SETS AND RELATIONS

JOHN VENN :

John Venn is remembered chiefly for his logical diagrams. Venn became critical of the methods used in diagrams in the nineteenth century, especially those of George Boole and Augustus de Morgan. So Venn wrote the book *Symbolic Logic* mostly to interpret and make his own personal corrections on Boole's work, but this was not the reason Venn became so famous. Venn wrote a paper entitled *On the Diagrammatic and Mechanical Representation of Prepositions and Reasonings* introducing diagrams known today as Venn diagrams. In *Symbollic Logic*, Venn further elaborated on these diagrams, which became the most important part of his books.

Venn extended Boole's mathematical logic and is best known to mathematicians and logicians for his diagrammatic way of representing sets, and their unions and intersections. Venn continued to improve his method for illustrating propositions by exclusive and inclusive circles. Venn's diagrams were the most consequential part of his logic trilogy, rather than his attempt to clarify what he believed to be inconsistencies and ambiguities in Boole's logic.

Later, he realized his diagrams were not sufficiently general so he extended his method by proposing a series of circles dividing the plane into compartments so that each successive circle would intersect all the compartments already existing. This idea was taken up and refined by Charles Dodgson who lived from 1832 to 1898. Dodgson's ideas led to the use of the closed compartment, or what is now known as the universal set.

‘‘A set is any collection of distinct and distinguishable objects of our intuition or thought.’’

By the term ‘distinct’ we mean that no object is repeated. By the term ‘distinguishable’ we mean that given an object, we can decide whether that object is in our collection or not.

A set is represented by listing all its elements between braces { } and by separating them from each other by commas (if there are more than one element).

Sets are usually denoted by capital Letters of English alphabet while the elements are denoted in general, by small latters.

If x is an element of a set A, we write $x \in$ A (read as ‘x belongs to A’). If x is not an element of A, we write $x \notin A$ (read as ‘x does not belong to A’). The symbol $\in$ is called the membership relation. Here are some examples :

Example 1**:** Let *A* = {1, 2, 5, 2, 3}. The elements of this collection are distinguishable but not distinct, hence A is not a set.

Example 2: Let *B* = collection of all vowels in English alphabets. Then *B* = {*a, e, i, o, u*}. Here elements of B are distinguishable as well as distinct. Hence B is a set.

Example 3: *C* = Collection of all intelligent persons of Delhi. Here elements are not distinguishable because if we select any person of Delhi, we cann't say with certainty whether he belongs to C or not, as there is no standard scale for evaluation of intelligence.

□ **Representation of a Set**

(*i*) **Tabular form or Roster Form**

Under this method elements are enclosed in curly brackets after separating them by commas.

Example:

If *A* is a natural number less than 5

A = {1, 2, 3, 4}

(*ii*) **Set builder method**

Under this method, set may be represented with the help of certain property or properties possessed by all the elements of that set.

A = {x | *P*(*x*)} or *A* = {*x* : *P*(*x*)}

This signifies, *A* is the set of element *x*, such that *x* has the property *P*.

Example:

The set *A* = {1, 2, 3, 4, 5} can be written as

A = {*x* | x □ *N* and *x* □ 5}

□ **Notations for Sets of Numbers**

1. The set of all natural numbers, or the set of all positive integers is represented by *N*.

2. Set of whole numbers is represented by *W*.

3. Set of all integers is represented by *Z* or *I*.

4. Set of rational numbers is represented by *Q*.

5. Set of irrational number is represented by Q^C or **Q(**

6. Set of real numbers is represented by R.

7. Set of complex numbers is by C.

4.1 Finite and Infinite Sets

- **Finite set**

A set having finite (definite) number of elements is called a finite set.

Example:

(*i*) Let $A = \{1, 2, 3\}$. Here A is a finite set as it has 3 elements (finite number of elements).

(*ii*) Let B = set of all odd positive Integers

$= \{1, 3, 5, 7, 9, \ldots\}$

Here B is not a finite set.

- **Infinite Set**

A set which is not a finite set is called an infinite set. Thus a set A is said to be an infinite set if the number of elements of A is not finite.

Examples:

(i) Let N = set of all positive integers = $\{1, 2, 3, 4, \ldots\}$

Here N is not a finite set and hence it is an infinite set.

(ii) Let Z = set of all integers = $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

Here Z is an infinite set.

(iii) Let Q = set of all rational numbers $= \left\{\frac{p}{q} : p, q \in Z, q \neq 0\right\}$

Here Q is an infinite set.

(iv) Let R = set of all real numbers.

Here R is an infinite et.

- **Cardinal Number of a Finite Set**

The number of elements in a finite set A is called the cardinal number of set A and is denoted by $n(A)$.

Example: Let $A = \{1, 3, 5\}$, then $n(A) = 3$.

4.2 Equivalent and Equal Sets

- **Equivalent Sets**

Two finite sets A and B are said to be equivalent if they have the same cardinal number. Thus sets A and B are equivalent iff $n(A) = n(B)$

If sets A and B are equivalent, we write A = B

Example: Let $A = \{1, 2, 3, 4, 5\}$, $B = \{a, e, i, o\ u\}$. Here $n(A) = n(B) = 5$. Therefore sets A and B are equivalent.

- **Equal Sets**

Two sets *A* and B are said to be equal if each element of A is an element of B and each element of B is an element of A. Thus two set A and B are equal if they have exactly the same elements but the order in which the elements in the two sets have been written may differ. If sets *A* and *B* are equal, we write *A* = *B*.

Example:

(*i*) Let A = {1, 2, 3}, B = {1, 2, 3}

Here *A* and *B* have exactly the same elements.

Therefore *A* = *B*

(*ii*) Let *A* = {1, 2, 3, 4, 5, 6}

B = {x : x N and 1 $\leq$ *x* < 7}

Here *A* and *B* are equal sets.

Note:

1. Two sets *A* and *B* are equal if $x \in A \Rightarrow x \in B$ and $x \Rightarrow B \Rightarrow x \in A$.
2. Equal sets are equivalent sets but equivalent sets may or may not be equal.

Example:

Let A = {1,2, 3}, B = {*a, b, c*}

Here *A* and *B* have same number of elements and hence they are equivalent but they are not equal set as 1 *A* but 1 $\in$ *B*.

4.3 Different Types of Sets

- **Null set (or empty set or void set)**

A set having no element is called a null set or an empty set or void set. It is denoted by ϕ or { }.

Examples:

(*i*) Let *A* = the set of even numbers not divisible by 2.

(*ii*) Let *C* = {*x* : *x* $\in$ *N* and 0 < *x* < 1}

Here *C* is a null set because there is no natural number lying between 0 and 1.

Note: The set {0} is not an empty set a it contains are element 0.

- **Singleton Set**

A set having single element is called a singleton set. It is represented by writing down the element within the braces.

Examples:

(*i*) *A* = The set of present prime minister of India.

(*ii*) {2}, {0}, {}.

□ Certain Useful Symbols

Symbols	Meaning
□	Implies
□	Belongs to
A □ *B*	*A* is a subset of *B*
□	Implies and is implied by
$\notin$	Does not belong to
s.t. (: or $\mid$)	Such that
$\forall$	For every
$\exists$	There exists
iff	If and only if
$a \mid b$ a is divisor of b	

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***Illustration* 1**

Represent the set $A = \{a, e, i, o, u\}$ in set-builder form.

Solution

Let x denote an arbitrary element of A.

Then x can be any vowel of English alphabet.

$\therefore$ A = {x : x is a vowel of English alphabet}.

***Illustration* 2**

Write the set $A = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in set-builder form.

Solution

We observe that in every element of set A, numerator is a natural number 1 to 6 and denominator is one more than the numerator.

Hence $A = \left\{x : x = \dfrac{n}{n+1}, n \in N, 1 \leq n \leq 6\right\}$.

***Illustration* 3**

Represent the set $A = \{x : x$ is an odd integer and $3 \leq x < 13\}$ in tabular form.

Solution

Element of A is x which must have properties: It is an odd integer and is among 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

So they are 3, 5, 7, 9, 11.

∴ A = {3, 5, 7, 9, 11}.

***Illustration* 4**

Match each of the sets on the left in the tabular (roster) form with the same set on the right given in the set-builder form :

(*i*) {5, 10, 15, ….}	(a) {x : x is a letter of work COLLEGE}
(*ii*) {C, O, L, E, G}	(b) {x : x is an odd natural number less than 10}
(*iii*) {1, 3, 5, 7, 9}	(c) {x : $x = 5n$, $n \in N$}
(*iv*) {2, 3}	(d) {x : x is a prime number and a divisor of 12}

Solution

In (c), given set is the set of all natural numbers which are multiples of 5 and therefore (i) matches (c).

In (a), there are seven letters in the word COLLEGE and L and E occur twice, therefore (ii) matches (a).

In (b), given set is th set of all odd natural numbers less than 10 and therefore (iii) matches (b).

Prime divisors of 12 are 2 and 3 and hence (iv) matches (d).

***Illustration* 5**

which of the following sets are empty, singleton, pair and which of them are equal.

$$A = \{x : x^2 = 9 \text{ and } 2x = 3\}$$

$$B = \{x : x^2 - 5x + 6 = 0, 2x = 6\}$$

$$C = \{x : x^2 - 4x + 3 = 0\}$$

$$D = \{x : x^2 = 25\}$$

$$E = \{x : 2x = 6 \text{ or } x = 1\}$$

Solution

$$A = \{x : x^2 = 9 \text{ and } 2x = 3\} = \{x : (x = 3 \text{ or } x = -3) \text{ and } x = \frac{3}{2}\}.$$

No x satisfies both the conditions. So there is no x in A i..e, $A = \phi$.

$$B = \{x : (x-3)(x-2) = 0 \text{ and } x = 3\}$$

$$= \{x : (x = 3 \text{ or } x = 2) \text{ and } x = 3\}$$

Only $x = 3$ satisfies both the predicates. So x has only one value 3.

∴ B = {3}

$C = \{x : (x - 3)(x - 1) = 0\} = \{x : x = 3 \text{ or } x = 1\}$

3 and 1 both satisfy the statement $x = 3$ or $x = 1$

∴ C = {1, 3}

Similarly, $D = \{-5, 5\}$ and $E = \{1, 3\}$.

Thus A is an empty set ; B is a singleton set, C, D, E are pair sets.

Also $\quad C = E$

***Illustration* 6**

$A = \{x : x = 4^n - 3n - 1$ and $n \in N\}$

$B = \{y : y = 9(n - 1)$ and $n \in N\}$

Prove that $A \subset B$

Solution

$x = 4^n - 3n - 1 = (1 + 3)^n - 3n - 1$

$= 1 + {}^nC_1 . 3 + {}^nC_2 . 3^2 + {}^nC_3 . 3^3 + ... {}^nC_n 3^n - 3n - 1$

$= 9[{}^nC_2 + {}^nC_3 . 3 + ... + 3^{n-2}]$

$y = 9(x - 1), \quad n \in N$

x is some multiple of 9

y is all the integral multiple of 9

$\therefore \quad A \subset B$

<u>PRACTICE EXERCISE</u>

1. Write the following sets in set-builder form :

(*i*) $A = \left\{\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \frac{6}{8}, \frac{7}{9}, \frac{8}{10}\right\}$

(*ii*) $B = \{1, 4, 9, 16, ...\}$

(*iii*) $C = \{2, 5, 10, ...\}$

(*iv*) $D = \{5, 7, 11, 13, 17, 19, 23, 29\}$

(*v*) $E = \{5, 9, 13, 17, 21, ...\}$

(*vi*) $F = \{14, 21, 28, 35, 42, ..., 98\}$

(*vii*) $G = \{1, 5, 10, 15, ...\}$

2. Describe the following sets in roster form (tabular form) :

$A = \{x : x \in Z$ and $|x| < 5\}$

$B = \{x : x$ is a letter of the word ‘proportion’$\}$

$C = \{x, x^3 = -1, x \in R\}$.

$D = \{x : x = 2n - 1 < 20$ and $n \in N\}$

$E = \left\{x : x = \frac{n}{n+1}, n \in N\right\}$

$F = \{x : x$ is a month of a year not having 31 days$\}$

3. Write the following sets in roster form :

(i) $A = (3^n - 2^n : n \in N \text{ and } 1 \le n \le 5\}$

(ii) $B = (a_n : n \in N,\ a_{n+1} = 2a_n \text{ and } a_1 = 3)$

(iii) $C = \{a_n : n \in N,\ a_{n+2} = a_{n+1} + a_n,\ a_1 = 1,\ a_2 = 2\}$

4. Determine the empty sets, singleton sets and pair sets in the following sets :

$A = \{x : x^2 = 16 \text{ and } 2x = 5\}$.

$B = \{x : x^2 = 2, x \text{ is a rational number}\}$.

C = {*x* : 3 □ *x* □□5 and 5 □□*x* □□6, where *x* is an integer}.

$D = \{x : x > 0 \text{ and } x^2 = 25\}$.

$E = \{x : x^3 + 1 = 0 \text{ and } x \text{ is an integer}\}$.

5. Which of the following sets are empty, singleton set and pair set. Which of them are equal?

$A = \{x : 2x = 10,\ x^2 - 7x + 10 = 0\}$

$B = \{x : x^2 - 16x + 55 = 0 \text{ and } x^2 = 25\}$

$C = \left\{x : -\frac{1}{2} \le x \le \frac{1}{2}\right\}$

$D = \{x : 0 \le 4x^2 \le 1\}$

6. Which of the following sets are infinite sets?

(i) The set of all lines parallel to *x*-axis.

(ii) The set of all circles passing through the origin.

(iii) The set of positive integers greater than 100.

(iv) The set of natural numbers which are multiples of 5.

