# STANDARD FORM OPERATIONS

## LARGE POWERS OF 10

**Task 1 – For each of the following, give your answers in standard form.**

1) $(7.2 \times 10^{68}) \times (4.5 \times 10^{93})$

$= (7.2 \times 4.5) \times (10^{68} \times 10^{93})$

$= 32.4 \times 10^{68+9}$

$= 32.4 \times 10^{161}$

$= \mathbf{3.24 \times 10^{162}}$

2) $(3.9 \times 10^{144}) \times (2.4 \times 10^{-37})$

$= (3.9 \times 2.4) \times (10^{144} \times 10^{-37})$

$= 9.36 \times 10^{144\ \ (-37)}$

$= \mathbf{9.36 \times 10^{107}}$

3) $(1.6 \times 10^{182}) \div (4 \times 10^{29})$

$= (1.6 \div 4) \times (10^{182} \div 10^{29})$

$= 0.4 \times 10^{182-2}$

$= 0.4 \times 10^{153}$

$= \mathbf{4 \times 10^{152}}$

4) $(9.5 \times 10^{211}) \div (3.8 \times 10^{-19})$

$= (9.5 \div 3.8) \times (10^{211} \div 10^{-19})$

$= 2.5 \times 10^{211-(-19)}$

$= \mathbf{2.5 \times 10^{230}}$

5) $(5.2 \times 10^{125}) \times (6.8 \times 10^{77})$

$= (5.2 \times 6.8) \times (10^{125} \times 10^{77})$

$= 35.36 \times 10^{125+77}$

$= 35.36 \times 10^{202}$

$= \mathbf{3.536 \times 10^{203}}$

6) $(2.4 \times 10^{101}) \times (7 \times 10^{56})$

$= (2.4 \times 7) \times (10^{101} \times 10^{56})$

$= 16.8 \times 10^{101+56}$

$= 16.8 \times 10^{157}$

$= \mathbf{1.68 \times 10^{158}}$

7) $(4.3 \times 10^{88}) \div (1.1 \times 10^{12})$

$= (4.3 \div 1.1) \times (10^{88} \div 10^{12})$

$= 3.909 \ldots \times (10^{88-12})$

$= \mathbf{3.91 \times 10^{76}\ \ (3\ sf)}$

8) $\frac{(2.7 \times 10^{64})^3}{(9 \times 10^{58})^2}$

$= \frac{19.683 \times 10^{192}}{81 \times 10^{116}}$

$= (19.683 \div 81) \times (10^{192} \div 10^{116})$

$= 0.243 \times 10^{192-116}$

$= 0.243 \times 10^{76}$

$= \mathbf{2.43 \times 10^{75}}$

9) An observatory records photon readings from a telescope. The size of each uncompressed reading is $4.6 \times 10^{173}$ units of data. Each reading is compressed by a factor of $2.3 \times 10^{-7}$. A storage block can hold $5 \times 10^{168}$ units of data. Work out how many compressed readings fit into one block.

Size of compressed readings:

$(4.6 \times 10^{173}) \times (2.3 \times 10^{-7})$

$= (4.6 \times 2.3) \times (10^{173} \times 10^{-7})$

$= 10.58 \times 10^{173+(-7)}$

$= 10.58 \times 10^{166}$

$= 1.058 \times 10^{167}$

**Block capacity:**

$(5 \times 10^{168}) \div (1.058 \times 10^{167})$

$= (5 \div 1.058) \times (10^{168} \div 10^{167})$

$= 4.7258 \ldots \times 10^{168-167}$

$= 4.7258 \ldots \times 10^{1}$

$= \mathbf{4.73 \times 10^{1}}$ ***or* 47.3 (3 *sf*)**

10) A data centre processes $3.2 \times 10^{152}$ bytes per second for 45 minutes. Work out the total number of bytes of data that was processed.

**Total time:**

$45 \times 60 = 2700$

$2.7 \times 10^{3}$ *seconds*

**Bytes produced:**

$(3.2 \times 10^{152}) \times (2.7 \times 10^{3})$

$= (3.2 \times 2.7) \times (10^{152} \times 10^{3})$

$= 8.64 \times 10^{152+3}$

$= \mathbf{8.64 \times 10^{155}}$

Task 2 – For each of the following, give your answers in standard form.

