A MEAN REVERTING TRADING STRATEGY TO BEAT BUY AND HOLD

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Abstract

Determining when an asset reverts toward its own average value is highly relevant for the portfolio management industry. This study aims to explore mean-reversion in financial markets and apply an active trading strategy to various types of securities, trough econometrics and technical analysis tools. Final goal is to compare this strategy with a simple Buy and Hold strategy on the S&P 500 index, and then with a momentum strategy. First of all, a third of securities used for this study can be defined mean-reverting considering one among the Augmented Dickey-Fuller test and a multiple linear regression. Results indicate that, maximizing for the profit factor, a performance indictor defined in this study, it is possible to reduce risk for half of the sample, obtaining a profitable strategy that beats the index, represented by an Exchange Traded Security: the 'SPDR S&P 500 ETF Trust'. Comparing every security before and after strategy application implies that the mean-reverting securities slightly under performs the other group, but considering the momentum strategy we are able to decrease risk for almost all securities and increasing profitability. Finally, in the validation stage, we can observe that the mean-reverting group outperform the other one, since both Out-Of-Sample validation and Monte Carlo simulation display better results on mean-reverting securities.

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Introduction

This study is divided into three main parts: identifying mean-reverting financial instruments, applying a trading strategy based on technical analysis, and validating the results through robustness checks. The first objective is to identify financial instruments that exhibit mean-reverting behavior. Mean reversion implies that an asset's price tends to return to its historical average, a phenomenon exploited by traders seeking to buy low and sell high. To detect mean-reverting assets, this study employs econometric techniques, including the Augmented Dickey-Fuller test and multiple linear regression. Past research provides various methods for detecting mean reversion, such as examining lagged correlations or testing for stationarity in prices. We use both the Augmented Dickey-Fuller test, which detects stationarity in log-price series, and regression techniques that analyze the impact of lagged log-returns on current series.

The second part of the study develops and tests a trading strategy designed to exploit mean-reverting behavior. The strategy is based on a moving average and employs two dynamic barriers, calculated by adjusting the moving average with a multiple of the historical returns volatility. The upper and lower barriers serve as thresholds for opening and closing positions: a long position is triggered when the price crosses above the lower threshold, and a short position is initiated when the price drops below the upper threshold. This approach captures deviations from the average, seeking to profit from mean reversion. Here the strategy is compared with a holding approach, both on index and on every single security, and with a momentum strategy, a widely discussed approach.

Finally, the strategy is validated to ensure its robustness. This phase includes two techniques: Out-Of-Sample strategy testing and Monte Carlo simulations. In the In-Sample period we can back test the strategy selecting its parameters, in order to use them in a new period and validate the strategy. Monte Carlo simulations has the goal to stress the strategy by applying it to multiple randomly generated price paths based on historical volatility and drift. Results show that, under certain conditions, the mean-reversion strategy outperforms Buy and Hold in terms of risk and profitability, particularly for assets that pass the mean-reversion tests. This study offers insights into how mean reversion can inspire active trading strategies and highlights the benefits of robustness checks in ensuring strategy sustainability.

Chapter 1

Preliminary tools and concepts

1.1 Literature Revision

The literature on mean reversion in financial markets has evolved from theoretical insights to practical applications, spanning across different asset classes. Mean reversion suggests that prices revert to their historical averages, a concept widely explored in econometric and trading research. One of the foundational studies on mean reversion in stock prices is by Poterba and Summers (1988), who analyzed stock prices to test for the presence of transitory components, suggesting that price shocks might revert to historical values over time. They used stationarity tests, regressions, and variance analysis to understand price movements. Subsequent work by Lothian and Taylor (1997) focused on real exchange rates, applying mean-reversion theories to currency markets and showing that exchange rates could exhibit mean-reverting characteristics over long horizons. Studies by Jorion and Sweeney (1996) also applied mean reversion to currency data, providing evidence that real exchange rates revert to long-run averages. Dehay and Leśkow (1996) extended these ideas, applying stationarity tests to stock market data to understand whether stock prices exhibit mean reversion, while Bessembinder et al. (1995) examined mean-reversion tendencies in commodity futures, where equilibrium asset prices often show a reversion tendency in the futures term structure.

Further work by Wooldridge (2016) in econometrics introduced various tools for analyzing time series, essential for detecting mean reversion. Wooldridge's techniques, alongside those in Tsay (2013) analysis of financial time series, inform the statistical methods applied in this study. Another line of research has addressed strategies for leveraging mean reversion in trading. Carcano, Falbo, and Stefani (2005) used a strategy based on dynamic barriers for mean-reverting markets, where positions were triggered when prices crossed certain thresholds relative to a moving average. Technical analysis has also been integrated into mean-reversion strategies, as seen in works by Defendi (2017), Bollinger (2009) and Giusti (2015), who discussed moving averages and indicators that can aid in identifying mean-reverting behavior, such as the Bollinger bands. Chu (2023) applied machine learning algorithms to test technical analysis rules in Out-Of-Sample, illustrating how these methods can enhance trading strategies by adapting to market dynamics.

About different investing approaches, Zakamulin and Giner (2023) incorporated transaction costs in their analysis of trend-following strategies, further informing this study's approach to realistic profitability assessments. Dichtl (2020) explored seasonal strategies, while Jegadeesh and Titman (2001) examined momentum strategies, showing that mean reversion could serve as a counter-strategy to momentum by exploiting price reversals. Additionally, Haug and Hirschey (2006) observed that specific seasonal patterns, such as the January Effect and September Swoon, could influence mean-reversion behavior in stock returns. Recent studies on the efficient market hypothesis, like Ţiţan (2015), have advanced the discussion on market predictability and strategy robustness. On the log-returns behavior we adopted a skewed t-distribution approach, as described by Jones and Faddy (2003), to capture the heavy-tailed and asymmetric nature of financial returns, which traditional Gaussian models may overlook. Together, these studies provide a framework for detecting, exploiting, and validating mean reversion in financial markets, highlighting the potential for mean-reverting strategies to outperform simple Buy and Hold approaches under certain conditions. Similarly, studies by Uhlenbeck and Ornstein (1930) and Robert and Salih (2015) on Brownian motion laid theoretical groundwork that was later applied to financial markets, establishing a basis for understanding random movements and potential mean-reversion behaviors. Raychaudhuri (2008) on Monte Carlo, useful for stress-testing trading strategies under varied market conditions, completes this framework providing insights into strategy performance across simulated scenarios simulation.

1.2 Efficient Market Hypothesis

One of the most debated theories in Finance literature is the Efficient Market Hypothesis. If this one was observed, it is impossible to realize profits diverging by a simple Buy and Hold strategy on financial markets. In according to Ţiţan (2015) there are three different types of efficiency, and this distinction is related to the type of information involved. Weak efficiency implies that every past information is included into price, making, for instance, technical analysis useless. Semi-strong efficiency implies instead that every public information is immediately reflected by the prices. Lastly a market is strongly efficient when even private information are already part of the price, making impossible realize profits from insider trading. Before to understand the markets efficiency it is necessary to identify different types of traders, basing on classification made by Arumugam (2023), because not every market operator has same targets. Portfolio managers have a long term vision, so they have to re balance clients' portfolios during the management period. These traders build up a typical hedging activity, their goal is not to realize profits by short term trading. Investment banks, hedge funds and aggressive market players in general try to realize profits every day exploiting prices oscillations or inefficiencies. Speculative activity implies to find a temporary difference between value and price. Positive returns of speculators are a prove that markets are not efficient.

1.3 Detection of mean reversion

A mean-reverting security has price that tends systematically to come back toward a value after a shock. So we have to consider a model that includes a deterministic value and shocks. The deterministic components of a stochastic process could be the time trend or a seasonality: many financial securities, especially stocks, have the tendency to increase their value over the time (Wooldridge (2016)), other, like currencies, can surge in some periods and fall in others, so it is necessary to represent these two features building models able to capture them. About the trend, a process used to capture it is:

$$y_t = \alpha_0 + \alpha_1 t + \epsilon_t,$$

where α_0 is the intercept. Parameter α_1 multiplying t, represents a linear time trend, so it measures change in y_t from one period to another, while ϵ_t is the time t error. A positive α_1 means that trend is positive, and vice versa. Many financial time series are approximated by an exponential trend (Tsay (2013)), so a price series has a constant growth rate over the time. Exponential trend can be showed by modeling natural logarithm of the original time series:

$$log(y_t) = \alpha_0 + \alpha_1 t + \epsilon_t. \tag{1.1}$$

Remember that $\Delta log(y_t) = log(y_t) - log(y_{t-1})$ is approximately, for small changes, equals to $(y_t - y_{t-1})/y_{t-1}$, the coefficient α_1 in (1.1) is equals to $\Delta log(y_t)$, for each t. This means that if α_1 is equal to 1 and t denotes months, price increases by 1% per month.
About seasonality we can write the following process:

$$y_t = \delta_0 + \delta_k D_t + u_t,$$

where δ is the intercept, δ_k is the coefficient for the $k - th$ dummy. D_t is a dummy variable representing a specific period: for example we can have a process with eleven dummy variables representing months. Every dummy is equal to one in the corresponding period, underlying time's behavior in that specific period. To avoid multi multicollinearity problems, one of the dummies is omitted. Analyzing a time series requires to exclude all deterministic components, trough de-trending and de-seasonalizing procedures. This because deterministic components could be correlated with the explanatory variable, showing a spurious relationship. A time series with trend or seasonality is not stationary, and this is the reason why working on returns series is better than the price series. So considering log prices instead of prices allows us to obtain returns trough differentiation: $r_t = ln(\frac{p_t}{p_{t-1}}) = ln(p_t) - ln(p_{t-1})$. Combining trend and seasonality we obtain:

$$y_t = \alpha_0 + \alpha_1 t + \delta_k D_{kt} + \epsilon_t,$$

where now the intercept is α_0, α_j represents the variation due to the passage of time, δ_k is the coefficient of the $k - th$ dummy variable D_{kt}, while ϵ_t is the error term on time t.
From an operating viewpoint there are different ways to study presence of mean reversion in a security path. In this study particular emphasis is settled on unit root analysis and regression, while autocorrelation is another way just described theoretically.

1.4 Stationarity and Random walk Hypothesis

A time series, in particular the log-price, p_t is defined as a random walk if:

$$p_t = p_{t-1} + a_t,$$

where p_{t-1} is the log-price of previous observation and a_t is a white noise process at time t. Value of p_{t+1} will be the sum between p_t and a random value that could be positive or negative with the same probability. If a_t has a symmetric distribution around zero, the expected value of p_t is p_{t-1}, and this implies that best prediction of the process is the previous value:

$$E[p_t] = E[p_{t-1} + a_t] = E[p_{t-1}] + E[a_t] = E[p_{t-1}] = p_{t-1}.$$

Random walk process can be viewed as an order one auto-regressive process, AR(1), with a coefficient ϕ equal to 1:

$$p_t = \phi p_{t-1} + \epsilon_t, \tag{1.2}$$

where ϵ_t is the error of the model. Including the intercept, ϕ_0, the model will be:

$$p_t = \phi_0 + \phi_1 p_{t-1} + \epsilon_t, \tag{1.3}$$

where now ϕ_1 represents the coefficient and ϕ_0 represents the time trend of the log price p_t, called drift. This constant term represents, in a random walk series, the time slope. So a positive slope implies that the log price eventually goes to infinite, while a negative drift implies that log price would converge to $-\infty$. A random walk is a non-stationary stochastic process, and Tsay (2013) suggests that for every weakly stationary and reversible stochastic process it is possible to forecast future values. A series is weakly stationary when mean and covariance are time independent: covariance only depends by l, number of lags. so, $E(r_t) = \mu$ and $Cov(r_t, r_{t-l}) = \gamma_l$. μ is a constant. Covariance has two features: $\gamma_0 = Var(r_t)$ and $\gamma_l = \gamma_{t-l}$. When stationarity is verified, we can affirm the series is mean-reverting: price will come back toward a deterministic component after every shock. To generalize time series modeling, we can interpret the process describing log-prices (p_t) behavior as a Finite Distributed Lag Model, in according with Wooldridge (2016):

$$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_k p_{t-k} + \epsilon_t,$$

where β_0 is the intercept, p_{t-k} is the log-price at time $t-k$, and β_k is the k-coefficient, ϵ_t is the model error.

1.4.1 Unit root test

A test used in past studies is the Dickey-Fuller on log prices. Lothian and Taylor 1997 found that in the exchange rates between US dollar and sterling, and between sterling and franc, there was a strong presence of mean reversion, estimated with a unit-root test on the logarithm of real exchange rate. In both cases root was lower a lot than unity. So, first method to detect mean reversion in a financial asset is to execute a Dickey-Fuller test. This test is widely used in finance literature, as suggested by Poterba and Summers (1988), Jorion and Sweeney (1996), Dehay and Leśkow (1996), Arias-Calluari et al. (2022). In all this papers, the Dickey-Fuller test is the main tool to study the stationarity. Considering the (1.2) and (1.3), in a non stationary time series, the ϕ_1 or ϕ values will be near to the unity. In both scenarios, this test considers the following Hypothesis:

- H_0: $\phi_1 = 1$
- H_1: $\phi_1 < 1$

The t-ratio, referred to as the Dickey-Fuller test is:

$$DF \equiv t\ ratio = \frac{\hat{\phi}_1 - 1}{std(\hat{\phi}_1)} = \frac{\sum_{t=1}^{T} p_{t-1} e_t}{\hat{\sigma}_\epsilon \sqrt{\sum_{t=1}^{T} p_{t-1}^2}} \tag{1.4}$$

If ϵ_t is a white noise series with finite moments of order greater than two, the DF statistic converges to a function of the standard Brownian motion as $T \to \infty$ .With the Least Squares method, Outputs for (1.4) are $\hat{\phi}_1 = \frac{\sum_{t=1}^{T} p_{t-1} p_t}{\sum_{t=1}^{T} p_{t-1}^2}$ and $\hat{\sigma_e}^2 = \frac{\sum_{t=1}^{T} (p_t - \hat{\phi}_1 p_{t-1})^2}{T-1}$. For many economic time series, $ARIMA(p,d,q)$ models might be more appropriate than the model described by equation 1.3. Since $AR(p)$ models are widely used in literature (Tsay (2013)), to verify presence of a unit root in an $AR(p)$ process, we may perform the test $H_0 : \phi = 1$ vs. $H_a : \phi < 1$ using the following regression:

$$\Delta p_t = \alpha + \beta t + \gamma p_{t-1} + \sum_{i=1}^{k} \delta_i \Delta p_{t-i} + \epsilon_t, \tag{1.5}$$

where $\Delta p_t = p_t - p_{t-1}$ represents the first difference of the series p_t, used to remove trends or seasonality. The parameter α is the intercept, capturing any fixed mean in the differenced series, while βt represents a deterministic trend component, where β is the slope of the trend over time. The coefficient γ corresponds to $\phi - 1$, where ϕ is the autoregressive coefficient of p_{t-1}. Under the null hypothesis H_0, $\gamma = 0$, indicating the presence of a unit root and non-stationarity in the series. The summation term $\sum_{i=1}^{k} \delta_i \Delta p_{t-i}$ includes lagged differences up to order k to correct for potential autocorrelation in the residuals ϵ_t, which are assumed to be independently and identically distributed with zero mean and constant variance. The Augmented Dickey-Fuller (ADF) test statistic is computed as the t-ratio of $(\hat{\phi} - 1)$, where $\hat{\phi}$ is the estimated value of the autoregressive coefficient. The formula for the ADF statistic is as follows:

$$ADF = \frac{\hat{\phi} - 1}{SE(\hat{\phi})}, \tag{1.6}$$

where $SE(\hat{\gamma})$ is the standard error of the estimated coefficient $\hat{\gamma}$. The ADF statistic is used to test the null hypothesis $H_0 : \gamma = 0$, indicating the presence of a unit root (and thus non-stationarity), against the alternative hypothesis $H_1 : \gamma < 0$, which suggests stationarity. This framework allows for the identification of whether a time series is stationary by evaluating the significance of γ.

1.4.2 Autocorrelation

Another important tool is the autocorrelation. Firstly, returns series can be defined as:

$$r_t = ln(p_t) - ln(p_{t-1}),$$

where p_t is the price at time t and p_{t-1} is the price at time t-1. Autocovariance is the covariance between the t *lagged* return and the $(t-k)$ *lagged* return:

$$\gamma_{t,t-l} = cov(r_t, r_{t-l}),$$

while the autocorrelation is:

$$\rho_l = \frac{Cov(r_t, r_t t - l}{\sqrt{Var(r_t)Var(r_{t-l})}} = \frac{Cov(r_t, r_{t-l})}{Var(r_t)} = \frac{\gamma_l}{\gamma_0},$$

where the property $Var(r_t) = Var(r_{t-l})$ for weakly stationary series is used. Portmanteau test is a test on the joint hypothesis that all autocorrelation are equals to zero:

- H_0: $\rho_1 = \rho_2 = \rho_m = 0$
- H_a: $\rho_i \neq 0$, with $i \in$ [1, m]

Mean reversion occurs when the correlation between returns is negative, this means that there is a negative relationship among returns in different lags. Portmanteau statistic $Q^*(m)$ is:

$$Q^*(m) = T\sum_{l=1}^{m} \hat{\rho_l^2},$$

where T is the dataset length. Under assumption that r_t is an *iid* (independent identically distributed) sequence, $Q^*(m)$ statistic is asymptotically a chi-squared random variable with m degree of freedom. The decision rule is to reject null hypothesis if $Q(m) > \chi^2_\alpha$ where χ^2_α is the $100(1-\frac{\alpha}{2})th$ percentile of the chi-squared distribution with m degrees of freedom and significance level α. The statistic $\hat{\rho}_l$ is called autocorrelation function (ACF), and it can be very useful to capture the dynamic of the series. It shows correlation values on different lags, and when it is higher than the standard error limit, that lag is significant. The two barriers represent significance level for autocorrelation presence. Usually they are built with the 95% confidence level, so their position is: $\pm z \cdot \frac{1}{\sqrt{N}}$m where N is observation number and z is the critical value from a standardized normal distribution, equal to 1.96. If there are not significant lag, we can affirm the series is a Random Walk.

