GENMATHS RESOURCES

SURDS

Task 1 – Write each of the following as a fully simplified surd.

1) $\sqrt{8} = \sqrt{4 \times 2} = \mathbf{2\sqrt{2}}$

2) $\sqrt{12} = \sqrt{4 \times 3} = \mathbf{2\sqrt{3}}$

3) $\sqrt{18} = \sqrt{9 \times 2} = \mathbf{3\sqrt{2}}$

4) $\sqrt{28} = \sqrt{4 \times 7} = \mathbf{2\sqrt{7}}$

5) $\sqrt{32} = \sqrt{16 \times 2} = \mathbf{4\sqrt{2}}$

6) $\sqrt{125} = \sqrt{25 \times 5} = \mathbf{5\sqrt{5}}$

7) $\sqrt{27} = \sqrt{9 \times 3} = \mathbf{3\sqrt{3}}$

8) $\sqrt{45} = \sqrt{9 \times 5} = \mathbf{3\sqrt{5}}$

9) $\sqrt{98} = \sqrt{49 \times 2} = \mathbf{7\sqrt{2}}$

10) $\sqrt{243} = \sqrt{81 \times 3} = \mathbf{9\sqrt{3}}$

11) $\sqrt{108} = \sqrt{36 \times 3} = \mathbf{6\sqrt{3}}$

12) $\sqrt{162} = \sqrt{81 \times 2} = \mathbf{9\sqrt{2}}$

13) $\sqrt{96} = \sqrt{16 \times 6} = \mathbf{4\sqrt{6}}$

14) $5\sqrt{28} = 5\sqrt{4 \times 7} = \mathbf{10\sqrt{7}}$

15) $3\sqrt{48} = 3\sqrt{16 \times 3} = \mathbf{12\sqrt{3}}$

16) $3\sqrt{300} = 3\sqrt{100 \times 3} = \mathbf{30\sqrt{3}}$

17) $7\sqrt{45} = 7\sqrt{9 \times 5} = \mathbf{21\sqrt{5}}$

18) $3\sqrt{32} = 3\sqrt{16 \times 2} = \mathbf{12\sqrt{2}}$

19) $9\sqrt{12} = 9\sqrt{4 \times 3} = \mathbf{18\sqrt{3}}$

20) $10\sqrt{50} = 10\sqrt{25 \times 2} = \mathbf{50\sqrt{2}}$

21) $6\sqrt{75} = 6\sqrt{25 \times 3} = \mathbf{30\sqrt{3}}$

Task 2 – Work out each of the following calculations and fully simplify your answer.

22) $\sqrt{3} \times \sqrt{3} = \mathbf{3}$

23) $\sqrt{5} \times \sqrt{5} = \mathbf{5}$

24) $\sqrt{7} \times \sqrt{4} = \sqrt{28} = \mathbf{2\sqrt{7}}$

25) $\sqrt{3} \times \sqrt{7} = \mathbf{\sqrt{21}}$

26) $\sqrt{6} \times \sqrt{12} = \sqrt{72} = \mathbf{6\sqrt{2}}$

27) $\sqrt{8} \times \sqrt{2} = \sqrt{16} = \mathbf{4}$

28) $\sqrt{32} \times \sqrt{3} = \sqrt{96} = \mathbf{4\sqrt{6}}$

29) $2\sqrt{5} \times 3\sqrt{6} = \mathbf{6\sqrt{30}}$

30) $3\sqrt{2} \times 4\sqrt{2} \times \sqrt{2} = \mathbf{24\sqrt{2}}$

31) $(\sqrt{4})^3 = 4\sqrt{4} = \mathbf{8}$

32) $2(\sqrt{2})^5 = \mathbf{8\sqrt{2}}$

33) $\sqrt{81} \div \sqrt{3} = \sqrt{27} = \mathbf{3\sqrt{3}}$

34) $\sqrt{38} \div \sqrt{2} = \mathbf{\sqrt{19}}$

35) $\sqrt{132} \div \sqrt{3} = \sqrt{44} = \mathbf{2\sqrt{11}}$

36) $\frac{\sqrt{30}}{\sqrt{5}} = \mathbf{\sqrt{6}}$

37) $10\sqrt{8} \div \sqrt{2} = 10\sqrt{4} = \mathbf{20}$

38) $\frac{3\sqrt{18}}{6} = \frac{\sqrt{18}}{2} = \mathbf{\frac{3\sqrt{2}}{2}}$

39) $\sqrt{2} \times \frac{\sqrt{72}}{\sqrt{3}} = \mathbf{4\sqrt{3}}$

40) $\sqrt{6} \times \sqrt{7} \div \sqrt{3} = \sqrt{42} \div \sqrt{3} = \mathbf{\sqrt{14}}$

41) $4\sqrt{98} \div 2\sqrt{7} = \mathbf{2\sqrt{14}}$

Task 3 – Work out the following calculations and fully simplify your answer.

