Technical Change, Task Allocation, and Labor Unions

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March 22, 2025

Abstract

We develop a general equilibrium model that combines the task approach, labor union wage setting, and search and matching frictions. We show that skill-biased technical change (SBTC) affects wage setting through changes in firms' task assignments to high- and low-skilled workers. The effect of this task reallocation depends on the shape of the relative task productivity schedule. In our framework, SBTC can harm low-skilled workers either by reducing real wages or increasing unemployment. We illustrate the mechanisms by calibrating the model using German and French data for the periods 1995–2005 and 2010–2017, and additionally estimate the relative task productivity schedule.

Keywords: task approach, labor unions, wage setting, skill-biased technical change, search and matching, labor demand

JEL Classification: J64, J51, E23, E24, O33

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We would like to thank the participants of the seminars at the Federal Reserve Bank of Philadelphia (2024), Federal Reserve Board in Washington, D.C. (2023), and University College Dublin (2023), and of the following conferences for helpful comments: the Society of Labor Economists (SOLE) Conference in Portland (2024), the Royal Economic Society Annual Conference in Belfast (2024), the AFSE Annual Congress in Paris (2023), the Warsaw International Economic Meeting (2023), and the conference of the German Economic Association in Basel (2022). We are grateful to Alice Krause for her support with the SOEP data selection and Audra Bowlus for kindly providing us with data for the calculation of the skill bias used in the calibration of our previous paper version. We declare that we have no relevant or material financial interests that relate to the research described in this paper.

1 Introduction

Skill-biased technical change (SBTC) has long been analyzed through the lens of conventional production theory, which views the production process as a "black box", largely unspecified beyond basic assumptions regarding the marginal products of production factors, returns to scale, and elasticity of substitution. For many economic applications, such coarse modeling of production has proven to be sufficient. However, in the context of SBTC, this risks overlooking possible adverse effects of technological progress on labor market outcomes. To shed light on the broader implications of SBTC, recent literature advocates conceptualizing the production process as a composition of various tasks. These tasks are allocated to production factors based on the principles of comparative advantage and optimal assignment theory. The distinction between skills and tasks becomes particularly important when, as in our paper, workers with a given skill level can potentially perform multiple tasks, and technological progress influences the equilibrium mapping between production tasks and workers of different skill groups, see Autor (2013) for an overview.

So far, the literature analyzing the impact of SBTC on the task allocation between low- and high-skilled workers, and its subsequent effects on the wage levels of both skill groups in general equilibrium, predominantly relies on perfect competition models. For the seminal work in this area, refer to Acemoglu and Autor (2011). These models often abstract from the frictions and imperfections inherent in real-world labor markets, such as search and matching frictions and the important role of collective bargaining in wage determination, which is particularly relevant within European contexts. In their concluding remarks, Acemoglu and Autor (2011, p. 1160) state that "Certain work practices, such as collective bargaining and unionized workplace arrangements, might have greater impact on the earnings distribution because of the way they impact the assignment of tasks to labor or capital." They conclude [p.1160]: "Further work tractably integrating various forms of labor market imperfections within a framework that incorporates the endogenous allocation of skills to tasks appears to be another fruitful area for research."

This paper seeks to bridge this gap in the literature and provides novel theoretical insights by adopting realistic modeling assumptions. Specifically, we assume that imperfect competition prevails in the labor market and that firms optimally adjust task assignments in response to changes in both the productivity and wages of high- and low-skilled workers. The proposed framework combines the task-based approach, union-influenced wage setting, and search and matching frictions. It distinguishes between two worker categories: low-skilled and high-skilled. The low-skilled category is meant to represent low- and medium-skilled workers in the real world, whereas the high-skilled category comprises highly educated individuals with advanced degrees. In the model, matching frictions only exist for low-skilled workers, who are also the sole group represented by labor unions. These assumptions are based on the observation that low- (and medium-) skilled workers often face greater challenges in finding suitable jobs due to a mismatch between their skills and job requirements, and labor unions often strive to reduce skill premia by acting more on behalf of low- and medium-skilled workers. Conversely, high-skilled workers are assumed to operate under perfect competition because they are usually well-informed about job opportunities and benefit from competitive wage determination. In the production process, both low- and high-skilled workers are, in principle, capable of performing all tasks, but high-skilled workers have a comparative advantage in more complex tasks. Each firm uses both skill groups in production and must decide how to allocate low- and high-skilled workers to tasks based on relative wages and relative productivity in the respective tasks. In the profit maximum, the representative firm determines a task threshold below which all tasks are performed by low-skilled workers, while tasks above the threshold are assigned to high-skilled workers.

The central focus of our paper is to investigate how labor unions respond to firms' incentives to modify the task allocation between low- and high-skilled workers due to SBTC. We model SBTC as an increase in the productivity of high-skilled workers across all tasks. Our analysis reveals two important channels through which SBTC-induced task reallocation impacts labor unions' behavior and consequently wage determination. These channels are absent in task-based perfect competition models. First, in a labor market with matching frictions, firms intending to change their task composition will likely adjust their posted vacancies, thereby affecting overall labor market tightness. This, in turn, influences the outside option of labor unions, the level of wage pressure in the economy, and labor market flows. Second, changes in task assignments trigger changes in the wage-setting behavior of labor unions by affecting the wage elasticity of labor demand, thereby impacting unions' wage markups. This channel has not yet been explored in the literature and deserves more attention, as it largely determines whether low-skilled workers are ultimately harmed by SBTC.

While it is well known that labor unions' wage markups inversely correlate with the labor demand elasticity, we offer a novel contribution by investigating the interplay between labor demand elasticity and task allocation. We demonstrate that even with a simple Cobb-Douglas structure at the task level, the labor demand elasticity is generally not constant but depends on the task threshold that divides the range of tasks assigned to low-skilled and high-skilled workers. How changes in the task threshold affect the wage elasticity of labor demand is determined by the shape of the relative task productivity schedule. This schedule describes the comparative advantage of the two skill groups in performing the various tasks. Since we allow for both concave and convex shapes of the relative task productivity schedule, the impact of changes in the task threshold on the labor demand elasticity is generally ambiguous. It can be explained by different degrees of substitutability between high- and low-skilled workers because, in general, the elasticity of substitution between low- and high-skilled workers depends on the task assignment, which may change due to SBTC. To provide some intuition, consider, for example, the case of a concave task productivity schedule. Such a shape implies that the increase in the comparative advantage of high-skilled workers decreases with increasing task complexity, or, in other words, both skill groups become more similar. Therefore, the substitutability of skills increases with increasing task complexity, and it declines with decreasing task complexity. The latter becomes relevant in the SBTC context—when SBTC increases the productivity of high-skilled workers across all tasks, firms *cet. par.* want to expand the range of tasks performed by high-skilled workers and lower the task threshold. With the declining threshold task complexity, low-skilled workers become less substitutable, which lowers the labor demand elasticity and increases the wage markup.

To establish the general equilibrium effects of SBTC, we derive a two-equation system from the model, analogous to standard search-and-matching models. This system describes the job creation activities of firms and the wage-setting behavior of labor unions for low-skilled workers. While the job creation curve, plotted with labor market tightness on the horizontal axis and the real wage on the vertical axis, has the usual negative slope, the wage curve can slope either upwards or downwards. With an upward-sloping wage curve, low-skilled workers benefit from an increase in the productivity of high-skilled workers, evidenced by both higher employment and higher real wages. Conversely, with a downward-sloping wage curve, low-skilled workers may be adversely affected by either an increase in unemployment or a reduction in real wages. The key factor determining the slope of the wage curve is the response of the labor demand elasticity to changes in the task threshold. The slope becomes negative if the labor demand elasticity increases with the task threshold, and if this reaction is strong, thus implying strong upward wage pressure when the task threshold decreases. Our results highlight that the task-allocation channel of SBTC, influencing the wage-setting behavior of labor unions through task allocation, is relevant for two reasons: (i) it can provide a technological explanation for varying degrees of wage rigidity across sectors and countries, as reflected in different slopes of the wage curve; and (ii) it may imply adverse labor-market effects of SBTC for low-skilled workers. The latter stands in contrast to the predictions of "canonical models", i.e., models using the conventional production function approach.[1]

To complement the theoretical analysis, we calibrate our model using data from Germany and France for the periods 1995–2005 and 2010–2017. We select these countries because both are comparably large economies with still high union coverage rates. The first objective of the quantitative analysis is to calibrate the model to align with real-world data, thereby identifying the unobserved parameters of the relative task productivity schedule. The second objective is to demonstrate that the effects of SBTC can vary across countries and periods, even when these countries have similar economic conditions. We interpret the mean values of selected labor market variables for each period and country as steady-state values. We then examine the consequences of a ten percent increase in the skill bias of production technologies on the respective steady states. We consider two different periods to illustrate that SBTC may have opposing impacts on variables such as labor market tightness within the same country, depending on the steady state.

According to the results, for the first period, SBTC leads to a significant decline in the labor demand elasticity, which substantially increases labor unions' wage markups. This contributes to decreased labor market tightness and increased unemployment among low-skilled workers in both countries. In Germany, a similar result is obtained for the second period, whereas in France, the impact of SBTC on the labor demand elasticity weakens, implying only a moderate increase in wage pressure. Consequently, low-skilled labor market tightness increases, and the unemployment rate declines. In both countries and periods, both low- and high-skilled workers experience real wage increases, but the

[1] In the canonical model, low-skilled workers usually benefit from SBTC in terms of higher real wages and higher labor demand. However, high-skilled workers benefit more from SBTC, as relative labor demand for high-skilled workers and the skill premium rise. For a full description of the implications of the canonical model, see Acemoglu and Autor (2011).

increase is significantly more pronounced for high-skilled workers, leading to a marked rise in the skill premium. We infer that the extent of the decline in the labor demand elasticity is essential in determining the labor market outcomes of SBTC in our simulation exercise. This decline depends on the characteristics of the initial steady state, specifically the task productivity schedule and the task allocation in this steady state.

Since the calibrated task productivity schedule plays a crucial role in driving the results regarding the SBTC effects, we aim to evaluate the plausibility of the calibration exercise by estimating a task productivity schedule. To achieve this, we derive a model-based estimation equation that allows us to obtain an (approximated) task productivity schedule compatible with real-world data. For the estimation, we utilize German microdata from the SOEP and approximate tasks using occupational data, resulting in task productivity schedules for Germany over two periods. The shapes of these estimated schedules align with those of the corresponding calibrated counterparts, which reinforces our confidence in the reliability of the SBTC effects derived from the calibration exercise.

The remainder of the paper is structured as follows: Section 2 reviews the literature related to our study. Section 3 introduces the model and discusses the implications of changes in the task threshold on the wage elasticity of labor demand for low-skilled workers and on labor unions' wage-setting behavior. Section 4 conducts a comparative-static analysis of the labor market consequences of SBTC. Section 5 calibrates the model using data from Germany and France. Section 6 provides empirical evidence on the shape of the relative task productivity schedule using German microdata from the SOEP. Section 7 summarizes the results and provides concluding remarks.

2 Related Literature

Our paper sheds new light on the labor market consequences of SBTC by incorporating the task approach into a model with labor union wage setting and matching frictions. It is shown that SBTC can adversely affect low-skilled workers through lower wages or higher unemployment, contrary to conventional models. The focus is on two transmission channels of SBTC that are not considered in models with fixed task allocation and perfect competition in labor markets. One channel examines the effects of SBTC on labor market tightness and labor unions' outside options, driven by the extensive margin reallocation of tasks to different worker categories (in addition to the intensive margin adjustment of labor inputs within tasks). The other channel investigates the effects of task reallocations on labor unions' wage markups, brought about by changes in the wage elasticity of labor demand, which, due to the task-based approach, can no longer be considered isoelastic. Our analysis complements the extensive literature on the effects of SBTC by revealing the important role of the curvature of the relative task productivity schedule in influencing unions' wage markups, the slope of the aggregate wage curve, and the comparative static results in general equilibrium.

Our paper is related to the literature modeling the production process with the task approach. In an early contribution, Rosen (1978) explains the structure of work activities within firms by developing a theory of optimum assignment of workers to tasks based on the principle of comparative advantage. Our paper is influenced by the seminal contribution of Acemoglu and Autor (2011) who demonstrate how the firms' optimum assignment decisions can be integrated into a general equilibrium model of SBTC and compare the consequences of SBTC in the canonical and task-based model. Follow-up papers, such as Acemoglu and Restrepo (2018a, 2018b, 2022), and Hémous and Olsen (2022), develop task-based approaches for the analysis of the labor-market consequences of automation. Assuming perfect competition in the labor market, these papers focus on the impact of SBTC or automation on wages and the labor share but do not consider the effects on collective bargaining, unemployment, or labor market tightness.[2]

Our paper focuses on the impact of SBTC on labor unions' wage setting brought about by changes in the firms' task allocation. A few papers also look at the impact of SBTC on labor unions but focus on the question of whether SBTC may be responsible for the deunionization observed in the US and other countries. In their theoretical analysis, Acemoglu et al. (2001) start with the well-known fact that unions compress the wage structure. If SBTC is limited, skilled workers accept to work at unionized firms because of the benefits provided by unions. However, with strong SBTC, the outside option, i.e. the competitive market return, for skilled workers increases. This weakens their incentive to join the unionized sector. In that sense, SBTC leads to deunionization which amplifies the original effect of SBTC on inequality. The hypothesis of Acemoglu et al. (2001) is supported by the model and calibration of Açıkgöz and Kaymak (2014) who use similar arguments to explain the deunionization in the US. Dinlersoz and Greenwood (2016) go

[2]The task-based approach has also been used in the literature on the labor-market effects of offshoring, see, e.g., Grossman and Rossi-Hansberg (2008) and Costinot and Vogel (2010) .

one step further and argue that SBTC is not only responsible for the deunionization observed in more recent decades but also for the increase in union density in the first half of the 20th century. Neto et al. (2019) consider the "reverse" question of whether labor unions influence the direction of technical change. They build an endogenous growth model of directed technical change in which only low-skilled workers are organized in labor unions. The unions' impact on SBTC depends on whether the unions are more wage-oriented or more employment-oriented. It turns out that in the first case firms have a higher incentive to invest in high-skilled technologies than in the second one.

