JULY 1958

PERT

PROGRAM
EVALUATION
RESEARCH
TASK

SUMMARY REPORT
Phase 1

SPECIAL PROJECTS OFFICE
BUREAU OF NAVAL WEAPONS
DEPARTMENT OF THE NAVY
WASHINGTON, D.C.

PREFACE

This report summarizes the work and results of the first phase of Project PERT (Program Evaluation Research Task). The project began on 27 January 1958 with the purpose of studying the application of statistical and mathematical methods to the planning, evaluation, and control of the program of the Navy Special Projects Office. The project team included people from the Special Projects Office, Booz, Allen & Hamilton, and Lockheed Missile Systems Division.

The task objectives, specified at the outset, were as follows:

1. To develop methodology for providing the Director, Special Projects Office (SP) and the top SP managers with continuous program evaluation, i.e., the integrated evaluation of

   (1) The progress to date and the progress outlook toward accomplishing the objectives of the Fleet Ballistic Missile (FBM) program

   (2) The changes in the validity of the established plans for accomplishing the program objectives, and

   (3) The effect of changes proposed for established plans

2. To establish procedures for applying the methodology as designed and tested to the over-all FBM program

The first phase of the PERT project has been completed. A methodology has been devised, its feasibility and implications have been examined, and its theoretical potential has been assessed. The Director of the Special Projects Office has approved the continuance of PERT into the second phase of activity—the systematic application of the methodology to selected subsystems of the Fleet Ballistic Missile Program. The second phase is now well under way.

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I. INTRODUCTION TO PERT

This research project seeks to develop an improved method for planning and evaluating progress of a major research and development program—in this case, the Fleet Ballistic Missile Program. The objective is laudable but certainly not new. Improvement in management methods is continually sought. A few obvious comments on the nature of the undertaking are in order if only to formally post our appreciation of the broad nature of the problem.

In its basic format the plan of a research and development program is similar to the format of any other type of program. A series of tasks are scheduled in a logical sequence building up to attainment of a final objective. Product performance is specified, resources are allocated, and time of achievement of each task as well as the final objective is presented.

Three factors, however, set research and development programming apart. First, we are attempting to schedule intellectual activity as well as the more easily measurable physical activity. Second, by definition, research and development projects are of a pioneering nature. Therefore previous, parallel experience upon which to base schedules of a new project is relatively unavailable. Third, the unpredictability of specific research results inevitably requires frequent change in program detail. These points are acknowledged by all experienced research people.

Yet, even though it be ridiculous to conceive of scheduling research and development with the split-second precision of an auto assembly line, it is clear that the farther reaching and more complex our projects become, the greater is the need for procedural tools to aid top managers to comprehend and control the project. At the very least, such a method can improve over the random "how goes it" examination if only by providing

- Orderliness and consistency in planning and evaluating all areas of the project
- Automatic identification of all potential trouble spots arising in a complex project as a result of failure in one area
- Speed in integrating progress evaluation

And throughout runs the requirement for faithful portrayal of the rapidly changing research endeavor.

The fundamental problem is size and complexity—and a means to handle this size and complexity in rapid evaluation and necessary reprogramming. This is the problem of PERT.

The basic approach of PERT is less novel than the execution of the approach. Grossly oversimplified, the approach involves

(1) The selection of specific, identifiable events which must occur along the way to successful conclusion of the project.
(2) The sequencing of these events and establishing of interdependencies between events so that a project network is developed.
(3) The estimate of time required to achieve these events together with a measurement of the uncertainties involved.
(4) The design of an analysis or evaluation procedure to process and manipulate these data.
(5) The establishing of information channels to bring actual achievement data and change data to the evaluation point.
(6) The application of electronic data processing equipment to the analysis procedure.

The end product is to be a periodic summary evaluation report across the top of the entire project backed by sub-summaries of the more detailed project areas. Where problems appear, alternate courses of action will be presented for consideration.

There are qualifying factors to be considered in any such method developed. These are clearly understood. First, the system will still be based on human judgment (events and times) at its very source. The system can integrate these judgments in an orderly, consistent and rapid manner—but the quality of these judgments is a constraint upon the method. Second, the system should not add a significant load on research people whose job is to carry on research and development—not to cater to an evaluation system. But technical people must do their forward planning, and such a system can be an element of this planning.

This last point introduces a most important matter in research administration. The people most qualified to speak on what they have done, are doing, can do, and might do in a development project are the development people themselves. To interpose a substantial layer of evaluation organization between top management and the development people stretches the time of progress reporting, risks distortion of reports through successive interpretation on the way to the top, and generally adds to the remoteness of top management to the tasks it is managing.

A system should be a close coupling between the laboratory and top management and should serve both the planning and evaluation interests of both, each at the proper level. This PERT seeks to do.

Since, in a very real sense, PERT is a major extension of the existing program evaluation system—in philosophy if not in specific procedure—the current project management pattern should be briefly examined as background.

1. CURRENT PROJECT MANAGEMENT OF THE FBM SYSTEM

The Director of the Special Projects Office is charged with the over-all management of the Fleet Ballistic Missile System. This general cognizance operates principally through two managerial divisions—the Plans and Programs Division and the Technical Division. These two divisions are charged with directing the activity of the FBM contractors toward the accomplishment of an over-all objective—the operational capability for the FBM System at a designated future point in time.

The Technical Division oversees the FBM contractors in the hardware developmental aspects of the system. The Plans and Programs Division is charged with management of resources, forward planning, and the evaluation of current progress as well as analysis of future ability to successfully accomplish the over-all objective. Both of these managerial divisions have action prerogatives. It is their responsibility to initiate and follow up on any remedial steps that are necessary to ensure the timely accomplishment of the FBM System objective.

The central repository of information depicting over-all progress on the FBM System is the SP Management Center, assigned organizationally to the Plans and Programs Division. Both managerial divisions transmit information to the Management Center for portrayal in an integrated progress report.

2. THE SP MANAGEMENT CENTER

The SP Management Center summarizes current and forecasted progress on the FBM System in an extensive graphical presentation. This presentation covers the entire FBM program across the top and extends in depth through the important subsystems and down to the level of principal components of the subsystems. This progress information is based upon a sequence of important milestones together with their scheduled dates for accomplishment. Indications of actual accomplishment, or slippage (observed or forecast), are noted. The detail analysis is then summarized in bar charts showing conditions as generally being in "good shape," with "minor weakness," etc.

Information is received in the Management Center and charts updated once a week. These form the basis of weekly briefings for the Director and others with a vital interest in the progress of the FBM System. Progress and reporting evaluations as described in the course of the weekly meetings form the basis for executive action designed to remedy undesirable situations as they arise.

The Management Center thus plays an important role in the SP effort. It is an excellent device for giving a "bird's-eye" view of the magnitude of the SP activity. It presents a broad picture of progress currently being achieved as well as insights into anticipations. The system thus aims at the same broad objectives posed in the opening discussion.

However, the SP managers recognize that the Management Center and its system, as presently constituted, can be improved. Points of potential improvement include the following.

(1) The Milestones that Appear on the Management Center Charts Frequently Represent Indefinite Accomplishments

In order to be effectual, milestones should be positive and tangible in nature so that they are definitely distinguishable as specific points in time. If they cannot be so identified, scheduled dates lose their meaning. Milestones now used make liberal use of terms such as "evaluate," "test," or "determination," which gives wide latitude in specifying the time of actual accomplishment.

(2) Milestones on the Management Center Charts Frequently Have Different Meanings to SP and to Contractors

Milestones should have commonly understood meanings as well as being identifiable as a point in time. In the absence of modifying statements, such milestones as "successful test" or "qualify by—" allow for differing interpretations on the part of SP and its contractors.

(3) SP Contractors Do Not Consistently Measure Progress against the Management Center Milestones

Effective communication and common understanding of the total evaluation require that the contractors measure their progress against scheduled milestones used by SP. This is not consistently the case. There need not be a complete identity, but events of importance should be included in both systems in a common manner. It is, of course, of mutual advantage to have but a single milestone system—with SP extracting from the contractors' lists those milestones which are of summary importance for top executive action.

(4) The Method of Analyzing and Presenting Progress in the Management Center Allows Some Slippages to Occur Unnoticed

In summarizing progress, the Management Center evaluation describes subsystem progress as being in "good shape," with "minor weakness," etc. The qualitative nature of this summary evaluation can obscure impending trouble as no means exists for highlighting the importance of an individual milestone to the over-all system. Thus, a subsystem may be shown in "good shape" because a slipped milestone of the subsystem is in itself relatively insignificant. However, the slipped, minor event may be unrecognized evidence of the future slippage of a most important event. This lack of recognition of impending trouble can stem from the absence of a formalized network which shows the time interactions and sequences of all events in the FBM System.

(5) The Management Center Portrayal Does Not Give Sufficient Help to Technical Personnel in Their Forward Planning Activity

In a complex, accelerated research program, it is inevitable that schedule slippages will occur. When such a situation comes about, the Technical Division must be in a position to evaluate the impact of the slippage on the whole program and suggest optional procedures if objectives are seriously jeopardized. The timely evaluation of optional future courses of action depends on a mechanism for speedily showing the consequences of such changes on the entire program. The Management Center analysis deals with relatively few milestones appraised in an "ad hoc" process. The results can, therefore, be inefficient and costly in terms of time involved. The ability of the technical people to make forward plans effectively suffers correspondingly.

These weaknesses in the present system—advanced as it is—have been pointed out by those who manage the system and those whose decisions are based on the output of the system. It is an aim of PERT to correct these weaknesses.

The SP Management Center has as its raw materials—milestones, program plans, schedules, and their analysis. These basic features will have an important role in any modified methodology which builds on the foundation of the present system. The PERT methodology seeks to exploit existing material and procedures to the greatest extent possible as long as their use does not prejudice the ultimate usefulness of the new methods.

3. THE SCHEDULING AND ANALYSIS PROCESS

A quick identification of the main elements in the process of scheduling and sebsequent performance analysis is in order to get certain broad definitions in common understanding.

Schedules and Scheduling

(1) A plan is nothing more than an ordered sequence of events (or activities) necessary to achieve a stated objective. To be complete, this ordered sequence of events must show *all* significant interrelationships that exist among those events beyond simple sequence.

A plan should be definitive in that it is developed in terms of the anticipated application of resources as well as the desired level of technical performance in the end item. A change in the objectives, in resources applied or performance desired, will ordinarily change the plan—the events, their interrelationships, or their sequence.

A schedule is the plan for action identified with calendar dates for planned accomplishment of explicit objectives. This stated or "plan" schedule becomes a formal benchmark for progress evaluation.

(2) Actual day-to-day happenings never follow the stated or "nominal" schedule exactly. They should bear a reasonable identity, but the "actual" schedule will continuously change and flex within the general limits of the nominal schedule.

It is important to note that the changing state of the "actual" schedule does not imply that the "nominal" schedule should similarly change. Only significant changes in the anticipated "actual" schedule should be reflected in the nominal schedule—otherwise we should never reach a stable schedule for common use and progress evaluation. The analysis procedure should aid technical people in arriving at conclusions as to when significant changes in anticipations have occurred so that the nominal or stated schedule is changed.

(3) At any point in time, an optimum "actual" schedule will exist. The existence of an optimum means that some criterion has been maximized or minimized. With a knowledge of the appropriate criterion, it might be possible to specify an optimum schedule. However, the real problem in fixing an optimum schedule lies in establishing a criterion that integrates time, resources, and technical performance meaningfully. No such satisfactory common criterion for optimization has been developed that could be usefully brought into the PERT method during Phase I.

(4) Since such a criterion is currently unavailable, an approach dealing with the time variable is the only practical procedure at this stage of development. Therefore, the PERT methodology has scheduled time (as a function of resource and performance) with only incidental attempts to deal with optimums. The proposed procedure is general in the sense that it can deal with a variety of different resource and performance combinations but only when these combinations are specifically designated by technical people.

The Analysis and Evaluation Process

The evaluation process must be capable of providing SP management with a valid picture of current progress at any point in time. To be effective, the process should also make for rapid, accurate analysis and be subject to ready interpretation. However, a good analysis and evaluation process must do more than merely fulfill these requirements.

(1) The process should designate events in a well-defined, positive sense so that there can be no question as to the time at which each event has been successfully accomplished.

(2) Potential slippages must be discovered before the fact so that remedial action can be instituted. The PERT procedure fulfills this requirement by citing probability statements for the accomplishment of all future events in the schedule.

Such knowledge of a potential slippage is important before-the-fact information. However, this knowledge alone is not enough. It is necessary for the managers to know the significance of such a slippage and its impact on other allied events. In some cases, a schedule slippage is important systemwise. In other cases, the slippage of an event may have little effect on the timely accomplishment of important future events. The PERT method identifies the significance of actual and potential slippages by showing the relative slack or rigidity in key scheduled dates.