(v) The set of prime numbers less than 99.

(vi) The set of straight lines passing through a fixed point.

(vii) The set of all natural numbers which divide 42.

(viii) $\{x : x \in Z \text{ and } x > 6\}$

(ix) {*x* : *x* is people of India speaking Hindi}

Answers

1. *(i)* $A = \left\{x : x = \frac{n}{n+2},\ n \in N \text{ and } n \le 8\right\}$ *(ii)* $B = (n^2 : n \in N\}$ *(iii)* $C = \{n^2 + 1; n \in N\}$

(iv) $D = \{x : x \text{ is a prime number and } 5 \le x \le 29\}$ *(v)* $E = \{x : x = 4n + 1, n \in N\}$

(vi) $F = \{x : x = 7n, n \in N \text{ and } 2 \le n \le 14\}$ *(vii)* $G = \{x : x = 5n, n \in N \text{ or } x = 1\}$

2. $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$, $B = \{p, r, o, t, i, n\}$, $C = \{-1\}$,

$D = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$, $E = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\right\}$

F = {February, April, June, September, November}

3. $A = \{1, 5, 19, 65, 211\}$, $B = \{3, 6, 12, 24, \ldots\}$, $C = \{1, 2, 3, 5, 8, 13, \ldots\}$

4. $A = \phi$, $B = \phi$, $C = \{5\}$, $D = \{5\}$, $E = \{-1\}$. A, B are empty sets and C, D, E are singleton sets.

5. $A = \{5\}$, $B = \{5\}$, $C = D$.

6. The sets in (i), (ii), (iii), (iv), (vi) and (viii) are infinite sets.

4.4 Subsets, Supersets and Proper Subsets

- **Subsets**

A set A is said to be a subset of a set B, if each element of A is also an element of B. If A is a subset of set B, we write $A \subseteq B$.

Thus $A \subseteq B \Leftrightarrow [x \in A \Rightarrow x \in B]$

Examples:

(*i*) Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 1, 5\}$. Then $A \subseteq B$.

(ii) Let A = {1, 2, 3}, B = {2, 3, 1}. Then $A \subseteq B$. Also $B \subseteq A$..

- **Proper Subsets**

A set A is said to be a proper subset of a set B, if

(*i*) every element of A is an element of B and

(*ii*) B has at least one element which is not an element of A.

This fact is expressed by writing $A \subset B$.

Examples:

(*i*) Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 1, 5\}$. Then $A \subset B$.

(*ii*) Let A = {1, 2, 3}, B = {2, 3, 1}. Then $A \not\subset B$.

- **Superset of a Set**

A set A is said to be a superset of a set B, if B is a subset of A i.e., each element of B is an element of A. If A is a superset of B, we write $A \supseteq B$.

Examples:

(*i*) Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 5, 4\}$

Here B is a subset of A, therefore A is a superset of B.

(*ii*) Let N = set of all natural numbers, and

Z = set of all integers.

- **Comparability of Sets**

  Two set A and B are said to be comparable if either $A \subset B$ or $B \subset A$ or $A = B$. If neither ($A \subset B$ or $B \subset A$) nor $A = B$, then A and B are said to be incomparable.

  Examples:

  (i) Sets {1, 2, 3} and {2, 3, 6, 7} are incomparable.

  (ii) Sets {1, 2, 3} and {1, 2} are comparable

  (iii) Sets {1, 2} and {2, 1} are comparable.

4.5 Power Set

Definition: The set or family of all the subsets of a given set A is said to the power set of A and is denoted by $P(A)$.

Symbolically, $P(A) = \{X : X \subseteq A\}$

Thus $X \in P(A) \Leftrightarrow X \subseteq A$

Also $\phi \in P(A)$ and $A \in P(A)$ for all sets A. Clearly the elements of $P(A)$ are the subsets of A.

Examples:

(i) If $A = \{1\}$, then $P(A) = \{\phi, \{1\}\}$.

(ii) If $A = \{1, 2\}$, then $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

(iii) If $A = \{1, 2, 3\}$, then $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$. If A has n elements then $P(A)$ has 2^n elements, for $n = 1, 2, 3$.

4.6 Universal Set

Any set which is superset of all the sets under consideration is called the universal set and is denoted by Ω or S or U.

Example: Let $A = \{1, 2, 3\}$, $B = \{3, 4, 6, 9\}$ and $C = \{0, 1\}$

We can take $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

or $S = \{0, 1, 2, 3, 4, 6, 8, 9\}$

as universal set.

ILLUSTRATIONS

***Illustration* 7**

Let $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3\}$ and $C = \{2, 4\}$. Find all sets X satisfying each pair of conditions

(i) $X \subseteq A$, $X \subseteq B$ and $X \subseteq C$

(ii) $X \subseteq A$ and $X \not\subset C$

Solution

(i) Since X is a subset of al of A, B and C, therefore, only those elements will be in X which are in all the three sets A, B and C.

$A \cap B \cap C = \{2\} \quad \therefore \quad X = \phi$ or $\{2\}$

(ii) Since X is a subset of A and X is not a subset of C, therefore, that element of A should belong to X which does not belong to C. Clearly such elements are 1 and 3.

Subsets of A containing 1 and not containing 3 are

$\{1\}, \{1, 2\}, \{1, 4\}, \{1, 2, 4\}$

Subsets of A containing 3 and not containing 1 are

$\{3\}, \{3, 2\}, \{3, 4\}, \{3, 2, 4\}$

Subsets of A containing both 1 and 3 are $\{1, 3\}, \{1, 3, 2\}, \{1, 3, 4\}$ $\{1, 2, 3, 4\}$

Hence $X = \{1\}, \{3\}, \{1, 2\}, \{1, 4\}, \{3, 2\}, \{3, 4\}, \{1, 3\}, \{1, 2, 4\}, \{2, 3, 4\}, \{1, 3, 2\}, \{1, 3, 4\}, \{1, 2, 3, 4\}$.

***Illustration* 8**

Find the power sets of the following sets

$A = \{ \{ \phi \} \}$, $B = \{\phi, \{ \phi \} \}$, $C = \{1, 2, \{3, 4 \} \}$

Solution

$P(A) = \{X : X \subseteq A\} = \{X : X = \phi, X = A\} = \{\phi, A\} = \{\phi, \{ \{ \phi \} \} \}$.

$P(B) = \{X : X \subseteq B\} = \{X : X = \phi, X = B, \mathrm{X} = \{\phi\}, \mathrm{X} = \{ \{ \phi \} \} \}$

$= \{\phi, \beta, \{\phi\}, \{ \{ \phi \} \} \} = \{\phi, \{ \phi \}, \{ \{ \phi \} \}, \{ \phi, \{ \phi \} \} \}$

$P(C) = \{X : X \subseteq C\}$

$= \{\phi, \{1\}, \{2\}, \{ \{ 3, 4 \} \}, \{1, 2\}, \{1, \{3, 4 \} \}, \{2, \{3, 4\} \}, C\}$.

***Illustration* 9**

Examine whether following statements are true or false

(i) $\phi \in \phi$ *(ii)* $o \in ($ *(iii)* $(= \{o\}$ *(iv)* $(= \{(\}$ *(v)* $\{ \{ 1 \}, 2, 3\} = \{1, \{2\}, 3\}$

(vi) $\{a\}$ ($\{a, \{a\}, \{ \{ a \} \} \}$. *(vii)* $\{a\}$ ($\{a, \{a\}, \{ \{ a \} \} \}$ *(viii)* $2 \subset \{1, 2, 4\}$

(ix) $\{ \{ 1 \} \} \supset \{ \{ 1, 2, 3 \} \}$

Solution

(i) False, because ϕ has no element

(ii) False, because ϕ has no element, $\therefore \ o \notin \phi$.

(iii) & *(iv)* are both false on the same ground

(v) False because $\{1\}$ is element of set on L.H.S. while it is not an element of the set on R.H.S.

(vi) True

(vii) True

(viii) False; because 2 is an element of the set {1, 2, 4} but it is not a subset.

(ix) False: because 1 is an element of the set on R.H.S> where as it is not an element of the set on L.H.S.

***Illustration* 10**

How many elements are in the set

$A = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

$B = \{x : x$ is even integer and $x < 19\}$

$C = \{x : 0 \le x \le 1$ and x is a rational number$\}$.

Solution

The elements of A are $\phi, \{\phi\}, \{\phi, \{\phi\}\}$.

So A has three elements.

$B = \{x : x = 0, \pm 2, \pm 4, \pm 6, \ldots$ and $x < 19\}$

$= \{\ldots, -4, -2, 0, 2, 4, 6, \ldots, 18\}$

$\therefore$ B is an infinite set and hence it has infinitely many elements. C is also an infinite set because $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ are all elements of C.

PRACTICE EXERCISE

7. If $A = \{x : x \in Q$ and $x^2 = 3x - 10 = 0\}$

   $B = \{x : x \in I$ and $-5 \le x < \frac{17}{8}\}$, then show that $A \subseteq B$.

8. Let $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3\}$ and $C = \{2, 4\}$. Find all sets X satisfying the following pair of conditions

   (i) $X \subseteq B$, $X \neq B$ and $X \not\subset C$ *(ii)* $X \subseteq B$, $X \not\subset C$.

9. Find the power sets of the following sets

   (i) {0} *(ii)* {1, {2, 3}} *(iii)* {2, 1, 7}

10. If $A = \{1, 2, (3, 4\}, \{5\}\}$, then test which of the following are true

    (i) $\phi \in A$ *(ii)* $\{3, 4\} \subset A$ *(iii)* $\{5\} \in A$ *(iv)* $\{2\} \in A$

11. Which of the following are true

    (i) Let A and B are two sets

    If $x \in A$ and $B \subseteq A$, then $x \in B$

    (ii) Let A, B, C be three sets.

    If $A \subset B$ and $B \in C$ then $A \in C$.

12. If $A = \{\phi, \{\phi\}, 1, \{1, \phi\}, 7\}$, then which of the following are true?

(i) $7 \subset A$ (ii) $\{\{\phi\}\} \subset A$ *(iii)* $\{\{7\}, \{1\}\} \subseteq A$ *(iv)* $\{\phi, \{\phi\}, \{1, \phi\}\} \subset A$

(v) $\{1\} \in A$

13. Which of the following statements are true?

(i) The set of all rectangles is contained in the set of all squares

(ii) The sets A = {x : x is a letter of the word 'LITTLE'}

B = {x : x is a letter of the word 'TITLE'} are equal

(iii) The set of all cats is contained in the set of all animals.

Answers

8. *(i)* $X = \{1\}, \{3\} \{1, 2\}, \{1, 3\}, \{2, 3\}$ *(ii)* $X = \{1\}, \{3\}, \{1, 2\}, \{1, 3\}. \{2, 3\}, \{1, 2, 3\}$

9. *(i)* $\{\phi, \{0\}\}$ *(ii)* $\{\phi, \{1\}, \{\{2, 3\}\}, \{1, \{2, 3\}\}\}$ *(iii)* $\{\phi, \{2\}, \{1\}, \{7\}, \{2, 1\}, \{2, 7\}, \{1, 7\}\}, \{2, 1, 7\}\}$

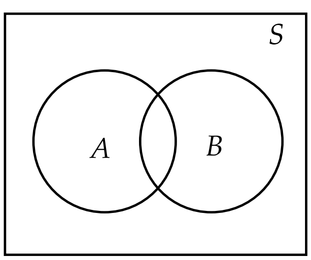
10. *(i)* False *(ii)* False *(iii)* True *(iv)* False

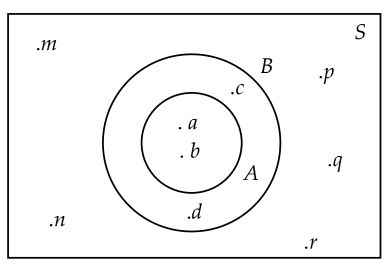
11. *(i)* False *(ii)* False **12.** *(i)* False *(ii)* True *(iii)* True *(iv)* True *(v)* False

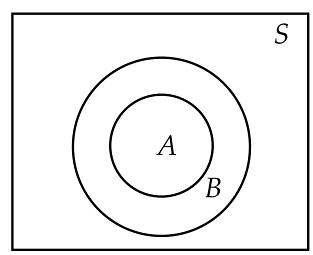
13. *(i)* False *(ii)* True *(iii)* True

4.7 Venn Diagrams

A Swiss mathematician Euler introduced the pictorial representation of sets in which a set is represented by a closed curve usually a circle and its elements by points within it. A statement involving sets can be easily understood with pictorial representation of the sets. The diagram showing these sets is called the Venn diagram of that statement. Venn diagram is named after British Logician John Venn (1834 – 1883).







Thus a set is represented by a circle or a closed geometrical figure inside the universal set. The universal set S is represented by a rectangular region. An element of a set A is represented by a point within the circle which represents A.

In the given figures is the universal set, B is a subset of S and A is a subset of B.

4.8 Operations on Sets

In algebra of numbers, the operations of addition (+) when applied on two numbers gives a third number $a + b$. Multiplication (.) and subtraction (–) give the numbers $a \,.\, b$ and $a - b$ respectively. Likewise, we will discuss three operations; Union ($\cup$). Intersection ($\cap$) and difference (–) applicable on any two sets. The laws satisfied by these operations from the 'algebra of sets'.

- **Union of Two Sets**

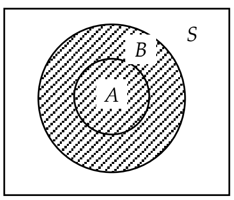
The union of two sets A and B is the set of all those elements which are either in A or in B or in both. This set is denoted by $A \cup B$ (read as 'A' union B').

