GROWTH & DECAY

Task 1

1) A new social media app launches with 500 users. If the number of users grows at a rate of 12% per month, how many users will there be after 18 months?

$$500\left(1+\frac{12}{100}\right)^{18}=3844.982\ldots$$

$$\mathbf{3845\ users}$$

2) A farmer has a population of 200 rabbits. The population decreases by 5% per month due to predators. How many rabbits will there be after 2 years?

$$200\left(1-\frac{5}{100}\right)^{24}=58.397\ldots$$

$$\mathbf{59\ rabbits}$$

3) An investment account starts with £1,200. The balance increases by 4% per year. How much will be in the account after 2 years?

$$1200\left(1+\frac{4}{100}\right)^{2}=£1297.92$$

$$\mathbf{£1297.92}$$

4) A car's value is £28,000 when new. The value depreciates by 18% each year. What will be its value after 6 years?

$$28000\left(1-\frac{18}{100}\right)^{6}=£8512.1868$$

$$\mathbf{£8512.19}$$

5) A country's population is currently 2 million and grows by 1.6% annually. How long will it take for the population to reach 3 million?

Trial and error:

$$2000000\left(1+\frac{1.6}{100}\right)^{25}=2974217.293$$

$$2000000\left(1+\frac{1.6}{100}\right)^{26}=3021804.769$$

26 years

6) A rare plant population in a nature reserve is 1,000. Due to climate change, its population decreases at a rate of 7% per year. How many years will it take until the population falls below 500?

Trial and error:

$$1000\left(1-\frac{7}{100}\right)^{8}=559.581\ldots$$

$$1000\left(1-\frac{7}{100}\right)^{9}=520.411\ldots$$

$$1000\left(1-\frac{7}{100}\right)^{10}=483.982\ldots$$

10 years

7) A music streaming service currently has 60 million subscribers. If it grows by 2.5% per quarter, how many subscribers will it have after 5 years?

$$60000000\left(1+\frac{2.5}{100}\right)^{20}=98316986.42$$

$$\mathbf{98,316,986\ subscribers}$$

8) The number of active volcanoes in a region is decreasing at 4% per decade. If there are currently 45 volcanoes, how many will there be in 100 years?

$$45\left(1-\frac{4}{100}\right)^{10}=29.917\ldots$$

$$\mathbf{30\ volcanoes}$$

9) A stock worth £120 grows by 1.5% per week. How many weeks will it take for the stock to double in value?

Trial and error:

$$120\left(1+\frac{1.5}{100}\right)^{45}=234.505\ldots$$

$$120\left(1+\frac{1.5}{100}\right)^{46}=238.023\ldots$$

$$120\left(1+\frac{1.5}{100}\right)^{47}=241.593\ldots$$

47 weeks

10) A scientist is studying bacteria that doubles in quantity every 5 hours. If there are 80 bacteria at the start, how many will there be after 1 day?

$$80(2)^{4.8} = 2228.609\ldots$$

$$\mathbf{2229\ bacteria}$$

Task 2

11) The function $P(t) = 200 \times 1.05^t$ models a town's population (in thousands) after t years.

   a. What is the initial population of the town?

   $P(0) = 200 \times 1.05^0 = 200$

   200,000

   b. What is the growth rate as a percentage?

   5%

   c. Estimate the town's population after 15 years.

   $P(15) = 200 \times 1.05^{15} = 415.7856\ldots$

   $\mathbf{415,786}$

12) The function $V(t) = 30{,}000 \times (0.88)^t$ represents the value of a car after t years.

   a. What does $V(0)$ represent?

   The initial value of the car.

   b. After 5 years, what is the value of the car?

   $V(5) = 30{,}000 \times (0.88)^5 = 15831.9575$

   $\mathbf{£15831.96}$

   c. What percentage does the car depreciate each year?

   12%

13) The graph of $y = ab^x$ passes through the points (0, 12) and (3, 96).
Work out the values of a and b.

When x = 0:

$$12 = ab^0$$
$$12 = a$$

When x = 3:

$$96 = 12b^3$$
$$b^3 = 8$$
$$b = 2$$

$$\boldsymbol{a = 12, \quad b = 2}$$

14) The graph of $y = ab^x$ passes through the points (0, 6) and (4, 1536).
Work out the values of a and b.

When x = 0:

$$6 = ab^0$$
$$6 = a$$

When x = 4:

$$1536 = 6b^4$$
$$256 = b^4$$
$$b = 4$$

$$\boldsymbol{a = 6, \quad b = 4}$$

15) The graph of $y = ab^x$ passes through the points (0, 8), (2, 18), and (5, p).
Work out the value of p.

When x = 0:

$$8 = ab^0$$
$$8 = a$$

When x = 2:

$$18 = 8b^2$$
$$\frac{9}{4} = b^2$$
$$b = \frac{3}{2}$$

$$y = 8\left(\frac{3}{2}\right)^x$$

When x = 5:

$$p = 8\left(\frac{3}{2}\right)^5 = \mathbf{\frac{243}{4}}$$

16) The graph of $y = ab^x$ passes through the points (0, 7), (2, 63), and (p, 5103).
Work out the value of p.

When x = 0:

$$7 = ab^0$$
$$7 = a$$

When x = 2:

$$63 = 7b^2$$
$$9 = b^2$$
$$b = 3$$

$$y = 7(3)^x$$

When y = 5103:

$$5103 = 7(3)^p$$
$$729 = 3^p$$
$$\boldsymbol{p = 6}$$

Challenge

17) A population of rare birds is modelled by $y = ab^x$, where y is the number of birds and x is the number of years since the study began.
The population is 240 when $x = 2$ and 1920 when $x = 5$.
Work out the values of a and b.

Form two equations:

$$240 = ab^2$$

$$1920 = ab^5$$

Divide the equations:

$$\frac{1920}{240} = \frac{ab^5}{ab^2}$$

$$8 = b^3$$
$$b = \sqrt[3]{8} = 2$$

$$240 = a \times 2^2$$
$$a = 60$$

$$\boldsymbol{a = 60}, \quad \boldsymbol{b = 2}$$

18) The value of a vintage car is modelled by $y = ab^x$ where y is the value in pounds and x is the number of years after 2010. The car was worth £12,000 in 2012 and £16,200 in 2014.
Work out the value of a to the nearest thousand and the value of b to 3 decimal places. Then, interpret what they mean in this context.

Form two equations:

$$12000 = ab^2$$

$$16200 = ab^4$$

Divide the equations:

$$\frac{16200}{12000} = \frac{ab^4}{ab^2}$$

$$1.35 = b^2$$
$$b = \sqrt{1.35} = 1.161895\ldots$$

$$12000 = a \times 1.161895\ldots^2$$
$$a = 8888.888\ldots$$

$$\boldsymbol{a = 9000}, \quad \boldsymbol{b = 1.162}$$

a represents the initial value of the car.

b is annual percentage increase of the value of the car by 16.2%.