Another related strand of literature integrates, similarly to our paper, union wage determination in the Mortensen-Pissarides matching model. In a matching model with monopoly unions, Pissarides (1986) shows that an efficient level of unemployment is obtained if unions only care about unemployed workers. The efficiency effects of firm-level collective bargaining in a search economy with concave production are analyzed in Bauer and Lingens (2014). Boeri and Burda (2009) show that preferences for rigid wages and collective bargaining may endogenously emerge if there are distortions of the separation decision in the form of a firing tax. Ebell and Haefke (2006) analyze how the bargaining regime affects the impact of product market competition on unemployment. Delacroix (2006) considers a segmented labor market with union and non-union sectors and analyses the interaction of unemployment benefits and union wage setting. Krusell and Rudanko (2016) discuss the holdup problem and show that without unions' credible commitment to future wages the firms' hiring is too low. Morin (2017) analyzes how unions affect the volatility of wages over the business cycle. As is evident from this overview, these papers are interested in research questions unrelated to SBTC and, in contrast to our paper, do not consider the impact of changes in the firms' task assignment on labor unions' wage claims.

Finally, some papers include automation in the Mortensen-Pissarides matching model. Similar to our model framework, Jaimovich et al. (2021) consider a model with a frictional labor market for low-skilled workers and perfect competition for high-skilled workers. In contrast to our paper, they assume that low-skilled workers are heterogeneous and that labor markets are fully segmented by ability and by type of produced goods. Segmented labor markets are also assumed in Cords and Prettner (2022), who show that robot adoption leads to falling wages and rising unemployment among low-skilled workers. Guimarães and Gil (2022) modify a standard matching model with endogenous job destruction and distinguish between firms using only workers and those using only machines. In Leduc and Liu (2024), firms draw an i.i.d. cost of automation and then decide whether to automate vacancies or not. The probability of automation, interpreted as automation threat, depends on the business cycle. During a business cycle expansion, the increased automation threat weakens workers' bargaining power and dampens wage increases. This automation channel therefore creates real wage rigidities that amplify labor market fluctuations. None of these papers, however, addresses the problem of assigning tasks to workers or the impact of task assignment on labor union wage setting and the aggregate wage curve.

3 The Model

3.1 Firms

There is a mass one of identical firms in the economy. Timing is discrete and will be explained in more detail below. At the end of period t the representative firm produces the final good Y_t by using the services of a continuum of tasks $y_t(i)$, measured on the unit interval, according to the Cobb-Douglas-function

$$Y_t = \exp\left[\int_0^1 \ln y_t(i)\mathrm{d}i\right]. \tag{1}$$

Index $0 < i < 1$ refers to the complexity of a particular task. The firm assigns L_t low-skilled workers and H_t high-skilled workers to the different tasks according to the task-specific production function

$$y_t(i) = A_{Lt}\,\alpha_L(i)\,l_t(i) + A_{Ht}\,\alpha_H(i)\,h_t(i), \tag{2}$$

where $l_t(i)$ and $h_t(i)$ denote the low-skilled and high-skilled labor input assigned to the task with index i in period t, respectively, and

$$L_t = \int_0^1 l_t(i)\mathrm{d}i \qquad \text{and} \qquad H_t = \int_0^1 h_t(i)\mathrm{d}i. \tag{3}$$

A_{Lt} and A_{Ht} denote factor-augmenting technology, whereas the functions $\alpha_H(i)$ and $\alpha_L(i)$ describe the productivity of high- and low-skilled workers in the task with index i, respectively. We make an important assumption regarding the task-related productivities.

We define $\bar{\alpha}(i) \equiv \alpha_H(i)/\alpha_L(i)$, hereafter also referred to as the relative task productivity schedule, and assume that $\bar{\alpha}'(i) > 0$. This implies that the comparative advantage of high-skilled (low-skilled) workers in performing the different tasks is increasing (decreasing) in the task index i.

The goods market and the labor market for high-skilled workers are competitive; consequently, high-skilled workers are always fully employed. This is because the labor market for high-skilled workers is typically a seller's market, where intense competition among companies to attract the best minds leads to competitive wages that reflect the productivity (and scarcity) of high-skilled workers. Firms are willing to pay market rates to attract the talent necessary for complex tasks. In contrast, the low-skilled labor market is characterized by matching frictions because low-skilled workers often face greater difficulties in finding jobs that match their skills due to lower mobility, fewer networking opportunities, and less access to information about job openings, making search and matching frictions a significant feature of their labor market.[3] The matching frictions are described by the linear homogeneous matching function $M_{Lt} = M(V_{Lt}, U_{Lt})$, where V_{Lt} denotes vacant jobs for low-skilled workers and U_{Lt} denotes low-skilled unemployed persons. Moreover, as will be explained in more detail in Section 3.2, all low-skilled workers are organized in firm-level labor unions that set the wage for their members in a monopolistic fashion.

The timing is as follows. At the beginning of period t there are L_{t-1} employed low-skilled workers and U_{Lt} unemployed workers. The total labor force is normalized to one, hence $U_{Lt} = 1 - L_{t-1} - H_{t-1}$. The representative labor union chooses a wage w_{Lt} anticipating that the respective firm may adjust the employment level by posting vacancies accordingly. This timing contrasts with that used in standard search and matching models but is in line with studies incorporating labor unions into the search and matching framework, such as Delacroix (2006) and Morin (2017). For simplicity, the inflow of unemployed workers into jobs and exogenous job separations happen simultaneously in such a way that a further change of a worker's employment/unemployment status within the same period is not possible. At the end of the period, production takes place as outlined above.

If the representative firm wants to increase the number of low-skilled workers it has to post vacant jobs first and bear the (constant) costs s_L for each vacant job. With rate

[3] A similar modeling assumption (perfect competition for high-skilled workers; frictional labor market for low-skilled workers) is made in Jaimovich et al. (2021).

$M_{Lt}/V_{Lt} \equiv m(\theta_{Lt})$ job vacancies are filled, where $\theta_{Lt} \equiv V_{Lt}/U_{Lt}$ describes labor market tightness in the low-skilled labor market in period t. The single firm considers labor market tightness and thus the job-filling rate as given. With the exogenous rate q_L low-skilled jobs are destroyed. The dynamics for low-skilled employment is therefore described by

$$L_t = (1 - q_L)L_{t-1} + m(\theta_{Lt})V_{Lt}. \tag{4}$$

Figure 1 illustrates the timing of the events during period t.

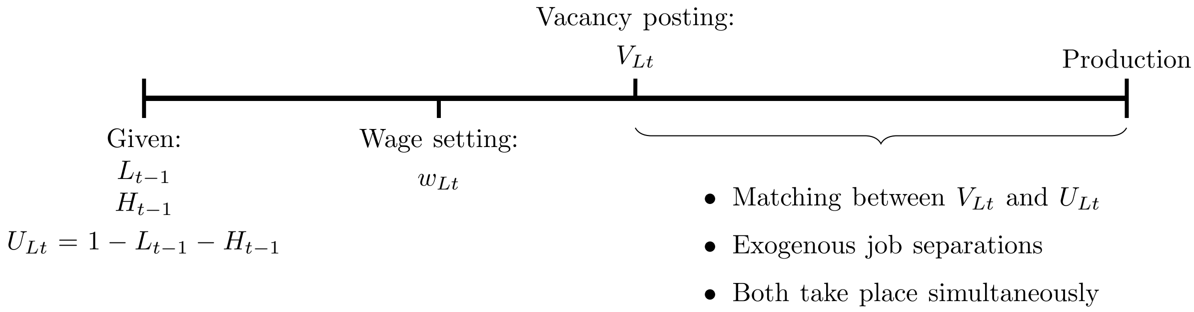


Figure 1: Timeline with events during period t

To simplify the analysis we follow Pissarides (2000, p. 68) in assuming that each firm is large enough to eliminate all uncertainty about the flow of labor. Moreover, the final good is chosen as the numeraire. The representative firm maximizes profits

$$\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} [Y_t - w_{Lt}L_t - w_{Ht}H_t - s_L V_{Lt}] \tag{5}$$

s.t. eqs. (1) – (4) and the conditions

$$l_t(i) \geq 0 \qquad \text{and} \qquad h_t(i) \geq 0, \tag{6}$$

where r is the constant real interest rate, and $l_0(i)$ and $h_0(i)$ are given. There are no productivity differences within the group of high- or low-skilled workers. Due to perfect competition in the high-skilled labor market and the fact that no high-skilled worker would supply labor to tasks paying lower wages, all high-skilled workers obtain the same real wage w_{Ht}. Similarly, the representative labor union sets a uniform real wage w_{Lt} for low-skilled workers.

The first-order conditions of this optimization problem are derived in Appendix A.1. The analysis focuses on the steady state in which $A_{Lt} = A_L$, $A_{Ht} = A_H$, $L_{t-1} = L_t = L$

and the time index on all variables can be omitted. Similar to the perfect competition model of Acemoglu and Autor (2011), there exists a task threshold $0 < I < 1$ such that unit labor costs of low-skilled workers are equal to those of high-skilled workers at I:

$$\frac{\widetilde{w}_L}{A_L \alpha_L(I)} = \frac{w_H}{A_H \alpha_H(I)}, \qquad \text{with} \quad \widetilde{w}_L \equiv w_L + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}. \tag{7}$$

The modified wage $\widetilde{w}_L$ represents the low-skilled labor costs relevant to the representative firm, i.e. the low-skilled real wage w_L plus the labor adjustment costs. In tasks $i < I$ only low-skilled workers are employed, i.e. $h(i) = 0$, whereas in tasks $i > I$ only high-skilled workers are employed, i.e. $l(i) = 0$. Eq. (7) can be written as:

$$\bar{\alpha}(I) = \frac{\widetilde{\omega}}{\bar{A}}, \quad \text{with} \quad \bar{\alpha}(I) = \frac{\alpha_H(I)}{\alpha_L(I)}, \quad \widetilde{\omega} \equiv \frac{w_H}{\widetilde{w}_L}, \quad \text{and} \quad \bar{A} \equiv \frac{A_H}{A_L}. \tag{8}$$

This leads to:

$$I = I\left(\widetilde{\omega}, \bar{A}\right), \qquad \text{with} \quad \frac{\partial I}{\partial \widetilde{\omega}} > 0 \quad \text{and} \quad \frac{\partial I}{\partial \bar{A}} < 0. \tag{9}$$

As shown in Appendix A.1, from the first-order conditions it follows that $\widetilde{w}_L l(i) = Y = w_H h(i)$. This has two important implications. First, the same labor input is used in all low-skilled and high-skilled tasks, respectively, i.e. $l(i) = l = L/I$ for $i < I$ and $h(i) = h = H/(1-I)$ for $i > I$. Second, it holds that $I = \widetilde{w}_L L/Y$ and $1 - I = w_H H/Y$ so that I represents the modified labor share of the low-skilled workers (as it refers to labor costs $\widetilde{w}_L$ and not w_L), and $1 - I$ corresponds to the high-skilled labor share. It follows that:

$$L = \frac{I}{1-I} \, \widetilde{\omega} \, H. \tag{10}$$

Wage changes have two effects on L: a direct effect at a given threshold I and an indirect effect due to a change of this threshold. Eq. (10) in combination with eq. (9) can be interpreted as the labor demand function for low-skilled workers for given H:

$$L = L\left(\widetilde{\omega}, I\left(\widetilde{\omega}, \bar{A}\right), H\right) \equiv L^d\left(\widetilde{\omega}, \bar{A}, H\right). \tag{11}$$

Moreover, taking into account the optimality conditions, the production function for the final good takes the following Cobb-Douglas form:

$$Y = B\left(A_L \frac{L}{I}\right)^I \left(A_H \frac{H}{1-I}\right)^{1-I},$$
$$B \equiv e^{\xi(I)}, \quad \xi(I) \equiv \int_0^I \ln \alpha_L(i)\mathrm{d}i + \int_I^1 \ln \alpha_H(i)\mathrm{d}i. \tag{12}$$

3.2 Labor Unions for Low-Skilled Workers

We consider firm-level labor unions and assume that all low-skilled workers within a firm are members of the respective labor union. Upon becoming unemployed, a worker exits the labor union. As explained in the introduction, "low-skilled workers" in the model are intended to encompass low- and medium-skilled workers in the real world. Following Lingens (2003, 2007) and Neto et al. (2019), the labor union only acts in the interest of low-skilled workers and does not bargain over high-skilled wages. The following arguments support this assumption: First, our model should be interpreted as describing firm and union behavior in the private sector. In a cross-country comparison, Blanchflower (2007) observes that the skill level is positively related to union membership in the public sector, but negatively in the private sector, and notices that the sectoral differences are large. According to Schnabel (2020), most empirical studies on union density across skill groups seem to have overlooked the crucial distinction between the private and public sectors. Ebbinghaus et al. (2011, p. 111) point out that "With the exception of the public sector and some well organized professions (particularly in Nordic countries), better educated employees, particularly those with tertiary (university) education tend to be less likely to join trade unions". Second, possible reasons why education is negatively related to unionism in the private sector are that "educated employees have greater individual bargaining power (and thus a lesser need for collective voice) and because sometimes they identify more with management than with the labour movement" (Schnabel, 2003, p. 30). According to Acemoglu et al. (2001) and Açıkgöz and Kaymak (2014), high-skilled workers have an incentive to leave the union because SBTC increases their outside option while labor unions compress wages in the interest of low- and medium-skilled workers. Checchi et al. (2007, p. 3) argue that "higher-earning people are more tolerant of inequality than those earning less and they are more likely to defend inequality as reward for effort or talent. Part of the explanation for a differential effect of relative earnings on the likelihood to join a trade union may therefore be found in different attitudes towards inequality". Third, studies focusing on specific countries find patterns that are in line with our assumption. For Germany, which is considered in the calibration below, Bonaccolto-Töpfer and Schnabel (2023) find that union members are more often educated to a lower level (having just basic secondary education). They are also more often plant and machine operators and assemblers and less often service and sales workers than non-union employees. Moreover, the union wage premium is only positive for workers with relatively little education, not for high-skilled workers. According to Bonaccolto-Töpfer and Schnabel (2023, p. 9), "This suggests that union membership may be particularly beneficial for disadvantaged workers".

