JEE MAINS 2024

Exam Solutions Shift-02

Session-01

29 JANUARY 2024

IIT JEE

Time- 03 Hrs **M.Marks : 300**

Topic Covered

Physics : Full Syllabus

Chemistry : Full Syllabus

Maths : Full Syllabus

GENERAL INSTRUCTION

1. Immediately fill in the particulars on this page of the test booklet.
2. The test is of 3 hours duration.
3. The test booklet consists of 90 questions. The maximum marks are 300.
4. There are Three Sections in the question paper, Section I, II & III consisting of Section-I (Physics), Section-II (Chemistry), Section-III (Mathematics) and having 30 questions in each part in which first 20 questions are compulsory and are of Objective Type and Last 10 questions are integers type in which you have to attempt 5 questions only.
5. There is only one correct response for each question.
6. Each correct answer will give 4 marks while 1 Mark will be deducted.
7. No student is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
8. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.

Paper Solutions Shift-02

(Memory Based Solutions)

Physics

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. In a simple pendulum of length 10 m, string is initially kept horizontal and the bob is released. 10% of energy is lost till the bob reaches lowermost position. Then find speed of bob at lowermost position.

(1) 6 m/s

(2) $6\sqrt{5}$ m/s

(3) $7\sqrt{5}$ m/s

(4) $4\sqrt{2}$ m/s

Answer (2)

Sol. $W_{total} = \Delta K$

$\Rightarrow 0.9mgl = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{1.8\times10\times10}$

$= 6\sqrt{5}$ m/s

2. The intensity at each slit are equal for a YDSE and it is maximum (I_{max}) at central maxima. If I is intensity for phase difference $\frac{7\pi}{2}$ between two waves (at screen).

Then $\frac{I}{I_{max}}$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $\frac{3}{8}$ (4) $\frac{1}{\sqrt{2}}$

Answer (1)

Sol. $I = I_{max}\cos^2\left(\frac{\Delta\phi}{2}\right)$

$\frac{I}{I_{max}} = \cos^2\frac{7\pi}{4}$ $\quad\because \Delta\phi = \frac{7\pi}{2}$

$\frac{I}{I_{max}} = \cos^2\left(\frac{\pi}{4}\right) = \frac{1}{2}$

3. An electromagnetic wave has electric field given by $\vec{E} = (9.6\hat{j})\sin\left[2\pi\left\{30\times10^6 t - \frac{1}{10}x\right\}\right]$, x and t are in SI units. The maximum magnetic field is

(1) 3.2×10^{-8}

(2) 9.6×10^{-8}

(3) 1.7×10^{-8}

(4) 10^{-7}

Answer (1)

Sol. $\frac{E}{B} = C$

$\Rightarrow B = \frac{E}{C} = 3.2\times10^{-8}$

4. A planet at distance r from sun takes 200 days to complete one revolution around sun. What will be time period for a planet at distance $\frac{r}{4}$ from the sun?

(1) 50 days

(2) 25 days

(3) 100 days

(4) 12.5 days

Answer (2)

Sol. $T^2 \propto R^3$

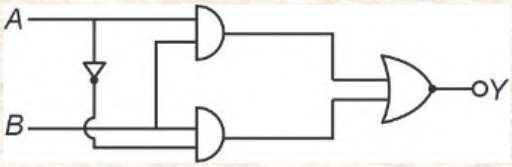
$\frac{200^2}{T^2} = \frac{r^3}{\left(\frac{r}{4}\right)^3}$

$\frac{200}{T} = (4)^{\frac{3}{2}}$

$\frac{200}{8} = T$

$\Rightarrow T = 25$ days

5. The truth table for the combination of logical gates



(1)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(3)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

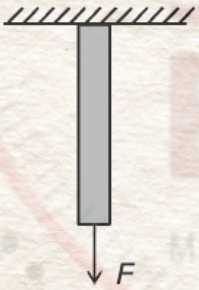
(4)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

Answer (3)

Sol. $Y = A\cdot B + \overline{A}\cdot B = B(A+\overline{A}) = B$

6. A uniform wire has length L and radius r. It is acted on by a force F as shown. The elongation is Δl. If F and r are both halved, the new elongation will be :



(1) $\frac{\Delta l}{2}$

(2) Δl

(3) $4\Delta l$

(4) $2\Delta l$

Answer (4)

Sol. $\Delta l = \frac{FL}{Ay} \propto \frac{F}{r^2}$

$$\Rightarrow \Delta l' = \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^2}\Delta l = 2\Delta l$$

7. Two forces F_1 and F_2 are applied on two rods P and Q of same materials such that elongation in rods are same. If ratio of their radii is $x : y$ and ratio of length is $m : n$, then ratio of $F_1 : F_2$ is

(1) $\left(\frac{y}{x}\right)^2 \frac{n}{m}$

(2) $\left(\frac{x}{y}\right)^2 . \frac{n}{m}$

(3) $\left(\frac{x}{y}\right)^2 . \frac{m}{n}$

(4) $\left(\frac{y}{x}\right)^2 . \left(\frac{m}{n}\right)$

Answer (2)

Sol. $\Delta l_1 = \frac{F_1 l_1}{YA_1}, \Delta l_2 = \frac{F_2 l_2}{YA_2}$

$$\frac{F_1}{F_2} = \frac{A_1}{A_2} \times \frac{l_2}{l_1} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{l_2}{l_1}\right) = \frac{x^2}{y^2} . \frac{n}{m}$$

8. Two charged particles A and B have charge q each while masses are m_1 & m_2. Both have same velocity v and enter into a transverse magnetic field B such that their radii are r_1 & r_2. Then the ratio $m_1 : m_2$ is

(1) $\frac{r_2}{r_1}$

(2) $\left(\frac{r_1}{r_2}\right)^2$

(3) $\frac{r_1}{r_2}$

(4) $\left(\frac{r_2}{r_1}\right)^2$

Answer (3)

Sol. $r = \frac{mv}{Bq}$

$$r \propto m \Rightarrow \frac{r_1}{r_2} = \frac{m_1}{m_2}$$

9. A liquid drop of radius R is divided into 27 identical drops. If surface tension of the drops is T, then find work done in this process.