42) $\sqrt{12}+\sqrt{27} = 2\sqrt{3}+3\sqrt{3} = \mathbf{5\sqrt{3}}$

43) $\sqrt{8}+\sqrt{18} = 2\sqrt{2}+3\sqrt{2} = \mathbf{5\sqrt{2}}$

44) $\sqrt{99}-\sqrt{11} = 3\sqrt{11}-\sqrt{11} = \mathbf{2\sqrt{11}}$

45) $\sqrt{72}-\sqrt{32} = 6\sqrt{2}-4\sqrt{2} = \mathbf{2\sqrt{2}}$

46) $\sqrt{48}+\sqrt{75} = 4\sqrt{3}+5\sqrt{3} = \mathbf{9\sqrt{3}}$

47) $\sqrt{200}+\sqrt{50} = 10\sqrt{2}+5\sqrt{2} = \mathbf{15\sqrt{2}}$

48) $3\sqrt{24}+7\sqrt{54} = 6\sqrt{6}+21\sqrt{6} = \mathbf{27\sqrt{6}}$

49) $4\sqrt{20}-\sqrt{45} = 8\sqrt{5}-3\sqrt{5} = \mathbf{5\sqrt{5}}$

50) $6\sqrt{20}-3\sqrt{45} = 12\sqrt{5}-9\sqrt{5} = \mathbf{3\sqrt{5}}$

51) $3\sqrt{12}+2\sqrt{75} = 6\sqrt{3}+10\sqrt{3} = \mathbf{16\sqrt{3}}$

52) $7\sqrt{18}-2\sqrt{50} = 21\sqrt{2}-10\sqrt{2} = \mathbf{11\sqrt{2}}$

53) $4\sqrt{63}+2\sqrt{28} = 12\sqrt{7}+4\sqrt{7} = \mathbf{16\sqrt{7}}$

54) $2\sqrt{80}+5\sqrt{45} = 8\sqrt{5}+15\sqrt{5} = \mathbf{23\sqrt{5}}$

55) $5\sqrt{50}-6\sqrt{8} = 25\sqrt{2}-12\sqrt{2} = \mathbf{13\sqrt{2}}$

56) $5\sqrt{12}+3\sqrt{27} = 10\sqrt{3}+9\sqrt{3} = \mathbf{19\sqrt{3}}$

57) $2\sqrt{175}-\sqrt{252} = 10\sqrt{7}-6\sqrt{7} = \mathbf{4\sqrt{7}}$

58) $2\sqrt{40}+3\sqrt{810} = 4\sqrt{10}+27\sqrt{10} = \mathbf{31\sqrt{10}}$

59) $5\sqrt{52}-2\sqrt{208} = 10\sqrt{13}-8\sqrt{13} = \mathbf{2\sqrt{13}}$

60) $4\sqrt{60}+10\sqrt{135} = 8\sqrt{15}+30\sqrt{15} = \mathbf{38\sqrt{15}}$

Task 4 – Expand and fully simplify.

61) $\sqrt{2}(3+\sqrt{5}) = \mathbf{3\sqrt{2}+\sqrt{10}}$

62) $\sqrt{7}(\sqrt{6}+2) = \mathbf{\sqrt{42}+2\sqrt{7}}$

63) $\sqrt{3}(4-\sqrt{12}) = 4\sqrt{3}-\sqrt{36} = \mathbf{4\sqrt{3}-6}$

64) $\sqrt{5}(3\sqrt{2}-\sqrt{6}) = \mathbf{3\sqrt{10}-\sqrt{30}}$

65) $(2+\sqrt{3})(3+\sqrt{3})$
$= 6+2\sqrt{3}+3\sqrt{3}+3$
$= \mathbf{5\sqrt{3}+9}$

66) $(2+\sqrt{2})(4-\sqrt{2})$
$= 8-2\sqrt{2}+4\sqrt{2}-2$
$= \mathbf{2\sqrt{2}+6}$

