

## LARGE POWERS OF 10

**Task 1 – For each of the following, give your answers in standard form.**

$$\begin{aligned}
 1) \quad & (7.2 \times 10^{68}) \times (4.5 \times 10^{93}) \\
 &= (7.2 \times 4.5) \times (10^{68} \times 10^{93}) \\
 &= 32.4 \times 10^{68+9} \\
 &= 32.4 \times 10^{161} \\
 &= \mathbf{3.24 \times 10^{162}}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & (3.9 \times 10^{144}) \times (2.4 \times 10^{-37}) \\
 &= (3.9 \times 2.4) \times (10^{144} \times 10^{-37}) \\
 &= 9.36 \times 10^{144+(-37)} \\
 &= \mathbf{9.36 \times 10^{107}}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & (1.6 \times 10^{182}) \div (4 \times 10^{29}) \\
 &= (1.6 \div 4) \times (10^{182} \div 10^{29}) \\
 &= 0.4 \times 10^{182-29} \\
 &= 0.4 \times 10^{153} \\
 &= \mathbf{4 \times 10^{152}}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & (9.5 \times 10^{211}) \div (3.8 \times 10^{-19}) \\
 &= (9.5 \div 3.8) \times (10^{211} \div 10^{-19}) \\
 &= 2.5 \times 10^{211-(-19)} \\
 &= \mathbf{2.5 \times 10^{230}}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & (5.2 \times 10^{125}) \times (6.8 \times 10^{77}) \\
 &= (5.2 \times 6.8) \times (10^{125} \times 10^{77}) \\
 &= 35.36 \times 10^{125+} \\
 &= 35.36 \times 10^{202} \\
 &= \mathbf{3.536 \times 10^{203}}
 \end{aligned}$$

$$\begin{aligned}
 6) \quad & (2.4 \times 10^{101}) \times (7 \times 10^{56}) \\
 &= (2.4 \times 7) \times (10^{101} \times 10^{56}) \\
 &= 16.8 \times 10^{101+56} \\
 &= 16.8 \times 10^{157} \\
 &= \mathbf{1.68 \times 10^{158}}
 \end{aligned}$$

$$\begin{aligned}
 7) \quad & (4.3 \times 10^{88}) \div (1.1 \times 10^{12}) \\
 &= (4.3 \div 1.1) \times (10^{88} \div 10^{12}) \\
 &= 3.909 \dots \times (10^{88-12}) \\
 &= \mathbf{3.91 \times 10^{76} \text{ (3 sf)}}
 \end{aligned}$$

$$\begin{aligned}
 8) \quad & \frac{(2.7 \times 10^{64})^3}{(9 \times 10^{58})^2} \\
 &= \frac{19.683 \times 10^{192}}{81 \times 10^{116}} \\
 &= (19.683 \div 81) \times (10^{192} \div 10^{116}) \\
 &= 0.243 \times 10^{192-116} \\
 &= 0.243 \times 10^{76} \\
 &= \mathbf{2.43 \times 10^{75}}
 \end{aligned}$$

- 9) An observatory records photon readings from a telescope. The size of each uncompressed reading is  $4.6 \times 10^{173}$  units of data. Each reading is compressed by a factor of  $2.3 \times 10^{-7}$ . A storage block can hold  $5 \times 10^{168}$  units of data. Work out how many compressed readings fit into one block.

**Size of compressed readings:**

$$\begin{aligned}
 & (4.6 \times 10^{173}) \times (2.3 \times 10^{-7}) \\
 &= (4.6 \times 2.3) \times (10^{173} \times 10^{-7}) \\
 &= 10.58 \times 10^{173+(-7)} \\
 &= 10.58 \times 10^{166} \\
 &= \mathbf{1.058 \times 10^{167}}
 \end{aligned}$$

**Block capacity:**

$$\begin{aligned}
& (5 \times 10^{168}) \div (1.058 \times 10^{167}) \\
&= (5 \div 1.058) \times (10^{168} \div 10^{167}) \\
&= 4.7258 \dots \times 10^{168-1} \\
&= 4.7258 \dots \times 10^1 \\
&= \mathbf{4.73 \times 10^1 \text{ or } 47.3 (3 sf)}
\end{aligned}$$

- 10) A data centre processes  $3.2 \times 10^{152}$  bytes per second for 45 minutes. Work out the total number of bytes of data that was processed.