(1) $4\pi R^2T$

(2) $3\pi R^2T$

(3) $8\pi R^2T$

(4) $\frac{1}{8}\pi R^2T$

Answer (3)

Sol. $W = T \times$ change in area (ΔS)

From volume conservation

$$\frac{4}{3}\pi R^3 = 27\pi r^3 \times \frac{4}{3}$$

$$R = 3r$$

$$r = \frac{R}{3}$$

$$\therefore \quad \Delta S = 4\pi r^2 \times 27 - 4\pi R^2$$

$$= 4\pi \times \frac{R^2}{9} \times 27 - 4\pi R^2 = 2\left(4\pi R^2\right)$$

$$W = 8\pi R^2T$$

10. Alternating voltage and current in circuit is given as

$V = (100 \sin\omega t)$ volt

$$I = 100\sin\left(\omega t + \frac{\pi}{3}\right)\text{mA}$$

Find average power dissipated in circuit.

(1) 2.5 w

(2) 5 w

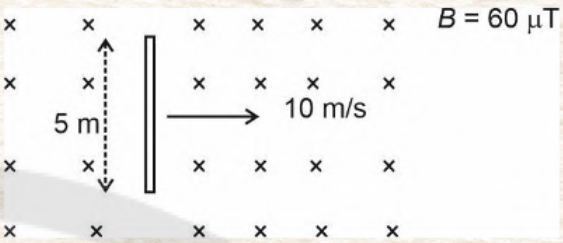
(3) 10 w

(4) 20 w

Answer (1)

Sol. $P_{avg} = IV\cos\phi = \frac{100}{\sqrt{2}} \times \frac{100\times10^{-3}}{\sqrt{2}}\cos 60° = 2.5$ w

11. Consider a rod moving in a magnetic field as shown:



The induced emf across the ends of the rod is

(1) 3 mV

(2) 6 mV

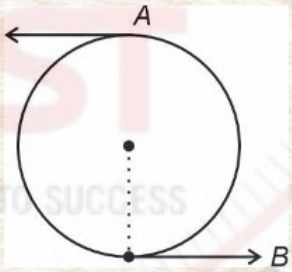
(3) 0 V

(4) 1 mV

Answer (1)

Sol. $\varepsilon = BlV = 3$ mV

12. A particle connected with light thread is performing vertical circular motion. Speed at point B (Lowermost point) is of just sufficient, so that it is able to complete its circular motion. Ignoring air friction, find the ratio of kinetic energy at A to that at B. (A being top-most point)



(1) 1 : 5

(2) 5 : 1

(3) $1:7\sqrt{2}$

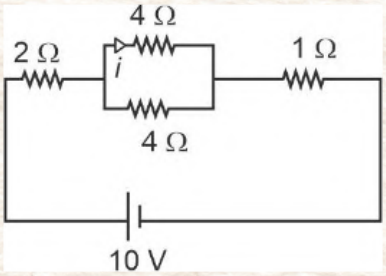
(4) $1:5\sqrt{2}$

Answer (1)

Sol. $v_A = \sqrt{gL}$

$$v_B = \sqrt{5gL}$$

$$\Rightarrow \quad \frac{k_A}{k_B} = \frac{1}{5}$$

13. 

In given circuit, an ideal battery is connected with four resistances as shown. Find current i as mentioned in diagram.

(1) 2 A (2) 1 A

(3) 4 A (4) 0.5 A

Answer (2)

Sol. req = 2 + 2 + 1 = 5 Ω

$$i_b = \frac{10}{5} = 2\ A$$

$$i = \frac{i_b}{2} = 1\ A$$

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. A physical quantity Q depends on other physical quantities a, b and c as

$$Q = \frac{a^4b^3}{c^2}$$

If maximum percentage error in measurement of a, b and c are 3%, 4% and 5% respectively, then find maximum percentage error in measurement of Q.

Answer (34)

Sol. $Q = \dfrac{a^4b^3}{c^2}$

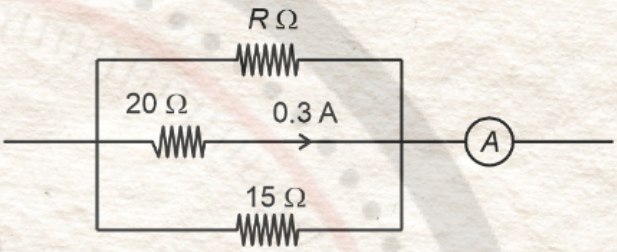
$$\frac{\Delta Q}{Q} = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + 2\frac{\Delta c}{c}$$

$$\frac{\Delta Q}{Q} \times 100 = 4(3) + 3(4) + 2(5)$$

$$= 12 + 12 + 10$$

$$\% \text{ error } \frac{\Delta Q}{Q}\% = 34\%$$

22. Consider the circuit shown :



The ammeter reads 0.9 A. Value of R is _____

Answer (30)

Sol. 20 Ω & 15 Ω in parallel

$\Rightarrow$ 20 × 0.3 = 15 × i

$\Rightarrow$ i = 0.4 A

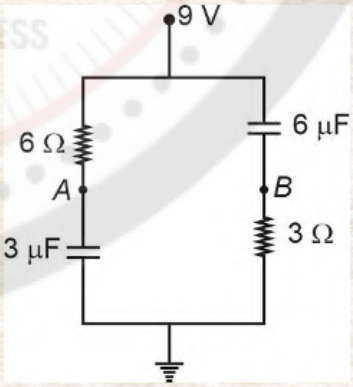
$\Rightarrow$ i_R = 0.9 – 0.3 – 0.4 A

= 0.2 A

$\Rightarrow$ R × 0.2 = 20 × 0.3

$\Rightarrow$ R = 30 Ω

23. Consider the circuit shown :



Charge on 6 μF when A and B are shorted is _____ μC.

Answer (36)

Sol. In steady state, 6 Ω and 3 Ω are in series.