The graph in the figure 1.2 indicates that, as opposed to graph in the figure 1.1, that $lag2$ and $lag5$ are below the significance barrier. That's a good starting point, there is a systematic behavior of the returns' series to revert after two and five months. Just for a theoretical purpose, we could even execute a test on individual autocorrelation. So:

- H_0: $\rho_l = 0$
- H_a: $\rho_l \neq 0$

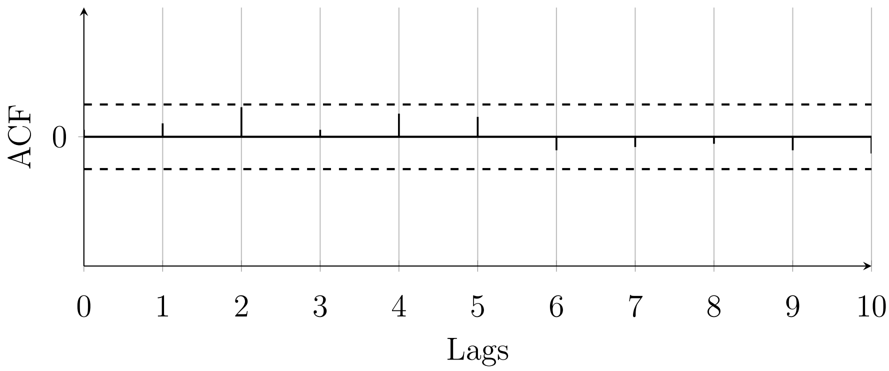

Figure 1.1: Autocorrelation function for a random walk series.

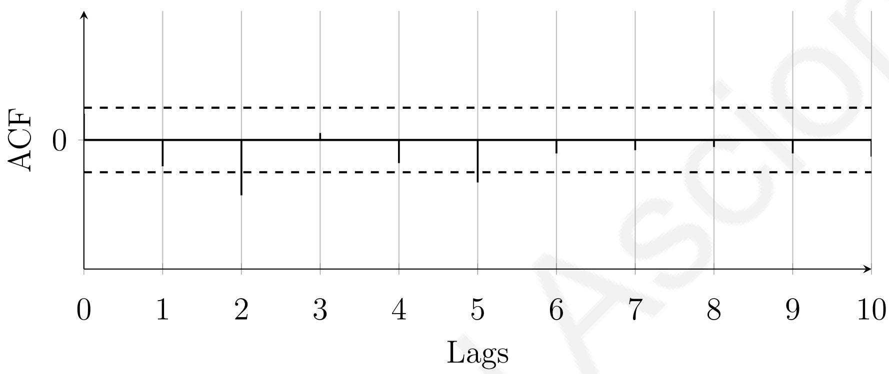

Figure 1.2: Autocorrelation function for a non random walk series

Test statistic is:

$$t\ ratio = \frac{\hat{\rho}_l}{\sqrt{(1 + 2\sum_{i=1}^{l-1}\hat{\rho_i^2})/T}}$$

If r_t is a stationary Gaussian series and $\rho_j = 0$ for $j < l$, the t-ratio is asymptotically distributed as a standard normal variable. The decision rule of the test is to reject H_0 when $|t\ ratio| > Z_{\frac{\alpha}{2}}$, where $Z_{\frac{\alpha}{2}}$ is the $100(1 - \frac{\alpha}{2})th$ percentile of the standard normal distribution.

1.4.3 Regression

Another important method is the linear regression applied on *k lag*. Dependent variable is return at time t, while the regressor is the *k lagged* return series:

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_k r_{t-k} + u_t,$$

where β_0 is the intercept, β_k is the regression coefficient for the *k lagged* return and u_t is the error at time t. Main problem in econometrics is to identify a causal effect in a relationship between two variables. Following Wooldridge (2016), Ordinary Least Square method is the way to reach this goal. Generally, considering a simple linear regression, with Y as the dependent variable and X as the explanatory variable, we can consider the following equation:

$$Y_i = \beta_0 + \beta_1 X_i + u_i,$$

that can be viewed as the regression line equation, for every observation i. So β_0 is the intercept, β_1 is the slope of the line. u is the error, so it includes all the features not captured by the model. Ordinary Least Square method consist in identifying coefficients (β_0 and β_1) that minimize sum of square errors:

$$\min_{b_0,b_1} \sum_{i=1}^{n} (Y_i - b_0 - b_1 X_i)^2, \tag{1.7}$$

where coefficients equations that allow to respect the 1.7 are $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}$ and $\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - \overline{X})^2}$. So, predicted values of the interest variable are: $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$, while residuals will be: $\hat{\eta}_i = Y_i - \hat{Y}_i$, Where Y_i is the $i-th$ observation of Y. Time series data are different from panel and cross-sectional data, because the variable we analyze is always the same, but it is observed in different times, while in panel data we observe different units of a population. So, it is important to remember that past can affect the future, while obviously the opposite is impossible. Ordinary Least Square method allows to obtain unbiased estimators if following assumptions are valid:

- **1) Linearity in parameters:** the process follows a linear model: $y_t = \beta_0 + \beta_1 x_{t1} + \beta_k x_{tk} + u_t$, where t is the time and k is a label to indicate the explanatory variable.
- **2) No perfect collinearity:** no independent variable is constant nor a perfect linear combination of the other variables. This means that every single independent variable can be correlated with others, but this assumption excludes perfect correlation.
- **3) Zero conditional mean:** $E(u_t|X) = 0, t = 1, 2, ...n$, so for each time t the expected value of the error, given the explanatory variable is zero. This assumption implies that the error is uncorrelated with every variable in the whole period of time. When the correlation between the variable and the error, in the same t is zero, we can say that the x_{tk} are contemporaneous exogenous, while if the correlation between u_t and x_{sk}, with $s \neq t$ is zero, the explanatory variable is strictly exogenous, and this last condition is essential to demonstrate unbiased-ness.

About this study, regression can be very useful when returning a negative and significant parameter, this means that with a high level of confidence it is possible to consider a causal effect between a lagged return and the return in t. Poterba and Summers (1988) used different methods to find mean reversion, and regression was one of those, in particular their test was based on whether the β_k was significantly different from zero. The probability limit of this statistic could be approximated as a linear combination of autocorrelations.

Chapter 2

Methodology

In according to Dichtl (2020), active investment strategies that attempt to outperform a Buy and Hold strategy can rely on three different factors: fundamental factors, seasonality and technical indicators. Fundamental strategies involve indicators from corporate analysis or macroeconomics. Dichtl applied the following regression model from Welch and Goyal (2008):

$$r_{t+1}^{\text{exc}} = \alpha_i + \beta_i x_{i,t} + \epsilon_{i,t+1},$$

where r_{t+1}^{exc} represents the log return on the S&P 500 index in excess of the log risk-free rate from period t to $t+1$. $x_{i.t}$ is a predictor variable, and α_i and β_i are regression parameters estimated using the Ordinary Least Square method. $\epsilon_{i,t+1}$ denotes the residual. As predictor variable there are different indicators useful for this model, such as the log dividend payout ratio, the log dividend yield, the log earnings-price ratio, the log book-to-market ratio, the volatility of the equity risk premium. About seasonality strategies, these are based on calendar effects: Haug and Hirschey (2006) studied for example the January effect, while Haug and Hirschey (2011) gave a contribution to the study on September swoon. Behind the first one there is a fundamental reason: investors tend to sell losing stocks in December for tax purposes and they buy again in January. The September swoon is an effect based on the negative stocks returns performance on September markets. Worst financial crisis, such as the Great Depression on October 1929, the Black Monday on October 1987, the Great Financial Crisis on September 2007 occurred between September and October. Dichtl and Drobetz (2015) analyzed market behavior on May to test the famous quote 'Sell in May and go away'. Technical analysis relies on historical prices and volumes to identify trading patterns. The proposed strategy of this study indeed uses a moving average as the pivot to open and close positions, as Carcano, Falbo, and Stefani (2005), but with an important difference: for every subperiod, defined by T, they found a fixed average value, and, as a consequence of that, their barriers were fixed. The concept behind the trading strategy is that if a security is mean-reverting, it is possible to buy when price is too low than the average, and, on the other hand, it is possible to sell when price tends to increase too much. So, a moving average can be an excellent starting point to identify hyper-bought and hyper-sold areas. A fundamental step is to understand when price is too low or too high, and for this goal it is used a rolling standard deviation calculated on returns series. So we created two barriers representing our trigger point, and overcame those, strategy produces operations in buy or sell side. These two barriers are simply calculated respectively as the sum and the difference between the moving average and the standard deviation, multiplied by an adjustment parameter.

2.1 Comparable strategies

Before to define the mean-reverting strategy, we introduce the Buy and Hold and the momentum strategy, that represents a benchmark in most past studies, like Hui and Chan (2019), or Dichtl (2020). A simple holding strategy involves buying a security to keep it in portfolio for a determined time period, in order to benefit from capital gains and dividends. This is the only doable one in a random-walk world, where there is no chance to gain an extra-profit from active trading strategies, implying total absence of market inefficiencies to exploit (Ţiţan (2015) and Dichtl and Drobetz (2014)). The momentum strategy can be viewed as the opposite of a mean-reverting approach, because it tries to exploit the tendency of a security to continue the past trend. First of all, Dichtl (2020) groups momentum approach in Technical analysis strategies, since they are based on past price behavior. In particular he tests different strategies with very short time horizon (one and three months) on S&P500, showing that they can offer a protection especially in bearish markets. This because they got better results than a simple holding strategy when index decreases its value. The performance is sensitive to the analysis period and transaction costs, so it doesn't represent an universal solution to beat the Standard and Poor index. Jegadeesh and Titman (1993) give us a framework of this approach testing a strategy in the 90's on American stocks markets. In particular they selected NYSE, NASDAQ and AMEX markets, showing the effective presence of a momentum effect. The built different portfolios using, as time window, three, six and twelve months. Their strategy involved buying the top-performing stocks and short-selling the worst-performing ones from the previous period. One of their explanations for this phenomenon is related to risk compensation: riskier stocks are expected to deliver higher returns. However, by building models that account for risk, they demonstrate that risk cannot fully explain momentum. Instead, they attribute part of the cause to behavioral biases and market inefficiencies.

2.2 Trading strategy

Technical indicator adopted for this strategy is the price Moving average, defined as:

$$MA_t = \frac{1}{N}\sum_{i=1}^{N} P_{t-i}, \tag{2.1}$$

where MA_t is the value of the moving average at time t, N is the moving average period, in other words the number of past observations considered until t, P_{t-i} is the asset price at time $(t-i)$. This is absolutely one of most famous and most used indicators by price analysts. It is reported in studies like Hilpisch (2021), Trombetta (2020) and Defendi (2017). Its main advantage is to smooth the price series, removing short-term noise, and then it approximates, especially for longer periods, the time trend. In its easiest version, it is an average of historical prices. It is called moving because for every observation at time t, the t-1 observation replaces the oldest. There also also other versions of the moving average, such as the weighted moving average and the exponentially moving average (Defendi (2017)). In his study, Dichtl (2020) used a very simple system based on two moving averages to outperform the market. It was long when a fast moving average (with a smaller N from equation 2.1), was higher than the slower one, and flat otherwise. Rolling standard deviation, estimated by Mele (2007) is instead defined as:

$$\sigma_t = \sqrt{\frac{\sum_{i=0}^{N-1} (r_i - \overline{r})^2}{N}},$$

where r_i is the log-return at time t_i and $\overline{r}$ is the average return until t-1. In a random walk world, the higher profit a trader can realize is derived from a Buy and Hold strategy. Obviously in some years market would go down, but in general there are not chances to make an extra-profit. But if price of any asset tends to come back to an average value, it is possible add some earn point to a holding strategy. This goal could be pursued by building a simple strategy including the moving average (2.1) as our reference point. Suppose now to insert an upper barrier and a lower barrier, that represent our trigger points: our algorithm produces a buy operation when price crosses above lower bound, and a sell short operation when it crosses below upper one. In order to represent the action of crossing, we need to consider the price and the two bands in different moments. A cross over implies that price is above the band, when in the previous month it was below, and vice versa for the cross under action. Only when both of these actions happened the strategy produces an operation, so trading conditions in t are:

$$\text{LONG} = \begin{cases} p_{t-2} > LOW_{t-2} \\ p_{t-1} < LOW_{t-1} \end{cases}$$

$$\text{SHORT} = \begin{cases} p_{t-2} > UPP_{t-2} \\ p_{t-1} < UPP_{t-1} \end{cases}$$

Where p_{t-2} and p_{t-1} are respectively close price observations at $t-2$ and t-1. LOW, that is the lower bound, is defined in a generic period t as:

$$LOW_t = MA_t(\theta) - \lambda\sigma_t^2(\omega) \tag{2.2}$$

UPP, that is the upper bound, is defined in a generic period t as:

$$UPP_t = MA_t(\theta) + \lambda\sigma_t^2(\omega) \tag{2.3}$$

Where θ is the period to calculate moving average, λ is the parameter to sum or subtract from the central moving average, and σ_t^2 is the standard deviation of returns' series, ω is the period used to calculate the returns' standard deviation. All three are expressed as a real, finite, positive, whole numbers. Noteworthy is that UPP and LOW are not fixed, but them self act like a moving average, and this indicators system we have built is very similar to a set of Bollinger bands, that is one of the most used indicators by technical analysts, it is well defined by Bollinger (2009). A standard Bollinger system is built using a moving average (2.1) calculated with $N = 20$ and two standard deviations to place the upper and lower barrier.

2.3 Optimization

Considering equations 2.2 and 2.3, we have to identify θ, λ and ω for each security. In order to to this is necessary apply the optimization procedure. Carcano, Falbo, and Stefani (2005) rely on this of procedure to identify levels of their triggers. Their strategy is a mean-reverting one, that open positions when the barrier are crossed. They describe a Ornstein-Uhlenbeck (Uhlenbeck and Ornstein (1930)) process for the price at time t, y_t as: $y_t = y_{t-1} + k(\phi - y_{t-1}) + \sigma\epsilon_t$, where y_{t-1} is value of the process at time t-1, k is the speed coefficient, ϕ is the long run average of the process, σ is the standard deviation and ϵ_t is the error term. To build the two trigger barriers, they de-trended the series and introduced a parameter, a, added and subtracted from ϕ. They try each combination of parameters for k, σ and T, the time-window size. This procedure was applied to two different commodities: Cocoa (a mean-reverting security) and wheat (a near random walk security). Comparing the two names in terms of Sharpe ratio, profit and standard deviation, they found that the Cocoa gained better results. About our optimization procedure, given three sets of parameters $A = [a, M]$, $B = [b, N]$ and $C = [c, O]$, and given i, j and l as steps between every $a \in A$, every $b \in B$ and every $c \in C$, the algorithm produces $(M * N * O)$ different outputs of our strategy for every single security. The strategy's output of parameters include: profit, standard deviation, number of operations and the profit factor. Profit at the end of the period (T) is calculated as:

$$profit_T = \sum_{t=1}^{T} sr_t, \tag{2.4}$$

where sr_t is the return of strategy on every t. To calculate sr it is necessary to multiply asset return with strategy position. So:

$$\text{sr}_t = \begin{cases} ar_i \times 1, & \text{when LONG position is active} \\ ar_i \times (-1), & \text{when SHORT position is active} \end{cases} \tag{2.5}$$

where ar_t is the asset return on time t. So to calculate profit for a Buy and Hold strategy we can apply equation 2.5 to ar_t. Standard deviation of strategy returns is instead calculated as:

$$\sigma_{sr} = \sqrt{\frac{\sum_{i=1}^{N}(sr_i - \overline{sr})^2}{N}}, \tag{2.6}$$

where N is number of observations. Finally, profit factor will be:

$$prf = \frac{\overline{r}_m}{\sigma_{sr}}, \tag{2.7}$$

where $\overline{r}_m$ is the average (monthly) return. These formulas are clearly applied to asset returns in order to obtain Buy and Hold stats. In the next chapter, only best combinations of parameters are selected in In-Sample for the whole period analyzed. Best a, b and c will correspond respectively to θ, λ and ω. Saying best we mean parameters that maximize profit factor for every security.

2.4 Robustness check

Testing a strategy in the past is not sufficient, this because what happened is not guaranteed in the future. Strategy could be too fit for the time series selected, or market conditions could change over the time, so it is necessary to make a further step: the validation. In this sub chapter we discussed two different techniques to validate a strategy: the In-Sample/Out-Of-Sample architecture and the Monte Carlo simulation. They are methods useful to stress the algorithm in order to understand how it can stand market difficulties. Trombetta (2020) gives us a framework on the validation in trading, highlighting its importance, introducing the Out-Of-Sample and the Monte Carlo validation. In particular, he designed a validation framework where In-Sample and Out-Of-Sample periods alternated multiple times. The reason behind this method is that he captured different changes in markets behavior, so the algorithm always changes target parameters in the optimization phase. Chu (2023) applied machine learning techniques in combination with technical analysis tools, leveraging classification algorithms to refine trading signals. To validate the strategy, he employed a robustness methodology, testing the model's parameters in Out-Of-Sample periods to evaluate its performance relative to the Buy and Hold benchmark.

2.4.1 Out-Of-Sample validation

First method analyzed here is the classical Out-Of-Sample validation, very spread in financial studies. For example Trombetta (2020) builds a system in which training set and testing set alternate more than one time, Chu (2023) explains the significance of testing on unknown data a strategy based on Machine learning algorithms. Testing a strategy in the In-Sample period allows to identify parameters that produce best results in terms of any performance indicator. The Out-Of-Sample period is instead necessary to test them and analyze the performance. Data in the testing set are unknown to the model, so the goal is to verify if parameters perform well on an unknown scenario. Generally the training set is longer than the testing set, and in this study the first period represents the 75% (nine years) of the total time span. In-Sample period starts on 01/01/2012, ending on 31/12/2012, while Out-Of-Sample period starts on 01/01/2021 ending on 31/12/2023.

