

Task 1 – Write each of the following as a fully simplified surd.

- 1) $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$
- 2) $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$
- 3) $\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$
- 4) $\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$
- 5) $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$
- 6) $\sqrt{125} = \sqrt{25 \times 5} = 5\sqrt{5}$
- 7) $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$
- 8) $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$
- 9) $\sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2}$
- 10) $\sqrt{243} = \sqrt{81 \times 3} = 9\sqrt{3}$
- 11) $\sqrt{108} = \sqrt{36 \times 3} = 6\sqrt{3}$
- 12) $\sqrt{162} = \sqrt{81 \times 2} = 9\sqrt{2}$
- 13) $\sqrt{96} = \sqrt{16 \times 6} = 4\sqrt{6}$
- 14) $5\sqrt{28} = 5\sqrt{4 \times 7} = 10\sqrt{7}$
- 15) $3\sqrt{48} = 3\sqrt{16 \times 3} = 12\sqrt{3}$
- 16) $3\sqrt{300} = 3\sqrt{100 \times 3} = 30\sqrt{3}$
- 17) $7\sqrt{45} = 7\sqrt{9 \times 5} = 21\sqrt{5}$
- 18) $3\sqrt{32} = 3\sqrt{16 \times 2} = 12\sqrt{2}$
- 19) $9\sqrt{12} = 9\sqrt{4 \times 3} = 18\sqrt{3}$
- 20) $10\sqrt{50} = 10\sqrt{25 \times 2} = 50\sqrt{2}$
- 21) $6\sqrt{75} = 6\sqrt{25 \times 3} = 30\sqrt{3}$

Task 2 – Work out each of the following calculations and fully simplify your answer.

- 22) $\sqrt{3} \times \sqrt{3} = 3$
- 23) $\sqrt{5} \times \sqrt{5} = 5$
- 24) $\sqrt{7} \times \sqrt{4} = \sqrt{28} = 2\sqrt{7}$
- 25) $\sqrt{3} \times \sqrt{7} = \sqrt{21}$
- 26) $\sqrt{6} \times \sqrt{12} = \sqrt{72} = 6\sqrt{2}$
- 27) $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$
- 28) $\sqrt{32} \times \sqrt{3} = \sqrt{96} = 4\sqrt{6}$
- 29) $2\sqrt{5} \times 3\sqrt{6} = 6\sqrt{30}$
- 30) $3\sqrt{2} \times 4\sqrt{2} \times \sqrt{2} = 24\sqrt{2}$
- 31) $(\sqrt{4})^3 = 4\sqrt{4} = 8$
- 32) $2(\sqrt{2})^5 = 8\sqrt{2}$
- 33) $\sqrt{81} \div \sqrt{3} = \sqrt{27} = 3\sqrt{3}$
- 34) $\sqrt{38} \div \sqrt{2} = \sqrt{19}$
- 35) $\sqrt{132} \div \sqrt{3} = \sqrt{44} = 2\sqrt{11}$
- 36) $\frac{\sqrt{30}}{\sqrt{5}} = \sqrt{6}$
- 37) $10\sqrt{8} \div \sqrt{2} = 10\sqrt{4} = 20$
- 38) $\frac{3\sqrt{18}}{6} = \frac{\sqrt{18}}{2} = \frac{3\sqrt{2}}{2}$
- 39) $\sqrt{2} \times \frac{\sqrt{72}}{\sqrt{3}} = 4\sqrt{3}$
- 40) $\sqrt{6} \times \sqrt{7} \div \sqrt{3} = \sqrt{42} \div \sqrt{3} = \sqrt{14}$
- 41) $4\sqrt{98} \div 2\sqrt{7} = 2\sqrt{14}$

Task 3 – Work out the following calculations and fully simplify your answer.

