NEGATIVE AND FRACTIONAL INDICES

Task 1 – Work out the value of each of the following.

1) $16^{\frac{1}{2}} = \mathbf{4}$

2) $4^{\frac{1}{2}} = \mathbf{2}$

3) $8^{\frac{1}{3}} = \mathbf{2}$

4) $64^{\frac{1}{3}} = \mathbf{4}$

5) $27^{\frac{1}{3}} = \mathbf{3}$

6) $125^{\frac{1}{3}} = \mathbf{5}$

7) $16^{\frac{1}{4}} = \mathbf{2}$

8) $27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2 = 3^2 = \mathbf{9}$

9) $8^{\frac{5}{3}} = \left(8^{\frac{1}{3}}\right)^5 = 2^5 = \mathbf{32}$

10) $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = \mathbf{27}$

11) $216^{\frac{2}{3}} = \left(216^{\frac{1}{3}}\right)^2 = 6^2 = \mathbf{36}$

12) $36^{\frac{3}{2}} = \left(36^{\frac{1}{2}}\right)^3 = 6^3 = \mathbf{216}$

13) $1000^{\frac{2}{3}} = \left(1000^{\frac{1}{3}}\right)^2 = 10^2 = \mathbf{100}$

14) $\left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\frac{8}{27}^{\frac{1}{3}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{\mathbf{4}}{\mathbf{9}}$

15) $\left(\frac{25}{36}\right)^{\frac{3}{2}} = \left(\frac{25}{36}^{\frac{1}{2}}\right)^3 = \left(\frac{5}{6}\right)^3 = \frac{\mathbf{125}}{\mathbf{216}}$

16) $\left(\frac{49}{81}\right)^{\frac{3}{2}} = \left(\frac{49}{81}^{\frac{1}{2}}\right)^3 = \left(\frac{7}{9}\right)^3 = \frac{\mathbf{343}}{\mathbf{729}}$

17) $\left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\frac{81}{16}^{\frac{1}{4}}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{\mathbf{27}}{\mathbf{8}}$

Task 2 – Work out the value of each of the following.

18) $4^{-1} = \frac{\mathbf{1}}{\mathbf{4}}$

19) $6^{-1} = \frac{\mathbf{1}}{\mathbf{6}}$

20) $3^{-2} = \frac{\mathbf{1}}{\mathbf{9}}$

21) $2^{-4} = \frac{\mathbf{1}}{\mathbf{16}}$

22) $3^{-3} = \frac{\mathbf{1}}{\mathbf{27}}$

23) $5^0 = \mathbf{1}$

24) $\left(\frac{1}{4}\right)^{-1} = \mathbf{4}$

25) $8^{-2} = \frac{\mathbf{1}}{\mathbf{64}}$

26) $\left(\frac{1}{6}\right)^{-2} = \mathbf{36}$

27) $\left(\frac{1}{3}\right)^{-1} = \mathbf{3}$

28) $\left(\frac{1}{4}\right)^{-2} = \mathbf{16}$

29) $a^{-4} = \frac{\mathbf{1}}{\boldsymbol{a^4}}$

30) $\left(\frac{2}{5}\right)^{-1} = \frac{\mathbf{5}}{\mathbf{2}}$

31) $\left(\frac{3}{4}\right)^{-3} = \frac{\mathbf{64}}{\mathbf{27}}$

32) $\left(\frac{4}{3}\right)^{-2} = \frac{\mathbf{9}}{\mathbf{16}}$

33) $\left(\frac{1}{5}\right)^{-3} = \mathbf{125}$

34) $\left(\frac{7}{11}\right)^{-2} = \frac{\mathbf{121}}{\mathbf{49}}$

Task 3 – Work out the value of each of the following.

35) $16^{-\frac{1}{2}} = \left(\frac{1}{16}\right)^{\frac{1}{2}} = \mathbf{\frac{1}{4}}$

36) $81^{-\frac{1}{4}} = \left(\frac{1}{81}\right)^{\frac{1}{4}} = \mathbf{\frac{1}{3}}$

37) $4^{-\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} = \mathbf{\frac{1}{2}}$

38) $64^{-\frac{3}{2}} = \left(\frac{1}{64}^{\frac{1}{2}}\right)^{3} = \left(\frac{1}{8}\right)^{3} = \mathbf{\frac{1}{512}}$

39) $25^{-\frac{3}{2}} = \left(\frac{1}{25}^{\frac{1}{2}}\right)^{3} = \left(\frac{1}{5}\right)^{3} = \mathbf{\frac{1}{125}}$

40) $4^{-\frac{5}{2}} = \left(\frac{1}{4}^{\frac{1}{2}}\right)^{5} = \left(\frac{1}{2}\right)^{5} = \mathbf{\frac{1}{32}}$