**Total time:**

$$\begin{aligned}
45 \times 60 &= 2700 \\
2.7 \times 10^3 &\text{ seconds}
\end{aligned}$$

**Bytes produced:**

$$\begin{aligned}
& (3.2 \times 10^{152}) \times (2.7 \times 10^3) \\
&= (3.2 \times 2.7) \times (10^{152} \times 10^3) \\
&= 8.64 \times 10^{152+3} \\
&= \mathbf{8.64 \times 10^{155}}
\end{aligned}$$

Task 2 – For each of the following, give your answers in standard form.

$$\begin{aligned}
11) \quad & (8 \times 10^{137}) + (3.4 \times 10^{137}) \\
&= 11.4 \times 10^{137} \\
&= \mathbf{1.14 \times 10^{138}}
\end{aligned}$$

$$\begin{aligned}
12) \quad & (6.1 \times 10^{159}) - (2.9 \times 10^{159}) \\
&= \mathbf{3.2 \times 10^{159}}
\end{aligned}$$

$$\begin{aligned}
13) \quad & (9.3 \times 10^{120}) + (1.7 \times 10^{121}) \\
&= 10^{120} \times (9.3 + (1.7 \times 10^1)) \\
&= 10^{120} \times (9.3 + 17) \\
&= 26.3 \times 10^{120} \\
&= \mathbf{2.63 \times 10^{121}}
\end{aligned}$$

$$\begin{aligned}
14) \quad & (4.8 \times 10^{-1}) + (7 \times 10^{-1}) \\
&= 10^{-189} \times ((4.8 \times 10^1) + 7) \\
&= 10^{-189} \times (48 + 7) \\
&= 55 \times 10^{-189} \\
&= \mathbf{5.5 \times 10^{-188}}
\end{aligned}$$

$$\begin{aligned}
15) \quad & (5.6 \times 10^{134}) - (4.2 \times 10^{133}) \\
&= 10^{133} \times ((5.6 \times 10^1) - 4.2) \\
&= 10^{133} \times (56 - 4.2) \\
&= 51.8 \times 10^{133} \\
&= \mathbf{5.18 \times 10^{134}}
\end{aligned}$$

$$\begin{aligned}
16) \quad & (3.1 \times 10^{76}) + (2.49 \times 10^{78}) \\
&= 10^{76} \times (3.1 + (2.49 \times 10^2)) \\
&= 10^{76} \times (3.1 + 249) \\
&= 252.1 \times 10^{76} \\
&= \mathbf{2.521 \times 10^{78}}
\end{aligned}$$

$$\begin{aligned}
17) \quad & (7.5 \times 10^{-162}) - (2.3 \times 10^{-163}) \\
&= 10^{-163} \times ((7.5 \times 10^1) - 2.3) \\
&= 10^{-16} \times (75 - 2.3) \\
&= 72.7 \times 10^{-163} \\
&= \mathbf{7.27 \times 10^{-162}}
\end{aligned}$$

$$\begin{aligned}
18) \quad & (2.2 \times 10^{92})^2 + (4.4 \times 10^{184}) \\
&= (2.2 \times 10^{92}) \times (2.2 \times 10^{92}) \\
&= 4.84 \times 10^{184}
\end{aligned}$$

$$\begin{aligned}
& (4.84 \times 10^{184}) + (4.4 \times 10^{184}) \\
&= \mathbf{9.24 \times 10^{184}}
\end{aligned}$$

- 19) A rare-event simulation logs  $6.02 \times 10^{146}$  events in Phase 1 and  $8.1 \times 10^{145}$  events in Phase 2. During validation,  $3.5 \times 10^{145}$  events are discarded. What is the final count in standard form?

**Phase 1 and phase 2:**

$$\begin{aligned} & (6.02 \times 10^{146}) + (8.1 \times 10^{145}) \\ &= 10^{145} \times ((6.02 \times 10^1) + 8.1) \\ &= 10^{145} \times (60.2 + 8.1) \\ &= 68.3 \times 10^{145} \\ &= 6.83 \times 10^{146} \end{aligned}$$

**Final count:**

$$\begin{aligned} & (6.83 \times 10^{146}) - (3.5 \times 10^{145}) \\ &= 10^{145} \times ((6.83 \times 10^1) - 3.5) \\ &= 10^{145} \times (68.3 - 3.5) \\ &= 64.8 \times 10^{145} \\ &= \mathbf{6.48 \times 10^{146}} \end{aligned}$$

- 20) A deep-sky survey estimates the mass of a cluster as  $2.7 \times 10^{45}$  kg with a correction of  $1.2 \times 10^{44}$  kg added for unseen matter, then subtracts  $8 \times 10^{43}$  kg for instrument bias. Find the corrected mass. Give your answer in standard form.

**Total with correction:**

$$\begin{aligned} & (2.7 \times 10^{45}) + (1.2 \times 10^{44}) \\ &= 10^{44} \times ((2.7 \times 10^1) + 1.2) \\ &= 10^{44} \times (27 + 1.2) \\ &= 28.2 \times 10^{44} \\ &= 2.82 \times 10^{45} \end{aligned}$$

**Final total (subtract discards):**

$$\begin{aligned} & (2.82 \times 10^{45}) - (8 \times 10^{43}) \\ &= 10^{43} \times ((2.82 \times 10^2) - 8) \\ &= 10^{43} \times (282 - 8) \\ &= 274 \times 10^{43} \\ &= \mathbf{2.74 \times 10^{45}} \end{aligned}$$

## Challenge

- 21) Given that,

$$(a \times 10^p) \times (b \times 10^q) = 3.15 \times 10^{210}$$

with,  $a = 4.5$ ,  $b = 7$ , and  $p - q = 19$

Work out the values of  $p$  and  $q$ .

