NISM Series X-A: Investment Adviser Level 1

Time Value of Money

Compounding · Discounting · Annuities · Perpetuity

LEARNING OBJECTIVES: TVM Concept • Present Value • Future Value • CAGR • Annuity • Perpetuity

1 Chapter Overview

01

Money today is worth **more** than the same future amount — it can earn returns. This is the foundation of TVM.

02

Discounting brings future cash flows to the present. **Compounding** grows present amounts to future values.

03

The later a cash flow is received, the **lower its present value**. A higher discount rate reduces PV further.

04

CAGR is the standard return measure — it equates the beginning and ending value of an investment over a period.

05

An **Annuity** is a series of equal cash flows at regular intervals. Ordinary = end of period; Annuity Due = start.

06

A **Perpetuity** is a cash flow that continues forever. PV = C/r — used to value perpetual bonds and pensions.

2 The Core Concept

Time Value of Money — Definition

Money available now is worth more than the same amount in the future because it can earn returns. This is the fundamental principle behind all financial decisions involving multi-period cash flows.

◆ Instinctive preference for current consumption over future consumption.

◆ Ability to invest now and earn a return — Rs.100 today grows to Rs.100.50 after one month at 6% p.a.

■ The later a cash flow, the lower its PV today. Key parameters: (a) Cash flows — single or stream, (b) Rate of interest, (c) Time period, (d) Frequency. PV uses discounting; FV uses compounding.

COMPOUNDING → grows at rate r

PV Today — Yr 1 — Yr 2 — Yr 3 — **FV** Future

← DISCOUNTING brings future back to today

3 Present Value (PV)

Present Value

The amount you would pay today for a future cash flow. Discounts future values to today's terms using the relevant rate. Higher discount rate or further cash flow = lower PV.

```
Single cash flow: PV = FV / (1 + r)^n
```
FV = Future Value | r = rate per period | n = number of periods

```
Regular cash flows: PV = C x [1 - 1/(1+r)^n] / r
```
C = regular payment per period

```
Excel: =PV(rate, nper, pmt, [fv], [type])
```
type=0 -> end of period (ordinary) | type=1 -> start of period (annuity due)

WORKED EXAMPLE — SHYAM'S ANNUITY

Shyam receives Rs.6,500/year for 8 years at 7% p.a. Equivalent lump sum today? PV = 6500/(1.07)^1 + ... + 6500/(1.07)^8 = **Rs.38,813** Formula: PV = 6500 x [1 - 1/(1.07)^8] / 0.07 = Rs.38,813 Excel: =PV(0.07, 8, -6500) = Rs.38,813

WORKED EXAMPLE — LUMP SUM RECEIPT

Rs.50,000 receivable after 5 years at 6% p.a. PV = 50,000 / (1.06)^5 = **Rs.37,363**

## 4 Future Value (FV) & Compounding

Future Value

What a present sum will be worth at a future date after growing at a compound rate. Greater compounding frequency = more interest-on-interest = higher FV.

```
FV = PV x (1 + r)^n
```
Quarterly: r = annual rate / 4, n = years x 4 | Excel: =FV(rate, nper, pmt, pv)

Scenario	Description	Calculation	Interest Earned
1 — Simple	Interest withdrawn each year; no compounding	Rs.5L x 8% x 5	Rs.2,00,000
2 — Annual	Interest re-invested; compounded yearly	5L x (1.08)^5 = Rs.7,34,664	Rs.2,34,664
3 — Quarterly	Compounded 4x per year (r=2%, n=20)	5L x (1.02)^20 = Rs.7,42,974	Rs.2,42,974

Rs.5 Lakh @ 8% for 5 years — Impact of Compounding Frequency

| Rs.200,000 | Rs.234,664 | Rs.242,974 |
| Simple Interest | Annual Compound | Quarterly Compound |
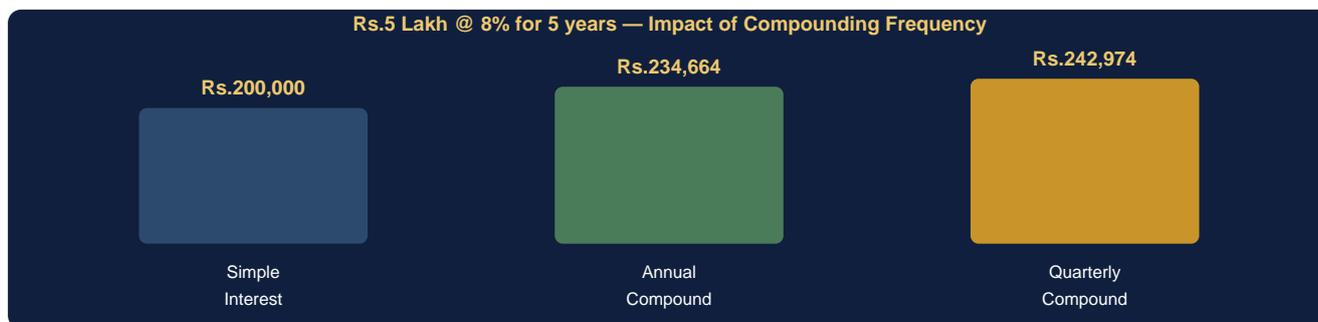
Quarterly compounding earns Rs.8,310 more than annual because interest is paid each quarter and itself earns interest for the remaining period.