- 42) $\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$
- 43) $\sqrt{8} + \sqrt{18} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$
- 44) $\sqrt{99} - \sqrt{11} = 3\sqrt{11} - \sqrt{11} = 2\sqrt{11}$
- 45) $\sqrt{72} - \sqrt{32} = 6\sqrt{2} - 4\sqrt{2} = 2\sqrt{2}$
- 46) $\sqrt{48} + \sqrt{75} = 4\sqrt{3} + 5\sqrt{3} = 9\sqrt{3}$
- 47) $\sqrt{200} + \sqrt{50} = 10\sqrt{2} + 5\sqrt{2} = 15\sqrt{2}$
- 48) $3\sqrt{24} + 7\sqrt{54} = 6\sqrt{6} + 21\sqrt{6} = 27\sqrt{6}$
- 49) $4\sqrt{20} - \sqrt{45} = 8\sqrt{5} - 3\sqrt{5} = 5\sqrt{5}$
- 50) $6\sqrt{20} - 3\sqrt{45} = 12\sqrt{5} - 9\sqrt{5} = 3\sqrt{5}$
- 51) $3\sqrt{12} + 2\sqrt{75} = 6\sqrt{3} + 10\sqrt{3} = 16\sqrt{3}$
- 52) $7\sqrt{18} - 2\sqrt{50} = 21\sqrt{2} - 10\sqrt{2} = 11\sqrt{2}$
- 53) $4\sqrt{63} + 2\sqrt{28} = 12\sqrt{7} + 4\sqrt{7} = 16\sqrt{7}$
- 54) $2\sqrt{80} + 5\sqrt{45} = 8\sqrt{5} + 15\sqrt{5} = 23\sqrt{5}$
- 55) $5\sqrt{50} - 6\sqrt{8} = 25\sqrt{2} - 12\sqrt{2} = 13\sqrt{2}$
- 56) $5\sqrt{12} + 3\sqrt{27} = 10\sqrt{3} + 9\sqrt{3} = 19\sqrt{3}$
- 57) $2\sqrt{175} - \sqrt{252} = 10\sqrt{7} - 6\sqrt{7} = 4\sqrt{7}$
- 58) $2\sqrt{40} + 3\sqrt{810} = 4\sqrt{10} + 27\sqrt{10} = 31\sqrt{10}$
- 59) $5\sqrt{52} - 2\sqrt{208} = 10\sqrt{13} - 8\sqrt{13} = 2\sqrt{13}$
- 60) $4\sqrt{60} + 10\sqrt{135} = 8\sqrt{15} + 30\sqrt{15} = 38\sqrt{15}$

Task 4 – Expand and fully simplify.

- 61) $\sqrt{2}(3 + \sqrt{5}) = 3\sqrt{2} + \sqrt{10}$
- 62) $\sqrt{7}(\sqrt{6} + 2) = \sqrt{42} + 2\sqrt{7}$
- 63) $\sqrt{3}(4 - \sqrt{12}) = 4\sqrt{3} - \sqrt{36} = 4\sqrt{3} - 6$
- 64) $\sqrt{5}(3\sqrt{2} - \sqrt{6}) = 3\sqrt{10} - \sqrt{30}$
- 65) $(2 + \sqrt{3})(3 + \sqrt{3})$
 $= 6 + 2\sqrt{3} + 3\sqrt{3} + 3$
 $= 5\sqrt{3} + 9$
- 66) $(2 + \sqrt{2})(4 - \sqrt{2})$
 $= 8 - 2\sqrt{2} + 4\sqrt{2} - 2$
 $= 2\sqrt{2} + 6$
- 67) $(4 + \sqrt{6})(4 - \sqrt{6})$
 $= 16 - 4\sqrt{6} + 4\sqrt{6} - 6$
 $= 10$
- 68) $(5 - \sqrt{6})(4 + \sqrt{2})$
 $= 20 + 5\sqrt{2} - 4\sqrt{6} - \sqrt{12}$
 $= 20 + 5\sqrt{2} - 4\sqrt{6} - 2\sqrt{3}$
- 69) $(1 + \sqrt{5})(3 - \sqrt{5})$
 $= 3 - \sqrt{5} + 3\sqrt{5} - 5$
 $= 2\sqrt{5} - 2$
- 70) $(5 + \sqrt{7})(3 - \sqrt{7})$
 $= 15 - 5\sqrt{7} + 3\sqrt{7} - 7$
 $= 8 - 2\sqrt{7}$
- 71) $(4 + \sqrt{7})^2$
 $= (4 + \sqrt{7})(4 + \sqrt{7})$
 $= 16 + 4\sqrt{7} + 4\sqrt{7} + 7$
 $= 8\sqrt{7} + 23$
- 72) $(5 - 2\sqrt{2})(3 + 3\sqrt{2})$
 $= 15 + 15\sqrt{2} - 6\sqrt{2} - 12$
 $= 9\sqrt{2} + 3$
- 73) $(4\sqrt{2} - \sqrt{6})(3 + 2\sqrt{6})$
 $= 12\sqrt{2} + 8\sqrt{12} - 3\sqrt{6} - 12$
 $= 12\sqrt{2} + 16\sqrt{3} - 3\sqrt{6} - 12$