Based on these findings and arguments, the firm-level union only acts on behalf of low- (and medium-) skilled workers but does not take the interests of high-skilled workers into account. With the timing assumption outlined in the last section, the present discounted utility of a low-skilled worker being employed at the end of period t is

$$\Psi_{EL,t} = w_{Lt} + \frac{1}{1+r}\left[q_L\,\Psi_{UL,t+1} + (1-q_L)\,\Psi_{EL,t+1}\right], \tag{13}$$

where $\Psi_{UL,t+1}$ denotes the present discounted utility of a low-skilled worker being unemployed at the end of period $t+1$. The job separation rate q_L refers to period $t+1$ but since it is assumed to be constant the time index is omitted. A low-skilled worker being unemployed at the end of period t has the present discounted utility

$$\Psi_{UL,t} = z_{Lt} + \frac{1}{1+r}\left[p_{L,t+1}\,\Psi_{EL,t+1} + (1-p_{L,t+1})\,\Psi_{UL,t+1}\right], \tag{14}$$

where z_L denotes net unemployment benefits and p_L is the exit rate from unemployment which positively depends on labor market tightness, i.e. $p_L \equiv M_L/U_L = \theta_L m(\theta_L)$. As we are not interested in the implications of different tax systems on the wage-setting process, it is assumed that unemployment benefits are financed by lump-sum taxes.

The union members, and hence the union itself, care about the excess of the value of a job in the firm, $\Psi_{EL,t}$, over the utility of an unemployed person, $\Psi_{UL,t}$. Thus, the utility function V_{Lt} of the labor union corresponds to the intertemporal analog of the rent maximizing union:

$$V_{Lt} = L_t\,R_{Lt}, \qquad \text{where} \qquad R_{Lt} \equiv (\Psi_{EL,t} - \Psi_{UL,t}). \tag{15}$$

To simplify the explanation of the model's mechanisms, it is assumed that the low-skilled wage is determined by the labor union in a monopolistic fashion.[4]

The dynamic analysis of labor union wage setting adheres closely to the assumptions and arguments made in Manning (1991). We focus on the determinants of steady state wage pressure and therefore assume that the determinants of labor market flows, q_L and p_L, as well as unemployment benefits z_L are constant over time. The probability of becoming unemployed, q_L, is assumed to be independent of the wage set in the respective period due to the observation that "except in exceptionally severe recessions, employment reduction seems to be through natural wastage rather than lay-offs, and the probability of moving from employment into unemployment seems remarkably constant (...). So, close to a steady state, current wages are unlikely to affect the probability of a worker being unemployed next period" (Manning, 1991, p. 329). With these assumptions and eqs. (13) and (14), the labor union utility function can be written

$$V_{Lt} = L_t \left(w_{Lt} - z_L + \delta R_{L,t+1} \right) \quad \text{with} \quad \delta \equiv \frac{1 - q_L - p_L}{1 + r} < 1. \tag{16}$$

If wages were only set for one period, the effect of the current wage on V_{Lt} would manifest through the term w_{Lt} and the wage impact on L_t. However, $R_{L,t+1}$ would remain unaffected since the wage is not a state variable in this model. In other words, future wages are not influenced by past wages. Of course, in a steady state future wages should be the same as the wage set in the current period even though the wage is only set for one period and future values are considered to be given and not influenced by today's wage setting.

As in Manning (1991), we consider the more general case where the wage is set for n periods. This implies that the wage w_{Lt} will affect the utility difference R_L from period t until period $t+n-1$, but not from period $t+n$ onwards. Running forward R_{Lt} for n periods leads to

$$R_{Lt} = \frac{1 - \delta^n}{1 - \delta} (w_{Lt} - z_{Lt}) + \delta^n R_{L,t+n}. \tag{17}$$

The representative labor union at the firm level maximizes $V_{Lt} = L_t R_{Lt}$ subject to the labor demand equation. As demonstrated in Appendix A.2, the rent-maximizing wage w_{Lt}

[4] This simplifying assumption of monopoly unions is also made in other studies, such as Pissarides (1986), Delacroix (2006), or Dinlersoz and Greenwood (2016).

leads to the following wage costs $\widetilde{w}_{Lt}$ relevant to the firm:

$$\widetilde{w}_{Lt} = \frac{\varepsilon_{L\widetilde{w}_L,t}}{\varepsilon_{L\widetilde{w}_L,t} + \delta^n - 1}\widetilde{z}_{Lt}, \quad \text{with} \quad \widetilde{z}_L \equiv z_L + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}, \tag{18}$$

where $\varepsilon_{L,\widetilde{w}_L}$ denotes the wage elasticity of the demand for low-skilled labor (in absolute values) and $\widetilde{z}_L$ consists of unemployment benefits and labor adjustment costs. Eq. (18) aligns with Manning's result that wage pressure increases with the duration of the wage contract. Since the focus of our paper is on the effect of changes in the task allocation on the labor demand elasticity, the expression in eq. (18) is simplified by restricting the analysis to the case $n \to \infty$ and therefore $\delta^n \to 0$.[5] Omitting time indices for steady-state values leads to

$$\widetilde{w}_L = \kappa_L \widetilde{z}_L, \quad \text{with} \quad \kappa_L \equiv \frac{\varepsilon_{L,\widetilde{w}_L}}{\varepsilon_{L,\widetilde{w}_L} - 1}, \tag{19}$$

where κ_L denotes the wage markup that is negatively related to the labor demand elasticity. As shown below, $\varepsilon_{L,\widetilde{w}_L} > 1$, implying $\kappa_L > 1$. The next subsection scrutinizes how changes in the task allocation influence the labor demand elasticity and hence union wage markups.

3.3 Task Reallocation and the Elasticity of Labor Demand

The wage elasticity of the demand for low-skilled labor (in absolute values) can be written as

$$\varepsilon_{L,\widetilde{w}_L} \equiv \left| \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{w}_L} \right| = 1 + \frac{1}{1-I} \frac{\partial \ln I}{\partial \ln \widetilde{\omega}} = 1 + \frac{1}{(1-I)\cdot \varepsilon_{\bar{\alpha},i}(I)} > 1, \tag{20}$$

where

$$\varepsilon_{\bar{\alpha},i}(I) \equiv \left. \frac{\mathrm{d} \ln \bar{\alpha}(i)}{\mathrm{d} \ln i} \right|_{i=I} > 0$$

denotes the elasticity of the relative task productivity schedule $\bar{\alpha}(i) \equiv \alpha_H(i)/\alpha_L(i)$ to a one-percent change in the task index i, where this elasticity is evaluated at the task threshold I. To simplify the notation, $\varepsilon_{\bar{\alpha},i}(I)$ is denoted $\varepsilon_{\bar{\alpha},I}$ in the following. We will look at this elasticity for different threshold values I, i.e. $\varepsilon_{\bar{\alpha},I}$ is considered to be a function of

[5] Other papers, such as Pissarides (1985), Layard and Nickell (1990), and Beissinger and Egger (2004), also simplify the analysis by considering wage contracts of infinite length.

I, where I is the profit maximizing task threshold obtained in general equilibrium.

The wage elasticity of the demand for low-skilled labor is the sum of a direct wage effect (equal to one) for a given task allocation, and a task reallocation effect caused by the change in the task threshold I due to a change in relative labor costs $\widetilde{\omega}$. The task reallocation effect implies that with an increase in $\widetilde{w}_L$ fewer tasks are allocated to low-skilled labor. The *strength* of this effect depends on the task threshold I in two ways. The more tasks are allocated to low-skilled labor the larger is $1/(1-I)$ which *cet. par.* increases the task reallocation effect and thereby $\varepsilon_{L,\widetilde{w}_L}$. However, the size of the task reallocation effect also negatively depends on $\varepsilon_{\bar{\alpha},I}$. In general, $\varepsilon_{\bar{\alpha},I}$ is a function of I with the sign of $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I$ depending on the functional form of $\bar{\alpha}$, i.e. $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I \lesseqgtr 0$ is possible.[6] This leads to

Proposition 1. *An increase in the task threshold I leads to the following change in the wage elasticity of the demand for low-skilled labor:*

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\begin{cases} >0, & \textit{if}\quad \mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I < I/(1-I)\\ =0, & \textit{if}\quad \mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I = I/(1-I)\\ <0, & \textit{if}\quad \mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I > I/(1-I).\end{cases}$$

Proof. Taking into account eq. (20) and the definition of κ_L in eq. (19), $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ can be written as:

$$\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}=\frac{1}{\kappa_L}\left(\frac{I}{1-I}-\frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I}\right). \tag{21}$$

Since $\kappa_L>0$, Proposition 1 immediately follows from eq. (21). □

Since the labor union's wage markup κ_L is negatively related to $\varepsilon_{L,\widetilde{w}_L}$, Proposition 1 can be directly applied to establish the effect of the threshold I on κ_L:

$$\varepsilon_{\kappa_L,I}\equiv\frac{\mathrm{d}\ln\kappa_L}{\mathrm{d}\ln I}=-(\kappa_L-1)\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I},\quad\text{with}\quad \mathrm{sgn}(\varepsilon_{\kappa_L,I})=-\mathrm{sgn}\left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}\right). \tag{22}$$

The important insight from this analysis is that changes in the task allocation have an impact on the effective wage-setting power of labor unions. The way in which a change

[6] As shown in Online Appendix B.1, the second-order condition for the optimization problem of the representative labor union puts a restriction on $\mathrm{d}\ln\bar{\alpha}/\mathrm{d}\ln I$ equivalent to $\bar{\alpha}''(I)\,I/\bar{\alpha}'(I)>-2$ so that the $\bar{\alpha}$ function must not be "too concave".

in the task allocation affects the labor demand elasticity and thus the labor union's wage markup crucially depends on the shape of the relative task productivity schedule.

To provide some intuition, consider, for example, a concave relative task productivity schedule which implies $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I < 0$. With the $\bar{\alpha}$ function being concave, at lower thresholds I the low- and high-skilled workers are more apart in terms of their productivities than at higher I. At a low I, a reduction of the task threshold leads to a pronounced drop in the relative high-skilled productivity $\bar{\alpha}$, thereby making the substitution of low-skilled by those with higher skills harder. Therefore, an increase in $\widetilde{w}_L$ requires only a small decline in I to induce such an increase in the relative productivity of low-skilled workers $1/\bar{\alpha}$ that the unit labor costs of both skill groups are equal again in optimum. As a result, labor demand decreases only slightly with increasing $\widetilde{w}_L$ at a low I. In contrast, with low-skilled workers being more substitutable at a high I, the response of labor demand to increasing $\widetilde{w}_L$ is more pronounced. This explains why a concave $\bar{\alpha}$ function leads to a rise in the labor demand elasticity with an increase in the task threshold I. The opposite applies if the $\bar{\alpha}$ function is convex and $\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}/\mathrm{d}\ln I$ is sufficiently large. These results can be relevant when comparing the outcomes of the wage-setting process in different sectors of the economy. Some sectors may encompass a range of tasks which rapidly increase in their complexity so that the relative productivity of high-skilled workers increases in a more exponential manner. Other sectors may display a task complexity profile that is prone to stronger substitution of different skills.

3.4 Solution of the Model in the Steady State

In the steady state the inflows into low-skilled employment are equal to the outflows from low-skilled employment. From eq. (4) follows $m(\theta_L)V_L = q_L L$. Equivalently, the inflows into low-skilled jobs are equal to the flows out of unemployment, i.e. $m(\theta_L)V_L = p_L(\theta_L)U_L$, where again $p_L(\theta_L) \equiv m(\theta_L)\theta_L$. With a mass one of individuals, $L + U_L + H = 1$. Hence,

$$q_L L = p_L(\theta_L)\,(1 - H - L). \tag{23}$$

Since the final good is chosen as numeraire, its price equals one. This implies

$$\int_0^I \ln\left(\frac{\widetilde{w}_L}{A_L\alpha_L(i)}\right)\mathrm{d}i + \int_I^1 \ln\left(\frac{w_H}{A_H\alpha_H(i)}\right)\mathrm{d}i = 0, \tag{24}$$

which closes the model. The solution of the model is described in

Definition (General Equilibrium)**.** *The general equilibrium values of the task threshold I, low-skilled employment L, labor market tightness θ_L, and the firm's wage costs $\widetilde{w}_L$ and w_H are determined by eqs. (7), (10), (19), (23) and (24). From the definition of $\widetilde{w}_L$ in eq. (7) the low-skilled wage w_L is obtained. The solution for output follows from eq. (12).*

4 Comparative Statics: Skill-biased Technical Change

In the following, we will analyze the implications of skill-biased technical change (SBTC) in the task-based matching model. To keep things as simple as possible, we consider a one-time increase in A_H while A_L remains constant ($\mathrm{d}\ln \bar{A} = \mathrm{d}\ln A_H$). The high-skilled and low-skilled labor force is assumed to be given. With perfect competition in the high-skilled labor market this assumption implies that high-skilled employment remains constant, i.e. $\mathrm{d}\ln H = 0$.

To ease the exposition, the matching function is assumed to be of the Cobb-Douglas type:

$$M_L = M(V_L, U_L) = V_L^{1-\beta_L} U_L^{\beta_L}, \qquad \text{with} \quad 0 < \beta_L < 1, \tag{25}$$

which implies $m(\theta_L) = \theta_L^{-\beta_L}$. Therefore, β_L is the (constant) elasticity of the job filling rate $m(\theta_L)$ with respect to labor market tightness θ_L (in absolute values). The elasticity of the job finding rate p_L with respect to θ_L is $(1 - \beta_L)$.

