*** Given section consists of questions of 3 marks each.** **[120]**

1. If α and β are the zeros of the quadratic polynomial f(x) = 6x^2 + x − 2, find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

   Ans. : $f(x) = 6x^2 - x - 2$
   Since α and β are the zeroes of the given polynomial
   $\therefore$ Sum of zeroes $[\alpha+\beta] = \frac{-1}{6}$
   Product of zeroes $(\alpha\beta) = \frac{-1}{3}$
   $= \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$
   $= \frac{\left(\frac{-1}{6}\right)^2-2\times\left(\frac{-1}{3}\right)}{-\frac{1}{3}} = \frac{\frac{1}{36}+\frac{2}{3}}{\frac{-1}{3}} = \frac{\frac{1+24}{36}}{\frac{-1}{3}}$
   $= \frac{\frac{25}{36}}{\frac{-1}{3}} = \frac{-25}{12}$

2. If α and β are the zeroes of the quadratic polynomial f(x) = ax^2 + bx + c, find evaluate: $a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

   Ans. : $a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$
   $= a\left(\frac{\alpha^3+\beta^3}{\alpha\beta}\right)+b\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$
   $= a\times\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}+b\times\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$
   $= a\times\frac{\left(\frac{-b}{a}\right)^3-3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)}{\left(\frac{c}{a}\right)}+b\frac{\left(\frac{-b}{a}\right)^2-2\times\frac{c}{a}}{\frac{c}{a}}$
   $= \frac{\frac{-b^3}{a^2}+\frac{3bc}{a}}{\frac{c}{a}}+\frac{\frac{b^3}{a^2}-\frac{2bc}{a}}{\frac{c}{a}}$
   $= \frac{\frac{-b^3}{a^2}+\frac{3bc}{a}+\frac{b^3}{a^2}-\frac{2bc}{a}}{\frac{c}{a}}$
   $= \frac{bc}{a}\times\frac{a}{c}$
   $= b$

3. If α and β are the zeroes of the quadratic polynomial f(x) = x^2 + px + q, find a quadratic polynomial whose zeroes are:
   i. $\alpha+2, \beta+2$
   ii. $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$.

   Ans. : $\because \alpha$ and β are the zeroes of the quadratic polynomial
   $f(x) = x^2 - 2x + 3$
   Here a = 1, b = -2, c = 3
   $\alpha+\beta = \frac{-b}{a} = -\left(\frac{-2}{1}\right) = 2$
   $\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$
   i. Zeros are $\alpha+2, \beta+2$,
      $\therefore$ Sum of zeros $= \alpha+2+\beta+2 = (\alpha+\beta)+4$
      $= 2+4 = 6$
      and product of zeros $= (\alpha+2)(\beta+2)$
      $= \alpha\beta+2\alpha+2\beta+4$
      $= \alpha\beta+2 = (\alpha+\beta)+4$
      $= 3+2\times2+4$
      $= 3+4+4 = 11$
      $\therefore$ Polynomial will be
      k[x^2 - (sumof zeroes)x + product of zeros]

$\Rightarrow\ k(x^2-6x+11)$

ii. Zeros are $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

Sum of zeros $= \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$

$= \frac{(\alpha-1)(\beta+1)+(\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\beta+\alpha-\beta-1+\alpha\beta+\beta-\alpha-1}{\alpha\beta+\alpha+\beta+1}$

$= \frac{2\alpha\beta-2}{\alpha\beta+(\alpha+\beta)+1} = \frac{2\times3-2}{3+2+1}$

$= \frac{6-2}{6} = \frac{4}{6} = \frac{2}{3}$

and product of zeros $\frac{\alpha-1}{\alpha+1} \times \frac{\beta-1}{\beta+1}$

$= \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\beta-\alpha-\beta+1}{\alpha\beta+\alpha+\beta+1} = \frac{\alpha\beta-(\alpha+\beta)+1}{\alpha\beta+(\alpha+\beta)+1}$

$= \frac{3-2+1}{3+2+1} = \frac{2}{3} = \frac{1}{3}$

$\therefore$ Polynomial will be

k[x^2 - (sum of zeros)x + product of zeros]

$\Rightarrow\ k\left[x^2-\frac{2}{3}x+\frac{1}{3}\right]$

4. If α and β are the zeroes of the quadratic polynomial such that $\alpha+\beta=24$ and $\alpha-\beta=8$, find a quadratic polynomial have α and β as its zeroes.

