

Exam Solutions Shift-02

Session-01

27 JANUARY 2024

IIT JEE

Time- 03 Hrs

M.Marks : 300

Topic Covered

Physics : Full Syllabus

Chemistry : Full Syllabus

Maths : Full Syllabus

GENERAL INSTRUCTION

1. Immediately fill in the particulars on this page of the test booklet.
2. The test is of 3 hours duration.
3. The test booklet consists of 90 questions. The maximum marks are 300.
4. There are Three Sections in the question paper, Section I, II & III consisting of Section-I (Physics), Section-II (Chemistry), Section-III (Mathematics) and having 30 questions in each part in which first 20 questions are compulsory and are of Objective Type and Last 10 questions are integers type in which you have to attempt 5 questions only.
5. There is only one correct response for each question.
6. Each correct answer will give 4 marks while 1 Mark will be deducted.
7. No student is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
8. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.



Paper Solutions **Shift-02**

(Memory Based Solutions)



CHEMISTRY

1. For 1st order reaction, time required for 99.9% completion is :

- (1) $2t_{1/2}$ (2) $4t_{1/2}$ (3) $5t_{1/2}$ (4) $10t_{1/2}$

Ans. (4)

Sol.
$$\frac{t_{99.9\%}}{t_{1/2}} = \frac{\frac{1}{k} \ln \left(\frac{100}{100 - 99.9} \right)}{\frac{1}{k} \ln 2} = \frac{\ln(10^3)}{\ln 2} = \frac{3}{0.3} = 10$$

$$t_{99.9\%} = 10t_{1/2}$$

2. Number of non polar molecules among following are :

HF, H₂O, CO₂, NH₃, SO₂, H₂, CH₄, BF₃

Ans. (4)

Sol. CO₂, H₂, CH₄, BF₃

3. 3M NaOH solution is to be prepared using 84 g NaOH, then the volume of solution in litre is

$$\underline{\hspace{2cm}} \times 10^{-1}$$

Ans. (7)

Sol.
$$3 = \frac{84 / 40}{V_{\text{sol(L.)}}}$$

$$\therefore V_{\text{solution}} = 0.7 \text{ L}$$

4. Select **incorrect** match :

- (1) Haber process : Fe
(2) Polythene : Ziegler-Natta catalyst [$\text{Al}_2(\text{CH}_3)_6 + \text{TiCl}_4$]
(3) Wacker's process : PtCl₂
(4) Photography : AgBr

Ans. (3)

Sol. Wacker's process : PdCl₂



Paper Solutions Shift-02

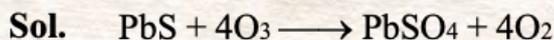
(Memory Based Solutions)



5. 1 mole PbS is oxidised by x mole O₃ liberating y mole O₂.

Determine (x + y).

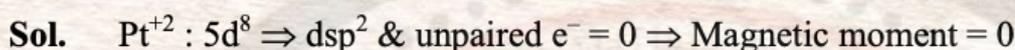
Ans. (8)



$$x = 4 ; y = 4$$

6. Spin only magnetic moment of [Pt(NH₃)₂Cl(CH₃NH₂)]Cl is :

Ans. (0)



7. S-1: Formation of Ce⁴⁺ is favoured by inert gas configuration.

S-2: Ce⁴⁺ acts as strong oxidising agent & converts to Ce³⁺.

Ans. Both S-1 & S-2 are correct.

8. Which of the following can't act as oxidising agent ?

- (1) MnO₄⁻ (2) N³⁻ (3) BrO₃⁻ (4) SO₄²⁻

Ans. (2)

Sol. In N³⁻, nitrogen is present in minimum O.N. & hence it cannot act as oxidising agent.

9. The quantity which changes with temperature is:

- (1) Molarity (2) Molality (3) Mole fraction (4) Mass %

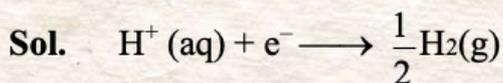
Ans. (1)

Sol. Quantities involving volume are temperature dependent.

10. Reduction potential of hydrogen electrode at pH = 3 is.....

$$\left(\frac{2.303RT}{F} = 0.059 \right)$$

Ans. (-0.177 volt)



$$\text{R.P.} = -\frac{0.059}{1} \log \left(\frac{1}{\text{H}^+} \right) = -0.059 \log(10^{-3})$$

$$= -0.059 \times 3 = -0.177 \text{ volt}$$



Paper Solutions **Shift-02**

(Memory Based Solutions)



11. Identify the species in which central atom is in d^2sp^3 hybridisation :

- (1) SF_6 (2) BrF_5 (3) $[PtCl_4]^{2-}$ (4) $[Co(NH_3)_6]^{3+}$

Ans. (4)

Sol. SF_6	:	sp^3d^2
BrF_5	:	sp^3d^2
$[PtCl_4]^{2-}$:	dsp^2
$[Co(NH_3)_6]^{3+}$:	d^2sp^3

12. $\Delta H^\circ = +77.2 \text{ kJ}$, $\Delta S^\circ = 122 \text{ J/mol-K}$, $T = 300 \text{ K}$, $\log K = ?$

Ans. (-7.07)

Sol. $\Delta G^\circ = -2.303RT \log k$

$$77.2 - \frac{300 \times 122}{1000} = \frac{-2.303 \times 8.314 \times 300 \log K}{1000}$$

$$\therefore \log K = -7.07$$

13. In group 16

Statement-I : Oxygen shows only -2 oxidation state.

Statement-II : On moving top to bottom, stability of $+4$ oxidation state decreases, whereas that of $+6$ oxidation state increases.

- (1) Both Statement I and Statement II are correct.
(2) Both Statement I and Statement II are incorrect.
(3) Statement I is correct but Statement II is incorrect.
(4) Statement I is incorrect but Statement II is correct.

Ans. (2)

Sol. Statement-I : Since electronegativity of oxygen is very high, it shows only negative oxidation state as -2 except in the case of OF_2 where its oxidation state is $+2$.

Statement-II : The stability of $+6$ oxidation state decreases down the group and stability of $+4$ oxidation state increases (inert pair effect).