2.4.2 Monte Carlo simulation

In finance, one of the most famous techniques is the Monte Carlo simulation, which is based on random sampling to compute results, as Raychaudhuri (2008) reports. The process starts from input data and, through a mathematical model, produces a multi-scenario analysis. Every prediction model can contain sources of error, but this uncertainty can be mitigated by repeating the experiment multiple times, and this is what a Monte Carlo analysis does. This type of simulation also allows to build advanced sensitivity analysis. So the starting point is to consider a model to use as simulation engine. Since we used the Geometric Brownian Motion in our algorithm, we can cite Hull (2022) and Glasserman (2003) that described this model for simulation purposes. In particular this equation is used in option pricing applications, because it allows to build different price paths, and this is our purpose. Since model to describe price evolution for this study is the Geometric Brownian Motion, we firstly need to identify the distribution of the input variables, to generate a set of random numbers. In this context, in according to the extension of the t-distribution proposed by Jegadeesh and Titman (1993), the log-returns distribution in this study is modeled as a skewed t-Student. This approach accounts for asymmetry and heavy tails often observed in financial data. To describe price movements of financial securities continuously, we can adopt the process equation followed by Robert and Salih (2015), Hull (2022) and Glasserman (2003) to describe the Geometric Brownian Motion:

$$dS = \mu S dt + \sigma S dz, \tag{2.8}$$

where dS represents the price variation over a specific time period, μ is the deterministic, constant component of the process, known as the drift, dt is the time increment, and dz is a random variable. In our model dz it follows a skewed t-distribution. Density function for a family of skewed t-students, described by Jones and Faddy (2003) is:

$$f(t) = f(t; a, b) = C_{a,b}^{-1} \left(1 + t \cdot \sqrt{a + b + t^2}\right)^{a+\frac{1}{2}} \left(1 - t \cdot \sqrt{a + b + t^2}\right)^{b+\frac{1}{2}}, \tag{2.9}$$

where t is our variable of interest, n is number of observation, a and b are parameters define the shape of the distribution. In particular, they control the weight of the tail and the influence of the skewness, respectively on the right side and on the left side of the distribution. When $a < b$ or $a > b$, distribution is negatively or positively skewed respectively. $C_{a,b} = 2^{a+b-1} \cdot B(a, b) \cdot (a + b)^{\frac{1}{2}}$, and $B(\cdot, \cdot)$ is the beta function. C is the normalization constant that guarantees the integral of the probability density function is equal to one, t is the return series in our case. The chart in the figure 2.1 shows probability density functions for a family of skewed t-students. A lower degrees of freedom value entails better fat tails, while increasing it approximates distribution to a normal one. This feature is caused by investors behavior: during panic selling volatility tends to increase more than other periods.

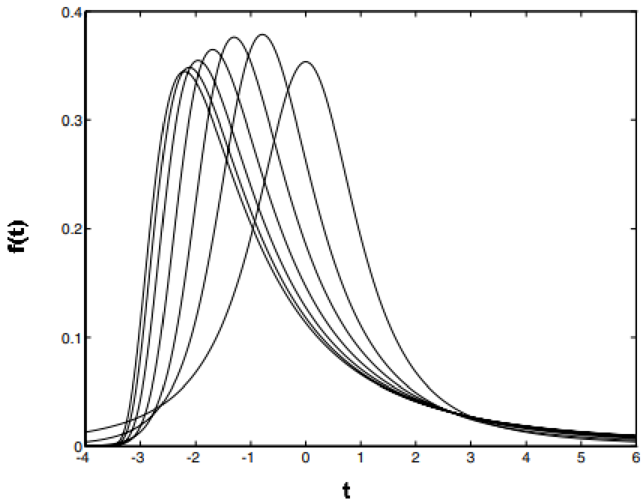

Figure 2.1: PDF for a family of skewed t-students - 't' on x-axis represents the variable of interest, in our case the log-returns, while 'f(t)' on y-axis represents the probability density function.

A famous test to understand wether a distribution is a normal one is the Jarque-Bera (Tsay (2013)): it verifies whether a data series is a normal one considering the skewness and kurtosis of the distribution. A normal distribution has skewness equal to zero and kurtosis equal to three. The test statistic, JB is defined as:

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right),$$

where n is the number of observations in the series, S is the sample skewness and K is the sample kurtosis. Under the null hypothesis (normality), the JB statistic follows an approximate chi-squared distribution with 2 degrees of freedom. If p-value of the test is lower than a certain α, the null hypothesis is rejected, indicating deviation from normality. The p-value of this test is the probability to observe realized values under the null hypothesis: observing the Skewness equal to zero and the Kurtosis equal to three, that correspond to a normal distribution. Jones and Faddy (2003) suggested a different way to identify the skewness and the degrees of freedom. They used indeed parameters a and b to define the skewness parameter, λ and number of degrees of freedom, v to model tail shape. In this case, $v = a + b$ and $\lambda = a - b$. The equation 2.8 describes continuous price movements, but it can be discretized:

$$\Delta S = \mu S \Delta t + \sigma S \Delta Z,$$

where ΔS is the price variation, μ represents the drift term, corresponding to the expected rate of return, and σ denotes the volatility term, which measures the standard deviation of the returns. The parameter Δt is the discrete time step over which the process is evaluated. The term ΔZ is the stochastic increment, sampled from a skewed t-distribution: $\Delta Z \sim \text{Skewed-}t_v(\lambda, 0, \Delta t)$, where v represents the degrees of freedom, determining the heaviness of the tails, λ is the skewness parameter that controls the asymmetry of the distribution, and Δt scales the stochastic term accordingly. Dividing both sides by S, we obtain the variation of the price relative to itself. This can be interpreted as the return in discrete time. In the Monte Carlo simulation, this equation is used with random numbers drawn from a skewed t-distribution to generate different price paths. To estimate the drift of the process, we rely on a regression of log-returns analysis of historical data, using time as independent variable:

$$r_t = \alpha_0 + \mu t + \epsilon_t, \tag{2.10}$$

where r_t is the log-returns, calculated as $ln(\frac{p_t}{p_{t-1}})$, α_0 is the intercept, μ is the drift and ϵ_t is the error of the regression. We use the intercept because we can't assume that at t_0 return is zero. To estimate the standard deviation, we isolate volatility extracting the drift out of the process: $\hat{r_t} = r_t - \mu$ and calculating it as $\hat{\sigma} = std(\hat{r_t})$. Once the drift and volatility are determined, it is possible to generate N simulated price paths starting from the last observed price in the time series. The trading strategy is applied to all these simulated price paths for the entire Out-Of-Sample period. At each time t, the simulated price S_t is computed as:

$$S_t = S_{t-1} e^{\mu \Delta t + \sigma \sqrt{\Delta t} \cdot Z_t}, \tag{2.11}$$

where S_{t-1} is the price in the previous time period, and Z_t is a random variable drawn from a skewed t-distribution to account for the asymmetry often observed in asset returns.

The standard deviation (σ) is multiplied by this random variable to model the stochastic component. The log-return at time t, r_t, is:

$$r_t \approx \ln\left(\frac{P_t}{P_{t-1}}\right) = \mu\Delta t + \sigma\sqrt{\Delta t} \cdot Z_t,$$

where r_t represents the log-return, which are modeled using the drift μ and volatility σ, with the randomness introduced by the skewed t-distributed variable Z_t. P_t and P_{t-1} are price respectively at time t and t-1, and Δt is the time changing. By applying this model, we generate multiple paths for future prices, in order to test the strategy over a wide range of possible outcomes.

Chapter 3

Results

3.1 Introduction to data

We selected a list of sixty-four financial securities among different categories. We have fifty stocks, selected randomly from US stock over 1 Billion$ of market capitalization. Then we have three different Exchange Traded Funds (ETF) composed by bonds and eleven futures divided by type: indexes, currencies and commodities. All time series are monthly, downloaded from Yahoo Finance. To represent futures, we took price of continuous futures, since every future contract has a monthly expiration. To realistically represent S&P 500 we worked on 'SPDR S&P 500 ETF Trust' time series, that we simply called SPY. The Entire time span goes from 01/01/2012 to 31/12/2023, but the training set starts on 01/01/2012 and ends on 31/12/2020 as reported in the figure 3.1.

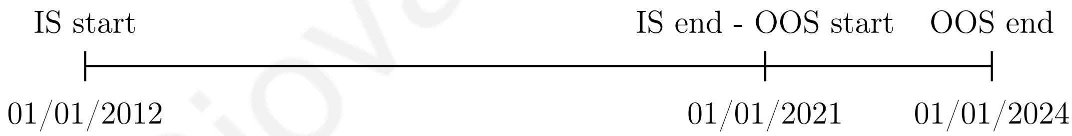

Figure 3.1: Timeline of the period division

Log-returns are calculated on adjusted close price series, to consider dividends for stocks. Returns series, fundamental for econometrics tests, are calculated as the logarithm of $\frac{P_t}{P_{t-1}}$, where P_t and P_{t-1} are respectively the price on time t and t-1. Using logarithm is an approximation, but is necessary to follow a homogeneous method. This calculus method is indeed used for all the applications. About transaction costs, nowadays it is possible to pay very low costs, especially for an institutional trading firm, for which costs are represented by small amounts charged by intermediaries. So we applied high transaction costs, from one to three percentage points even to consider the worst case scenario. About futures we didn't consider any cost over the transactional ones, so margins and overnight costs are set to zero. On ETF's too are considered only the operative expenses. table 3.1 lists all different securities, grouping them by category.

Ticker	Full Name	Category
AAPL	Apple Inc.	Stock
ABBV	AbbVie Inc.	Stock
ABT	Abbott Laboratories	Stock
ADBE	Adobe Inc.	Stock
AMGN	Amgen Inc.	Stock
AMZN	Amazon.com, Inc.	Stock
BA	The Boeing Company	Stock
BLK	BlackRock, Inc.	Stock
BMY	Bristol-Myers Squibb Company	Stock
BRK-B	Berkshire Hathaway Inc.	Stock
C	Citigroup Inc.	Stock
CMCSA	Comcast Corporation	Stock
COST	Costco Wholesale Corporation	Stock
CSCO	Cisco Systems, Inc.	Stock
CVX	Chevron Corporation	Stock
DIS	The Walt Disney Company	Stock
GOOGL	Alphabet Inc.	Stock
GS	The Goldman Sachs Group, Inc.	Stock
HD	The Home Depot, Inc.	Stock
INTC	Intel Corporation	Stock
JNJ	Johnson & Johnson	Stock
JPM	JPMorgan Chase & Co.	Stock
KO	The Coca-Cola Company	Stock
LLY	Eli Lilly and Company	Stock
MA	Mastercard Incorporated	Stock
MCD	McDonald's Corporation	Stock
MDT	Medtronic plc	Stock
META	Meta Platforms, Inc.	Stock
MO	Altria Group, Inc.	Stock
MRK	Merck & Co., Inc.	Stock
MSFT	Microsoft Corporation	Stock
NFLX	Netflix, Inc.	Stock
NKE	NIKE, Inc.	Stock
NVDA	NVIDIA Corporation	Stock
ORCL	Oracle Corporation	Stock
PEP	PepsiCo, Inc.	Stock
PFE	Pfizer Inc.	Stock
PG	The Procter & Gamble Company	Stock
PYPL	PayPal Holdings, Inc.	Stock
QCOM	QUALCOMM Incorporated	Stock
SBUX	Starbucks Corporation	Stock
SPGI	S&P Global Inc.	Stock
T	AT&T Inc.	Stock
TMO	Thermo Fisher Scientific Inc.	Stock
TSLA	Tesla, Inc.	Stock
UNH	UnitedHealth Group Incorporated	Stock

Ticker	Full Name	Category
V	Visa Inc.	Stock
VZ	Verizon Communications Inc.	Stock
WMT	Walmart Inc.	Stock
XOM	Exxon Mobil Corporation	Stock
CL=F	Crude Oil Future	Commodity
GC=F	Gold Future	Commodity
NG=F	Natural Gas Future	Commodity
SI=F	Silver Future	Commodity
6A=F	Australian Dollar Future	Currency
6B=F	British Pound Future	Currency
6E=F	Euro Future	Currency
6J=F	Yen Future	Currency
NQ=F	Nasdaq 100 Index Future	Index
RTY=F	Russell 2000 Index Future	Index
YM=F	Dow Jones Industrial Average Index Future	Index
AGG	iShares Core U.S. Aggregate Bond	ETF
BIL	SPDR Bloomberg 1-3 Month T-Bill	ETF
TLT	iShares 20+ Year Treasury Bond	ETF

Table 3.1: Securities list.

3.2 Augmented Dickey-Fuller Test

Testing stationarity to identify mean-reversion is a widely adopted approach in financial literature. Several studies, including Lothian and Taylor (1997), Jorion and Sweeney (1996), and Poterba and Summers (1988), have employed stationarity tests to investigate whether financial time series exhibit mean-reverting behavior. Specifically, they implemented order-one autoregressive models to test for the presence of a unit root. Lothian and Taylor (1997) and Jorion and Sweeney (1996), by applying these models to real exchange rates, found evidence of mean-reversion, rejecting the null hypothesis of the Dickey-Fuller test. In this analysis, the Augmented Dickey-Fuller test was applied to the monthly log-price series of all sixty-four securities, covering the entire period from 01/01/2012 to 31/12/2023. The goal of the test is to verify whether the series are stationary, a necessary condition to confirm mean-reversion, in according to the already cited studies. The null hypothesis assumes the presence of a unit root, indicating non-stationarity. To reject the null hypothesis, the p-value of the test must be lower than the significance level of 10% (or 0.1). The p-value represents the probability of obtaining the observed test statistic, or a more extreme value, under the assumption that the null hypothesis is true. Rejecting null hypothesis supports the conclusion that the series is stationary. The Augmented Dickey-Fuller test on an order one autoregressive model is based on the following equation:

$$\Delta p_{it} = \alpha_i + \gamma_i p_{it-1} + \sum_{l=1}^{k} \delta_{il} \Delta p_{it-j} + \epsilon_{it}$$

In this formulation, for every security i, $\Delta pi_t = p_{it} - p_{it-1}$ represents the first difference of the log-price series at time t, which is used to eliminate trends. The term α_i represents the intercept of the process. The parameter γ_i, equal to $(\phi_i - 1)$, is the coefficient of interest, where ϕ_i is the autoregressive coefficient of the lagged value p_{it-1}. If $\gamma_i = 0$, the series has a unit root and is therefore non-stationary, while $\gamma_i < 0$ suggests stationarity. To account for potential autocorrelation in the residuals, the model includes lagged differences of the series, represented by $\sum_{l=1}^{k} \delta_{ij} \Delta p_{t-j}$, where j is the lag and k is the maximum number of lags, defined the Bayesian (BIC) Information criteria, (Chakrabarti and Ghosh (2011)). The term ϵ_{it} is a white noise error term with zero mean and constant variance. The ADF statistic is calculated as the ratio between the estimated value of γ_i and its standard error. A negative and significant value of γ_i, indicated by a low p-value, provides evidence against the null hypothesis of non-stationarity. The results of the test, reported in table 3.2, show that six securities exhibit stationary log-price series. Among these, four are stocks, namely Citigroup (C), Walt Disney (DIS), Tesla (T), and Verizon (VZ), while two are commodity futures: Silver (SI=F) and Natural Gas (NG=F). These findings suggest that these securities are mean-reverting. In table 3.2, we reported the ADF statistic, the coefficient γ_i, and the p-value.

Tick	**ADF Statistic**	**Coefficient γ**	**P-value**
6A=F	-2.2882	-0.0451	0.1758
6B=F	-1.6017	-0.0361	0.4826
6E=F	-1.9139	-0.0476	0.3255
6J=F	-1.5542	-0.0278	0.5065
AAPL	-0.1823	-0.0014	0.9405
ABBV	-1.1601	-0.0149	0.6904
ABT	-1.3192	-0.0112	0.6202
ADBE	-1.1155	-0.0076	0.7088
AGG	-1.4432	-0.0191	0.5613
AMGN	-1.7754	-0.0218	0.3927
AMZN	-1.3469	-0.0103	0.6074
BA	-1.7644	-0.0295	0.3982
BIL	1.9862	0.0262	0.9986
BLK	-1.3340	-0.0157	0.6134
BMY	-2.5406	-0.0479	0.1058
BRK-B	-1.1204	-0.0108	0.7068
C	**-2.8520**	**-0.0849**	**0.0512**
CL=F	-2.5514	-0.0940	0.1034
CMCSA	-2.5532	-0.0347	0.1030
COST	-0.2702	-0.0018	0.9296
CSCO	-1.2645	-0.0162	0.6452
CVX	-1.2475	-0.0273	0.6528
DIS	**-2.7709**	**-0.0455**	**0.0625**
GC=F	-0.7678	-0.0165	0.8283
GOOGL	-1.0505	-0.0089	0.7343
GS	-1.2364	-0.0206	0.6577
HD	-1.7589	-0.0133	0.4010
INTC	-1.2818	-0.0217	0.6374
JNJ	-2.2354	-0.0186	0.1936

Tick	ADF Statistic	Coefficient γ	P-value
JPM	-1.1630	-0.0135	0.6892
KO	-0.8428	-0.0189	0.8062
LLY	0.9802	0.0066	0.9940
MA	-1.4849	-0.0105	0.5409
MCD	0.1231	-0.0003	0.9676
META	-1.3983	-0.0179	0.5830
MDT	-2.3000	-0.0294	0.1719
MO	-2.4680	-0.0407	0.1234
MRK	-0.9169	-0.0104	0.7823
MSFT	-0.0059	-0.0000	0.9580
NFLX	-1.5544	-0.0158	0.5064
NG=F	**-2.7198**	**-0.1177**	**0.0706**
NKE	-1.4831	-0.0154	0.5418
NQ=F	-0.4190	-0.0036	0.9068
NVDA	0.3045	0.0018	0.9775
ORCL	-0.4937	-0.0063	0.8932
PEP	-1.2929	-0.0106	0.6323
PFE	-2.1734	-0.0329	0.2160
PG	-0.9431	-0.0093	0.7734
PYPL	-1.4263	-0.0243	0.5695
QCOM	-0.9296	-0.0170	0.7780
RTY=F	-1.9062	-0.1019	0.3291
SBUX	-1.6518	-0.0180	0.4560
SI=F	**-2.6273**	**-0.0824**	**0.0874**
SPGI	-1.1348	-0.0085	0.7009
T	**-3.0555**	**-0.0782**	**0.0300**
TMO	-1.5359	-0.0102	0.5156
TLT	-1.5681	-0.0309	0.4995
TSLA	-1.1420	-0.0106	0.6980
UNH	-0.8774	-0.0058	0.7953
V	-2.1930	-0.0134	0.2087
VZ	**-2.6169**	**-0.0504**	**0.0895**
WMT	-0.7376	-0.0084	0.8367
XOM	-1.1439	-0.0262	0.6972
YM=F	-0.9907	-0.0099	0.7566

Table 3.2: Augmented Dickey-Fuller test results - 'Tick' is the security ticker.

