*** Given section consists of questions of 2 marks each.** **[64]**

1. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

Ans. : Given quadratic equation: $x^2 - 3$
Recall the identity $a^2 - b^2 = (a - b)(a + b)$. Using it,
we can write:
$x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$
So, the value of $x^2 - 3$ is zero when $x = \sqrt{3}$ or $x = -\sqrt{3}$
Therefore, the zeroes of $x^2 - 3$ are $\sqrt{3}$ and $-\sqrt{3}$
Now, the sum of zeroes = $\sqrt{3} - \sqrt{3} = 0 = \frac{-(\text{ Coefficient of } x)}{\text{Coefficient of } x^2}$
and the product of zeroes = $(\sqrt{3})(-\sqrt{3}) = -3 = \frac{-3}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

2. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Ans. : We have,
$x^2 + 7x + 10 = (x + 2)(x + 5)$
So, the value of $x^2 + 7x + 10$ is zero when $x + 2 = 0$ or $x + 5 = 0$, i.e., when $x = -2$ or $x = -5$. Therefore, the zeroes of $x^2 + 7x + 10$ are -2 and -5.
Now,
sum of zeroes = $-2 + (-5) = -(7) = \frac{-(7)}{1} = \frac{-(\text{ Coefficient of } x)}{\text{Coefficient of } x^2}$
product of zeroes = $(-2) \times (-5) = 10 = \frac{10}{1} = \frac{\text{Constant term}}{\text{coefficient of } x^2}$

3. If the zeroes of the polynomial $f(x) = x^3 - 3x^2 + x + 1$ are a - b, a, a + b, find a and b.

Ans. : $f(x) = x^3 - 3x^2 + x + 1$
It is given that a - b, a and a + b are the zeroes of f(x).
Now, Sum of the zeroes $= -\frac{\text{Coeff. of } x^2}{\text{Coeff. of } x^3}$
⇒ a - b + a + a +b = $-\frac{-3}{1}$
⇒ 3a = 3
⇒ a = 1
and, Product of zeros $= -\frac{\text{Constant term}}{\text{Coeff. of } x^3}$
⇒ (a -b)(a) (a + b) $= -\frac{1}{1}$
⇒ $a(a^2 - b^2) = -1$
⇒ $1 - b^2 = -1$ (∵ a = 1)
⇒ $b^2 = 2$
⇒ b = $\pm\sqrt{2}$
Hence the value of a = 1 and b = $\pm\sqrt{2}$

4. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}, \frac{1}{3}$ respectively.

Ans. : Let the polynomial be $ax^2 + bx + c$.
and its zeroes be α and β.
Then, $\alpha + \beta = \sqrt{2} = -\frac{b}{a}$ and $\alpha\beta = \frac{1}{3} = \frac{c}{a}$
If a = 3, then $b = -3\sqrt{2}$ and c = 1.
So, one quadratic polynomial which fits the given conditions is $3x^2 - 3\sqrt{2}x + 1$.
It is given that $\alpha + \beta = \sqrt{2}$ and $\alpha\beta = \frac{1}{3}$
Now, standard form of quadratic polynomial is given by $x^2 - (\alpha + \beta)x + \alpha\beta$
$= x^2 - (\alpha + \beta)x + \alpha\beta$
$= x^2 - \sqrt{2}x + \frac{1}{3}$
$= \frac{1}{3}(3x^2 - 3\sqrt{2}x + 1)$
Hence the required quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$

5. Find the zeroes of quadratic polynomial $t^2 - 15$ and verify the relationship between the zeroes and their coefficients.

Ans. : We have quadratic polynomial as $t^2 - 15$

$= t^2 - (\sqrt{15})^2$

$= (t - \sqrt{15})(t + \sqrt{15})$ [As, $x^2 - y^2 = (x - y)(x + y)$]

The value of $t^2 - 15$ is zero when $(t - \sqrt{15}) = 0$ or $(t + \sqrt{15}) = 0$,

i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15}+(-\sqrt{15})=0=\frac{-0}{1}=\frac{-(\text{coefficient of } t)}{\text{coefficient of } t^2}$

Product of zeroes = $(\sqrt{15})(-\sqrt{15})=-15=\frac{-15}{1}=\frac{\text{constant term}}{\text{coefficient of } t^2}$

Hence verified.