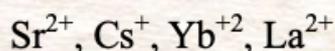


Paper Solutions Shift-02

(Memory Based Solutions)



14. How many of following has/have noble gas configuration ?



Ans. (2)

Sol. (Sr^{2+} , Cs^+)

15. Which of the following has d^{10} configuration ?

(1) Cr, Cd, Cu, Ag

(2) Cd, Cr, Ag, Zn

(3) Ag, Cr, Cu, Zn

(4) Cu, Cd, Zn, Ag

Ans. (4)

Sol. Cr : $[\text{Ar}] 3d^5 4s^1$

Cu : $[\text{Ar}] 3d^{10} 4s^1$

Ag : $[\text{Kr}] 4d^{10} 5s^1$

Zn : $[\text{Ar}] 3d^{10} 4s^2$

Cd : $[\text{Kr}] 4d^{10} 5s^2$

16. Which of the following is used to identify the phenolic group test?

(1) Carbylamine test

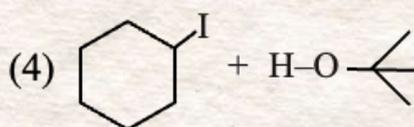
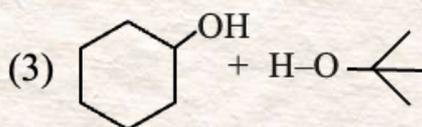
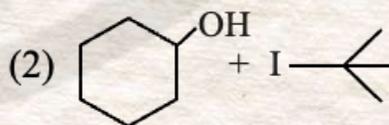
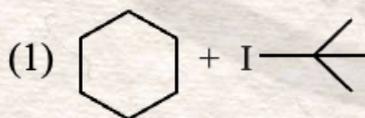
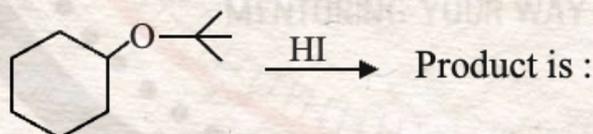
(2) Lucas test

(3) Tollen's test

(4) Phthalein dye test

Ans. (4)

17.



Ans. (2)

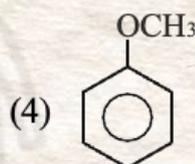
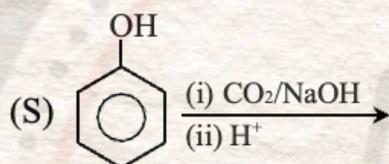
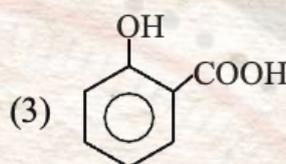
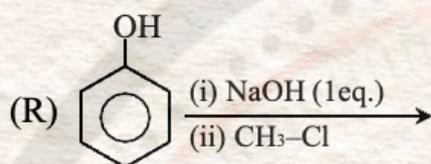
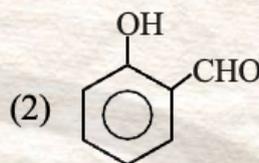
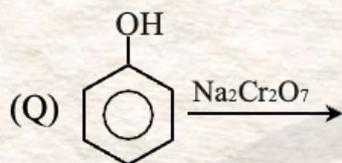
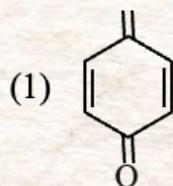
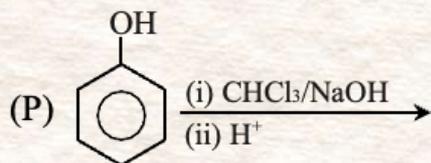


Paper Solutions Shift-02

(Memory Based Solutions)



18. Match the column



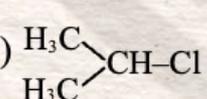
Ans. (P) – (2) ; (Q) – (1) ; (R) – (4) ; (S) – (3)

19. When egg is boiled then which of the following structure of protein remains intact?

- (1) Quaternary structure (2) Primary structure
(3) Secondary structure (4) Tertiary structure

Ans. (2)

20. Which of the following compound will not give S_N1 reaction?

- (1) $CH_2=CH-CH_2Cl$ (2) $Ph-CH_2-Cl$
(3)  (4) $CH_3-CH=CH-Cl$

Ans. (4)

21. The second homologue of monocarboxylic acid is

- (1) $HCOOH$ (2) CH_3COOH (3) CH_3CH_2COOH (4) $CH_3CH_2CH_2-COOH$

Ans.

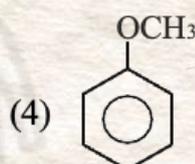
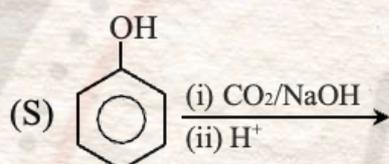
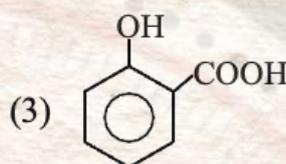
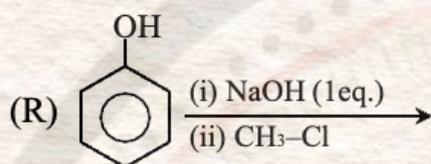
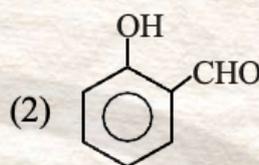
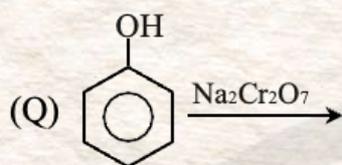
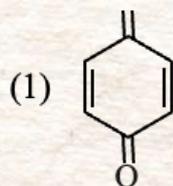
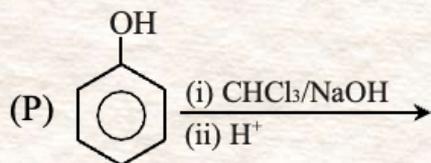


Paper Solutions Shift-02

(Memory Based Solutions)



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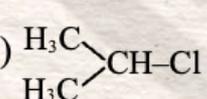
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Ans. (2)

20. Which of the following compound will not give S_N1 reaction?

- (1) $\text{CH}_2=\text{CH}-\text{CH}_2\text{Cl}$ (2) $\text{Ph}-\text{CH}_2-\text{Cl}$
(3)  (4) $\text{CH}_3-\text{CH}=\text{CH}-\text{Cl}$

Ans. (4)

21. The second homologue of monocarboxylic acid is

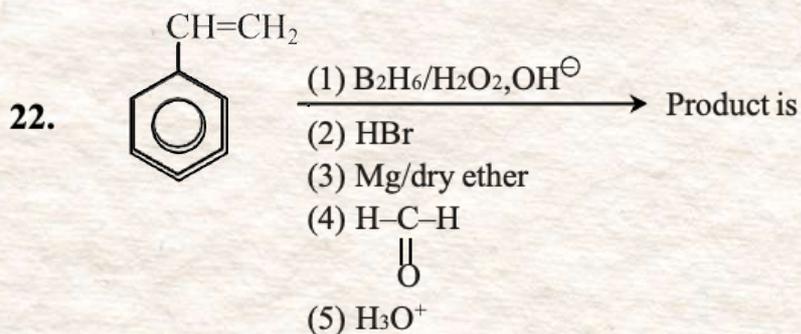
- (1) HCOOH (2) CH_3COOH (3) $\text{CH}_3\text{CH}_2\text{COOH}$ (4) $\text{CH}_3\text{CH}_2\text{CH}_2-\text{COOH}$

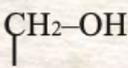
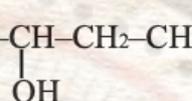
Ans.



