

PARTIAL FRACTIONS

1.3.1 DEFINITION

An expression of the form $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial in x , is called a rational fraction.

(1) **Proper rational functions:** Functions of the form $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomials and $g(x) \neq 0$, are called rational functions of x .

If degree of $f(x)$ is less than degree of $g(x)$, then $\frac{f(x)}{g(x)}$ is called a proper rational function.

Example: $\frac{x+2}{x^2+2x+4}$ is a proper rational function.

(2) **Improper rational functions :** If degree of $f(x)$ is greater than or equal to degree of $g(x)$, then $\frac{f(x)}{g(x)}$ is called an improper rational function.

For example: $\frac{x^3}{(x-1)(x-2)}$ is an improper rational function.

(3) **Partial fractions :** Any proper rational function can be broken up into a group of different rational fractions, each having a simple factor of the denominator of the original rational function. Each such fraction is called a partial fraction.

If by some process, we can break a given rational function $\frac{f(x)}{g(x)}$ into different fractions, whose denominators are the factors of $g(x)$, then the process of obtaining them is called the resolution or decomposition of $\frac{f(x)}{g(x)}$ into its partial fractions.

1.3.2 DIFFERENT CASES OF PARTIAL FRACTIONS

(1) **When the denominator consists of non-repeated linear factors:** To each linear factor $(x-a)$ occurring once in the denominator of a proper fraction, there corresponds a single partial fraction of the form $\frac{A}{x-a}$, where A is a constant to be determined.

If $g(x) = (x-a_1)(x-a_2)(x-a_3)\dots\dots(x-a_n)$, then we assume that, $\frac{f(x)}{g(x)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \dots\dots + \frac{A_n}{x-a_n}$

Where $A_1, A_2, A_3, \dots\dots, A_n$ are constants, can be determined by equating the numerator of L.H.S. to the numerator of R.H.S. (after L.C.M.) and substituting $x = a_1, a_2, \dots\dots, a_n$.

Note : $\square$ Remainder of polynomial $f(x)$, when divided by $(x-a)$ is $f(a)$.

e.g., Remainder of $x^2 + 3x - 7$, when divided by $x - 2$ is $(2)^2 + 3(2) - 7 = 3$.

$$\square \frac{px+q}{(x-a)(x-b)} = \frac{pa+q}{(x-a)(a-b)} + \frac{pb+q}{(b-a)(x-b)}$$

Example: 1 The remainder obtained when the polynomial $x^{64} + x^{27} + 1$ is divided by $(x+1)$ is

- (a) 1 (b) -1 (c) 2 (d) -2

Solution: (a) Remainder of $x^{64} + x^{27} + 1$, when divided by $x+1$ is $(-1)^{64} + (-1)^{27} + 1 = 1 - 1 + 1 = 1$.

Example: 2 If $\frac{2x+3}{(x+1)(x-3)} = \frac{a}{x+1} + \frac{b}{x-3}$, then $a+b$

- (a) 1 (b) 2 (c) $\frac{9}{4}$ (d) $\frac{-1}{4}$

Solution: (b) $2x+3 = a(x-3) + b(x+1)$

$$\text{Put } x = -1; \quad 2(-1)+3 = a(-1-3) \Rightarrow 1 = -4a \Rightarrow a = \frac{-1}{4}$$

$$\text{Now put } x = 3; \quad 2(3)+3 = b(3+1) \Rightarrow 9 = 4b \Rightarrow b = \frac{9}{4}$$

$$\text{Therefore, } a+b = \frac{-1}{4} + \frac{9}{4} = 2.$$

Example: 3 If $\frac{3x+a}{x^2-3x+2} = \frac{A}{x-2} - \frac{10}{x-1}$, then

- (a) $a=7$ (b) $a=-7$ (c) $A=-13$ (d) $A=13$

Solution: (a, d) $\frac{3x+a}{x^2-3x+2} = \frac{A}{x-2} - \frac{10}{x-1}$

$$\Rightarrow (3x+a) = A(x-1) - 10(x-2) \Rightarrow 3 = A-10, \quad a = -A+20 \quad (\text{On equating coefficients of } x \text{ and constant term})$$

$$\Rightarrow A = 13, \quad a = 7.$$

(2) **When the denominator consists of linear factors, some repeated:** To each linear factor $(x-a)$ occurring r times in the denominator of a proper rational function, there corresponds a sum of r partial fractions.

