of $y = \frac{2}{x^2}$ on the grid below.

Explain why $\frac{2}{x^2} > 0$ for all values of x, where $x \neq 0$.

For all $x \neq 0$, x^2 is positive because the square of any non-zero number is positive.

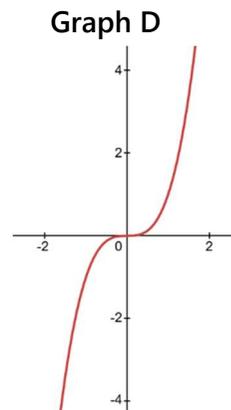Given that the numerator 2 is positive and the denominator x^2 is also positive, the fraction $\frac{2}{x^2} > 0$ for all $x \neq 0$.
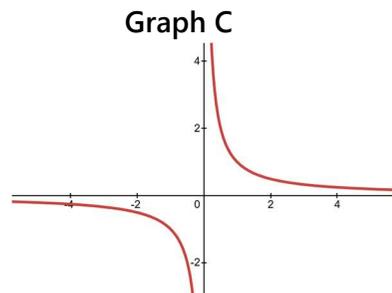
Task 3

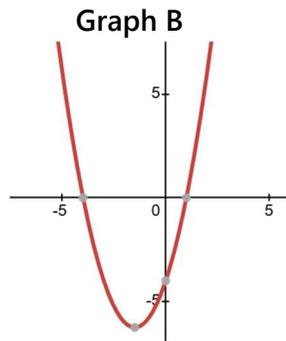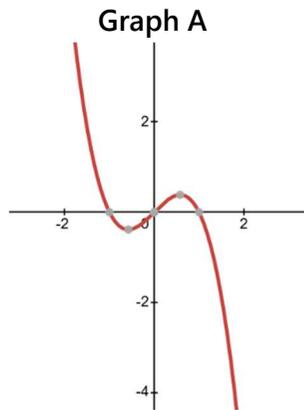8) Which of the following graphs could represent $y = x^3 - 2x$? Explain your reasoning.
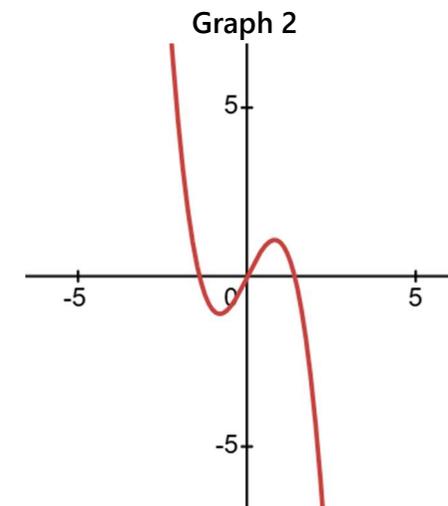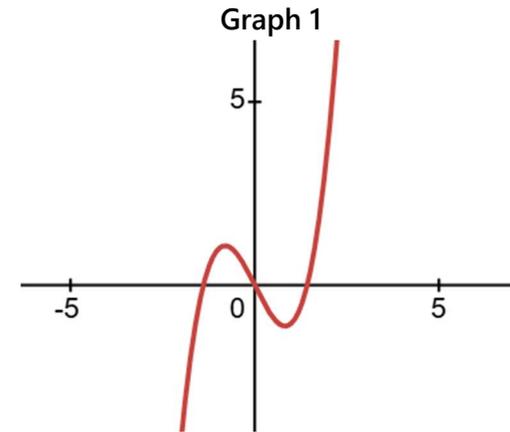
Graph 1

Graph 2

Graph 1, because the coefficient of $x^3 > 0$.

9) Match each of the following equations with a graph shown below.

Graph	Letter
$y = x^3$	D
$y = \dfrac{1}{x}$	C
$y = -x^3 + x$	A

Graph A

Graph B

Graph C

Graph D

10) Which of the following equations represents a reciprocal equation?
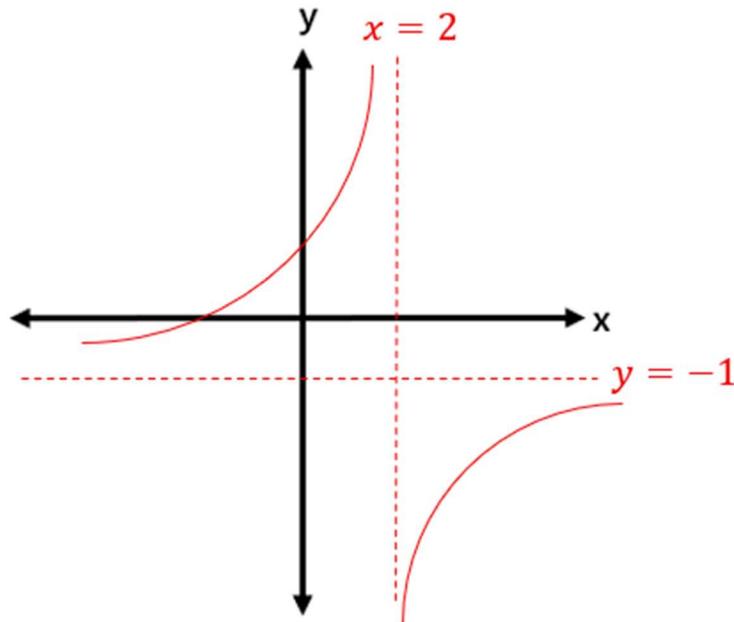
a) $y = x^3 - 2x$

b) $y = \dfrac{5}{x}$

c) $y = x^2 - 4$

Equation b

11) Use the space below to sketch the general shape of the graph of $y = x^3$

Challenge

12) A reciprocal equation is of the form $y = \frac{a}{x+b} + c$, where $a < 0$. Use the axes below to sketch the general shape of this graph, with a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = -1$.

13) A cubic function $y = ax^3 + bx^2 + c$ passes through the points: (0, 2), (-1, 4), and (2, 22). Work out the values of a, b, and c. You must demonstrate clear algebraic working. Do not use trial and error.

When $x = 0, y = 2$:
$$2 = a(0)^3 + b(0)^2 + c$$
$$c = 2$$

When $x = -1, y = 4$:
$$4 = a(-1)^3 + b(-1)^2 + 2$$
$$-a + b = 2$$

When $x = 2, y = 22$:
$$22 = a(2)^3 + b(2)^2 + 2$$
$$8a + 4b = 20$$

Solve simultaneously (multiply first equation by 4):
$$8a + 4b = 20$$
$$-(-4a + 4b = 8)$$
$$12a = 12$$
$$a = 1$$

$$-a + b = 2$$
$$-1 + b = 2$$
$$b = 3$$

$$a = 1, \quad b = 3, \quad c = 2$$

14) A reciprocal function is defined as $y = \frac{k}{x+1} + 2$. The graph passes through the point (1, -1). Work out the value of k.

Use algebraic substitution $x = 1, y = -1$:

$$-1 = \frac{k}{1+1} + 2$$

$$-1 = \frac{k}{2} + 2$$

$$-3 = \frac{k}{2}$$

$$\boldsymbol{k = -6}$$