Paper Solutions Shift-02

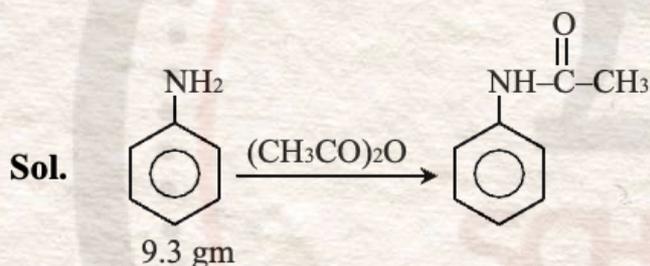
(Memory Based Solutions)



- (1)  (2) $\text{Ph}-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{OH}$
 (3) $\text{Ph}-\text{CH}_2-\text{CH}_2-\text{O}-\text{CH}_3$ (4) 

Ans. (1)

23. When 9.3 gm of aniline is reacted with acetic anhydride then mass of acetanilide formed is [X] gm. Report your answer as 10X.



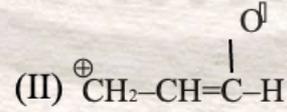
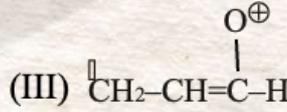
$$\text{Mole of Aniline} = \frac{9.3}{93} = 0.1$$

$$\text{Mole of acetanilide} = 0.1$$

$$\text{Mass of acetanilide} = 0.1 \times 135 = 13.5 \text{ gm}$$

$$10x = 13.5 \times 10 = 135 \text{ gm}$$

24. The correct stability order of following resonating structures is

- (I) $\text{CH}_2=\text{CH}-\text{CH}=\text{O}$ (II)  (III) 
 (1) II > III > I (2) I > II > III (3) I > III > II (4) III > II > I

Ans. (2)



Paper Solutions Shift-02

(Memory Based Solutions)

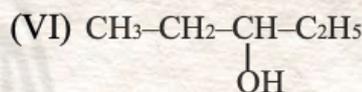
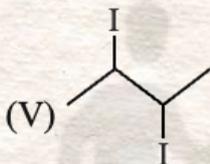
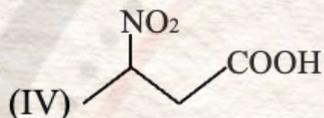
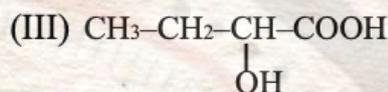
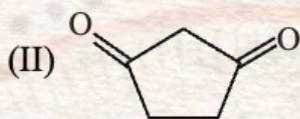
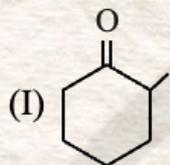


25. Steam volatile and water immiscible substances are separated by

- (1) Steam distillation (2) Fractional distillation under reduced pressure
(3) Fractional distillation (4) Distillation.

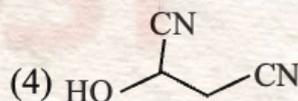
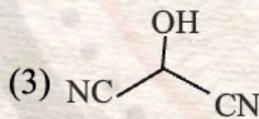
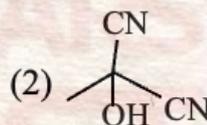
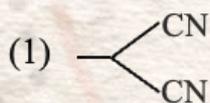
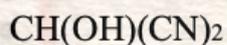
Ans. (1)

26. How many of the following compounds contain chiral centre ?



Ans. 4 (I, III, IV, V)

27. The bond line representation of following compound is



Ans. (3)



PHYSICS

1. An equation of real gas $\left(p - \frac{a}{V^2}\right)(V - b) = RT$

then dimension of $\left(\frac{a}{b^2}\right)$ is

P : Pressure

V = Volume

R = Gas constant

T = Temperature

- (1) $[ML^{-1}T^{-2}]$ (2) $[MLT^{-2}]$ (3) $[ML^2T^{-2}]$ (4) $[MLT^{-1}]$

Ans. (1)

Sol. Basic theory

2. **Assertion:** There can be positive zero error in vernier calliper.

Reason: Due to mishandling or rough handling of instrument

- (1) Assertion true, reason true and reason is correct explanation of assertion
(2) Assertion true, reason true and reason is not correct explanation of assertion
(3) Assertion true, reason false
(4) Assertion false, reason true

Ans. (1)

3. In a RLC series circuit $R = 10\Omega$, $L = \frac{100}{\pi}mH$, $C = \frac{10^{-3}}{\pi}F$ and frequency is 50 Hz. Find power factor.

Ans. 1

Sol. $X_L = \frac{100}{\pi} \times 2\pi \times 50 \times 10^{-3} = 10\Omega$

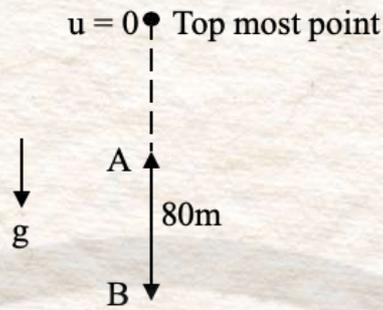
$$X_C = \frac{1}{2\pi \times 50 \times \frac{10^{-3}}{\pi}} = 10\Omega$$

$$X_L = X_C$$

$$\cos\phi = 1$$

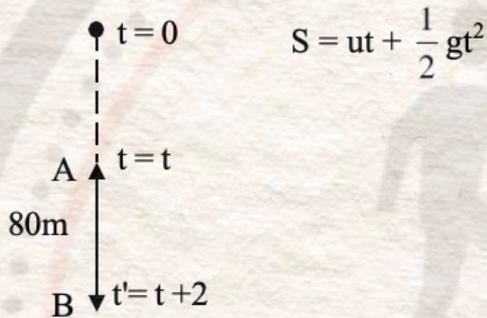


4. A stone is released and while free-fall stone covers 80 m distance in last 2 sec. Find distance of point A from top most point.