67) $(4+\sqrt{6})(4-\sqrt{6})$
$= 16-4\sqrt{6}+4\sqrt{6}-6$
$= \mathbf{10}$

68) $(5-\sqrt{6})(4+\sqrt{2})$
$= 20+5\sqrt{2}-4\sqrt{6}-\sqrt{12}$
$= \mathbf{20+5\sqrt{2}-4\sqrt{6}-2\sqrt{3}}$

69) $(1+\sqrt{5})(3-\sqrt{5})$
$= 3-\sqrt{5}+3\sqrt{5}-5$
$= \mathbf{2\sqrt{5}-2}$

70) $(5+\sqrt{7})(3-\sqrt{7})$
$= 15-5\sqrt{7}+3\sqrt{7}-7$
$= \mathbf{8-2\sqrt{7}}$

71) $(4+\sqrt{7})^2$
$= (4+\sqrt{7})(4+\sqrt{7})$
$= 16+4\sqrt{7}+4\sqrt{7}+7$
$= \mathbf{8\sqrt{7}+23}$

72) $(5-2\sqrt{2})(3+3\sqrt{2})$
$= 15+15\sqrt{2}-6\sqrt{2}-12$
$= \mathbf{9\sqrt{2}+3}$

73) $(4\sqrt{2}-\sqrt{6})(3+2\sqrt{6})$
$= 12\sqrt{2}+8\sqrt{12}-3\sqrt{6}-12$
$= \mathbf{12\sqrt{2}+16\sqrt{3}-3\sqrt{6}-12}$

74) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$= 5+\sqrt{10}-\sqrt{10}-2$

$= \mathbf{3}$

75) $(2\sqrt{3}-2\sqrt{7})(\sqrt{3}+2\sqrt{7})$

$= 6+4\sqrt{21}-2\sqrt{21}-28$

$= \mathbf{2\sqrt{21}-22}$

76) $(1+\sqrt{8})(2+\sqrt{2})$

$= 2+\sqrt{2}+2\sqrt{8}+\sqrt{16}$

$= 2+\sqrt{2}+4\sqrt{2}+4$

$= \mathbf{5\sqrt{2}+6}$

77) $(3-\sqrt{8})(5+\sqrt{8})$

$= 15+3\sqrt{8}-5\sqrt{8}-8$

$= 7-2\sqrt{8}$

$= \mathbf{7-4\sqrt{2}}$

78) $(2+\sqrt{32})(4-3\sqrt{32})$

$= 8-6\sqrt{32}+4\sqrt{32}-96$

$= -88-2\sqrt{32}$

$= \mathbf{-88-8\sqrt{2}}$

Task 5 – Rationalise the denominator of each of the following expressions. Fully simplify your answers.

79) $\frac{2}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \mathbf{\sqrt{2}}$

80) $\frac{5}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \mathbf{\sqrt{5}}$

81) $\frac{7}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} = \mathbf{\sqrt{7}}$

82) $\frac{10}{\sqrt{10}}\times\frac{\sqrt{10}}{10} = \mathbf{\sqrt{10}}$

83) $\frac{3}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} = \mathbf{\frac{3\sqrt{7}}{7}}$

84) $\frac{8}{\sqrt{8}}\times\frac{\sqrt{8}}{\sqrt{8}} = \sqrt{8} = \mathbf{2\sqrt{2}}$

85) $\frac{2\sqrt{6}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{12}}{2} = \mathbf{2\sqrt{3}}$

86) $\frac{5+\sqrt{10}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \mathbf{\frac{5\sqrt{3}+\sqrt{30}}{3}}$

87) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \mathbf{\frac{\sqrt{6}+\sqrt{15}}{3}}$

88) $\frac{2}{1+\sqrt{2}} = \frac{2(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{2-2\sqrt{2}}{1-\sqrt{2}+\sqrt{2}-2} = \frac{2-2\sqrt{2}}{-1}$

$= \mathbf{2\sqrt{2}-2}$

89) $\frac{3}{1-\sqrt{7}} = \frac{3}{1-\sqrt{7}}\times\frac{1+\sqrt{7}}{1+\sqrt{7}} = \frac{3+3\sqrt{7}}{1+\sqrt{7}-\sqrt{7}-7} = \frac{3+3\sqrt{7}}{-6}$

$= \mathbf{\frac{-1-\sqrt{7}}{2}}$

90) $\frac{4}{3+\sqrt{7}} = \frac{4}{3+\sqrt{7}}\times\frac{3-\sqrt{7}}{3-\sqrt{7}} = \frac{12-4\sqrt{7}}{9-3\sqrt{7}+3\sqrt{7}-7} = \frac{12-4\sqrt{7}}{2}$