$\Rightarrow$ $\Delta V_{6\Omega} = 6\ V = \Delta V_{6\mu F}$

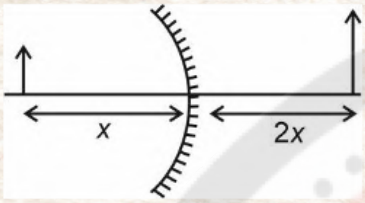
$\Rightarrow$ $\phi = CV = 36\ \mu C$

24. Distance between twice-magnified virtual image of an object placed in front of mirror is 15 cm. Find focal length of spherical mirror in cm.

Answer (10)

Sol. Magnified virtual image of real object

$\Rightarrow$ Concave mirror



$\left(\frac{v}{u}\right) = 2$

$\Rightarrow 2x + x = 15$

$x = 5$ cm

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{10} - \frac{1}{5} = \frac{1}{f}$

$\frac{1-2}{10} = \frac{-1}{10} = \frac{1}{f}$

$\Rightarrow f = -10$

25. The displacement of a particle changing with time as $x = 6t^3 - 12t^2 + 20t + 30$. Find velocity (in m/s) of particle when it's acceleration became zero. (t is time in s)

Answer (12)

Sol. $v = \frac{dx}{dt} = 20$

$= 18t^2 - 24t + 20$

$a = \frac{dv}{dt} = 36t - 24$

At $a = 0$

$t = \frac{24}{36} = \frac{2}{3}$ sec

Then,

$v = 18 \times \frac{4}{9} - 24 \times \frac{2}{3} + 20$

$= 8 - 16 + 20 = 12$ m/s

26. Electric field in a region is given by $\vec{E} = \left(6\hat{i} + 7\hat{j} + 8\hat{k}\right)$ units. An area of 30 units is considered in y-z plane. Calculate the electric flux through this area.

Answer (180)

Sol. $\phi = \vec{E} \cdot \vec{A} = \left(6\hat{i} + 7\hat{j} + 8\hat{k}\right) \cdot 30\hat{i} = 180$

27. N moles of non-linear polyatomic gas (degree of freedom 6) is mixed with 2 moles of monoatomic gas. The resultant mixture has molar specific heat equal to that of a diatomic gas, then N is

Answer (4)

Sol. $\frac{n_1\frac{f_1}{2}R + n_2\frac{f_2}{2}R}{n_1 + n_2} = \frac{5}{2}R$

$\frac{2 \times \frac{3}{2}R + N \times \frac{6}{2}R}{N + 2} = \frac{5}{2}R$

$\frac{6 + 6N}{N + 2} = 5$

$6 + 6N = 5N + 10$

$N = 4$

28. A particle starts oscillation from origin on x-axis with period of oscillation (6) sec and amplitude A. If time taken by particle to reach from $x = A$ to $x = \frac{\sqrt{3}}{2}A$ for the first time is τ then. Value of 6τ is _____ sec.

Answer (3)

Sol. $x = A\sin\left(\omega t + \frac{\pi}{2}\right)$

$x = A\cos\omega t$

$\frac{\sqrt{3}}{2}A = A\cos\left(\frac{2\pi}{\tau}t\right)$

$\frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{3}t\right)$

$\frac{\pi}{6} = \frac{t}{3}\pi$

$t = \frac{1}{2} = 0.5$

$6\tau = 3$

29.

30.

Paper Solutions Shift-02

(Memory Based Solutions)

Chemistry

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Best reducing agent among the given ions is

   (1) Ce^{4+} (2) Gd^{2+}

   (3) Lu^{3+} (4) Nd^{3+}

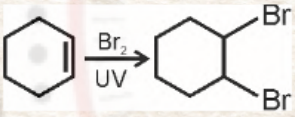
Answer (2)

Sol. Gd^{2+} : [Xe] $5d^1 4f^7$

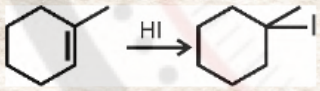
Gd^{2+} would get converted into Gd^{3+} as Gd^{3+} has stable electronic configuration

2. Choose the correct reaction.

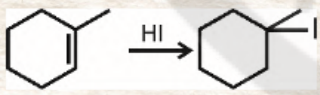
(1) $CH_3-CH_2-\overset{\overset{O}{\|}}{C}-NH_2 \xrightarrow[NaOH]{Br_2} CH_3-CH_2-CH_2-NH_2$

(2) 

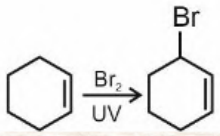
(3) $CH_3-CH_2-CH_2-CH_2-NH_2 \xrightarrow[(0-5)°C]{HNO_2} CH_3-CH_2-CH_2-CH_2-OH$

(4) 

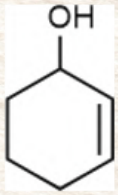
Answer (4)

Sol. 

$CH_3-CH_2-\overset{\overset{O}{\|}}{C}-NH_2 \xrightarrow[NaOH]{Br_2} CH_3-CH_2-NH_2$



$CH_3-CH_2-CH_2-CH_2-CH_2-NH_2 \xrightarrow[(0-5)°C]{HNO_2} CH_3-CH_2-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3$

3. IUPAC name of compound  is

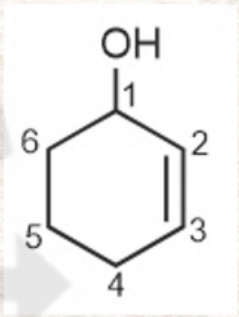
   (1) Hex-2-en-1-ol

   (2) Cyclohex-2-en-1-ol

   (3) 3-hydroxy cyclohexene

   (4) Cyclohex-1-en-3-ol

Answer (2)

Sol. 

Cyclohex-2-en-1-ol

4. Why does oxygen shows anomalous behaviour?

   (1) Large size, high electronegativity

   (2) Small size, small electronegativity

   (3) Small size, high electronegativity absence of vacant d-orbital

   (4) Large size, high electronegativity presence of vacant d-orbital

Answer (3)

Sol. Oxygen shows anomalous behaviour due to small size, high electronegativity and absence of vacant d-orbital.