Ans. 45

Sol.



$$5(t+2)^2 - 5t^2 = 80 \Rightarrow t = 3\text{sec}$$

$$S_A = 0 + \frac{1}{2} \times 10 \times 3^2 = 45\text{m}$$

5. A person is standing on horizontal ground. A rod of mass 12 kg is touching a shoulder of person and other end is resting on ground. Angle made by rod with horizontal is 60° . Reaction force applied by person on rod is

(1) 60 N

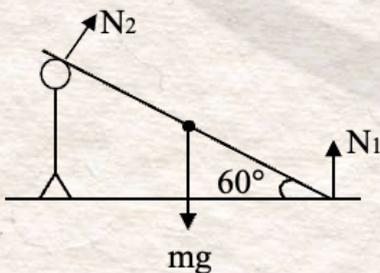
(2) 30 N

(3) 90 N

(4) 120 N

Ans. (2)

Sol.



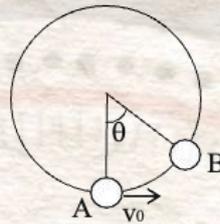


Taking torque about N_1 .

$$mg \cdot \frac{\ell}{2} \cos 60^\circ = N_2 \ell$$

$$N_2 = 30 \text{ N}$$

6. Point 'B' is at highest point of trajectory of object. Magnitude of acceleration at 'A' and 'B' is equal. Find the angle ' θ ' as shown.



(1) $2 \tan^{-1}(1/2)$

(2) $\tan^{-1}(1/2)$

(3) $\tan^{-1}(1/4)$

(4) $\tan^{-1}(2)$

Ans. (1)

Sol. Apply work energy theorem

$$mg(1 - \cos\theta) = \frac{1}{2} m v_0^2$$

$$\frac{v_0^2}{\ell} = 4g \sin^2\left(\frac{\theta}{2}\right) \quad \dots(1)$$

$$g \sin\theta = \frac{v_0^2}{\ell} \quad \dots(2)$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{2}$$

$$\theta = 2 \tan^{-1}\left(\frac{1}{2}\right)$$

7. In an adiabatic process, pressure is proportional to cube of temperature. Find the ratio C_p/C_v .

Ans. $3/2$

Sol. $PT^{\gamma/1-\gamma} = \text{constant}$

$$P \propto T^3$$

$$PT^{-3} = C$$

$$\frac{\gamma}{1-\gamma} = -3$$

$$\gamma = -3 + 3\gamma$$

$$2\gamma = 3$$

$$\gamma = 3/2$$



8. **Assertion:** Angular velocity of earth around sun is lesser than the angular velocity of moon about earth

Reason: Time taken by moon revolve around earth is less than time taken by earth to revolve around sun

(1) Both Assertion (A) and Reason (R) are true & correct explanation of Assertion 'A'

(2) Both 'A' and 'R' are correct but 'R' is not correct explanation of 'A'

(3) 'A' is correct and 'R' is false

(4) 'A' is false and 'R' is correct

Ans. (1)

Sol.
$$T = \frac{2\pi}{\omega} \quad T_{\text{earth}} = 365 \text{ days}$$

$$T_{\text{moon}} = 27 \text{ days}$$

9. If wave function of a metal is 6.68eV. Find threshold frequency.

(1) $8 \times 10^{15} \text{ Hz}$

(2) $1.6 \times 10^{15} \text{ Hz}$

(3) $10 \times 10^{15} \text{ Hz}$

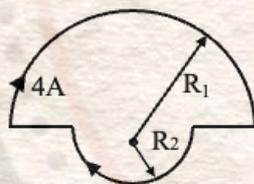
(4) $4 \times 10^{15} \text{ Hz}$

Ans. (2)

Sol. $6.68 \times 1.6 \times 10^{-19} = 6.626 \times 10^{-34} \text{ v}_0$

$$1.6 \times 10^{15} \text{ Hz} = \text{v}_0$$

10. Find magnetic field strength at the centre of loop.



$$R_1 = \frac{\pi}{2}$$

$$R_2 = \frac{\pi}{4}$$

Ans. $24 \times 10^{-7} \text{ Tesla}$

Sol.
$$B_{\text{centre}} = \frac{\mu_0 (i)}{4R_1} + \frac{\mu_0 i}{4R_2}$$

$$= \frac{\mu_0 \times 4}{4} \left[\frac{1}{R_1} + \frac{1}{R_2} \right]$$

$$= \mu_0 \left[\frac{2}{\pi} + \frac{4}{\pi} \right]$$

$$= 4\pi \times 10^{-7} \left[\frac{6}{\pi} \right]$$

$$= 24 \times 10^{-7} \text{ Tesla}$$



11. **Assertion:** If external force is removed, then body will try to regain its actual shape, this is called elasticity.

Reason: Due to intermolecular force, this happens

- (1) Assertion True, Reason True & Reason is correct explanation of assertion
- (2) Assertion True, Reason True & Reason is not correct explanation of assertion
- (3) Assertion True, Reason false
- (4) Assertion false, Reason True

Ans. (1)

12. A bullet gets embedded in a fixed target. It is found that bullet losses $1/3^{\text{rd}}$ of its velocity in traveling 4 cm into target and losses remaining kinetic energy while traveling further $d \times 10^{-3}$ m. Find d.

Ans. 32

Sol. $v^2 = u^2 + 2ax$

$$\left(\frac{2u}{3}\right)^2 = u^2 + 2(-a)(4\text{cm}) \quad \dots(1)$$

for next

$$0 = \left(\frac{2u}{3}\right)^2 + 2(-a)(x) \quad \dots(2)$$

using equation (i) &(ii)

$$x = 32 \times 10^{-3} \text{ m}$$

So $d = 32$

13. 1 mole of an ideal O_2 gas is at 27°C . Find its total kinetic energy?

- (1) 1250 J
- (2) 6250 J
- (3) 645 J
- (4) 1025 J

Ans. (2)

Sol. Kinetic Energy = $\frac{n}{2}fRT$

$$\text{KE} = \frac{1}{2} \times 5 \times \frac{25}{3} \times 300$$

$$= 6250 \text{ J}$$



14. Light of intensity $I = 6 \times 10^8 \frac{W}{m^2}$ is incident on an object kept in medium of refractive index, $\mu = 3$ assuming 100% absorption. Find radiation pressure (N/m^2)?