Ans. : $\because \alpha$ and β are the zeroes of a quadratic polynomial

$\alpha+\beta=24$ and $\alpha-\beta=8$

$\therefore\ 4\alpha\beta=(\alpha+\beta)^2-(\alpha-\beta)^2$

$=(24)^2-(8)^2=576-64=512$

$\alpha\beta=\frac{512}{4}=128$

$\therefore$ Quadratic polynomial will be

k[x^2 - (sum of zeroes)x + product of zero]

$\Rightarrow k[x^2-24x+128]$

5. If α and β are the zeroes of the quadratic polynomial f(x) = x^2 - x - 2, find the value of $\frac{1}{\alpha}-\frac{1}{\beta}$

Ans. : $f(x)=x^2+x-2$

Here $a=1, b=1, c=-2$

$\because\ \alpha$ and β are the zeroes of f(x)

$\therefore \alpha+\beta=\frac{-b}{a}=\frac{-1}{1}=-1$

and $\alpha\beta=\frac{c}{a}=\frac{-2}{1}=-2$

Now $\frac{1}{\alpha}-\frac{1}{\beta}=\frac{\beta-\alpha}{\alpha\beta}-\frac{\alpha-\beta}{\alpha\beta}$

$=\frac{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}{\alpha\beta}=\frac{\sqrt{(-1)^2-4(-2)}}{-2}$

$=\frac{\sqrt{1+8}}{-2}=\frac{\sqrt{9}}{-2}=\frac{-3}{2}$

6. If one zero of the quadratic polynomial f(x) = $4x^2$ - 8kx - 9 is negative of the other, find the value of k.

Ans. : Since α and $-\alpha$ are the zeros of the quadratic polynomial f(x) = $4x^2$ - 8kx - 9

$\alpha-\alpha=0$

$\frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2}=0$

$\frac{-8k}{4}=0$

$-8k=0\times4$

$k=\frac{0}{-8}$

$-8k=0$

$k=0$

Hence, the Value of k is 0.

7. Find the zeros of the following quadratic polynomial and verify the relationship between the zeros and their coefficients:

$p(x) = x^2 + 2\sqrt{2x} - 6$

Ans. : $p(x) = x^2 + 2\sqrt{2}x - 6 \ = x^2 + 3\sqrt{2}x - \sqrt{2}x - \sqrt{2} \times 3\sqrt{2}$

$= x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2}) \ = (x - \sqrt{2})(x + 3\sqrt{2})$

Zeroes of the polynomial are $\sqrt{2}$ and $-3\sqrt{2}$

Sum of the zeroes $= \frac{-2\sqrt{2}}{1}$

$\sqrt{2} - 3\sqrt{2} = -2\sqrt{2}$

$-2\sqrt{2} = -2\sqrt{2}$

product of zeroes $\Rightarrow \ \sqrt{2} \times -3\sqrt{2} = -\frac{6}{1}$

$-6 = -6$

Hence the relatioship varified.

8. If α and β are the zeros of the quadratic polynomial f(t) = t^2 - 4t + 3, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

Ans. : We have,

$F(t) = t^2 - 4t + 3$

given that α and β are zeroes of F(t)

$\therefore \ \alpha + \beta = \frac{-(-4)}{1} = 4$

$\alpha\beta = \frac{3}{1} = 3$

Now $\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta)$

$= (\alpha\beta)^3(\alpha + \beta)$

$= (3)^3 \times 4$

$= 27 \times 4$

$= 108$

9. If α and β are the zeroes of the quadratic polynomial p(s) = $3s^2$ - 6s + 4, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$.