(3) The utility of a good analysis and evaluation process lies in the activity which it gives rise to—not to the mere divulging of facts. A good evaluation process notes potential schedule disturbances and then guides positive action in such a way as to obviate the disturbances it has identified.

The PERT procedure is designed to fulfill this most important requirement.

(4) An evaluation and analysis process for a research and development program should reflect the inherent uncertainty of scheduled dates.

Even should all future steps in a plan be firmly set, the uncertainties in estimating future dates require that recognition be given to the random nature of the actual accomplishment times for each step. Randomness does not mean a lack of predictability. It does mean that there is an indefiniteness to a prediction. A good evaluation process will take explicit account of the magnitude of this indefiniteness. Engineers in the FBM development have indicated that the only realistic way of estimating R&D accomplishment is through the citing of a range of time in which it is predicted that the accomplishment will take place.

(5) When an existent schedule has been shown to be infeasible, the analysis and evaluation process should be able to give aid in the formulation of a more appropriate schedule. In the PERT methodology, the analysis process can provide this service through itemizing new dates which have a high probability of being met.

(6) The development of the FBM incorporates a tremendously complex system of event achievement. It is estimated that there may be upwards of 5,000 events which should be portrayed in the evaluation process. The computations that must be undertaken for each event as well as the interactions between events require something more than unabetted human contemplation. For this reason, the PERT procedure has been laid out so as to be compatible with processing on modern electronic computers.

* * * * *

This chapter has considered in broad outline the background against which the PERT team approached its task. Having noted the broad objectives of the activity, the nature of specific requirements, and the structure of the current systems foundations, it is now possible to turn to a detailed examination of the PERT approach. The following chapter will describe the elements of the proposed procedure as it has been designed to accommodate the specifications of an efficient analysis and evaluation system.

II. THE PERT APPROACH

1. DATA STRUCTURE

The basic structure of the PERT approach rests on a flow plan and detailed elapsed-time estimates.

(1) The Flow Plan

It is assumed that an ordered sequence of events (with associated resources and performance) can constitute a valid model of the FBM research and development schedule.

The flow plan is a sequence of events that are necessary to the accomplishment of the end objective. The events themselves are distinguishable points in time that coincide with the beginning and/or end of specific tasks in the R&D activity. The flow plan identifies the events and casts them into a pattern which shows their interrelationships and time precedences. A flow plan is at least implicitly associated with specified performance specifications and a given rate of resource application.

Exhibit A presents an idealized version of a typical flow plan which will form the basis of subsequent analytical treatment. In Exhibit A the numbered circles represent events in the development system. The arrows between events represent the times necessary for accomplishing activities. The configuration of the event pattern is the result of a specified sequence of activities. Thus, event #50 might conceivably be the objective of the idealized system. It must occur in time after events #51 and #54, i.e., event #50 is the culmination of two separate activities that start with events #51 and #54. It can be observed that some events depend only on a single prior event while others can depend on more. The activities (represented by the arrows)

SYSTEM FLOW PLAN

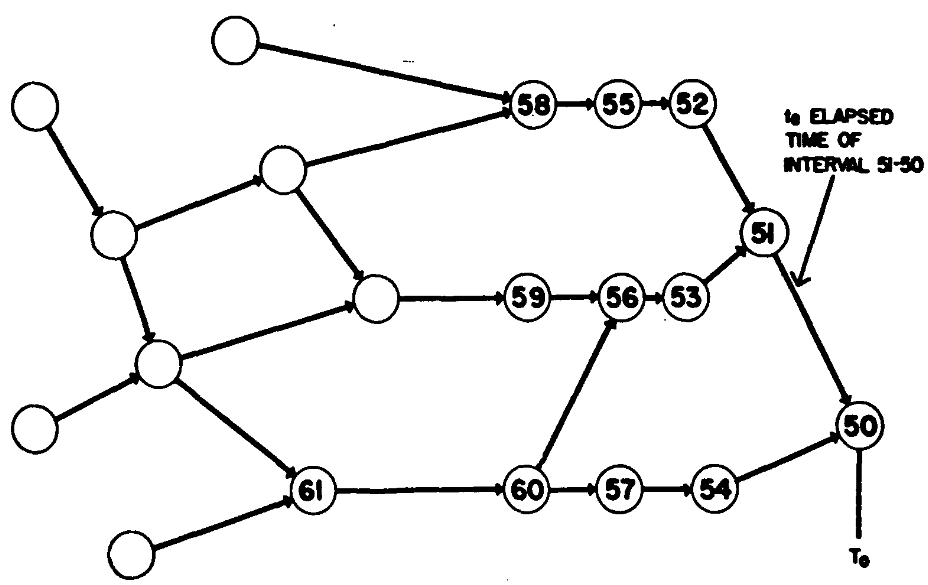


EXHIBIT A

cannot be initiated until the immediately preceding event has been accomplished.

(2) Elapsed Time Estimates

Elapsed-time estimates obtained from competent engineers form the basis of inferring the uncertainties involved in the accomplishment of the events as well as time expectations.

Exhibit A indicates that there is a value t_e associated with the interval between events #50 and #51. Similar values exist for each arrow on the flow chart. These t_e values are obtained from data given by engineers responsible for performing the indicated activity. Raw data are obtained in the form of estimates of time necessary for performing the activities under assumptions of optimistic, pessimistic, and most likely conditions. Statistical techniques are then applied to the raw data in order to put them in a form amenable to analysis.

The following sections will broadly discuss the sequence of steps that will be necessary in translating the engineers' estimates into measures that are descriptive of expected times for an activity and the uncertainty involved in that expectation. The appendixes to this report detail the mathematical justification for the techniques applied.

(3) The Time Distribution Estimate

The engineer's three estimates of necessary time are graphically portrayed on the top line of Exhibit B. An activity is started and its completion is estimated at some future point in time depending upon how fortuitously the program develops. Thus the points a, m, and b will correspond to the optimistic, likely, and pessimistic estimates of the engineer.

The lower portion of Exhibit B demonstrates the belief of the PERT team as to the general characteristics of the probability distribution of the time involved in performing the activity. It is felt that the distribution will have but one peak, and that this peak is the most likely time for completion. Thus, the point m is representative of the most probable time. Similarly, it is assumed that there is relatively little chance that either the optimistic or pessimistic estimates will be realized. Hence, small probabilities are associated with the points a and b. No assumption is made about the position of the point m relative to a and b. It is free to take any position between the two extremes—depending entirely on the estimator's judgment.

ESTIMATING THE TIME DISTRIBUTION

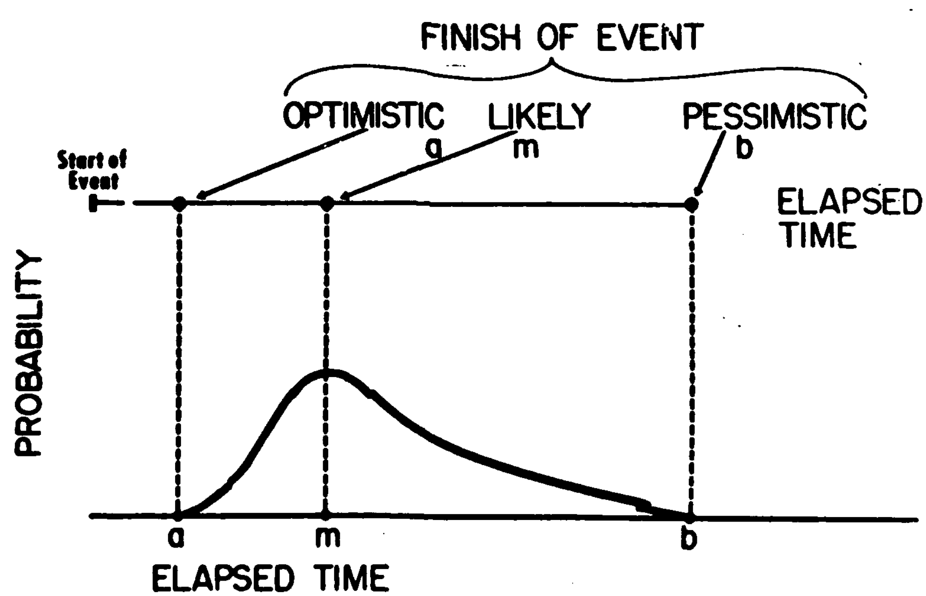


EXHIBIT B

Utilizing mathematical techniques, it is possible to fashion an analytical expression which will fill in the entire curve. This is typically shown in Exhibit B. Appendix B shows the detailed derivation of the functional form of the probability distribution. The flexibility of this expression is demonstrated in the series of figures in Exhibit C. Here it can be seen that the expression can suitably reflect a variety of situations. The most likely time can fall midway between the extremes regardless of whether or not they are close together—curves B and C, Exhibit C. With equal facility, the analytical expression can portray the cases where the most likely time is close to either the optimistic or pessimistic times.

(4) Determination of Expected Value and Variance of the Time Intervals between Events

In order to make statistical inferences about the times at which future events will be accomplished, it is necessary to typify the intervals between adjacent events in terms of their expected values and variances. The expected value is a statistical term that corresponds to "average" or "mean" in common parlance. The variance is a term that is descriptive of the uncertainty associated with the process. If the variance is large, there is great uncertainty timewise in an event's accomplishment. If the variance is small, the estimate is fairly precise as to the time at which the activity will be completed—i.e., the optimistic and pessimistic estimates are close together.

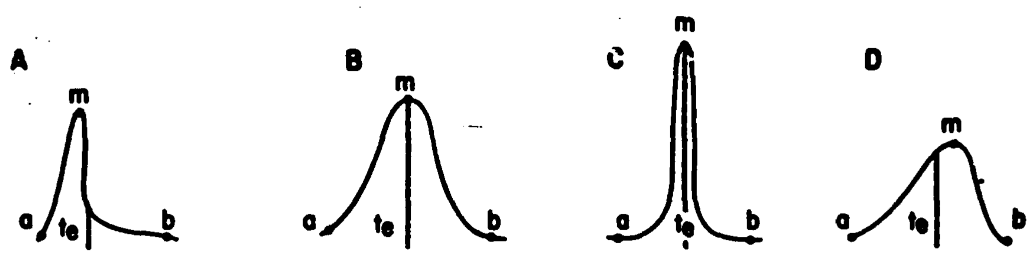
Mathematical investigation of the various distribution types yields the simple estimating equations shown at the bottom of Exhibit C. Solution of these two equations for each interval between events will yield the necessary values of "mean" time and variance.

(5) Data Organization in List of Sequenced Events

Once the raw data estimates have been translated into usable form, it is necessary to structure the information into a pattern which will lend itself to analytical treatment.

The first step in organizing the data is to order the events in a particular sequence. Starting with the last event (the final objective), the events are placed sequentially on a list until all future events are noted and the present is reached. The rationale of the ordering of the events is such that no event

DETERMINING "EXPECTED" VALUE & VARIANCE OF TIME INTERVALS



PROBLEM: GIVEN THREE ESTIMATES OF ELAPSED TIME, FIND t_e, THE EXPECTED VALUE (MEAN) AND $\sigma_{t_e}^2$, (VARIANCE) OF DISTRIBUTION WHEN DISTRIBUTION FORM VARIES AS SHOWN ABOVE

a - OPTIMISTIC ESTIMATE OF INTERVAL
m - MOST LIKELY TIME OF INTERVAL
b - PESSIMISTIC ESTIMATE OF INTERVAL
} OBTAINED FOR EACH INTERVAL

SOLUTION: AN ESTIMATING EQUATION WAS DEVELOPED WHICH GIVES ESTIMATE OF MEAN AND VARIANCE FOR RANGE OF DISTRIBUTIONS TO BE ENCOUNTERED

$$t_e = \left[\frac{a+4m+b}{6}\right]$$

$$\sigma_{t_e}^2 = \left(\frac{b-a}{6}\right)^2$$

APPLY TO EACH INTERVAL

EXHIBIT C

LIST OF SEQUENCED EVENTS

WITH ELAPSED TIME ESTIMATES

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	IMMEDIATE PRECEDING EVENTS			IMMEDIATELY FOLLOWING EVENTS		
	EVENT	ELAPSED TIME ESTIMATES		EVENT	ELAPSED TIME ESTIMATES	
EVENT NO	NO.(S)	MEAN t_e	VARIANCE $\sigma_{t_e}^2$	NO.(S)	MEAN t_e	VARIANCE $\sigma_{t_e}^2$
To 50	51 54	7 10	3 4			
51	52 53	11 15	4 4	50	7	3
54	57	18	6	50	10	4
52	55	12	7	51	11	4
53	56	10	5	51	15	4
55	58	7	4	52	12	7
56	59 60	11 22	5 7	53	10	5
57	60	18	4	54	18	6
60	61	10	5	56 57	22 18	7 4
• • • NOW						

EXHIBIT D

is placed on the list until all of its successors have been listed. The events of the illustrative example (Exhibit A) have been ordered in this sequence in the first column of Exhibit D.