3.3 Regression

Poterba and Summers (1988) applied a multiple linear regression considering quarterly lags, to capture long-term behavior of returns time series. Their goal was to find negative and significant parameters to find presence of mean-reversion. In this study we followed this methodology, applying the regressions on monthly observations of every single security. As for the Augmented Dickey-Fuller test, we considered the entire time period, so from 01/01/2012 to 31/12/2023. For each security i, every equation is:

$$r_{it} = \beta_{0i} + \sum_{k=1}^{24} \beta_{ki} r_{it-k} + \epsilon_{it},$$

where for every observation t and for every security i, r_{it} is the return series on time t, r_{it-k} is the return series on time t-k, and ϵ_{it} is the regression error. The value of k goes from one to twenty-four, in order to cover a two years time span. This is a choice applied also for the optimization in section 3.3. To consider a mean-reverting behavior on every lag, it is necessary to observe a β_{ki} negative and significant, so p-value has to be lower than 10% (or 0.1), where the p-value represents the probability to obtain the observed coefficient value under the null hypothesis H_0: $\beta_{ki} = 0$. Fourteen securities worth to be considered, and three of these (Netflix, Oracle and Nike) show significance considering more than one lag. In the table 3.3 we reported only the securities that passed the regression test, highlighting the corresponding significant lag, the Coefficient β_{ik}, the standard error of the coefficient, the R-squared of the regression and the p-value. The R-squared, or R^2 of a multiple linear regression measures the proportion of the variance in the dependent variable Y that is explained by the independent variables in the model. It is defined as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}.$$

where y_i are the observed values of the dependent variable, $\hat{y}_i$ are the predicted values from the regression model, $\bar{y}$ is the mean of the observed values, $\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$ represents the Sum of Squared Residuals (SSR), and $\sum_{i=1}^{n}(y_i - \bar{y})^2$ represents the Total Sum of Squares (SST). We can observe that following this method thirteen stocks and two currency futures (British Pound and Euro) are mean-reverting. Grouping both tests results, twenty-one securities are mean-reverting: two currencies futures, two commodities futures and seventeen stocks.

Tick	**Lag**	**Coefficient**	**SE**	**R-Squared**	**P-value**
6B=F	3	-0.1683	0.0936	0.0278	0.0748
6E=F	6	-0.1980	0.0927	0.0409	0.0350
AMGN	3	-0.2002	0.0845	0.0399	0.0193
BRK-B	3	-0.1818	0.0850	0.0328	0.0343
COST	3	-0.3158	0.0820	0.0991	0.0002
CSCO	18	-0.2564	0.0951	0.0647	0.0082
HD	18	-0.2075	0.1126	0.0313	0.0682
MA	24	-0.2864	0.1368	0.0450	0.0391
MSFT	18	-0.2892	0.1011	0.0723	0.0051
NFLX	3	-0.1753	0.0810	0.0336	0.0321
NFLX	18	-0.2357	0.0908	0.0604	0.0108
NKE	3	-0.1609	0.0830	0.0271	0.0548
NKE	18	-0.4052	0.1114	0.1119	0.0004
ORCL	3	-0.2439	0.0852	0.0572	0.0049
ORCL	18	-0.2732	0.1219	0.0456	0.0271
PG	3	-0.1409	0.0847	0.0201	0.0984

Tick	Lag	Coefficient	SE	R-Squared	P-value
V	24	-0.2430	0.1328	0.0348	0.0705
XOM	6	-0.1671	0.0888	0.0267	0.0620

Table 3.3: Regression results - 'Lag' is the lag shows significance, 'SE' is the standard error of the regression.

3.4 Buy and hold

Holding a security in portfolio implies executing a single long operation on the first dataset day. We applied a Buy and Hold strategy to the 'SPDR S&P 500 ETF Trust' and to every single security, in order to have a comparison with the Index, and a to observe what happens before and after the application of the proposed strategy. Profit, standard deviation and profit factor, at final time T are calculated as in the equations 2.4, 2.6 and 2.7. So first of all, our reference series is the cumulative returns series. For Buy and Hold and in general for all trading strategies, we can calculate this one, on every time t, as following:

$$R_{it} = \sum_{i=1}^{t} r_{im}, \tag{3.1}$$

where r_{im} is the $m-th$ monthly return for the $i-th$ security. Profit on time T is the sum of every r_{im}. Our target stat, the profit factor, is calculated as ratio between $\bar{r_i}$ and the standard deviation of strategy returns, $\text{std}(r_i)$, where $\bar{r_i}$ is the average strategy outcome, and both are calculated monthly. We have chosen to build a own indicator following Hui and Chan (2019). In particular they built the Zhourayev Index, defined as ratio between the extra-return on a Buy and Hold strategy and their strategy standard deviation. This index is similar to another one, very famous in financial literature: the Sharpe ratio (Sharpe (1966)), defined as:

$$SR_i = \frac{R_i - rf}{\sigma_i},$$

where SR_i is the Sharpe ratio for the security i, R_i is the annualized return and σ_i is the annualized standard deviation for the $i-th$ security. This indicator can be applied to a single security or to a strategy. For example Carcano, Falbo, and Stefani (2005) used Sharpe ratio to compare outcomes of their strategy with different parameters. In this study we have chosen, as risk free measure, rf, the yearly yield of the 10-years US T-note on 01/01/2012 (3.588%). To compute transaction costs, since we calculated log-returns and not simple returns, we have calculated a product of every net return (return multiplied by (1-c), where c is the transaction cost). When mean-reverting and momentum strategies are applied, transaction costs are doubled and then multiplied by number of operations. They are doubled because an operation is opened and closed, so we have two different operations. In the tables 3.4 and 3.5 we reported results of the holding strategy on the SPY. In the first one we have the standard deviation and the profit calculated in different transactional scenarios, while in the second one we reported the profit factor and the Sharpe ratio, considering different transaction costs. This strategy executes just the operations of opening (on 01/01/2012) and closing (on 31/12/2021).

Tick	**Std**	**$P_{tc=0\%}$**	**$P_{tc=1\%}$**	**$P_{tc=2\%}$**	**$P_{tc=3\%}$**
SPY	0.039	1.22	1.20	1.18	1.16

Table 3.4: Buy & Hold stats - 'Std' is the standard deviation, 'P' is the profit, and 'tc' is the transaction costs %.

Tick	**$prf_{tc=0\%}$**	**$prf_{tc=1\%}$**	**$prf_{tc=2\%}$**	**$prf_{tc=3\%}$**	**$sr_{tc=0\%}$**	**$sr_{tc=1\%}$**	**$sr_{tc=2\%}$**	**$sr_{tc=3\%}$**
SPY	0.292	0.287	0.282	0.277	0.736	0.720	0.703	0.687

Table 3.5: Buy & Hold stats - 'prf' is the profit factor, 'sr' is the sharpe ratio and 'tc' is the transaction costs %.

We then applied the holding strategy on every security, to study, before and after the application of our strategy, how the standard deviation changes and then in order to make a comparison in terms of profit factor and Sharpe ratio. In tables 3.6 and 3.7 we reported all stats for the Buy and Hold strategy applied on all sixty-four securities, so we calculated profit, profit factor and Sharpe ratio, considering different transaction costs, from 0% to 3%. All currencies futures produce a loss in the holding period, while among other futures, Crude oil (CL=F), Silver (SI=F) and Natural gas (NG=F) don't produce any positive result. Exxon (XOM) is the only stocks with a negative final outcome. About stocks and Exchange Traded Funds, they always produce profit, even applying higher transaction costs, while the BIL is the only to lose money when the 3% transaction cost is applied. Considering the extra-return produced by every security on the risk-free return at 3.588%, we can look at the Sharpe ratio index: with no transaction costs applied, fifty-two securities have a positive extra-return, while applying extreme transaction costs they become fifty-one.

Tick	**Std**	**$P_{tc=0\%}$**	**$P_{tc=1\%}$**	**$P_{tc=2\%}$**	**$P_{tc=3\%}$**
6A=F	0.031	-0.333	-0.353	-0.373	-0.393
6B=F	**0.026**	**-0.153**	**-0.173**	**-0.193**	**-0.213**
6E=F	**0.022**	**-0.087**	**-0.107**	**-0.127**	**-0.147**
6J=F	0.025	-0.241	-0.261	-0.281	-0.301
AAPL	0.080	2.244	2.224	2.204	2.184
ABBV	0.078	1.378	1.358	1.338	1.318
ABT	0.053	1.628	1.608	1.588	1.568
ADBE	0.059	2.782	2.762	2.742	2.722
AGG	0.009	0.285	0.265	0.245	0.225
AMGN	**0.064**	**1.436**	**1.416**	**1.396**	**1.376**
AMZN	0.080	2.818	2.798	2.778	2.758
BA	0.102	1.260	1.240	1.220	1.200
BIL	0.001	0.046	0.026	0.006	-0.014

Tick	Std	$P_{tc=0\%}$	$P_{tc=1\%}$	$P_{tc=2\%}$	$P_{tc=3\%}$
BLK	0.063	1.608	1.588	1.568	1.548
BMY	0.069	0.921	0.901	0.881	0.861
BRK-B	**0.043**	**1.085**	**1.065**	**1.045**	**1.025**
C	**0.094**	**0.816**	**0.796**	**0.776**	**0.756**
CL=F	0.151	-0.792	-0.812	-0.832	-0.852
CMCSA	0.059	1.536	1.516	1.496	1.476
COST	**0.047**	**1.758**	**1.738**	**1.718**	**1.698**
CSCO	**0.070**	**1.094**	**1.074**	**1.054**	**1.034**
CVX	0.068	0.161	0.141	0.121	0.101
DIS	**0.065**	**1.649**	**1.629**	**1.609**	**1.589**
GC=F	0.044	0.102	0.082	0.062	0.042
GOOGL	0.060	1.798	1.778	1.758	1.738
GS	0.078	0.997	0.977	0.957	0.937
HD	**0.054**	**1.984**	**1.964**	**1.944**	**1.924**
INTC	0.068	0.903	0.883	0.863	0.843
JNJ	0.044	1.128	1.108	1.088	1.068
JPM	0.071	1.477	1.457	1.437	1.417
KO	0.044	0.767	0.747	0.727	0.707
LLY	0.053	1.707	1.687	1.667	1.647
MA	**0.061**	**2.357**	**2.337**	**2.317**	**2.297**
MCD	0.044	1.034	1.014	0.994	0.974
MDT	0.052	1.300	1.280	1.260	1.240
META	0.100	2.173	2.153	2.133	2.113
MO	0.061	0.817	0.797	0.777	0.757
MRK	0.049	1.043	1.023	1.003	0.983
MSFT	**0.057**	**2.222**	**2.202**	**2.182**	**2.162**
NFLX	**0.133**	**3.450**	**3.430**	**3.410**	**3.390**
NG=F	**0.134**	**-0.030**	**-0.050**	**-0.070**	**-0.090**
NKE	**0.062**	**1.799**	**1.779**	**1.759**	**1.739**
NQ=F	0.049	1.592	1.572	1.552	1.532
NVDA	0.104	3.649	3.629	3.609	3.589
ORCL	**0.052**	**0.959**	**0.939**	**0.919**	**0.899**
PEP	0.040	1.071	1.051	1.031	1.011
PFE	0.052	0.921	0.901	0.881	0.861
PG	**0.041**	**1.069**	**1.049**	**1.029**	**1.009**
PYPL	0.073	1.901	1.881	1.861	1.841
QCOM	0.094	1.209	1.189	1.169	1.149
RTY=F	0.073	0.340	0.320	0.300	0.280
SBUX	0.061	1.647	1.627	1.607	1.587
SI=F	**0.084**	**-0.273**	**-0.293**	**-0.313**	**-0.333**
SPGI	0.065	2.135	2.115	2.095	2.075
T	**0.050**	**0.634**	**0.614**	**0.594**	**0.574**
TLT	0.035	0.487	0.467	0.447	0.427
TMO	0.054	2.216	2.196	2.176	2.156
TSLA	0.164	4.799	4.779	4.759	4.739
UNH	0.055	2.051	2.031	2.011	1.991

Tick	**Std**	$\mathbf{P_{tc=0\%}}$	$\mathbf{P_{tc=1\%}}$	$\mathbf{P_{tc=2\%}}$	$\mathbf{P_{tc=3\%}}$
V	**0.052**	**2.227**	**2.207**	**2.187**	**2.167**
VZ	**0.046**	**0.850**	**0.830**	**0.810**	**0.790**
WMT	0.050	1.065	1.045	1.025	1.005
XOM	**0.066**	**-0.359**	**-0.379**	**-0.399**	**-0.419**
YM=F	0.041	0.858	0.838	0.818	0.798

Table 3.6: Buy & Hold strategy stats - 'Std' is the standard deviation for every single security, 'P' stands for profit and 'tc' represents transaction costs - Mean-reverting securities are highlighted.

Tick	$\mathbf{prf_{tc=0\%}}$	$\mathbf{prf_{tc=1\%}}$	$\mathbf{prf_{tc=2\%}}$	$\mathbf{prf_{tc=3\%}}$	$\mathbf{sr_{tc=0\%}}$	$\mathbf{sr_{tc=1\%}}$	$\mathbf{sr_{tc=2\%}}$	$\mathbf{sr_{tc=3\%}}$
6A=F	-0.122	-0.129	-0.137	-0.144	-0.686	-0.707	-0.728	-0.749
6B=F	**-0.066**	**-0.074**	**-0.083**	**-0.092**	**-0.583**	**-0.608**	**-0.632**	**-0.657**
6E=F	**-0.044**	**-0.054**	**-0.064**	**-0.074**	**-0.589**	**-0.617**	**-0.646**	**-0.675**
6J=F	-0.108	-0.117	-0.126	-0.135	-0.724	-0.750	-0.776	-0.801
AAPL	0.262	0.260	0.257	0.255	0.770	0.762	0.754	0.746
ABBV	0.187	0.184	0.181	0.179	0.431	0.423	0.415	0.407
ABT	0.285	0.282	0.278	0.275	0.785	0.773	0.761	0.749
ADBE	0.444	0.441	0.438	0.435	1.348	1.337	1.326	1.316
AGG	0.285	0.265	0.245	0.225	-0.129	-0.197	-0.266	-0.334
AMGN	**0.209**	**0.206**	**0.203**	**0.200**	**0.556**	**0.546**	**0.536**	**0.526**
AMZN	0.330	0.328	0.326	0.323	1.003	0.995	0.987	0.979
BA	0.116	0.114	0.112	0.110	0.295	0.289	0.282	0.276
BIL	0.471	0.267	0.063	-0.141	-9.692	-10.393	-11.093	-11.794
BLK	0.237	0.234	0.231	0.228	0.651	0.640	0.630	0.620
BMY	0.124	0.121	0.119	0.116	0.277	0.268	0.258	0.249
BRK-B	**0.235**	**0.230**	**0.226**	**0.222**	**0.566**	**0.551**	**0.536**	**0.521**
C	**0.081**	**0.079**	**0.077**	**0.075**	**0.169**	**0.162**	**0.155**	**0.148**
CL=F	-0.059	-0.060	-0.062	-0.063	-0.236	-0.241	-0.245	-0.249
CMCSA	0.244	0.241	0.238	0.235	0.662	0.651	0.640	0.629
COST	**0.350**	**0.346**	**0.342**	**0.338**	**0.981**	**0.967**	**0.954**	**0.940**
CSCO	**0.146**	**0.144**	**0.141**	**0.138**	**0.354**	**0.345**	**0.336**	**0.327**
CVX	0.022	0.019	0.016	0.014	-0.076	-0.086	-0.095	-0.104
DIS	**0.235**	**0.233**	**0.230**	**0.227**	**0.650**	**0.640**	**0.630**	**0.620**
GC=F	0.026	0.021	0.016	0.011	-0.161	-0.175	-0.190	-0.204
GOOGL	0.280	0.277	0.274	0.270	0.788	0.777	0.766	0.756
GS	0.120	0.117	0.115	0.112	0.277	0.269	0.261	0.253
HD	**0.343**	**0.339**	**0.336**	**0.332**	**0.984**	**0.972**	**0.960**	**0.949**
INTC	0.125	0.122	0.119	0.117	0.275	0.266	0.256	0.247
JNJ	0.238	0.234	0.230	0.225	0.583	0.569	0.554	0.540
JPM	0.195	0.193	0.190	0.187	0.523	0.514	0.505	0.496
KO	0.163	0.159	0.155	0.150	0.324	0.310	0.295	0.280
LLY	0.300	0.297	0.293	0.290	0.836	0.824	0.812	0.800
MA	**0.362**	**0.359**	**0.356**	**0.353**	**1.072**	**1.061**	**1.050**	**1.040**
MCD	0.217	0.213	0.209	0.205	0.513	0.499	0.484	0.470

Tick	$\mathbf{prf_{tc=0\%}}$	$\mathbf{prf_{tc=1\%}}$	$\mathbf{prf_{tc=2\%}}$	$\mathbf{prf_{tc=3\%}}$	$\mathbf{sr_{tc=0\%}}$	$\mathbf{sr_{tc=1\%}}$	$\mathbf{sr_{tc=2\%}}$	$\mathbf{sr_{tc=3\%}}$
MDT	0.232	0.228	0.225	0.221	0.598	0.586	0.574	0.561
META	0.213	0.211	0.209	0.207	0.594	0.587	0.581	0.574
MO	0.126	0.123	0.120	0.117	0.261	0.251	0.240	0.230
MRK	0.199	0.195	0.191	0.187	0.470	0.457	0.444	0.431
MSFT	**0.366**	**0.363**	**0.359**	**0.356**	**1.074**	**1.062**	**1.051**	**1.040**
NFLX	**0.243**	**0.241**	**0.240**	**0.239**	**0.755**	**0.750**	**0.745**	**0.741**
NG=F	**-0.002**	**-0.004**	**-0.006**	**-0.008**	**-0.084**	**-0.089**	**-0.094**	**-0.099**
NKE	**0.273**	**0.270**	**0.267**	**0.264**	**0.769**	**0.759**	**0.748**	**0.738**
NQ=F	0.366	0.361	0.357	0.352	0.832	0.819	0.806	0.793
NVDA	0.329	0.327	0.325	0.323	1.029	1.022	1.016	1.010
ORCL	**0.173**	**0.170**	**0.166**	**0.163**	**0.395**	**0.382**	**0.370**	**0.358**
PEP	0.251	0.246	0.242	0.237	0.602	0.586	0.570	0.554
PFE	0.165	0.161	0.158	0.154	0.367	0.355	0.343	0.330
PG	**0.246**	**0.242**	**0.237**	**0.232**	**0.590**	**0.574**	**0.558**	**0.542**
PYPL	0.404	0.400	0.396	0.392	0.689	0.680	0.672	0.663
QCOM	0.120	0.118	0.116	0.114	0.301	0.295	0.288	0.281
RTY=F	0.141	0.132	0.124	0.116	0.008	-0.001	-0.010	-0.019
SBUX	0.254	0.251	0.248	0.245	0.700	0.690	0.679	0.668
SI=F	**-0.036**	**-0.039**	**-0.042**	**-0.044**	**-0.227**	**-0.235**	**-0.243**	**-0.250**
SPGI	0.305	0.302	0.299	0.296	0.889	0.879	0.869	0.859
T	**0.119**	**0.115**	**0.111**	**0.107**	**0.200**	**0.187**	**0.174**	**0.161**
TLT	0.131	0.125	0.120	0.114	0.151	0.132	0.114	0.096
TMO	0.382	0.379	0.375	0.372	1.121	1.109	1.097	1.086
TSLA	0.274	0.273	0.272	0.271	0.877	0.873	0.869	0.865
UNH	0.351	0.347	0.344	0.341	1.015	1.003	0.991	0.979
V	**0.401**	**0.397**	**0.393**	**0.390**	**1.176**	**1.163**	**1.151**	**1.138**
VZ	**0.171**	**0.167**	**0.163**	**0.159**	**0.364**	**0.350**	**0.336**	**0.323**
WMT	0.198	0.194	0.190	0.187	0.473	0.460	0.448	0.435
XOM	**-0.051**	**-0.053**	**-0.056**	**-0.059**	**-0.330**	**-0.340**	**-0.350**	**-0.359**
YM=F	0.238	0.232	0.227	0.221	0.423	0.407	0.391	0.375

Table 3.7: Buy & Hold strategy stats - 'prf' is the profit factor, 'sr' is the Sharpe ratio - Mean-reverting securities are highlighted.

