

BOUND CALCULATIONS

- 1) A rectangle has a length of 7 m and a width of 4 m, each rounded to the nearest metre.

a. What is the minimum possible perimeter of the rectangle?

Minimum perimeter

$$= 2 \times (6.5 + 3.5)$$

$$= 2 \times 10$$

$$= \mathbf{20\text{ m}}$$

b. What is the maximum possible area of the rectangle?

Maximum area

$$= 7.5 \times 4.5$$

$$= \mathbf{33.75\text{ m}^2}$$

- 2) Jupiter's diameter is 142,000 km to the nearest thousand. Saturn's diameter is 120,500 km to the nearest 500 km. Work out the minimum difference in their diameters.

Minimum difference

$$= 141,500 - 120,750$$

$$= \mathbf{20,750\text{ km}}$$

- 3) A cyclist travelled 48 km to the nearest kilometre at a speed of 16 km/h, correct to the nearest km/h. Work out the maximum time travelled. Give your answer in hours and minutes, to the nearest minute.

Maximum time

$$= \text{maximum distance} \div \text{minimum speed}$$

$$= 48.5 \div 15.5$$

$$= 3.12903\dots\text{ h}$$

$$= \mathbf{3\text{ h } 8\text{ min}}$$

- 4) The radius of a circle is 6 cm to the nearest cm.

a. Work out the maximum circumference of the circle. Give your answer in terms of π .

Maximum circumference

$$= 2 \times \pi \times 6.5$$

$$= \mathbf{13\pi\text{ cm}}$$

- b. Work out the minimum area. Give your answer to 2 decimal places.

Minimum area

$$= \pi \times 5.5^2$$

$$= 30.25\pi\text{ cm}^2$$

$$= \mathbf{95.03\text{ cm}^2\text{ (2 dp)}}$$

- 5) The height of plants measure: 7.2 cm, 5.8 cm, 6.5 cm, 6.9 cm, 7.0 cm, each to the nearest tenth.

a. Work out the greatest possible mean.

Max heights: 7.25, 5.85, 6.55, 6.95, 7.05

Mean

$$= \frac{7.25 + 5.85 + 6.55 + 6.95 + 7.05}{5}$$

$$= 33.65 \div 5 = \mathbf{6.73\text{ cm}}$$

b. Work out the smallest possible median.

Min heights: 5.75, 6.45, 6.85, 6.95, 7.15

6.85

- 6) A frame has a length of 3.2 m and a width of 1.5 m, both to 1 decimal place. Work out the maximum possible area of the frame.

Maximum area

$$= 3.25 \times 1.55$$

$$= \mathbf{5.0375\text{ m}^2}$$

- 7) The length of a square field is measured as 40 m to the nearest metre. Find the minimum possible area of the field.

Minimum area

$$= 39.5^2$$

$$= \mathbf{1560.25\text{ m}^2}$$

- 8) A circular pond has a radius of 8 m, correct to the nearest metre. Work out the maximum circumference. Give your answer in terms of π .

Maximum circumference

$$= 2 \times \pi \times 8.5$$

$$= \mathbf{17\pi\text{ m}}$$

- 9) A parcel has a mass of 2.4 kg, correct to the nearest 0.1 kg. Work out the maximum mass of the parcel.

$$\text{Maximum mass} = 2.45 \text{ kg}$$

- 10) A string is measured as 12.5 m, correct to the nearest 0.1 m. Find the minimum possible length.

$$\text{Minimum length} = 12.45 \text{ m}$$

- 11) A cube has side length 6 cm, correct to the nearest cm. Work out the maximum possible volume.

$$\begin{aligned}\text{Maximum volume} &= 6.5^3 \\ &= 274.625 \text{ cm}^3\end{aligned}$$

- 12) $a = b - c$

$b = 3.54$ correct to 2 decimal places

$c = 2.168$ correct to 3 decimal places

Work out the upper bound of a .