$$74) (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$$

$$= 5 + \sqrt{10} - \sqrt{10} - 2$$

$$= 3$$

$$75) (2\sqrt{3} - 2\sqrt{7})(\sqrt{3} + 2\sqrt{7})$$

$$= 6 + 4\sqrt{21} - 2\sqrt{21} - 28$$

$$= 2\sqrt{21} - 22$$

$$76) (1 + \sqrt{8})(2 + \sqrt{2})$$

$$= 2 + \sqrt{2} + 2\sqrt{8} + \sqrt{16}$$

$$= 2 + \sqrt{2} + 4\sqrt{2} + 4$$

$$= 5\sqrt{2} + 6$$

$$77) (3 - \sqrt{8})(5 + \sqrt{8})$$

$$= 15 + 3\sqrt{8} - 5\sqrt{8} - 8$$

$$= 7 - 2\sqrt{8}$$

$$= 7 - 4\sqrt{2}$$

$$78) (2 + \sqrt{32})(4 - 3\sqrt{32})$$

$$= 8 - 6\sqrt{32} + 4\sqrt{32} - 96$$

$$= -88 - 2\sqrt{32}$$

$$= -88 - 8\sqrt{2}$$

$$85) \frac{2\sqrt{6}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{12}}{2} = 2\sqrt{3}$$

$$86) \frac{5+\sqrt{10}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}+\sqrt{30}}{3}$$

$$87) \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}+\sqrt{15}}{3}$$

$$88) \frac{2}{1+\sqrt{2}} = \frac{2(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{2-2\sqrt{2}}{1-\sqrt{2}+\sqrt{2}-2} = \frac{2-2\sqrt{2}}{-1} = 2\sqrt{2} - 2$$

$$89) \frac{3}{1-\sqrt{7}} = \frac{3}{1-\sqrt{7}} \times \frac{1+\sqrt{7}}{1+\sqrt{7}} = \frac{3+3\sqrt{7}}{1+\sqrt{7}-\sqrt{7}-7} = \frac{3+3\sqrt{7}}{-6} = \frac{-1-\sqrt{7}}{2}$$

$$90) \frac{4}{3+\sqrt{7}} = \frac{4}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} = \frac{12-4\sqrt{7}}{9-3\sqrt{7}+3\sqrt{7}-7} = \frac{12-4\sqrt{7}}{2} = 6 - 2\sqrt{7}$$

$$91) \frac{2}{5+\sqrt{3}} = \frac{2}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}} = \frac{10-2\sqrt{3}}{25-5\sqrt{3}+5\sqrt{3}-3} = \frac{10-2\sqrt{3}}{22} = \frac{5-\sqrt{3}}{11}$$

$$92) \frac{5}{4-\sqrt{6}} = \frac{5}{4-\sqrt{6}} \times \frac{4+\sqrt{6}}{4+\sqrt{6}} = \frac{20+5\sqrt{6}}{16+4\sqrt{6}-4\sqrt{6}-6} = \frac{20+5\sqrt{6}}{10} = \frac{4+\sqrt{6}}{2}$$