5. Match the following

(A) Lyman	(i) IR
(B) Balmer	(ii) IR
(C) Paschen	(iii) Visible
(D) Pfund	(iv) UV

   (1) A → (iv), B → (iii)
   C → (i), D → (ii)

   (2) A → (i), B → (iii)
   C → (ii), D → (iv)

(3) A → (iv), B → (ii)
C → (iii), D → (iv)

(4) A → (i), B → (ii)
C → (iii), D → (iv)

Answer (1)

Sol. Lyman → UV

Balmer → Visible

Paschen → IR

Pfund → IR

6. IUPAC name of K_2MnO_4 is

(1) Potassium tetraoxomanganate(VI)

(2) Potassium tetraoxomanganate(III)

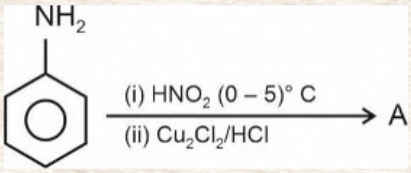
(3) Potassium tetraoxomanganese(VI)

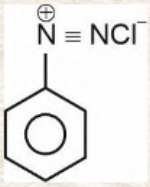
(4) Tetraoxomanganese(VI) potassium

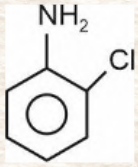
Answer (1)

Sol. Correct IUPAC name of K_2MnO_4 is Potassium tetraoxomanganate(vi)

7. Find out final product (A)



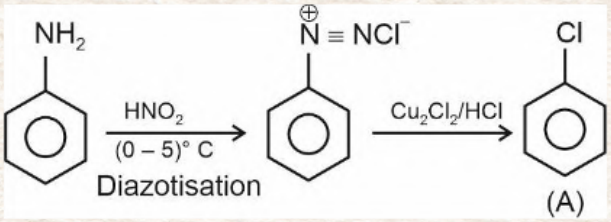
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

8. Which of the following element has highest 1st Ionization energy?

(1) N (2) C

(3) Si (4) Al

Answer (1)

Sol. N has highest 1st Ionization energy among C, Si, N and Al.

For, N = 1402 kJ mol^{-1} (IE_1)

C = 1086 kJ mol^{-1} (IE_1)

Al = 577 kJ mol^{-1} (IE_1)

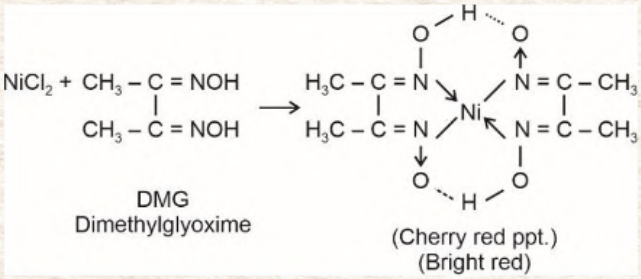
Si = 786 kJ mol^{-1} (IE_1)

9. Which reagent gives bright red ppt with Ni^{2+} in basic medium?

(1) DMG (2) Nessler's reagent

(3) KCNS (4) $K_4[Fe(CN)_6]$

Answer (1)

Sol. 

10. Match the following List-I and List-II

	List-I (Polymer)		List-II (Monomer)
(A)	Starch	(i)	β-glucose
(B)	Cellulose	(ii)	Nucleotide
(C)	Nucleic acid	(iii)	α-glucose
(D)	Protein	(iv)	α-Amino acid

(1) A → (i); B → (iii); C → (ii); D → (iv)

(2) A → (iii); B → (i); C → (ii); D → (iv)

(3) A → (iii); B → (i); C → (iv); D → (ii)

(4) A → (ii); B → (iii); C → (i); D → (iv)

Answer (2)

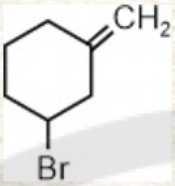
Sol. Starch is polymer of α-D-glucose. Cellulose is polymer of β-D-glucose. Nucleic acid is polymer of nucleotide. Proteins are polymer of α-aminoacids.

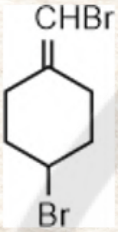
Paper Solutions Shift-02

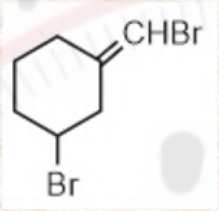
(Memory Based Solutions)

11. Which of the following can show geometrical isomerism?

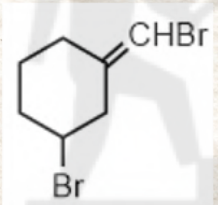
(1) 

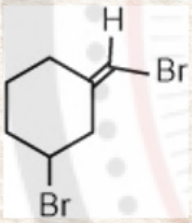
(2) 

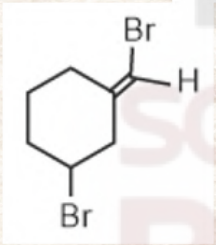
(3) 

(4) 

Answer (4)

Sol. The two geometrical isomers of 

are 

and 

12. Which reagent is used to convert alkyl halide into alkyl isocyanide?

(1) KCN (2) AgCN

(3) KNO_2 (4) $AgNO_2$

Answer (2)

Sol. $R-X+AgCN \rightarrow R-N\equiv C+AgX$

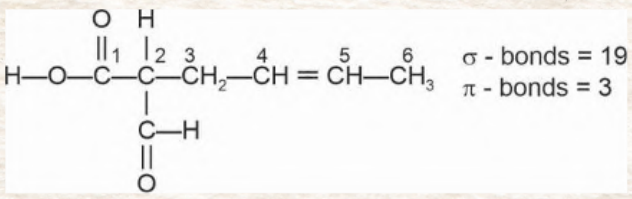
13. Find the total number of sigma (σ) and π bonds in 2-formylhex-4-enoic acid.