6. Find the zeroes of quadratic polynomial $4u^2 + 8u$ and verify the relationship between the zeroes and their coefficients.

Ans. : The quadratic equation is given as: $4u^2 + 8u$

it can be written in the standard form as:

$= 4u^2 + 8u + 0$

$= 4u(u + 2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$,

i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and −2

Sum of zeroes = $0+(-2)=-2=\frac{-(8)}{4}=\frac{-(\text{ coefficient of } u)}{\text{coefficient of } u^2}$

Product of zeroes = $0\times(-2)=0=\frac{0}{4}=\frac{\text{constant torm}}{\text{coefficient of } u^2}$

Hence verified

7. Find the zeroes of quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and their coefficients.

Ans. : We have given the quadratic equation as: $6x^2 - 3 - 7x$

First of all we will write it into standard form as: $6x^2 - 7x - 3$

(Now we will factorize 7 such that the product of the factors is equal to - 18 and the sum is equal to - 7)

It can be written as

$= 6x^2 + 2x - 9x - 3$

$= 2x(3x + 1) - 3(3x + 1)$

$= (3x + 1)(2x - 3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$,

i.e. $X=\frac{-1}{3}$ or $\frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$

Sum of zeroes = $\frac{-1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(\text{ coefficient of } x)}{\text{coefficient of } x^2}$

Product of zeroes $=\frac{-1}{3}\times\frac{3}{2}=\frac{-1}{2}=\frac{-3}{6}=\frac{\text{constant term}}{\text{coefficient of } x^2}$

Hence, verified

8. If the product of zeros of the quadratic polynomial $f(x) = x^2 - 4x + k$ is 3, find the value of k.

Ans. : Product of zeros $=\frac{c}{a}$

$\alpha\cdot\beta=\frac{K}{1}$

$3=\frac{K}{1}$

$3=K$

$K=3$

9. If the sum of the zeros of the quadratic polynomial $f(x) = kx^2 - 3x + 5$ is 1, write the value of k.

Ans. : $f(x) = kx^2 - 3x + 5$

Here $a = k$, $b = -3$, $c = 5$

$\therefore$ Sum of zeros $= \frac{-b}{a} = -\left(\frac{-3}{k}\right) = \frac{3}{k}$

$\therefore \frac{3}{k} = 1 \Rightarrow k = 3$

10. If α, β are the zeros of the polynomial $2y^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$.

Ans. : α and β are the zeros of the polynomial $2y^2 + 7y + 5$

$\therefore\ \alpha + \beta = \frac{-b}{a} = \frac{-7}{2}$

$\alpha\beta = \frac{c}{a} = \frac{5}{2}$

$\therefore\ \alpha + \beta + \alpha\beta = \frac{-7}{2} + \frac{5}{2} = \frac{-2}{2} = -1$

11. What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$?

	$x^2 + 6x + 8$
$x^2 - 4x + 3$	$x^4 + 2x^3 - 13x^2 - 12x + 21$
	$x^4 - 4x^3 + 3x^2$
	$6x^3 - 16x^2 - 12x$
	$6x^3 - 24x^2 - 18x$
	$8x^2 - 30x + 21$
	$8x^2 - 32x + 21$
	$2x - 2$

Ans. :

We must subtract [2x - 2] + 10m the given polynomial so as to get the resulting polynomial exactly divisible by $x^2 - x + 3$

12. Write the family of quadratic polynomials having $-\frac{1}{4}$ and 1 as its zeros.

Ans. : We know that, if x = a is a zero of a polynomial then x - 2 is a factor of quadratic polynomials.