3.5 Application of trading strategy

Firstly, the time span in which we tested the discussed strategy is the In-Sample period, from 01/01/2012 to 31/12/2020. The strategy relies on the use of a moving average to build upper and lower barriers that act as triggers for trading decisions. This system is very similar to a Bollinger bands system (Bollinger (2009)), in which starting from the twenty-period moving average, he added and subtracted two times the standard deviation to obtain two moving bands. For each security i, the moving average on time t is calculated as follows:

$$MA_{it} = \frac{1}{\theta_i} \sum_{l=1}^{\theta_i} P_{it-l},$$

where P_{it-l} represents the price at time $t-l$, and θ_i is the moving average period for the $i-th$ security. This value, along with other two parameters, is optimized to maximize the profit factor. To construct the upper and lower barriers, the rolling standard deviation of the return series is required, which is calculated using ω_i and the adjustment parameter λ_i. The rolling standard deviation for the security i, at time t is defined as:

$$\sigma_{it} = \sqrt{\frac{\sum_{t=1}^{\omega_i} (r_{it} - \overline{r_i})^2}{\omega_i}},$$

where r_{it} represents the return at time t, and $\overline{r_i}$ is the average return over the same period. As shown in equations 2.2 and 2.3, the parameters θ_i and ω_i are chosen to maximize the profit factor, which is the target statistic for this optimization. The optimization process, as seen in Carcano, Falbo, and Stefani (2005), involves executing strategy applying all possible combinations of these parameters, for each security i. This includes the adjustment parameter λ, which defines the width of the barriers and can assume values between zero and five. The applicable moving average periods are set at three months, six months, twelve months, eighteen months (one year and a half), and twenty-four months (two years). The rolling standard deviation is calculated over periods ranging from three to twenty-four months. So in total, 660 parameter combinations are tested for each security For each combination of parameters, the strategy determines entry and exit points for long and short positions. The strategy works as follow: a long position is opened when the price crosses above the lower barrier, which requires two conditions to be met: $p_{it-2} > LOW_{it-2}$ and $p_{it-1} < LOW_{it-1}$. Here, p_{it-2} and p_{it-1} represent the prices at times $t-2$ and $t-1$, respectively, while LOW_{it} is the lower barrier, defined as $MA_{it} - \lambda_i \sigma_{it}$. Conversely, a short position is opened when the price crosses below the upper barrier, requiring $p_{it-2} < UPP_{it-2}$ and $p_{it-1} > UPP_{it-1}$, where UPP_{it} is the upper barrier, defined as $MA_{it} + \lambda_i \sigma_{it}$.

While applying the strategy, the analysis does not focus on profits, as investment decisions are influenced by the investor's utility curve. For instance, a portfolio manager might prefer investing in a less risky security, even if it results in lower profits. This is the reason why we have chose a different performance indicator to maximize. However to evaluate the strategy we consider: profit, calculated as the cumulative sum of the monthly returns generated by strategy; standard deviation calculated on monthly strategy returns; the profit factor, which is the ratio between the average monthly return and the standard deviation of the strategy return and finally the Sharpe ratio, calculated as ratio between the yearly extra-profit and yearly standard deviation of the strategy. The extra-profit is the difference between the strategy yearly return and the risk-free return. During the optimization, only the parameter combination that maximizes the profit factor is selected for each security. As a result, each security has 660 combinations to evaluate, but only one is ultimately chosen. table 3.8 reports the parameters that maximize the profit factor in the second column, including, the moving average period, the adjustment parameter, and the rolling standard deviation period in this order. First results indicate that fifty-seven out of sixty-four securities generate profits when the strategy is applied, without considering transaction costs. This outcome is noteworthy because the optimization process was not designed to maximize profit directly. Considering instead the extreme scenario in terms of costs (3%), we have twenty-eight profitable names. Additionally, thirty-one securities experience a reduction in their standard deviation after applying the strategy, in particular twenty-five of stocks and Exchange Traded Funds, all the commodities futures, three currency futures out of four and half of the index futures.

A comparison with the Buy and Hold strategy for the SPY index reveals that four mean-reverting securities have a Sharpe ratio higher than the index: Costco (COST), Mastercard (MA), Microsoft (MSFT), and Visa (V), representing the 19.05% of the group. Except for Mastercard, these securities also display a higher profit factor (14.28% of the mean-reverting group). Among the non-mean-reverting securities, eight (18.60%) exhibit both a profit factor and a Sharpe ratio that surpass the SPY's statistics. However, when considering a transaction cost of 3% per trade, the number of non-mean-reverting securities beating the index drops to six (13.95%), while the results for the mean-reverting securities remain unchanged, with Costco, Microsoft, and Visa continuing to outperform the market. So stocks in the mean-reverting slightly outperforms the other group absorbing high transaction costs when these are applied. There is a double interpretation, basing on the index we chose. Considering the Sharpe ratio, the mean-reverting group returns a higher value, while the profit factor is higher when transaction costs are applied. However the difference between the two groups in both situations is small.

Further analysis compares the mean-reverting strategy to the Buy and Hold strategy on individual securities. Four out of twenty-one mean-reverting securities outperform the index in terms of both profit factor and Sharpe ratio, representing 19.05% of the mean-reverting group. These securities are the British Pound future (6B=F), the Silver future (SI=F), Microsoft (MSFT), and Exxon (XOM). Microsoft is the only security capable of absorbing transaction costs while still outperforming the index. Among the forty-three non-mean-reverting securities, ten achieve a higher profit factor than the index, while twelve surpass the index in Sharpe ratio, representing 23.26% and 27.9%, respectively. When applying high transaction costs (3%), five securities from the non mean-reverting group still maintain a higher profit factor, and seven retain a higher Sharpe ratio than the index, while Microsoft (MSFT) is the only mean-reverting stock to beat own Holding strategy. These findings suggest that one mean-reverting security out of five performs better when the proposed strategy is applied, although the non mean-reverting group seems to perform better. Tables 3.8 and 3.9 summarize the results for all securities, including their standard deviation, profit, profit factor, and Sharpe ratio, under various transaction cost scenarios.

Tick	**Par**	**N**	**Std**	$\mathbf{P_{tc=0\%}}$	$\mathbf{P_{tc=1\%}}$	$\mathbf{P_{tc=2\%}}$	$\mathbf{P_{tc=3\%}}$
6A=F	(18, 5, 7)	4	0.029	-0.188	-0.268	-0.348	-0.428
6B=F	**(12, 3, 16)**	**12**	**0.026**	**-0.042**	**-0.282**	**-0.522**	**-0.762**
6E=F	**(3, 2, 22)**	**6**	**0.019**	**-0.087**	**-0.207**	**-0.327**	**-0.447**
6J=F	(12, 1, 9)	2	0.023	-0.009	-0.049	-0.089	-0.129
AAPL	(12, 0, 9)	11	0.076	1.893	1.673	1.453	1.233
ABBV	(12, 0, 3)	12	0.079	0.541	0.301	0.061	-0.179

Tick	**Par**	**N**	**Std**	$\mathbf{P_{tc=0\%}}$	$\mathbf{P_{tc=1\%}}$	$\mathbf{P_{tc=2\%}}$	$\mathbf{P_{tc=3\%}}$
ABT	(6, 0, 17)	10	0.051	0.871	0.671	0.471	0.271
ADBE	(18, 0, 3)	2	0.060	2.392	2.352	2.312	2.272
AGG	(24, 0, 4)	6	0.010	0.202	0.082	-0.038	-0.158
AMGN	**(24, 0, 5)**	**8**	**0.068**	**0.337**	**0.177**	**0.017**	**-0.143**
AMZN	(24, 0, 14)	2	0.076	2.218	2.178	2.138	2.098
BA	(24, 0, 8)	11	0.110	0.967	0.747	0.527	0.307
BIL	(24, 0, 3)	5	0.001	0.047	-0.053	-0.153	-0.253
BLK	(6, 0, 4)	24	0.064	0.342	-0.138	-0.618	-1.098
BMY	(18, 0, 3)	8	0.071	0.115	-0.045	-0.205	-0.365
BRK-B	**(6, 0, 3)**	**22**	**0.044**	**0.561**	**0.121**	**-0.319**	**-0.759**
C	**(3, 0, 4)**	**37**	**0.091**	**0.000**	**-0.740**	**-1.480**	**-2.220**
CL=F	(3, 0, 23)	25	0.148	1.087	0.587	0.087	-0.413
CMCSA	(18, 0, 3)	12	0.061	0.486	0.246	0.006	-0.234
COST	**(24, 0, 4)**	**2**	**0.047**	**1.359**	**1.319**	**1.279**	**1.239**
CSCO	**(18, 0, 6)**	**10**	**0.064**	**0.471**	**0.271**	**0.071**	**-0.129**
CVX	(24, 0, 17)	6	0.069	0.309	0.189	0.069	-0.051
DIS	**(12, 0, 3)**	**12**	**0.068**	**0.605**	**0.365**	**0.125**	**-0.115**
GC=F	(3, 0, 3)	28	0.043	0.727	0.167	-0.393	-0.953
GOOGL	(24, 0, 22)	6	0.055	0.479	0.359	0.239	0.119
GS	(6, 0, 4)	22	0.076	0.579	0.139	-0.301	-0.741
HD	**(24, 0, 4)**	**4**	**0.056**	**1.023**	**0.943**	**0.863**	**0.783**
INTC	(3, 0, 4)	37	0.067	0.329	-0.411	-1.151	-1.891
JNJ	(24, 0, 23)	6	0.041	0.323	0.203	0.083	-0.037
JPM	(24, 1, 14)	8	0.064	0.168	0.008	-0.152	-0.312
KO	(6, 1, 23)	18	0.040	0.249	-0.111	-0.471	-0.831
LLY	(3, 0, 3)	32	0.054	1.235	0.595	-0.045	-0.685
MA	**(24, 0, 7)**	**4**	**0.061**	**1.466**	**1.386**	**1.306**	**1.226**
MCD	(24, 0, 21)	6	0.045	0.605	0.485	0.365	0.245
MDT	(6, 0, 9)	20	0.052	0.457	0.057	-0.343	-0.743
META	(12, 0, 3)	12	0.077	1.204	0.964	0.724	0.484
MO	(24, 0, 3)	7	0.063	0.566	0.426	0.286	0.146
MRK	(12, 0, 3)	16	0.049	-0.052	-0.372	-0.692	-1.012
MSFT	**(24, 0, 21)**	**2**	**0.048**	**1.736**	**1.696**	**1.656**	**1.616**
NFLX	**(24, 0, 13)**	**4**	**0.102**	**2.083**	**2.003**	**1.923**	**1.843**
NG=F	**(3, 1, 3)**	**34**	**0.131**	**-0.558**	**-1.238**	**-1.918**	**-2.598**
NKE	**(18, 0, 3)**	**12**	**0.062**	**1.007**	**0.767**	**0.527**	**0.287**
NQ=F	(24, 0, 18)	2	0.046	1.121	1.081	1.041	1.001
NVDA	(12, 4, 4)	4	0.105	3.761	3.681	3.601	3.521
ORCL	**(18, 5, 3)**	**14**	**0.051**	**0.106**	**-0.174**	**-0.454**	**-0.734**
PEP	(24, 0, 3)	4	0.041	0.644	0.564	0.484	0.404
PFE	(24, 5, 6)	14	0.054	-0.557	-0.837	-1.117	-1.397
PG	**(18, 0, 23)**	**5**	**0.036**	**0.715**	**0.615**	**0.515**	**0.415**
PYPL	(12, 0, 5)	4	0.075	1.525	1.445	1.365	1.285
QCOM	(3, 0, 17)	28	0.092	0.949	0.389	-0.171	-0.731
RTY=F	(24, 0, 10)	2	0.076	0.251	0.211	0.171	0.131
SBUX	(6, 0, 6)	18	0.055	1.428	1.068	0.708	0.348

Tick	Par	N	Std	$P_{tc=0\%}$	$P_{tc=1\%}$	$P_{tc=2\%}$	$P_{tc=3\%}$
SI=F	**(12, 0, 6)**	**13**	**0.077**	**0.381**	**0.121**	**-0.139**	**-0.399**
SPGI	(24, 0, 5)	4	0.061	1.257	1.177	1.097	1.017
T	**(3, 0, 22)**	**33**	**0.047**	**0.277**	**-0.383**	**-1.043**	**-1.703**
TLT	(3, 0, 23)	29	0.030	0.414	-0.166	-0.746	-1.326
TMO	(24, 0, 22)	2	0.050	1.355	1.315	1.275	1.235
TSLA	(6, 0, 6)	24	0.167	2.915	2.435	1.955	1.475
UNH	(12, 0, 4)	10	0.055	1.346	1.146	0.946	0.746
V	**(24, 0, 5)**	**2**	**0.051**	**1.509**	**1.469**	**1.429**	**1.389**
VZ	**(24, 1, 24)**	**9**	**0.042**	**0.179**	**-0.001**	**-0.181**	**-0.361**
WMT	(18, 0, 24)	3	0.046	0.834	0.774	0.714	0.654
XOM	**(6, 4, 9)**	**22**	**0.066**	**0.141**	**-0.299**	**-0.739**	**-1.179**
YM=F	(24, 0, 19)	4	0.039	0.525	0.445	0.365	0.285

Table 3.8: In-Sample strategy results - 'Par' represent parameters combination maximizing profit factor, 'N' is operations number, 'Std' is the strategy standard deviation, 'P' is the profit and 'tc' stands for transaction costs - Mean-reverting securities are highlighted.