After the events have been listed in the appropriate order, they are associated with the pertinent information that relates to their performance. Columns (2), (3), and (4) of Exhibit D deal with the events that immediately precede those in column (1). For the activities indicated (arrows on the flow plan), the mean and variance of the distribution of times are listed. Columns (5), (6), and (7) show similar information for those events which immediately follow the event indicated in column (1). With the events organized in this fashion, it is possible to proceed to the analysis. At this point, it can be noted that the listing procedure can be carried out efficiently within an electronic computer.

2. ANALYSIS OF THE DATA

The PERT analysis is designed to highlight certain events in the FBM that are of importance either because of their critical position or their danger of slipping schedule.

(1) Computation of "Earliest Times" for Events

The earliest time at which an event can be accomplished is found by examining the form of the planned research and development. Implicit in a discussion of earliest times should be the recognition of a distribution of earliest times. The procedure for identifying the important values of this distribution is described in the following paragraphs.

Considering the events in inverse order from their appearance on the list of sequenced events, the activities currently under way are examined. From all the activities that lead to the first event on the list (the last one on the list of sequenced events), choose the one with the longest expected time. The expected time and its associated variance are listed in columns (2) and (3) of the output sheet—see Exhibit E. Sequentially, each of the events (in their inverse order) are handled in this fashion. As the process moves beyond activities that are currently under way, the procedure inte-

SAMPLE OUTPUT SHEET

(1) EVENT NO	(2) EARLIEST TIMES T_E EXPECTED	(3) EARLIEST TIMES VARIANCE	(4) LATEST TIMES $T_{OE} = T_{OS}$ T_L EXPECTED	(5) LATEST TIMES $T_{OE} = T_{OS}$ VARIANCE	(6) $t' = T_L - T_E$	(7) PROB OF "NO SLACK"	(8) ORIGINAL SCHED	(9) PROB OF MEET'G SCHED	(10) LATEST TIMES $T_{OS} = 90$	(11) NEW SCHED	(12) PROB OF "NO SLACK"
50	92	38	92	0	0	.50	82	.05	90	90	.63
51	85	35	85	3	0	.50	77	.09	83	82	.70
54	74	29	82	4	8	.08	73	.42	80	77	.05
52	47	25	74	7	27	.00+	70	1.00-	72	62	.00+
53	70	31	70	7	0	.50	60	.04	68	67	.69
55	35	18	62	14	27	.00+	55	1.00-	60	55	.09
56	60	26	60	12	0	.50	50	.02	58	58	.65
57	56	23	64	10	8	.08	55	.42	62	61	.35
•	•	•	•	•	•	•	•	•	•	•	•
X-NOW	0	0	•	•	•	•	•	•	•	•	•

(TIME IS SHOWN IN WEEKS FROM X OR TIME "NOW")

EXHIBIT E

grates means and variances previously considered. As an illustration of this process of integration, the computation of event #51 will be described in detail.

Exhibit E shows that event #51 is immediately preceded by events #52 and #53.

This exhibit also shows the activity between events #51 and #52 has a mean estimated time of 11 weeks—while that between #51 and #53 has 15 weeks.

Adding the 11 weeks to the 47 weeks which is the earliest expected time for event #52 (see Exhibit F, column (2)), yields an expected earliest time for event #51 of 58 weeks—insofar as event #51 depends on the activity that was initiated with event #52. A similar calculation is made for the earliest expected time for event #51 as constrained by the activity starting with event #53. This calculation yields a mean (expectation) of 85 weeks.

Noting that the greater time span is required (on the average) by the activity starting with event #53, the choice of 85 weeks is made as the earliest time for event #51. The associated variance for event #51 is found by adding the variance for #53 to that for the activity between #51 and #53—or, 31 + 4 = 35.

(2) Computation of the "Latest Times" for Events

The latest time at which an event can be accomplished is found by fixing the objective event at some future date and working backwards through the earlier events.

The latest time for an event, like the earliest, exists in the form of a distribution which is described in terms of its expectation (mean) and variance. Latest times are predicated upon a designation of some future date as being desirable (or satisfactory) for accomplishing the final objective. The latest time for an interim event is located at a point such that, if the following events are accomplished according to anticipations, the objective will then be completed precisely on the desired date.

The procedure for arriving at the latest dates for events is performed in the same general fashion as that for the earliest times. However, the events are taken sequentially in the same order as they appeared on the original listing—Exhibit D. The objective event is assigned a mean that corresponds to its desirable date with zero variance. Then, utilizing the information in the last two columns of Exhibit D, earlier events are specified by subtracting their activity times from the expected times of the succeeding events. For the illustrative example the results of the calculations are shown in columns (4) and (5) of Exhibit E.

(3) Computation of "Slack" in the System

Examination of columns (2) and (4) in Exhibit E shows that in some cases there are differences between the earliest and the latest times at which an event will occur (i.e., their mean times). Slack can be taken as a measure of the scheduling flexibility that is present in a flow plan. Thus, granting that the forward anchor point in the latest times computation is a desirable one, then the slack period for an event represents the time interval in which it might reasonably be scheduled.

Slack exists in a system as a consequence of multiple path junctures that arise when two or more activities contribute to a third. This condition is illustrated in the simplified flow plan shown in Exhibit F. This exhibit shows the earliest times (T_E) for a small complex of events together with the time intervals between them (t_e). It can be seen that of the three paths that lead from event #8 to #6, the longest expected time of event #6 is at the 65th week.

If the 65th week is satisfactory for accomplishing the performance of event #6, the system can be anchored at this point and latest times computed in backwards computation, discussed above. Exhibit G shows the time relationship of the earliest and latest times for these events—the dashed circles represent the latest times (T_L). This comparison illustrates that events #44 and #33 have slack, i.e., events #44 and #33 could be scheduled anywhere within their slack range and still not disturb the expectation of timely accomplishment of the final event at week 65.

The slack for each event in the illustrative example appears in the sixth column of Exhibit E. It can be noted that for some of the events a zero slack condition exists. This indicates that the earliest and latest times for these events are identical. If the zero slack events are joined together, they will form a path that will extend from

DETERMINATION OF SLACK

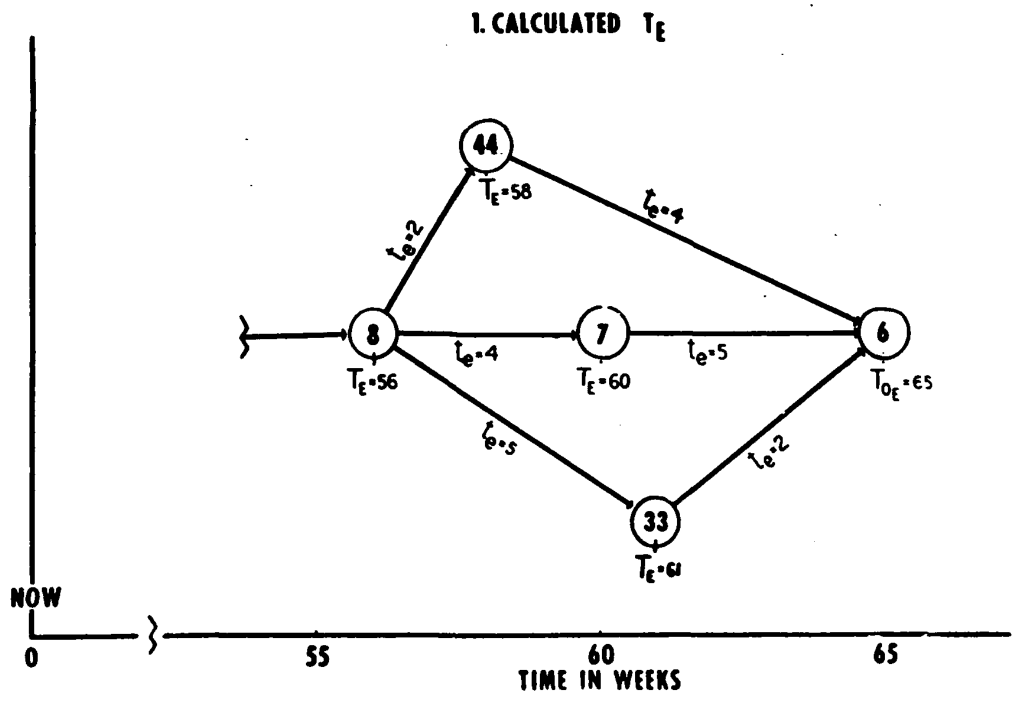


EXHIBIT F

DETERMINATION OF SLACK

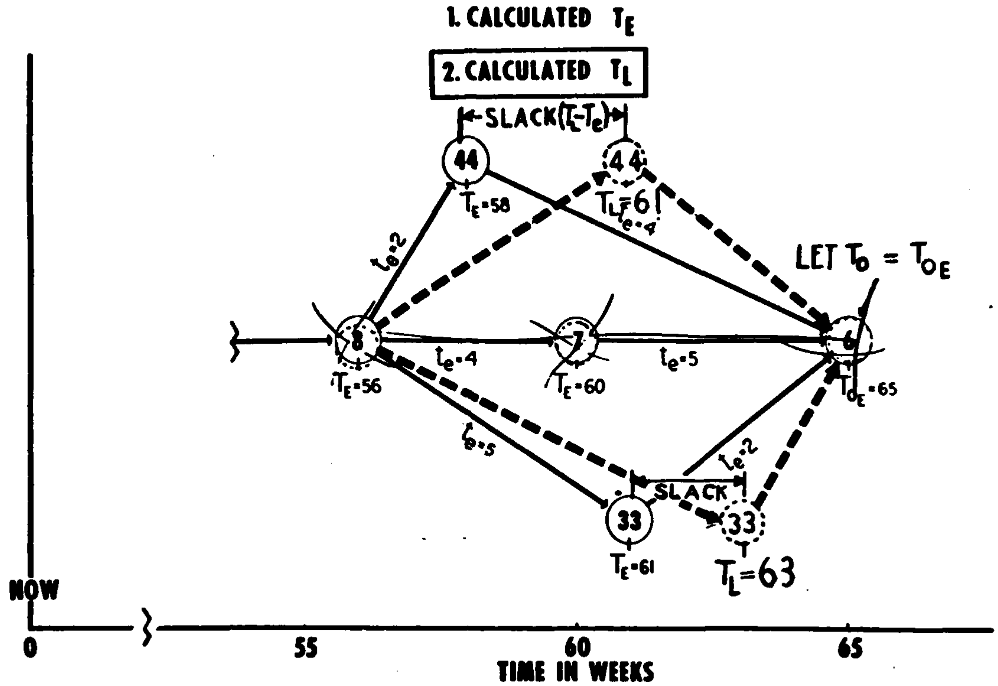


EXHIBIT G

the present to the final event. This path can be looked upon as "the critical path." Should any event on the critical path slip beyond its expected date of accomplishment, then the final event can be expected to slip a similar amount.

(4) The Chance Aspects of Slack

The slack computation discussed in the previous section describes the expected slack for future events. Actual slack will generally be different from its expectation—it will assume some actual value depending on the exigencies of the R&D process and their probabilities. Theoretical analysis allows inferences to be made about the actual slack that will develop.

Exhibit H graphically shows the expected slack in the top figures. However, chance factors are at work in the situation so that the observed slack may turn out to be smaller than anticipated. This is shown in the middle row of the exhibit. (In the same sense, actual slack may turn out to be larger than expected.) Chance may work perversely and make the latest time for an event fall earlier than the earliest time. Such a situation is portrayed in the bottom row of Exhibit H.

In spite of the wide range of possible values that slack may actually take, many of the values are highly improbable. When a "no slack" condition exists, there is a need for examination and control of the situation. Therefore, statements can be made as to the probability of the "no slack" condition. The seventh column of Exhibit E shows such probability statements for the illustrative example. Where these probabilities are in the neighborhood of .5, it is important to closely monitor the events in question as slippages in these events can jeopardize the timely accomplishment of the objective.