(1) 20 (2) 22

(3) 18 (4) 24

Answer (2)

Sol. The structure of 2-formylhex-4-enoic acid is



14. A gas 'X' is added to Nessler's reagent then brown precipitate is formed, gas X is

(1) NH_3 (2) SO_2

(3) Cl_2 (4) Br_2

Answer (1)

Sol. $\underset{\text{Nessler's reagent}}{2K_2HgI_4+3KOH+NH_3} \longrightarrow \underset{\text{Brown ppt}}{[OHg_2\cdot NH_2]I}+7KI+2H_2O$

Ammonia gas on reaction with Nessler's reagent to form brown ppt. Brown ppt formed is also called iodide of million's base ($H_2N-Hg-O-Hg-I$)

15. Match the following

I (compounds)		II (pKa)	
(a)	p-nitrophenol	(i)	10
(b)	m-nitrophenol	(ii)	16
(c)	Ethanol	(iii)	7.1
(d)	Phenol	(iv)	8.3

(1) (a)→(i); (b)→(ii); (c)→(iii); (d)→(iv)

(2) (a)→(iii); (b)→(iv); (c)→(ii); (d)→(i)

(3) (a)→(iv); (b)→(iii); (c)→(ii); (d)→(i)

(4) (a)→(iii); (b)→(iv); (c)→(i); (d)→(ii)

Answer (2)

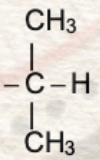
Sol. Acidic strength order:

p-nitrophenol > m-nitrophenol > Phenol >> ethanol

16. We have given some hydrocarbons

(A) $HC\equiv CH$

(B) $H_2C=CH_2$

(C) 

(D) $CH_3-CH_2-CH_2-H$

Correct order of acidic strength of above hydrocarbons.

(1) A > B > C > D (2) A > B > D > C

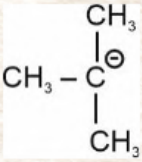
(3) C > D > B > A (4) A > C > B > D

Answer (2)

Sol. More the stability of conjugate base of given acids, more will be the acidic strength.

(A) $HC\equiv C^{\ominus}$ (more % s character more will be stability of anion)

(B) $H_2C=CH^{\ominus}$

(C)  (Alkyl group increases electron density on carbon so stability decreases)

(D) $CH_3 - CH_2 - \overset{\ominus}{C}H_2$

Order of stability of conjugate base

A > B > D > C

So order of acidic strength

A > B > D > C

17. In chromatographic techniques, which of the following follows preferential adsorption?

(A) Column chromatography

(B) Thin layer chromatography

(C) Paper chromatography

(1) A only (2) B only

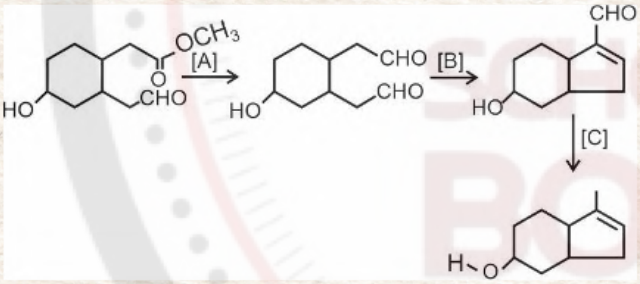
(3) C only (4) A and B both

Answer (4)

Sol. Column chromatography, Thin layer chromatography] Separation based on absorption of substance

Paper chromatography→ Partition chromatography

18. Consider the following sequence of reactions



Fina A, B and C

(1) A: DiBAL-H

B: NaOH (dil)

C: Zn – Hg/HCl

(2) A: $LiAlH_4$

B: KOH (alcoholic)

C: $NH_2 - NH_2/KOH$

(3) A: DiBAL – H

B: NaOH (dil)

C: $NH_2 - NH_2/KOH$

(4) A: $NaBH_4$

B: KOH (aqueous)

C: Zn – Hg/HCl

Answer (3)

Sol. (A) DiBALH – Convert ester to aldehyde

(B) dil NaOH – Aldol condensation

(C) $NH_2 - NH_2/KOH$ – Wolff Kishner reduction

19. The correct statement about Zn, Cd, Hg are

(1) All are solid metals at room temperature

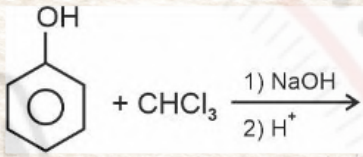
(2) They have high enthalpy of atomization

(3) All are paramagnetic

(4) Zn, Cd cannot show variable oxidation state but Hg can show variable oxidation state

Answer (4)

Sol. Hg can show +1 and +2 O.S.

20. 

The major product in the above reaction is

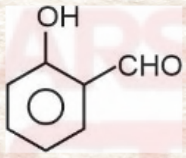
(1) 2-hydroxybenzaldehyde

(2) 2-hydroxybenzoic acid

(3) 4-hydroxybenzaldehyde

(4) 3-hydroxybenzaldehyde

Answer (1)

Sol. 

is the major product in Reimer-Tiemann reaction

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Oxidation state of Fe (Iron) in complex formed in brown ring test.

Answer (1)

Sol. Complex formed during brown ring test is $[Fe(H_2O)_5NO]SO_4$.

NO is present as NO^+ here.

$x + 5 \times 0 + 1 = +2$

$\boxed{x = +1}$

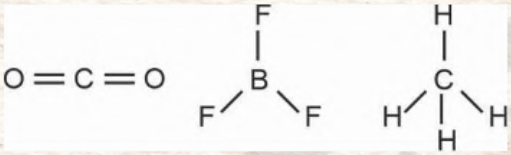
Oxidation state of Fe is +1

22. How many of the following compounds have zero dipole moment?

NH_3, H_2O, HF, CO_2, SO_2, BF_3, CH_4

Answer (3)

Sol. CO_2, BF_3 and CH_4 have symmetrical structures leading to $\mu = O$



23. Calculate equilibrium constant for the given following reaction at 500K.

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

Given molarity of $NH_3(g)$, $N_2(g)$ and $H_2(g)$ at equilibrium is 1.5×10^{-2}M, 2×10^{-2} M and 3×10^{-2} M respectively.

Answer (417)

Sol. $$K_C = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

$$K_C = \frac{(1.5\times10^{-2})^2}{(2\times10^{-2})\times(3\times10^{-2})^3}$$

$$K_C = \frac{2.25\times10^{-4}}{2\times10^{-2}\times27\times10^{-6}}$$

$$K_C = 0.04167\times10^4$$

$$K_C = 416.7 \approx 417$$

24. 50 ml of 0.5 M oxalic acid is completely Neutralised by 25 ml of NaOH solution. Find out amount of NaOH (in gm) present in 25 ml of given NaOH solution.

