GENMATHS RESOURCES

MATHEMATICAL REASONING

Task 1 – State whether each of the following are always, sometimes or never true.

1) A multiple of 10 is even.
   Always (multiples of 10 end in 0)

2) The sum of two odd numbers is odd.
   Never (odd + odd = even)

3) The product of three even numbers is even. Always

4) The factors of a prime number are all prime.
   Never. Prime numbers all have a factor of 1, which is not prime.

5) The product of two odd numbers is odd. Always (odd × odd = odd)

6) A square number is also a cube number. Sometimes (e.g. $64 = 8^2 = 4^3$)

7) The difference between two square numbers is even. Sometimes
   (e.g. 9 – 1 = 8 even, 16 – 9 = 7 odd)

8) The sum of three consecutive numbers is a multiple of 3.
   $n + (n + 1) + (n + 2) = 3n + 3$
   Always

9) The quotient of two different even numbers is even.
   (8 ÷ 2 = 4 even, 6 ÷ 2 = 3 odd)
   Sometimes

10) The product of an even number and an odd number is odd.
    Never (even × any integer = even)

Task 2

11) A is an even number and B is an odd number.
    Explain why 3(A + B) is always odd.
    even + odd = odd
    3 is odd
    odd × odd = odd

12) A is an odd number. Give three examples to support, A^2 is always odd.
    Sample answer:
    1 × 1 = 1
    3 × 3 = 9
    5 × 5 = 25

13) N is a multiple of 6. Explain why N must also be a multiple of 2 and 3.
    6 is divisible by both 2 and 3

14) P is a square number and Q is a cube number. Maya says that P + Q is always odd. Give a counterexample example to prove her wrong.
    1 is both square and cube
    1 + 1 = 2 (even)

Task 3

15) A rectangle has integer side lengths. Its area is odd. What can you say about its side lengths?
    The side lengths must both be odd

16) The sum of two prime numbers is always even. True or false? Give three examples.
    False
    2 + 3 = 5
    3 + 5 = 8
    5 + 7 = 12

17) The product of two consecutive even numbers is divisible by 8. Always, sometimes, or never true? Explain.

2 × 4 = 8

4 × 6 = 24

6 × 8 = 48

Always; one number is divisible by 2 and one number is divisible by 4, hence always divisible by 8.

18) The difference between two consecutive integers is always, sometimes or never odd?

(n + 1) – n = 1

Always

19) A fraction in the form $\frac{odd}{even}$ can be simplified to an integer. Always, sometimes, or never?

Never. Odd numbers divided by even numbers will always have a remainder.

20) A sequence is formed by multiplying the prior term by 2. If the first term is odd, explain why every term after will be even.

odd × 2 = even

even × 2 = even

Hence, every remaining term will be even.