It is useful to write the model equations in log differences. They can be condensed into a three-equations system for θ_L, $\widetilde{w}_L$ and I summarized in

Proposition 2 (Comparative Statics)**.** *Let u_L denote the low-skilled unemployment rate, and let $\varepsilon_{\widetilde{z}_L,\theta_L}$ be the elasticity of $\widetilde{z}_L$ with respect to θ_L, where $\widetilde{z}_L$ is defined in eq. (19) and $0 < \varepsilon_{\widetilde{z}_L,\theta_L} < \beta_L$. Moreover, $\varepsilon_{\kappa_L,I}$ denotes the elasticity of the wage markup κ_L with respect to the task threshold I, as defined in eq. (22). Then*

$$\mathrm{d}\ln \theta_L = \frac{1}{(1-\beta_L)\, u_L} \left[\mathrm{d}\ln A_H - \frac{1}{1-I} \mathrm{d}\ln \widetilde{w}_L + \frac{1}{1-I} \mathrm{d}\ln I \right], \tag{26}$$

$$\mathrm{d}\ln \widetilde{w}_L = \varepsilon_{\widetilde{z}_L,\theta_L} \mathrm{d}\ln \theta_L + \varepsilon_{\kappa_L,I}\, \mathrm{d}\ln I, \tag{27}$$

$$\mathrm{d}\ln I = -(\varepsilon_{L,\widetilde{w}_L} - 1)\, \mathrm{d}\ln \widetilde{w}_L. \tag{28}$$

Proof. See Appendix A.3.

Eq. (26) represents the job creation condition, eq. (27) is the wage equation for low-skilled workers, and eq. (28) can be interpreted as the "task allocation" equation (respectively in log differences). It is evident that both, job creation and wage setting, are influenced by changes in the task threshold I. The equations in Proposition 2 represent general equilibrium relationships in which the adjustment of the high-skilled wage necessary for full employment of high-skilled workers has already been taken into account. In eq. (28), changes in the task threshold depend solely on changes in low-skilled labor costs. This is because, in general equilibrium, the high-skilled wage increases in proportion to increases in high-skilled productivity. For the detailed formal analysis see Appendix A.3.

According to the job creation equation, an increase in A_H *cet. par.* leads to higher labor market tightness. The reason is that an increase in A_H increases the marginal product of low-skilled workers in final goods production, hence each firm has the incentive to increase the labor input l in each low–skilled task $i < I$ at given wages $\widetilde{w}_L$.[7] In general equilibrium, the increase in A_H also increases the high-skilled wage, thereby increasing the relative cost of high-skilled workers. This, in turn, leads to a higher demand for low-skilled workers in each task and, hence, to a higher labor market tightness. As can be seen from the wage equation, an increase in labor market tightness leads to higher wage pressure and *cet. par.* increases $\widetilde{w}_L$. However, this increase in labor costs induces firms to reduce the range of tasks allocated to low-skilled labor, which reduces labor market tightness and has ambiguous effects on wage setting as explained above. Inserting the task allocation equation in the other two equations leads to

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)\,u_L}\left[\mathrm{d}\ln A_H - \frac{\varepsilon_{L,\widetilde{w}_L}}{1-I}\,\mathrm{d}\ln\widetilde{w}_L\right], \tag{29}$$

$$\mathrm{d}\ln\widetilde{w}_L = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}}\,\mathrm{d}\ln\theta_L, \tag{30}$$

where for the latter expression eq. (22) and the definition of κ_L in eq. (19) have been used, and $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I \neq 1$ must hold. In this version of the job creation and wage-setting equation, the adjustment of the task threshold I due to a change in the firm's low-skilled labor costs is already taken into account. This two-equations system can be graphically represented by a job creation curve (JC) and wage curve (WC) in $\theta_L - \widetilde{w}_L$ space. As can be seen from eq. (29), increases in A_H lead to a rightward shift of the JC. Moreover, the

[7]The marginal product of low-skilled workers in the production of final goods is Y/l.

JC is downward sloping, i.e.

$$\Phi \equiv \left.\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln \theta_L}\right|_{\mathrm{JC}} = -\frac{(1-I)(1-\beta_L)u_L}{\varepsilon_{L,\widetilde{w}_L}} < 0. \tag{31}$$

As regards the WC described by eq. (30), the relationship between θ_L and $\widetilde{w}_L$ is not unambiguous. The slope of the WC is

$$\Gamma \equiv \left.\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln \theta_L}\right|_{\mathrm{WC}} = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1-\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}} \gtrless 0. \tag{32}$$

Quite similar to standard matching models, the slope of the WC positively depends on $\varepsilon_{\widetilde{z}_L,\theta_L}$ which is a function of r, q_L, β_L, and s_L, as shown in eq. (A.13) in Appendix A.3. In addition to these parameters, the slope of the WC in the task-based model also depends on $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$, i.e. on how changes in the task allocation affect the wage elasticity of labor demand.

In a conventional matching model an increase in labor market tightness leads to higher wage claims of workers, implying an upward–sloping WC in θ_L–$\widetilde{w}_L$ space. In eq. (30) this situation arises if $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$. However, as is evident from Proposition 1, the case $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ is also possible, implying that the corresponding WC would be downward-sloping. In that case, two situations can be distinguished depending on whether the JC is steeper or flatter than the WC, i.e. depending on whether $|\Phi| \gtrless |\Gamma|$. In Online Appendix B.2 we first demonstrate that, irrespectively of the slope of the WC, a steady state equilibrium exists in all situations. Moreover, we show that all steady state equilibria can be in principle (saddle-path) stable so that we cannot rule out the possibility of a downward-sloping WC in a general comparative-static analysis. The results of this analysis are summarized in

Proposition 3 (Comparative-Static Results)**.** *High-skilled labor-augmenting technical change has the following effects on the labor market equilibrium:*

(i) Low-skilled labor market tightness:

$$\frac{\mathrm{d}\ln \theta_L}{\mathrm{d}\ln A_H}\begin{cases} > 0, & \text{if} \quad \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \ \vee \ \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \ \wedge \ |\Phi| > |\Gamma|\right) \\ < 0, & \text{otherwise.}\end{cases}$$

(ii) Low-skilled labor costs:

$$\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln A_H} \begin{cases} > 0, & \textit{if} \quad \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \quad \vee \quad \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \quad \wedge \quad |\Phi| < |\Gamma|\right) \\ < 0, & \textit{otherwise.} \end{cases}$$

(iii) Task threshold:

$$\frac{\mathrm{d}\ln I}{\mathrm{d}\ln A_H} \begin{cases} < 0, & \textit{if} \quad \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \quad \vee \quad \left(\frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \quad \wedge \quad |\Phi| < |\Gamma|\right) \\ > 0, & \textit{otherwise.} \end{cases}$$

Proof. Solving eqs. (29) and (30) leads to:

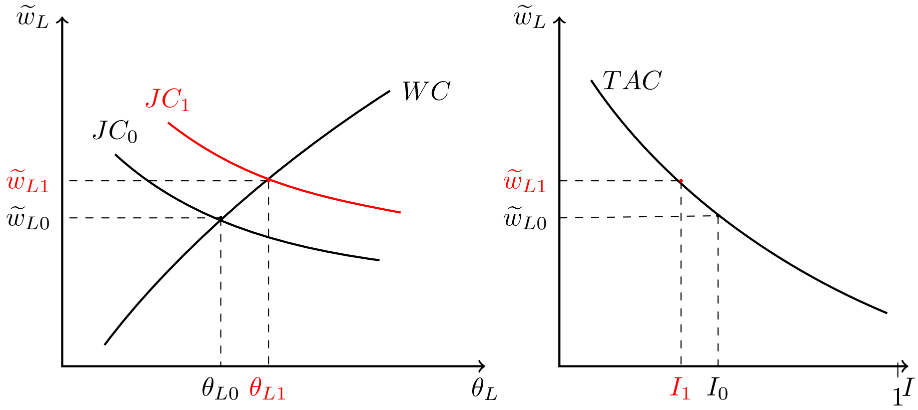
$$\frac{\mathrm{d}\ln \theta_L}{\mathrm{d}\ln A_H} = \frac{1-I}{\varepsilon_{L,\widetilde{w}_L}} \frac{1}{|\Phi| + \Gamma},$$
$$\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln A_H} = \frac{\varepsilon_{\widetilde{z}_L,\theta_L}}{1 - \frac{\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}} \frac{\mathrm{d}\ln \theta_L}{\mathrm{d}\ln A_H} = \frac{(1-I)\,\Gamma}{\varepsilon_{L,\widetilde{w}_L}} \frac{1}{|\Phi| + \Gamma}.$$

Because of eq. (28) it holds:

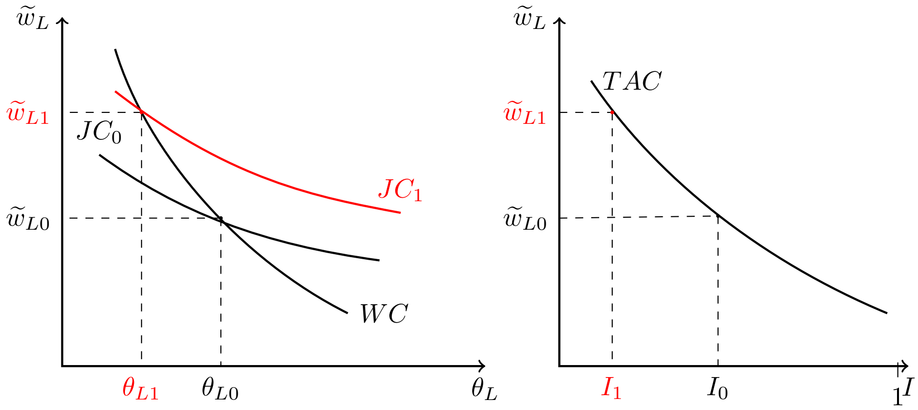
$$\frac{\mathrm{d}\ln I}{\mathrm{d}\ln A_H} = -(\varepsilon_{L,\widetilde{w}_L} - 1)\frac{\mathrm{d}\ln \widetilde{w}_L}{\mathrm{d}\ln A_H} = -\frac{(1-I)\,\Gamma}{\kappa_L} \frac{1}{|\Phi| + \Gamma}.$$

□

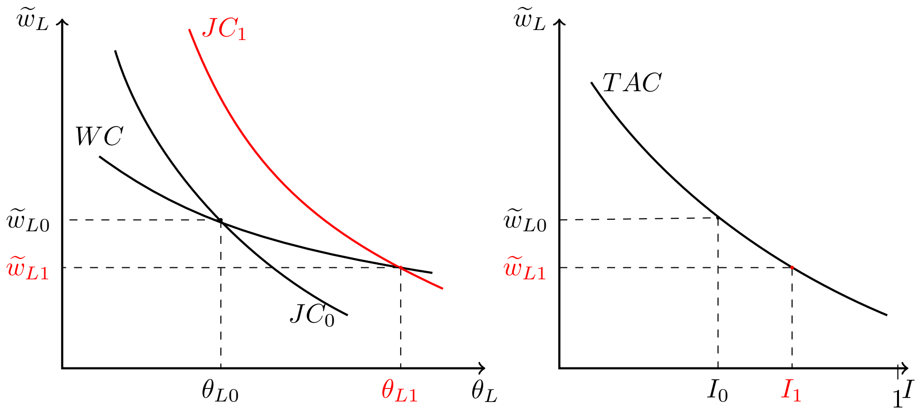
Figure 2 illustrates the comparative-static results. If $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$, the WC is upward-sloping (see Figure 2a). In this case, an increase in A_H leads to an increase in labor market tightness θ_L and in firms' labor costs $\widetilde{w}_L$. Figure 2b depicts the situation where both curves are downward-sloping and the WC is steeper than the JC. In that case, an increase in A_H still leads to an increase in $\widetilde{w}_L$, but θ_L is declining. If the JC is steeper than the WC, as depicted in Figure 2c, the opposite results are obtained, i.e. θ_L increases whereas $\widetilde{w}_L$ decreases.



(a) Case 1: $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$



(b) Case 2: $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$; WC steeper than JC



(c) Case 3: $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$; JC steeper than WC

Figure 2: Effects of skill-biased technological progress (increase in the productivity of high-skilled workers A_H) on labor market outcomes and task allocation

Notes: The effects depend on the size of $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ and the relative slopes of the job creation curve (JC) and the wage curve (WC). Graphical illustration of the JC, WC, and TAC (task allocation curve) follows from the formal analysis of the slopes and curvatures of these curves; see Online Appendix B.3 for detailed derivations. The diagrams should nevertheless be interpreted as a sketch. The axes scale is allowed to differ across cases and may encompass different ranges of values for $\widetilde{w}_L$, θ_L, and I.

To provide some intuition for the comparative-static results, it is useful to distinguish between firm-level decisions and general equilibrium results. At the firm level, the high-skilled wage and labor market tightness are considered as given. An increase in A_H increases the marginal product of low-skilled workers, inducing firms to increase the labor input in each low-skilled task. At the same time, the increase in A_H also lowers the unit labor costs for high-skilled workers relative to low-skilled workers at the old task threshold I. Hence, the firm has an incentive to reduce the range of tasks performed by low-skilled workers and to assign high-skilled workers to those tasks instead. The firm's labor union chooses a wage w_L that implies $\widetilde{w}_L = \kappa_L \widetilde{z}_L$, where $\widetilde{z}_L$ is taken as given. Depending on the curvature of the task productivity schedule $\bar{\alpha}$, the decline in I affects the labor demand elasticity as outlined in Proposition 1. In response to that, the labor union's wage claims may rise, remain unchanged or fall.

In the general equilibrium, the increase in the firms' demand for high-skilled workers *cet. par.* leads to a rise in high-skilled wages and in the firms' relative wage costs $\widetilde{\omega} = w_H/\widetilde{w}_L$.[8] The increase in $\widetilde{\omega}$ *cet. par.* increases labor demand for low-skilled workers. As shown in Appendix A.3,

$$\mathrm{d}\ln L = \frac{1}{1-I}\,\mathrm{d}\ln I + \mathrm{d}\ln\widetilde{\omega}, \tag{33}$$

$$\mathrm{d}\ln \theta_L = \frac{1}{(1-\beta_L)u_L}\mathrm{d}\ln L. \tag{34}$$

Hence, whether L and therefore θ_L increase relative to the initial equilibrium depends on whether the positive effect on labor demand caused by the increase in $\widetilde{\omega}$ is larger than the negative effect caused by the decline in the task threshold I. Of course, changes in θ_L lead to changes in $\widetilde{z}_L$ which lead to further adjustments in labor unions' wage claims for low-skilled workers.