Ans. : Since α and β are the zeroes of the quadratic polynomial p(s) = $3s^2$ - 6s + 4

$\alpha + \beta = \frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2}$

$\alpha + \beta = \frac{-(-6)}{3}$

$\alpha + \beta = \frac{6}{3}$

$\alpha + \beta = 2$

$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\alpha\beta = \frac{4}{3}$

We have, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$

$= \frac{(\alpha + \beta^2) - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$

By substituting $\alpha + \beta = 2$ and $\alpha\beta = \frac{4}{3}$ we get,

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta \ = \frac{(2)^2 - 2\left(\frac{4}{3}\right)}{\frac{4}{3}} + 2\frac{(2)}{\frac{4}{3}} + 3\left(\frac{4}{3}\right)$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta \ = \frac{4 - \frac{8}{3}}{\frac{4}{3}} + \frac{4}{\frac{4}{3}} + \frac{12}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta \ = \frac{\frac{4 \times 3}{1 \times 3} - \frac{8}{3}}{\frac{4}{3}} + \frac{4}{\frac{4}{3}} + \frac{12}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta \ = \frac{\frac{12 - 8}{3}}{\frac{4}{3}} + \frac{4}{\frac{4}{3}} + \frac{12}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta \ = \frac{\frac{4}{3}}{\frac{4}{3}} + \frac{4}{\frac{4}{3}} + \frac{12}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta \ = \frac{4}{3} \times \frac{3}{4} + \frac{4 \times 3}{4} + \frac{12}{3}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta \ =1+\frac{12}{4}+\frac{12}{3}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta \ =\frac{1\times 12}{1\times 12}+\frac{12\times 3}{4\times 3}+\frac{12\times 4}{3\times 4}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta \ =\frac{12+36+48}{12}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{48+48}{12}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{96}{12}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=8$

Hence, the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta \text{ is } 8$

10. For what value of k, is -2 a zero of the polynomial $3x^2$ + 4x + 2k?

Ans. : We know if $x=\alpha$ is zero polynomial then $x-\alpha$ is a factor of f(x)
Since -2 is factor of f(x). Therefore x + 2 is a factor of f(x)
Now, we divide f(x) = $3x^2$ + 4x +2k by g(x) = x + 2 to find the value of k

$$\begin{array}{r|l} & 3x+2 \\ x+2) & 3x^2+4x+2k \\ & \underline{\overset{-}{3x^2}\ \overset{-}{+}2x} \\ & \qquad 2x+2k \\ & \qquad \underline{\overset{-}{2x}\ \overset{+}{-}4} \\ & \qquad\qquad 2k+4 \end{array}$$

Now, Remainder = 0
2k + 4 = 0
2k = -4
$k=\frac{-4}{2}$
k = -2
Hence, the value of k is -2

11. If α and β are the zeros of the quadratic polynomial f(x) = x^2 - p(x + 1) - c, show that $(\alpha+1)(\beta+1)=1-c$

Ans. : $f(x)=x^2-p(x+1)-c=x^2-px-p-c$
Sum of zeroes $=\alpha+\beta=p$
Product of zeroes $=-p-c=\alpha\beta$
$(\alpha+1)(\beta+1)=\alpha\beta+\alpha+\beta+1=-p-c+p+1$
$=1-c=\text{R.H.S}$
$\therefore$ Hence proved.

12. If α and β are the zeroes of the quadratic polynomial f(x) = ax^2 + bx + c, then evaluate:
$\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$

Ans. : $\frac{1}{a\alpha+b}+\frac{1}{\alpha\beta+b}$

$\Rightarrow \frac{a\beta+b+a\alpha+b}{(a\alpha+b)(\alpha\beta+b)}$

$=\frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$=\frac{a(\alpha+\beta)+b}{a^2\alpha\beta+\alpha\beta(\alpha^2\beta)+b^2}$

$=\frac{a\times\frac{a+2b}{a}}{a\times\frac{c}{a}+\frac{abc(-b)+b^2}{a}}=\frac{b}{ac-b^2+b^2}=\frac{b}{ac}$