Answer (2)

Sol. $M_1V_1N_1 = M_2V_2N_2$

$(50)\ (0.5)\ (2) = (M_2)\ (25)\ (1)$

$M_2 = 2$

Moles of NaOH = $\frac{2\times25}{1000} = \frac{1}{20}$

Mass of NaOH = $\frac{1}{20}\times40 = 2$gm

25. If standard enthalpy of vaporization of CCl_4 is 30.5 kJ/mol, find heat absorbed for vaporization of 294 gm of CCl_4. [Nearest integer] [in kJ]

Answer (58)

Sol. Vaporization of 1 mole CCl_4 requires 30.5 kJ

294 gm is $\frac{294}{154} = 1.91$ moles

Vaporization of 1.91 moles of CCl_4 will require 30.5×1.91 kJ = 58.255 kJ

26. Find out molality of 0.8 M H_2SO_4 solution having density of solution equal to 1.02 gm/ml (Nearest integer)

Answer (1)

Sol. $$m = \frac{1000\,M}{10008 - M(\mu)}$$

$$= \frac{1000\,(0.8)}{1000\,(1.02) - (0.8)\,(98)} = \frac{800}{1020 - 78.4}$$

$$= \frac{800}{941.6} = 0.849$$

$$\approx 1$$

27. Aqueous solution of $[AuCl_4]^-$ on electrolysis by passing current for 10 minutes, the mass of Au deposited at Cathode is 1.97 gm. Find out current required (in A) (Nearest integer)

Answer (5)

Sol. $Au^{3+} + 3e^- \longrightarrow Au(s)$

1.97 gm

0.03 mole $\qquad \frac{1.97}{197} = 0.01$mole

Charge = 0.03×96500

Current = $\frac{0.03\times96500}{10\times60}$

$= 4.825$ A

≈ 5A

28. If half life of radioactive bromine (Br-82) is 36 hr, find percentage remaining after one day. [nearest integer]

Answer (63)

Sol. $$\ln\frac{N_0}{N} = \lambda t = \frac{\ln2}{36}\times24$$

$$= \frac{2}{3}\ln2$$

$$\Rightarrow \frac{N_0}{N} = 2^{2/3}$$

$$\Rightarrow \frac{N}{N_0} = \frac{1}{2^{2/3}}$$

% age remaining = $100\,\frac{N}{N_0} = \frac{100}{2^{2/3}} = 62.99$

29.

30.

Mathematics

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Given set = {1, 2, 3, …, 50}

One number is selected randomly from set. Find probability that number is multiple of 4 or 6 or 7.

(1) $\frac{21}{50}$ (2) $\frac{18}{50}$

(3) $\frac{8}{25}$ (4) $\frac{21}{25}$

Answer (1)

Sol. Take $P(A)$ = Probability that number is multiple of 4

$P(B)$ = Probability that number is multiple of 6

$P(C)$ = Probability that number is multiple of 7

$P(A) = \frac{12}{50}$, $P(B) = \frac{8}{50}$, $P(C) = \frac{7}{50}$

$P(A \cap B) = \frac{4}{50}$ (Multiple of 12)

$P(B \cap C) = \frac{1}{50}$ (Multiple of 42)

$P(A \cap C) = \frac{1}{50}$ (Multiple of 28)

$P(A \cap B \cap C) = 0$ (Multiple of 84)

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} + 0$

$= \frac{21}{50}$

2. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}\,dx$ is

(1) $\sqrt{2} - \sqrt{3} + 1$ (2) $2\sqrt{2} - \sqrt{3} - 1$

(3) $2\sqrt{2} + \sqrt{3} - 1$ (4) $\sqrt{2} + \sqrt{3} - 1$

Answer (2)

Sol. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |\sin x - \cos x|\,dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x - \cos x)\,dx$

$= (\sin x + \cos x)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} + (-\sin x - \cos x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(\sin\frac{\pi}{6} + \cos\frac{\pi}{6}\right)\right] + \left[\left(-\sin\frac{\pi}{3} - \cos\frac{\pi}{3}\right) - \left(-\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)\right]$

$= \left[\sqrt{2} - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)\right] + \left[-\frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$

$= 2\sqrt{2} - \sqrt{3} - 1$

3. A = {1, 2, 3, 4} minimum number of elements added to make it equivalence relation on set A containing (1, 3) and (1, 2) in it.

(1) 8 (2) 9

(3) 12 (4) 16

Answer (1)

Sol. Set A = {1, 2, 3, 4}

For reflexive relation

We need to have (1, 1), (2, 2), (3, 3), (4, 4).

For symmetric,

(1, 3) $\in$A

So (3, 1) should be added

And (1, 2) $\in$A

So (2, 1) should be added

set has become {(1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (3, 1), (1, 2), (2, 1)}

Now (3, 1) $\in$A

(1, 2) $\in$A

So (3, 2) should be added (for transitive)

Then (2, 3) should be added (for symmetric)

So set becomes

{(1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (3, 1), (1, 2), (2, 1), (3, 2), (2, 3)}

So minimum 8 elements are added

4. If $\ln a, \ln b, \ln c$ are in AP and $\ln a - \ln 2b, \ln 2b - \ln 3c, \ln 3c - \ln a$ are in AP then $a : b : c$ is

(1) 1 : 2 : 3 (2) 7 : 7 : 4

(3) 9 : 9 : 4 (4) 4 : 4 : 9

Answer (3)

Sol. $\ln a, \ln b, \ln c \rightarrow$ AP

$\Rightarrow b^2 = ac$ ...(i)

$\ln\frac{a}{2b}, \ln\frac{2b}{3c}, \ln\frac{3c}{a}$ $\rightarrow$ AP

$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b}\times\frac{3c}{a}$

$\frac{4b^2}{9c^2} = \frac{3c}{2b}$

$8b^3 = 27c^3$

$\boxed{2b = 3c}$ ...(ii)$\Rightarrow$ $\boxed{4b = 9c}$

$4b^2 = 9c^2$

$4ac = 9c^2$

$\Rightarrow \boxed{4a = 9c}$ ...(iii)