11) $(8 \times 10^{137}) + (3.4 \times 10^{137})$

$= 11.4 \times 10^{137}$

$= \mathbf{1.14 \times 10^{138}}$

12) $(6.1 \times 10^{159}) - (2.9 \times 10^{159})$

$= \mathbf{3.2 \times 10^{159}}$

13) $(9.3 \times 10^{120}) + (1.7 \times 10^{121})$

$= 10^{120} \times (9.3 + (1.7 \times 10^{1}))$

$= 10^{120} \times (9.3 + 17)$

$= 26.3 \times 10^{120}$

$= \mathbf{2.63 \times 10^{121}}$

14) $(4.8 \times 10^{-188}) + (7 \times 10^{-189})$

$= 10^{-189} \times ((4.8 \times 10^{1}) + 7)$

$= 10^{-189} \times (48 + 7)$

$= 55 \times 10^{-189}$

$= \mathbf{5.5 \times 10^{-188}}$

15) $(5.6 \times 10^{134}) - (4.2 \times 10^{133})$

$= 10^{133} \times ((5.6 \times 10^{1}) - 4.2)$

$= 10^{133} \times (56 - 4.2)$

$= 51.8 \times 10^{133}$

$= \mathbf{5.18 \times 10^{134}}$

16) $(3.1 \times 10^{76}) + (2.49 \times 10^{78})$

$= 10^{76} \times (3.1 + (2.49 \times 10^{2}))$

$= 10^{76} \times (3.1 + 249)$

$= 252.1 \times 10^{76}$

$= \mathbf{2.521 \times 10^{78}}$

17) $(7.5 \times 10^{-162}) - (2.3 \times 10^{-163})$

$= 10^{-163} \times ((7.5 \times 10^{1}) - 2.3)$

$= 10^{-163} \times (75 - 2.3)$

$= 72.7 \times 10^{-163}$

$= \mathbf{7.27 \times 10^{-162}}$

18) $(2.2 \times 10^{92})^2 + (4.4 \times 10^{184})$

$(2.2 \times 10^{92}) \times (2.2 \times 10^{92})$

$= 4.84 \times 10^{184}$

$(4.84 \times 10^{184}) + (4.4 \times 10^{184})$

$= \mathbf{9.24 \times 10^{184}}$

19) A rare-event simulation logs $6.02 \times 10^{146}$ events in Phase 1 and $8.1 \times 10^{145}$ events in Phase 2. During validation, $3.5 \times 10^{145}$ events are discarded. What is the final count in standard form?

Phase 1 and phase 2:

$(6.02 \times 10^{146}) + (8.1 \times 10^{145})$

$= 10^{145} \times ((6.02 \times 10^{1}) + 8.1)$

$= 10^{145} \times (60.2 + 8.1)$

$= 68.3 \times 10^{145}$

$= 6.83 \times 10^{146}$

Final count:

$(6.83 \times 10^{146}) - (3.5 \times 10^{145})$

$= 10^{145} \times ((6.83 \times 10^{1}) - 3.5)$

$= 10^{145} \times (68.3 - 3.5)$

$= 64.8 \times 10^{145}$

$= \mathbf{6.48 \times 10^{146}}$

20) A deep-sky survey estimates the mass of a cluster as $2.7 \times 10^{45}$ kg with a correction of $1.2 \times 10^{44}$ kg added for unseen matter, then subtracts $8 \times 10^{43}$ kg for instrument bias. Find the corrected mass. Give your answer in standard form.

Total with correction:

$(2.7 \times 10^{45}) + (1.2 \times 10^{44})$

$= 10^{44} \times ((2.7 \times 10^{1}) + 1.2)$

$= 10^{44} \times (27 + 1.2)$

$= 28.2 \times 10^{44}$

$= 2.82 \times 10^{45}$

Final total (subtract discards):

$(2.82 \times 10^{45}) - (8 \times 10^{43})$

$= 10^{43} \times ((2.82 \times 10^{2}) - 8)$

$= 10^{43} \times (282 - 8)$

$= 274 \times 10^{43}$

$= \mathbf{2.74 \times 10^{45}}$

Challenge

21) Given that,

$(a \times 10^{p}) \times (b \times 10^{q}) = 3.15 \times 10^{210}$

with, $a = 4.5$, $b = 7$, and $p - q = 19$

Work out the values of p and q.