Since $\frac{-1}{4}$ and 1 are zeros of polynomial.

Therefore $\left(x + \frac{1}{4}\right)(x - 1)$

$= x^2 + \frac{1}{4}x - x - \frac{1}{4}$

$= x^2 + \frac{1}{4}x - \frac{1 \times 4}{1 \times 4}x - \frac{1}{4}$

$= x^2 + \frac{1-4}{4}x - \frac{1}{4}$

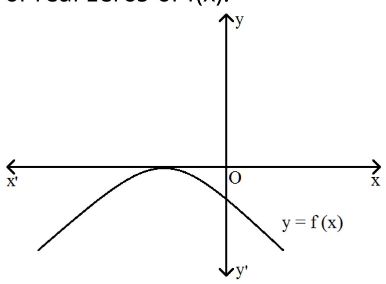
$= x^2 - \frac{3}{4}x - \frac{1}{4}$

Hence, the family of quadratic polynomials is $f(x) = k\left(x^2 - \frac{3}{4}x - \frac{1}{4}\right)$, where k is any non-zeros real number

13. If a quadratic polynomial f(x) is factorizable into linear distinct factors, then what is the total number of real and distinct zeros of f(x)?

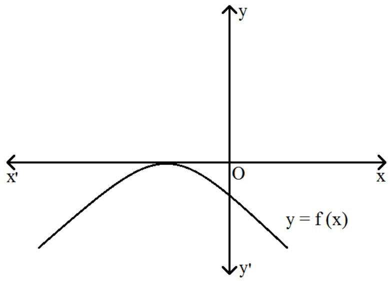
Ans. : In a quadratic polynomial f(x) its degree is 2 and it can be factorised in to two distinct linear factors. f(x) has two distinct zeros.

14. The graph of the polynomial $f(x) = ax^2 + bx + c$ is as shown in Fig. Write the value of $b^2 - 4ac$ and the number of real zeros of f(x).



Ans. : The graph of the polynomial $f(x) = ax^2 + bx + c$ or the curve touches x-axis at point $\left(\frac{-b}{2a}, 0\right)$.

The x-coordinate of this point given two equal zeros of the polynomial and $b^2 - 4ac = 0$.



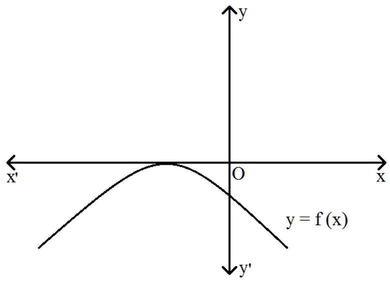
Hence the number of real zeros of f(x) is 2 and $b^2 - 4ac = 0$

15. Write the standard form of a quadratic polynomial with real coefficients.

Ans. : $ax^2 + bx + c$ is a standard form of quadratic polynomial with real co-efficients and $a \neq 0$.

16. The graph of the polynomial $f(x) = ax^2 + bx + c$ is as shown in Fig. Write the value of $b^2 - 4ac$ and the number of real zeros of f(x), write the sign of c.

Ans. : The parabola $y = ax^2 + bx + c$ cuts y-axis at point P which lies on OY.



Putting x = 0 in $y = ax^2 + bx + c$, We get y = c.
So, the coordinates of P are (0, c).
Clearly, P lies on OY'
Therefore c < 0

17. If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate: $\frac{1}{\alpha} - \frac{1}{\beta}$

Ans. : $\Rightarrow \frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta-\alpha}{\alpha\beta}$

$\Rightarrow \frac{1}{\alpha} - \frac{1}{\beta} = \frac{-(\alpha-\beta)}{\alpha\beta}$

$\Rightarrow \frac{1}{\alpha} - \frac{1}{\beta} = \frac{-\frac{1}{a}\sqrt{b^2-4ac}}{\frac{c}{a}}$