$$\begin{aligned}a_{\text{upper}} &= b_{\text{upper}} - c_{\text{lower}} \\ &= 3.545 - 2.1675 \\ &= 1.3775\end{aligned}$$

- 13) $x = \frac{y}{z}$

$y = 10.2$ correct to 3 significant figures

$z = 0.5$ correct to 1 decimal place

Work out the lower bound of x .

$$\begin{aligned}x_{\text{lower}} &= \frac{y_{\text{lower}}}{z_{\text{upper}}} \\ &= \frac{10.15}{0.55} \\ &= 18.454545 \dots \\ &= 18.5 \text{ (3 sf)}\end{aligned}$$

- 14) $m = np$

$n = 5.46$ correct to 2 decimal places

$p = 10$ correct to 2 significant figures

Work out the lower bound of m .

$$\begin{aligned}m_{\text{lower}} &= n_{\text{lower}} \times p_{\text{lower}} \\ &= 5.455 \times 9.5 \\ &= 51.8225\end{aligned}$$

- 15) A rectangular swimming pool is measured 10 m \times 4 m, both to the nearest metre. Work out the maximum possible perimeter.

$$\begin{aligned}\text{Maximum perimeter} &= 2 \times (10.5 + 4.5) \\ &= 30 \text{ m}\end{aligned}$$

- 16) A cylinder has radius 4.0 cm to 1 decimal place and a height of 12 cm to the nearest centimetre. Given that the cylinder weighs 200 g to the nearest gram, work out the minimum possible density to 2 decimal places.

$$D = \frac{m}{v}$$

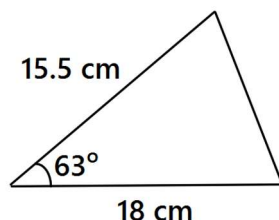
$$\begin{aligned}\text{Maximum volume} &= \pi \times 4.05^2 \times 12.5 \\ &= 205\frac{1}{32}\pi\end{aligned}$$

$$\begin{aligned}\text{Minimum density} &= \frac{199.5}{205\frac{1}{32}\pi} \\ &= 0.3097 \dots \\ &= 0.31 \text{ g/cm}^3 \text{ (2 dp)}\end{aligned}$$

- 17) A car travels 150 km, correct to the nearest km, in 2 hours, correct to the nearest tenth. Work out the minimum possible average speed. Give your answer to 1 decimal place.

$$\begin{aligned}\text{Minimum speed} &= \text{minimum distance} \div \text{maximum time} \\ &= 149.5 \div 2.05 \\ &= 72.9268 \dots \\ &= 72.8 \text{ km/h (1 dp)}\end{aligned}$$

- 18) A triangle is pictured below. The length 15.5 cm has been rounded to 1 decimal place. The length 18 cm has been rounded to 2 significant figures. The angle 63° has been rounded to the nearest degree. Work out the lower bound of the area of the triangle. Give your answer to 1 decimal place.



$$\begin{aligned}
 \text{Area} &= \frac{1}{2}ab \sin C \\
 &= \frac{1}{2} \times 15.45 \times 17.5 \times \sin (62.5) \\
 &= 119.9127 \dots \\
 &= \mathbf{119.9 \text{ cm}^2 \text{ (1 dp)}}
 \end{aligned}$$

- 19) $y = \frac{3x}{k}$
 $x = 12.4$ correct to 3 significant figures
 $k = 7.01$ correct to 3 significant figures
 By considering bounds, work out the value of y to a suitable degree of accuracy. Give a reason for your answer.

$$\begin{aligned}
 y_{\text{lower}} &= \frac{3x_{\text{lower}}}{k_{\text{upper}}} \\
 &= \frac{3 \times 12.35}{7.015} \\
 &= 5.28153 \dots
 \end{aligned}$$

$$\begin{aligned}
 y_{\text{upper}} &= \frac{3x_{\text{upper}}}{k_{\text{lower}}} \\
 &= \frac{3 \times 12.45}{7.005} \\
 &= 5.33190 \dots
 \end{aligned}$$

$y = 5.3 \text{ (1 dp)}$
Both round to the same value to 1 decimal place (or to 2 significant figures).