41) $9^{-\frac{3}{2}} = \left(\frac{1}{9}^{\frac{1}{2}}\right)^{3} = \left(\frac{1}{3}\right)^{3} = \mathbf{\frac{1}{27}}$

42) $\left(\frac{1}{16}\right)^{-\frac{1}{2}} = 16^{\frac{1}{2}} = \mathbf{4}$

43) $\left(\frac{1}{125}\right)^{-\frac{1}{3}} = 125^{\frac{1}{3}} = \mathbf{5}$

44) $\left(\frac{27}{64}\right)^{-\frac{1}{3}} = \left(\frac{64}{27}\right)^{\frac{1}{3}} = \mathbf{\frac{4}{3}}$

45) $\left(\frac{81}{16}\right)^{-\frac{1}{4}} = \left(\frac{16}{81}\right)^{\frac{1}{4}} = \mathbf{\frac{2}{3}}$

46) $\left(\frac{8}{27}\right)^{-\frac{2}{3}} = \left(\frac{27}{8}\right)^{\frac{2}{3}} = \left(\frac{27}{8}^{\frac{1}{3}}\right)^{2} = \mathbf{\frac{9}{4}}$

47) $\left(\frac{16}{25}\right)^{-\frac{3}{2}} = \left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{25}{16}^{\frac{1}{2}}\right)^{3} = \mathbf{\frac{125}{64}}$

48) $\left(\frac{27}{64}\right)^{-\frac{4}{3}} = \left(\frac{64}{27}\right)^{\frac{4}{3}} = \left(\frac{64}{27}^{\frac{1}{3}}\right)^{4} = \mathbf{\frac{256}{81}}$

49) $\left(\frac{16}{49}\right)^{-\frac{3}{2}} = \left(\frac{49}{16}\right)^{\frac{3}{2}} = \left(\frac{49}{16}^{\frac{1}{2}}\right)^{3} = \mathbf{\frac{343}{64}}$

50) $\left(\frac{25}{9}\right)^{-\frac{3}{2}} = \left(\frac{9}{25}\right)^{\frac{3}{2}} = \left(\frac{9}{25}^{\frac{1}{2}}\right)^{3} = \mathbf{\frac{27}{125}}$

Challenge

51) Work out the value of

$$8^{\frac{2}{3}} + 25^{\frac{1}{2}}$$
$$= 4 + 5$$
$$= \mathbf{9}$$

52) Work out the value of

$$25^{-\frac{1}{2}} \times 40$$
$$= \frac{1}{5} \times 40$$
$$= \mathbf{8}$$

53) Work out the value of

$$4^{\frac{3}{2}} \div 8^{2}$$
$$= 8 \div 64$$
$$= \mathbf{\frac{1}{8}}$$

54) Fully simplify

$$(16x^2)^{\frac{1}{2}}$$
$$= \mathbf{4x}$$

55) Fully simplify

$$(25y^8)^{\frac{1}{2}}$$
$$= \mathbf{5y^4}$$

56) Fully simplify

$$(64m^3)^{\frac{2}{3}}$$
$$= (4m)^2$$
$$= \mathbf{16m^2}$$

57) Fully simplify

$$(243u^{10})^{-\frac{3}{5}}$$
$$= \left(\frac{1}{243u^{10}}^{\frac{1}{5}}\right)^{3}$$
$$= \left(\frac{1}{3u^2}\right)^{3}$$
$$= \mathbf{\frac{1}{27u^6}}$$

58) Work out the value of $\sqrt[4]{3 \times 27 \times 10^{40}}$.
Give your answer in standard form.

$$\sqrt[4]{81 \times 10^{40}}$$

$$= \mathbf{3 \times 10^{10}}$$

59) Work out the value of $\sqrt[3]{3^2 \times 9 \times 9 \times 10^{21}}$.

$$\sqrt[3]{729 \times 10^{21}}$$

$$= 9 \times 10^7$$

$$= \mathbf{90,000,000}$$

60) Given that $5 \times \sqrt{5} = 5^x$, work out the value of x.

$$5^1 \times 5^{\frac{1}{2}} = 5^{\frac{3}{2}}$$

$$\boldsymbol{x = \frac{3}{2}}$$

61) Given that $2^{-n} = 0.4$, work out the value of $(2^n)^6$.

$$2^n = \frac{1}{0.4} = \frac{5}{2}$$

$$(2^n)^6 = \left(\frac{5}{2}\right)^6 = \mathbf{\frac{15625}{64}}$$

62) Given that $\left(\frac{64}{27}\right)^m = \frac{9}{16}$, work out the value of m.

$$\boldsymbol{m = -\frac{2}{3}}$$