From (ii) & (iii)

$4a = 9c = 4b = k$

$a = \frac{k}{4}\ b = \frac{k}{4}\ c = \frac{k}{9}$

$a : b : c = \frac{1}{4} : \frac{1}{4} : \frac{1}{9}$

$a : b : c = 9 : 9 : 4$

5. If $r = |z|$, $\theta = \arg(z)$ and $z = 2 - 2i\tan\left(\frac{5\pi}{8}\right)$ then find (r, θ)

(1) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$ (2) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

(3) $\left(2\tan\frac{3\pi}{8}, \frac{5\pi}{8}\right)$ (4) $\left(2\tan\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

Answer (2)

Sol. $z = 2 - 2i\dfrac{\sin\frac{5\pi}{8}}{\cos\frac{5\pi}{8}}$

$= \dfrac{2}{\cos\frac{5\pi}{8}}\left(\cos\frac{5\pi}{8} - i\sin\frac{5\pi}{8}\right)$

$= \dfrac{2}{\cos\frac{5\pi}{8}}e^{i\frac{(-5\pi)}{8}}$

$= 2\sec\left(\frac{5\pi}{8}\right)e^{i\frac{(-5\pi)}{8}}$

$= 2\sec\left(\frac{3\pi}{8}\right)e^{ir}\ e^{i\frac{(-5\pi)}{8}}$

$= 2\sec\frac{3\pi}{8}e^{i\frac{(3\pi)}{8}}$

$\theta = \frac{3\pi}{8}, r = 2\sec\frac{3\pi}{8}$

6. In which interval the function $f(x) = \dfrac{x}{x^2 - 6x - 16}$ is increasing?

(1) ϕ (2) $\left[1, \frac{3}{4}\right)\cup\left(\frac{5}{4}, \infty\right)$

(3) $\left(\frac{5}{4}, \infty\right)$ (4) $\left[\frac{3}{4}, \frac{5}{4}\right]$

Answer (1)

Sol. $f(x) = \dfrac{x}{x^2 - 6x - 16}$

$f'(x) = \dfrac{(x^2 - 6x - 16) - (x)(2x - 6)}{(x^2 - 6x - 16)^2}$

$\Rightarrow \dfrac{-x^2 - 16}{(x^2 - 6x - 16)^2} < 0\ \forall\ x \in D_f$

$\therefore\ x \in \phi$

7. (α, β) lie on the parabola $y^2 = 4x$ and (α, β) also lie on chord with mid-point $\left(1, \frac{5}{4}\right)$ of another parabola $x^2 = 8y$, then value of $|(8 - \beta)(\alpha - 28)|$ is

(1) 192 (2) 92

(3) 64 (4) 128

Answer (1)

Sol. Chord with point, $T = S_1$

$\Rightarrow\ xx_1 - 4(y + y_1) = x_1^2 - 8y_1$

$(x_1, y_1) \equiv \left(1, \frac{5}{4}\right) \Rightarrow x - 4\left(y + \frac{5}{4}\right) = \frac{1 - 8\times 5}{4}$

$x - 4y - 5 = -9$

$\Rightarrow x - 4y + 4 = 0$ $(L1)$

(α, β) lie on $(L1)$ and also $y^2 = 4x$

$\Rightarrow \alpha - 4\beta + 4 = 0$

$\beta^2 = 4\alpha$

$\beta^2 = 4(4\beta - 4)$

$\beta^2 - 16\beta + 16 = 0$

$\Rightarrow (\beta-8)^2 = 64-16 = 48$

$\Rightarrow \beta = 8 \pm 4\sqrt{3}$

$\alpha = 4\beta - 4$

$= 28 \pm 16\sqrt{3}$

$(28+16\sqrt{3},\ 8+4\sqrt{3})$ and $(28-16\sqrt{3},\ 8-4\sqrt{3})$

$(8-\beta)(\alpha-28)$

$\Rightarrow (-4\sqrt{3})(16\sqrt{3})$

$= -192$

8. Unit vector $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$ makes angles $\frac{\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$ with $\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}\right), \left(\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right), \left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right)$ respectively and $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}$ find $\vec{u} - \vec{v}$.

(1) $\sqrt{\frac{5}{2}}$ (2) $\sqrt{\frac{7}{2}}$

(3) $\sqrt{\frac{2}{5}}$ (4) $\sqrt{\frac{2}{7}}$

Answer (1)

Sol. $\frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$ ...(1)

$\frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2}$ ...(2)

$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{-1}{2}$ ...(3)

$\Rightarrow y = 0,\ z = \frac{1}{\sqrt{2}},\ x = \frac{-1}{\sqrt{2}}$

$v - u = \sqrt{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

$|\vec{v} - \vec{u}| = \sqrt{2 + \frac{1}{2}}$

$= \sqrt{\frac{5}{2}}$

9. If first term of non-constant GP be $\frac{1}{8}$ and every term is AM of next two, then $\sum_{r=1}^{20} T_r - \sum_{r=1}^{18} T_r$ is

(1) 2^{15} (2) -2^{15}

(3) -2^{18} (4) 2^{18}

Answer (2)

Sol. $a_1 = \frac{1}{8}$

$a, ar, ar^2, ar^3 \ldots\ldots$

$2ar = ar^2 + ar^3$

$2 = r + r^2$

$r^2 + r - 2 = 0$

$(r+2)(r-1) = 0$

$r \neq 1$

$\Rightarrow r = -2$

$\sum_{r=1}^{20} T_r - \sum_{r=1}^{18} T_r$

$= \frac{a(1-r^{20})}{1-r} - \frac{a(1-r^{18})}{1-r}$

$= \frac{1}{8}\left[\frac{1}{3}\left[1 - r^{20} - 1 + r^{18}\right]\right]$

$= \frac{1}{24}2^{18}[1-4]$

$= -\frac{2^{18}}{8} \Rightarrow -2^{15}$

10. The mean of 5 observations is $\frac{24}{5}$ and variance is $\frac{194}{25}$. If the mean of first four observations is $\frac{7}{2}$ then the variance of first four observations is