(5) The Effect of an Existent Schedule

The analysis of the slack that has been discussed above did not explicitly take into account any scheduled dates save that of the objective. The actual situation will show scheduled dates for many of the interim events. If activity has been programmed in accordance with such a schedule, the slack analysis should take a different form. Before

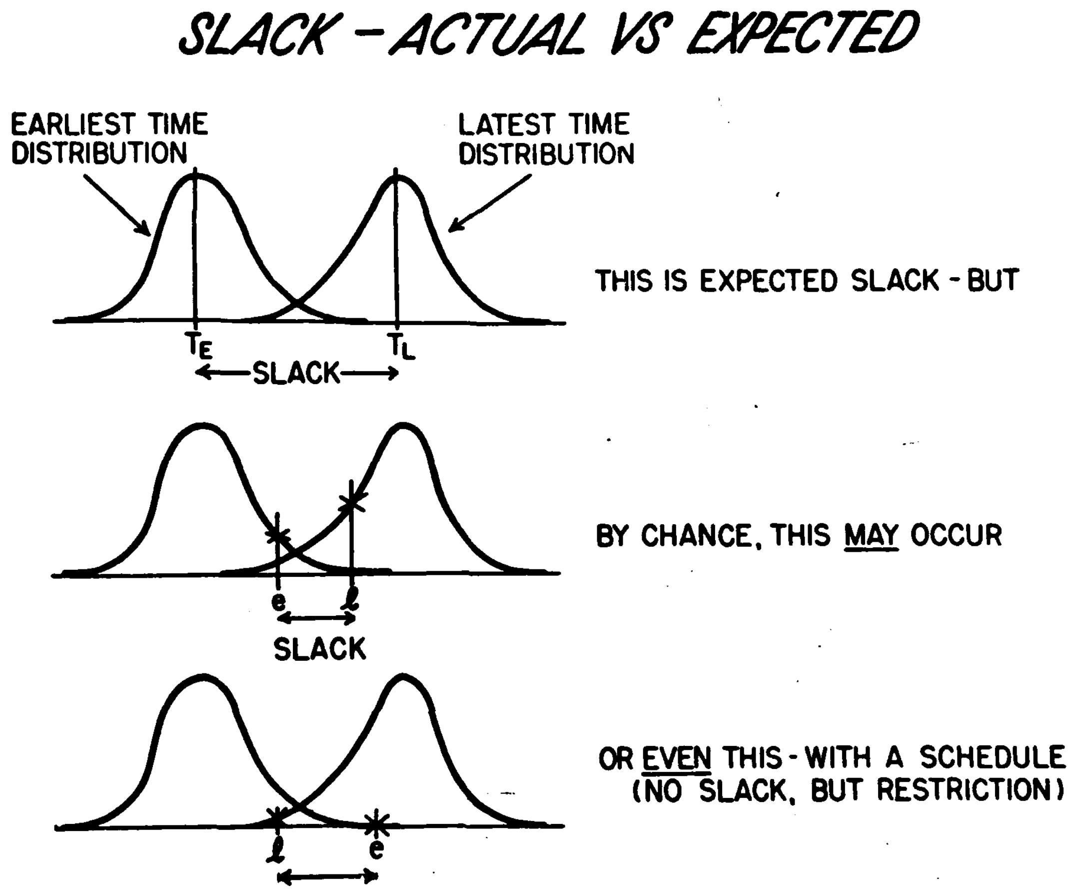


EXHIBIT H

such an analysis is undertaken, an appraisal of the feasibility of the existent schedule should be made.

Decisions as to the feasibility of a scheduled date for accomplishing an event rest ultimately on a subjective basis. As a guide for developing criteria for making such decisions, it is possible to estimate the probability of actually meeting scheduled dates. The uncertainties of the future prevent a forecast of the precise time at which an event will be accomplished. However, there is knowledge of expected times, and a procedure is also available for giving the probability of any given deviation from that expectation.

Exhibit I graphically illustrates the uncertainties involved in predicting the precise time at which an event will occur. The exhibit shows the last few events in the illustrative example. Event #50 might have been scheduled at time T_{O_S}. However, an earliest time analysis could indicate that the event is expected to occur at time T_{o_E} with variance, $\sigma T_E{}^2$. Statistical theory shows that the probability distribution of times for accomplishing an event can be closely approximated with the normal probability density. It is, therefore, possible to calculate the probability that the event will have occurred by any future date. The probability that event #50 will have occurred by time T_{o_s} is represented in Exhibit I by the shaded area under the curve.

Similar calculations can be made for each scheduled date. A hypothetical list of dates is shown in column (8) of Exhibit F. This series of dates might represent an existent scheduled arrived at by any means. Column (9) of this exhibit shows the probabilities of meeting each scheduled date if all activities are carried out as soon as they can be. Where the probabilities assume low values, it is reasonable to assume that the schedule is infeasible. High values indicate the opposite—that the

ESTIMATE OF PROBABILITY OF MEETING SCHEDULED DATE-T_{O_S}

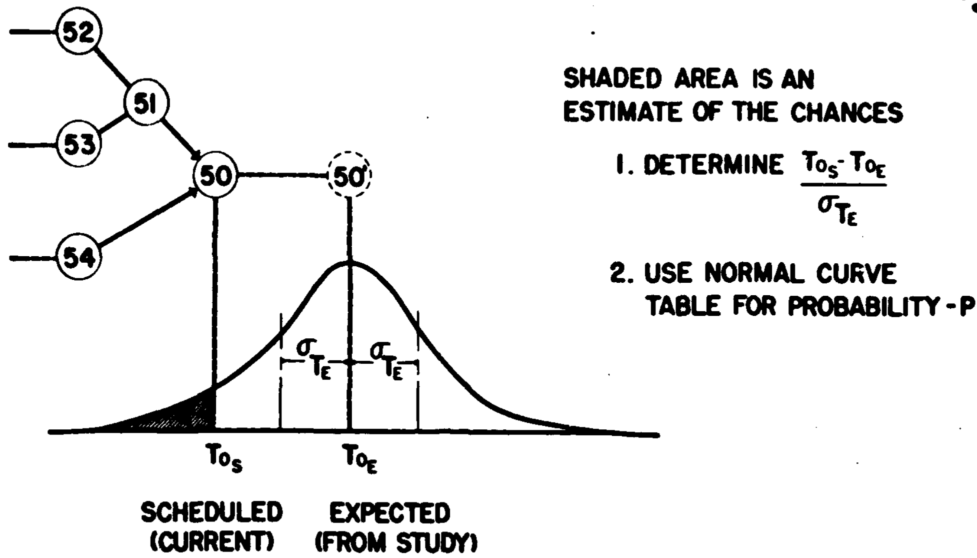


EXHIBIT I

schedule will be met with a high probability. Technical managers can reappraise a given schedule in the light of the probabilities here cited. If it is decided that a scheduled date is infeasible, then resources and/or performance must be altered or a rescheduling must take place. If the first option is chosen here, then competent technical advices will provide a new plan and appropriate estimates will be acquired from the estimating sources. If the decision is to reschedule, then a rational means of changing the schedule should be found.

(6) A Method of Rescheduling

The procedure for obtaining an optimum schedule is both complicated and time consuming. An optimum would have to define costs of slipping a scheduled date and costs of scheduling before an activity can be started, for all events individually and collectively. The problem appears to be insurmountable within the available time frame. An optional procedure has been worked out, however, which provides a reasonable if sub-optimal approach to the question of rescheduling. The procedure can be detailed in terms of the following steps:

- If it is possible to extend the scheduled date of the objective event, this should be done. Exhibit J(1) gives an indication of the range of dates that are suggested as appropriate if they are satisfactory to the FBM management. Appendix D specifies the procedure for identifying the location of the points c and d in Exhibit J(1).

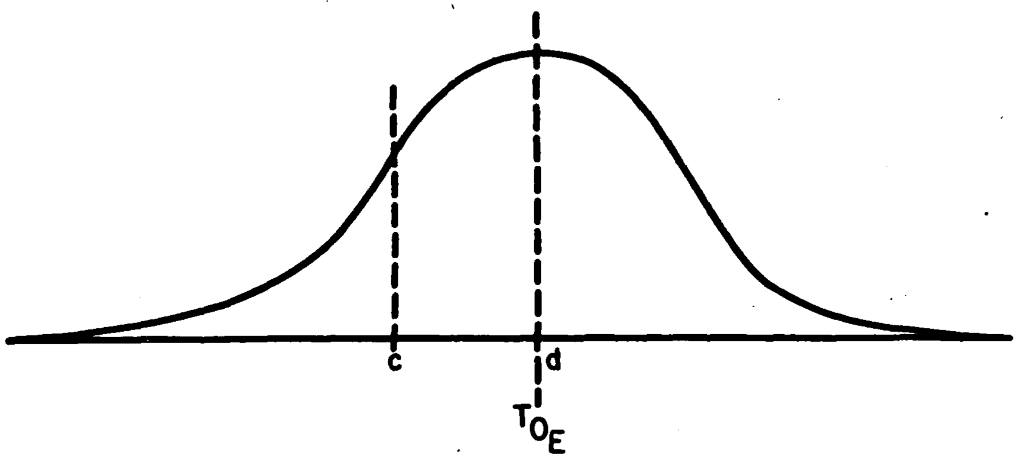
If the scheduled date of the objective event has been changed, it is necessary to recompute the latest times for all events. (Latest dates are based upon the precise location of the objective event.) Column (10) on Exhibit E shows what a new set of latest dates would be if the objective event was slipped from week 82 to week 90.

The earliest and latest times for an event are then compared. Each event is categorized according to whether or not its latest time precedes its earliest time. When the latest time falls before the earliest, set the scheduled date at the latest time. When the latest time does not fall before the earliest, then set the scheduled date for the event at the point that maximizes the probability that it will actually be met and fall within the slack interval. (See Exhibit J(2)). The techniques for making this calculation are shown in Appendix D. The dates arrived at through the application of the procedure to the illustrative example are shown in column (11) of Exhibit E.

After obtaining the new schedule, it is appropriate to appraise again future possibilities by recomputing the probabilities of "no slack" in the system. If the probability of "no slack" is much in excess of .5, then serious consideration should be brought to bear on the advisability of redeployment of resources and/or changes in performance. If the

A RESCHEDULING PROCEDURE

1. SLIP LATEST SCHEDULED DATE



TO A POINT BETWEEN c & d

AT c ; Pr [ACCOMPLISHMENT] = 25 (1 CHANCE IN 4)
AT d ; Pr [ACCOMPLISHMENT] = .50 (1 CHANCE IN 2)

EXHIBIT J(1)

probability is approximately .5, then management should closely monitor progress on the events. If the probability of "no slack" is much less than .5, then progress on the activity leading to the event should receive but routine checking. The probabilities for the illustrative example are shown in the last column of Exhibit E.

2. RECOMPUTE ALL LATEST TIMES

3. FOR SCHEDULE OF INTERIM EVENTS -

WHERE $T_L < T_E$ SET $T_S = T_L$

$T_L \geq T_E$ SET T_S TO MAXIMIZE

$Pr[e \leq T_S \leq \ell]$

EXHIBIT J(2)

III. FIELD ACTIVITY IN PHASE I

In the course of developing the methodology, the PERT team visited at LMSD, Sunnyvale and Aerojet General, Sacramento. During the course of these visits, flow charts of the missile subsystem and the propulsion component were made. The events that appeared on these constructions were extracted from several sources. The SP Program Management Plans, and Lockheed Master Development Plan, and the Lockheed Master Test Plan all contributed to the itemization of the events. The system flow diagrams were constructed with some haste, but attempts were made to delete non-definitive and redundant milestones from the list.

Time estimates were gathered from engineers, planners, and the various evaluation staffs. In some cases, estimates were tendered conditionally with the understanding that they had not been subject to sufficient consideration nor were they authenticated by the contractor managements.

1. THE MISSILE SUBSYSTEM

Exhibit K portrays the nature of the system flow chart for the missile. Time data were gathered in units of a month. Earliest and latest times were computed, as was the slack. The various components of the subsystem were coded and the critical path indicated. The objective of this exercise was to ascertain the availability of the basic data and to develop insights into the magnitudes of the numbers involved.

Exhibit K appears in censored form in order to prevent disclosure of classified details. However, the complex form of the interactions of events can

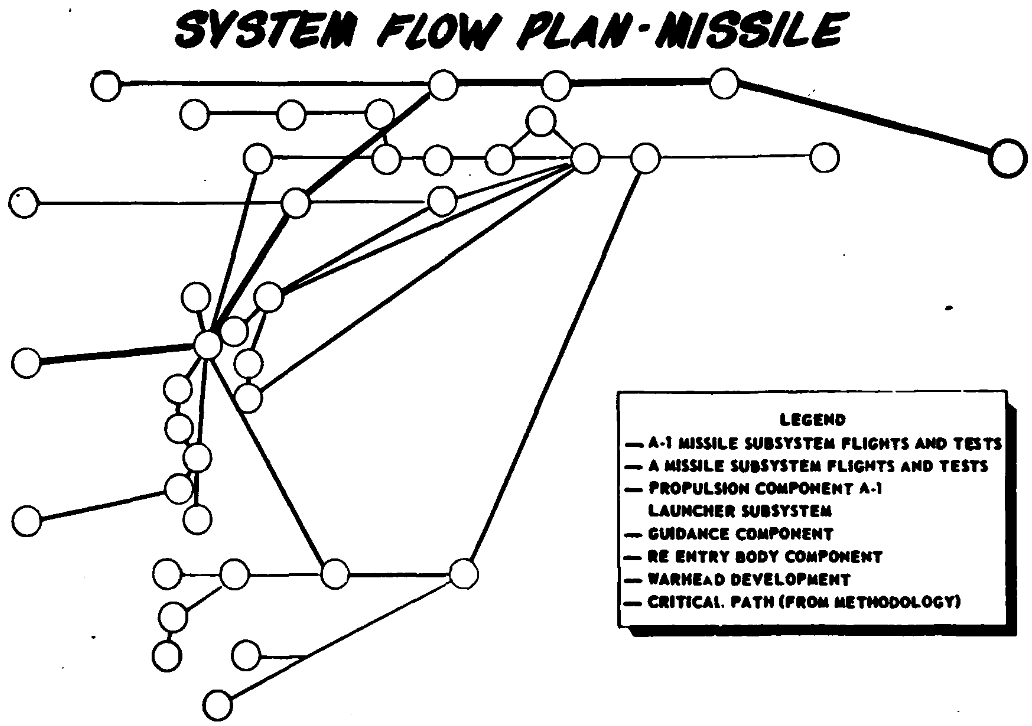


EXHIBIT K

be observed. In addition, it has been possible to color-code activities so as to highlight the sequence of activities that develop in any of the missile components. These are indicated in the legend on Exhibit K.