Let $g(x) = (x-a)^k(x-a_1)(x-a_2)\dots(x-a_r)$. Then we assume that

$$\frac{f(x)}{g(x)} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \dots + \frac{A_k}{(x-a)^k} + \frac{B_1}{(x-a_1)} + \dots + \frac{B_r}{(x-a_r)}$$

Where $A_1, A_2, \dots, A_k$ are constants. To determine the value of constants adopt the procedure as above.

Example: 4 If $\frac{3x+4}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$, then $A=$

- (a) $\frac{-1}{2}$ (b) $\frac{15}{4}$ (c) $\frac{7}{4}$ (d) $\frac{-1}{4}$

Solution: (c) We have, $\frac{3x+4}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$$\Rightarrow 3x+4 = A(x+1)^2 + B(x+1)(x-1) + C(x-1)$$

Putting $x=1$, we get $7 = A(2)^2 \Rightarrow A = \frac{7}{4}$.

Example: 5 The partial fraction of $\frac{x^2}{(x-1)^3(x-2)}$ are

(a) $\frac{-1}{(x-1)^3} + \frac{3}{(x-1)^2} - \frac{4}{(x-1)} + \frac{4}{(x-2)}$

(b) $\frac{-1}{(x-1)^3} - \frac{3}{(x-1)^2} + \frac{4}{(x-1)} + \frac{4}{(x-2)}$

(c) $\frac{-1}{(x-1)^3} + \frac{-3}{(x-1)^2} + \frac{-4}{(x-1)} + \frac{4}{(x-2)}$

(d) None of these

Solution: (c) Put the repeated factor $(x-1)=y \Rightarrow x=y+1$

$$\therefore \frac{x^2}{(x-1)^3(x-2)} = \frac{(1+y)^2}{y^3(y-1)} = \frac{1+2y+y^2}{y^3(-1+y)}$$

Dividing the numerator, $1+2y+y^2$ by $-1+y$ till y^3 appears as factor, we get

$$\frac{1+2y+y^2}{-1+y} = (-1-3y-4y^2) + \frac{4y^3}{-1+y}$$

$$\text{Given expression} = \frac{-1}{y^3} - \frac{3}{y^2} - \frac{4}{y} + \frac{4}{-1+y} = \frac{-1}{(x-1)^3} + \frac{-3}{(x-1)^2} + \frac{-4}{(x-1)} + \frac{4}{(x-2)}.$$

(3) **When the denominator consists of non-repeated quadratic factors:** To each irreducible non repeated quadratic factor $ax^2 + bx + c$, there corresponds a partial fraction of the form $\frac{Ax+B}{ax^2+bx+c}$, where A and B are constants to be determined.

Example : $\frac{4x^2+2x+3}{(x^2+4x+9)(x-2)(x+3)} = \frac{Ax+B}{x^2+4x+9} + \frac{C}{x-2} + \frac{D}{x+3}$

Note : $\square \frac{px+q}{x^2(x-a)} = \frac{-q}{ax^2} - \frac{pa+q}{a^2x} + \frac{pa+q}{a^2(x-a)}$

$\square \frac{px+q}{x(x-a)^2} = \frac{q}{a^2x} - \frac{q}{a^2(x-a)} + \frac{pa+q}{a(x-a)^2}$

$\square \frac{px+q}{x(x^2+a^2)} = \frac{q}{a^2x} + \frac{pa^2-qx}{a^2(x^2+a^2)}$