13. For the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also, find the zeroes of these polynomials by factorization.
$-\frac{8}{3}, \frac{4}{3}$

Ans. : Given that, sum of zeroes $(S)=-\frac{8}{3}$
and product of zeroes $(P)=\frac{4}{3}$

Required quadratic expression,
$f(x) = x^2 - Sx + P$
$= x^2 + \frac{8}{3}x + \frac{4}{3} = 3x^2 + 8x + 4$
Using factorisation method,
$= 3x^2 + 6x + 2x + 4$
$= 3x(x+2) + 2(x+2) = (x+2)(3x+2)$
Hence, the zeroes of f(x) are -2 and $-\frac{2}{3}$

14. For what value of k, is -3 a zero of the polynomial $x^2 + 11x + k$?

Ans. : -3 is a zero of polynomial $f(x) = x^2 + 11x + k$
It will satisfy the polynomial
f(x) = 0 ⇒ f(-3) = 0
Now $x^2 + 11x + k = 0$
⇒ $(-3)^2 + 11x(-3) + k = 0$
⇒ 9 - 33 + k = 0
⇒ -24 + k = 0
⇒ k = 24

15. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then find the value of a.

Ans. : $p(x) = ax^2 - 3(a - 1)x - 1$
∵ 1 is its zero
∴ (x - 1) will be its factor
Now, $p(1) = a(1)^2 - 3(a - 1) \times 1 - 1$
= a - 3a + 3 - 1
= -2a + 2
∴ (x - 1) is its factor
∴ Remainder = 0
∴ p(1) = 0
⇒ -2a + 2 = 0 ⇒ 2a = 2
⇒ $a = \frac{2}{2} = 1$
∴ a = 1

16. If α and β are the zeros of the quadratic polynomial $p(x) = 4x^2 - 5x - 1$, find the value of $\alpha^2\beta + \alpha\beta^2$.

Ans. : We have,
$P(x) = 4x^2 - 5x - 1$
give that α and β are zeroes of p(x)
$\therefore \alpha + \beta = \frac{-(-5)}{4} = \frac{5}{4},\ \alpha\beta = \frac{(-1)}{4} = \frac{-1}{4}$
Now $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$
$= \left(\frac{-1}{4}\right)\left(\frac{5}{4}\right)$
$= \frac{-5}{16}$

17. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 1$, find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

Ans. : $f(x) = x^2 - 1$
sum of zeroes $\alpha + \beta = 0$
Product of zeroes $\alpha\beta = -1$
Sum of zeroes $= \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}$
$= \frac{2\left((\alpha+\beta)^2 - 2\alpha\beta\right)}{\alpha\beta}$
$= \frac{2\left[(0)^2 - 2 \times -1\right]}{-1}$
$= \frac{2(2)1}{-1}$
$= -4$
Product of zeroes $= \frac{2\alpha \times 2\beta}{\alpha\beta} = \frac{4\alpha\beta}{\alpha\beta}$

Hence the quadratic equation is x^2 - (sum of zeroes)x + product of zeroes
$= k(x^2 + 4x + 14)$

18. If α and β are the zeroes of the quadratic polynomial f(x) = x^2 - x - 4, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$

Ans. : Since $\alpha + \beta$ are the zeroes of the polynomial: x^2 - x - 4
Sum of the roots $(\alpha + \beta) = 1$
Product of the roots $(\alpha\beta) = -4$
$= \frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$
$= \frac{1}{-4} + 4 = \frac{-1}{4} + 4 = \frac{-1+16}{4} = \frac{15}{4}$

19. If the sum of the zeros of the quadratic polynomial f(t) = kt^2 + 2t + 3k is equal to their product, find the value of k.