2. THE PROPULSION COMPONENT

Whereas the events in the missile subsystem were gathered at a broad level with little detail, the propulsion component was typified with 160 lower level events. The date of the exercise was 12 March; hence, this point marks the origin of the time scale. The form of the propulsion flow chart with the critical path (zero slack) indicated is given in a separate appendix (classified). The detailed description of the events and the computed times and standard deviations appear in the appendix. It may be noted that according to the preliminary time estimates as given in this appendix, the date of event #37 falls in the latter part of January, 1959. This outlook has undoubtedly changed since the date of the study, as remedial action has been undertaken by both the contractor and the SP staff.

In obtaining the system flow for the propulsion component, recourse was made to what may be called a principle of analysis and synthesis. The process was analyzed into many events for purposes of providing detailed control in case such would be of interest at some point in time. This detailed itemization of events should not be construed as implying that all events should be so laid out for top management's attention. All of the events may hold interest for some individuals while others may be concerned with only the broader elements. All points of view can be accommodated by synthesizing the detailed findings and reporting only those facts that will necessarily call for decision. Thus, many different outputs can be produced according to the various requirements and interests of the particular consumer.

An additional benefit arises from breaking the initial analysis into many detailed events. By utilizing this procedure, it is possible to diminish the impact of existent schedules on the estimates. When estimates are elicited on the basis of detailed activities, the individual estimators are most apt to respond independent of schedules—as schedules are generally published in fairly gross detail. In taking advantage of this independence, PERT hopes to obtain the most valid estimates possible.

At the present time, the PERT team is actively engaged in obtaining authentic data for decision purposes. In order to obtain both depth and breadth of outlook, analyses are being made of the missile subsystem and three of its components. Thus, it will be possible to gain important insights into the interactions among the missile components as well as the impact of the component's progress on the missile subsystem itself. Coincidental with the field activity, work is in progress on the programming of the NORC computer to handle the calculations necessary for analysis and evaluation. A continuing flow of information is to be provided by contractor personnel. Eventually, it is planned that the entire FBM System can be integrated into a single network so that total system evaluations can successfully and automatically be made—rather than appraisals of isolated parts.

APPENDIX A

THE STOCHASTIC MODEL AND ITS SPECIALIZATION TO THE FBM PROGRAM

1. THE STOCHASTIC MODEL

Appendix A describes the abstract mathematical model that is employed.

Let there be given some *events* A, B, ..., and *activities* a, b, ...

In this appendix "events" and "activities" will designate certain events and activities, but in Part 1 the discussion is purely formal. The events and activities are finite in number. Each activity has a unique event as its *start* and a unique second event as its *immediate successor*. If P and Q are the start and the immediate successor of an activity, respectively, Q is said to be an *immediate successor* of P, and P is an *immediate predecessor* of Q. If A, B, ..., C is a sequence of events such that each, except A, is an immediate successor of its predecessor in the sequence, C is said to be a *successor* of A, and A is a *predecessor* of C.

No event is a successor of itself.

There is a special event P, called the present event, which has no predecessor, and which is the predecessor of every other event.

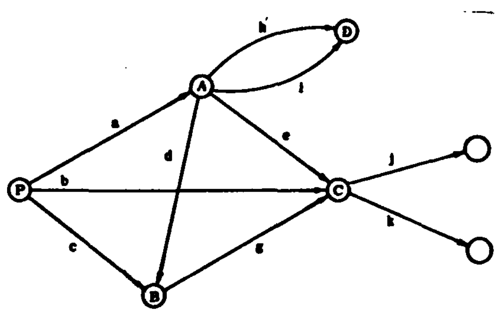


Figure 1

These events, activities, and relations constitute the *geometric model*, or *topological model*. The geometric model can be represented geometrically as suggested in Figure 1. The circles represent events, and each arrow represents an activity from its start to its immediate successor.

In this appendix the events will be interpreted as achievements or milestones in a process that unfolds in time, and activities will represent actions that must be performed. This motivates the extension of the geometric model to the stochastic model.

Each activity has a *time*. The time is stochastic and normally distributed; however, the special case of a certain time must be included, and in this memorandum the term normal distribution will include this special case of zero variance.

One can think of an activity as a process or procedure that requires time for its performance. The time of an activity is the time between the start and finish of the action. We shall introduce the time of an event which can be thought of as the instant at which the event occurs.

The activity times determine a stochastic time of each event in accordance with the following definition. The present event P has the certain time zero. Assume (as an induction hypothesis) that the time is defined for every predecessor of an event X. Let the activities with X as immediate successor be a, ..., b. Consider t(a) the random variable which is the sum of the following two random variables: the time of a and the time of the start of a. Similarly, consider t(b) and the corresponding random variable for all immediate predecessors of X. The "time" of X is the (random) greatest of the times t(a), ..., t(b). This definition is valid (the induction can be carried out) because the present event P precedes every event, no event precedes itself, and the events can be arranged in a linear sequence such that if A precedes B in the geometric model, A precedes B in the linear sequence. The proof is not given in this part; all proofs are deferred to Appendix C.

The *stochastic model* consists of the geometric model together with the activity times and event times.

This completes the description of the model as now employed. The following paragraphs discuss some complexities which may need to be introduced into the model. These complexities have not yet been required.

The model as described above corresponds to a process in which every activity must be carried out. One might encounter a situation in which two or more alternative activities are carried on in parallel with the understanding that as soon as one activity is completed, the parallel activities will be abandoned. Such a situation would involve the small adjustment of computing the shortest of two or more times rather than the longest.

It is conceivable that the time of an activity is conditioned by the outcome of preceding activities. For example, in Figure 1 the activity g may be required only if activity c "fails." In this case the distribution of g would be constructed from the probability of failure of c and the two conditional distributions of g that correspond to failure and success, respectively, of c.

2. SPECIALIZATION OF THE MODEL TO THE FBM PROGRAM

In this part a realization of the stochastic model is extracted from the planned development of the FBM and the associated systems.

Many things must be done to achieve the purposes with which SP is concerned. These things include decision making, research, development, design, fabrication, production, testing, etc. The activities leading to the desired results must be planned. One reason for this is that the activities must be sequenced in some feasible manner. The sequences are optional to some extent. One might develop a missile before thinking about a ship and then adapt the ship to the missile characteristics. Alternatively, one might conceivably set the ship characteristics and accept resulting constraints on the missile. Actually, under pressure of time it is necessary to have numerous parallel developments with coordination.

The present analysis assumes that the objectives of SP have been planned. Necessarily, the plan must designate what we call events. Indeed, if an activity is planned to commence after certain other accomplishments have been consummated, it must be necessary to recognize when the prerequisite accomplishments have been realized. This requires some check point, or event in our terminology, which will trigger off the activity in question. Furthermore, usually action is desired just as soon as the prerequisites are achieved; hence, an event occurs instantaneously—at the first instant when certain conditions prevail.

To construct a reasonable plan one would take into account the times required to carry out the various activities. However, a feasible plan could be made without such consideration. A feasible plan is one that could be carried out, and hence would be void of inconsistencies such as a requirement that both activity a be completed before b is started and activity b be completed before a is started. If one is merely sure that all prerequisites of activities are planned for completion before the activities start, the plan is feasible.

However, in spite of the logical redundancy of activity times in the construction of a feasible plan, estimates of these times must be considered in making an intelligent plan. In the estimation of the time required to carry out an activity, it is often useful to break up the activity into portions or subactivities, to estimate for the subactivities, and to total the times for the parts. If an activity is to be divided into parts, it is necessary to designate accurately the points of division. This means that, when the activity is carried out, one must be able to recognize the instant at which one subactivity terminates and a succeeding one starts. The state of the process at this critical instant is an event.

Another utility of events is found in the monitoring of progress towards objectives. Events can serve as milestones, and one can analyze progress by checking off the events at the instants they occur.

The entire development of the FBM and its associated subsystems is an activity. But this over-all activity naturally breaks up into large subactivities concerned with the missile, guidance, ship, etc. Furthermore, these subactivities subdivide further until eventually one might have thousands of subactivities. These subactivities are the *activities* of the analysis.

In order to structure the thousands of activities into an orderly system, events are required. An *event* is a state or condition in future development which will be clearly recognized at the instant the state or condition occurs.

Let us particularize these concepts by a few very simple, hypothetical examples. Suppose that a second test program is to start as soon as a first test program is successful. We can consider the first program as an activity a and the second program as an activity b. A third activity c consists of the preparation of the facilities of the second test program. If A designates the instant in time at which the second test program can start, the relations among A and the activities can be represented as in Figure 2. In the situation of Figure 2, the completions of the activities a and b are not events. The event A represents the instant at which both a and b are completed. Hence, if X and Y are the instants at which a and c are completed, respectively, then A is the later of X and Y. The events X and Y could be introduced as in Figure 3. In this

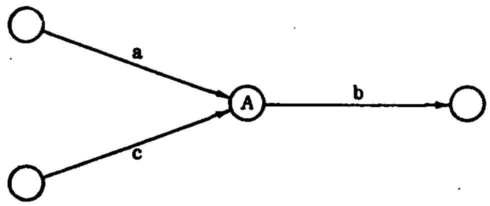


Figure 2

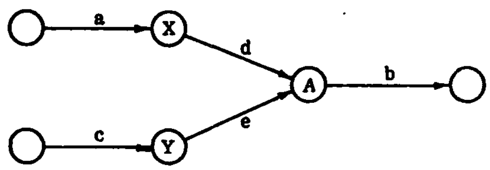


Figure 3

figure the activities d and e have times with zero expected value as well as zero variance. Thus, there is nothing to be done in the activities d and e; these activities are completed instantaneously. The event A is not the completion time of each activity separately. The time of A is the later of the times of X and Y.

To avoid possible misunderstanding, we emphasize that in an actual application the analysis will be much more detailed than suggested by Figure 3. The activities a, b, and c would be broken down into many subactivities. Furthermore, there would most likely be much detail between event A and the start of activity b; this detail would cover administrative action, planning and scheduling which might be required prior to the start of activity b. Of course, it would be a matter of definition which determined whether these activities were prior to or part of activity b.

In the stochastic process an event must never succeed itself. Figures 4 and 5 are not allowed. If an event succeeded itself, and if the intervening

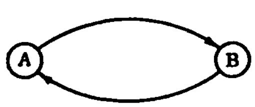


Figure 4

activity time were not zero, the realization of the process in time would be impossible (because a predecessor must precede its successor; this is true even when the time of the intervening activity is zero).

Consider Figure 5. Suppose that this figure represents a situation in which each of three activi-

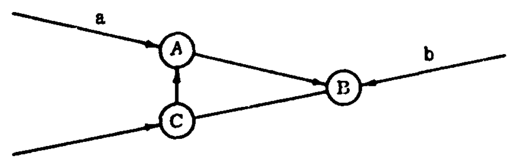


Figure 5

ties a, b, and c cannot be completed until final adjustments are made in the light of the results of the other two activities. If continual interactions among the three activities must take place, it may be necessary to reduce Figure 5 to Figure 6, in which r is the simultaneous unfolding of a, b, and c, and R is the completion of a, b, and c. In some

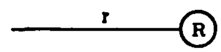


Figure 6

cases more detailed planning might lead to Figure 7, the activities d, e, and f are parts of a, b, and c that can be realized prior to the start, X, of a coordinating activity g; the coordination is completed in the event Y, and the three activities proceed to their conclusions.

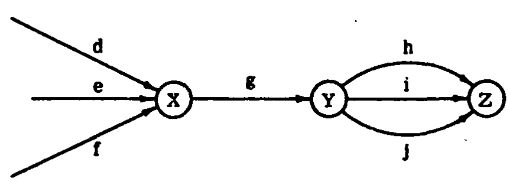


Figure 7

Suppose that the stochastic process were that indicated in Figure 1. The event P is the time at which a computation is made. The event A will occur at the time a subsequent to the present (here as elsewhere we use the symbol for an activity to denote the time of the activity.) This time a will be the time required to carry out the corresponding activity (since this activity starts at the present, most likely the activity is the termination of some action which started in the past.) As soon as the activity is completed (at time a subsequent to the present), the event A occurs. The occurrence of an event is instantaneous. At the occurrence of A the activities d, e, h, and i are started. Similarly the activity g starts at the time of the occurrence of B. The event C will occur as soon as e, b, and g are completed. Hence, the time of C subsequent to the present will be the greater of the three times a plus e, b, and c plus g.