Symbolically, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

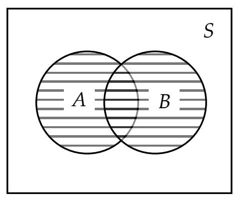
or $\quad A \cup B = \{x : x \in A \vee x \in B\}$

Clearly, $x \in A \cup B \Leftrightarrow x \notin A$ and $x \notin B$.

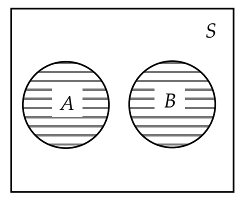
The union of two sets can be represented by a Venn diagram as shown in following figures. The shaded region represents $A \cup B$.



$A \cup B$
(when $A \subset B$)



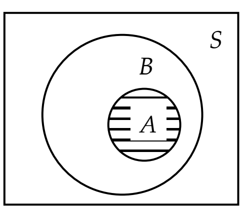
$A \cup B$ when neither
$A \subset B$ nor $B \subset A$



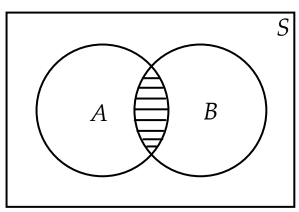
$A \cup B$ when A and B
are disjoint sets

- **Intersection of two Sets**

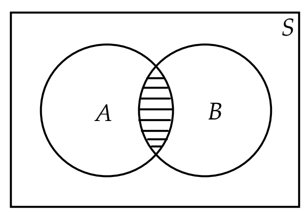
The intersection of two sets A and B is the set of all those elements which are common in A and B. This set is denoted by $A \cap B$ (read as 'A intersection B').



$A \cup B$
(when $A \subset B$)



$A \cup B$ when neither
$A \subset B$ nor $B \subset A$



$A \cup B$ when neither
$A \subset B$ nor $B \subset A$

Symbolically, $\quad A \cap B = \{x : x \in A \text{ and } x \in B\}$

or $\quad A \cap B = \{x : x \in A \wedge x \in B\}$ [$\wedge$ denotes 'and']

Clearly $\quad x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$

But $\quad x \notin A \cap B \Leftrightarrow x \notin B$.

The intersection of two sets can be presented by a Venn diagram as shown in the given figures. The shaded region represents $A \cap B$.

Disjoint Sets

Two sets A and B are said to be disjoint sets if they have no common element i.e., $A \cap B = \phi$.

The disjoint sets can be represented by Venn diagram as shown in the figure.

ILLUSTRATIONS

***Illustration* 11**

Let $A = \{x : x \text{ is a prime number less than } 10\}$

$B = \{x : x \in Z, -5 < x < 5\}$.

Solution

Then $A = (2, 3, 5, 7)$ and $B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$\therefore$ $A \cup B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 7\}$

Note:

(1) $x \notin A \cup B \Leftrightarrow x \notin A$ and $x \notin B$

(2) If $A \subseteq B$, then $A \cup B = B$.

***Illustration* 12**

Let $A = \{1, 2, 3\}$ and $B = \{2, 1, 5, 6\}$

Solution

Then $A \cap B = \{1, 2\}$.

Note: (1) If $A \subseteq B$, then $A \cap B = A$

(2) For any two sets A and B

(i) $(A \cup B) \cap A = A$ and $(A \cap B) \cap B = B$

(ii) $(A \cap B) \cup A = A$ and $(A \cap B) \cup B = B$

***Illustration* 13**

Let A = set of all rational numbers

B = set of all irrational numbers.

Solution

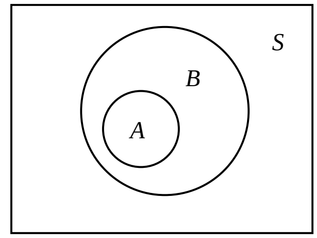
Then $A \cap B = \phi$, as no number can be both rational and irrational.

Here A and B are disjoint sets.

4.9 DIFFERENCE AND COMPLEMENTS

- **Difference of Two Sets**

The difference of two sets A and B in this order (also called 'relative complement' of B in A) is the set of all those elements of A which are not elements of B. It is denoted by $A - B$ and is read as "A minus B".

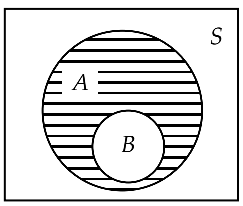


Symbolically, $A - B = \{x : x \in A \text{ and } x \notin B\}$

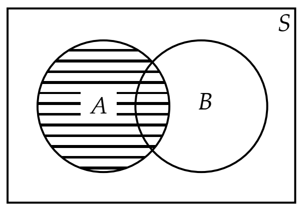
Thus $\quad x \in A - B \Leftrightarrow x \in A$ in this order is

$B - A = \{x : x \in B \text{ and } x \notin A\}$.

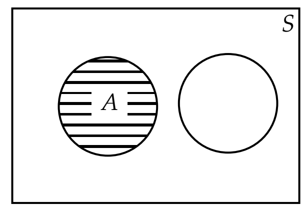
A – B can be represented by Venn diagram as shown in the given figure. The shaded region represents $A - B$.



$A - B$ when $B \subset A$



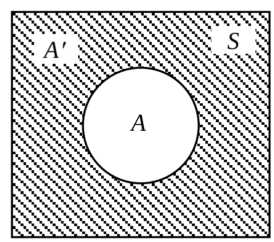
$A -\ B$ when neither $A \subset B$ nor $B \subset A$



$A \cup\ B$ when A and B are Disjoint sets. Clearly $A - B = A$

- **Complement of a Set**

The complement of a set A (also called 'absolute complement' of A) is the set of all those elements of the universal set S which are not elements of A. It is denoted by A' or A^c.



Clearly $\quad A'$ or $A^c = S - A$

Symbolically, $\quad A'$ or $A^c = \{x : x \in S \text{ and } x \notin A\}$

Thus $\quad x \in A' \Leftrightarrow x \notin A$.

Complement of a set can be represented by Venn diagram as shown in the given picture. The shaded region represents A'.

ILLUSTRATIONS

***Illustration* 14**

Let $A = \{1, 3, 5, 6, 7\}$, $B = \{2, 3, 4, 5\}$

Solution

$A - B = \{1, 6, 7\}$, $B - A = \{2, 4\}$

Note:

Clearly $A - B \neq B - A$ (as evident from above examples). Hence difference of two is not commutative.

***Illustration* 15**

Let $S = N = \{1, 2, 3, \ldots\}$

and $A = \{1, 3, 5, \ldots\}$.

Solution

Then $A' = \{2, 4, 6, 8, \ldots\} = B$ (say)

Clearly $B' = \{1, 3, 5, \ldots\} = A$

Thus $A' = B \Leftrightarrow B' = A$

Note:

Since every element is supposed to be an element of S, so the statement $x \in S$ is superfluous in the sentence $x \in A' \Leftrightarrow x \in S$ and $x \notin A$.

Hence $x \in A' \Leftrightarrow x \notin A$

and $x \in A \Leftrightarrow x \notin A'$.

***Illustration* 16**

If $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 8)$, $C = \{2, 3, 4, 5, 6, 7\}$. Then verify that

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $(A \cup B)' = A' \cap B'$

(iii) $A - (B \cup C) = (A - B) \cap (A - C)$

(iv) $A - C = A \cap C'$

Solution

(i) $B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A \cap B = \{1, 8\}$, $A \cap C = \{2, 4, 6\}$

Now $A \cap (B \cup C) = \{x : x \in A \text{ and } x \in B \cup C\} = \{1, 2, 4, 6, 8\}$.

$(A \cap B) \cup (A \cap C) = \{1, 2, 4, 6, 8\}$

$\therefore \quad A \cap (B \cup C) = (A \cap B) \cap (A \cap C)$

(ii) $A \cup B = \{x : x \in A \text{ or } x \in B\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

$(A \cup B)' = \{x : x \in S$ and $x \notin A \cup B\} = \{9\}$

$A' = \{x : x \in S$ and $x \notin A\} = \{3, 5, 7, 9\}$

$B' = \{x : x \in S$ and $x \notin B\} = \{2, 4, 6, 9\}$

$A' \cap B' = \{9\}$ $\therefore$ $(A \cup B)' = A' \cap B'$

(iii) $A - (B \cup C) = \{1, 2, 4, 6, 8\} - \{1, 2, 3, 4, 5, 6, 7, 8\} = \phi$.

$A - B = \{1, 2, 4, 6, 8\} - \{1, 3, 5, 7, 8\} = \{2, 4, 6\}$

$A - C = \{1, 2, 4, 6, 8\} - \{2, 3, 4, 5, 6, 7\} = \{1, 8\}$

$\therefore$ $(A - B) \cap (A - C) = \phi$

$\therefore$ $A - (B \cup C) = (A - B) \cap (A - C)$

(iv) $C' = S - C = \{1, 8, 9\}$

$A \cap C' = \{1, 2, 4, 6, 8\} \cap \{1, 8, 9\} = \{1, 8\}$

and $A - C = \{1, 8\}$ $\therefore$ $A - C = A \cap C'$.

***Illustration* 17**

Taking the set of natural numbers as the universal set, write down the complements of the following sets

(i) $\{x : x \in N$ and $x + 5 = 8\}$

(ii) $\{x : x \in N$ and $x = 3n$ for some $n \in N)$

(iii) $\{x : x \in N$ and x is divisible by 3 and 5$\}$

(iv) $\{x : x \in N$ and x is a perfect square$\}$

Solution

Here universal set $S = N$ = set of all natural numbers.

(i) Let $A = \{x : x \in N$ and $x + 5 = 8\} = \{x : x \in N$ and $x = 3\} = \{3\}$

$\therefore$ $A' = \{x : x \in N$ and $x \neq N\}$

(ii) Let A $\{x : x \in N$ and $x = 3n$ for some $n \in N\}$

= set of all natural numbers which are multiple of 3.

$\therefore$ A' = set of all natural numbers which are not multiple of 3.

(iii) Let $A = \{x : x \in n$ and x is divisible by 3 and 5$\{$

$= \{x : x \in N$ and x is divisible by 15$\}$

$\therefore$ $A' = \{x : x \in n$ and x is not divisible by 15$\}$

$= \{x : x \in N$ and x is not divisible by 3 and 5$\}$

(iv) Let $A = \{x : x \in N$ and x is a perfect square$\}$

$\therefore$ $A' = \{x : x \in N$ and x is not a perfect square$\}$

***Illustration* 18**

Shade the following sets in figure

(i) $A' \cap (B \cup C)$ *(ii)* $A' \cap (C - B)$

Solution

(i) In figure (i), shaded region represents $A' \cap (B \cap C)$

(ii) In figure (ii), shaded region represents $A' \cap (C - B)$

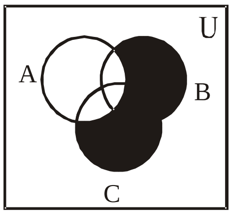


Fig. (i)

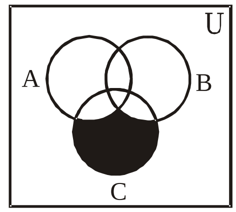


Fig.(ii)

PRACTICE EXERCISE

14. If $S = \{0, 1, 2, 3, \ldots, 9\}$, $A = \{0, 1, 2, 3, 4\}$, $B = \{1, 2, 3\}$
    $C = \{5, 6, 7\}$, $D = \{5, 7, 8, 9\}$, then find *(i)* $B - A$ *(ii)* A' *(iii)* $(C \cup D)'$ *(iv)* $(D - C)'$
    (v) $C \cup S$
    (vi) $A - (B - C)'$
15. Let $A = \{3, 6, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$ and $D = \{5, 10, 15, 20)$. Find
    (i) $A \cap B$ *(ii)* $A \cup C$ *(iii)* $A - D$ *(iv)* $C \cap A$ *(v)* $D - A$
    (vi) $B \cup C$
16. If $A = \{x : x$ is a positive integer < 8 and x is a multiple of 3 or 5$\}$,
    $B = \{x : x^3 - 6x^2 + 11x - 6 = 0\}$, $C = (x : x$ is even number $\leq 7)$, then show that
    (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
    (ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
    (iii) $A \cup (B \cap C) = (A \cap B) \cap C$

Answers

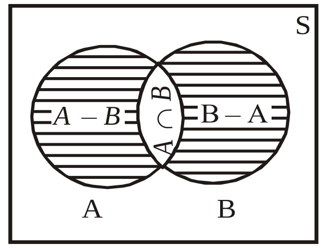
14. *(i)* ϕ *(ii)* $\{5, 6, 7, 8, 9\}$ *(iii)* $(0, 1, 2, 3, 4\}$ *(iv)* $\{0, 1, 2, 3, 4, 5, 6, 7\}$ *(v)* $\{0, 1, 2, 3, \ldots 9\}$ *(vi)* $\{0, 4\}$

15. *(i)* $\{12\}$ *(ii)* $\{2, 4, 6, 8, 10, 12, 14, 16, 3, 15, 18, 21\}$ *(iii)* $\{3, 6, 12, 18, 21\}$ *(iv)* $\{6, 12\}$ *(v)* $\{5, 10, 20\}$

(vi) $\{2, 4, 6, 8, 10, 12, 14, 16, 20\}$

4.10 Some Theorems on Number of Elements of Sets

Use the following results whichever are required



(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

(iii) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

(iv) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A \cap B = \phi$

(v) $n(A) = n(A - B) + n(A \cap B)$

(vi) $n(B) = n(B - A) + n(A \cap B)$

(vii) Number of elements belonging to exactly one of the sets A and B

$= n(A) + n(B) - 2n(A \cap B)$

$= n(A - B) + n(B - A)$

$= n(A \cup B) - n(A \cap B)$

(viii) Number of elements belonging to exactly two of A, B and C

$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$.