In Figure 2a, the increase in $\mathrm{d}\ln\widetilde{\omega}$ dominates, and θ_L and $\widetilde{w}_L$ increase. Despite the decline in the task threshold I, labor demand for low-skilled workers is higher in the new equilibrium because more workers are employed in each of the remaining low-skilled tasks. The WC is relatively steep for $0 < \mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$ because the decline in I then leads to a lower labor demand elasticity and hence higher wage pressure. Vice versa, for $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 0$ the WC is relatively flat because the increase in labor unions' wage claims (due to higher θ_L) is dampened by the increase in the labor demand elasticity. For $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I = 0$ the slope of the resulting WC lies in between the other two cases. Since the slope of the WC is related to the concept of real wage rigidity, this

[8]By how much w_H and $\widetilde{\omega}$ rise also depends on the change in low-skilled wage costs $\widetilde{w}_L$.

analysis offers additional explanations for different degrees of real wage rigidity between countries or industries. In the literature, the degree of real wage rigidity is often explained by institutional factors such as the unemployment compensation system or the degree of centralization of wage bargaining; see, e.g., Layard, Nickell, and Jackman (1991), Chapter 9. According to our analysis, changes in the task composition also affect the real wage response to changes in labor market tightness depending on the curvature of the task productivity schedule. In that sense, the production technology may also influence the extent of real wage rigidity in an industry or country.

In Figure 2b, $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ and the slope of the JC is smaller than the slope of WC (in absolute values). Because of eq. (31), a relatively small $|\Phi|$ arises if the low-skilled unemployment rate u_L is relatively low and I is relatively high, i.e. many tasks are allocated to low-skilled workers, implying that the labor share of low-skilled workers is relatively high. The firm's reduction in I leads to a strong decline in the labor demand elasticity and thus to a strong increase in wage pressure. Since the labor share of low-skilled workers is high, the increase in low-skilled wages raises each firm's labor costs significantly, implying a relatively small increase in output, the labor demand for high-skilled workers, the high-skilled wage, and $\widetilde{\omega}$. As a consequence, in eq. (33) the effect on L due to a decline in I dominates. The resulting decline in θ_L would *cet. par.* lead to lower wage pressure. However, this effect is overcompensated by the decline in the labor demand elasticity and thus the rising wage markup.

In Figure 2c, $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ and the slope of the JC is larger than the slope of the WC (in absolute values). In comparison to the previous figure, the situation is now reversed. The slope $|\Phi|$ is the larger, the lower the task threshold I, i.e. the lower the low-skilled labor share in the initial equilibrium. Despite the initial increase in $\widetilde{w}_L$ caused by the decline in I, the rise of the firm's labor cost is this time comparably small, leading to a relatively strong increase in production, the demand for high-skilled workers, the high-skilled wage and $\widetilde{\omega}$. These general equilibrium effects lead to a strong increase in L and θ_L and even to an increase in the task threshold I that reduces wage pressure. The rising wage pressure due to higher θ_L is overcompensated by the strongly declining wage pressure as the response of the labor demand elasticity and thus the markup is very strong. As a consequence, $\widetilde{w}_L$ even falls in comparison to the initial equilibrium.

Appendix A.4 summarizes how an increase in A_H affects other variables of the model. If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$, SBTC will unambiguously increase $\widetilde{\omega}$, w_H, L, and Y, and decrease

u_L. In most of the cases, for example always when $1 > \mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 0$, low-skilled workers will benefit from an increase in A_H in terms of their wages w_L as well. However, for negative responses of the labor demand elasticity it is even possible that w_L will decline implying a pronounced increase in wage inequality. If $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$ the relative sizes of the slopes of the JC and WC again play a key role. In the majority of the cases, the effects for w_L match qualitatively those for $\widetilde{w}_L$ discussed above. Therefore, if the low-skilled workers experience higher unemployment rates due to SBTC the rise in inequality is less pronounced than in a situation of declining unemployment rates.

The analysis of this section shows that, other than in a canonical model with conventional modeling of a production process, SBTC can in our model harm low-skilled workers. This is always the case if the labor demand elasticity increases with higher I and this response is strong. Whether low-skilled workers lose in such a case in terms of their wages or their labor market tightness is to a large extent governed by the initial task composition. This insight could be relevant in a sectoral context when different sectors may face similarly strong responses to increasing task thresholds but different divisions of tasks between low- and high-skilled workers.

5 Calibration

The first objective of the quantitative analysis is to calibrate the model to align with real-world data, thereby identifying the unobserved parameters of the relative task productivity schedule $\bar{\alpha}(i)$. The second objective is to demonstrate that the effects of SBTC can vary across countries and time periods, even when these countries have similar economic conditions. Quantifying the effects of SBTC, we illustrate the most important mechanisms of our model: *(i)* how task reallocation affects the labor demand elasticity and thus the wage pressure in the economy, and *(ii)* how these resulting changes in the labor demand elasticity impact labor market outcomes.

5.1 Design

We calibrate the model to German and French data for two time periods: 1995 to 2005 and 2010 to 2017. We select Germany and France for our quantitative analysis as both countries represent large economies with high union coverage rates. We exclude the period

2006–2009, which covers the Great Recession. This is in line with Iftikhar and Zaharieva (2019), who argue that the financial crisis in Germany was relatively quickly overcome. We treat the two periods under consideration as separate steady states. More specifically, we interpret the mean values of the labor market variables used as calibration targets as steady-state values for the respective periods.[9] We consider two different steady states to demonstrate that even for the same country SBTC can have opposing effects on some labor market variables depending on the initial steady state.

Since our model distinguishes between only two skill levels, the low-skilled group in our calibration comprises both low-skilled and medium-skilled workers. We follow Battisti et al. (2018), Iftikhar and Zaharieva (2019), and Krusell et al. (2000) in defining high-skilled workers as workers with at least a Bachelor's degree. To improve the model's fit to the data, we introduce a scale parameter ζ in the Cobb-Douglas matching function, $M(V_L, U_L) = \zeta\, V_L^{1-\beta_L}\, U_L^{\beta_L}$, which indexes the efficiency of the matching process.

To calibrate the model, we need to select a specific functional form for the relative task productivity schedule $\bar{\alpha}(i)$. For simplicity, we employ the following isoelastic function: $\bar{\alpha}(i) = i^{\rho}$ for $0 < i < 1$. Despite its simplicity, this form can generate any of the scenarios described in Figures 2a–2c, without imposing restrictions on the curvature of the relative task productivity schedule. The parameterization of the model then identifies the explicit form of the relative task productivity schedule for both countries and across the two time periods, thereby determining which scenario in Figures 2a–2c occurs.

The model is characterized by 10 exogenous parameters: $\{\beta_L, q_L, \zeta, r, z_L, s_L, H, A_H, A_L, \rho\}$. We derive six parameters from the data or the literature (see Table 1), and calculate four parameters to match German and French data during the periods 1995–2005 and 2010–2017 (see Tables 2 and 3). One period in the model corresponds to one quarter, so all parameters are interpreted quarterly.

The first two parameters remain constant across countries and periods. We set the matching elasticity β_L to 0.5, which falls within the range of estimates reported in Petrongolo and Pissarides (2001). The quarterly separation rate q_L is set to 0.0873, converted from a monthly separation rate of 0.03 as calibrated by Battisti et al. (2018).[10]

[9]Even though the period 1995–2005 includes the 2001/2002 recession in Germany and France, this recession was less severe than the Great Recession. Including both years in the first period allows us to consider a longer period as a steady-state period.

[10]Battisti et al. (2018) provide the average low-skilled separation rate for 20 countries. We are aware that Germany and France have slightly different quarterly separation rates (0.0315 vs. 0.0338), as shown in Hobijn and Şahin (2009). Because the difference between the separation rates of low- and high-skilled

Table 1: Parameter values for Germany (DE) and France (FR)

Parameter	**Country**	**Value**		**Source**
		95–05	10–17	
Parameters without country variation				
Matching elasticity: β_L	–	0.5	0.5	Petrongolo and Pissarides (2001)
Separation rate: q_L	–	0.0873	0.0873	Battisti et al. (2018)
Parameters with country variation				
Real interest rate: r	DE	0.0117	0.0016	Deutsche Bundesbank, FRED
	FR	0.0104	0.0047	Banque de France, FRED
Unemployment benefits: z_L	DE	0.41	0.32	Values are set to match net replacement rates from the OECD
	FR	0.42	0.40	
Share of high-skilled: H	DE	0.24	0.285	EU-LFS
	FR	0.263	0.359	
Skill bias: $\frac{A_H}{A_L}$	DE	1.121	1.344	Own calculations based on data from EU-LFS, EU-SILC, Destatis
	FR	1.237	1.332	

Notes: For detailed data description see Online Appendix B.5.

The real interest rate r and the share of high-skilled workers H are derived from the data and vary across time and countries. Unemployment benefits are set to match the average net replacement rates calculated using OECD data (OECD Benefits, Taxes and Wages Dataset) for each country and period.[11] Lastly, we calculate the time-dependent skill bias A_H/A_L for Germany and France. For a detailed description of the data and the corresponding calculations, see Online Appendix B.5.

For each country, we calibrate the remaining four parameters by matching four targets calculated with German and French data, respectively, over the two periods. The targets are summarized in Table 2, and the parameters obtained in the calibration are shown in Table 3. For the model equations used in the calibration, see Online Appendix B.6. The most important target is the task threshold I, which is calculated as the relative share of low-skilled and medium-skilled labor compensation in total labor compensation. The comparative-static results depend on whether $\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I$ is below or above one, i.e., whether the task threshold I is smaller or larger than the specific boundary value

workers is much more pronounced (0.0873 vs. 0.0472), we deliberately decided to use the values calibrated by Battisti et al. (2018). Nevertheless, we checked that using the country-specific separation rates from Hobijn and Şahin (2009) would have no impact on the comparative static results.

[11] The calibrated low-skilled wage in Germany and France in 1995–2005 (2010–2017) is 0.668 (0.639) and 0.673 (0.689), respectively. To obtain net average replacement rates from OECD data, we first compute the average replacement rates for two previous in-work earnings and two different family situations, including social assistance benefits. Then, we calculate the weighted average of the net replacement rates for short-term (less than one year) and long-term unemployed (more than one year). The final replacement rates for Germany and France in 2001–2005 (2010–2017) are 0.61 (0.50) and 0.62 (0.58), respectively.

Table 2: Matched targets for Germany (DE) and France (FR)

Target	Country	Value		Source
		95–05	10–17	
Low-skilled unemployment rate: u_L	DE	0.106	0.062	EU-LFS
	FR	0.112	0.113	
Low-skilled labor market tightness: θ_L	DE	0.204	0.471	IAB-JVS, EU-LFS, BA
	FR	0.279	0.301	DARES, Pôle emploi, EU-LFS, IAB-JVS
Skill premium: $\frac{w_H}{w_L}$	DE	1.32	1.49	EU-SILC, Destatis
	FR	1.43	1.41	EU-SILC
Task threshold: I	DE	0.6499	0.6229	WIOD SEA Release 2013, EU Klems Release 2017
	FR	0.6333	0.5436	

Notes: For detailed data description see Online Appendix B.5.

$I_b = 1 - (\sqrt{1+\rho} - 1)/\rho$. The value for I_b for each country and period is derived from the calibrated ρ.

5.2 Steady-State Productivity Functions

In this section, we use the calibrated parameters of the relative task productivity schedule $\bar{\alpha}(i)$ to obtain the steady-state productivity functions for Germany and France. By setting $\bar{\alpha}(i) = i^{\rho}$, the total relative productivity in our quantitative analysis can be described by the function $\bar{A}\,\bar{\alpha}(i) = \bar{A}\,i^{\rho}$. Figure 3 depicts the calibrated productivity functions for both steady states and countries. The calibrated total relative productivity depends on the skill bias $\bar{A} = A_H/A_L$ calculated from the data and on the calibrated ρ value. Differences in the positions and/or curvatures of the total relative productivity curves in each country do not represent a simulated increase in $\bar{A}$, but instead reflect differences in technology parameters in the two different initial steady states. Simulated effects of an increase in $\bar{A}$ on labor market variables in each of the steady states will be covered in the next subsection.

The productivity functions for both steady states and countries are concave, reflecting a calibrated ρ value below one. The German and French productivity curves in the initial steady state exhibit a similar degree of concavity, driven by nearly identical calibrated values of ρ (0.694 for Germany vs. 0.671 for France). Additionally, the skill bias is higher in France, resulting in a steeper initial productivity curve compared to Germany. This difference can be largely attributed to the composition of skill groups in each country. The share of medium-skilled workers within the group of low- and medium-skilled workers is

Table 3: Calibrated parameters

Parameter	Germany		France	
	95–05	10–17	95–05	10–17
Matching efficiency parameter: ζ	0.452	0.686	0.528	0.549
Vacancy cost: s_L	3.197	4.369	3.363	4.014
High-skill biased technology: A_H	1.121	1.344	1.237	1.332
Parameter of function $\bar{\alpha}(I)$: ρ	0.694	0.915	0.671	0.681

Notes: Calibration has been done by matching the targets in Table 2. The model parameters satisfy the stability conditions described in Online Appendix B.2.

higher in Germany, amounting to 79.9% in the initial period compared to 59.1% in France, according to data from EU-LFS. Combined with the fact that medium-skilled workers obtain more education and are generally more productive than less educated workers, this implies a higher A_L in Germany than in France. Moreover, both countries differ in how the educational system is linked to the work organization. Traditionally, France is considered as "organizational" space and Germany as "qualificational" space—this classification has been introduced in Maurice et al. (1986). In France, hiring criteria are based on workers' general education, whereas in Germany, specific jobs require qualifications tailored to those jobs. These qualifications are primarily obtained by medium-skilled workers through vocational education, which is more prevalent in Germany than in France. This type of education creates stronger school-to-work linkages, leading to higher productivity, see, e.g., Elbers et al. (2021) and DiPrete et al. (2017). This factor additionally contributes to a higher A_L and ultimately lower skill bias in Germany than in France.