One can think of an event simply as the time of the completion of one set of activities and the time of the commencement of another set. Thus, event C is the time of the completion of activity e, b, or g, whichever is later, and C is also the time at which activities j and k both commence.

An activity can involve simply waiting. Suppose that some project is to commence at time 5 from the present, and that it is not necessary for any action to take place prior to the start of the project (except the decision, already made, to start the project at the specified time of 5 from the present). If the event X is the start of the project, there would be an activity between P and X designated by an arrow x, and the time x would have the expected value 5 and variance zero.

The activity of waiting explains another aspect of the process which some people find paradoxical. Suppose we have the situation of Figure 8. In the

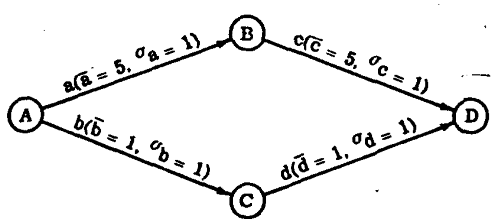


Figure 8

figure we have indicated along with each activity time the expected value and standard deviation of the time. Someone will object that event C will not occur as soon as activity b is completed for the following reason. Event D cannot occur until roughly 10 units of time subsequent to A (because $\bar{a} + \bar{c} = 10$). If d starts as soon as b is completed, then d will be completed roughly 8 time units before c is completed. In many such cases there will be administrative action which will delay the start of d. But if there is any such administrative contraints on the stochastic process, they must appear in the analysis. Suppose that in the present case it has been decided that d will not start until two time units subsequent to B (perhaps because

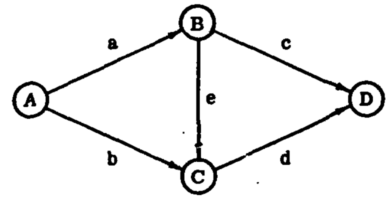


Figure 9

some "schedule" has been set). Then Figure 8 must be augmented as in Figure 9. The activity e has the expected time 2 with variance zero.

Furthermore, it is necessary that the activity e (of waiting) appear in the analysis. Without the constraint of e, it is suggested that we could determine, at least roughly, some "latest time" at which C could occur without producing delay beyond that already inherent in the times of activities a and c. Then it is proposed that one could "schedule" C between the earliest and latest times. Suppose that we try to do this. In view of the expected value and standard deviations of the times c and d (as indicated in Figure 8) it would seem fairly safe to schedule d to start 2 units of time subsequent to the occurrence of B. But there would be a very small probability that d will not be completed until after c has been completed. Hence, the expected time of event D will be very slightly later than it would be if d were started immediately at the completion of b. In many cases it would be obvious that the delay would be insignificant. However, in some cases one would wish to delay the start of d as long as possible, and the question would arise as to just what risk of delaying D would result from various delays in the start of d. Such a question could be answered if the activity e is introduced into the analysis as indicated in Figure 9.

To repeat, if the analysis produced only earliest and latest times, and if scheduled times for events were set in the light of earliest and latest times, the feasibility and implications of the schedule would be determined by the analysis suggested in the case of Figure 9. This is due to the fact that any constraint imposed by a "schedule" would alter "earliest" and "latest" times.

The stochastic process should include all constraints including all restrictions implied by "scheduling" or any other administrative action. Then the times of the events are the times at which they will occur, not earliest times at which they could occur.

APPENDIX B

THE ANALYSIS OF ACTIVITY TIME ESTIMATES AND THE MATHEMATICAL COMPUTATIONS

The model of the FBM program consists of designated activities, events, relations of precedence and succession, and distributions of activity times. The analysis of the model involves two major items. Estimates of activity times must be obtained. Then event times must be calculated. These two parts of the analysis are discussed in this appendix.

1. THE ANALYSIS OF ACTIVITY TIME ESTIMATES

We have defined activities and we have stated that it is necessary to estimate the expected value and variance of each activity in the FBM program. This part discusses the sources of these estimates and the procedures for obtaining them.

Activities are of diverse natures. They include production, fabrication, testing, development, analysis, administration, research, and decision making. Specific examples are the following: the fabrication which takes place between the delivery of a chamber to Aerojet and the availability of the specific test vehicle for firing; the test program which takes place between the first development test and the qualifications of the *A* motor for firing. This last activity can be broken down into subactivities which would most likely include development, analysis, administration, and decision making.

The diversity of activities makes a general discussion difficult. But we are at present facing the generalities.

First, let us consider appropriate sources of information concerning the possible time that an activity might take. In this matter it is essential not to accept uncritically any times that appear in plans and contracts. Rather, one must probe to the bases upon which the plans and contract schedules are made. One reason is that schedules (other than the ones we shall set) are not adequately responsive to changing conditions and prospects. Secondly, present schedules do not estimate uncertainties, and thirdly, they are made under pressures which involve matters other than the accurate estimate of times; one such pressure is haste.

Hence, preferably we should get information from people who are to perform the activities or to directly supervise the performance. These people are technicians in spirit, even if they have administrative responsibility.

Since it will be difficult to get more than a single authoritative estimate, the problem of personal bias is serious. These biases will depend partly on the weight of company policies, schedules, and commitments. In some cases it will be possible to get supplementary estimates from technicians not employed by the responsible contractor. For example, the estimates of an Aerojet employee can also be made by some Lockheed technician who is intimately concerned with the activity and its results; also, an estimate can be obtained within SP.

This leads to the question of the combination of two or more estimates. This is discussed below.

We turn then from the question of sources of time estimates to the parameters estimated. The estimates by a technician (or other person, if necessary) should be made after careful explanations by a highly qualified interviewer. After this first interview the estimates may be made and transmitted to SP on forms.

We propose that the technical person be asked to estimate the following for an activity. First, a "likely" time which technically we shall interpret as the mode of the activity time. Synonyms which can be offered are "most probable time," and "time you would expect." Even "expected time" makes an impression on persons who are not statisticians, but this term must be avoided when there is any possibility of confusion with the statistical meaning of this term.

Secondly, the technician is asked for an "optimistic" time. There should be practically no hope of completing the activity in less than the optimistic time, but good luck might give a time close to the optimistic.

Thirdly, we ask for a "pessimistic" time. This concept is not as sharp and clear as we would like. Although experience to date has not produced any serious difficulty with this concept, we must handle the estimation with great care. This difficulty is the obvious one that in research or development it is difficult to set a date within which an achievement can be guaranteed.

As a first description of the pessimistic time, we ask for a time which will not be exceeded, barring "acts of God," but which might conceivably be approached.

An "act of God" is an unexpected, unforeseen event whose occurrence would not be anticipated in any reasonable planning. Examples are wrecks and unusual strikes. If all experts anticipate the success of a research project, and if the project fails, the failure is included as an "act of God." If, however, expert opinion is divided, failure is not an "act of God," and in setting a pessimistic estimate the estimator should allow for time to complete the activity in case of failure and a fresh start. (An alternative deserving serious consideration is a time estimate assuming success and an estimated probability of success.)

The pessimistic estimate does not involve simply a listing of foreseeable difficulties and an estimate of the total time to overcome each. A technical man might say that of ten possible but unlikely difficulties, it would be unreasonable to anticipate all ten; he might say that he is reasonably certain that it would be foolish to expect more than three of the ten to materialize. Or the man may know from experience that delays are usually caused by unforeseeable developments; this man might base his estimates more on intuition than analysis.

Perhaps we should say that a pessimistic estimate is a time which is longer than expected, but which might be required in one out of a hundred similar activities. Furthermore, although one cannot set an absolute time limit which will never be exceeded, no normal person would hedge against a time greater than the pessimistic time.

In case of a refusal by a technician to state a pessimistic estimate, one would go to the person or body that decided to include the activity. This decision maker must justify some upper limit to the accomplishment (the upper limit could include the time to accomplish some substitute achievement, and it might involve a prospect of foregoing the activity and of accepting some performance degradation).

In Appendix A it was assumed that the activity times are normally distributed. It will be clear that in many cases the times are not normal. However, when event times are calculated from activity times, we compute sums of activity times. Hence, the event times will be roughly normal even if the activity times are not normal (because of the central limit theorem). This will not be the case for immediate successors of the present event. These special cases can be handled directly, and we shall not discuss them here. For the general case we shall use only the expected values and variances of the activity times; from these we shall calculate expected values and variances of event times. Hence, our problem is to estimate the expected value and variance of an activity time from the likely, optimistic, and pessimistic times discussed above. Furthermore, we feel free to use a non-normal model of the distribution of activity times as a tool in this estimation.

We recall that for unimodal frequency distributions, the standard deviation can be estimated roughly as one-sixth of the range. Hence, it seems reasonable to estimate the standard deviation of an activity time as one-sixth of the difference between the pessimistic and optimistic time estimates.

The estimate of the expected value of an activity time is more difficult. We do not accept the likely time as the expected value. We feel that an activity time will more often exceed than be less than an estimated likely time. Hence, if likely times were accepted as expected values, an undesirable bias would be introduced. Our apprehension of this bias is supported by estimates already obtained. In many cases the likely time is nearer the optimistic than the pessimistic time. In such a situation one feels that the expected time should exceed the likely time.

We shall introduce an estimate of the expected time which would seem to adjust at least crudely for the bias that would be present if likely times were accepted as expected times. As a model of the distribution of an activity time, we introduce the beta distribution whose mode is at the likely time, whose range is the interval between the optimistic and pessimistic times, and whose standard deviation is one-sixth of the range. The probability density of this distribution is

$$f(t) = (\text{constant})\,(t - a)^{\alpha}\,(b - t)^{\gamma} \qquad \text{(see Figure 10)}$$

in which the optimistic and pessimistic time estimates are respectively $\underline{a}$ and $\underline{b}$ (the left- and right-hand ends of the range); the "constant," α and γ are functions of $\underline{a}$, $\underline{b}$, and the likely time M (i.e., the modal time). To reduce this probability density function to the standard form of the beta distribution, we introduce the random variable $\underline{x}$ as related to $\underline{t}$, as follows:

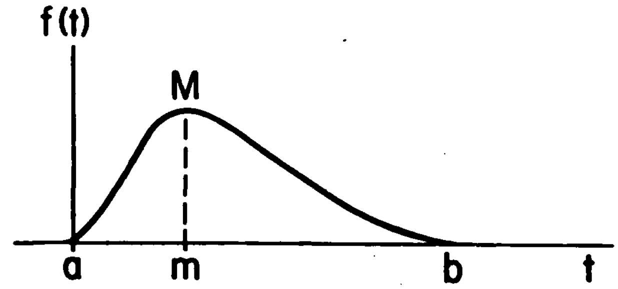
$$x = \frac{t - a}{b - a}.$$

The probability density of $\underline{x}$ is

$$f^*(x) = \{B(\alpha + 1, \gamma + 1)\}^{-1} x^{\alpha}(1 - x)^{\gamma}$$

β DISTRIBUTION

$$f(t) = K\,(t-a)^{\alpha}\,(b-t)^{\gamma}$$



VALUES OF α & γ OBTAINED BY

1. SETTING M·m OF ESTIMATES
2. SETTING σ (BETA DISTRIBUTION) = $\dfrac{b-a}{6}$

Figure 10

Since the modal value of t is M, if r denotes the mode of x,

$$r = \frac{M-a}{b-a}$$

If E(x) and V(x) are respectively the expected value and variance of x, straightforward computation leads to

$$r = \frac{\alpha}{\alpha+\gamma}$$

$$E(x) = \frac{\alpha+1}{\alpha+\gamma+2}$$

$$V(x) = \frac{(\alpha+1)\quad(\gamma+1)}{(\alpha+\gamma+2)^2(\alpha+\gamma+3)}$$

Since the variance of t is $(b-a)^2/36$, the variance of x is 1/36. We eliminate γ from the equations for r and V(x) after substituting 1/36 for V(x), and we obtain

$$\alpha^3 + (36r^3 - 36r^2 + 7r)\alpha^2 - 20r^2\alpha - 24r^3 = 0$$

We can now compute the parameters in f(t). Given M, a, and b, the above fomulas enable us to calculate in succession r, α, γ, (using the relation between r, α, and γ, E(x), and finally E(t) (because the equation of transformation between t and x implies that $E(t) = a + (b-a)\,E(x)$.

Let us study the relation between r and E(x). Numerical calculation with use of the above formulas gives the first two columns of Table 1.