5 Rate of Return & CAGR

CAGR — Compounded Annual Growth Rate

The compound rate that equates the beginning value of an investment with its ending value. CAGR is the accepted standard return measure in financial markets.

```
CAGR = (End Value / Beginning Value)^(1/n) - 1

n = years | Fractional: convert days to years (e.g. 450/365) Excel: =RATE(years, , -PV, FV)
```

Example	Begin Value	End Value	Period	CAGR
Simple growth	Rs.100	Rs.120	2 years	9.54%
Mutual fund NAV	Rs.10.50	Rs.12.25	3 years	5.27%
Fractional period	Rs.11 (NAV)	Rs.13.50	450 days	18.07%

For 450 days: CAGR = (13.5/11)^(365/450) - 1 = 18.07%. Always convert days to years. CAGR is standard except for sub-annual periods.

6 Periodic Payments — PMT & NPER

Key Excel Functions for Loan Planning

Function	Purpose	Key Inputs	Example
PMT	Calculate EMI	rate/12, nper x 12, -PV	Rs.30L @6.5% for 20y = Rs.22,367/mo
NPER	Periods to repay	rate/12, -PMT, PV	Rs.5L @8%, EMI=12,000 -> 49 months
PV	Present value	rate, nper, -pmt	Rs.6,500 x 8y @7% -> Rs.38,813
FV	Future value	rate, nper, pmt, -pv	Rs.5,000 @8% for 5y -> Rs.7,347
RATE	Solve for CAGR	nper, , -pv, fv	Rs.100 -> Rs.120 in 2y -> 9.54%

> **WORKED EXAMPLE — HOME LOAN EMI**
>
> Satish: Rs.30 lakh @6.5% p.a. for 20 years, monthly reset. =PMT(0.065/12, 240, -3000000) = **Rs.22,367/month** Rate falls to 6.25%: Rs.21,928/month. Saving = Rs.439/month on a 0.25% rate cut.

7　Annuity — Ordinary vs Due

> **Annuity**
>
> A sum of money paid at regular intervals. Fixed annuity pays a predetermined return; Floating annuity is benchmarked to inflation or an index. Common examples: pensions, EMIs, SIPs.

ORDINARY ANNUITY	ANNUITY DUE
Payment at END of period	Payment at START of period
Excel type = 0	Excel type = 1
PV = Rs.15,849	**PV = Rs.17,434**

Feature	Ordinary Annuity	Annuity Due
Payment timing	End of each period	Start of each period
Excel type param	0 (or blank)	1
Common examples	Bond coupon, loan EMI	Insurance premium, advance rent
PV (Rs.5,000 x 4y @10%)	Rs.15,849	Rs.17,434 — higher (money received sooner)
Better for recipient?	✘ Lower PV	✔ Higher PV — get money earlier, invest longer

> **ANNUITY TABLE METHOD**
>
> PV can also be computed using an Annuity Table (PV factor table). Rs.12,000/year for 10 years at 5%: Annuity Factor = 7.7217 PV = 12,000 x 7.7217 = **Rs.92,660** (same as =PV(0.05, 10, -12000))

8　Perpetuity

> **Perpetuity**
>
> A cash flow that continues forever with no finite end date. Perpetual bonds and lifetime pensions are real-life examples. The infinite series converges to a simple formula.

```
PV of Perpetuity = C / r
```
C = constant annual cash flow | r = discount rate (decimal)

> **WORKED EXAMPLE — PERPETUAL BOND**
>
> A perpetual bond pays Rs.10,000 annual interest. Discount rate = 8%. PV = 10,000 / 0.08 = **Rs.1,25,000** If rates rise to 10%: PV = 10,000 / 0.10 = Rs.1,00,000 Bond prices and interest rates move inversely.

9　Quick Revision Sheet — Chapter 2

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KEY FORMULAS

$\rightarrow$ PV (single) = FV / (1+r)^n

$\rightarrow$ PV (annuity) = C x [1 - 1/(1+r)^n] / r

$\rightarrow$ FV = PV x (1+r)^n

$\rightarrow$ CAGR = (End/Begin)^(1/n) - 1

$\rightarrow$ PV (perpetuity) = C / r

$\rightarrow$ EMI = PMT(r/12, n x 12, -Loan)