(ix) Number of elements belonging to exactly one of A, B and C

$= n(A) + n(B) + n(C) - 2n(A \cap B)$

$- 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(x) $n(A' \cap B') = n(S) - n(A \cup B)$

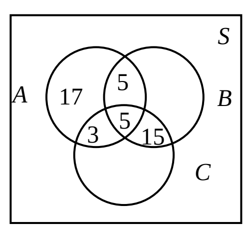
(xi) $n(A' \cup B') = n(S) - n(A \cap B)$.

ILLUSTRATIONS

***Illustration* 19**

The report of one survey of 100 students stated that the number of students studying the various languages were: Sanskrit, Hindi and Tamil–5; Hind and Sanskrit–10; Tamil and Sanskrit–8; Hindi and Tamil–20; Sanskrit30; Hindi–23; Tamil–50. The surveyor who prepared this report was fired. . Why?

Solution



Let $A = \{x : x \text{ studies Sanskrit}\}$

$B = \{x : x \text{ studies Hindi}\}$

$C = \{x : x \text{ studies Tamil}\}$

Given, $n(A) = 30$, $n(B) = 23$, $n(C) = 50$, $n(S) = 100$,

$n(A \cap B \cap C) = 5.$

$n(A \cap B) = 10$, $n(B \cap C) = 20$

$\therefore$ Number of students in $A \cap B$ but not in $C = 10 - 5 = 5$.

Similarly, $n[B - (A \cup C)] = n(B) - n[B \cap (A \cup C)]$

$= n(B) - n[(B \cap A) \cup (B \cap C)]$

$= n(B) - \{n(B \cap A) + n(B \cap C) - n[B \cap A \cap C]\}$

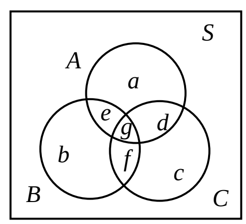
$= 23 - \{10 + 20 - 5\} = 2,$

Which is impossible because number of students can't be –ve. So, the number of students studying has been wrongly reported. That is why he was fired.

***Illustration* 20**

A class has 175 students. Following is the description showing the number of students studying one or more of the following subjects in this class. Mathematics 100, Physics 70, Chemistry 46, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics, Physics and Chemistry 18. How many students are enrolled in Mathematics alone, Physics alone and Chemistry alone? Are there students who have not offered any of these three subjects

Solution



Let A, B and C denote the sets of students studying Mathematics. Physics and Chemistry respectively.

Let us denote the number of elements (students) contained in the bounded region as shown in the diagram by a, b, c, d, e, f and g respectively.

Then, $a + d + g + e = 100$

$b + f + g + e = 70$

$c + f + g + d = 46$

$g + e = 30$

$g + d = 28$

$g + f = 23$

$g = 18.$

Solving these, we get $g = 18$, $f = 5$, $d = 10$, $e = 12$, $c = 13$, $b = 35$, $a = 60$.

$\therefore$ $a + b + c + d + e + f + g = 153.$

So, the number of students who have not offered any of these three subjects = 175 – 153 = 22.

Number of students studying Mathematics only = a = 60

Number of students studying Physics only = b = 35

Number of students studying Chemistry only = c = 13.

***Illustration* 21**

In a certain city, only two newspapers A and B are published. It is known that 25% of the city population reads A and 20% reads B, while 8% reads both A and B. It is also known that 30% of those who read A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements, 2hilw 50% of those who read both A and B, look into advertisements. What % of the population read an advertisement?

Solution

Let L = Set of people who read paper A

M = Set of people who read paper B.

Let the total number of persons in the city = 100

Then, $n(L) = 25$, $n(M) = 20$, $n(L \cap M) = 100$

$n(L - M) = n(L) - n(L \cap M) = 25 - 8 = 17$

$n(M - L) = n(M) - n(L \cap M) = 20 - 8 = 12.$

$\therefore$ % of people reading an advertisement

= (30% of 17) + (40% of 12) + (50% of 8)

$= \frac{51}{10} + \frac{24}{5} + 4 = 13.9\%$.

PRACTICE EXERCISE

17. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many can speak both Hindi and English?
18. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?
19. If 45% of the students of a class have offered Mathematics and 85% of the Biology, find the percentage of students who offered Biology only?
20. In a survey of 600 students in a school, 150 students were found to be drinking Tea and 225 drinking Coffee, 100 were drinking both Tea and Coffee. Find how many students were drinking neither Tea nor Coffee?
21. An investigator interviewed 100 students to determine their preferences for the three drinks: Milk (M), Coffee (C) and Tea (T). He reported the following: 10 students had all the three drinks M, C,T, 20 had M and C, 30 had C and T; 25 had M and T; 12 had M only; 5 had C only; 8 had T only. Using a venn diagram, find how many did not take any the three drinks?

22. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students who had taken *(i)* only chemistry *(ii)* only mathematics *(iii)* only physics *(iv)* physics and chemistry but not mathematics *(v)* mathematics and physics but not chemistry *(vi)* only one of the subjects *(vii)* at least one of the three subjects *(viii)* none of three subjects.

Answers

17. 50 **18.** 30 **19.** 55% **20.** 325 **21.** 20

22. *(i)* 5 *(ii)* 4 *(iii)* 2 *(iv)* 1 *(v)* 6 *(vi)* 11 *(vii)* 23 *(viii)* 2

4.11 Number of Elements in the Cartesian Product

If A and B are two finite sets, then $n(A \times B) = n(A) . n(B)$.

That is if A has m elements and B has n elements, then $A \times B$ has m, n elements.

Let $A = \{x_1, x_2, x_3, \ldots, x_m)$

and $B = \{y_1, y_2, y_3, \ldots., y_n)$

Then, $A \times B = \{(x_1, y_1), (x_1, y_2), (x_1, y_3), \ldots, (x_1, y_n); (x_2, y_1), (x_2, y_2), (x_2, y_3), \ldots, (x_m, y_n)\}$.

Clearly each row has n ordered pairs and there are m such rows. So $A \times B$ has $m . n$ elements.

Hence, $n(A \times B) = n(A). n(B)$

Similarly, $n(A \times B \times C) = n(A) . n(B) . n(C)$.

Note:

The elements of $A \times B$ are also called ordered pairs or 2-tuples. The elements of $A \times B \times C$ are called ordered triplets or 3-tuples.

ILLUSTRATIONS

***Illustration* 22**

Let A and B be two sets such that $n(A) = 5$ and $n(B) = 2$. If a, b, c, d, e are distinct and $(a, 2)$, $(b, 3)$, $(c, 2)$, $(d, 3)$, $(e, 2)$ are in $A \times B$. Find A and B.

Solution

Since a, b, c, d, e are distinct and$(a, 2)$, $(b, 3)$, $(c, 2)$, $(d, 3)$, $(e, 2)$ are elements of $A \times B$, therefore

$a, b, c, d, e \in A$.

But $n(A) = 5 \therefore A = \{a, b, c, d, e$

Again $n(B) = 2$ and $(a, 2), (b, 3) \in A \times B \quad \therefore \quad B = \{2, 3\}$.

***Illustration* 23**

The Cartesian product $A \times A$ has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of $A \times A$.

Solution

Let $n(A) = p$

Given, $n(A \times A) = 9$

$\Rightarrow$ $n(A) . n(A) = 9$

$\Rightarrow$ $p . p = 9 \Rightarrow p = 3 \ \therefore \ n(A) = 3$

Now, $(-1, 0) \in A \times A \Rightarrow -1 \in A$ and $0 \in A$

Again, $(0, 1) \in A \times A \Rightarrow 0 \in A$ and $1 \in A$

Thus $-1 \in A, 0 \in A$ and $1 \in A$

$\therefore$ $-1, 0, 1 \in A$.

But A has exactly three elements, therefore $A = \{-1, 0, 1\}$

Remaining elements of $A \times A$ are

(–1, –1), (–1,1), (0, –1), (0,0), (1, –1), (1, 0), (1,1).

PRACTICE EXERCISE

23. If $A = \{1, 2, 3\}$, $B = (3, 4\}$, $C = \{4, 5, 6\}$, then find

(i) $(A \times B) \cup (B \times C)$ *(ii)* $(A \times B) \cap (B \times C)$ *(iii)* $(A \times B) \cup (A \times C)$

(iv) $(A \times B) \cap (A \times C)$ *(v)* $A \times (B \cap C)$ *(vi)* $A \times (B \cup C)$

24. Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$.Verify that

(i) $A \times C \subseteq B \times D$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

25. If the ordered pairs $(x, -1)$ and $(5, y)$ belong to the set $\{(a, b): b = 2a - 3\}$, find the values of x and y.

26. *(i)* If $(x + 2, 4) = (5, 2x + y)$, find x and y

(ii) If $(x - 2, 2y + 1) = (y - 1, x + 2)$, find x and y

(iii) If $(x + 1, y - 2) = (3, 1)$, find x and y.

(iv) If $\left(\frac{a}{3}, b+5\right) = (-1, -2)$, find a and b.

27. If $A = \{a, b, c, d\}$ and B is equivalent to A, then find the number of elements in *(i)* $A \times B$ *(ii)* $B \times B$

28. Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B, where x, y, z are distinct elements.

29. If $A = \{a, b, c\}$ and some elements, of $A \times B$ are (a, p), (b, q), (c, p). Write down the remaining elements of $A \times B$ if $n(A \times B) = 6$.

30. If $B = \{2, 3, 5\}$ and $(a, 2)$, $(b, 3)$, $(c, 5)$ are in $A \times B$, find A and the remaining elements of $A \times B$ such that $n(A \times B)$ is least.

Answers

23. *(i)* {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6) *(ii)* {(3, 4)

(iii) {(1, 3), (1, 4), (1, 5),(1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}

(iv) {(1, 4), (1, 5), (1, 6), (2, 4),(2, 5), (2, 6),(3,), (3, 5), (3, 6)}

(v) {(1, 4), (2, 4), (3, 4)}

(vi) {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}

25. $x = 1, y = 7$ **26.** *(i)* $x = 3, y = –2$ *(ii)* $x = 3, y = 2$ *(iii)* $x = 2, y = 3$

(iv) $a = –3, b = –7$

27. *(i)* 16 *(ii)* 16 **28.** $A = \{x, y, z\}$, $B = \{1, 2\}$ **29.** $\{(a, q), (b, p), (c, q)\}$

30. $A = \{a, b, c\}$ $\{(a, 3), (a, 5), (b, 2), (b, 5), (c, 2), (c, 3)\}$

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RELATIONS

Introduction

Let A and B be two sets. Then a relation R from A to b is a subset of $A \times B$

Thus, R is a relation from A to $B \Leftrightarrow R \subseteq A \times B$.

- **Total Number of Relations**

  Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So, total number of subsets of $A \times B$ is 2^{mn}. Since each subset of $A \times B$ defines a relation from A to B, so total number of relations from A to B is 2^{mn}.

ILLUSTRATIONS

***Illustration* 24**

If $A = \{a, b, c, d\}$. $B = \{p, q, r, s\}$, then which of the following are relations from A to B? Give reasons for your answer

(a) $R_2 = \{(q, b), (c, s), (d, r)\}$

(b) $R_3 = \{(a, p), (a, q), (d, p), (c, r), (b, r)\}$

(c) $R_4 = \{(a, p), (q, a), (b, s), (s, b)\}$

Solution

(a) Since $(q, b) \in$ iRi$_2$ but $(q, b) \notin A \times B$. So, $R_2 \not\subset A \times B$.

Thus, R_2 is not a relation from A to B.

(b) Clearly, $R_3 \subseteq A \times B$. So R_3 is a relation from A to B.

(c) R_4 is not a relation from A to B, because (q, a) and (s, b) are elements of *Ri*$_4$ *but (*q, a*) and (*s, b*)* are not in $A \times B$. As such $R_4 \not\subset A \times B$

- **Domain and Range of a Relation**

  Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R.

  Thus, Dom $(R) = \{a : (a, b) \in R\}$

  and Range $(R) = \{b : (a, b) \in R\}$

ILLUSTRATIONS

***Illustration* 25**

If R is a relation from set $A = \{2, 4, 5\}$ to set $B = \{1, 2, 3, 4, 6, 8\}$ defined by $xRy \Leftrightarrow x$ divides y

(i) Write R as a set of ordered pairs

(ii) Find the domain and the range of R.

Solution

(i) Clearly, $2R2$, $2R4$, $2R8$, $4R4$ and $4R8$

$\therefore \quad R = \{(2, 2), (2, 4), (2, 6), (2, 8), (4, 4), (4, 8)\}$

(ii) Clearly, Dom(R) = {2, 4} and Range 9R) = {2, 4, 6, 8}

- **Relation on a Set**

  Let A be a non-void set. Then, a relation from A to itself i.e. a subset of $A \times A$, is called a relation on set A.

- **Inverse Relation**

  Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}$$

  Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$.

ILLUSTRATIONS

***Illustration* 26**

Let A be the set of first ten natural numbers and let R be a relation on a defined by $(x, y) \in R \Leftrightarrow x + 2y = 10$, i.e. $R = \{(x, y) : x \in A, y \in A \text{ and } x + 2y = 10\}$. Express R and R^{-1} as sets of ordered pairs. Determine also *(i)* domains of R and R^{-1} *(ii)* ranges of R and R^{-1}.

Solution

We have,

$$(x, y) \in R \Leftrightarrow x + 2y = 10 \Leftrightarrow y = \frac{10 - x}{2}, \ x, y \in A$$

Where $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Now, $\quad x = 1 \Rightarrow y = \dfrac{10-1}{2} = \dfrac{9}{2} \notin A$.