Ans. : Let, the two zeroes of the f(t) = kt^2 + 2t + 3k be α and β
Sum of the zeroes $(\alpha + \beta) = \frac{-2}{k}$
Product of the zeroes $\alpha\beta = \frac{3k}{k} = 3$
According to question,
$\frac{-2}{k} = 3$
$-2 = 3k$
$k = \frac{-2}{3}$

20. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

Ans. : Let $f(x)$ $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$ and given that $\sqrt{2}$ is one of the zeroes of f(x)
i.e., $(x - \sqrt{2})$ is one of the factor of given cubic polynomial.
Now, using divison algorithm,

Quotient: $6x^2 + 7\sqrt{2}x + 4$
$(x - \sqrt{2})\overline{)6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}}$
$6x^3 - 6\sqrt{2}x^2$
$-\quad +$
$7\sqrt{2}x^2 - 10x - 4\sqrt{2}$
$7\sqrt{2}x^2 - 14x$
$-\quad +$
$4x - 4\sqrt{2}$
$4x - 4\sqrt{2}$
$-\quad -$
$\times$

21. Write the coefficient of the polynomial p(z) = z^5 - $2z^2$ + 4.

Ans. : p(z) = z^5 + oz^4 + oz^3 – $2z^2$ + oz + 4
Coefficient of z^5 = 1
Coefficient of z^4 = 0
Coefficient of z^3 = 0
Coefficient of z^2 = -2
Coefficient of z = 0
Constant = 4

22. If α and β are the zeroes of the quadratic polynomial f(x) = x^2 - 5x + 4, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Ans. : $f(x) = x^2 - 5x + 4$
Here $a = 1, b = -5,\ c = 4$
$\because$ α and β are the zeroes of f(x)
$\therefore \alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{1} = 5$
and $\alpha\beta = \frac{c}{a} = \frac{4}{1} = 4$
Now, $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\beta\alpha} - 2\alpha\beta$

$= \frac{\alpha+\beta}{\alpha\beta} - 2\alpha\beta$

$= \frac{5}{4} - 2 \times 4 = \frac{5}{4} - 8$

$= \frac{5-32}{4} = \frac{-27}{4}$

23. Apply division algorithm to find the quotient q(x) and remainder r(x) in dividing f(x) by g(x) in the following:
$f(x) = 4x^3 + 8x + 8x^2 + 7$, $g(x) = 2x^2 - x + 1$

Ans. : $f(x) = 4x^3 + 8x + 8x^2 + 7$, $g(x) = 2x^2 - x + 1$
or $f(x) = 4x^3 + 8x^2 + 8x + 7$, $g(x) = 2x^2 - x + 1$
$\because$ Degree of f(x) = 3 and degree of g(x) = 2
$\therefore$ Degree of q(x) = will be = 3 - 2 = 1
and degree of r(x) will be less than 2
Now using division algorithm,
f(x) = g(x) × q(x) + r(x)
$\Rightarrow 4x^3 + 8x^2 + 8x + 7 = (2x^2 - x + 1)(ax + b) + (cx + d)$
$= 2ax^3 + (2b - a)x^2 + (a - b + c)x + (b + d)$
Equating the corresponding co-effecients
2a = 4 ⇒ a = 2
2b - a = 8 ⇒ 2b - 2 = 8
⇒ 2b = 8 + 2 = 10
$\Rightarrow b = \frac{10}{2} = 5$
a - b + c = 8
2 - 5 + c = 8 ⇒ c = 8 - 2 + 5 = 11
b + d = 7 ⇒ 5 + d = 7
⇒ d = 7 - 5 = 2
q(x) = (ax + b) = 2x + 5
r(x) = cx + d = 11x + 2

24. For what value of k, is 3 a zero of the polynomial $2x^2 + x + k$?

Ans. : We know that is $x = \alpha$ is zero polynomial, and then $x - \alpha$ is a factor of f(x)
Since 3 is zero of f(x)
Therefore x - 3 is a factor of f(x)
Now, we divide $f(x) = 2x^2 + x + k$ by g(x) = x - 3 to find the value of k

$$\begin{array}{r|l} & 2x + 7 \\ x - 3 & +2x^2 + x \ +k \\ & \underset{-}{+}2x^2 \underset{+}{-} 6x \\ \hline & + \ 7x + k \\ & \underset{-}{+} \ 7x \quad \underset{+}{-} 21 \\ \hline & k + 21 \end{array}$$