In comparing both periods, the slope of the French productivity function increases in the more recent period due to an increasing skill bias in France. This shift is driven by a strong increase in the share of high-skilled workers and a pronounced decrease in the share of low-skilled workers, while the proportion of medium-skilled workers remains constant over time (see data from EU-LFS). However, the concavity of the productivity function remains largely unchanged due to a similarly calibrated ρ in the second period. Our calibration indicates that the gap in the relative productivity of high-skilled workers between the two periods widens as task complexity increases. For Germany, the pattern is different—the relative productivity of high-skilled workers in the period 2010–2017 is lower than in the period 1995–2005 for tasks with lower complexity, with the gap decreasing up to a task index i of around 0.4. Beyond this value, the relative productivity of high-skilled workers is higher in the more recent period and the gap widens with increasing

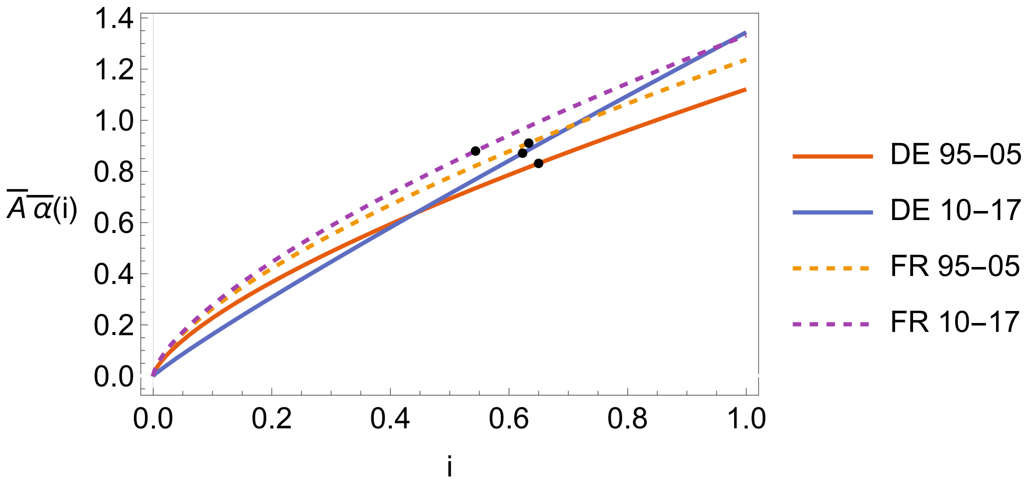


Figure 3: Calibrated productivity functions for Germany (DE) and France (FR) in periods 1995–2005 and 2010–2017.

Notes: The relative productivity function is described by $\bar{A}\bar{\alpha}(i) = \bar{A}i^{\rho}$. The circle marker on each of the plotted curves indicates the position of the corresponding task threshold $i = I$ set as a target in the calibration; see Table 2.

task complexity.

Figure 4 provides a more detailed picture of the differences in the productivity profile across periods for Germany. In the second period, a higher skill bias in Germany shifts the entire relative productivity profile upwards, leading to a growing discrepancy between the old and new productivity functions. This effect is illustrated by the violet dashed line, where ρ is kept at its initial level, and only the higher skill bias in the second period is taken into account. The increase in skill bias can be attributed to a change in skill composition, similar to the trends observed in France, according to data from EU-LFS. In the second period, the higher calibrated ρ results in a lower degree of concavity, as illustrated by the solid yellow line, where $\bar{A}$ remains at the level of the period 1995–2005. This induces an increase in the relative productivity of low-skilled workers that could alternatively be interpreted as a shift in task complexity i at the given relative productivity $\bar{\alpha}$. This shift is consistent with the empirically documented increase in occupational complexity in Germany, especially among less educated workers. For instance, Pikos and Thomsen (2016) document an increase in the categories of performed tasks, and Spitz-Oener (2006) highlight the change in task composition of occupations towards more non-routine tasks. Bachmann et al. (2022) demonstrate that jobs previously intensive in routine tasks but becoming more intensive in cognitive tasks are associated with increased training, which in turn enhances the productivity of workers in these jobs.[12] In contrast, for France,

[12] Caines et al. (2017) provide similar evidence of a complex-task-biased technical change for the US.

Bittarello et al. (2024) show that low- and medium-skilled workers experienced a similar increase in routine and social tasks as high-skilled workers from 1991 to 2013, but either no change (low- plus medium-skilled) or a decrease (high-skilled) in cognitive tasks.[13] This may justify why the calibrated French relative productivity function does not exhibit a similar shift as the German one.

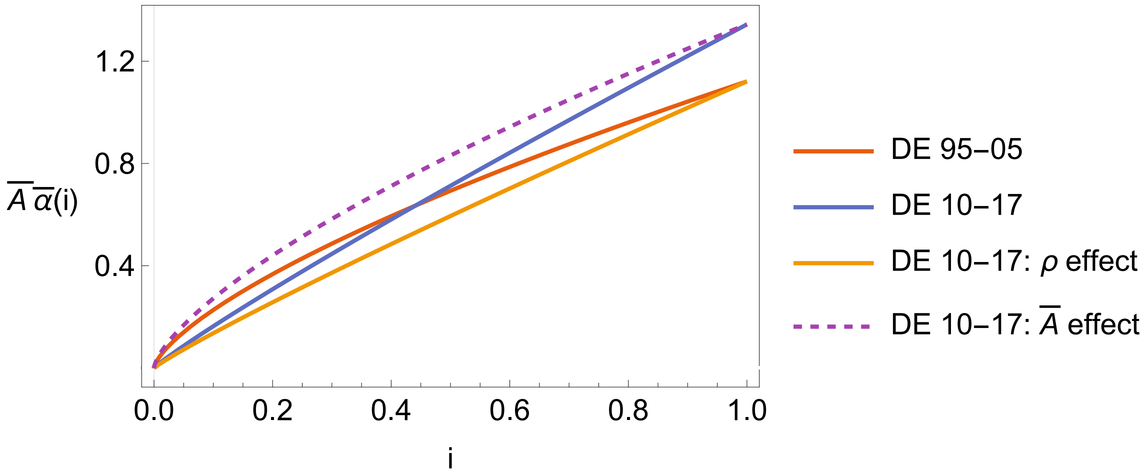


Figure 4: Calibrated productivity functions for Germany (DE) in two periods 1995–2005 and 2010–2017.

Notes: The relative productivity function is described by $\bar{A}\bar{\alpha}(i) = \bar{A}i^{\rho}$. The solid yellow line (dashed violet line) corresponds to the productivity function for the period 2010–2017 when keeping $\bar{A}$ (ρ) at the level of the period 1995–2005.

5.3 Comparative Statics: Increase in A_H

In this section, we simulate a 10% increase in A_H across both countries and time periods to analyze the varying effects of SBTC on labor market outcomes. We illustrate how task reallocation influences the labor demand elasticity and consequently the wage pressure in the economy, and how these resulting changes in the labor demand elasticity impact labor market outcomes. The simulated results are presented in Table 4.

In the period 1995–2005, the impact of a 10% increase in A_H on θ_L is negative in both countries, with the decrease being more pronounced in Germany than in France. The negative effect on θ_L corresponds to an increase in the unemployment rate of low-skilled workers in both countries and occurs due to two factors. First, for both countries, the calibrated values of the task productivity schedule parameter ρ result in the target values of I lying above the boundary value I_b in this period. This implies $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I > 1$,

[13]With expanding higher education, the skill premium has decreased, and highly educated workers have been assigned more routine tasks. Bittarello et al. (2024) illustrate this using a comparison of task categories of a bank clerk in 1991 and 2013 as an example.

Table 4: Labor market effects of an increase in high-skill productivity A_H by 10% (changes in variables expressed in percent).

Variable	**Germany**		**France**	
	95-05	10-17	95-05	10-17
Low-skilled labor market tightness: θ_L	–1.57	–0.81	–1.30	0.41
Low-skilled unemployment rate: u_L	0.70	0.38	0.57	–0.18
Task threshold: I	–2.84	–2.81	–2.96	–3.47
Low-skilled wage: w_L	1.38	1.80	1.39	1.55
High-skilled wage: w_H	8.72	8.40	8.74	8.71
Skill premium: $\frac{w_H}{w_L}$	7.34	6.59	7.35	7.17

Notes: Changes in u_L in percentage points are for Germany 0.075 (period 1995–2005) and 0.023 (2010–2017); the corresponding numbers for France are 0.064 and –0.021.

meaning that when firms reallocate tasks towards more high-skilled workers due to higher A_H the labor demand elasticity decreases significantly, thereby leading to a substantial improvement in the wage-setting power of labor unions. In this scenario, the wage curve (WC) is downward-sloping, as explained in Section 4. Second, given the model parameters and targets in this period, it holds for both countries that $|\Phi| < |\Gamma|$ so that the wage curve is steeper than the job creation curve (JC); see also Figure 2b.

In the period 2010–2017, the impact on θ_L remains negative for Germany but is weaker than in the previous period. Correspondingly, also in this period the unemployment rate increases due to SBTC in Germany, albeit to a lesser extent (0.023 percentage points compared to 0.075 percentage points in the previous period). In contrast, in France, the impact on θ_L (u_L) becomes positive (negative). The boundary value I_b in France remains constant from the first to the second period. However, a significant reduction in I, reflecting educational expansion in France and thus an increasing share of high-skilled income in total income, results in this case in $I < I_b$, for which we have $\mathrm{d}\ln \varepsilon_{L,\widetilde{w}_L}/\mathrm{d}\ln I < 1$. Similar to the previous case, the wage-setting power of labor unions increases due to the task reallocation towards more high-skilled workers. However, the increase in the wage markup is more moderate this time. The wage curve is upward-sloping and the SBTC effect on θ_L corresponds to the scenario in Figure 2a.

Concerning the effect of an increase in A_H on the task threshold, there is a stronger reallocation of tasks towards high-skilled workers in France than in Germany in both periods. Low-skilled wages rise due to SBTC in both countries and periods. This is due to a lower labor demand elasticity, and thus stronger wage-setting power of labor unions, induced by the decline in I. However, the increase in low-skilled wages is moderate in both countries and periods compared to the significant increase in wages of high-skilled workers. This translates into a pronounced increase in the skill premium in both countries and periods.

Our simulation shows that two factors determine the direction and the strength of the labor demand elasticity reaction in response to changes in I: the relative task productivity schedule and the task threshold in the economy determined by the share of high-skilled income in total income. Since these factors can vary across different steady states, the effects of SBTC on labor market variables can be completely different as well. The opposed signs of SBTC effects on labor market tightness in France in 1995–2005 and 2010–2017 steady states perfectly exemplify this insight.

6 Empirical Task Productivity Schedule

In this section, we aim to validate an important result of the calibration exercise. Both the theoretical analysis and the calibration results indicate that the shape of the relative task productivity schedule $\bar{\alpha}(i)$ significantly influences labor market outcomes for low-skilled workers in the context of SBTC. Unfortunately, the function $\bar{\alpha}(i)$ cannot be directly observed in real-world data. This limitation makes the carefully calibrated version of our theoretical framework a central tool for quantifying the effects of SBTC through the task reallocation channel. For the calibration, we employed a flexible functional form for $\bar{\alpha}(i) = i^{\rho}$, which allows for both concave and convex shapes of the relative task productivity schedule. One of the results of this calibration is that we obtained different values for ρ for Germany and France in both considered periods, resulting in distinct calibrated task productivity schedules, as shown in Figure 3. Specifically, for Germany, we found that the relative task productivity of high-skilled workers declined in lower-indexed tasks from the first to the second period, while it increased significantly in higher-indexed tasks. In the following, we utilize German microdata from the Socio-Economic Panel (SOEP) and approximate tasks using occupational data. We estimate the approximate task productivity schedules for Germany for two periods and examine how the estimated shapes and changes over time compare to their calibrated counterparts. We focus on Germany for illustrative purposes, and because of our comparative advantage in handling German microdata.

6.1 Deriving the empirical task productivity schedule

Our approach to estimating the relative task productivity schedule is based on the methodology suggested by Ales et al. (2015).[14] The point of departure for deriving an estimable task productivity schedule is the functional form of $\bar{\alpha}(i)$ used in the calibration (in logs):

$$\ln \bar{\alpha}(i) = \rho \ln i.$$

Whereas the theoretical function $\bar{\alpha}(i) = \alpha_H(i)/\alpha_L(i)$ takes values for all task complexity indices i, in the real world only the equilibrium task threshold would be observed and, therefore, only the value $\bar{\alpha}(I)$. This equilibrium value is determined in our model by eq. (7), i.e., the task allocation condition. From this, it follows:

$$\frac{A_H \alpha_H(I)}{A_L \alpha_L(I)} = \frac{w_H}{\widetilde{w}_L}.$$

With the simplifying assumption of no search-and-matching frictions, and defining $a_H(I) \equiv A_H \alpha_H(I)$ and $a_L(I) \equiv A_L \alpha_L(I)$, the above condition can be written in logs as

$$\ln a_H(I) - \ln a_L(I) = \ln w_H - \ln w_L. \tag{35}$$

Using the chosen specification for $\bar{\alpha}(i)$, the left-hand-side of eq. (35) is

$$\ln a_H(I) - \ln a_L(I) = \ln \bar{A} + \ln \bar{\alpha}(I) = \bar{a} + \rho \ln I,$$

and, therefore:

$$\ln w_H - \ln w_L = \bar{a} + \rho \ln I.$$

This equation holds for one specific value of i, which makes it clear that we cannot estimate the task productivity function using observed wages for low- and high-skilled workers alone. One natural way to address this problem is to extend the theoretical approach by considering the heterogeneity in ability or talent levels among low-skilled and high-skilled workers. Introducing $K > 2$ talent levels results in $(K-1)$ different thresholds. The firm's assignment of workers with a given talent to tasks is done analogously to the results in

[14] Ales et al. (2015) use a task assignment model to consider the implications of technical change for tax policy design. Their focus is on the slope rather than the curvature of the task productivity schedule, and they therefore simply estimate a linear function.

the basic theoretical model, i.e., the workers with the lowest talent index are assigned to the lowest task interval, and so on. In line with Ales et al. (2015), for the empirical analysis, we translate this setup to continuous talent types. Using k as the talent index, we can then formulate the following function that serves as an approximation for the total productivity function $\bar{A}\bar{\alpha}(i)$ in our two-skill case:[15]

$$\frac{\mathrm{d}\ln a(k,i)}{\mathrm{d}k} = \bar{a} + \rho \ln i.$$

Extending condition (35) to the continuous talent-types case gives an equation that is used for the estimation of $\bar{A}\bar{\alpha}(i)$:

$$\frac{\mathrm{d}\ln w(k)}{\mathrm{d}k} = \bar{a} + \rho \ln i. \tag{36}$$

6.2 Data

In the empirical analysis, we use the 1991–2017 waves of the German Socio-Economic Panel (SOEP), a high-quality and representative dataset of nearly 15,000 households and about 30,000 individuals in Germany.[16] For more information about the SOEP, see Goebel et al. (2019). In the first step, we perform a careful data selection process, which is described in detail in Online Appendix B.7.1. For the estimation of eq. (36), we consider two periods, 1991–1997 (period 1) and 2011–2017 (period 2), which roughly correspond to the two periods in the calibration exercise. The gap between periods 1 and 2 is slightly longer than in the calibration. By making the periods more distant from each other, we increase the likelihood of identifying potential changes in the real-world task productivity schedule. In the following, we outline the relevant variables required for the estimation.