$= \mathbf{6-2\sqrt{7}}$

91) $\frac{2}{5+\sqrt{3}} = \frac{2}{5+\sqrt{3}}\times\frac{5-\sqrt{3}}{5-\sqrt{3}} = \frac{10-2\sqrt{3}}{25-5\sqrt{3}+5\sqrt{3}-3} = \frac{10-2\sqrt{3}}{22}$

$= \mathbf{\frac{5-\sqrt{3}}{11}}$

92) $\frac{5}{4-\sqrt{6}} = \frac{5}{4-\sqrt{6}}\times\frac{4+\sqrt{6}}{4+\sqrt{6}} = \frac{20+5\sqrt{6}}{16+4\sqrt{6}-4\sqrt{6}-6} = \frac{20+5\sqrt{6}}{10}$

$= \mathbf{\frac{4+\sqrt{6}}{2}}$

93) $\frac{4}{2-\sqrt{8}} = \frac{4}{2-\sqrt{8}}\times\frac{2+\sqrt{8}}{2+\sqrt{8}} = \frac{8+4\sqrt{8}}{4+2\sqrt{8}-2\sqrt{8}-8} = \frac{8+8\sqrt{2}}{-4}$

$= \mathbf{-2-2\sqrt{2}}$

94) $\frac{10}{\sqrt{3}+\sqrt{5}} = \frac{10}{\sqrt{3}+\sqrt{5}}\times\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}} = \frac{10\sqrt{3}-10\sqrt{5}}{3-\sqrt{15}+\sqrt{15}-5} = \frac{10\sqrt{3}-10\sqrt{5}}{-2}$

$= \mathbf{5\sqrt{5}-5\sqrt{3}}$

95) $\frac{\sqrt{4}+\sqrt{5}}{2+\sqrt{5}} = \frac{\sqrt{4}+\sqrt{5}}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{2\sqrt{4}-\sqrt{20}+2\sqrt{5}-5}{4-2\sqrt{5}+2\sqrt{5}-5}$

$= \frac{4-2\sqrt{5}+2\sqrt{5}-5}{-1}$

$= \frac{-1}{-1}$

$= \mathbf{1}$

Challenge

96) Express $(3+\sqrt{2})^2-(3-\sqrt{2})^2$ in the form $a+b\sqrt{c}$ where a, b and c are integers to be found.

$$(3+\sqrt{2})(3+\sqrt{2})-(3-\sqrt{2})(3-\sqrt{2})$$

$$=(9+3\sqrt{2}+3\sqrt{2}+2)-(9-3\sqrt{2}-3\sqrt{2}+2)$$

$$=(11+6\sqrt{2})-(11-6\sqrt{2})$$

$$=\mathbf{12\sqrt{2}}$$

97) Fully simplify $\frac{a}{a+\sqrt{a}}$

$$\frac{a}{a+\sqrt{a}}\times\frac{a-\sqrt{a}}{a-\sqrt{a}}$$

$$=\frac{a^2-a\sqrt{a}}{a^2-a\sqrt{a}+a\sqrt{a}-a}$$

$$=\frac{a^2-a\sqrt{a}}{a^2-a}$$

$$=\frac{\boldsymbol{a-\sqrt{a}}}{\boldsymbol{a-1}}$$

98) Fully simplify $(5+\sqrt{8})^3$

$$(5+\sqrt{8})(5+\sqrt{8})(5+\sqrt{8})$$

$$=(25+5\sqrt{8}+5\sqrt{8}+8)(5+\sqrt{8})$$

$$=(33+10\sqrt{8})(5+\sqrt{8})$$

$$=165+33\sqrt{8}+50\sqrt{8}+80$$

$$=245+83\sqrt{8}$$

$$=\mathbf{245+166\sqrt{2}}$$

99) Rationalise the denominator of $\frac{4}{1+\frac{1}{\sqrt{4}}}$

$$\frac{4}{1+\frac{1}{\sqrt{4}}}$$

Multiply all terms by $\sqrt{4}$

$$\frac{4\sqrt{4}}{\sqrt{4}+1}$$

$$=\frac{4\sqrt{4}}{\sqrt{4}+1}\times\frac{\sqrt{4}-1}{\sqrt{4}-1}$$

$$=\frac{16-4\sqrt{4}}{4-\sqrt{4}+\sqrt{4}-1}$$

$$=\frac{16-8}{3}$$

$$=\frac{\mathbf{8}}{\mathbf{3}}$$