Table 1

r	E(x)	(4r + 1)/6
0	.2053	.1667
1/8	.2539	.2500
1/4	.3228	.3333
3/8	.4075	.4167
1/2	.5000	.5000

If E(x) is plotted as a function of r, it is seen that the relation between these variables is approximately linear. A simple linear approximation, namely

$$E(x) = (4r + 1)/6$$

is given in the third column. We shall accept this approximation because it is accurate enough for our purposes, and because the prior computation requires among other operations the solution of a cubic equation. From the relations between E(x) and E(t) and between r and M, the linear approximation reduces to

$$E(t) = (a + 4M + b)/6.$$

This formula will be used in the FBM system analysis.

This result was derived under the assumption that the beta distribution is an adequate model of the distribution of an activity time. The choice of the beta distribution was dictated by intuition because empirical evidence is lacking. Hence it will be appropriate to consider the reasonableness of the result.

The result can be reduced to

$$E(t) = \frac{1}{3}(2M + \frac{a + b}{2}).$$

This means that E(t) is the weighted mean of M and the mid-range (a + b)/2, with weights 2 and 1 respectively. In other words, E(t) is located one-third of the way from the likely time M to the mid-range.

We can consider whether the weights 2 and 1 in the above formula seem appropriate. Perhaps it would be intuitively better to move the likely time only one-fourth of the way towards the mid-range. Perhaps one-half or one-tenth. Our own opinion is that one-third seems reasonable. However, we intend to code into the computer routine this ratio as a parameter. Hence the weights can be changed if authoritative judgment dictates. Furthermore, by making computer runs with various parameter values one can test the sensitivity of the computed result to this choice of weights.

As experience with the development of the FBM accumulates, we can compare actual activity times with estimates of M, a, and b. This will enable us to reconsider the weights used.

As further evidence of the appropriateness of the beta distribution model, we have presented the graph of $f^*(x)$ for the case in which r = 1/4 (Figure 11). This represents the case in which the likely time is one-fourth of the way from the optimistic to the pessimistic time. This graph reveals that most of the activity times will be symmetrically distributed about r = 1/4 between r = 0 and r = 1/2, but that there is enough probability above r = 1/2 to bring the expected time up to .32. In our opinion this graph is as reasonable as that obtained from any other commonly occurring probability density function.

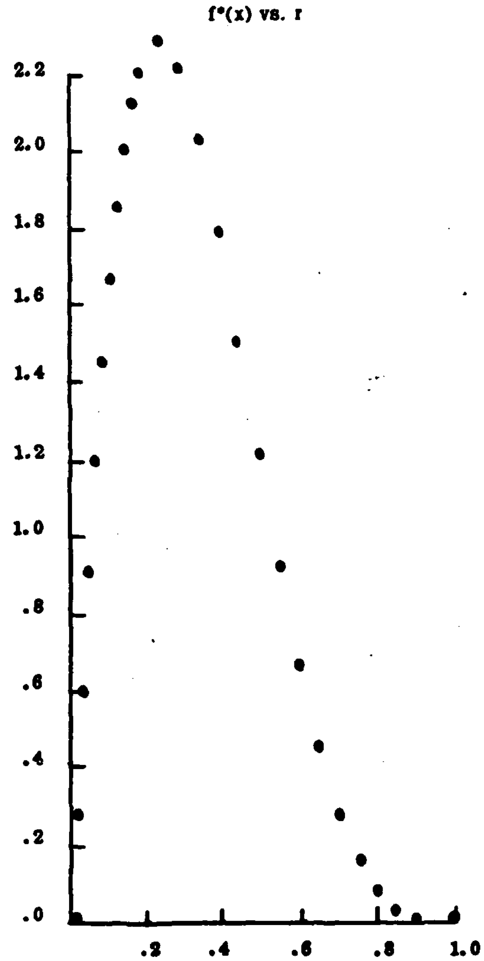


Figure 11

2. THE MATHEMATICAL COMPUTATIONS

This part discusses the calculation of the expected values and variances of event times. A description is given of the mathematical problems that are presently unsolved.

We consider first some analytic difficulties. One serious difficulty is indicated in Figure 12. The elapsed times between $\underline{A}$ and $\underline{D}$ is the greater of

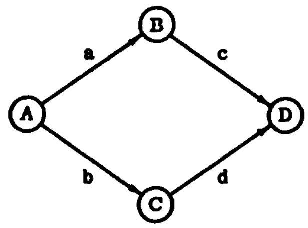


Figure 12

the two times $\underline{a} + \underline{c}$ and $\underline{b} + \underline{d}$. In case the expected values and variances of $\underline{a} - \underline{c}$ and $\underline{b} - \underline{d}$ were equal (in practice this is unlikely), the expected value and variance of the time from $\underline{A}$ to $\underline{D}$ would be estimated without serious difficulty. In fact, this could be done for $\underline{n}$ activities as in Figure 13 by means of tabulated results (see Kendall, *Advanced Theory of Statistics*, Vol. I, for references) or formulas (in a

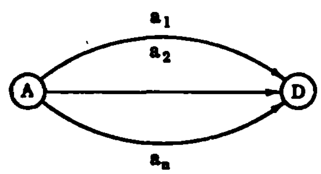


Figure 13

paper by Clark and Williams to appear shortly in the *Annuals of Mathematical Statistics*). However, it is assumed that $a_1, \ldots, a_n$ are identically distributed. As soon as one departs from the case of identically distributed variables, there are no numerical results or formulas available.

However, we could develop a basis for handling the situation of Figure 12 in the following way. The problem, normalized, becomes that of the expected value and variance of the greater of two independent normal variables, one with expected value zero and variance one. (The distribution of the greater of two normal variables is not normal, but we shall not worry about this partly because we shall consider only the expected value and variance.) For this two parameter problem (the two parameters being the expected value and variance of the second normal distribution after normalization reduces the first expected value to zero and variance to one), we would construct by numerical integration a two-way table of values. The resulting table could be fitted by some formula convenient for a computer.

With this result we could handle the case of Figure 13 with adequate accuracy even when the activity times are not identically distributed. We could get the expected value and variance of the greater of a_1 and a_2, etc. Then we could compute the expected value and variance of the greater of a_3 and the greater of a_1 and a_2, etc.

However, presently available information reveals that we encounter much greater complexity. Figure 14 represents one such situation which

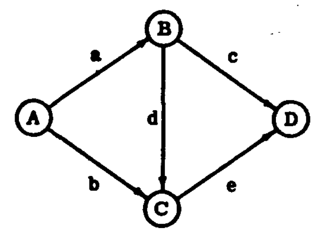


Figure 14

occurs. In this case the elapsed time for $\underline{A}$ to $\underline{D}$ is the greater of the three times $\underline{a} + \underline{c}$, $\underline{a} + \underline{d} + \underline{e}$, and $\underline{b} + \underline{e}$. Furthermore, these three times (sums of activity times) are correlated, and there seems no way to circumvent computation with correlated variables. The correlations involved in the example of Figure 13 are easily calculated. The covariance of $\underline{a} + \underline{c}$ and $\underline{a} + \underline{d} + \underline{e}$ is the variance of $\underline{a}$, and the other covariance is obtained in the same way (two of the times are independent). The expected value and variance of the greatest of the three times can be calculated by the numerical evaluation of a single integral, but the numerical values to be summed are obtained from the distribution functions of two-dimensional correlated normal variables. These distribution functions are tabulated (see Karl Pearson's tables, volume 2). The tables have three parameters, and consist of a two-way table for each of several values of the correlation coefficient. These tables could be stored in a computer with large storage capacity; possibly the tables could be fitted by formulas. A high speed computer (such as the IBM 700 series or the Remington 1103) could calculate the expected value and variance of the greatest of three normal variables in a reasonably short time.

However, since even greater complexities are encountered, and since we would like to use a medium speed computer, we should investigate the possibility of obtaining simple approximations. Let us discuss this prospect at some length. Consider for the moment the problem of the greater of two correlated normal variables. This is a three parameter problem, the parameters being the difference of the expected values, the ratio of the standard deviations, and the coefficient of correlation. One could construct tables to handle this problem there would be two-way tables for selected values of the correlation. Possibly the tables could be fitted by relatively simple analytic expressions.

Consider the situation of Figure 14. If we restrict consideration to the two constraints $\underline{a} + \underline{c}$ and $\underline{b} + \underline{e}$, we could determine the expected value and variance of the greater of these two times. Then we could combine this constraint with that of the third time $\underline{a} + \underline{d} + \underline{e}$, and this again involves the greater of two time constraints. The difficulty is that one of these two constraints is itself a complicated computed result, and it is difficult to calculate the correlation between $\underline{a} + \underline{d} + \underline{e}$ and the greater of the other two times. We know the covariance of $\underline{a} + \underline{d} + \underline{e}$ with each of $\underline{a} + \underline{c}$ and $\underline{b} + \underline{e}$, but what relation is there between the two simpler correlations and the more complicated correlation involving the greater of two times? The problem is not simple. However, it is likely that study based upon numerical evaluation of special cases would produce *some rule of* thumb for estimating the desired correlation at least roughly.

With such a rule of thumb we could handle many complex situations in the following manner. Suppose that the time of some event were subject to several constraints (i.e., the time is the greatest of several times). We would find the expected value and variance of the greater of the first two times. Then using the rule of thumb, we would estimate the correlation between the time of the third constraint and the greater of the first two times. Then the tables mentioned above would permit us to estimate the expected value and variance of the greatest of the first three constraints. The fourth and succeeding constraints would be introduced similarly.

If this possibility should be realized, another difficulty would arise, namely that of arranging the analysis in a computer. The computer could determine all time constraints on each event (this amounts to listing all paths along activity arrows from the present event to each event, and totaling the times and variances of the activities). This might require too much time even on a high speed computer. Moreover, it might be unreasonable to expect the computer to apply properly the rule of thumb for combining correlations (however, given two specific paths from the present to a specific event, it would be easy to compute the correlation of the two time constraints). The difficulty would not be with the rule of thumb but with isolating the various paths and their interrelationships (which determine the correlations).

It is likely that the best procedure with respect to the computer would be something like the following. When the computer calculates the expected value and variance of an event time, the computer must know the constraints on this time (there is a constraint for each path of activities from the present to the event in question, and the constraint of the path is the sum of the activity times along the path). It might take the computer a relatively long time to isolate all the paths to an event and to accurately compute the expected value and variance of the event time. Hence, it may be desirable to make a preliminary analysis off the computer. For each event we would examine all the activity paths, select the most important ones (rejecting those paths whose activity times are relatively small), and tell the computer how to compute (i.e., tell the computer what constraints to consider, in what order, and with what simplifying approximations.)

At best, it will be a non-trivial task to implement the above analysis. Hence, we should consider alternatives. The most obvious alternative is the one now employed. The time constraints of all paths leading up to an event are considered, and one assigns to the event the expected value and variance of the greatest of these constraints.

This simplification gives biased estimates and the bias is such that the estimated expected times of events are too small. To get some idea of the bias, we can consider two independently distributed normal variables with the same parameters μ and σ. The expected value of the greater of the two is $\mu + .56\sigma$ (see Kendall, loc. cit.). Thus, if two parallel activities are each expected to take a year with a standard deviation of one month (which is roughly equivalent to a range from 9 to 15 months), the expected time to the completion of both activities is 12 + .56 (1) = 12 1/2 months roughly. In the presently used simplifications the half month is lost. As a very rough guess, we might fear that the presently used simplification is biased to the extent of a couple of months.

However, the presently used simplification is appealing. For each event there is a longest time activity path from the present to the event. This path can be called a "critical path." Of special interest is the critical path to some future event $\underline{Z}$

of paramount importance. Each activity time along this path contributes directly to the elapsed time from the present to Z. Slippages in the completion times of these activities cause slippages of equal amounts in Z. Furthermore, for events or activities not on the critical path, some delay can be tolerated without changing the expected time of Z. For each "non-critical" event (not on the critical path) one can set a "latest" expected time for the event such that if all expected times were constrained to occur at this latest time (variances being unaltered), the expected time of Z would be unchanged *as computed by the presently used method*. However, this concept introduces the biases of the present computations in a very severe form. But, ignoring the biases, we have a way of designating critical events whose slippages are serious, and non-critical events each with its possible slippage. Such analysis is useful in spite of the biases.

Let us next face the problem of some rigorous analysis of "latest" times at which events can occur without jeopardizing the time of some event Z of paramount importance. Rigorously there seems to be no rigid latest times at which events can occur. To illustrate, suppose that X and Y are immediate predecessors of Z and that the intermediate activities are x and y respectively. Suppose that both x and y have the expected value μ and variance σ. Then if a time for Z is set rigidly, and if X and Y occur at times $\mu + .56\sigma$ before Z (the term $.56\sigma$ is clarified above), then the expected time of Z will be the time rigidly set. One is inclined to accept the latest times for X and Y as $\mu + .56\sigma$ prior to Z. But note that if X precedes Z by $\mu + t\sigma$ where t is say 4 or 5, and if Y precedes Z by only μ, the expected time of Z is still as set. This example reveals that there is some freedom to adjust "latest times" among two or more events.