This shows that 1 is not related to any element in A. Similarly we can observe that 3, 5, 7, 9 and 10 are not related to any element of A under the defined relation.

Further we find that

For $x = 2$, $y = \frac{10-2}{2} = 4 \in A$ $\therefore$ $(2, 4) \in R$

For $x = 4$, $y = \frac{10-4}{2} = 3 \in A$ $\therefore$ $(4, 3) \in R$

For $x = 6$, $y = \frac{10-6}{2} = 2 \in A$ $\therefore$ $(6, 2) \in R$

For $x = 8$, $y = \frac{10-8}{2} = 1 \in A$ $\therefore$ $(8, 1) \in R$

Thus, $R = \{(2, 4), (4, 3), (6, 2), (8, 1)\}$

$\Rightarrow$ $R^{-1} = \{(4, 2), (3, 4), (2, 6), (1, 8)\}$

Clearly, Dom$(R) = \{2, 4, 6, 8\}$ = Range (R^{-1})

and, Range $(R) = \{4, 3, 2, 1\}$ = Dom (R^{-1}).

***Illustration* 27**

A relation R is defined on the set Z of integers as follows

$(x, y) \in R \Leftrightarrow x^2 + y^2 = 25$

Express R and R^{-1} as the sets of ordered pairs and hence find their respective domains.

Solution

We have,

$(x, y) \in R \Leftrightarrow x^2 + y^2 = 25 \Leftrightarrow y = \pm\sqrt{25 - x^2}$

We observe that

$x = 0 \Rightarrow y = \pm 5$

$\therefore$ $(0, 5) \in R$ and $(0, -5) \in R$

$x = \pm 3 \Rightarrow y = \sqrt{25-9} = \pm 4$

$\therefore$ $(3, 4) \in R$, $(-3, 4) \in R$, $(3, -4) \in R$ and $(-3, -4) \in R$

$x = \pm 4 \Rightarrow y = \sqrt{25-16} = \pm 3$

$\therefore$ $(4, 3) \in R$, $(-4, 3) \in R$, $(4, -3) \in R$ and $(-4, -3) \in R$

$x = \pm 5 \Rightarrow y = \sqrt{25-25} = 0$

$\therefore$ $(5, 0) \in R$ and $(-5, 0) \in R$

We also notice that for any other integral value of x, y is not an integer

$\therefore$ $R = \{(0, 5). (0, -5), (3, 4), (-3, 4), (3, -4), (-3, -4), (4, 3), (-4, 3), (4, -3), (-4, -3), (5, 0), (-5, 0)\}$

$R^{-1} = \{(5, 0), (-5, 0), (4, 3), (4, -3), (-4, 3), (-4, -3), (3, 4), (3, -4), (-3, 4), (-3, -4), (0, 5), (0, -5)\}$

Clearly, domain (R) = {0, 3, –3, 4, –4, 5, –5} = domain (R^{-1}).

4.12 Types of Relations

In this section we intend to define various types of relations on a given set A.

- **Void Relation**

  Let A be a set. Then $\phi \subseteq A \times A$ and so it is a relation on A. This relation is called the void or empty relation on A.

- **Universal Relation**

  Let A be a set. Then $A \times A \subseteq A \times A$ and so it is a relation on A. This relation is called the universal relation on A.

Note:

It is to note here that the void and the universal relations on a set A are respectively the smallest and the largest relations on A.

- **Identity Relation**

  Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A.

  In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only.

  Example:

  The relation $I_A = \{(1, 1), (2, 2), (3, 3)\}$is the identity relation on set $A = \{1, 2, 3\}$. But relations $R_1 = \{(1, 1), (2, 2)\}$ and $R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$ are not identity relations on A, because $(3, 3) \notin R_1$ and in R_2 element 1 is related to elements 1 and 3.

- **Reflexive Relation**

  A relation R on a set A is said to be reflexive if every element of A is related to itself.

  Thus, R is reflexive $\Leftrightarrow$ $(a, a) \in R$ for all $a \in A$.

  A relation Ron a set A is not reflexive if there exists an element $a \in A$ such that $(a, a) \notin R$.

  Example:

  Let $A = \{1, 2, 3\}$ be a set. Then $R = \{(1, 1), (2, 2), (3, 3), (1, 3), (2, 1)$ is a reflexive relation on A. But $R_1 = \{(1, 1), (3, 3), (2, 1), (3, 2)\}$ is not a reflexive relation on A, because $2 \in A$ but $(2, 2) \notin R_1$.

Note:

The identity relation on a non-void set A is always reflexive relation on A. However, a reflexive relation on A is not necessarily the identity relation on A. The relation $R = \{(a,a), (b, b), (c, c), (a,b)\}$ is a reflexive relation on set $A = \{a, b, c\}$ but it is not the identity relation on A.

- **Symmetric Relation**

  A relation R on a set A is said to be a symmetric relation iff

  $(a, b) \in R \;\Rightarrow\; (b, a) \in R$ for all $a, b \in A$

  i.e. $aRb \Rightarrow bRa$ for all $a, b \in A$.

Note:

The identity and the Universal relations on a non-void set are symmetric relations.

Example:

Let $A = \{1, 2, 3, 4\}$ and let R_1 and R_2 be relations on A given by $R_1 = \{(1, 3), (1, 4), (3, 1), (2, 2) (4, 1)$ and $R_2 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Clearly, R_1 is a symmetric relation on A. However, R_2 is not so, because $(1, 3) \in R_2$ but $(3, 1) \notin R_2$.

Note:

A reflexive relation on a set A is not necessarily symmetric. For example, the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$ is a reflexive relation on set $A = \{1, 2, 3\}$ but it is not symmetric.

- **Transitive Relation**

  Let A be any set. A relation R on A is said to be a transitive relation iff

  $(a, b) \in R$ and $(b, c) \in R \;\Rightarrow\; (a, c) \in R$ for all $a, b, c \in A$.

  i.e. aRb and $bRc \;\Rightarrow\; aRc$ for all $a, b\ c \in A$

Note:

The identity and the universal relations on a non-void set are transitive.

Example:

On the set N of natural numbers, the relation R defined by $xRy \;\Rightarrow\; x$ is less than y is transitive, because for any $x, y, z \in N$

$x < y$ and $y < z \;\Rightarrow\; x < z \;\Rightarrow\; xRy$ and $yRz \Rightarrow xRz$

- **Equivalence Relation**

  A relation R on a set A is said to be an equivalence relation on A iff

  (i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$

  (ii) it is symmetric i.e. $(a, b) \in R \;\Rightarrow\; (b, a) \in R$ for all $a, b \in A$

  (iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \;\Rightarrow\; (a, c) \in R$ for all $a, b, c \in A$.

ILLUSTRATIONS

***Illustration* 28**

Three relations R_1, R_2 and R_3 are defined on set $A = \{a, b, c\}$ as follows

(i) $R_1 = \{a, a), (a, b), (a, c), (b, b), (b, c), (c, a) (c, b), (c, c)\}$

(ii) $R_2 = \{(a, b), (b, a), (a, c), (c, a)\}$

(iii) $R_3 = \{(a, b), (b, c), (c, a)\}$

(iv) Find whether each of R_1, R_2 and R_3 is reflexive, symmetric and transitive.

Solution

(i) Reflexive: Clearly, $(a, a), (b, b), (c, c), \in R_1$.

So, R_1 is reflexive on A.

Symmetric: We observe that $(a, b) \in R_1$ but $(b, a) \notin R_1$.

So, R_1 is not symmetric on A.

Transitive: We find that $(b, c) \in R_1$ and $(c, a) \in R_1$ but $(b, a) \notin R_1$. So, R is not transitive on A.

(ii) Reflexive: Since (a, a), (b, b) and (c, c) are not in R_2. So, it is not a reflexive relation on A.

Symmetric: We find that the ordered pairs obtained by interchanging the components of ordered pairs in R_2 are also in R_2. So, R_2 is a symmetric relation on A.

Transitive: Clearly $(a, b) \in R_2$ and $(b, a) \in R_2$ but $(a, a) \notin R_2$. So, it is not a transitive relation on R_2.

(iii) Reflexive: Since non of (a, a), (b, b) and (c, c) is an element of R_3. So, R_3 is not reflexive on A.

Symmetric: Clearly, $(b, c) \in R_3$ but $(c, b) \notin R_3$. So, R_3 is not symmetric on A.

Transitive: Clearly, $(a, b) \in R_3$ and $(b, c) \in R_3$ but

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MISCELLANEOUS PROBLEMS

OBJECTIVE TYPE

Example 1

The set $A = [x : x \in R, x^2 = 16 \text{ and } 2x = 6]$ equals

(a) ϕ (b) [14, 3, 4] (c) [3] (d) [4]

Solution

$x^2 = 16 \Rightarrow x = \pm 4$

$2x = 6 \Rightarrow x = 3$

There is no value of x which satisfies both the above equations.

Thus, $A = \phi$

$\therefore$ **Ans. (a)**

Example 2

Let $A = [x : x \in R, |x| < 1]$; $B = \{x : x \in R, |x - 1| \geq 1$ and $A \cup B = R - D$, then the set D is

(a) $[x : 1 < x \leq 2]$ (b) $[x : 1 \leq x < 2]$

(c) $[x : 1 \leq x \leq 2]$ (d) None of these

Solution

$A = [x : x \in R, -1 < x < 1]$

$B = [x : x \in R : x - 1 \leq -1 \text{ or } x - 1 \geq 1]$

$= [x : x \in R : x \leq 0 \text{ or } x \geq 2]$

$\therefore \quad A \cup B = R - D$

Where $D = [x : x \in R, 1 \leq x < 2]$

$\therefore$ **Ans. (b)**

Example 3

If $A = \{1, 2, 3\}$ and $B = \{1, 3, 5, 7\}$, then $A \cup B =$

(a) {1, 3, 5, 7} (b) {1, 2, 3, 5, 7} (c) {1, 2, 3, 7} (d) None of these

Solution

$A \cup B = \{1, 2, 3, 5, 7\}$

$\therefore$ **Ans. (b)**

Example 4

If $A = \{1, 2, 3, 4\}$, $B = \{4, 5, 6, 7\}$, then $A \cap B =$

(a) $\{1, 2, 3, 4\}$ (b) $\{6, 7\}$ (c) $\{1, 2\}$ (d) $\{ 4\}$

Solution

$A \cap B = \{4\}$

$\therefore$ **Ans. (d)**

Example 5

If $A = \{1, 2, 3, 4\}$, $B = \{4, 5, 6, 7\}$, then $A - B =$

(a) $\{1, 2, 3, 4\}$ (b) $\{5, 6, 7\}$ (c) $\{4\}$ (d) $\{1, 2, 3, 4\}$

Solution

$A - B = \{1, 2, 3\}$

$\therefore$ **Ans. (a)**

Example 6

Let $A = \{1, 2, 3, 4\}$, then total number of subsets of $A =$

(a) 4 (b) 8 (c) 12 (d) 16

Solution

Total number of subsets = $2^4 = 16$

$\therefore$ **Ans. (d)**

Example 7

If $aN = \{ax : x \in N\}$, then the set $6N \cap 8N$ is equal to

(a) $8N$ (b) $48\ N$ (c) $12\ N$ (d) $24\ N$

Solution

$6N = \{6, 12, 18, 24, 30, \ldots\}$

$8N = \{8, 16, 24, 32, \ldots\}$

$\therefore$ $6N \cap 8N = \{24, 48, \ldots\} = 24N$

$\therefore$ **Ans. (d)**

Example 8

If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 3, 6, 7\}$, then the number of elements in the set $(A \times B) \cap (B \times A)$ is equal to

(a) 4 (b) 5 (c) 10 (d) 20

Solution

$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

$= (2, 3) \times (2, 3)$

$= \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

$\therefore$ number of elements = 4.

$\therefore$ **Ans. (a)**

Example 9

In a class of 55 students, the number of students studying different subjects are 23 in mathematics, 24 in physics, 19 in chemistry, 12 in mathematics and physics, 9 is mathematics and chemistry, 7 in physics and chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is

(a) 6 (b) 9 (c) 7 (d) all of these

Solution

$n(M) = 23, n(P) = 24, n(C) = 19$

$n(M \cap P) = 12, n(M \cap C) = 9, n(P \cap C) = 7$

$n(M \cap P \cap C) = 4.$

We have to find

$n(M \cap P' \cap C'), n(P \cap M' \cap C'), n(C \cap M' \cap P')$

Now $n(M \cap P' \cap C') = n[M \cap (P \cap C'),]$

$= n(M) - n(M \cap (P \cup C))$

$= n(M) - n[(M \cap P) \cup (M \cap C)]$

$= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C)$

$= 23 - 12 - 9 + 4 = 27 - 21 = 6$

$n(P \cap M' \cap C') = n[P \cap (M \cup C)']$

$= n(P) - n[P \cap (M \cup C)]$

$= n(P) - n[(P \cap M) \cup (P \cap C)]$

$= n(P) - n(P \cap M) - n(P \cap C) + n(P \cap M \cap C)$

$= 24 - 12 - 7 + 4 = 9$

$n(C \cap M' \cap P') = n(\mathrm{C}) - n(C \cap P)$

$- n(C \cap M) + n(C \cap P \cap M)$

$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

$\therefore$ **Ans. (d)**

Example 10

If P, Q and R are subsets of a set A, then $R \times (P^c \cup Q^c)^c =$

(a) $(R \times P) \cap (R \times Q)$ (b) $(R \times Q) \cap (R \times P)$ (c) $(R \times P) \cup (R \times Q)$ (d)None of these

Solution

$R \times (P^c \cup Q^c)^c = R \times [(P^c)^c \cap (Q^c)^c]$

$= R \times (P \cap Q) = (R \times P) \cap (R \times Q)$

$\therefore$ **Ans. (a)**

Example 11

With reference to a universal set, the inclusion of a subset in another, is relation, which is

(a) symmetric only (b) equivalence relation (c)reflective only (d) None of these

Solution

Since $A \subseteq A$ $\therefore$ relation '$\subseteq$' is reflexive.