Ans. 6

Sol. Radiation = $\frac{IA}{\left(\frac{h\nu}{\lambda}\right)} \left(\frac{h}{\lambda}\right) \frac{1}{A} = \frac{I}{v} = \frac{I}{C\mu} = \frac{6 \times 10^8 \times 3}{3 \times 10^8} = 6$

15. A ring and a solid sphere of same mass and radius are released from same point of inclined plane. Find the ratio of their KE when they reach to bottom without slipping

- (1) 1 : 7 (2) 1 : 3 (3) 1 : 5 (4) 1 : 1

Ans. (4)

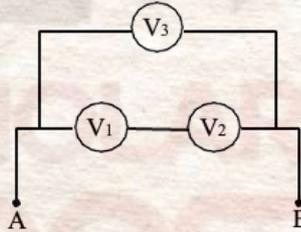
Sol. EC

$$mgh = k_f - k_i$$

$$k_f = mgh$$

$$\text{so } KE_{\text{Ring}} = K.E._{\text{solid sphere}}$$

16. Three voltmeters of different internal resistances are connected as shown in figure and a certain voltage is applied across AB. State which is true?



- (1) $V_1 + V_2 > V_3$ (2) $V_1 + V_2 \neq V_3$ (3) $V_1 + V_2 = V_3$ (4) $V_1 + V_2 < V_3$

Ans. (3)

Sol. By series and parallel combination

$$V_1 + V_2 = V_3$$

17. Specific resistance S is given as $S = \frac{RA}{\ell}$. If length is doubled, find corresponding change in S.

- (1) S is halved
(2) S is doubled
(3) S is quadrupled
(4) No change in S

Ans. (4)



18. **Assertion :** Static friction depends on area of contact but independent of material.

Reason : Kinetic friction is independent of area of contact but depends on material.

- (1) Assertion true, reason true and reason is correct explanation of assertion.
- (2) Assertion true, reason true and reason is not correct explanation of assertion.
- (3) Assertion true, reason false.
- (4) Assertion false, reason true.

Ans. (4)

19. **Assertion :** Work done by electrostatics force on an object when moved on equipotential surface is always zero.

Reason : Electric field lines falls perpendicular to the equipotential surface

- (1) Assertion true, reason true and reason is correct explanation of assertion.
- (2) Assertion true, reason true and reason is not correct explanation of assertion.
- (3) Assertion true, reason false.
- (4) Assertion false, reason true.

Ans. (1)

Sol. Assertion is true and reason is true and correct explanation.

20. A nucleus of C^{13} breaks into C^{12} and neutron. Find energy released.

Atomic mass of $C^{12} = 12.000 \text{ u}$

$C^{13} = 13.013975 \text{ u}$

$n = 1.008665 \text{ u}$

- (1) 3.04 MeV (2) 4.1 MeV (3) 4.94 MeV (4) 6 MeV

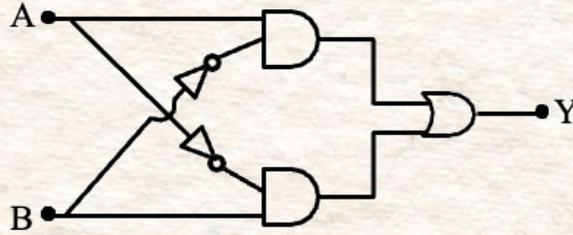
Ans. (3)

Sol. Mass defect = $13.013975 - (12 + 1.008665) = 0.00531 \text{ U}$

Energy released = $0.00531 \times 931 = 4.94 \text{ MeV}$



21. For given logic circuit. The truth table will be



A	B	Y	A	B	Y	A	B	Y	A	B	Y
0	0	0	0	0	1	0	0	0	0	0	0
0	1	1	0	1	1	0	1	0	0	1	1
1	0	1	1	0	1	1	0	0	1	0	1
1	1	1	1	1	0	1	1	1	1	1	0

Ans. (4)

Sol. Using Boolean algebra

$$Y = A\bar{B} + \bar{A}B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

22. In a transformer, ratio of turns in primary to secondary coil is 10 : 1. If primary side voltage is 230 volt and frequency is 50 Hz and resistance of secondary side is 46Ω then find power output.

- (1) 11.5 W (2) 12 W (3) 12.5 W (4) 23 W

Ans. (1)

Sol.
$$\frac{N_1}{N_2} = \frac{V_1}{V_2}$$

$$\frac{10}{1} = \frac{230}{V_2}$$

$$V_2 = 23 \text{ V}$$

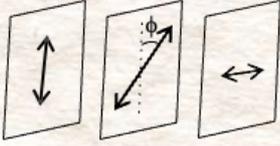
$$P_2 = \frac{V_2^2}{R_2} = \frac{23 \times 23}{46} = \frac{23}{2} = 11.5 \text{ W}$$



23. Between two polaroid placed in crossed position, a third polaroid is introduced. By what angle (in degree) the introduced polaroid placed should be rotated to get maximum intensity of the out coming light.

Ans. 45

Sol.



$$I = I_0 \cos^2 \phi \sin^2 \phi$$

$$I_{\max} \text{ at } \phi = 45^\circ$$

24. If their fundamental frequencies are sounded together, beat frequency is 7 Hz. Find velocity (in m/s) of sound in air?



Ans. 24

Sol. $f_1 = \frac{v}{4\ell_1}$ $f_2 = \frac{v}{2\ell_2}$

$$f_1 = \frac{v \times 100}{4 \times 150} \quad f_2 = \frac{v \times 100}{2 \times 350}$$

$$f_1 = \frac{v}{6} \quad f_2 = \frac{v}{7}$$

$$\frac{v}{6} - \frac{v}{7} = 7$$

$$\frac{v}{42} = 7, \quad v = 42 \times 7$$

$$v = 294 \text{ m/sec}$$

25. For 200 μA current galvanometer deflects by $\pi/3$ radians. For what value of current, it will deflect by $\pi/10$ radians ?

Ans. 60

Sol. $i \propto \theta$ (angle of deflection)

$$\frac{i_1}{i_2} = \frac{\theta_1}{\theta_2}$$

$$\frac{200 \mu\text{A}}{i_2} = \frac{\pi/3}{\pi/10} = \frac{10}{3}$$

$$60 \mu\text{A} = i_2$$



26. Two charges of magnitude $-4 \mu\text{C}$ kept at $(1, 0, 4)$ and another charge of $+4 \mu\text{C}$ kept at $(2, -1, 5)$ in the presence of external electric field $E = 0.2 \hat{i} \text{ V/cm}$. The torque on the system of charges is $8\sqrt{\alpha} \times 10^{-5} \text{ N-m}$. Find α .