Tick	$prf_{tc=0\%}$	$prf_{tc=1\%}$	$prf_{tc=2\%}$	$prf_{tc=3\%}$	$sr_{tc=0\%}$	$sr_{tc=1\%}$	$sr_{tc=2\%}$	$sr_{tc=3\%}$
6A=F	-0.090	-0.129	-0.167	-0.205	-0.675	-0.808	-0.941	-1.073
6B=F	**-0.021**	**-0.139**	**-0.258**	**-0.376**	**-0.475**	**-0.886**	**-1.296**	**-1.706**
6E=F	**-0.052**	**-0.125**	**-0.198**	**-0.270**	**-0.733**	**-0.984**	**-1.236**	**-1.487**
6J=F	-0.005	-0.028	-0.050	-0.073	-0.479	-0.557	-0.635	-0.713
AAPL	0.257	0.227	0.197	0.167	0.753	0.649	0.546	0.443
ABBV	0.081	0.045	0.009	-0.027	0.151	0.026	-0.099	-0.223
ABT	0.164	0.127	0.089	0.051	0.368	0.237	0.107	-0.024
ADBE	0.438	0.431	0.424	0.416	1.346	1.320	1.295	1.270
AGG	0.250	0.102	-0.046	-0.195	-0.221	-0.736	-1.250	-1.764
AMGN	**0.058**	**0.031**	**0.003**	**-0.025**	**0.050**	**-0.046**	**-0.143**	**-0.239**
AMZN	0.345	0.339	0.333	0.326	1.058	1.037	1.015	0.994
BA	0.104	0.080	0.057	0.033	0.265	0.183	0.102	0.020
BIL	0.567	-0.627	-1.821	-3.015	-8.548	-12.685	-16.821	-20.958
BLK	0.052	-0.021	-0.093	-0.166	0.018	-0.233	-0.485	-0.736
BMY	0.018	-0.007	-0.032	-0.057	-0.084	-0.170	-0.256	-0.342
BRK-B	**0.122**	**0.026**	**-0.070**	**-0.166**	**0.191**	**-0.142**	**-0.475**	**-0.808**
C	**0.000**	**-0.077**	**-0.154**	**-0.231**	**-0.114**	**-0.381**	**-0.648**	**-0.915**
CL=F	0.083	0.045	0.007	-0.032	0.219	0.086	-0.047	-0.180
CMCSA	0.087	0.044	0.001	-0.042	0.133	-0.016	-0.166	-0.315
COST	**0.342**	**0.332**	**0.322**	**0.312**	**0.964**	**0.929**	**0.894**	**0.859**
CSCO	**0.081**	**0.046**	**0.012**	**-0.022**	**0.118**	**-0.000**	**-0.119**	**-0.237**
CVX	0.053	0.032	0.012	-0.009	0.032	-0.038	-0.109	-0.180
DIS	**0.092**	**0.055**	**0.019**	**-0.017**	**0.165**	**0.039**	**-0.087**	**-0.213**
GC=F	0.194	0.045	-0.105	-0.255	0.429	-0.089	-0.607	-1.125
GOOGL	0.102	0.077	0.051	0.025	0.167	0.078	-0.011	-0.100
GS	0.074	0.018	-0.038	-0.094	0.120	-0.074	-0.269	-0.463
HD	**0.213**	**0.197**	**0.180**	**0.163**	**0.555**	**0.497**	**0.440**	**0.382**

Tick	$\mathbf{prf_{tc=0\%}}$	$\mathbf{prf_{tc=1\%}}$	$\mathbf{prf_{tc=2\%}}$	$\mathbf{prf_{tc=3\%}}$	$\mathbf{sr_{tc=0\%}}$	$\mathbf{sr_{tc=1\%}}$	$\mathbf{sr_{tc=2\%}}$	$\mathbf{sr_{tc=3\%}}$
INTC	0.046	-0.058	-0.161	-0.265	0.006	-0.353	-0.712	-1.071
JNJ	0.092	0.058	0.024	-0.010	0.068	-0.050	-0.169	-0.287
JPM	0.031	0.001	-0.028	-0.057	-0.055	-0.157	-0.258	-0.360
KO	0.060	-0.027	-0.114	-0.202	-0.050	-0.353	-0.656	-0.958
LLY	0.215	0.103	-0.008	-0.119	0.553	0.168	-0.218	-0.603
MA	**0.284**	**0.269**	**0.253**	**0.238**	**0.813**	**0.760**	**0.706**	**0.652**
MCD	0.159	0.128	0.096	0.064	0.320	0.211	0.101	-0.009
MDT	0.086	0.011	-0.064	-0.139	0.097	-0.163	-0.423	-0.683
META	0.170	0.136	0.102	0.068	0.454	0.337	0.219	0.102
MO	0.106	0.080	0.054	0.027	0.202	0.111	0.020	-0.070
MRK	-0.011	-0.078	-0.145	-0.212	-0.248	-0.481	-0.713	-0.945
MSFT	**0.428**	**0.418**	**0.408**	**0.398**	**1.265**	**1.231**	**1.197**	**1.163**
NFLX	**0.240**	**0.231**	**0.222**	**0.213**	**0.731**	**0.699**	**0.667**	**0.635**
NG=F	**-0.048**	**-0.107**	**-0.166**	**-0.225**	**-0.246**	**-0.450**	**-0.654**	**-0.858**
NKE	**0.179**	**0.137**	**0.094**	**0.051**	**0.454**	**0.306**	**0.157**	**0.009**
NQ=F	0.361	0.348	0.335	0.323	1.028	0.983	0.938	0.894
NVDA	0.369	0.361	0.354	0.346	1.181	1.154	1.126	1.099
ORCL	**0.023**	**-0.037**	**-0.097**	**-0.157**	**-0.123**	**-0.330**	**-0.537**	**-0.744**
PEP	0.183	0.160	0.138	0.115	0.384	0.305	0.226	0.147
PFE	-0.121	-0.182	-0.244	-0.305	-0.613	-0.824	-1.035	-1.247
PG	**0.220**	**0.190**	**0.159**	**0.128**	**0.473**	**0.366**	**0.260**	**0.153**
PYPL	0.378	0.358	0.338	0.318	1.170	1.102	1.033	0.964
QCOM	0.098	0.040	-0.018	-0.075	0.225	0.026	-0.174	-0.373
RTY=F	0.302	0.253	0.205	0.157	0.907	0.741	0.574	0.408
SBUX	0.251	0.188	0.125	0.061	0.683	0.463	0.244	0.025
SI=F	**0.063**	**0.020**	**-0.023**	**-0.066**	**0.083**	**-0.066**	**-0.215**	**-0.364**
SPGI	0.242	0.227	0.211	0.196	0.669	0.616	0.563	0.509
T	**0.056**	**-0.077**	**-0.210**	**-0.343**	**-0.028**	**-0.489**	**-0.949**	**-1.409**
TLT	0.128	-0.051	-0.231	-0.410	0.104	-0.518	-1.139	-1.761
TMO	0.320	0.311	0.301	0.292	0.902	0.869	0.836	0.803
TSLA	0.169	0.141	0.113	0.086	0.524	0.427	0.331	0.235
UNH	0.250	0.213	0.176	0.139	0.680	0.551	0.422	0.293
V	**0.346**	**0.337**	**0.328**	**0.318**	**0.996**	**0.965**	**0.933**	**0.901**
VZ	**0.051**	**-0.000**	**-0.051**	**-0.102**	**-0.074**	**-0.250**	**-0.427**	**-0.603**
WMT	0.200	0.185	0.171	0.157	0.466	0.416	0.367	0.317
XOM	**0.021**	**-0.044**	**-0.109**	**-0.174**	**-0.085**	**-0.310**	**-0.535**	**-0.760**
YM=F	0.200	0.170	0.139	0.109	0.429	0.323	0.217	0.111

Table 3.9: In-Sample strategy results - 'prf' is the profit factor, 'sr' is the Sharpe ratio and 'tc' stands for transaction costs - Mean-reverting securities are highlighted.

3.6 Comparison with a momentum strategy

Recovering previous results from the In-Sample period, we can compare our strategy with one of the most debated portfolio management strategies: the momentum Investing. As cited in section 2.1, Jegadeesh and Titman (2001) applied their strategy using three, six and twelve months as time window, showing presence of a momentum effect on American stock markets. Following this method, our momentum algorithm is based on the quarter, q. It produces a Buy order when in then previous quarter, $q-1$ the security i grew, while it short-sells when in then previous quarter, $q-1$ the security i lost value. For every security we calculated profit, as the cumulative sum of the returns generated, strategy standard deviation, the profit factor, which is the ratio between the average monthly return and the standard deviation of the strategy return and finally the Sharpe ratio, calculated as ratio between the yearly extra-profit and yearly standard deviation of the strategy. The extra-profit is the difference between the strategy yearly return and the risk-free return, equal to 3.588%. First interesting result comparing the two strategies, is that every security, except Exxon, shows a lower standard deviation when the mean-reverting strategy is applied. Considering the profitability, for the mean-reverting group, we have eleven securities out of twenty-one (52.38% of the mean-reverting group) with a higher profit factor, and twelve out of twenty-one (57.14%) with a higher Sharpe ratio. On the other group twenty-two out of forty-three (51.16% of the non mean-reverting group) have a better profit factor, while twenty-three (53.48%) have a better Sharpe ratio. Considering now transaction costs at 3%, we can observe that these have a huge impact on momentum strategy, because for both indicators (profit factor and Sharpe ratio), mean-reverting strategy beats the momentum one on nineteen mean-reverting securities out of twenty-one. Furthermore when the discussed strategy is applied, thirty-seven non mean-reverting securities out of forty-three perform better. In table 3.10 we reported, for every ticker, the number of operations, the strategy standard deviation and the profit, including transaction costs. In the table 3.11 we reported the profit factor and the Sharpe ratio considering costs.

Tick	**N**	**Std**	$\mathbf{P_{tc=0\%}}$	$\mathbf{P_{tc=1\%}}$	$\mathbf{P_{tc=2\%}}$	$\mathbf{P_{tc=3\%}}$
6A=F	16	0.089	0.087	-0.233	-0.553	-0.873
6B=F	**16**	**0.074**	**0.185**	**-0.135**	**-0.455**	**-0.775**
6E=F	**16**	**0.068**	**0.249**	**-0.071**	**-0.391**	**-0.711**
6J=F	26	0.095	-0.431	-0.951	-1.471	-1.991
AAPL	13	0.299	1.648	1.388	1.128	0.868
ABBV	17	0.238	-0.381	-0.721	-1.061	-1.401
ABT	15	0.178	0.268	-0.032	-0.332	-0.632
ADBE	13	0.209	1.650	1.390	1.130	0.870
AGG	14	0.034	0.165	-0.115	-0.395	-0.675
AMGN	**21**	**0.223**	**-0.873**	**-1.293**	**-1.713**	**-2.133**
AMZN	15	0.304	1.245	0.945	0.645	0.345
BA	18	0.346	1.501	1.141	0.781	0.421
BIL	9	0.004	0.050	-0.130	-0.310	-0.490
BLK	16	0.221	0.701	0.381	0.061	-0.259
BMY	20	0.214	-0.781	-1.181	-1.581	-1.981
BRK-B	**16**	**0.138**	**0.486**	**0.166**	**-0.154**	**-0.474**

Tick	N	Std	$P_{tc=0\%}$	$P_{tc=1\%}$	$P_{tc=2\%}$	$P_{tc=3\%}$
C	**20**	**0.257**	**-0.108**	**-0.508**	**-0.908**	**-1.308**
CL=F	22	0.355	0.639	0.199	-0.241	-0.681
CMCSA	14	0.164	0.525	0.245	-0.035	-0.315
COST	**16**	**0.187**	**0.179**	**-0.141**	**-0.461**	**-0.781**
CSCO	**22**	**0.201**	**-0.447**	**-0.887**	**-1.327**	**-1.767**
CVX	24	0.177	-1.175	-1.655	-2.135	-2.615
DIS	**18**	**0.205**	**0.133**	**-0.227**	**-0.587**	**-0.947**
GC=F	18	0.166	0.053	-0.307	-0.667	-1.027
GOOGL	24	0.186	-0.361	-0.841	-1.321	-1.801
GS	22	0.237	-0.410	-0.850	-1.290	-1.730
HD	**19**	**0.188**	**0.267**	**-0.113**	**-0.493**	**-0.873**
INTC	23	0.252	-0.643	-1.103	-1.563	-2.023
JNJ	15	0.139	0.067	-0.233	-0.533	-0.833
JPM	20	0.217	0.292	-0.108	-0.508	-0.908
KO	18	0.125	0.106	-0.254	-0.614	-0.974
LLY	15	0.199	0.215	-0.085	-0.385	-0.685
MA	**13**	**0.187**	**1.052**	**0.792**	**0.532**	**0.272**
MCD	24	0.151	-0.467	-0.947	-1.427	-1.907
MDT	14	0.152	0.516	0.236	-0.044	-0.324
META	14	0.296	0.645	0.365	0.085	-0.195
MO	19	0.183	-0.555	-0.935	-1.315	-1.695
MRK	17	0.159	-0.060	-0.400	-0.740	-1.080
MSFT	**17**	**0.204**	**0.763**	**0.423**	**0.083**	**-0.257**
NFLX	**15**	**0.495**	**2.303**	**2.003**	**1.703**	**1.403**
NG=F	**14**	**0.322**	**0.979**	**0.699**	**0.419**	**0.139**
NKE	**20**	**0.215**	**0.789**	**0.389**	**-0.011**	**-0.411**
NQ=F	18	0.165	0.053	-0.307	-0.667	-1.027
NVDA	10	0.405	2.371	2.171	1.971	1.771
ORCL	**22**	**0.151**	**-0.135**	**-0.575**	**-1.015**	**-1.455**
PEP	22	0.127	-0.205	-0.645	-1.085	-1.525
PFE	20	0.160	-0.259	-0.659	-1.059	-1.459
PG	**16**	**0.158**	**0.610**	**0.290**	**-0.030**	**-0.350**
PYPL	12	0.355	-0.275	-0.515	-0.755	-0.995
QCOM	20	0.260	0.414	0.014	-0.386	-0.786
RTY=F	10	0.226	-0.403	-0.603	-0.803	-1.003
SBUX	16	0.204	0.807	0.487	0.167	-0.153
SI=F	**18**	**0.261**	**0.170**	**-0.190**	**-0.550**	**-0.910**
SPGI	15	0.216	0.611	0.311	0.011	-0.289
SPY	14	0.112	0.462	0.182	-0.098	-0.378
T	**21**	**0.132**	**-0.008**	**-0.428**	**-0.848**	**-1.268**
TLT	17	0.114	0.155	-0.185	-0.525	-0.865
TMO	16	0.205	0.418	0.098	-0.222	-0.542
TSLA	20	0.713	2.629	2.229	1.829	1.429
UNH	16	0.186	0.617	0.297	-0.023	-0.343
V	**17**	**0.178**	**0.273**	**-0.067**	**-0.407**	**-0.747**
VZ	**20**	**0.118**	**0.206**	**-0.194**	**-0.594**	**-0.994**

Tick	N	Std	$P_{tc=0\%}$	$P_{tc=1\%}$	$P_{tc=2\%}$	$P_{tc=3\%}$
WMT	20	0.185	-0.293	-0.693	-1.093	-1.493
XOM	**21**	**0.153**	**-0.684**	**-1.104**	**-1.524**	**-1.944**
YM=F	20	0.120	-0.200	-0.600	-1.000	-1.400

Table 3.10: Momentum strategy results - 'Par' represent parameters combination maximizing profit factor, 'N' is operations number, 'Std' is the strategy standard deviation, 'P' is the profit and 'tc' stands for transaction costs - Mean-reverting securities are highlighted.

Tick	$prf_{tc=0\%}$	$prf_{tc=1\%}$	$prf_{tc=2\%}$	$prf_{tc=3\%}$	$sr_{tc=0\%}$	$sr_{tc=1\%}$	$sr_{tc=2\%}$	$sr_{tc=3\%}$
6A=F	0.094	-0.251	-0.596	-0.941	-0.076	-1.271	-2.467	-3.662
6B=F	**0.240**	**-0.176**	**-0.592**	**-1.009**	**0.347**	**-1.095**	**-2.537**	**-3.980**
6E=F	**0.354**	**-0.101**	**-0.555**	**-1.010**	**0.696**	**-0.878**	**-2.452**	**-4.027**
6J=F	-0.434	-0.959	-1.483	-2.007	-1.881	-3.697	-5.513	-7.329
AAPL	0.531	0.447	0.363	0.279	1.718	1.428	1.138	0.848
ABBV	-0.154	-0.291	-0.429	-0.566	-0.683	-1.159	-1.635	-2.112
ABT	0.145	-0.018	-0.180	-0.343	0.300	-0.263	-0.826	-1.389
ADBE	0.758	0.639	0.519	0.400	2.455	2.041	1.627	1.214
AGG	0.468	-0.328	-1.123	-1.918	0.561	-2.194	-4.950	-7.705
AMGN	**-0.376**	**-0.557**	**-0.738**	**-0.919**	**-1.464**	**-2.091**	**-2.718**	**-3.344**
AMZN	0.394	0.299	0.204	0.109	1.248	0.919	0.589	0.260
BA	0.418	0.318	0.217	0.117	1.343	0.996	0.649	0.302
BIL	1.237	-3.206	-7.649	-12.092	-4.918	-20.310	-35.701	-51.092
BLK	0.306	0.166	0.027	-0.113	0.896	0.413	-0.071	-0.554
BMY	-0.351	-0.531	-0.711	-0.891	-1.385	-2.009	-2.632	-3.255
BRK-B	**0.339**	**0.116**	**-0.107**	**-0.330**	**0.914**	**0.142**	**-0.631**	**-1.403**
C	**-0.040**	**-0.190**	**-0.339**	**-0.489**	**-0.279**	**-0.796**	**-1.314**	**-1.832**
CL=F	0.174	0.054	-0.065	-0.185	0.500	0.086	-0.327	-0.741
CMCSA	0.308	0.144	-0.020	-0.185	0.850	0.280	-0.289	-0.858
COST	**0.092**	**-0.073**	**-0.237**	**-0.402**	**0.127**	**-0.443**	**-1.013**	**-1.584**
CSCO	**-0.214**	**-0.424**	**-0.634**	**-0.845**	**-0.918**	**-1.647**	**-2.376**	**-3.105**
CVX	-0.639	-0.899	-1.160	-1.421	-2.415	-3.319	-4.222	-5.126
DIS	**0.062**	**-0.107**	**-0.276**	**-0.446**	**0.041**	**-0.546**	**-1.133**	**-1.719**
GC=F	0.031	-0.178	-0.386	-0.594	-0.110	-0.831	-1.552	-2.274
GOOGL	-0.187	-0.435	-0.684	-0.933	-0.840	-1.701	-2.563	-3.424
GS	-0.166	-0.345	-0.524	-0.702	-0.728	-1.347	-1.966	-2.584
HD	**0.136**	**-0.058**	**-0.252**	**-0.446**	**0.281**	**-0.391**	**-1.063**	**-1.736**
INTC	-0.245	-0.421	-0.596	-0.772	-0.992	-1.600	-2.208	-2.816
JNJ	0.047	-0.161	-0.369	-0.577	-0.097	-0.817	-1.537	-2.258
JPM	0.130	-0.048	-0.226	-0.403	0.284	-0.332	-0.948	-1.563
KO	0.082	-0.195	-0.472	-0.749	-0.003	-0.963	-1.922	-2.882
LLY	0.104	-0.041	-0.186	-0.330	0.180	-0.322	-0.823	-1.325
MA	**0.540**	**0.407**	**0.273**	**0.140**	**1.680**	**1.217**	**0.755**	**0.292**
MCD	-0.298	-0.604	-0.911	-1.217	-1.271	-2.331	-3.392	-4.453
MDT	0.327	0.150	-0.028	-0.205	0.897	0.282	-0.333	-0.948