Another logical difficulty is the following. If the expected time of any activity whatsoever is increased, or if the expected time of any event is set at a later time, there is an increase in the expected elapsed time to each succeeding event. (Except in unusual situations involving activities with certain times.) For example, suppose that there are two activities between events A and B. Suppose that the expected value and variance of the first activity time is 10 and 1 month respectively, and the corresponding parameters for the second event are 1 and 1 respectively. Then if the expected time of the second activity changes from 1 to 2, there will be practically no change in the expected time of the second event B; but rigorously the expected elapsed time from A to B will be increased by a tiny fraction of a second.

A precise notion of a latest time for events is the following. Let Z be some event of paramount importance. Let some delay D in Z be selected. This delay D might be a week, or it could be a day or a month. Then for a given event X one could compute the increase in the expected elapsed time from the present to X which would result in an increase of D in the expected elapsed time to Z. The result would indicate how much the event X could be slipped without a resulting serious delay.

The unfortunate aspect of this definition is that it seems computationally impossible. To show why this is so, let X^1 be the latest time of X as defined in the last paragraph. To compute X^1, it would seem necessary to make one or two runs of the analysis with the constraints augmented by a single constraint which would delay X (an artificial activity between X and X^1.) A couple such trial runs would enable one to interpolate for the delay in X which would set back the expected time of Z by the amount D. The trouble is that this computation must be carried out for every event and activity of interest. The time required for such computation would seem to be excessive. However, it would be very easy to perform such analysis for selected problems.

As we maneuver from an intellectually sound definition of latest time to one that can be made operational, it seems necessary to introduce some ultimately illogical procedure. The most obvious thing to do is to run time backwards. There are logical difficulties with this procedure. However, it seems rational to introduce the fiction of a reverse development of the process which starts with some event Z and works backward in time to the present. If such an analysis is made by the method employed for the forward computation of the times at which events will occur, then the expected time of an event could be interpreted as a latest time for the event. Such an analysis need not proceed merely from a fixed time for the final event, but could take into account the uncertainty of the final event.

APPENDIX C

THE PRESENTLY EMPLOYED APPROXIMATE COMPUTATION

This part describes the presently used method of approximating the event times by hand computation.

To facilitate reference, the events are numbered 1, 2, ...etc. For each event a card is made up which records the immediate predecessors and immediate successors of the event. This card has the format of Figure 15. Figure 15 designates the event numbered 28 and indicates that the immediate

28	
58	3
7	167
13	45

Figure 15

predecessors of 28 are 58, 7, and 13, and that the immediate successors are 3, 167, and 45.

The cards are arranged in numerical order. The card of Figure 15 would follow the cards for the events numbered between 1 and 27 inclusive. There could be fewer than 27 predecessors of 28 because there may be gaps in the sequence of integers used.

The major step in the following computation is that of rearranging the cards into a sequence such that if one event precedes another in the stochastic model, its card will precede the other's card in the sequence to be constructed.

The rearrangement will involve the removal of cards from the sequence in which they start. As a card is removed it is placed at the end (or bottom) of a second sequence (or pile). Furthermore, certain steps in the operation will involve the removal of the card at the beginning of the second sequence and the placing of the card at the end of a third sequence. At the end of the operation all cards will be in the third sequence, and the third sequence will have the desired property that if one event precedes another in the stochastic model, its card precedes the other's in the third list.

The card for the present event is removed from the first sequence and placed in the second sequence. Suppose that the present event has the number 1 and that its card is given in Figure 16. We observe that 10 is an immediate successor of 1.

1	
	10
	11
	12
	148
	1234

Figure 16

We look up the card 10 which is in the first sequence. We will find the notation that event 1 is an immediate predecessor of 10. We check the event 1 on the card of event 10, obtaining, say, Figure 17. In Figure 17 we have used the hyphen as a check mark. After we check 1 on the card for

10	
1-	16
	44

Figure 17

event 10, we observe whether all the immediate predecessors of 10 have been checked. In the case of Figure 17 this is so. Since this is so, the card for 10 is placed at the end of the second sequence. We return to the card for 1 and note that the second immediate successor of 1 is 11. We find the card for 11 which will be in the first sequence. On this card we check the 1 which indicates that 1 is an immediate predecessor of 11, and we obtain, say, Figure 18. After checking 1, we observe that all

11	
1-	66
4	

Figure 18

the immediate predecessors of 11 have not yet been checked. Hence, we leave the card for 11 in its place in sequence 1. (When at some later time in the operation 4 is checked, the card for 11 will be placed at the end of the second sequence of cards.) We continue until all the immediate successors of 1 have been handled in the same way. Then the card for 1 is placed at the end of the third sequence (actually, 1 will be the first card in sequence 3).

The next step is to take the first card in sequence 2 and to handle it in the same way that 1 was handled. Suppose that this first card in sequence 2 is 10 as given in Figure 17. Since Figure 17 indicated that 16 is an immediate successor of 10, we look for 16 in the first sequence. If we do not find 16 (because it has been removed from the first sequence), we take no action as regards 16. If 16 is still in sequence 1, we check on the card for 16 the immediate predecessor 10. If, after 10 is checked, we find that all the immediate predecessors of 16 have been checked, we would place 16 at the end of sequence 2. Otherwise 16 is left in sequence 1. We continue until all the immediate successors of 10 have been handled. Then 10 is placed at the end of sequence 3. The first card in sequence 2 is processed, and the operation continues as indicated.

We shall now prove that the process will continue until all the cards are in sequence 3, and that sequence 3 will have the desired property.

In the first place we note that a card is not placed in sequence 2 until all its immediate predecessors are already in sequence 2 or 3. Hence, when a card is placed in sequence 3, all its predecessors, immediate or not, are in sequence 3. Hence, the final sequence 3 will have the required property.

Finally, we must show that during the operation, sequence 2 will never be empty. Otherwise the operation might come to a halt before all cards were in sequence 3. Suppose that we reach a point in the operation at which there is no card is sequence 2, but there are still cards in sequence 1. Each event in sequence 1 has at least one immediate predecessor still in sequence 1 (otherwise the event would have been removed from sequence 1 at some step). Select an arbitrary event $\underline{A}$ in sequence 1. Choose one of the immediate predecessors of $\underline{A}$, say $\underline{B}$, which is still in sequence 1. Choose an immediate predecessor or $\underline{B}$, say $\underline{C}$, which is still in sequence 1. This process can go on forever, and an infinite sequence of events is generated. No event can appear twice in this infinite sequence because an event cannot be a predecessor of itself. But this is a contradiction; we have an infinite sequence with all different terms, and the terms are selected from a finite set of events.

We shall now work with sequence 3, and compute event times. The first event in sequence 3 is the present event. Its time is zero with zero variance. Next we consider the second event $\underline{B}$ in sequence 3.

Its sole predecessor is the present event. However, there might be two or more activities with $\underline{B}$ as immediate successor (each of these activities would start with the present event). We select the activity with the greatest expected value; in case of a tie, among the events with maximum expected value we choose one with maximum variance. The expected value and variance of this most severe time constraint is taken as the expected value and variance of the time of this second event.

In general, suppose that expected values and variances have been determined for all events that precede $\underline{X}$ in sequence 3. Let the activities with $\underline{X}$ as immediate successor be $\underline{a}$, ..., $\underline{z}$. Let the starts of these activities be the events $\underline{A}$, ...$\underline{Z}$, respectively. We add the expected times and variances, respectively, of $\underline{A}$ and $\underline{a}$, and we get a first constraint on the expected time and variance of $\underline{X}$. In the same way we get an expected time and variance for each activity with $\underline{X}$ as its immediate successor. We choose the greatest of these expected times, or in case of a tie we choose the greatest of one of the expected times with maximum variance. This expected time chosen is obtained from one of the activities $\underline{a}$, ..., $\underline{z}$, say $\underline{y}$. We assigned to $\underline{X}$ as expected time and variance the sum of the expected times and variances, respectively, of $\underline{Y}$ and $\underline{y}$. This operation is appropriate because in sequence 3, each event is preceded by all events which precede it in the stochastic model.

APPENDIX D

A RESCHEDULING PROCEDURE

1. FINAL EVENT (OBJECTIVE)

(1) As a first criterion of schedule feasibility, follow the rule that the scheduled date of the final event must have a probability of accomplishment in the range .25 to .50. (Note that if the probability is .5, then in regard to the final event $T_{E_o} = T_{L_o} = T_{s_o}$; if the probability is greater than .5, then the scheduled date for final completion is later than its expected earliest value.)

(2) If the probability of accomplishing the final event is below the stipulated minimum, the first step is to note the slippage necessary in the final event to bring it within the range of minimum interval cited above.
The necessary schedule date of the final event can be computed as follows:

S^1 = schedule date with approximately .25 probability of achievement

K_ϵ = normal deviate exceeded with probability, ϵ

T_{E_o} = earliest expected time for final accomplishment

S^{11} = scheduled date with approximately .5 probability of achievement

1. $T_{E_o} + X = S^1$; $\frac{X}{\sigma_{T_E}} = K_{.75}$ $K_{.75} = -.67$

2. $T_{E_o} = S^{11}$

E.g., if $T_{E_o} = 92$ $\sigma_{T_E} = 6.2$

$\frac{X}{6.2} = -.67$; $X = (-.67)(6.2) = -4.15$

Taking the next larger integral value of X

$S^1 = T_{E_o} + X = + 92 + (-4) = 88$

$S^{11} = T_{E_o} = 92$

Therefore, the values of S_O in the interval 88 - 92 will satisfy the criterion of number 1 above.

(3) The interval 88 – 92 can be referred to as a "feasible" range. It should first be examined as to its propriety as a scheduled time for the final event.

(4) Assume that it is reasonable to have a final scheduled date fall within this feasible interval. Choose this date (a highly desirable one) as week 90.

(5) Recompute all T_{L_i} based upon $T_{L_o} = 90$.

(6) Return and compute $PR[e_2 < S_2 \leq l_i]$ for all events based on the new values of the T_{L_i}.

(7) The scheduled time of the final event has been changed to a date that is deemed feasible (by the foregoing definition).
Some of the intervening events must be rescheduled in order to allow for proper dovetailing of the events in constructing the end product.
Interim events can be of a second kind. When succeeding events are of a nature such that they follow in a natural sequence regardless of scheduled date, there is no need to reschedule (such events might be primarily of a progress measuring nature.)
However, a priori, there will probably not be any designation which will automatically separate the events (interim) into the categories "reschedulable" and "not reschedulable." Therefore, all events should be rescheduled periodically (nominally, at least). If the probabilities of accomplishment are smaller than technical personnel deem wise, then they can change the real scheduled dates as necessary.

(8) Methods for rescheduling an interim event are outlined in this appendix under the title "Procedures for Scheduling."

(9) In appraising the new schedule, it should be noted that difficulties can arise in the face of either of two exigencies:
1. The latest time, l, can come before <u>S</u>
2. The earliest time, <u>e</u>, can come after <u>S</u>
The probabilities of these two situations occurring are given respectively by:
1. P (no slack)
2. P (restriction)
If the probabilities are high that either of these two situations will occur, then action or close observation is called for. In the example (Exhibit E), action would probably be called for in the cases of events 69, 67, and 66. Close observation should be devoted to 61, 60, 56, 53, 51, and 50. The remainder of the events need not be subjected to the same close scrutiny.

(10)Using the procedure indicated in (9) above, it is possible to designate three different classes of events—which are ordered in respect to their probable relative criticalness. Such a list should only be suggestive. The technical personnel upon review in the light of their full knowledge should make the actual designation of real criticalness and provide for the necessary action and control.

2. INTERIM EVENTS

(1) An expected value exists for the earliest and latest times at which an event can occur—as does the variance for each of the two distributions.

(2) Define E and σ_e^2 as mean and variance at earliest times
L and σ_l^2 as mean and variance at latest times
S^1 as the suggested time for scheduling the event.

$$S^1 = \max_s [\Pr(e \le s)\ \Pr(l \ge s)]$$

(3) It is noted that the foregoing designation of the suggested schedule time does not explicitly take into account the costs associated with various options.
When one sets a schedule by maximizing the joint probability, a premium is set on having $\underline{e} \le l$ and also on having the interval overlap the scheduled date. The particular premiums that are set depend in some fashion on the variances of the latest and earliest time distributions.
The costs of not meeting the requirements are not explicit and do not differentiate between the case where S^1 falls before $\underline{e}$ and the case where S^1 falls after l.

(4) *Owing to the neglecting of cost functions, use of this procedure will occasionally give values of* S^1 that are larger than $\underline{E}$ when $\underline{E} > \underline{L}$. Such an outcome appears illogical, thus this procedure should only be used when $\underline{E} < \underline{L}$. When $\underline{E} > \underline{L}$, the suggested value S^1 can be equated to $\underline{L}$. This procedure will make $S^1 = L$, and thus take advantage of fortuitous early completions at the cost of occasionally scheduling an event at too early a time.

[redacted]