Form two equations:

$(4.5 \times 10^{p}) \times (7 \times 10^{q})$

$= 31.5 \times 10^{p+q}$

$= 3.15 \times 10^{p+q+1}$

$p + q + 1 = 210$ (1)

$p - q = 19$

$p = q + 19$ (2)

Sub equation (2) into equation (1):

$p + q + 1 = 210$

$q + 19 + q + 1 = 210$

$2q + 20 = 210$

$2q = 190$

$q = 95$

$p - 95 = 19$

$p = 114$

$\mathbf{p = 114, q = 95}$

22) The numbers

$X = k \times 10^{190}$ and $Y = 8.0 \times 10^{188}$ satisfy

$X + Y = 8.56 \times 10^{190}$.

Work out the value of k.

$Y = 0.08 \times 10^{190}$

$k + 0.08 = 8.56$

$\mathbf{k = 8.48}$

23) A student writes:

$$(4.0 \times 10^{172}) + (5.0 \times 10^{170})$$
$$= 9.0 \times 10^{172}$$

a) Explain the error.

b) Give the correct result in standard form.

**Corrected working:**

$$(4.0 \times 10^{172}) + (5.0 \times 10^{170})$$
$$= 10^{170} \times \left((4.0 \times 10^{2}) + 5.0\right)$$
$$= 10^{170} \times (400 + 5)$$
$$= 405 \times 10^{170}$$
$$= 4.05 \times 10^{172}$$

a) The student did not consider that the numbers are being multiplied by different powers of 10. The student should align powers before adding.

b) See above for working

$$\mathbf{4.05 \times 10^{172}}$$

24) Without completing a full calculation, order these numbers from least to greatest, justifying briefly:

$$A = (2.9 \times 10^{155}) \times (4 \times 10^{77})$$
$$B = (1.1 \times 10^{233})$$
$$C = (6 \times 10^{78})^3$$

$$A = 11.6 \times 10^{232} = 1.16 \times 10^{233}$$
$$C = 216 \times 10^{234} = 2.16 \times 10^{236}$$

**Correct order: B, A, C**

(See above for justification)

25) Construct two different pairs $(U, V)$ in standard form with $U > V$ such that $U + V = 1 \times 10^{161}$ and $U \times V = 9 \times 10^{320}$

$$V = \frac{9 \times 10^{320}}{U}$$

$$U + \frac{9 \times 10^{320}}{U} = 1 \times 10^{161}$$

**Multiply all terms by U:**

$$U^2 + (9 \times 10^{320}) = (1 \times 10^{161})U$$
$$U^2 - (1 \times 10^{161})U + (9 \times 10^{320}) = 0$$

$$a = 1, b = (-1 \times 10^{161}), c = (9 \times 10^{320})$$

**Solve the quadratic using the formula:**

$$U = \frac{(1 \times 10^{161}) \pm \sqrt{(-1 \times 10^{161})^2 - (4 \times 1 \times 9 \times 10^{320})}}{2(1)}$$

$$\frac{(10 \times 10^{160}) \pm \sqrt{(1 \times 10^{322}) - (36 \times 10^{32}\ )}}{2}$$

$$\frac{(10 \times 10^{160}) \pm \sqrt{(100 \times 10^{32}\ ) - (36 \times 10^{320})}}{2}$$

$$\frac{(10 \times 10^{160}) \pm \sqrt{(100 - 36) \times 10^{320}}}{2}$$

$$\frac{(10 \times 10^{160}) \pm \sqrt{64 \times 10^{320}}}{2}$$

$$\frac{(10 \times 10^{160}) \pm \left(\left(\sqrt{64} \times \sqrt{10^{320}}\right)\right)}{2}$$

$$\frac{(10 \times 10^{160}) \pm (8 \times 10^{160})}{2}$$

**Solution set 1:**

$$U = \frac{(10 \times 10^{160}) + (8 \times 10^{160})}{2}$$
$$= \frac{18 \times 10^{160}}{2}$$
$$= 9 \times 10^{160}$$

$$V = \frac{9 \times 10^{320}}{9 \times 10^{160}}$$
$$= (9 \div 9) \times (10^{320} \div 10^{16}\ )$$
$$= 1 \times 10^{160}$$

**Solution set 2:**

$$U = \frac{(10 \times 10^{160}) - (8 \times 10^{160})}{2}$$
$$= \frac{2 \times 10^{160}}{2}$$
$$= 1 \times 10^{160}$$

$$V = \frac{9 \times 10^{320}}{1 \times 10^{160}}$$
$$= (9 \div 1) \times (10^{320} \div 10^{160})$$
$$= 9 \times 10^{160}$$

**However, U > V**

**Therefore:**

$$\boldsymbol{U = 9 \times 10^{160} \quad and \quad V = 1 \times 10^{160}}$$