Now, remainder = 0
k + 21 = 0
k = -21
Hence, the value of k is -21

25. Write the zeros of the polynomial $x^2 - x - 6$.

Ans. : $f(x) = x^2 - x - 6$
$= x^2 - 3x + 2x - 6$
$\left\{ \begin{array}{l} -6 = -3 \times 2 \\ -1 = -3 + 2 \end{array} \right\}$
= x(x - 3) + 2(x - 3)
= (x - 3)(x + 2)
$\therefore$ Zeros of f(x) will be if (x - 3)(x + 2) = 0
Either x - 3 = 0, then x = 3
or x + 2 = 0, then x = 3
$\therefore$ Zeros are 3, -2

26. For the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also, find the zeroes of these polynomials by factorization.
$\frac{-3}{2\sqrt{5}}, -\frac{1}{2}$

Ans. : Give that,$S = -\frac{3}{2\sqrt{5}}$ and $P = -\frac{1}{2}$
$\therefore$ Required quadratic expression,
$f(x) = x^2 - Sx + P = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$
$= 2\sqrt{5}x^2 + 3x - \sqrt{5}$
Using factorisation method,
$= 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5}$
$= \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5})$
$= (2x + \sqrt{5})(\sqrt{5}x - 1)$
Hence, the zeroes of f(x) are $-\sqrt{\frac{5}{2}}$ and $\frac{1}{\sqrt{5}}$

27. For the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also, find the zeroes of these polynomials by factorization.
$-2\sqrt{3}, -9$

Ans. : Given that, $S = -2\sqrt{3}$ and $P = -9$
$\therefore$ Required quadratic expression,
$f(x) = x^2 - Sx + P = x^2 + 2\sqrt{3}x - 9$
$= x^2 + 3\sqrt{3}x - \sqrt{3}x - 9$
[using factorisation method]
$= x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3})$
$= (x + 3\sqrt{3})(x - \sqrt{3})$
Hence, the zeroes of f(x) are $-3\sqrt{3}$ and $\sqrt{3}$

28. If the squared difference of the zeroes of the quadratic polynomial f(x) = x^2 + px + 45 is equal to 144, find the value of p.

Ans. : $f(x) = x^2 + P(x) + 45$
Here $a = 1, b = p, c = 45$
Let α and β be the zeroes of f(x),their
$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1} = -p$
and $\alpha\beta = \frac{c}{a} = \frac{45}{1} = 45$
and $(\alpha - \beta)^2 = 144$
$\because (\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$
$\Rightarrow (-p)^2 = 144 + 4 \times 45$
$\Rightarrow p^2 = 144 + 180 = 324$
$\Rightarrow p^2 = (\pm 18)^2$
$\therefore\ p = \pm 18$

29. Find the cubic polynomial with the sum, sum of the product of its zeros taken two at a time, and product of its zeros as 3, -1 and -3 respectively.

Ans. : Let α, β and γ be the zrose of cubic polynomial p(x).
Given:
$\alpha + \beta + \gamma = 3\ \ldots(1)$
$\alpha\beta + \beta\gamma + \gamma\alpha = -1\ \ldots(2)$
$\alpha\beta\gamma = -3\ \ldots(3)$
Thus, required cubic polynomial is
$p(x) = k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$
$p(x) = k(x^3 - 3x^2 - x + 3)$
Where is any non-zerorcal number.

30. If α and β are the zeroes of the quadratic polynomial p(y) = $5y^2$ - 7y + 1, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$

Ans. : $p(y) = 5y^2 - 7y + 1$
here, $a = 5, b = -7,\ c = 1$
$\because\ \alpha$ and β are the zeroes of p(y)

$\therefore \alpha$ and $\beta = \frac{-b}{a} = -\left(\frac{-7}{5}\right) = \frac{7}{5}$

and $\alpha\beta = \frac{c}{a} = \frac{1}{5}$

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{\frac{7}{5}}{\frac{1}{5}} = \frac{7}{5} \times \frac{5}{1} = 7$