Since $A \subseteq B, B \subseteq C \Rightarrow A \subseteq C$

$\therefore$ relation '$\subseteq$' is transitive

But $A \subseteq B \Rightarrow B \subseteq C$

$\therefore$ relation is not symmetric

$\therefore$ **Ans. (d)**

Example 12

Two finite sets have m and n elements. Then total number of subsets of the first set is 56 more than that of the total number of subsets of the second. The values of m and n are

(a) 7, 6 (b) 6, 3 (c) 5, 1 (d) 8, 7

Solution

Since the two finite sets have m and n elements.

$\therefore$ no. of subsets of these sets will be 2^m and 2^n respectively.

$\therefore$ $2^m - 2^n = 56$

Putting $m = 6$, $n = 3$, we get

$2^6 - 2^3 = 64 - 8 = 56$

Hence $m = 6$, $n = 3$

$\therefore$ **Ans. (b)**

Example 13

Let R be the relation on the set R of all real numbers defined by $a\ R\ b$ iff $|a - b| \leq 1$.

Then R is

(a) reflexive and symmetric (b)symmetric only

(c) transitive only (d) anti-symmetric only

Solution

$|a - a| = 0 < 1 \quad \therefore \quad a\,R\,a \;\forall\; a \in R$

$\therefore$ R is symmetric

Again $a\,R\,b \Rightarrow |a - b| \le 1 \Rightarrow |b - a| \le 1$

$\Rightarrow$ $b\,R\,a$

$\therefore$ R is symmetric.

Again $1R\frac{1}{2}$ and $\frac{1}{2}$ $R1$ but $\frac{1}{2} \neq 1$

$\therefore$ R is not anti-symmetric.

Further, $1R2$ and $2R3$ but $1\ R\ 3$

[Q $|1 - 3| = 2 > 1$]

$\therefore$ R is not transitive

$\therefore$ **Ans. (a)**

Example 14

The relation "less than" in the set of natural numbers is

(a) only symmetric (b) only transistive (c) only reflexive (d) equivalence relation

Solution

Since $x < y, y < z \Rightarrow x < z \;\forall\; x, y, z \in N$

$\therefore$ $x\,R\,y, y\,R\,z \Rightarrow x\,R\,z$

$\therefore$ relation is transistive.

Q $x < y$ does not give $y < x$

$\therefore$ relation is not symmetric

and Q $x < x$ does not hold, $\therefore$ relation is not reflexive

$\therefore$ **Ans. (b)**

Example 15

Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) $R(c, d)$ if $ad\,(b + c) = bc\,(a + d)$, then R is

(a) symmetric only (b) reflexive only

(c) transitive only (d) an equivalence relation

Solution

For $(a, b), (c, d) \in N \times N$

$(a, b)\ R\ (c, d) \Rightarrow ad\ (b + c) = bc\ (a + d)$

Reflexive:

Since $ab\ (b + a)$

$= ba\ (a + b)\ \forall\ ab \in N$

$\therefore\ (a, b)\ R\ (a, b)\ \therefore\ R$ is reflexive.

Symmetric:

For $(a, b), (c, d) \in N \times N$, let $(a, b)\ R\ (c, d)$

$\therefore\ ad\ (b + c) = bc\ (a + d)$

$\Rightarrow\ bc\ (a + d) = ad\ (b + c)$

$\Rightarrow\ cb\ (d + a) = da\ (c + b)$

$\Rightarrow\ (c, d)\ R\ (a, b)\ \therefore\ R$ is symmetric

Transitive:

For

$(a, b), (c, d), (e, f) \in N \times N$, let

$(a, b)\ R\ (c, d), (c, d)\ R\ (e, f)$

$\therefore\ ad\ (b + c) = bc\ (a + d)$

$cf\ (d + e) = de\ (c + f)$

$\Rightarrow\ adb + adc = bca + bcd$ …(i)

and $cfd + cfe = dec + def$ …(ii)

(i) $\times\ ef$ + (ii) $\times\ ab$ gives

$adbef + adcef + cfdab + cfeab$

$=\ bcaef + bcdef + decab + defab$

$\Rightarrow\ adcf\ (b + e) = bcde\ (a + f)$

$\Rightarrow\ af\ (b + e) = be\ (a + f)$

$\Rightarrow\ (a, b)\ R\ (e, f)$

$\therefore\ R$ is transitive

Hence R is an equivalence relation.

$\therefore$ **Ans. (d)**

SUBJECTIVE TYPE

Example 1

Let A and B be two sets such that $A \times b$ consists of 6 elements. If three elements of $A \times B$ are: (1, 4), (2, 6), (3, 6). Find $A \times B$ and $B \times A$.

Solution

Since (1, 4), (2, 6) and (3, 6) are elements of $A \times B$, it follows that 1, 2, 3 are elements of A and 4, 6 are elements of B. It is given that $A \times B$ has 6 elements. So, $A = \{1, 2, 3\}$ and $B = \{4, 6\}$.

Hence,

$$A \times B = \{1, 3, 3\} \times \{4, 6\}$$
$$= \{(1, 4), (1, 6), (2, 4), (2, 6), (3, 4), (3, 6)\}$$

and $B \times A = \{4, 6\} \times \{1, 2, 3\}$

$$= \{(4, 1), (4, 2), (4, 3), (6, 1), (6, 2), (6, 3)\}$$

Example 2

If $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$, which of the following are relations from A to B? Give reason in support of your answer

(i) $R_1 = \{(1, 4), (1, 5), (1, 6)\}$

(ii) $R_2 = \{(1, 5), (2, 4), (3, 6)\}$

(iii) $R_3 = \{(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)\}$

(iv) $R_4 = \{(4, 2), (2, 6), (5, 1), (2, 4)\}$

Solution

(i) Clearly $R_1 \subseteq A \times B$. So, it is relation from A to B.

(ii) Clearly $R_2 \subseteq A \times B$. So, it is a relation from A to B.

(iii) Clearly $R_3 \subseteq A \times B$. So, it is a relation from A to B.

(iv) Since $(4, 2) \in R_4$ but $(4, 2) \notin A \times B$. So, R_4 is not a relation from A to B.

Example 3

A relation R is defined from a set $A = \{2, 3, 4, 5\}$ to a set $B = \{3, 6, 7, 10\}$ as follows $(x, y) \in R \Leftrightarrow x$ divides y. Express R as a set of ordered pairs and determine the domain and range of R. Also, find R^{-1}.

Solution

Recall that $a|b$ stands for a divides b. For the elements of the given sets A and B, we find that 2|6, 2|10, 3|3, 3|6, and 5|10.

$\therefore$ $(2, 6) \in R$, $(2, 10) \in R$, $(3, 3) \in R$,$(3, 6) \in R$ and $(5, 10) \in R$.

Thus, R = {(2, 6), (2, 10), (3, 3), (3, 6), (5, 10).

Clearly, Dom (R) = {2, 3, 5} and Range (R) = {3, 6, 10}

Also, R^{-1} = {(6, 2), (10, 2), (3, 3), (6, 3), (10, 5)}

Example 4

If R is the relation "less than" from A = {1, 2, 3, 4, 5} to B = {1, 4, 5}, write down the set of ordered pairs corresponding to R. Find the inverse of R.

Solution

It is given that $(x, y) \in R$

$\Leftrightarrow$ $x < y$, where $x \in A$ and $y \in B$.

For the elements of the given sets A and B, we find that

$1 < 4,\ 1 < 5,\ 2 < 4,\ 2 < 5,\ 3 < 4,\ 3 < 5$ and $4 < 5$

$\therefore$ $(1, 4) \in R$, $(1, 5) \in R$, $(2, 4) \in R$ $(2, 5) \in R$, $(3, 4) \in R$, $(3, 5) \in R$ and $(4, 5) \in R$.

Thus, R = {(1, 4), (1, 5) (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)}.

This shows that

R^{-1} = {(4, 1), (5, 1), (4, 2), (5, 2), (4, 3), (5, 3), (5, 4)}

= $\{(x, y): x \in B, y \in A \text{ and } x > y\}$

Example 5

Let R be a relation on the set of all lines in a plane defined by $(l_1, l_2) \in R \Leftrightarrow$ line l_1 is parallel to line l_2. Show that R is an equivalence relation.

Solution

Let L be the given set of all lines in a plane. Then, we observe the following properties.

Reflexive: For each line $l \in L$, we have

$L \parallel l \Rightarrow (l, l) \in R$ for all $l \in L \Rightarrow R$ is reflexive

Symmetric: Let $l_1, l_2 \in L$ such that $(l_1, l_2) \in R$. Then

$(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R$.

So, R is symmetric on L.

Transitive: Let $l_1, l_2, l_3 \in L$ such that $(l_1, l_2) \in R$ and $(l_2, l_3) \in R$. Then,

$(l_1, l_2) \in R$ and $(l_2, l_3) \in R \Rightarrow l_1 \parallel l_2$ and $l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3$

$\Rightarrow (l_1, l_3) \in R$

Example 6

Check the following relations R and S for reflexivity, symmetry and transitivity

(i) $a\ R\ b$ iff b is divisible by a, $a, b \in N$

(ii) $l_1\ S\ l_2$ iff $l_1 \perp l_2$ are straight lines in a plane.

Solution

(i) We have, $aRb \Rightarrow a \mid b$ for all $a, b \in N$.

Reflexivity: For any $a \in N$, we have $a \mid a \Rightarrow aRa$.

Thus, aRa for all $a \in N$. So, R is reflexive on N.

Symmetry: R is not symmetric because if a,b, then b may not divide a. For example, 2|6 but 6 + 2

Transitivity: Let $a, b, c \in N$ such that aRb and bRc. Then aRb and $bRc \Rightarrow a|b$ and $b|c \Rightarrow a|c \Rightarrow aRc$. So, R is a transitive relation on N.

(ii) Let L be the set of all lines in a plane. We are given that $l_1\ S\ l_2 \Leftrightarrow l_1 \perp l_2$ for all $l_1, l_2 \in L$.

Reflexivity: S is not reflexive because a line cannot be perpendicular to itself i.e. $l \perp l$ is not true.

Symmetry: Let $l_1, l_2 \in L$ such that $l_1\ S\ l_2$. Then $l_1\ S\ l_2 \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_2 \Rightarrow l_2\ S\ l_1$. So, S is symmetric on L.

Transitive: S is not transitive, because $l_1 \perp l_2$ and $l_2 \perp l_1$ does not imply that $l_1 \perp l_3$.

Example 7

Prove that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow x - y$ is divisible by n is an equivalence relation on Z.

Solution

We observe the following properties

Reflexivity: For any $a \in N$, we have

$a - a = 0 = 0 \times n \Rightarrow a - a$ is divisible by $n \Rightarrow (a, a) \in R$

Thus, $(a, a) \in R$ for all $a \in Z$.

So, R is reflexive on$Z$

Symmetry: Let $(a, b) \in R$. Then,

$(a, b) \in R \Rightarrow (a - b)$ is divisible by n

$\Rightarrow (a - b) = np$ for some $p \in Z$

$\Rightarrow b - a = n(-p)$

$\Rightarrow b - a$ is divisible by n $\qquad [\text{Q}\ \ p \in Z \Rightarrow -p \in Z]$

$\Rightarrow (b, a) \in R$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in$ iZ.

So, R is symmetric on Z.

Transitivity: Let $a, b, c \in Z$ such that $(a, b) \in R$ and $(b, c) \in R$. Then,

$(a, b) \in R \Rightarrow (a - b)$ is divisible by n

$\Rightarrow a - b = np$ for some $p \in Z$

$(b, c) \in R \Rightarrow (b - c)$ is divisible by n

$\Rightarrow b - c = nq$ for some $q \in Z$

$\therefore$ $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $a - b = np$ and $b - c - nq$

$\Rightarrow$ $(a - b) + (b - c) = nn + na$

$\Rightarrow$ $a - c = n\,(p + q)$

$\Rightarrow$ $a - c$ is divisible by n [Q $p, q, \in Z \Rightarrow p + q \in Z$]

$\Rightarrow$ $(a, c) \in R$

Thus, $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R$ for all $a, b, c \in Z$. So, R is transitive relation on Z.

Thus, R being reflexive, symmetric and transitive, is an equivalence relation on Z.

Example 8

Prove that the relation R on the set $N \times N$ defined by $(a, b)\, R\, (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation.

Solution

We observe the following properties

Reflexivity: Let (a, b) be an arbitrary element of $N \times N$.

Then,

$(a, b) \in N \times N$

$\Rightarrow$ $a, b \in N$

$\Rightarrow$ $a + b = b + a$ [by commutativity of add. on N]

$\Rightarrow$ $(a, b)\, R(a, b)$

Thus, $(a, b)\, R\, (a, b)$ for all $(a, b) \in N \times N$.

So, R is reflexive on $N \times N$.

Symmetry: Let $(a, b), (c, d) \in N \times N$ be such that $(a, b)\, R\, (c, d)$. Then,

$(a, b)\, R\, (c, d) \Rightarrow a + d = b + c \Rightarrow c + b = d + a$

$\Rightarrow$ $(c, d)\, R\, (a, b)$ [by def. of R]

Thus,

$(a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b)$ for all $(a, b), (c, d) \in N \times N$.

So, R is symmetric on $N \times N$.