**Form two equations:**

$$\begin{aligned} & (4.5 \times 10^p) \times (7 \times 10^q) \\ &= 31.5 \times 10^{p+q} \\ &= 3.15 \times 10^{p+q+1} \\ & p + q + 1 = 210 \quad (1) \end{aligned}$$

$$p - q = 19$$

$$p = q + 19 \quad (2)$$

**Sub equation (2) into equation (1):**

$$\begin{aligned} & p + q + 1 = 210 \\ & q + 19 + q + 1 = 210 \\ & 2q + 20 = 210 \\ & 2q = 190 \\ & q = 95 \\ & p - 95 = 19 \\ & p = 114 \end{aligned}$$

$$\mathbf{p = 114, q = 95}$$

- 22) The numbers

$X = k \times 10^{190}$  and  $Y = 8.0 \times 10^{188}$  satisfy

$$X + Y = 8.56 \times 10^{190}.$$

Work out the value of  $k$ .

$$Y = 0.08 \times 10^{190}$$

$$k + 0.08 = 8.56$$

$$\mathbf{k = 8.48}$$

23) A student writes:

$$(4.0 \times 10^{172}) + (5.0 \times 10^{170}) \\ = 9.0 \times 10^{172}$$

a) Explain the error.

b) Give the correct result in standard form.

**Corrected working:**

$$(4.0 \times 10^{172}) + (5.0 \times 10^{170}) \\ = 10^{170} \times ((4.0 \times 10^2) + 5.0) \\ = 10^{170} \times (400 + 5) \\ = 405 \times 10^{170} \\ = 4.05 \times 10^{172}$$

a) The student did not consider that the numbers are being multiplied by different powers of 10. The student should align powers before adding.

b) See above for working

$$4.05 \times 10^{172}$$

24) Without completing a full calculation, order these numbers from least to greatest, justifying briefly:

$$A = (2.9 \times 10^{155}) \times (4 \times 10^{77})$$

$$B = (1.1 \times 10^{233})$$

$$C = (6 \times 10^{78})^3$$

$$A = 11.6 \times 10^{232} = 1.16 \times 10^{233}$$

$$C = 216 \times 10^{234} = 2.16 \times 10^{236}$$

**Correct order: B, A, C**

(See above for justification)

25) Construct two different pairs  $(U, V)$  in standard form with  $U > V$  such that  $U + V = 1 \times 10^{161}$  and  $U \times V = 9 \times 10^{320}$

$$V = \frac{9 \times 10^{320}}{U}$$

$$U + \frac{9 \times 10^{320}}{U} = 1 \times 10^{161}$$

**Multiply all terms by U:**

$$U^2 + (9 \times 10^{320}) = (1 \times 10^{161})U$$

$$U^2 - (1 \times 10^{161})U + (9 \times 10^{320}) = 0$$

$$a = 1, b = (-1 \times 10^{161}), c = (9 \times 10^{320})$$

**Solve the quadratic using the formula:**

$$U = \frac{(1 \times 10^{161}) \pm \sqrt{(-1 \times 10^{161})^2 - (4 \times 1 \times 9 \times 10^{320})}}{2(1)}$$

$$\frac{(10 \times 10^{160}) \pm \sqrt{(1 \times 10^{322}) - (36 \times 10^{320})}}{2}$$

$$\frac{(10 \times 10^{160}) \pm \sqrt{(100 \times 10^{320}) - (36 \times 10^{320})}}{2}$$

$$\frac{(10 \times 10^{160}) \pm \sqrt{(100 - 36) \times 10^{320}}}{2}$$

$$\frac{(10 \times 10^{160}) \pm \sqrt{64 \times 10^{320}}}{2}$$

$$\frac{(10 \times 10^{160}) \pm ((\sqrt{64} \times \sqrt{10^{320}}))}{2}$$

$$\frac{(10 \times 10^{160}) \pm (8 \times 10^{160})}{2}$$

**Solution set 1:**

$$U = \frac{(10 \times 10^{160}) + (8 \times 10^{160})}{2}$$

$$= \frac{18 \times 10^{160}}{2}$$

$$= 9 \times 10^{160}$$

$$\begin{aligned}
 V &= \frac{9 \times 10^{320}}{9 \times 10^{160}} \\
 &= (9 \div 9) \times (10^{320} \div 10^{160}) \\
 &= 1 \times 10^{160}
 \end{aligned}$$

**Solution set 2:**

$$\begin{aligned}
 U &= \frac{(10 \times 10^{160}) - (8 \times 10^{160})}{2} \\
 &= \frac{2 \times 10^{160}}{2} \\
 &= 1 \times 10^{160}
 \end{aligned}$$

$$\begin{aligned}
 V &= \frac{9 \times 10^{320}}{1 \times 10^{160}} \\
 &= (9 \div 1) \times (10^{320} \div 10^{160}) \\
 &= 9 \times 10^{160}
 \end{aligned}$$

**However,  $U > V$**

**Therefore:**

$$\mathbf{U = 9 \times 10^{160} \quad and \quad V = 1 \times 10^{160}}$$