Tick	$\mathbf{prf_{tc=0\%}}$	$\mathbf{prf_{tc=1\%}}$	$\mathbf{prf_{tc=2\%}}$	$\mathbf{prf_{tc=3\%}}$	$\mathbf{sr_{tc=0\%}}$	$\mathbf{sr_{tc=1\%}}$	$\mathbf{sr_{tc=2\%}}$	$\mathbf{sr_{tc=3\%}}$
META	0.210	0.119	0.028	-0.064	0.605	0.289	-0.026	-0.341
MO	-0.291	-0.491	-0.690	-0.889	-1.204	-1.895	-2.586	-3.277
MRK	-0.037	-0.243	-0.449	-0.656	-0.353	-1.068	-1.783	-2.497
MSFT	**0.360**	**0.200**	**0.039**	**-0.121**	**1.071**	**0.515**	**-0.041**	**-0.597**
NFLX	**0.448**	**0.389**	**0.331**	**0.273**	**1.479**	**1.277**	**1.075**	**0.872**
NG=F	**0.293**	**0.209**	**0.125**	**0.041**	**0.903**	**0.612**	**0.322**	**0.032**
NKE	**0.353**	**0.174**	**-0.005**	**-0.184**	**1.056**	**0.436**	**-0.183**	**-0.803**
NQ=F	0.031	-0.179	-0.389	-0.598	-0.111	-0.837	-1.563	-2.289
NVDA	0.563	0.515	0.468	0.421	1.862	1.697	1.533	1.368
ORCL	**-0.086**	**-0.367**	**-0.649**	**-0.930**	**-0.537**	**-1.511**	**-2.485**	**-3.459**
PEP	-0.155	-0.488	-0.821	-1.154	-0.819	-1.973	-3.126	-4.279
PFE	-0.155	-0.396	-0.636	-0.876	-0.762	-1.594	-2.425	-3.257
PG	**0.372**	**0.177**	**-0.018**	**-0.213**	**1.061**	**0.385**	**-0.290**	**-0.966**
PYPL	-0.075	-0.140	-0.205	-0.270	-0.359	-0.585	-0.810	-1.035
QCOM	0.153	0.005	-0.143	-0.291	0.393	-0.120	-0.632	-1.144
RTY=F	-0.171	-0.256	-0.341	-0.426	-0.752	-1.046	-1.341	-1.635
SBUX	0.380	0.229	0.079	-0.072	1.142	0.619	0.097	-0.426
SI=F	**0.063**	**-0.070**	**-0.203**	**-0.335**	**0.079**	**-0.380**	**-0.840**	**-1.299**
SPGI	0.272	0.139	0.005	-0.129	0.777	0.314	-0.149	-0.612
SPY	0.398	0.157	-0.084	-0.325	1.057	0.223	-0.612	-1.446
T	**-0.006**	**-0.313**	**-0.620**	**-0.927**	**-0.292**	**-1.356**	**-2.420**	**-3.484**
TLT	0.131	-0.157	-0.445	-0.733	0.139	-0.860	-1.858	-2.857
TMO	0.196	0.046	-0.104	-0.254	0.504	-0.015	-0.535	-1.055
TSLA	0.355	0.301	0.247	0.193	1.178	0.991	0.804	0.617
UNH	0.320	0.154	-0.012	-0.178	0.914	0.339	-0.236	-0.811
V	**0.147**	**-0.036**	**-0.220**	**-0.403**	**0.308**	**-0.327**	**-0.962**	**-1.597**
VZ	**0.168**	**-0.158**	**-0.484**	**-0.811**	**0.278**	**-0.852**	**-1.982**	**-3.112**
WMT	-0.152	-0.360	-0.568	-0.776	-0.721	-1.441	-2.161	-2.881
XOM	**-0.429**	**-0.693**	**-0.956**	**-1.220**	**-1.721**	**-2.634**	**-3.547**	**-4.460**
YM=F	-0.160	-0.480	-0.800	-1.120	-0.851	-1.960	-3.068	-4.177

Table 3.11: Momentum strategy results - 'prf' is the profit factor, 'sr' is the Sharpe ratio and 'tc' stands for transaction costs - Mean-reverting securities are highlighted.

3.7 Validation

3.7.1 Out-Of-Sample validation

In the Out-Of-Sample period, parameters from previous procedure are kept to apply the strategy on period from 01/01/2021 to 31/12/2023. We recovered parameters values for the second column of table 3.8. Main goal of this kind of validation is to test the strategy performance on an unknown dataset with parameters that maximize profit factor. In the In-Sample period we indeed trained the algorithm producing parameters that fitted dataset for every security, so it is necessary to test those value on a different period. For every security we extracted profit, calculated as sum of monthly returns. standard deviation, profit factor and Sharpe ratio. On all sixty-four securities, twenty-five (39.6%) are profitable in the testing period, divided in ten mean-reverting (47.62% of the mean-reverting group) and fifteen non mean-reverting (34.88% of the non mean-reverting group). In table 3.12 we can observe that all four currency futures produce profit in this period, while in the training period, as we can see in table 3.8, they got a loss. In particular, among the mean-reverting securities that passed this validation there are seven stocks and three futures: the British pound (6B=F), the Euro (6E=F), the Natural gas (NG=F) futures. The Out-Of-Sample validation shows that half of the securities that passed the Augmented Dickey-Fuller test or the regression, successfully passed the validation period when strategy is applied with no transaction costs. This last is an interesting result, especially compared to the non mean-reverting group. If we considered the extra-return on risk-free asset instead of the profit, we should observe the Sharpe ratio sign: in this case there is some change, because without transaction costs applied, profitable names become seventeen, and seven of these belong to the mean-reverting group (33.33% on the twenty-one mean-reverting securities), while the ten non mean-reverting represent the 23.25% of the non mean-reverting set. So the first group continues to outperform the other one, even if the comparison is made in terms of extra-profit. Considering 3% transaction costs, profitable securities become nine (better than the In-Sample back testing), while eight securities perform better than the risk-free asset even with so high costs. In tables 3.12 and 3.13 we reported, for every ticker, number of operations, the strategy standard deviation, and results in terms of profit in the first one, while in the second one we reported profit factor and Sharpe ratio, considering all transaction costs scenarios. In table 3.13 we can observe missing data due to the fact that our strategy opened a single position, no closing it. So we just reported opening cost when calculated profit.

Tick	**N**	**Std**	$\mathbf{P_{tc=0\%}}$	$\mathbf{P_{tc=1\%}}$	$\mathbf{P_{tc=2\%}}$	$\mathbf{P_{tc=3\%}}$
6A=F	2	0.017	0.020	-0.010	-0.030	-0.100
6B=F	**2**	**0.012**	**0.004**	**-0.036**	**-0.076**	**-0.116**
6E=F	**2**	**0.011**	**0.017**	**-0.0203**	**-0.063**	**-0.103**
6J=F	2	0.029	0.053	0.013	-0.0207	-0.067
AAPL	3	0.065	0.107	0.047	-0.0103	-0.073
ABBV	8	0.059	-0.423	-0.583	-0.743	-0.903
ABT	9	0.041	-0.382	-0.562	-0.742	-0.922
ADBE	3	0.119	0.244	0.184	0.124	0.064
AGG	3	0.019	-0.010	-0.080	-0.140	-0.200
AMGN	**4**	**0.051**	**0.071**	**-0.009**	**-0.089**	**-0.169**
AMZN	1	-	-	-0.010	-0.020	-0.030
BA	2	0.086	0.087	0.047	0.007	-0.033
BIL	2	0.002	0.036	-0.004	-0.044	-0.084
BLK	11	0.091	-0.553	-0.773	-0.993	-1.213
BMY	3	0.051	0.242	0.182	0.122	0.062
BRK-B	**10**	**0.058**	**-0.004**	**-0.204**	**-0.404**	**-0.604**
C	**17**	**0.080**	**-0.940**	**-1.280**	**-1.620**	**-1.960**
CL=F	5	0.051	-0.319	-0.419	-0.519	-0.619
CMCSA	3	0.083	-0.071	-0.131	-0.191	-0.251

Tick	N	Std	$P_{tc=0\%}$	$P_{tc=1\%}$	$P_{tc=2\%}$	$P_{tc=3\%}$
COST	**2**	**0.038**	**0.290**	**0.250**	**0.210**	**0.170**
CSCO	**5**	**0.050**	**-0.185**	**-0.285**	**-0.385**	**-0.485**
CVX	1	-	-	-0.010	-0.020	-0.030
DIS	**6**	**0.104**	**0.139**	**0.019**	**-0.101**	**-0.221**
GC=F	12	0.039	0.036	-0.204	-0.444	-0.684
GOOGL	1	-	-	-0.010	-0.020	-0.030
GS	10	0.078	-0.306	-0.506	-0.706	-0.906
HD	**5**	**0.065**	**-0.0107**	**-0.117**	**-0.217**	**-0.317**
INTC	13	0.098	-0.059	-0.319	-0.579	-0.839
JNJ	1	-	-	-0.010	-0.020	-0.030
JPM	1	-	-	-0.010	-0.020	-0.030
KO	6	0.021	-0.120	-0.240	-0.360	-0.480
LLY	10	0.083	0.442	0.242	0.042	-0.158
MA	**2**	**0.037**	**0.084**	**0.044**	**0.004**	**-0.036**
MCD	1	-	-	-0.010	-0.020	-0.030
MDT	9	0.062	-0.202	-0.382	-0.562	-0.742
META	3	0.132	0.892	0.832	0.772	0.712
MO	3	0.042	-0.038	-0.098	-0.158	-0.218
MRK	4	0.056	0.277	0.197	0.117	0.037
MSFT	**1**	**-**	**-**	**-0.010**	**-0.020**	**-0.030**
NFLX	**1**	**-**	**-**	**-0.020**	**-0.080**	**-0.120**
NG=F	**11**	**0.221**	**0.054**	**-0.166**	**-0.386**	**-0.606**
NKE	**5**	**0.097**	**-0.437**	**-0.537**	**-0.637**	**-0.737**
NQ=F	1	-	-	-0.010	-0.020	-0.030
NVDA	4	0.170	0.492	0.412	0.332	0.252
ORCL	**3**	**0.100**	**0.269**	**0.209**	**0.149**	**0.089**
PEP	4	0.035	-0.087	-0.167	-0.247	-0.327
PFE	2	0.028	0.243	0.203	0.163	0.123
PG	**1**	**-**	**-**	**-0.010**	**-0.020**	**-0.030**
PYPL	2	0.107	0.359	0.319	0.279	0.239
QCOM	10	0.074	-0.411	-0.611	-0.811	-1.011
RTY=F	1	-	-	-0.010	-0.020	-0.030
SBUX	7	0.078	-0.004	-0.144	-0.284	-0.424
SI=F	**7**	**0.071**	**-0.704**	**-0.844**	**-0.984**	**-1.124**
SPGI	5	0.060	-0.263	-0.363	-0.463	-0.563
T	**5**	**0.039**	**0.167**	**0.067**	**-0.033**	**-0.133**
TLT	6	0.032	0.054	-0.066	-0.186	-0.306
TMO	1	-	-	-0.010	-0.020	-0.030
TSLA	12	0.169	-1.029	-1.269	-1.509	-1.749
UNH	6	0.043	-0.100	-0.220	-0.340	-0.460
V	**2**	**0.041**	**0.118**	**0.078**	**0.038**	**-0.002**
VZ	**1**	**-**	**-**	**-0.010**	**-0.020**	**-0.030**
WMT	1	-	-	-0.010	-0.020	-0.030
XOM	**9**	**0.072**	**-0.404**	**-0.584**	**-0.764**	**-0.944**
YM=F	1	-	-	-0.010	-0.020	-0.030

Tick	**N**	**Std**	$\mathbf{P_{tc=0\%}}$	$\mathbf{P_{tc=1\%}}$	$\mathbf{P_{tc=2\%}}$	$\mathbf{P_{tc=3\%}}$

Table 3.12: Out-Of-Sample strategy results - 'Par' represent parameters combination maximizing profit factor, 'N' is operations number, 'Std' is the strategy standard deviation, 'P' is the profit and 'tc' stands for transaction costs - Mean-reverting securities are highlighted.

Tick	$\mathbf{prf_{tc=0\%}}$	$\mathbf{prf_{tc=1\%}}$	$\mathbf{prf_{tc=2\%}}$	$\mathbf{prf_{tc=3\%}}$	$\mathbf{sr_{tc=0\%}}$	$\mathbf{sr_{tc=1\%}}$	$\mathbf{sr_{tc=2\%}}$	$\mathbf{sr_{tc=3\%}}$
6A=F	0.076	-0.079	-0.234	-0.389	-0.339	-0.875	-1.411	-1.947
6B=F	**0.016**	**-0.143**	**-0.301**	**-0.460**	**-0.807**	**-1.356**	**-1.905**	**-2.454**
6E=F	**0.050**	**-0.070**	**-0.190**	**-0.310**	**-0.760**	**-1.175**	**-1.590**	**-2.006**
6J=F	0.087	0.021	-0.045	-0.111	-0.057	-0.285	-0.514	-0.743
AAPL	0.065	0.029	-0.008	-0.045	0.068	-0.060	-0.187	-0.315
ABBV	-0.289	-0.398	-0.507	-0.616	-1.177	-1.555	-1.933	-2.311
ABT	-0.298	-0.439	-0.579	-0.720	-1.283	-1.770	-2.257	-2.744
ADBE	0.108	0.082	0.055	0.028	0.287	0.195	0.103	0.011
AGG	-0.081	-0.327	-0.572	-0.817	-0.832	-1.682	-2.531	-3.381
AMGN	**0.107**	**-0.013**	**-0.133**	**-0.253**	**0.168**	**-0.248**	**-0.663**	**-1.079**
AMZN	-	-	-	-	-	-	-	-
BA	0.078	0.042	0.007	-0.029	0.150	0.026	-0.098	-0.222
BIL	1.323	-0.128	-1.579	-3.030	-0.302	-5.328	-10.354	-15.379
BLK	-0.196	-0.273	-0.351	-0.429	-0.791	-1.061	-1.330	-1.600
BMY	0.251	0.189	0.127	0.064	0.666	0.450	0.235	0.019
BRK-B	**-0.002**	**-0.113**	**-0.224**	**-0.336**	**-0.186**	**-0.571**	**-0.956**	**-1.341**
C	**-0.347**	**-0.472**	**-0.598**	**-0.723**	**-1.331**	**-1.766**	**-2.201**	**-2.636**
CL=F	-0.207	-0.272	-0.337	-0.402	-0.920	-1.145	-1.370	-1.595
CMCSA	-0.045	-0.083	-0.121	-0.159	-0.281	-0.412	-0.544	-0.676
COST	**0.589**	**0.508**	**0.426**	**0.345**	**1.766**	**1.485**	**1.203**	**0.922**
CSCO	**-0.197**	**-0.303**	**-0.409**	**-0.516**	**-0.891**	**-1.259**	**-1.627**	**-1.995**
CVX	-	-	-	-	-	-	-	-
DIS	**0.054**	**0.007**	**-0.039**	**-0.085**	**0.086**	**-0.075**	**-0.235**	**-0.396**
GC=F	0.031	-0.176	-0.383	-0.590	-0.160	-0.878	-1.595	-2.312
GOOGL	-	-	-	-	-	-	-	-
GS	-0.126	-0.208	-0.290	-0.372	-0.567	-0.852	-1.137	-1.422
HD	**-0.020**	**-0.139**	**-0.258**	**-0.377**	**-0.229**	**-0.641**	**-1.053**	**-1.465**
INTC	-0.018	-0.096	-0.174	-0.253	-0.167	-0.439	-0.710	-0.981
JNJ	-	-	-	-	-	-	-	-
JPM	-	-	-	-	-	-	-	-
KO	-0.183	-0.365	-0.548	-0.731	-1.122	-1.755	-2.388	-3.020
LLY	0.156	0.085	0.015	-0.056	0.416	0.172	-0.073	-0.317
MA	**0.176**	**0.092**	**0.008**	**-0.076**	**0.327**	**0.037**	**-0.254**	**-0.544**
MCD	-	-	-	-	-	-	-	-
MDT	-0.105	-0.198	-0.291	-0.385	-0.529	-0.853	-1.176	-1.499
META	0.270	0.252	0.234	0.216	0.858	0.795	0.732	0.669
MO	-0.069	-0.178	-0.287	-0.396	-0.483	-0.861	-1.238	-1.616
MRK	0.199	0.142	0.084	0.027	0.504	0.305	0.105	-0.094

Tick	$prf_{tc=0\%}$	$prf_{tc=1\%}$	$prf_{tc=2\%}$	$prf_{tc=3\%}$	$sr_{tc=0\%}$	$sr_{tc=1\%}$	$sr_{tc=2\%}$	$sr_{tc=3\%}$
MSFT	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
NFLX	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
NG=F	**0.008**	**-0.025**	**-0.058**	**-0.091**	**-0.019**	**-0.134**	**-0.249**	**-0.364**
NKE	**-0.237**	**-0.292**	**-0.346**	**-0.400**	**-0.930**	**-1.118**	**-1.306**	**-1.494**
NQ=F	-	-	-	-	-	-	-	-
NVDA	0.116	0.097	0.078	0.059	0.341	0.275	0.210	0.145
ORCL	**0.142**	**0.111**	**0.079**	**0.047**	**0.389**	**0.279**	**0.169**	**0.059**
PEP	-0.194	-0.372	-0.549	-0.727	-0.970	-1.586	-2.202	-2.818
PFE	0.674	0.563	0.452	0.341	1.962	1.577	1.192	0.808
PG	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
PYPL	0.135	0.120	0.105	0.090	0.369	0.317	0.265	0.213
QCOM	-0.163	-0.242	-0.322	-0.401	-0.705	-0.980	-1.255	-1.530
RTY=F	-	-	-	-	-	-	-	-
SBUX	-0.002	-0.060	-0.118	-0.176	-0.139	-0.340	-0.541	-0.742
SI=F	**-0.473**	**-0.567**	**-0.661**	**-0.755**	**-1.785**	**-2.111**	**-2.437**	**-2.763**
SPGI	-0.336	-0.464	-0.591	-0.719	-1.335	-1.778	-2.220	-2.663
T	**0.125**	**0.050**	**-0.025**	**-0.099**	**0.170**	**-0.090**	**-0.349**	**-0.608**
TLT	0.050	-0.061	-0.173	-0.285	-0.153	-0.540	-0.927	-1.313
TMO	-	-	-	-	-	-	-	-
TSLA	-0.197	-0.243	-0.289	-0.335	-0.743	-0.902	-1.061	-1.220
UNH	-0.094	-0.207	-0.320	-0.433	-0.570	-0.961	-1.352	-1.743
V	**0.219**	**0.145**	**0.071**	**-0.004**	**0.509**	**0.252**	**-0.005**	**-0.262**
VZ	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
WMT	-	-	-	-	-	-	-	-
XOM	**-0.180**	**-0.260**	**-0.340**	**-0.421**	**-0.767**	**-1.044**	**-1.322**	**-1.600**
YM=F	-	-	-	-	-	-	-	-

Table 3.13: Out-Of-Sample strategy results - 'prf' is the profit factor, 'sr' is the Sharpe ratio and 'tc' stands for transaction costs - Mean-reverting securities are highlighted.