31. What must be added to the polynomial f(x) = $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$?

Ans. : We know that,
f(x) = g(x) × q(x) + r(x)
f(x) - r(x) = g(x) × q(x)
f(x) + {- r(x)} = g(x) × q(x)
Clearly, Right hand side is divisible by g(x).
Therefore, Left hand side is also divisible by g(x). Thus, if we add - r(x) to f(x), then the resulting polynomial is divisible by g(x).
Let us now find the remainder when f(x) is divided by g(x).

$$\begin{array}{r|l} & x^2 + 1 \\ x^2 + 2x - 3 & +x^4 + 2x^3 - 2x^2 + x - 1 \\ & \underset{-}{+}x^4 \underset{-}{+} 2x^3 \underset{+}{-} 3x^2 \\ \hline & + x^2 + x - 1 \\ & \underset{-}{+}x^2 \underset{-}{+}2x \underset{+}{-} 3 \\ \hline & - x + 2 \end{array}$$

Hence, we shuold add -r(x) = x - 2 to f(x) so that the resulting polynomial is divisible by g(x).

32. If a - b, a and b are zeros of the polynomial f(x) = $2x^3 - 6x^2 + 5x - 7$, write the value of a.

Ans. : Let a - b, a and a + b be the zeros of the polynomial f(x) = $2x^3 - 6x^2 + 5x - 7$ then

Sum of the zeros $= \frac{-\text{Coefficiemt of } x^2}{\text{Coefficiemt of } x^3}$

$(a - d) + a + (a + b) = -\left(\frac{-6}{2}\right)$

$a + a + a - d + d = \frac{6}{2}$

$3a = 3$

$a = \frac{3}{3}$

$a = 1$

Hence, the value of a is 1.

33. For the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also, find the zeroes of these polynomials by factorization.
$\frac{21}{8}, \frac{5}{16}$

Ans. : Given that $S = \frac{21}{8}$ and $P = \frac{5}{16}$
Required quadratic expression,
$f(x) = x^2 - Sx + P$
$= x^2 - \frac{21}{8}x + \frac{5}{16} = 16x^2 - 42x + 5$
Using factrisation method
$= 16x^2 - 40x - 2x + 5$
$= 8x(2x - 5) - 1(2x - 5) = (2x - 5)(8x - 1)$
Hence, the zeroes of f(x) and $\frac{5}{2}$ and $\frac{1}{8}$

34. If α and β are the zeroes of the quadratic polynomial f(x) = $ax^2 + bx + c$, then evaluate:
$\alpha^4 + \beta^4$

Ans. : Now, $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$
$= \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2(\alpha\beta)^2$
$= \left\{\left(\frac{-b}{a}\right)^2 - \frac{2c}{a}\right\}^2 - 2(\alpha\beta)^2$

$= \left(\frac{b^2}{a^2} - \frac{2c}{a}\right)^2 - 2(\alpha\beta)^2$

$= \frac{b^4}{a^4} - \frac{4b^2c}{a^3} + \frac{4c^2}{a^2} - 2\left(\frac{c}{a}\right)^2$