Transitivity: Let $(a, b), (c, d), (e, f) \in N \times N$ such that

$(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$. Then,

$$\left.\begin{array}{r}(a,b)R(c,d) \Rightarrow a+d=b+c \\ \text{and} \quad (c,d)R(e,f) \Rightarrow c+f=d+e\end{array}\right\}$$

$\Rightarrow \quad (a + d) + (c + f) = (b + c) + (d + e)$

$\Rightarrow \quad a + f = b + e \;\Rightarrow\; (a, b)\ R\ (e, f)$

Thus, $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f) \;\Rightarrow\; (a, b)\ R\ (e, f)$ for all $(a, b), (c, d), (e, f) \in N \times N$.

So, R is transitive on $N \times N$.

Hence, R being reflexive, symmetric and transitive, is an equivalence relation on $N \times N$.

Example 9

Let N be the set of all natural numbers and let R be a relation on $N \times N$, defined by $(a, b)\ R\ (c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$. Show that R is an equivalence relation on $N \times N$.

Solution

We observe the following properties

Reflexivity: Let (a, b) be an arbitrary element of $N \times N$. Then,

$(a, b) \in N \times N \;\Rightarrow\; ab = ba$ [By commutativity of multiplication on N]

$\Rightarrow\; (a, b)\ R\ (a, b)$

Thus, $(a, b)\ R\ (a, b)$ for all $(a, b) \in N \times N$

So, R is reflexive on $N \times N$.

Symmetry: Let $(a, b), (c, d) \in N \times N$ be such that $(a, b)\ R\ (c,d)$. Then,

$(a, b)\ R(c, d) \;\Rightarrow\; ad = bc$

$\Rightarrow cb = da$ [By commutativity of multiplication on N]

$\Rightarrow (c, d)\ R\ (a, b)$

Thus, $(a, b)\ R\ (c, d)$

$\Rightarrow \quad (c, d)\ R\ (a, b)$ for all $(a, b), (c, d) \in N \times N$.

So, R is symmetric on $N \times N$.

Transitivity: Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b)\ R\ (c, d)$ and $(c,d)\ R\ (e, f)$. Then,

$$\left.\begin{array}{l}(a,b)\ R\ (c,d) \Rightarrow ad = bc \\ (c,d)\ R\ (e,f) \Rightarrow cf = de\end{array}\right\} \Rightarrow (ad)\,(cf) = (bc)\,(de)$$

$\Rightarrow\; af = be$

$\Rightarrow\; (a, b)\ R\ (e, f)$

Thus, $(a, b)\ R\ (c,d)$ and $(c, d)\ R\ (e,f)$

$\Rightarrow$ $(a, b)\ R\ (e, f)$ for all $(a, b), (c, d), (e, f) \in N \times N$.

So, R is transitive on $N \times N$.

Hence, R being reflexive, symmetric and transitive, is an equivalence relation on $N \times N$.

Example 10

Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by

$$(a, b)\ R\ (c, d) \Leftrightarrow ad\,(b + c) = bc\,(a + d).$$

Check whether R is an equivalence relation on $N \times N$.

Solution

We observe the following properties

Reflexivity: Let (a, b) be an arbitrary element of $N \times N$. Then, $(a, b) \in n \times N \Rightarrow a, b \in N$

$\Rightarrow$ $ab\,(b + a) = ba\,(a + b)$

$\Rightarrow$ $(a, b)\ R\ (a, b)$

Thus, $(a, b)\ R\ (a, b)$ for $(a, b) \in N \times N$.

So, R is reflexive on $N \times N$.

Symmetry: Let $(a, b), (c, d) \in N \times N$ be such that $(a, b)\ R\ (c, d)$. Then,

$(a, b)\ R\ (c, d) \Rightarrow ad\,(b + c) = bc\,(a + d)$

$\Rightarrow cb\,(d + a) = da(c + b)$ [By comm.. of add. and mult. on N]

$\Rightarrow (c, d)\ R\ (a, b)$

Thus,

$(a, b)\ R\ (c, d) \Rightarrow (c, d)\ R\ (a, b)$ for all $(a, b), (c, d) \in N \times N$.

So, R is symmetric on $N \times N$.

Transitivity: Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$. Then

$(a, b)\ R\ (c, d) \Rightarrow ad\,(b + c) = bc\,(a + d)$

$$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad}$$

$$\Rightarrow \frac{1}{b} + \frac{1}{c} - \frac{1}{a} + \frac{1}{d} \quad \ldots\text{(i)}$$

and, $(c, d)\ R\ (e, f) \Rightarrow cf\,(d + e) = de\,(c + f)$

$$\Rightarrow \frac{d+e}{de} = \frac{c+f}{cf}$$

$$\Rightarrow \frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f} \quad \ldots\text{(ii)}$$

Adding (i) and (ii), we get

$$\left(\frac{1}{b}+\frac{1}{c}\right)+\left(\frac{1}{d}+\frac{1}{e}\right)=\left(\frac{1}{a}+\frac{1}{d}\right)+\left(\frac{1}{c}+\frac{1}{f}\right)$$

$\Rightarrow \quad \frac{1}{b}+\frac{1}{e}=\frac{1}{a}+\frac{1}{f}$

$\Rightarrow \quad \frac{b+e}{be}=\frac{a+f}{af}$

$\Rightarrow \quad af(b+e)=be\,(a+f)$

$\Rightarrow \quad (a, b)\ R\ (e, f)$

Thus, $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\Rightarrow \quad (a, b)\ R\ (e, f)$ for all $(a, b), (c, d), (e, f) \in N \times N$.

So, R is transitive on $N \times N$.

Hence, R being reflexive, symmetric and transitive, is an equivalence relation on $N \times N$.

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Exercise – I

OBJECTIVE TYPE QUESTIONS

Multiple choice questions with ONE option correct

1. Let $A = \{(x, y)| y = e^x, x \in R\}$ and $B = \{(x, y) | y = x, x \in R\}$, then
   (a) $B \subset A$ (b) $A \subset B$ (c) $A \cap B = \phi$ (d) $A \cup B = A$
2. If $a N = \{a x|x \in N\}$ and $b N \cap c N = d N$, where $b, c \in N$ are relatively prime, then
   (a) $d = bc$ (b) $c = bd$ (c) $b = cd$ (d) None of these
3. If X and Y are two sets, then $X \cap (Y \cup X)'$ equals
   (a) X (b) Y (c) ϕ (d) None of these
4. Let $A = \{x : x$ is a multiple of $3\}$ and $B = \{x : x$ is a multiple of $5\}$. Then $A \cap B$ is given by
   (a) $\{3, 6, 9,..\}$ (b) $\{5, 10, 15, 20,\ldots\}$ (c) $\{15, 30, 45,\ldots\}$ (d) None of these
5. If $A = \{1, 2, 3\}$ and $B = \{3, 8\}$, then $(A \cup B) \times (A \cap B)$ is
   (a) $\{(3, 1),(3, 2), (3, 3), (3, 8)\}$ (b) $\{(1, 3), (2, 3), (3, 3), (8, 3)\}$
   (c) $\{(1, 2), (2, 2), (3, 3), (8, 8)\}$ (d) $\{(8, 3), (8, 2), (8, 1), (8, 8)\}$
6. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is
   (a) 2^{mn} (b) $2^{mn} - 1$ (c) $2mn$ (d) m^n
7. If R is a relation on a finite set having n elements, then the number of relations on A is
   (a) 2^n (b) 2^{n^2} (c) n^2 (d) n^n
8. Let R be a reflexive relation on a finite set A having n-elements, and let there be m ordered pairs in R. Then
   (a) $m \geq n$ (b) $m \leq n$ (c) $m = n$ (d) None of these
9. Let $R = \{(a, a)\}$ be a relation on a set A. Then R is
   (a) symmetric (b) anti-symmetric
   (c) symmetric and anti-symmetric (d) neither symmetric nor anti-symmetric
10. Let $P = \{(x, y) + x^2 + y^2 = 1, x, y \in R\}$. Then P is
    (a) reflexive (b) symmetric (c) transitive (d) anti-symmetric
11. Let X be a family of sets and R be a relation on X defined by 'A is disjoint from B'. Then R is
    (a) reflexive (b) symmetric (c) anti-symmetric (d) transitive

12. Let R and S be two non-void relations on a set A. Which of the following statements is false?

(a) R and S are transitive $\Rightarrow$ $R \cup S$ is transitive

(b) R and S are transitive $\Rightarrow$ $R \cap S$ is transitive

(c) R and S are symmetric $\Rightarrow$ $R \cup S$ is symmetric

(d) R and S are reflexive $\Rightarrow$ $R \cap S$ is reflexive

13. If R be a relation < from $A = \{1, 2, 3, 4\}$ to $B = \{1, 3, 5\}$ i.e. $(a, b) \in R \Leftrightarrow a < b$, then $R\ o\ R^{-1}$ is

(a) $\{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$

(b)$\{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$

(c) $\{(3, 3), (3, 5), (5,3), (5, 5)\}$ (d) $\{(3, 3), (3, 4), (4, 5)\}$

14. If $R \subset A \times B$ and $S \subset B \times C$ be two relations, then $(S\ o\ R)^{-1} =$

(a) $S^{-1}\ o\ R^{-1}$ (b) $R^{-1}\ o\ S^{-1}$ (c) $S\ o\ R$ (d) $R\ o\ S$

15. Let R be a relation on the set N of natural numbers defined by $n\ R\ m \Leftrightarrow n$ s a factor of m (i.e. $n \mid m$). Then R is

(a) reflexive and symmetric (b) transitive and symmetric

(c) equivalence (d) reflexive, transitive but not symmetric

Multiple choice questions with ONE or MORE THAN ONE option correct

1. If $a\ N = \{a\ x \mid x \in N\}$ and $b\ N \cap c\ N = d\ N$, where $b, c \in N$ are relatively prime, then

(a) $d = bc$ (b) $c = bd$ (c) $b = cd$ (d) None of these

2. Let R and S be two relations on a set A. Then

(a) R and S are transitive, then $R \cup S$ is also transitive

(b) R and S are transitive, then $R \cap S$ is also transitive

(c) R and are reflexive, then $R \cap S$ is also reflexive

(d) R and S are symmetric, then $R \cup S$ is also symmetric

3. Let L be the set of all straight lines in the Euclidean plane. Two lines l_1 and l_2 are said to be related by the relation R iff l_1 is parallel to l_2. Then the relation R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

4. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is/are relations from X to Y?

(a) $R_1\ \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$ (b) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$

(c) $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7\}$ (d) $R_4 = \{(1, 3),(2, 5), (2, 4), (7, 9)\}$

5. Let n be a fixed positive integer. Define a relation R on the set Z of integers by $a \, R \, b \Leftrightarrow n \mid a - b$. Then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

6. Let R be the relation over the set of straight lines of a plane such that $l_1 R \, l_2 \Leftrightarrow l_1 \perp l_2$. Then R is

(a) symmetric (b) reflexive

(c) transitive (d) an equivalence relation

7. Let R be the relation over the set of integers such that $m \, R \, n$ if and only if m is a multiple of n. Then R is

(a) reflexive (b) symmetric

(c) transitive (d) an equivalence relation

8. Let $A = \{1, 2, 3, 4\}$ and R be a relation in A given by $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1), (1, 3)\}$

Then R is

(a) reflexive (b) symmetric

(c) transitive (d) an equivalence relation

9. Let $f : R \to R$ be a mapping such that $f(x) = \frac{x^2}{1+x^2}$. Then f is

(a) many-one (b) one-one (c) into (d) onto

10. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. If f is a function from A to B and g is a one-one function from A to B then the maximum number of definitions of

(a) f is 9 (b) g is 9 (c) f is 27 (d) g is 6

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Exercise - II

COMPREHENSIVE / MATCHING TYPE

Passage Based Questions

Passage – I

For any non empty sets A, B and C the following laws hold.

Important Laws: *(i)* $A \cup A = A$ *(ii)* $A \cap A = A$

Identity Laws: *(i)* $A \cup \phi = A$ *(ii)* $A \cap U = A$

Commutative Laws: *(i)* $A \cup B = B \cup A$ *(ii)* $A \cap B = B \cap A$

Associative Laws: *(i)* $(A \cup B) \cup C = A \cup (B \cup C)$ *(ii)* $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive Laws: *(i)* $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ *(ii)* $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De-Morgan's Laws: *(i)* $(A \cup B)' = A' \cap B'$ *(ii)* $(A \cap B)' = A' \cup B'$

1. If X and Y are two sets, $X \cap (Y \cup X)^C$
   (a) X (b) Y (c) ϕ (d) 18
2. Which of the following is not correct?
   (a) $A \subseteq A^C$ if and only if $A = \phi$
   (b) $A^C \subseteq A$ if and only if $A = X$, where X is a universal set
   (c) If $A \cup B = A \cup C$ the $B = C$
   (d) $A = B$ is equivalent to $A \cup C = B \cup C$ and $A \cap C = B \cap C$
3. The set $(A \cap B^C)^C \cup (B \cap C)$ is equal to
   (a) $A^C \cup B \cup C$ (b) $A^C \cup B$ (c) $A^C \cup C^C$ (d) None of these
4. The set $(A \cup B \cup C) \cap (A \cap B^C \cap C^C)^C \cap C^C$ is equal to
   (a) $B \cap C^C$ (b) $A \cap C$ (c) $B \cap C^C$ (d) None of these

Passage – II

Let A be a non-empty set. Then, a relation R on A is said to be

(i) Reflexive: If $aRa \ \forall \ a \in A$ i.e. if $(a, a) \in R \ \forall \ a \in A$.

(ii) Symmetric: If $aRb \Rightarrow bRa$ i.e. if $(a, b) \in R \Rightarrow (b, a) \in R$.

