Sample Integer Program

Problem Statement: A media company has five spots (S_1, S_2, S_3, S_4 and S_5) that it wants to assign to three breaks (B_1, B_2 and B_3). The company knows the estimated impressions for each spot and break combination. How can the company find and assign a subset of spots to breaks so as to maximize the total no. of impressions while ensuring the following constraints?

1. A break can have no more than one spot.
2. Spots S_1 and S_2 belong to a prominent advertiser and so at least one of them must be picked.

Table 1: Estimated Impressions

Spot/Break	**B_1**	**B_2**	**B_3**
S_1	7	8	1
S_2	3	1	5
S_3	2	9	3
S_4	1	7	2
S_5	6	2	7

Let us define decision variables y_{ij} where $1 \leq i \leq 5$ and $1 \leq j \leq 3$ such that y_{ij} is one if spot i is assigned to break j, and is zero otherwise.

1. The objective function z is given by Equation 1.

$$\begin{aligned} \text{Max. } z = \quad & 7y_{11} + 8y_{12} + y_{13} \\ & + 3y_{21} + y_{22} + 5y_{23} \\ & + 2y_{31} + 9y_{32} + 3y_{33} \\ & + y_{41} + 7y_{42} + 2y_{43} \\ & + 6y_{51} + 2y_{52} + 7y_{53} \end{aligned} \tag{1}$$

2. The following constraints ensure that a spot can be assigned to no more than one break.

$$y_{11} + y_{12} + y_{13} \leq 1 \tag{2}$$

$$y_{21} + y_{22} + y_{23} \leq 1 \tag{3}$$

$$y_{31} + y_{32} + y_{33} \leq 1 \tag{4}$$

$$y_{41} + y_{42} + y_{43} \leq 1 \tag{5}$$

$$y_{51} + y_{52} + y_{53} \leq 1 \tag{6}$$

3. The following constraints ensure that a break can have no more than one spot.

$$y_{11} + y_{21} + y_{31} + y_{41} + y_{51} \leq 1 \quad (7)$$

$$y_{12} + y_{22} + y_{32} + y_{42} + y_{52} \leq 1 \quad (8)$$

$$y_{13} + y_{23} + y_{33} + y_{43} + y_{53} \leq 1 \quad (9)$$

4. The following constraints ensures that either S_1 or S_2 must be selected.

$$y_{11} + y_{12} + y_{13} + y_{21} + y_{22} + y_{23} \geq 1 \quad (10)$$

5. Binary variable constraints

$$y_{ij} \in \{0, 1\} \text{ for } 1 \leq i \leq 5 \text{ and } 1 \leq j \leq 3 \quad (11)$$