$\Rightarrow \frac{1}{\alpha} - \frac{1}{\beta} = \frac{-\sqrt{b^2-4ac}}{c}$

Thus, $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{-\sqrt{b^2-4ac}}{c}$

18. If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate: $\alpha - \beta$

Ans. : The two zerose of the polynomials are,

$\frac{-b+\sqrt{b^2-4ac}}{2a} - \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$

$= \frac{-b+\sqrt{b^2-4ac}+b+\sqrt{b^2-4ac}}{2a}$

$= \frac{2\sqrt{b^2-4ac}}{2a} = \frac{\sqrt{b^2-4ac}}{a}$

19. If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate: $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

Ans. : $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

$\Rightarrow \left[\frac{\alpha+\beta}{\alpha\beta}\right] - 2\alpha\beta$

$\Rightarrow \frac{-b}{a} \times \frac{a}{c} - 2\frac{c}{a} = -2\frac{c}{a} - \frac{b}{c} = \frac{-ab-2c^2}{ac} - \left[\frac{b}{c} + \frac{2c}{a}\right]$

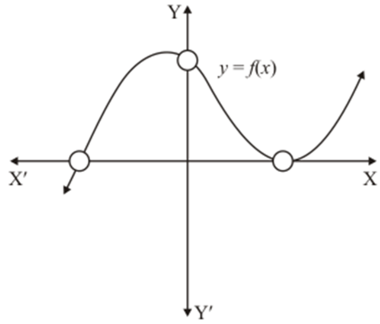
20. Write a quadratic polynomial, sum of whose zeros is $2\sqrt{3}$ and their product is 2.

Ans. : Sum of zeros $= 2\sqrt{3}$
and product of zeros = 2
Quadratic polynomial will be f(x) = x^2 - (sum of zeros) x + product of zeros $= x^2 - 2\sqrt{3}x + 2$

21. If the graph of quadratic polynomial ax^2 + bx + c cuts negative direction of y-axis, then what is the sign of c?

Ans. :
Since graph of quadratic polynomial f(x) = ax^2 + bx + c cuts negative direction of y-axis
So, put x = 0 to find the intersection point on y-axis y = 0 + 0 + c = c
So, the point is (0, c)
Now it is given that the quadratic polynomial cuts negative direction of y
So, c < 0

22. The graph of a polynomial y = f(x), shown in Fig. Find the number of real zeros of f(x).



Ans. : The curve touches x-axis at one point and also intersects at one point So number of zeros will be 3, two equal and one distinct.

23. The Sum and product of the zeros of a quadratic polynomial are $-\frac{1}{2}$ and -3 respectively. What is the quadratic polynomial?

Ans. : Sum of zeros $(\alpha + \beta) = \frac{-1}{2}$
and product of zeros $(\alpha \cdot \beta) = -3$
∴ Quadratic polynomial with be
k(x^2 - {sum of zeros} x + product of zeros}
$\Rightarrow k\left[x^2 - \left(-\frac{1}{2}\right)x + (-3)\right]$
$\Rightarrow k\left[x^2 + \frac{1}{2}x - 3\right]$

24. Very-Short-Answer Question:
If α and β are the zeroes of a polynomial $2x^2$ + 7x + 5, write the value of $\alpha + \beta + \alpha\beta$.

Ans. : Let α, β are the zeros of $2x^2$ + 7x + 5
Then we have
$\alpha + \beta = -\frac{7}{2}$
$\alpha\beta = \frac{5}{2}$
Hence,
$\alpha + \beta + \alpha\beta = (\alpha + \beta) + \alpha\beta$
$= -\frac{7}{2} + \frac{5}{2}$
$= \frac{-7+5}{2}$
$= \frac{-2}{2}$
$= -1$

25. Very-Short-Answer Question:
If 3 is a zero of the polynomial $2x^2$ + x + k, find the value of k.

Ans. : Since 3 is a zero of f(x) = $2x^2$ + x + k, we have
f(3) = 0
⇒ $2(3)^2$ + 3 + k = 0

⇒ 18 + 3 + k = 0
⇒ 21 + k = 0
⇒ k = -21

26. Very-Short-Answer Question:
If the sum of the zeros of the quadratic polynomial kx^2 - 3x + 5 is 1, write the value of k.