Ans. 2

Sol. $\vec{\tau} = \mathbf{P} \times \mathbf{E}$

$$\vec{P} = Pr = 4 \times 10^{-6} \times \sqrt{3} \frac{(\hat{i} - \hat{j} + \hat{k})}{\sqrt{3}}$$

$$\mathbf{P} = 4 \times 10^{-6} (\hat{i} - \hat{j} + \hat{k})$$

$$\mathbf{E} = 0.2 \times 10^2 \hat{i} = 20 \hat{i} \text{ V/m}$$

$$\vec{\tau} = 4 \times 10^{-6} \times 20 [(\hat{i} - \hat{j} + \hat{k}) \times \hat{i}]$$

$$\vec{\tau} = 8 \times 10^{-5} (\hat{k} + \hat{j})$$

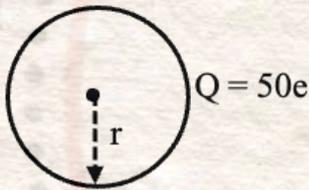
$$|\vec{\tau}| = 8\sqrt{2} \times 10^{-5} \text{ Nm}$$

$$\alpha = 2$$

27. A nucleus with atomic number '50' and having radius of nucleus is $9 \times 10^{-13} \text{ cm}$. Calculate the potential (in MV) at the surface of the nucleus.

Ans. 8

Sol.



$$V_{\text{surface}} = \frac{kQ}{r} = \frac{9 \times 10^9 \times 50 \times 1.6 \times 10^{-19}}{9 \times 10^{-15}}$$

$$= 80 \times 10^5 \text{ volt}$$

$$V_{\text{surface}} = 8 \text{ MV}$$

28. A pressure inside wall pipe before hole is $4.5 \times 10^4 \text{ N/m}^2$. When a small hole is made in pipe, pressure is changed to $2.0 \times 10^4 \text{ N/m}^2$. If speed of water flux after hole is $\sqrt{v} \text{ m/s}$. Find out v :

Ans. 50

Sol. $\Delta P = \frac{1}{2} \rho v^2$

$$2.5 \times 10^4 = \frac{1}{2} \times 10^3 v_0^2$$

$$v_0 = \sqrt{50} \text{ m/s}$$

$$v = 50$$



MATHEMATICS

1. Find number of common terms in the two given series

4, 9, 14, 19, up to 25 terms and

3, 9, 15, 21, up to 37 terms

(1) 4

(2) 7

(3) 5

(4) 3

Ans. (1)

Sol. 4, 9, 14, 19, 124 $\rightarrow d_1 = 5$

3, 9, 15, 21, 219 $\rightarrow d_2 = 6$

1st common term = 9 and common difference of common terms = 30

Common terms are 9, 39, 69, 99

4 common terms

2. Let $8 = 3 + \frac{3+p}{4} + \frac{3+2p}{4^2} + \dots \infty$, then p is

(1) 9

(2) $\frac{5}{4}$

(3) 3

(4) 1

Ans. (1)

Sol. $8 = 3 + \frac{3+p}{4} + \frac{3+2p}{4^2} + \dots$ (i)

multiply both sides by $\frac{1}{4}$, we get

$$2 = \frac{3}{4} + \frac{3+p}{4^2} + \dots$$
 (ii)

Equation (i) – equation (ii)

$$\Rightarrow 6 = 3 + \frac{p}{4} + \frac{p}{4^2} + \dots$$

$$\Rightarrow 3 = \frac{p}{4\left(1 - \frac{1}{4}\right)} \Rightarrow p = 9$$

3. For $\frac{x^2}{25} + \frac{y^2}{16} = 1$, find the length of chord whose mid point is $P\left(1, \frac{2}{5}\right)$

(1) $\frac{\sqrt{1681}}{5}$

(2) $\frac{\sqrt{1481}}{5}$

(3) $\frac{\sqrt{1781}}{5}$

(4) $\frac{\sqrt{1691}}{5}$

Ans. (4)



Sol. By $T = S_1$

$$\Rightarrow \frac{x}{25} + \frac{y}{16} = \frac{1}{25} + \frac{4}{25} \cdot \frac{1}{16}$$

$$\Rightarrow \frac{x}{25} + \frac{y}{40} = \frac{4+1}{100}$$

$$\Rightarrow \frac{x}{25} + \frac{y}{40} = \frac{1}{20}$$

$$\Rightarrow 8x + 5y = 10$$

$$\Rightarrow \frac{x^2}{25} + \left(\frac{10-8x}{5}\right)^2 \frac{1}{16} = 1$$

$$\Rightarrow \frac{x^2}{25} + \frac{4}{25} \left(\frac{5-4x}{16}\right)^2 = 1$$

$$\Rightarrow x^2 + \frac{(5-4x)^2}{4} = 25$$

$$\Rightarrow 4x^2 + (5-4x)^2 = 100$$

$$\Rightarrow 20x^2 - 8x - 15 = 0$$

$$x_1 + x_2 = 2$$

$$x_1 x_2 = \frac{-15}{4}$$

$$\begin{aligned} \text{length of chord} &= |x_1 - x_2| \sqrt{1+m^2} \\ &= \frac{\sqrt{1691}}{5} \end{aligned}$$

4. If $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, then find $f'(10)$.

Ans. (202)

Sol. $f'(x) = 3x^2 + 2x f'(1) + f'(2)$

$$f''(x) = 6x + 2f'(1)$$

$$f'''(3) = 6$$

$$f'(1) = -5$$

$$f''(2) = 2$$

$$\Rightarrow f(10) = 300 + 20(-5) + 2$$

$$= 202$$

5. Let $\int_0^1 \frac{dx}{\sqrt{x+3} + \sqrt{x+1}} = A + B\sqrt{2} + C\sqrt{3}$ then the value of $2A + 3B + C$ is

(1) 3

(2) 4

(3) 5

(4) 6

Ans. (1)



Sol. On rationalising

$$\int_0^1 \frac{(\sqrt{x+3} - \sqrt{x+1})}{2} dx$$
$$= \frac{2}{3.2} \left\{ (x+3)^{3/2} - (x+1)^{3/2} \right\}_0^1$$
$$= \frac{1}{3} \{8 - 3\sqrt{3} - (2\sqrt{2} - 1)\}$$
$$= \frac{1}{3} \{9 - 3\sqrt{3} - 2\sqrt{2}\}$$
$$= \left(3 - \sqrt{3} - \frac{2\sqrt{2}}{3} \right) : A = 3, B = -\frac{2}{3}, C = -1$$

$$\therefore 2A + 3B + C = 6 - 2 - 1 = 3$$

6. If $|z - i| = |z - 1| = |z + i|$, $z \in C$, then the numbers of z satisfying the equation are

- (1) 0 (2) 1 (3) 2 (4) 4

Ans. (2)

Sol. z is equidistant from 1, i , & $-i$

only $z = 0$ is possible

\therefore number of z equal to 1

7. If sum of coefficients in $(1 - 3x + 10x^2)^n$ and $(1 + x^2)^n$ is A and B respectively then

- (1) $A^3 = B$ (2) $A = B^3$ (3) $A = 2B$ (4) $A = B$

Ans. (2)