(iii) Transitive: If aRb and $bRc \Rightarrow aRc$ i.e. if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

(iv) Antisymmetric: If aRb and $bRa \Rightarrow a = b$.

(v) Equivalence Relation: A relation which is reflexive, symmetric and transitive is called on an equivalence relation.

1. Let S be the set of integers. For $a, b \in S$, $a\ R\ b$ if and only if $|a - b| < 1$, then
   (a) R is not reflexive
   (b) R is not symmetric
   (c) $R = \{(a, a); a \in I\}$
   (d) R is not an equivalence relation

2. If $n|m$ means that n is a factor of m, the relation | is
   (a) reflexive and symmetric
   (b) transitive and symmetric
   (c) reflexive, transitive and symmetric
   (d) reflexive, transitive and not symmetric

3. Let X be a nonempty set and $P(X)$ be the set of all subsets of X. For $A, B \in P(X)$, ARB if and only if $A \cap B = \phi$ then the relation
   (a) R is reflexive
   (b) R is symmetric
   (c) R is not transitive
   (d) R is an equivalence relation

4. For real numbers x and y, define a relation R, $x\ R\ y$ if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is
   (a) reflexive
   (b) symmetric
   (c) transitive
   (d) an equivalence relation

Matching Type Questions

1. Let A be the set of all children in the world and R be a relation on A.

Column I	**Column II**
(A) $x\ R\ y$ if x and y have the same sex	(P) R is reflexive but not symmetric
(B) $x\ R\ y$ if x and y have at least one of Two parents common	(Q) R is reflexive
	(R) R is an equivalence relation
	(S) R is symmetric but not reflexive

(a) A-S, B-R
(b) A-Q, S, B-R
(c) A-Q, B-R, S
(d) A-R, B-Q

2. If $A = \{x : x \in I, -2 \leq x \leq 2\}$, $B = \{x \in I; 0 \leq x \leq 3\}$, $C = \{x : x \in N, 1 \leq x \leq 2\}$ and $D = \{(x, y) \in N \times N; x + y = 8\}$

Column I	**Column II**
(A) $n(A \cup (B \cup C))$	(P) 7
(B) $n(D)$	(Q) 6
(C) $n(B \cup C)$	(R) 5
	(S) 4

(a) A-Q, B-P, C-S, R
(b) A-Q, B-P, C-S
(c) A-Q, R, B-P, C-S
(d) A-Q, B-P, R, C-S

3. The relations on N in column 1 are

Column I	**Column II**
(A) $\{(x, y) \mid x < y$	(P) Reflexive
(B) $\{(x, y) \mid x + y = 10\}$	(Q) symmetric
(C) $\{(x, y) \mid x = y$ or $x - y = 1\}$	(R) transitive
	(S) equivalence relation

(a) A-R, B-Q, C-P
(b) A-R, S, B-Q, C-P
(c) A-R B-Q, R, C-P
(d) A-R, B-Q, C-P, R

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Exercise - III

SUBJECTIVE TYPE

1. If $A = \{1, 2, 3, 4\}$, $B = \{4, 6, 7, 8\}$ and $C = \{2, 4, 6, 8\}$ verify that $(A \cup B) \cup C = A \cup (B \cup C)$.

2. If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$, $C = \{3, 4, 5, 6\}$, verify that $A \cap (B \cap C) = (A \cap B) \cup C$.

3. Prove that $A^C - B^C = B - A$.

4. Prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$

5. Prove that $A - (B \cup C) = (A - B) - (A - C)$.

6. Prove that $(A - B) \cup (B - A') = (A \cup B) - (A \cap B)$

7. If A, B and C are three sets and U is the universal set such that $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Find $n(A' \cap B')$.

8. A survey shows that 76% of the Indians like oranges, whereas 62% like bananas. What percentage of the Indians like both oranges and bananas?

9. In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find

   (i) How many drink tea and coffee both

   (ii) How many drink coffee but not tea

10. For a certain test, a candidate could offer English or Hindi or both the subjects. Total number of students was 500, of whom 350 appeared in English and 90 in both the subjects. Use set operations to show

    (i) How many appeared in English only?

    (ii) How many appeared in Hindi?

    (iii) How many appeared in Hindi only?

11. *(i)* In a group of people, 50 speak both English and Hindi and 30 people speak English but not Hindi. All the people speak at least one of the two languages. How many people speak English?

    (ii) In a survey of 100 persons it was found that 28 read magazine A. 30 read magazine B, 42 read magazine C, 8 read magazines A and B, 10 read magazines A and C, 5 read magazines B and C and 3 read all the three magazines. Find.

    (a) How many read none of the three magazines?

    (b) How many read magazine C only?

12. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper *T*, 26 read newspaper 1, 9 read both *H* and *I*, *II* read both *H* and *T*, 8 read both *T* and 1, 3 read all the three newspapers. Find

(i) The number of people who read at least one of the newspapers.

(ii) The number of people who read exactly one newspaper

13. *(i)* In a survey of 600 students in a school 150 students were found to be drinking Tea and 225 drinking Coffee, 100 were drinking both Tea and Coffee. Find how many students were drinking neither Tea nor Coffee.

(ii) In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students know either Hindi or English. How many students are there in the group?

14. In a town of 10,000 families in was found that 40% family buy newspaper *A*, 20% families buy newspaper *B* and 10% families buy newspaper *C*, 5% families buy *A* and *b*, 3% buy *B* and *C* and 4% buy *A* and *C*. If 2% families buy all the three newspapers, find the number of families which buy *(i) A* only *(ii) B* only (iii) none of *A*, *B* and *C*.

15. A survey of 500 television viewers produced the following information; 285 watch football, 195 watch hockey, 115 watch basketball, 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the three games. How many watch all the three games? How many watch exactly one of the three games?

16. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students that had taken

(i) only chemistry *(ii)* only mathematics *(iii)* only physics *(iv)* physics and chemistry but not mathematics *(v)* mathematics and physics but not chemistry *(vi)* only one of the subjects *(vii)* atleast one of the three subjects *(viii)* none of the three subjects.

17. Determine the domain and the range of the relation R, where $R = \{(x, x^3) : x$ is a prime number less than $10\}$

18. Which of the following relations on the set R of real numbers are equivalence relations

(i) $R = \{(a, b) : |a| = |b|\}$

(ii) $R = \{(a, b) : |a| \geq |b|\}$

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Exercise - IV

PROBLEMS ASKED IN COMPETITION

Multiple Choice Questions with ONE correct answer

1. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is

   (a) Reflexive (b) Transitive (c) Not Symmetric (d) A function

2. Let R_1 be a relation defined by $R_1 = \{(a, b) | a \geq b\ a, b \in R\}$. Then R_1 is

   (a) an equivalence relation on R (b) Reflexive, transitive but not symmetric

   (c) Symmetric, transitive but not reflexive

   (d) Neither transitive nor Reflexive but symmetric

3. If $A = \{1, 2, 3, 4\}$; $B = \{a, b\}$ and f is a mapping such that $f : A \to B$, then $A \times B$ is

   (a) $\{(a, 1), (3, b)\}$

   (b) $\{(a, 2), (4, b)\}$

   (c) $\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b), (4, a), (4, b)\}$

   (d) None of these

4. Let W denote the words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W|$ the words x and y have at least one letter in common$\}$ Then R is

   (a) reflexive, symmetric and not transitive

   (b) reflexive, symmetric and transitive

   (c) reflexive, not symmetric and transitive

   (d) not reflexive, symmetric and transitive

5. If a set A has n elements, then the total number of subsets of A is

   (a) n (b) n^2 (c) 2^n (d) $2n$

6. If A, B, C be three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then

   (a) $A = B$ (b) $B = C$ (c) $A = C$ (d) $A = B = C$

7. Given the sets $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is

   (a) {3} (b) (1, 2, 3, 4} (c) {1, 2, 4, 5) (d) {1, 2, 3 4, 5, 6}

8. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?

   (a) 3 (b) 6 (c) 9 (d) 18

9. Two finite sets have m and n elements. The total number of subsets of the first set is 48 more than the total number of subsets of the second set. The values of m and n are

(a) 7, 6 (b) 6, 3 (c) 6, 4 (d) 7, 4

10. If A, B and C are non-empty sets, then $(A - B) \cup (B - A)$ equals

(a) $(A \cup B) - B$ (b) $A - (A \cap B)$ (c) $(A \cup B) - (A \cap B)$ (d)$(A \cap B) \cup (A \cup B)$

11. In a class of 100 students, 55 students have passed in Mathematics and 67 students have passed in Physics. Then the number of students who have passed in Physics only is

(a) 22 (b) 33 (c) 10 (d) 45

12. In a class of 100 students, 55 students have passed in Mathematics and 67 students have passed in Physics. Then the number of students who have passed in Physics only is

(a) 22 (b) 33 (c) 10 (d) 45

13. Let $A = \{x : -1 \leq x \leq 1\} = B$ and S be the subset of $A \times b$ defined by $S = \{(x, y) \in A \times B : x^2 + y^2 = 1\}$. This defines

(a) a one-0ne function from A into B (b) a many-one function from A into B

(c) a bijective mapping from A into B (d) not a function.

14. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is

(a) 128 (b) 216 (c) 240 (d) 160

15. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is

(a) at least 30 (b) at most 20 (c) exactly 25 (d) none of these

16. If $A = \{x \{ x$ is a multiple of 3} and $B = \{x : x$ is a multiple of 5}, then $A - B$ is ($\bar{A}$ means complement of A)

(a) $\bar{A} \cap B$ (b) $A \cap \bar{B}$ (c) $\bar{A} \cap \bar{B}$ (d) $\overline{A \cap B}$

17. In a certain town 25% families own a phone and 15% own a car, 65% families own neither a phone nor a car. 2000 families own both a car and a phone.

Consider the following statements in this regard

1. 10% families own both a car and a phone.
2. 35% families own either a car or a phone.
3. 40,000 families live in the town. Which of the above statements are correct?

(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2 and 3

18. If A and B are two sets such that $n(A) = 70$, $n(B) = 60$ and $n(A \cup B) = 110$, then $n(A \cap B)$ is equal to

(a) 240 (b) 50 (c) 40 (d) 20

19. If the set A has p elements, B has q elements, then the number of elements in $A \times B$ is

(a) $p + q$ (b) $p + q + 1$ (c) pq (d) p^2

20. If A and B are not disjoint sets, then $n(A \cup B)$ is equal to

(a) $n(A) + n(B)$ (b) $n(A) + n(B) - n(A \cap B)$

(c) $n(A) + n(B) + n(A \cap B)$ (d) $n(A)\, n(B)$

21. If $A = \{c : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$, then $A \times (B \cap C)$ is

(a) $\{(2, 4), (3, 4)\}$ (b) $\{(4, 2), (4, 3)\}$

(c) $\{(2, 4), (3, 4), (4, 4)\}$ (d) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

22. In a city 20 percent of the population travels by car 50 percent travels by bus and 10 percent travels by both car and bus. Then persons traveling by car or bus is

(a) 80 percent (b) 40 percent (c) 60 percent (d) 70 percent

23. If $A = [(x, y) : x^2 + y^2 = 25]$ and $B = [(x, y) : x^2 + 9y^2 = 144]$ then $A \cap B$ contains

(a) one point (b) three points (c) two points (d) four points

24. Let R i $\{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is

(a) not symmetric (b) transitive (c) a function (d) reflexive

25. A class has 175 students. The following data shows the number of students obtaining one or more subjects. Mathematics 100; Physics 80 ; Chemistry 40 ; Mathematics and Physics 30 ; Mathematics and Chemistry 28 ; Physics and Chemistry 23 ; Mathematics, Physics and Chemistry 18. How many students have offered Mathematics alone?

(a) 35 (b) 48 (c) 60 (d) 22

26. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is

(a) 2^9 (b) 9^2 (c) 3^2 (d) $2^9 - 1$

27. If two sets A and B are having 99 elements in common, then the number of elements common to each of the sets $A \times B$ and $B \times A$ are

(a) 2^{99} (b) 99^2 (c) 100 (d) 18

28. Given $n(U) = 20$, $n(A) = 12$, $n(B) = 9$, $n(A \cap B) = 4$ where U is the universal set, A and B are subsets of U, then $n((A \cup B)^C) =$

(a) 17 (b) 9 (c) 11 (d) 3

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Answers

Exercise - I

Only One Option is correct

1.	(b)	2.	(a)	3.	(c)	4.	(c)	5.	(b)
6.	(a)	7.	(b)	8.	(a)	9.	(c)	10.	(b)
11.	(b)	12.	(a)	13.	(c)	14.	(b)	15.	(d)

More Than One Choice Correct

1.	(a)	2.	(b, c, d)	3.	(a, b, c, d)	4.	(a, b, c)	5.	(a, b, c, d)
6.	(a)	7.	(a, c)	8.	(a, b)	9.	(a, c)	10.	(c, d)

Exercise - II

Passage – I

1.	(c)	2.	(c)	3.	(b)	4.	(c)

Passage – II

1.	(a)	2.	(d)	3.	(a, b, c)	4.	(a)

Matching Type Questions

1.	(d)	2.	(b)	3.	(a)

Exercise - IV

Problems asked in competition

Multiple Choice Questions with ONE correct answer

1.	(c)	2.	(b)	3.	(c)	4.	(a)	5.	(c)
6.	(b)	7.	(b)	8.	(b)	9.	(c)	10.	(c)
11.	(d)	12.	(d)	13.	(d)	14.	(c)	15.	(b)
16.	(c)	17.	(s)	18.	(c)	19.	(b)	20.	(c)
21.	(a)	22.	(c)	23.	(d)	24.	(a)	25.	(c)
26.	(a)	27.	(b)	28.	(d)				

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