3.7.2 Monte Carlo simulation

The second validation procedure is the Monte Carlo simulation, a widely used approach in financial literature for modeling an strategy validation. Trombetta (2020) describes the method as a versatile tool for stress-testing strategies under a range of simulated market scenarios. In particular, it can be useful to submit strategy to different market conditions, changing, for example transaction costs, stop loss or price observations, in order to identify strategy behavior. Similarly, Neaime (2015) applies Monte Carlo simulations to analyze mean-reverting behaviors in emerging MENA stock markets, showcasing its utility in evaluating the robustness of trading models. Following the foundational frameworks outlined in Raychaudhuri (2008) and Glasserman (2003) , which highlight the relevance of stochastic processes for simulating financial systems, we identified the Geometric Brownian Motion as the baseline model for price dynamics. As discussed in section 2.4.2 and based on the extension of the t-distribution proposed by Jegadeesh and Titman (1993), the log-returns distribution is modeled as a skewed t-Student. This approach accounts for asymmetry and heavy tails often observed in financial data. To accurately model the return distribution for each security, we employed the skewed t-distribution, described by equation 2.11. This equation depends on the parameters a and b, which define the shape of the distribution by controlling the weight of the tails and the impact of skewness. Our objective was to identify the degrees of freedom of the distribution by maximizing the likelihood function:

$$L(a, b; r_1, r_2, \ldots, r_n) = \prod_{i=1}^{n} f(r_i; a, b), \tag{3.2}$$

where r_i is the log-return series, a and b are parameters define the shape of the distribution. In particular, they control the weight of the tail and the influence of the skewness, respectively on the right side and on the left side of the distribution. n is number of observations. Since the equation 3.2 is the product of different probability density for each observation r_k, we worked with the log-likelihood function, maximizing:

$$\mathcal{L}(a, b; r_1, r_2, \ldots, r_n) = \sum_{i=1}^{n} \log f(r_i; a, b).$$

Expanding the equation 3.2 we obtain:

$$\begin{aligned}\mathcal{L}(a, b; r_1, r_2, \ldots, r_n) = \sum_{i=1}^{n} \Bigg[& - \log C_{a,b} \\ & + \left(a + \frac{1}{2}\right) \log \left(1 + r_i \cdot \sqrt{a + b + r_i^2}\right) \\ & + \left(b + \frac{1}{2}\right) \log \left(1 - r_i \cdot \sqrt{a + b + r_i^2}\right) \Bigg],\end{aligned} \tag{3.3}$$

where r_n is the $n - th$ log-return, $C_{a,b}$ represents a normalization constant that ensures the skewed t-distribution integrates to 1, thereby making it a valid probability density function. The normalization constant $C_{a,b}$ is defined as:

$$C_{a,b} = \frac{\Gamma(a+b)\sqrt{a+b}}{\Gamma(a)\Gamma(b)\sqrt{\pi}},$$

where Γ denotes the Gamma function, a continuous extension of the factorial function. $C_{a,b}$ is essential to properly scale the probability density function $f(x)$, ensuring that the total probability across the distribution's domain sums to 1. We can observe that degrees of freedom are not included into this equation, but Jones and Faddy suggested to re-parametrize a and b. Defining v as degrees of freedom and λ as the parameter to set skewness of distribution, Jones and Faddy defined $v = a + b$ and $\lambda = a - b$. Replacing these values in the equation 3.3, we can identify number of degrees of freedom, v by maximizing the function. The equation 3.4 describes instead the sample skewness for every security i, in according to Tsay (2013).

$$S_i = \frac{N}{(N)(N-2)} \sum_{l=1}^{N} \left(\frac{r_{il} - \bar{r}_i}{s_i}\right)^3, \tag{3.4}$$

where $N = 107$, since we have 108 price observations in nine years (the In-Sample life), r_{il} is the return observation on time l, $\bar{r_i}$ is the sample mean and s_i is the sample standard deviation, both for the $i-th$ security. The equation 3.4 is an alternative way to calculate Skewness, because we identified parameters v and λ, maximizing the equation 3.3. We established limits to the degrees of freedom, to avoid unrealistic values: the lower limit is three, while the upper one is fifty. Next step is to extract the drift and the standard deviation from the In-Sample period, in order to model the log-return distribution. It is necessary a preamble: we reported drift for every security in table 3.14, but in the model we did not consider the time trend, to avoid that the past performance could have influenced results. Conduct analysis on de-trended time series is a very spread analysis method: Poterba and Summers (1988) applied a de-trending procedure to isolate stationary components and conduct their analysis on stock prices. Jensen, Johansen, and Simonsen (2003) studied presence of skewness in profit and loss from different investment approaches, on de-trended time series. Although we did not consider drift thanks to the skewed-t Student, we keep distribution features in the simulation: for each Geometric Brownian Motion equation, the chosen distribution has the Skewness and degrees of freedom extracted in the In-Sample period, for each security. To identify the drift, μ we can follow Wooldridge (2016) and write the equation 2.10 for every security log-returns series i:

$$r_{it} = \alpha_{0i} + \mu_i t + \epsilon_{it},$$

where r_{it} is the log-return, α_{0i} is the intercept of the model and μ_i is the process drift. The residuals series $\hat{r_{it}}$ is defined, for every security i, as $\hat{r}_{it} = r_{it} - (\alpha_{0i} + \mu_i t)$. Once taken drift out, we can calculate the residuals standard deviation, equal to $\hat{\sigma_i} = std(\hat{r_{it}})$. Drifts μ_i and standard deviations σ_i for each security are on monthly basis and we reported them in table 3.14. Next step is to build, in Out-Of-Sample period, 1.000 different price paths for all securities, and then apply to them the strategy. Equation 2.11 adopted for every security i will be:

$$S_{it} = S_{it-1} e^{\sigma_i \sqrt{\Delta t} \cdot Z_{it}}. \tag{3.5}$$

This equation is repeated 1.000 times, and the starting point of every series, S_{0i}, is the price for every security, i, on the first month of testing set period: 01/01/2021. Every price on the generic time t is calculated with the equation 3.5,where Z_{iT} is a random number extracted from the skewed t-distribution, and σ_i is the residuals standard deviation. In total we have 1.000 different price paths, and on every single path we applied our strategy, with no transaction costs. So, since Trombetta (2020) suggests to use this type of simulation to let a specific parameter change, we adopted the price evolution as variable element, using a model as the Geometric Brownian Motion. At the end of the experiment, on time T (21/12/2023) we calculated, for every security i, an average on the 1.000 output profits, P_{iT} as:

$$P_{iT} = \frac{\sum_{n=1}^{1000} p_{inT}}{1.000},$$

where p_{inT} is the profit of the $n-th$ path on time T for security i. Profit is calculated, like for every application we made, as the sum of monthly returns realized by the proposed strategy. Then we counted how many times strategy produces a positive outcome, in percentage form. We also reported in table 3.14 the highest and the lowest outcome profit. About significant results: among the mean-reverting securities, we can observe that eight of them have a positive average profit (38.09%). They are Costco (COST), Walt Disney

(DIS), Mastercard (MA), Microsoft (MSFT), Nike (NKE), Procter & Gamble (PG), Visa (V) and Verizon (VZ). Considering the non mean-reverting group, the eleven securities with a positive average profit represent the 25.58% of their group.

Tick	**Sk**	**Df**	μ	**Std**	**P**	**max**	**min**	**W%**
6A=F	1.8423	10.1588	0.00020	0.030230	-0.00227	0.43700	-0.58859	48.9
6B=F	**-0.9234**	**9.9670**	**0.00006**	**0.02615**	**-0.00379**	**0.45564**	**-0.40232**	**48.2**
6E=F	**-0.6476**	**9.9352**	**0.00011**	**0.02214**	**-0.00166**	**0.28594**	**-0.27107**	**47.4**
6J=F	0.1000	10.0000	0.00021	0.02435	-0.00394	0.50098	-0.42854	48.3
AAPL	-2.3107	50.0000	0.00034	0.07936	-0.00545	1.19486	-1.36978	49.3
ABBV	-0.6894	10.0047	-0.00009	0.07839	-0.00256	1.35501	-1.39999	49.2
ABT	-0.7237	7.3331	0.00005	0.05327	-0.01468	0.69933	-0.85208	49.8
ADBE	-0.6275	50.0000	0.00009	0.05844	0.00363	0.83666	-0.839280	51.3
AGG	0.9734	10.0956	0.00004	0.00927	-0.00027	0.12326	-0.15290	49.3
AMGN	**-0.2550**	**10.0036**	**-0.00018**	**0.06397**	**-0.00601**	**0.75726**	**-1.03363**	**49.1**
AMZN	0.0549	16.7647	0.00009	0.07972	0.00363	0.92408	-0.97924	51.3
BA	-0.1277	3.0000	-0.00029	0.10151	-0.02049	1.60259	-1.67288	47.8
BIL	10.0000	10.4195	0.00001	0.00080	-0.00002	0.01012	-0.01298	48.1
BLK	-1.0824	10.0827	0.00000	0.06338	-0.01435	1.06539	-1.25399	48.7
BMY	0.1043	3.7234	-0.00018	0.06913	0.00070	1.59189	-2.01876	50.7
BRK-B	**0.2620**	**10.0464**	**-0.00011**	**0.04305**	**-0.01026**	**0.83300**	**-0.88240**	**48.0**
C	**-0.9031**	**3.7614**	**-0.00016**	**0.09357**	**-0.01608**	**2.13197**	**-11.21016**	**48.4**
CL=F	-0.7018	3.0000	0.00025	0.15105	0.00340	33.98696	-27.16349	47.7
CMCSA	-1.4013	6.6179	-0.00014	0.05862	0.01630	0.79266	-1.05628	51.0
COST	**-1.2863**	**50.0000**	**0.00004**	**0.04691**	**0.00640**	**0.58068**	**-0.56991**	**50.9**
CSCO	**-0.4154**	**10.0002**	**-0.00004**	**0.06986**	**-0.00428**	**0.97498**	**-1.16897**	**50.3**
CVX	-0.2946	3.6930	-0.00014	0.068280	-0.00070	1.27905	-1.13750	49.7
DIS	**0.4745**	**3.1541**	**-0.00010**	**0.06538**	**0.00638**	**2.97088**	**-2.30187**	**51.3**
GC=F	1.0006	50.0000	0.00042	0.04276	0.00253	0.84214	-0.80342	49.6
GOOGL	0.3254	10.0262	-0.00005	0.06003	-0.00205	0.81245	-0.91843	48.9
GS	-0.7203	5.5738	-0.00006	0.07795	-0.01154	1.68194	-1.65636	50.2
HD	**-0.9469**	**9.9782**	**-0.00016**	**0.05390**	**-0.00542**	**0.62192**	**-0.84081**	**48.8**
INTC	-1.7795	10.0176	0.00000	0.06755	0.00890	1.29497	-1.36713	50.3
JNJ	-0.7667	10.0175	-0.00008	0.04421	-0.00074	0.57699	-0.70743	49.3
JPM	-0.9725	3.9257	-0.00006	0.07068	0.00272	1.24010	-1.14856	50.2
KO	-5.3779	9.8692	-0.00000	0.04396	-0.01099	0.45855	-0.51000	48.2
LLY	-0.6729	10.0017	0.00003	0.05313	0.00385	1.18483	-1.09071	49.0
MA	**-0.8514**	**4.7159**	**-0.00002**	**0.06089**	**0.00580**	**0.79176**	**-1.18547**	**52.5**
MCD	-0.8311	9.8643	0.00012	0.04428	-0.00497	0.48076	-0.70949	49.0
MDT	-1.2149	50.0000	-0.00009	0.05235	-0.00933	0.74868	-1.01010	47.9
META	0.2980	4.1110	-0.00013	0.09991	-0.00939	3.46509	-4.02550	48.9
MO	-2.0242	50.0000	-0.00034	0.05967	0.00769	0.72701	-0.71275	51.0
MRK	-1.9753	50.0000	-0.00007	0.04902	-0.00319	0.75929	-0.87296	48.9
MSFT	**-0.4311**	**6.2137**	**0.00022**	**0.05634**	**0.00064**	**0.70324**	**-0.80474**	**50.4**
NFLX	**-0.2467**	**3.5094**	**-0.00010**	**0.13278**	**-0.01895**	**1.92114**	**-2.18361**	**48.3**
NG=F	**0.0877**	**3.0000**	**-0.00020**	**0.13419**	**-0.03618**	**6.24448**	**-6.40458**	**46.2**
NKE	**-1.5928**	**46.5904**	**0.00012**	**0.06145**	**0.00844**	**0.86335**	**-0.80942**	**52.2**
NQ=F	-1.4881	10.0870	0.00022	0.04857	-0.00169	0.61428	-0.74362	49.3

Tick	Sk	Df	μ	Std	P	max	min	W%
NVDA	-0.0268	3.6874	0.00039	0.10301	0.02251	3.31795	-2.19319	50.2
ORCL	**-0.7594**	**50.0000**	**0.00002**	**0.05167**	**0.00275**	**0.74028**	**-0.74227**	**50.2**
PEP	-0.7419	10.0469	-0.00002	0.03984	-0.00020	0.52161	-0.64124	49.5
PFE	0.6170	9.9072	-0.00013	0.05206	-0.00581	0.64570	-0.73683	48.2
PG	**-1.5284**	**50.0000**	**0.00007**	**0.04050**	**0.00569**	**0.50187**	**-0.49261**	**51.6**
PYPL	1.7621	50.0000	0.00061	0.07258	-0.00738	1.20722	-1.34962	50.1
QCOM	-0.4084	3.9873	0.00038	0.09348	-0.01366	1.84383	-1.69729	49.8
RTY=F	-1.0258	3.0000	0.00089	0.07279	-0.01676	2.66944	-4.41774	47.6
SBUX	-0.9577	9.9614	0.00001	0.06065	-0.01930	1.20794	-1.18822	47.5
SI=F	**1.6438**	**8.2971**	**0.00072**	**0.08189**	**-0.00143**	**1.65803**	**-1.64116**	**49.5**
SPGI	-1.4112	9.9653	-0.00001	0.06542	-0.00778	0.71550	-0.80632	48.7
T	**-1.2036**	**9.9267**	**-0.00018**	**0.04960**	**-0.00044**	**0.63314**	**-0.6645**	**49.4**
TLT	1.5177	9.9858	0.00009	0.03475	-0.00291	0.41074	-0.44241	49.5
TMO	-0.1696	10.0639	0.00002	0.05416	-0.00246	0.71520	-0.88342	49.3
TSLA	2.1081	22.4992	0.00041	0.16321	-0.01344	3.57509	-4.33069	49.1
UNH	-0.9594	10.0616	-0.00001	0.05462	-0.00099	0.94944	-0.98577	48.4
V	**-0.8380**	**50.0000**	**-0.00012**	**0.05181**	**0.00700**	**0.64259**	**-0.63077**	**51.0**
VZ	**-1.1457**	**50.0000**	**-0.00008**	**0.04636**	**0.00571**	**0.52873**	**-0.56839**	**50.8**
WMT	-0.4128	9.9995	0.00012	0.05022	-0.00827	0.56568	-0.74167	47.6
XOM	**-0.3670**	**3.3058**	**-0.00026**	**0.06568**	**-0.01532**	**1.79094**	**-1.45674**	**50.2**
YM=F	-0.5869	5.1116	0.00003	0.04054	-0.00501	0.61191	-0.63105	49.8

Table 3.14: Strategy application on 1.000 simulated price paths - 'Sk' is the log-returns skewness, 'Df' stands for degrees of freedom, 'μ' is the In-Sample drift, 'Std' is the In-Sample standard deviation, 'P' is the profit calculated with no transaction costs, 'max' is the highest profit produced by strategy, 'min' is the lowest profit produced by strategy, 'W%' is the percentage of profitable paths - Mean-reverting securities are highlighted.

3.8 Final remarks

This thesis has shown that a mean-reverting trading strategy, based on moving averages and dynamic bands, can serve as a valid alternative to traditional approaches such as Buy and Hold and momentum investing. Firstly, the discussed strategy is able to decrease the standard deviation for twenty-five of stocks and Exchange Traded Funds, for all the commodities futures, for three currency futures out of four and for half of the index futures. In the training period, results indicated that maximizing the profit factor, mean-reverting instruments systematically outperformed non-mean-reverting ones in terms of profitability, especially when transaction costs are applied. Furthermore, the real challenge emerged during the Out-Of-Sample validation and Monte Carlo simulation, where the difference in the robustness among the two groups of securities is wider in favor of the mean-reverting one. The comparison with the quarterly momentum strategy revealed that mean reversion can be a more effective approach, decreasing volatility of returns fo every security except for Exxon and outperform the momentum strategy for one of the four currency futures, two commodities futures, two index futures and thirty-one stock out of the fifty-three stocks and Exchange Traded Funds (more than the half). Moreover, the mean-reverting group slightly outperforms the other group in terms of both profit factor and Sharpe ratio. Transaction costs have a very high impact on momentum approach.

Conclusion

This chapter offered a way to identify mean-reverting securities and exploit this feature to build a trading strategy. Starting from using statistical tools as Augmented Dickey-Fuller test and Regression, we identify twenty-one securities to which apply our algorithm. To identify hyper-bought and hyper-sold areas we built two moving bands trough a price moving average and a rolling standard deviation calculated on returns. Optimizing parameters in order to get the highest profit factor. We compared then the strategy with the Buy and Hold approach and the momentum investing, looking at the profit factor, the Sharpe ratio and the strategies risk. The choice to use monthly data reduces noise in the series and allows to pay lower transaction costs, but obviously working with daily data could have returned a more long and robust series to work on. Then we validate the strategy in an Out-Of-Sample period and recurring to the Monte Carlo simulation. Since we created different price path, we were able to apply the strategy to each of them, calculating the profit, the average, the highest and the lowest ones. In the Monte Carlo simulation we extracted, trough the Geometric Brownian Motion, 1.000 path, hypothesizing the skewed t-distribution for returns and removing drift from the model. This because we wanted to simulate the price paths without considering the time trend in the In-Sample period. In the options pricing applications, number of simulation is far higher than that we used. Results reinforce the notion that active strategies based on mean-reversion can offer a competitive edge over simpler as well as generalized methods.

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