Sol. $A = 8^n$ $B = 2^n$

(B) $\therefore A = B^3$

8. Let a_1, a_2, \dots, a_{10} are 10 observations such that $\sum_{i=1}^{10} a_i = 50$ and $\sum_{i \neq j}^{10} a_i \cdot a_j = 1100$, then their

standard deviation will be

- (1) $\sqrt{5}$ (2) $\sqrt{30}$ (3) $\sqrt{15}$ (4) $\sqrt{10}$

Ans. (1)

Sol. $(a_1 + a_2 + \dots + a_{10})^2 = 50^2$

$$\Rightarrow \sum a_i^2 + 2 \sum_{i \neq j} a_i a_j = 2500$$

$$\Rightarrow \sum a_i^2 = 300$$

$$\sigma^2 = \frac{\sum a_i^2}{10} - \left(\frac{\sum a_i}{10} \right)^2$$

$$\Rightarrow \sigma^2 = 5 \Rightarrow \text{S.D} = \sqrt{5}$$



9. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then

Statement-1 : $f(-x)$ is inverse of $f(x)$

Statement-2 : $f(x + y) = f(x)f(y)$

(1) Both are true

(2) Both are false

(3) Only statement 1 is true

(4) Only statement 2 is true

Ans. (1)

Sol. $f(x)f(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= f(x+y)$$

$$\therefore f(x) f(-x) = f(0)$$

$$= I$$

10. If $a = \lim_{x \rightarrow 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$ and $b = \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$ find $a \cdot b^3$

(1) 16

(2) 32

(3) -16

(4) 48

Ans. (2)

Sol. $a = \lim_{x \rightarrow 0} \frac{\sqrt{1+x^4} - 1}{x^4 \left[\sqrt{1+\sqrt{1+x^4}} + \sqrt{2} \right]}$

$$= \lim_{x \rightarrow 0} \frac{x^4}{x^4 \left[\sqrt{1+\sqrt{1+x^4}} + \sqrt{2} \right] \left[\sqrt{1+x^4} + 1 \right]}$$

$$= \frac{1}{2\sqrt{2} \times 2} = \frac{1}{4\sqrt{2}}$$

$$b = \lim_{x \rightarrow 0} \frac{\sin^2 x}{(1 - \cos x)} (\sqrt{2} + \sqrt{1 + \cos x})$$

$$= 2 \times (\sqrt{2} + \sqrt{2}) = 4\sqrt{2}$$

$$\therefore ab^3 = (4\sqrt{2})^3 = 32$$



11. If the minimum distance of centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ from any point on the parabola $y^2 = 4x$ is d , find d^2

Ans. (20)

Sol. Normal to parabola is $y = mx - 2m - m^3$

$$\text{centre } (2, 8) \rightarrow 8 = 2m - 2m - m^3$$

$$\Rightarrow m = -2$$

$$\therefore p \text{ is } (m^2, -2m) = (4, 4)$$

$$\Rightarrow d^2 = 4 + 16 = 20$$

12. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$, $\vec{a} \times \vec{c} = \vec{b}$ & $\vec{a} \cdot \vec{c} = 3$ find $\vec{a} \cdot (\vec{c} \times \vec{b} - \vec{b} \times \vec{c})$

(1) 24

(2) -24

(3) 18

(4) 15

Ans. (1)

Sol. $[\vec{a} \ \vec{c} \ \vec{b}] = (\vec{a} \times \vec{c}) \cdot \vec{b} = |\vec{b}|^2 = 27$

$$\therefore \text{we need} = 27 - 0 - 3 = 24$$

13. Consider the line $L : 4x + 5y = 20$. Let two other lines are L_1 and L_2 which trisect the line L and pass through origin, then tangent of angle between lines L_1 and L_2 is

(1) $\frac{20}{41}$

(2) $\frac{30}{41}$

(3) $\frac{40}{41}$

(4) $\frac{10}{41}$

Ans. (2)

Sol. Let line L intersect the lines L_1 and L_2 at P and Q

$$P\left(\frac{10}{3}, \frac{4}{3}\right), Q\left(\frac{5}{3}, \frac{8}{3}\right)$$

$$\therefore \text{mOA} = \frac{2}{5}$$

$$\text{mOQ} = \frac{8}{5}$$

$$\tan\theta = \left| \frac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{16}{25}} \right|$$

$$= \left(\frac{6}{5} \times \frac{25}{41} \right)$$

$$= \frac{30}{41}$$



Paper Solutions Shift-02

(Memory Based Solutions)



14. If ${}^{n-1}C_r = (k^2 - 8) {}^nC_{r+1}$, then the range of 'k' is
(1) $k \in (2\sqrt{2}, 3]$ (2) $k \in (2\sqrt{2}, 3)$ (3) $k \in [2, 3)$ (4) $k \in (2\sqrt{2}, 8)$

Ans. (1)

Sol. ${}^{n-1}C_r = (k^2 - 8) \frac{n}{r+1} \cdot {}^{n-1}C_r$

$$\Rightarrow k^2 - 8 = \frac{r+1}{n}$$

here $r \in [0, n-1]$

$$\Rightarrow r+1 \in [1, n]$$

$$\Rightarrow k^2 - 8 \in \left[\frac{1}{n}, 1 \right]$$

$$\Rightarrow k^2 \in \left[8 + \frac{1}{n}, 9 \right]$$

$$\Rightarrow k \in (2\sqrt{2}, 3]$$

15. If $\alpha x + \beta y + 9 \ln|2x + 3y - 8\lambda| = x + C$ is the solution of $(2x + 3y - 2)dx + (4x + 6y - 7)dy = 0$, then $\alpha + \beta + \gamma =$

- (1) 18 (2) 19 (3) 20 (4) 21

Ans. (1)

Sol. Let $2x + 3y = t$

$$\Rightarrow 2 + 3 \frac{dy}{dx} = \frac{dt}{dx}$$

$$\text{Now } (t-2) + (2t-7) \left(\frac{dt}{dx} - 2 \right) \times \frac{1}{3} = 0$$

$$\Rightarrow -\frac{(3t-6)}{2t-7} = \frac{dt}{dx} - 2$$

$$\Rightarrow \frac{dt}{dx} = \frac{t-8}{2t-7}$$

$$\Rightarrow \int \frac{2t-7}{t-8} dt = \int dx$$

$$\Rightarrow \int 2 + \frac{9}{t-8} dt = \int dx$$

$$\Rightarrow 2t + |9 \ln|t-8|| = x + C$$

$$\Rightarrow 2(2x + 3y) + 9 \ln|2x + 3y - 8| = x + C$$

$$\alpha = 4, \beta = 6, \gamma = 8$$



Paper Solutions Shift-02

(Memory Based Solutions)



Sol. $\phi \cap \phi = \phi \Rightarrow (\phi, \phi) \notin R \Rightarrow$ not reflexive.