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$

(3) $\frac{5}{4}$ (4) $\frac{2}{3}$

Answer (3)

Sol. $\sum_{i=1}^{5} x_i = 24$

$\frac{\sum x_i^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$

$\Rightarrow \sum x_i^2 = \frac{770}{25} \times 5 = 154$

5^{th} observation $= 24 - \frac{7}{2} \times 4 = 10$

New variance $= \frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{7}{2}\right)^2$

$= \frac{154 - 100}{4} - \frac{49}{4}$

$= \frac{5}{4}$

Pending

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The remainder when $64^{32^{32}}$ is divided by 9 is

Answer (1)

Sol. $64 \equiv 1 \pmod 9$

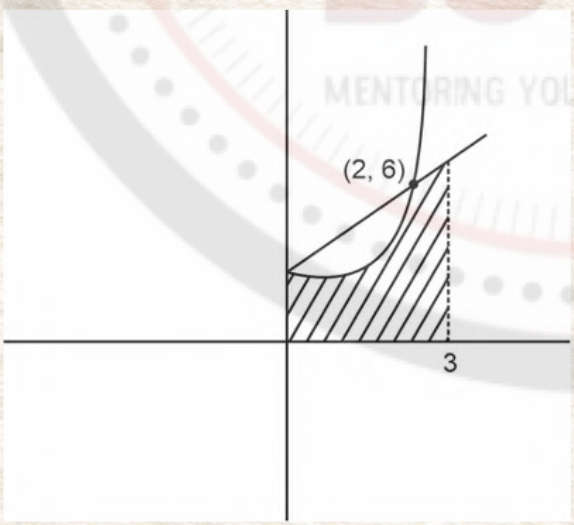
$64^{32^{32}} \equiv 1^{32^{32}} \pmod 9$

$\Rightarrow$ Remainder = 1

22. Area bounded by $0 \le y \le \min\{x^2 + 2, 2x + 2\}$, $x \in [0, 3]$ is A, then 12 A is

Answer (164)

Sol. $\min\{x^2+2,\ 2x+2\} \begin{cases} x^2+2 & 0 \le x \le 2 \\ 2x+2 & 2 \le x \le 3 \end{cases}$



Area = $A = \int_0^2 (x^2+2)dx + \frac{1}{2}[6+8]\times 1$

$= \frac{x^3}{3} + 2x \Big]_0^2 + 7$

$\frac{8}{3} + 4 + 7 = \left(\frac{8}{3} + 11\right)$ unit

$12A = 12\left(\frac{8}{3} + 11\right) = 164$

23. The number of ways to distribute 8 identical books into 4 distinct bookshelf is (where any bookshelf can be empty)

Answer (165)

Sol. $x_1 + x_2 + x_3 + x_4 = 8$

Number of ways = $\binom{8+4-1}{4-1}$

$= \binom{11}{3}$

$= 165$

24. If $f(x) = \ln\left(\frac{1-x^2}{1+x^2}\right)$ then value of $225(f'(x) - f''(x))$ at $x = \frac{1}{2}$

Answer (736)

Sol. $f(x) = \ln(1 - x^2) - \ln(1 + x^2)$

$f'(x) = \frac{-2x}{1-x^2} - \frac{2x}{1+x^2}$

$= -2x\left[\frac{2}{1-x^4}\right]$

$f'(x) = \frac{4x}{x^4-1}$

$f''(x) = 4\left[\frac{(x^4-1) - 4x^4}{(x^4-1)^2}\right]$

$= 4\left[\frac{-3x^4-1}{(x^4-1)^2}\right]$

$f'(x) - f''(x) = 4\left[\frac{x}{x^4-1} + \frac{3x^4+1}{(x^4-1)^2}\right]$

At $x = \frac{1}{2}$

$225[f'(x) - f''(x)] = 736$

25. $\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x}=x^3-x^2+6$, then find sum of roots,

Answer (1)

Sol. $\therefore$

$$\frac{\cos 2x(3+\cos^2 2x)}{(\cos^2 x-\sin^2 x)[\sin^4 x+\cos^4 x+\sin^2 x\cos^2 x]},$$

$\cos^2x-\sin^2x=\cos 2x$

$$=\frac{3+\cos^2 2x}{1-\sin^2 x\cos^2 x}=4\left(\frac{3+\cos^2 2x}{4-\sin^2 2x}\right)=4$$

$\Rightarrow x^3-x^2+6=4$

$\Rightarrow x^3-x^2+2=0$

$\therefore$ therefore sum of roots = 1

26. $x\left(\cos\left(\frac{y}{x}\right)\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$

where $\sin\left(\frac{y}{x}\right)=\ln|x|+\frac{\alpha}{2}$ and $f(1)=\frac{\pi}{3}$

Find α^2.

Answer (3)

Sol. $\left(\cos\frac{y}{x}\right)\frac{dy}{dx}=\frac{y}{x}\cos\frac{y}{x}+1$

Putting $y=vx$

$$\Rightarrow \frac{dy}{dx}=x\frac{dv}{dx}+v$$

$$\Rightarrow \cos v\left(x\frac{dv}{dx}+v\right)=v\cos v+1$$

$$\Rightarrow \int\cos v\,dv=\int\frac{dx}{x}$$

$$\Rightarrow \sin\frac{y}{x}=\ln|x|+c$$

where $c=\frac{\alpha}{2}$

putting initial condition,

$$2\sin\frac{\pi}{3}=\alpha$$

$$\Rightarrow \alpha=\sqrt{3}$$

$$\Rightarrow \alpha^2=3$$

27. If $OA=a$, $\vec{OC}=b$, and area of ΔOAC is S and a parallelogram with sides parallel to OA and OC and diagonal $OB=12\vec{a}+4b$, has area equal to B, then $\frac{B}{S}$ is equal to

Answer (96)

Sol. $S=\frac{1}{2}|\vec{a}\times\vec{b}|$

$$B=|12\vec{a}\times 4\vec{b}|$$

$$\Rightarrow \frac{B}{S}=\frac{48|\vec{a}\times\vec{b}|}{\frac{1}{2}|\vec{a}\times\vec{b}|}=96$$

28.

29.

30.

Rough Work

THE END