Wages

First, we construct median real hourly wages at the occupational level for each period. In the benchmark scenario, we use the German Classification of Occupations 1992 (KldB92) at the 3-digit level and restrict each occupation to have at least 50 workers, resulting in a total of 166 occupations. Using the median instead of the mean, along with this

[15] Of course, $\bar{A}$ is only a shift parameter. The curvature of the total productivity schedule depends on the shape of the relative task productivity schedule $\bar{\alpha}(i)$.

[16] Socio-Economic Panel (SOEP), data for years 1984-2021, SOEP-Core v38.1, EU Edition–Update, 2023, `doi:10.5684/soep.core.v38.1eu`

size requirement, should ensure that we obtain reliable figures for average occupational wages. Online Appendix B.7.2 additionally describes alternative scenarios that use the International Standard Classification of Occupations 1988 (ISCO88) and a different size requirement.

Empirical task indices

We treat occupations as the empirical analogs of tasks. Following Ales et al. (2015), we consider the average wage level of an occupation as indicative of its complexity. We rank occupations according to their complexity based on occupational wages. In the benchmark scenario, we use wages from period 1 to rank the occupations and keep this ranking fixed for period 2. By using the same ranking for both 1991–1998 and 2011–2017, we assume that the complexity of occupations remains time-invariant. This approach prevents potential differences in the estimated period-specific task productivity schedules from being attributed to any change in wage rankings. Online Appendix B.7.2 discusses an alternative period considered for the occupational ranking. The ordered occupations are assigned an index i from a normalized interval $[0, 1]$ such that the indices are evenly spaced within this unit-length interval.

Empirical talent indices

The theoretical underpinning of our empirical strategy, outlined in the previous subsection, requires continuous talent types. In the empirical application, we consider discrete talent indices k distributed across the normalized interval $[0, 1]$. We assume thereby that talents are assigned to task thresholds and this assignment function is strictly increasing. Talent indices are obtained as follows: First, we calculate the shares of low- and medium-skilled workers (low-skill intensities) in the ranked occupations and apply the LOWESS smoother to this data. Then, we reweight the smoothed low-skilled intensities so that they sum up to one across all occupations. Finally, we compute talent ranks k as cumulative low-skill intensities in the ranked occupations. They increase with increasing i, but unlike task indices, the k values are not evenly spaced within the unit-length interval. The smoothed low-skilled intensities decrease with increasing occupational ranking, which strengthens our confidence in the plausibility of occupational ordering based on wages. For a graphical representation of low-skill intensities and talent ranks k, we refer to Online Appendix B.7.2.

Wages as a function of talent indices

The ranked occupational wages are smoothed using the LOWESS smoother. Since talent indices k are an increasing function of i, ranked wages can be represented as a function of k. Figure 5 depicts log smoothed wages over talent indices for both periods. These plots show that wages of workers at the lower and higher talent spectrum increased in period 2 compared to period 1, whereas they remained roughly the same for workers in the middle of the talent distribution. Moreover, returns to talent, as reflected by the slope of each wage graph, are decreasing for lower k, constant at middle k, and increasing at higher k. This suggests an increasing pattern of returns to talent with increasing task complexity and thus corroborates the crucial comparative advantage assumption that the productivity of higher-skilled workers increases in more complex tasks. Returns to talent, which are the dependent variable in eq. (36), are calculated using numerical methods and evaluated at each k.

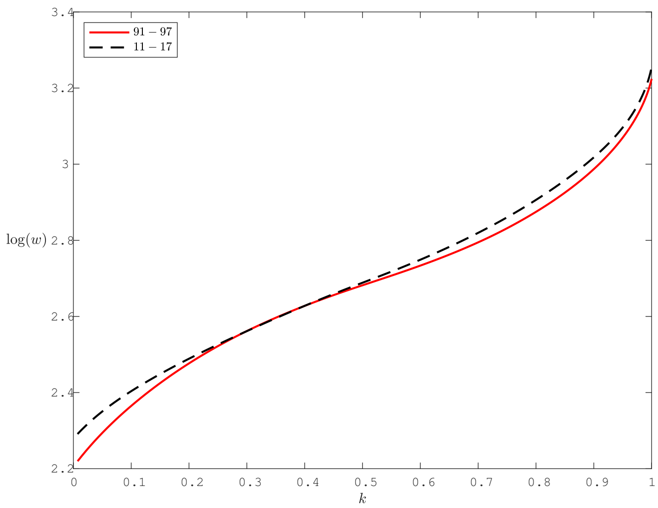


Figure 5: Log wages as a function of talents in period 1 (1991–1997) and period 2 (2011–2017)

6.3 Results

Table 5 summarizes the estimation results.[17] These findings show that the estimated values for the parameter ρ (0.72 in period 1 and 1.37 in period 2) are very similar to

[17]We consider the interval $[1, 2]$ for i in the estimation rather than $[0, 1]$. This is because the logarithmic transformation of values between 0 and 1 induces large spaces between $\ln(i)$ at lower i which become smaller towards the end of the interval. Since $\ln(i)$ is the explanatory variable in eq. (36), such extremely uneven spacing artificially overemphasizes returns to skills at lower k. In contrast, due to the property $\ln(1 + i) \approx i, i \in [0, 1]$ values of log task indices from the interval $[1, 2]$ are close to evenly spaced.

the calibrated values (0.69 in period 1 and 0.92 in period 2). Figure 6 depicts the estimated task productivity schedules in both periods, which closely resemble the calibrated counterparts for Germany in Figure 3. As in the calibration exercise, the estimated function $\bar{A}\bar{\alpha}(i)$ is concave in period 1 and becomes almost linear in period 2. Both curves intersect in the middle range of task complexity.[18] With this empirical evidence, we can be confident that the results of the calibration exercise are reliable. To ensure that the empirical results are not sensitive to alternative data selection specifications, we perform several robustness checks, which are reported in Online Appendix B.7.2. According to these findings, the pattern of estimated task productivity schedules remains robust if we consider *(a)* 1998–2010 (between periods 1 and 2) as the period used for the ordering of occupations, *(b)* ISCO88 as an alternative occupation classification, *(c)* a less restrictive requirement for a minimum occupation size.

Table 5: Estimation results for the task productivity function for period 1 (1991–1997) and period 2 (2011–2017); confidence intervals for estimated parameters are given in parentheses.

Period	$\bar{a}$	ρ	Estimated $\bar{A}\bar{\alpha}(i)$
1991–1997	0.77 (0.56, 0.99)	0.72 (0.18, 1.26)	$2.17\, i^{0.72}$
2011–2017	0.52 (0.32, 0.72)	1.37 (0.85, 1.88)	$1.68\, i^{1.37}$

Notes: $N = 166$; estimated $\bar{A}\bar{\alpha}$-function is recovered from the estimated equation $\mathrm{d}\ln w(k)/\mathrm{d}k = \bar{a} + \rho \ln i$ and takes the form $e^{\bar{a}}\, i^{\rho}$.

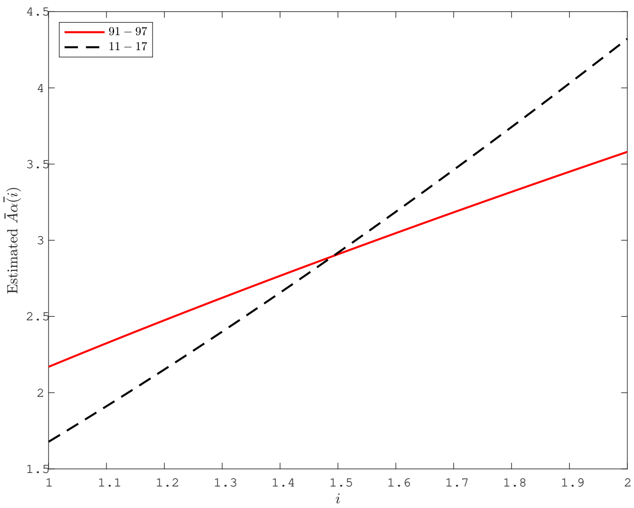


Figure 6: Estimated task productivity function for period 1 (1991–1997) and period 2 (2011–2017)

[18]The intercept in this diagram represents the relative productivity between the workers with the lowest skill and the next-lowest talent and, therefore, cannot be directly interpreted as the relative productivity of high-skilled workers (skill bias), $\bar{A}$.

7 Summary and Conclusions

This paper combines the task approach, labor union wage setting, and the matching framework to analyze the impact of skill-biased technical change (SBTC) on labor union wage setting brought about by changes in firms' assignment of tasks to low- and high-skilled workers. In labor union models, the wage elasticity of labor demand plays a crucial role in the extent of wage pressure in the economy. We show that this elasticity is influenced by the task threshold that divides the range of tasks performed by low- and high-skilled workers. Specifically, we demonstrate that the labor demand elasticity for low-skilled workers consists of a direct wage effect and a task reallocation effect. The latter implies that with an increase in low-skilled labor costs, fewer tasks are allocated to low-skilled labor. The strength of the task reallocation effect depends on the intensity with which low-skilled workers are used in the production process and on the shape of the relative task productivity schedule that reflects the substitutability of high- and low-skilled workers. Since both convex and concave shapes of the relative task productivity schedule are theoretically possible, the effect of a change in the task allocation on the labor demand elasticity remains ambiguous.

This ambiguity carries over to the general equilibrium, condensed into a two-equation system reflecting job creation by firms and wage claims of labor unions. Whereas in standard matching models an increase in labor market tightness leads to higher wage pressure along a positively sloped wage curve, in our model the wage curve can also be downward sloping. This has significant implications for the effects of SBTC. In contrast to the standard result that an increase in the factor productivity of high-skilled workers also positively impacts the employment and wages of low-skilled workers, in our model, low-skilled workers may instead experience higher unemployment or lower real wages.

Calibrating the model to German and French data for the periods 1995–2005 and 2010–2017, we find that the impact of SBTC may even change its sign over time. With the parameterization for the first period, SBTC increases low-skilled unemployment in both countries. With the parameterization for the second period, SBTC still increases low-skilled unemployment in Germany but reduces it in France. In both countries and periods, real wages of high-skilled workers increase more than those of low-skilled workers, thus increasing the skill premium. The driving force behind these outcomes is the decline in the labor demand elasticity due to SBTC, particularly the strength of this decline. It depends on the shape of the relative task productivity schedule and the task allocation of low- and high-skilled workers, which differ across countries and periods. To ensure the credibility of our calibration results for the task productivity schedule, we utilize German microdata from the SOEP and approximate tasks using occupational data. The estimated shape of this approximated task productivity schedule, as well as its changes over time, closely matches our quantitative findings, thereby bolstering our confidence in the calibration results.

These results highlight several interesting extensions for future research. For example, our modeling framework could be applied to compare the outcomes of the wage-setting process in different sectors of the economy. Some sectors may entail a range of tasks that rapidly increase in complexity, leading to more exponential increases in the relative productivity of high-skilled workers. Other sectors may exhibit a task complexity profile conducive to stronger skill substitution. Different sectors may also have varying task allocations among skill groups. Applying insights from our model could in this case explain differing sectoral real wage developments for workers with the same skill level facing the same extent of SBTC. Additionally, including capital, especially automation capital, into the model would allow to analyze the impact of automation on the bargaining power of labor unions and unemployment. These possible extensions of our model would deepen our understanding of the broader implications of technological change on labor market outcomes.