If $A \cap B \neq \phi \Rightarrow B \cap A \neq \phi \Rightarrow$ Symmetric

If $A \cap B \neq \phi$ and $B \cap C \neq \phi \Rightarrow A \cap C = \phi$

for example $A = \{1, 2\}$

$B = \{2, 3\}$

$C = \{3, 4\}$

20. If four points $(0, 0), (1, 0), (0, 1), (2k, 3k)$ are concyclic, then k is

(1) $\frac{4}{13}$

(2) $\frac{5}{13}$

(3) $\frac{7}{13}$

(4) $\frac{9}{13}$

Ans. (2)

Sol. Equation of circle is

$$x(x-1) + y(y-1) = 0$$

$$x^2 + y^2 - x - y = 0$$

$B(2k, 3k)$

$$\Rightarrow 4k^2 + 9k^2 - 2k - 3k = 0$$

$$\Rightarrow 13k^2 = 5k$$

$$\Rightarrow k = 0, \frac{5}{13}$$

$$\therefore k = \frac{5}{13}$$

21. If $f(x)$ is differentiable function satisfying $f(x) - f(y) \geq \log \frac{x}{y} + x - y$, then find $\sum_{N=1}^{20} f' \left(\frac{1}{N^2} \right)$

Ans. (2890)

Sol. Let $x > y$

$$\lim_{y \rightarrow x} \frac{f(x) - f(y)}{x - y} \geq \frac{\log x - \log y}{x - y} + 1$$

$$f'(x^-) \geq \frac{1}{x} + 1$$

$\Rightarrow f'(x^-) = f'(x^+)$ as $f(x)$ is differentiable function

$$f'(x) = \frac{1}{x} + 1$$

$$f' \left(\frac{1}{N^2} \right) = N^2 + 1$$

$$\sum_{N=1}^{20} f' \left(\frac{1}{N^2} \right) = \sum (N^2 + 1) = \frac{20 \times 21 \times 41}{6} + 20 = 2890$$



Paper Solutions Shift-02

(Memory Based Solutions)



22. Let $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ where $y(0) = 1$, $x(0) = 2$, and $x(t) = y(t)$, then t is

(1) $\frac{\ln 3}{a-b}$

(2) $\frac{\ln 2}{b-a}$

(3) $\frac{\ln 2}{a-b}$

(4) $\frac{\ln 3}{b-a}$

Ans. (3)

Sol. $\frac{dx}{dt} + ax = 0$

$$\Rightarrow \ln x = -at + c$$

$$x(0) = 2 \Rightarrow c = \ln 2$$

$$\therefore x = 2e^{-at}$$

$$\frac{dy}{dt} + by = 0 \Rightarrow y = e^{-bt}$$

$$x(t) = y(t)$$

$$2e^{-at} = e^{-bt}$$

$$\Rightarrow t = \frac{\ln 2}{a-b}$$

23. If $H(a, b)$ is the orthocentre of ΔABC where $A(1, 2)$, $B(2,3)$ & $C(3, 1)$, then find $\frac{36I_1}{I_2}$ if

$$I_1 = \int_a^b x \sin(4x - x^2) dx \text{ and } I_2 = \int_a^b \sin(4x - x^2) dx$$

Ans. (72)

Sol. ΔABC is isosceles

$\Rightarrow H$ lies on angle bisector passing through $(3, 1)$ which is $x + y = 4$

$$\therefore a + b = 4$$

Now apply $(a + b - x)$ in I_1

$$2I_1 = \int_a^b 4 \sin(4x - x^2) dx$$

$$\Rightarrow 2I_1 = 4I_2$$

$$\Rightarrow \frac{I_1}{I_2} = 2$$

$$\therefore \frac{36I_1}{I_2} = 72$$



24. $f(x) = \begin{cases} \frac{\sin(x-3)}{2^{x-[x]}}, & x > 3 \\ \frac{a(x^2 - 7x + 12)}{b|x^2 - 7x + 12|}, & x < 3 \\ b, & x = 3 \end{cases}$. Find number of ordered pairs (a, b) so that f(x) is continuous

at $x = 3$

Ans. (1)

Sol. LHL = RHL = f(3)

$$-\frac{a}{b} = 2^1 = b$$

$$\Rightarrow b = 2 \text{ and } a = -4$$

$$\Rightarrow (a, b) = (-4, 2)$$

25. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix}$, $B = [B_1 \ B_2 \ B_3]$ where B_1, B_2, B_3 are column matrices such that

$$AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AB_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

α = sum of diagonal elements of B

$\beta = |B|$, then find $|\alpha^3 + \beta^3|$

Ans. (1.125)

Sol. $A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{3}{2} & \frac{1}{2} \\ 1 & -2 & 0 \end{bmatrix}$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 2 \end{bmatrix}, B_3 = \begin{bmatrix} 2 \\ -5 \\ 2 \\ -1 \end{bmatrix}$$

$$\text{Tr}(B) = -\frac{1}{2}$$

$$|B| = -1$$

$$\therefore a = -\frac{1}{2}, b = -1$$

$$|\alpha^3 + \beta^3| = \frac{9}{8} = 1.125$$



Paper Solutions **Shift-02**

(Memory Based Solutions)



26. If $\cos(2x) - a \sin x = 2a - 7$ has a solution for $a \in [p, q]$ and $r = \tan 9^\circ + \tan 63^\circ + \tan 81^\circ + \tan 27^\circ$, then $p \cdot q \cdot r = ?$

- (1) $40\sqrt{5}$ (2) $32\sqrt{5}$ (3) $30\sqrt{5}$ (4) $48\sqrt{5}$

Ans. (4)

Sol. $2(\sin^2 x - 4) + a(\sin x + 2) = 0$

$$2(\sin x - 2) + a = 0$$

$$\Rightarrow a = 4 - 2 \sin x$$

$$a \in [2, 6]$$

$$\text{Also, } r = \left(\tan 9^\circ + \frac{1}{\tan 9^\circ} \right) + \left(\tan 27^\circ + 1 \frac{1}{\tan 27^\circ} \right)$$

$$= \frac{2}{\sin 18^\circ} + \frac{2}{\sin 54^\circ}$$

$$= \frac{2 \times 4}{\sqrt{5} - 1} + \frac{2 \times 4}{\sqrt{5} + 1}$$

$$= \frac{8 \times 2\sqrt{5}}{4} = 4\sqrt{5}$$

$$\therefore pqr = 48\sqrt{5}$$

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