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EXCEL FUNCTIONS

$\rightarrow$ PV(rate, nper, pmt) — present value

$\rightarrow$ FV(rate, nper, pmt, pv) — future value

$\rightarrow$ RATE(nper, , -pv, fv) — CAGR / interest rate

$\rightarrow$ PMT(rate, nper, pv) — periodic payment / EMI

$\rightarrow$ NPER(rate, pmt, pv) — number of periods

$\rightarrow$ type=0 = end of period; type=1 = start of period

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TVM PRINCIPLES

$\rightarrow$ Earlier money is worth more than later money

$\rightarrow$ Higher discount rate -> lower PV of future cash flows

$\rightarrow$ More frequent compounding -> higher FV, same rate

$\rightarrow$ Annuity Due PV > Ordinary Annuity PV (same cash flows)

$\rightarrow$ Perpetuity PV = C/r (converges from infinite series)

$\rightarrow$ CAGR is standard; not used for sub-annual periods

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EXAM TIPS

$\rightarrow$ Always match rate frequency to period frequency

$\rightarrow$ Quarterly: divide rate by 4, multiply periods by 4

$\rightarrow$ Fractional CAGR: n = days/365 (e.g. 450/365)

$\rightarrow$ Annuity Due PV = Ordinary Annuity PV x (1+r)

$\rightarrow$ Higher interest rate = lower perpetuity price

$\rightarrow$ For EMI: always use monthly rate and total months

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MCQ — Practice Questions — Chapter 2

Test your understanding of Chapter 2. The correct answer and explanation are shown below each question.

Q1 Which of the following best describes the time value of money?

(A) Money loses value only due to inflation	**(B) Money available today is worth more than the same amount in the future**
(C) Future cash flows are always more valuable than present ones	(D) The value of money is constant over time

Answer: (B) — Money today can be invested to earn returns, making it more valuable than the same amount received later.

Q2 Rs.1,000 is to be received after 3 years. The discount rate is 5% p.a. What is the present value?

(A) Rs.863	(B) Rs.1,157
(C) Rs.950	(D) Rs.900

Answer: (A) — PV = 1000 / (1.05)^3 = 1000 / 1.1576 = Rs.863.

Q3 Krishna invests Rs.5 lakh at 8% compounded quarterly for 5 years. Which option gives the highest future value?

(A) Simple interest	(B) Annual compounding
(C) Quarterly compounding	(D) All give the same FV

Answer: (C) — More frequent compounding means more periods of interest-on-interest, giving a higher FV.

Q4 An investment grows from Rs.10.50 to Rs.12.25 in 3 years. What is the CAGR?

(A) 5.27%	**(B)** 6.14%
(C) 4.95%	**(D)** 5.60%

Answer: (A) — CAGR = (12.25/10.50)^(1/3) - 1 = (1.1667)^0.333 - 1 = 5.27%.

Q5 Satish takes a Rs.30 lakh home loan at 6.5% p.a. for 20 years. Which Excel formula gives the monthly EMI?

(A) =PMT(0.065, 20, -3000000)	**(B) =PMT(0.065/12, 240, -3000000)**
(C) =PMT(0.065/12, 20, -3000000)	**(D)** =PV(0.065/12, 240, 3000000)

Answer: (B) — Monthly rate = 6.5%/12; total months = 20 x 12 = 240. PMT gives the equal monthly payment.

Q6 An annuity of Rs.5,000/year for 4 years at 10% — Annuity Due vs Ordinary Annuity:

(A) Both have the same PV	**(B)** Ordinary Annuity has higher PV (Rs.17,434)
(C) Annuity Due has higher PV (Rs.17,434)	**(D)** Annuity Due has lower PV because payments start earlier

Answer: (C) — Annuity Due pays at the start of each period. Earlier receipts can be re-invested, giving higher PV (Rs.17,434 vs Rs.15,849).

Q7 A perpetual bond pays Rs.10,000/year. Discount rate is 8%. If rates rise to 10%, what happens to the bond price?

(A) Price rises to Rs.1,25,000	**(B) Price falls from Rs.1,25,000 to Rs.1,00,000**
(C) Price stays at Rs.1,25,000	**(D)** Price rises from Rs.1,00,000 to Rs.1,25,000

Answer: (B) — PV = C/r. At 8%: Rs.1,25,000. At 10%: Rs.1,00,000. Bond prices and rates move inversely.

Q8 For quarterly compounding at 8% p.a., what are the correct r and n for a 5-year deposit?

(A) r = 8%, n = 5	**(B)** r = 2%, n = 5
(C) r = 2%, n = 20	**(D)** r = 8%, n = 20

Answer: (C) — Quarterly: r = 8%/4 = 2% per quarter; n = 5 years x 4 quarters = 20 periods.

Q9 Which Excel function finds how many months it takes to repay a loan?

(A) PMT	**(B)** PV
(C) NPER	**(D)** RATE

Answer: (C) — NPER (Number of Periods) calculates how many payment periods are required to fully repay a loan.

Q10 Rs.6,500 is received annually for 8 years at 7% p.a. The PV (annuity formula) is approximately:

(A) Rs.42,000	(B) Rs.38,813
(C) Rs.35,500	(D) Rs.52,000

Answer: (B) — PV = 6500 x [1 - 1/(1.07)^8] / 0.07 = Rs.38,813. Also =PV(0.07, 8, -6500) in Excel.