A Appendix

A.1 The Firm's Optimization Problem

Combining eqs. (1)–(5), and considering the restrictions (6) for $l_t(i)$ and $h_t(i)$, the firm's optimization problem can be written as

$$\begin{aligned}
\max_{\{l_t(i),h_t(i),V_{Lt},\mu_{Lt}\}} \mathcal{L} = \sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1} & \Bigg\{\exp\left[\int_0^1 \ln\left(A_{Lt}\alpha_L(i)l_t(i) + A_{Ht}\alpha_H(i)h_t(i)\right)\mathrm{d}i\right] \\
& -w_{Lt}\int_0^1 l_t(i)\mathrm{d}i - w_{Ht}\int_0^1 h_t(i)\mathrm{d}i - s_L V_{Lt}\Bigg\} \\
+\sum_{t=0}^{\infty}\mu_{Lt}\left(\frac{1}{1+r}\right)^{t-1} & \left[m(\theta_{Lt})V_{Lt} + (1-q_L)\int_0^1 l_{t-1}(i)\mathrm{d}i - \int_0^1 l_t(i)\mathrm{d}i\right] \\
\text{s.t.} \quad l_t(i) \geq 0, \qquad & h_t(i) \geq 0, \qquad \text{and} \qquad l_0(i), h_0(i) \text{ given,}
\end{aligned}$$

where μ_{Lt} denotes the shadow price of L_t in period t. The single firm takes aggregate labor market tightness θ_{Lt} as given. The first-order conditions are $\partial\mathcal{L}/\partial\mu_{Lt} = 0$, $\partial\mathcal{L}/\partial V_{Lt} = 0$ (which gives $\mu_{Lt} = s_L/m(\theta_{Lt})$), and the complementary slackness conditions

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial h_t(i)} \leq 0, \qquad & h_t(i) \geq 0, \qquad \frac{\partial\mathcal{L}}{\partial h_t(i)}h_t(i) = 0, \\
\frac{\partial\mathcal{L}}{\partial l_t(i)} \leq 0, \qquad & l_t(i) \geq 0, \qquad \frac{\partial\mathcal{L}}{\partial l_t(i)}l_t(i) = 0.
\end{aligned}$$

This leads to

$$\frac{Y_t}{y_t(i)}A_{Ht}\alpha_H(i) \leq w_{Ht}, \qquad h_t(i) \geq 0, \tag{A.1}$$

$$\frac{Y_t}{y_t(i)}A_{Lt}\alpha_L(i) \leq \widetilde{w}_{Lt} \equiv w_{Lt} + \frac{s_L}{m(\theta_{Lt})} - \frac{s_L}{m(\theta_{L,t+1})}\frac{(1-q_L)}{(1+r)}, \qquad l_t(i) \geq 0. \tag{A.2}$$

Due to complementary slackness in each equation, only one inequality can hold at the same time. As can be seen from eq. (A.2), the low-skilled labor costs relevant to the firm, $\widetilde{w}_{Lt}$, consist of the wage w_{Lt} plus the search costs incurred in period t, which are reduced by the vacancy posting costs that are saved in period $t+1$ if the employment relationship continues. For the discussion of the different cases we focus on the steady state in which $\theta_{L,t+1} = \theta_{Lt} = \theta_L$. In that case, $\widetilde{w}_{Lt} = \widetilde{w}_L$, where

$$\widetilde{w}_L \equiv w_L + \frac{(q_L+r)}{(1+r)}\frac{s_L}{m(\theta_L)}. \tag{A.3}$$

Case 1: $l(i) > 0$ and $h(i) = 0$. Due to eq. (2) in the main text $y(i) = A_L\alpha_L(i)l(i)$, implying $Y/l(i) = \widetilde{w}_L$ in eq. (A.2). The marginal contribution of unskilled labor in task i to total output Y equals the low-skilled labor costs relevant to the firm. It follows that $l(i) = l$, i.e. the same labor input l is chosen in all low-skilled tasks. From eq. (A.1) follows

$$\frac{\widetilde{w}_L}{A_L\alpha_L(i)} < \frac{w_H}{A_H\alpha_H(i)},$$

if the constraint on $h(i)$ is binding. Hence, low-skilled workers are employed in those tasks in which their unit labor costs are lower than those of high-skilled workers. At the margin where $\partial\mathcal{L}/\partial h(i) = 0$, there is a specific task $i = I$ for which $\widetilde{w}_L/(A_L\alpha_L(I)) = w_H/(A_H\alpha_H(I))$.

Case 2: $h(i) > 0$ and $l(i) = 0$. From eq. (2) follows $y(i) = A_H\alpha_H(i)h(i)$, implying $Y/h(i) = w_H$ in eq. (A.1) which is interpreted analogously. It follows that $h(i) = h$, i.e. the same labor input h is chosen in all high-skilled tasks. From eq. (A.2) follows

$$\frac{\widetilde{w}_L}{A_L\alpha_L(i)} > \frac{w_H}{A_H\alpha_H(i)},$$

if the constraint on $l(i)$ is binding. Hence, high-skilled workers are employed in those tasks in which their unit labor costs are lower than those of low-skilled workers. At the margin where $\partial\mathcal{L}/\partial l(i) = 0$, there is a specific task $i = I$ for which $\widetilde{w}_L/(A_L\alpha_L(I)) = w_H/(A_H\alpha_H(I))$.

Case 3: $h(i) > 0$ and $l(i) > 0$. Because of eq. (2) $y(i) = A_L\alpha_L(i)l(i) + A_H\alpha_H(i)h(i)$. In eq. (A.2) it holds that $(Y/y(i))A_L\alpha_L(i) = \widetilde{w}_L$. In eq. (A.1) it holds that $(Y/y(i))A_H\alpha_H(i) = w_H$. Hence,

$$\frac{\widetilde{w}_L}{A_L\alpha_L(I)} = \frac{w_H}{A_H\alpha_H(I)}.$$

Case 4: $h(i) = 0$ and $l(i) = 0$. In that case, $y(i) = 0$ which due to the production function in eq. (1) is not possible.

In cases 1-3 the task threshold I is defined as the task where unit labor costs for high-and low-skilled workers are equal. This condition can be written as

$$\bar{\alpha}(I) \equiv \frac{\alpha_H(I)}{\alpha_L(I)} = \frac{A_L\, w_H}{A_H \widetilde{w}_L}. \tag{A.4}$$

Since $\bar{\alpha}'(i) > 0$, there is only one task $i = I$ where unit labor costs of both worker types are equal. It must hold that $I < 1$, because values $I \geq 1$ would imply that no high-skilled workers are used in the production process, in contradiction to our assumption that high-skilled workers are fully employed. Moreover, if unemployment benefits are not too high, it is never optimal for labor unions to demand such high wages that no unskilled workers are employed. In that case, it must also hold that $I > 0$. As a consequence, $0 < I < 1$.

A.2 Wage Setting of Labor Unions

The representative labor union chooses w_{Lt} to maximize $V_{Lt} = R_{Lt}\, L_t$ s.t. to the expression for R_{Lt} in eq. (17) and the labor demand equation (11), $L_t = L^d\left(\widetilde{\omega}_t, \cdot\right)$, where $\widetilde{\omega}_t \equiv w_{Ht}/\widetilde{w}_{Lt}$. In line with steady-state considerations, aggregate labor market tightness is considered to be given and constant. Therefore, $\widetilde{w}_{Lt}$ corresponds to the expression in eq. (A.3). The first-order condition $\mathrm{d}V_{Lt}/\mathrm{d}w_{Lt} = 0$ gives

$$\frac{\partial R_{Lt}}{\partial w_{Lt}} L_t + R_{Lt} \frac{\partial L^d}{\partial \widetilde{\omega}_t} \frac{\partial \widetilde{\omega}_t}{\partial w_{Lt}} = 0. \tag{A.5}$$

Multiplying by $\widetilde{w}_{Lt}/L_t$ and defining

$$\varepsilon_{L\widetilde{w}_L,t} \equiv \left| \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{w}_{Lt}} \right| = \frac{\partial \ln L^d(\cdot)}{\partial \ln \widetilde{\omega}_t} \tag{A.6}$$

leads to

$$\left(\frac{1-\delta^n}{1-\delta}\right) \widetilde{w}_{Lt} - R_{Lt}\, \varepsilon_{L\widetilde{w}_L,t} = 0.$$

Defining

$$\widetilde{z}_{Lt} \equiv z_{Lt} + \frac{(q_L + r)}{(1+r)} \frac{s_L}{m(\theta_L)}$$

and noting that $w_{Lt} - z_{Lt} = \widetilde{w}_{Lt} - \widetilde{z}_{Lt}$ gives

$$\frac{(1-\delta^n)}{1-\delta} \widetilde{w}_{Lt} - \left[\frac{(1-\delta^n)}{1-\delta} (\widetilde{w}_{Lt} - \widetilde{z}_{Lt}) + \delta^n R_{L,t+n} \right] \varepsilon_{L\widetilde{w}_L,t} = 0.$$

Therefore, the wage w_{Lt} set in period t for n periods implies the following wage costs $\widetilde{w}_{Lt}$ in period t:

$$\widetilde{w}_{Lt} = \frac{\varepsilon_{L\widetilde{w}_L,t}}{\varepsilon_{L\widetilde{w}_L,t} - 1}\left(\widetilde{z}_{Lt} - \frac{(1-\delta)\delta^n}{1-\delta^n} R_{L,t+n}\right). \tag{A.7}$$

In the steady state, $R_{L,t+n} = (\widetilde{w}_{Lt} - \widetilde{z}_{Lt})/(1-\delta)$. Hence,

$$\widetilde{w}_{Lt} = \frac{\varepsilon_{L\widetilde{w}_L,t}}{\varepsilon_{L\widetilde{w}_L,t} + \delta^n - 1}\widetilde{z}_{Lt}. \tag{A.8}$$

A.3 Proof of Proposition 2

From eqs. (7) and (10) follows

$$\mathrm{d}\ln I = \frac{1}{\varepsilon_{\bar{\alpha},I}}\left(\mathrm{d}\ln\widetilde{\omega} - \mathrm{d}\ln\bar{A}\right) \tag{A.9}$$

and

$$\mathrm{d}\ln L = \frac{1}{1-I}\,\mathrm{d}\ln I + \mathrm{d}\ln\widetilde{\omega}, \tag{A.10}$$

where it has been taken into account that $\mathrm{d}\ln H = 0$, and

$$\mathrm{d}\ln\widetilde{\omega} = \mathrm{d}\ln w_H - \mathrm{d}\ln\widetilde{w}_L. \tag{A.11}$$

From eq. (19) follows

$$\mathrm{d}\ln\widetilde{w}_L = \varepsilon_{\widetilde{z}_L,\theta_L}\mathrm{d}\ln\theta_L + \varepsilon_{\kappa_L,I}\,\mathrm{d}\ln I, \tag{A.12}$$

where

$$\varepsilon_{\widetilde{z}_L,\theta_L} \equiv \frac{\mathrm{d}\ln\widetilde{z}_L}{\mathrm{d}\ln\theta_L} = \beta_L\frac{\widetilde{z}_L - z_L}{\widetilde{z}_L} = \beta_L\kappa_L\frac{\widetilde{w}_L - w_L}{\widetilde{w}_L} = \beta_L\frac{\frac{(q_L+r)}{1+r}s_L\theta_L^{\beta_L}}{z_L + \frac{(q_L+r)}{1+r}s_L\theta_L^{\beta_L}} < \beta_L \tag{A.13}$$

and

$$\varepsilon_{\kappa_L,I} \equiv \frac{\mathrm{d}\ln\kappa_L}{\mathrm{d}\ln I} = -(\kappa_L - 1)\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} = -\frac{\kappa_L - 1}{\kappa_L}\left(\frac{I}{1-I} - \frac{\mathrm{d}\ln\varepsilon_{\bar{\alpha},I}}{\mathrm{d}\ln I}\right). \tag{A.14}$$

Because of eq. (23)

$$\mathrm{d}\ln\theta_L = \frac{1}{(1-\beta_L)u_L}\mathrm{d}\ln L, \tag{A.15}$$

where $u_L \equiv (1-H-L)/(1-H)$ denotes the low-skilled unemployment rate.

The price index equation (24) can be written as

$$I(\ln\widetilde{w}_L - \ln A_L) + (1-I)(\ln w_H - \ln A_H) - \xi(I) = 0,$$

where

$$\xi(I) \equiv \int_0^I \ln\alpha_L(i)\mathrm{d}i + \int_I^1 \ln\alpha_H(i)\mathrm{d}i.$$

The total differential of this equation is

$$\begin{aligned} &I(\mathrm{d}\ln\widetilde{w}_L - \mathrm{d}\ln A_L) + (1-I)(\mathrm{d}\ln w_H - \mathrm{d}\ln A_H) \\ &- [(\ln w_H - \ln\widetilde{w}_L) - (\ln A_H - \ln A_L) - \ln\bar{\alpha}(I)]\,\mathrm{d}I = 0, \end{aligned} \tag{A.16}$$

where it has been taken into account that $\xi'(I) = -\ln\bar{\alpha}(I)$. Since the task threshold is endogenously determined from profit maximization, eq. (7) must hold, implying $\ln\bar{\alpha}(I) = (\ln w_H - \ln\widetilde{w}_L) - (\ln A_H - \ln A_L)$. Hence, the term in brackets in the second line is zero, leading to

$$\mathrm{d}\ln w_H = \mathrm{d}\ln A_H + \frac{I}{1-I}\mathrm{d}\ln A_L - \frac{I}{1-I}\mathrm{d}\ln\widetilde{w}_L. \tag{A.17}$$

The job creation equation (26) in Proposition 2 is obtained by combining eqs. (A.17), (A.11), (A.10), and (A.15), and by assuming $\mathrm{d}\ln A_L = 0$. The wage-setting equation corresponds to eq. (A.12) and the task allocation equation follows from eqs. (A.9), (A.11) and (A.17), where $\mathrm{d}\ln\bar{A} = \mathrm{d}\ln A_H$ if $\mathrm{d}\ln A_L = 0$. This concludes the proof of Proposition 2. □

A.4 Comparative Statics: Effects on Other Variables

$$\frac{\mathrm{d}\ln\widetilde{\omega}}{\mathrm{d}\ln A_H} = \left(|\Phi| + \frac{1}{\kappa_L}\Gamma\right)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \vee \\ & \quad \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < \frac{1}{\kappa_L}|\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln w_H}{\mathrm{d}\ln A_H} = \left(|\Phi| + \frac{\varepsilon_{L,\widetilde{w}_L} - I}{\varepsilon_{L,\widetilde{w}_L}}\Gamma\right)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \vee \\ & \quad \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < \frac{\varepsilon_{L,\widetilde{w}_L} - I}{\varepsilon_{L,\widetilde{w}_L}}|\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln w_L}{\mathrm{d}\ln A_H} = \left(\frac{\kappa_L}{1 - \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I}} - 1\right)\frac{\varepsilon_{\widetilde{z}_L,\theta_L}\,\widetilde{z}_L}{\varepsilon_{L,\widetilde{w}_L}\,w_L}(1-I)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } 1 > \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 - \kappa_L \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < |\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln L}{\mathrm{d}\ln A_H} = \frac{|\Phi|}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln u_L}{\mathrm{d}\ln A_H} = -\frac{1-u_L}{u_L}\frac{|\Phi|}{|\Phi|+\Gamma}$$

$$\begin{cases} < 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \\ > 0, & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln A_H} = \Big(|\Phi| + (1-I)\Gamma\Big)\frac{1}{|\Phi|+\Gamma}$$

$$\begin{cases} > 0, & \text{if } \frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} < 1 \;\vee\; \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| > |\Gamma|\right) \vee \\ & \quad \left(\frac{\mathrm{d}\ln\varepsilon_{L,\widetilde{w}_L}}{\mathrm{d}\ln I} > 1 \;\wedge\; |\Phi| < (1-I)|\Gamma|\right) \\ < 0, & \text{otherwise} \end{cases}$$

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