Ans. : Let f(x) = kx^2 - 3x + 5
Sum of zeros $= \frac{-(\text{coefficient of } x^2)}{\text{coefficient of x}}$
$= \frac{-(-3)}{k} = \frac{3}{k}$
Given, sum of zeros = 1
$\therefore \frac{3}{k} = 1$
$\Rightarrow k = 3$

27. Very-Short-Answer Question:
If -2 is a zero of the polynomial $3x^2$ + 4x + 2k then find the value of k.

Ans. : Since -2 is a zero of f(x) = $3x^2$ + 4x + 2K, we have
f(2) = 0
⇒ $3(-2)^2$ - 4(-2) + 2k = 0
⇒ 12 - 8 + 2k = 0
⇒ 4 + 2k = 0
⇒ 2k = -4
⇒ k = -2

28. If one zero of the polynomial $(a^2 + 9)x^2$ + 13x + 6a is the reciprocal of the other, find the value of a.

Ans. : $(a^2 + 9)x^2$ + 13x + 6a = 0
Here, A = $(a^2 + 9)$, B = 13 and C = 6a
Let α and $\frac{1}{\alpha}$ be the two zeros.
Then, product of the zeros $= \frac{C}{A}$
$\Rightarrow \alpha, \frac{1}{\alpha} = \frac{6a}{a^2+9}$
$\Rightarrow 1 = \frac{6a}{a^2+9}$
⇒ a^2 + 9 = 6a
⇒ a^2 - 6a + 9 = 0
⇒ a^2 - 2 × a × 3 + 3^2 = 0
⇒ $(a - 3)^2$ = 0
⇒ a - 3 = 0
⇒ a = 3

29. Very-Short-Answer Question:
If one zero of the quadratic polynomial kx^2 + 3x + k is 2 then find the value of k.

Ans. : Since 2 is a zero of f(x) = kx^2 + 3x + k, we have
f(2) = 0
⇒ $k(2)^2$ + 3(2) + k = 0
⇒ 4k + 6 + k = 0
⇒ 5k + 6 = 0
⇒ 5k = -6
$\Rightarrow k = -\frac{6}{5}$

30. Very-Short-Answer Question:
If 1 is a zero of the polynomial ax^2 - 3(a - 1)x - 1, then find the value of a.

Ans. : Since 1 is a zero of f(x) = ax^2 - 3(a - 1)x - 1, we have
f(1) = 0
⇒ $a(1)^2$ - 3(a - 1)1 - 1 = 0
⇒ a - 3a + 3 - 1 = 0
⇒ -2a + 2 = 0

$\Rightarrow 2a = 2$
$\Rightarrow a = 1$

31. Very-Short-Answer Question:

Write the zeros of the polynomial $x^2 - x - 6$

Ans. : We have,
$f(x) = x^2 - x - 6$
$= x^2 - 3x + 2x - 6$
$= x(x - 3) + 2(x - 3)$
$= (x + 2)(x - 3)$
$\therefore f(x) = 0$
$\Rightarrow (x + 2)(x - 3) = 0$
$\Rightarrow x + 2 = 0$ or $x - 3 = 0$
$\Rightarrow x = -2$ or $x = 3$
So, the zeros of f(x) are -2 and 3

32. Very-Short-Answer Question:

If one zero of the polynomial $x^2 - 4x + 1$ is $2+\sqrt{3}$. Write the other zero.

Ans. : Let the other zero of the polynomial $(x^2 - 4x + 1)$ be α.
Then, sum of roots $= \frac{-(-4)}{1}$
$\therefore 2+\sqrt{3}+\alpha = 4$
$\Rightarrow \alpha = 2-\sqrt{3}$
Thus, the other zero is $(2-\sqrt{3})$.

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