$$93) \frac{4}{2-\sqrt{8}} = \frac{4}{2-\sqrt{8}} \times \frac{2+\sqrt{8}}{2+\sqrt{8}} = \frac{8+4\sqrt{8}}{4+2\sqrt{8}-2\sqrt{8}-8} = \frac{8+8\sqrt{2}}{-4} = -2 - 2\sqrt{2}$$

$$94) \frac{10}{\sqrt{3}+\sqrt{5}} = \frac{10}{\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}} = \frac{10\sqrt{3}-10\sqrt{5}}{3-\sqrt{15}+\sqrt{15}-5} = \frac{10\sqrt{3}-10\sqrt{5}}{-2} = 5\sqrt{5} - 5\sqrt{3}$$

$$95) \frac{\sqrt{4}+\sqrt{5}}{2+\sqrt{5}} = \frac{\sqrt{4}+\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{2\sqrt{4}-\sqrt{20}+2\sqrt{5}-5}{4-2\sqrt{5}+2\sqrt{5}-5} = \frac{4-2\sqrt{5}+2\sqrt{5}-5}{-1} = \frac{-1}{-1} = 1$$

Task 5 – Rationalise the denominator of each of the following expressions. Fully simplify your answers.

$$79) \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$80) \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$$

$$81) \frac{7}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \sqrt{7}$$

$$82) \frac{10}{\sqrt{10}} \times \frac{\sqrt{10}}{10} = \sqrt{10}$$

$$83) \frac{3}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

$$84) \frac{8}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \sqrt{8} = 2\sqrt{2}$$

Challenge

96) Express $(3 + \sqrt{2})^2 - (3 - \sqrt{2})^2$ in the form $a + b\sqrt{c}$ where a, b and c are integers to be found.

$$\begin{aligned} & (3 + \sqrt{2})(3 + \sqrt{2}) - (3 - \sqrt{2})(3 - \sqrt{2}) \\ &= (9 + 3\sqrt{2} + 3\sqrt{2} + 2) - (9 - 3\sqrt{2} - 3\sqrt{2} + 2) \\ &= (11 + 6\sqrt{2}) - (11 - 6\sqrt{2}) \\ &= \mathbf{12\sqrt{2}} \end{aligned}$$

97) Fully simplify $\frac{a}{a+\sqrt{a}}$

$$\begin{aligned} & \frac{a}{a + \sqrt{a}} \times \frac{a - \sqrt{a}}{a - \sqrt{a}} \\ &= \frac{a^2 - a\sqrt{a}}{a^2 - a\sqrt{a} + a\sqrt{a} - a} \\ &= \frac{a^2 - a\sqrt{a}}{a^2 - a} \\ &= \frac{a - \sqrt{a}}{a - 1} \end{aligned}$$

98) Fully simplify $(5 + \sqrt{8})^3$

$$\begin{aligned} & (5 + \sqrt{8})(5 + \sqrt{8})(5 + \sqrt{8}) \\ &= (25 + 5\sqrt{8} + 5\sqrt{8} + 8)(5 + \sqrt{8}) \\ &= (33 + 10\sqrt{8})(5 + \sqrt{8}) \\ &= 165 + 33\sqrt{8} + 50\sqrt{8} + 80 \\ &= 245 + 83\sqrt{8} \\ &= \mathbf{245 + 166\sqrt{2}} \end{aligned}$$

99) Rationalise the denominator of $\frac{4}{1 + \frac{1}{\sqrt{4}}}$

$$\frac{4}{1 + \frac{1}{\sqrt{4}}}$$

Multiply all terms by $\sqrt{4}$

$$\frac{4\sqrt{4}}{\sqrt{4} + 1}$$

$$\begin{aligned} &= \frac{4\sqrt{4}}{\sqrt{4} + 1} \times \frac{\sqrt{4} - 1}{\sqrt{4} - 1} \\ &= \frac{16 - 4\sqrt{4}}{4 - \sqrt{4} + \sqrt{4} - 1} \\ &= \frac{16 - 8}{3} \\ &= \mathbf{\frac{8}{3}} \end{aligned}$$