$\frac{b^4}{a^4} - \frac{4b^2c}{a^3} + \frac{4c^2}{a^2} - 2\frac{c^2}{a^2}$

$= \frac{b^4}{a^4} - \frac{4b^2c}{a^3} + \frac{2c^2}{a^2}$

$= \frac{b^4 - 4ab^2c + 2a^2c^2}{a^4}$

$= \frac{(b^2-2ac)^2 - 2a^2c^2}{a^4}$

Thus, $\alpha^4 + \beta^4 = \frac{(b^2-2ac)^2 - 2a^2c^2}{\alpha^4}$

35. Give an example of polynomials f(x), g(x), q(x) and r(x) satisfying f(x) = g(x), q(x) + r(x), where degree r(x) = 0.

Ans. : Using division algorithm, we have
f(x) = g(x) × q(x) + r(x)
$x^5 - 4x^3 + x^2 + 3x + 1 = (x^3 - 3x + 1)(x^2 - 1) + 2$
$x^5 - 4x^3 + x^2 + 3x + 1 = x^5 - 3x^3 + x^2 - x^3 + 3x - 1 + 2$
$x^5 - 4x^3 + x^2 + 3x + 1 = x^5 - 3x^3 - x^3 + x^2 + 3x - 1 + 2$
$x^5 - 4x^3 + x^2 + 3x + 1 = x^5 - 4x^3 + x^2 + 3x + 1$
Hence an example for polynomial f(x), g(x), q(x) and r(x) satisfying f(x) = g(x) × q(x) + r(x) are
$f(x) = x^5 - 4x^3 + x^2 + 3x + 1$
$g(x) = (x^3 - 3x + 1)$
$q(x) = (x^2 - 1)$
r(x) = 2

36. If α, β are the zeros of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

Ans. : Given: $f(x) = x^2 - 5x + k$
The co-efficients are a = 1, b = -5 and c = k.
$\therefore \alpha + \beta = \frac{-b}{a}$
$\Rightarrow \alpha + \beta = -\frac{-5}{1}$
$\Rightarrow \alpha + \beta = 5 \ \ldots(1)$
Also, $\alpha - \beta = 1 \ \ldots(2)$
From (1) & (2), we get:
$2\alpha = 6$
$\Rightarrow \alpha = 3$
Putting the value of α in (1), we get $\beta = 2$
Now, $\alpha\beta = \frac{c}{a}$
$\Rightarrow 3 \times 2 = \frac{k}{1}$
$\therefore k = 6$

37. Very-Short-Answer Question:
If the product of the zeros of the quadratic polynomial $x^2 - 4x + k$ is 3 then write the value of k.

Ans. : Let $f(x) = x^2 - 4x + k$
Product of zeros $= \frac{\text{Constant term}}{\text{Coefficient of x}}$
$= \frac{k}{1} = k$
Given, Product of zeros = 3
$\therefore k = 3$

38. Using remainder theorem, find the remainder when $p(x) = x^3 + 3x^2 - 5x + 4$ is divided by (x - 2).

Ans. : Given: $p(x) = x^3 + 3x^2 - 5x + 4$
Now, $p(2) = 2^3 + 3(2)^2 - 5(2) + 4$
= 8 + 12 - 10 + 4
= 14

39. Very-Short-Answer Question:
If (x + a) is a factor of $(2x^2 + 2ax + 5x + 10)$, find the value of a.

Ans. : Let f(x) = $2x^2 + 2ax + 5x + 10$
Since (x + a) is a factor of f(x), we have
f(-a) = 0
⇒ $2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$
⇒ $2a^2 - 2a^2 - 5a + 10 = 0$
⇒ 5a = 10
⇒ a = 2

40. Find the zeros of the following quadratic polynomial and verify the relationship between the zeros and the coefficients:
$2\sqrt{3}x^2 - 5x + \sqrt{3}$

Ans. : We have,
$f(x) = 2\sqrt{3}x^2 - 5x + \sqrt{3}$
$= 2\sqrt{3}x^2 - 2x - 3x + \sqrt{3}$
$= 2x(\sqrt{3}x - 1) - \sqrt{3}(\sqrt{3}x - 1)$
$= (\sqrt{3} - 1)(2x - \sqrt{3})$
$\therefore f(x) = 0$
$\Rightarrow (\sqrt{3}x - 1)(2x - \sqrt{3}) = 0$
$\Rightarrow \sqrt{3} - 1 = 0$ or $2x - \sqrt{3} = 0$
$\Rightarrow x = \frac{1}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{2}$

So, the zeros of f(x) are $\frac{1}{\sqrt{3}}$ and $\frac{\sqrt{3}}{2}$

Sum of zeros $= \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2} = \frac{2+3}{2\sqrt{3}}$
$= \frac{5}{2\sqrt{3}} = \frac{-(\text{Coefficient of x})}{(\text{Coeficient of } x^2)}$

Product of zeros $= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$
$= \frac{\sqrt{3}}{2\sqrt{3}} = \frac{\text{Constant term}}{(\text{Coefficient of } x^2)}$

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