- 20) The area of a circle is given by the formula $A = \pi \times r^2$. The radius of the circle is measured as 4.00 cm, correct to 2 decimal places. By considering bounds, work out the value of the area of the circle to a suitable degree of accuracy. Give a reason for your answer.

$$\begin{aligned}
 A_{\text{lower}} &= \pi \times r_{\text{lower}}^2 \\
 &= \pi \times 3.995^2 \\
 &= 50.1398 \dots
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{upper}} &= \pi \times r_{\text{upper}}^2 \\
 &= \pi \times 4.005^2 \\
 &= 50.3912 \dots
 \end{aligned}$$

$A = 50 \text{ (2 sf)}$
Both round to the same value to 2 significant figures (or to the nearest unit).

- 21) A runner completes a distance of 1500 m in a recorded time of 4.8 minutes, correct to 1 decimal place. By considering bounds, work out the runner's speed in m/s to a suitable degree of accuracy. Give a reason for your answer.

$$\begin{aligned}
 s_{\text{lower}} &= d \div t_{\text{upper}} \\
 &= 1500 \div (4.85 \times 60) \\
 &= 5.15463 \text{ m/s} \dots
 \end{aligned}$$

$$\begin{aligned}
 s_{\text{upper}} &= d \div t_{\text{lower}} \\
 &= 1500 \div (4.75 \times 60) \\
 &= 5.26315 \text{ m/s} \dots
 \end{aligned}$$

$s = 5 \text{ m/s (1 sf)}$
Both round to the same speed to 1 significant figure (or to the nearest unit).

22) The kinetic energy of a meteor is given by

$$KE = \frac{1}{2}mv^2$$

The mass of the meteor, m , is 3.62 kg, correct to 3 significant figures. The velocity of the meteor, v , is 1000 m/s, correct to 4 significant figures. By considering bounds, work out the value of KE to a suitable degree of accuracy. Give a reason for your answer.

$$\begin{aligned} KE_{lower} &= \frac{1}{2} \times m_{lower} \times v_{lower}^2 \\ &= \frac{1}{2} \times 3.615 \times 999.5^2 \\ &= 1805692.952 \dots \end{aligned}$$

$$\begin{aligned} KE_{upper} &= \frac{1}{2} \times m_{upper} \times v_{upper}^2 \\ &= \frac{1}{2} \times 3.625 \times 1000.5^2 \\ &= 1814312.953 \dots \end{aligned}$$

$$KE = 1,810,000 \text{ (3 sf)}$$

Both round to the same value to 3 significant figures (or to the nearest 10,000)

Challenge

23) In triangle DEF

$$d^2 = e^2 + f^2 - (2ef \cos D)$$

where d is the side opposite angle D .

$e = 7.6 \text{ cm}$, correct to 1 decimal place

$f = 5.42 \text{ cm}$, correct to 3 significant figures

$D = 38^\circ\text{C}$, correct to the nearest degree

By considering bounds, work out the value of d to a suitable degree of accuracy. Give a reason for your answer.

Note that:

$$\cos(38.5) = 0.782608 \dots$$

$$\cos(37.5) = 0.79335 \dots$$

$$\begin{aligned} d_{lower} &= \sqrt{7.55^2 + 5.415^2 - (2 \times 7.55 \times 5.415 \times \cos(38.5))} \\ &= 4.72584 \dots \end{aligned}$$

$$\begin{aligned} d_{upper} &= \sqrt{7.65^2 + 5.425^2 - (2 \times 7.65 \times 5.425 \times \cos(37.5))} \\ &= 4.70136 \dots \end{aligned}$$

$$d = 4.7 \text{ (1 dp)}$$

Both round to the same value to 1 decimal place (or 2 significant figures)