Example: 6 The partial fractions of $\frac{3x-1}{(1-x+x^2)(2+x)}$ are

(a) $\frac{x}{(x^2-x+1)} + \frac{1}{x+2}$

(b) $\frac{1}{x^2-x+1} + \frac{x}{x+2}$

(c) $\frac{x}{x^2-x+1} - \frac{1}{x+2}$

(d) $\frac{-1}{x^2-x+1} + \frac{x}{x+2}$

Solution: (c) $\frac{3x-1}{(1-x+x^2)(2+x)} = \frac{Ax+B}{x^2-x+1} + \frac{C}{x+2}$

$$\Rightarrow (3x-1) = (Ax+B)(x+2) + C(x^2-x+1)$$

Comparing the coefficient of like terms, we get $A+C=0$, $2A+B-C=3$, $2B+C=-1$

$$\Rightarrow A=1, B=0, C=-1$$

$$\therefore \frac{3x-1}{(1-x+x^2)(2+x)} = \frac{x}{x^2-x+1} - \frac{1}{x+2}.$$

Example: 7 If $\frac{(x+1)^2}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$, then $\sin^{-1}\left(\frac{A}{C}\right) =$

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

Solution: (a) $\frac{(x+1)^2}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$$\Rightarrow (x+1)^2 = A(x^2+1) + (Bx+C)x \Rightarrow A+B=1, C=2, A=1 \Rightarrow B=0$$

$$\text{Therefore } \sin^{-1}\left(\frac{A}{C}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}.$$

(4) **When the denominator consists of repeated quadratic factors:** To each irreducible quadratic factor ax^2+bx+c occurring r times in the denominator of a proper rational fraction there corresponds a sum of r partial fractions of the form.

$$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots + \frac{A_rx+B_r}{(ax^2+bx+c)^r}$$

Where, A 's and B 's are constants to be determined.

Example: 8 If $\frac{x}{(x-1)(x^2+1)^2} = \frac{1}{4}\left[\frac{1}{(x-1)} - \frac{x+1}{x^2+1}\right] + y$ then $y =$

(a) $\frac{(1-x)}{2(x^2+1)^2}$

(b) $\frac{(1-x)}{3(x^2+1)}$

(c) $\frac{1+x}{2(x^2-1)^2}$

(d) None of these

Solution: (a) $\frac{x}{(x-1)(x^2+1)^2} = \frac{1}{4}\left[\frac{1}{(x-1)} - \frac{x+1}{x^2+1}\right] + y$

$$\Rightarrow \frac{x}{(x-1)(x^2+1)^2} = \frac{1}{4}\left[\frac{1}{(x-1)} - \frac{x+1}{x^2+1}\right] + \frac{Ax+B}{(x^2+1)^2}$$

$$\Rightarrow 4x = (x^2+1)^2 - (x+1)(x-1)(x^2+1) + 4(Ax+B)(x-1)$$

$$\Rightarrow 4A+2=0, 4B-4A=4 \Rightarrow A=-\frac{1}{2}, B=\frac{1}{2}$$

$$\therefore y = \frac{Ax+B}{(x^2+1)^2} = \frac{1}{2} \frac{(1-x)}{(x^2+1)^2}$$

1.3.3 PARTIAL FRACTIONS OF IMPROPER RATIONAL FUNCTIONS

If degree of $f(x)$ is greater than or equal to degree of $g(x)$, then $\frac{f(x)}{g(x)}$ is called an improper rational function and every rational function can be transformed to a proper rational function by dividing the numerator by the denominator.

We divide the numerator by denominator until a remainder is obtained which is of lower degree than the denominator.

$$\text{i.e., } \frac{f(x)}{g(x)} = Q(x) + \frac{R(x)}{g(x)}, \text{ where degree of } R(x) < \text{degree of } g(x).$$

For example, $\frac{x^3}{x^2 - 5x + 6}$ is an improper rational function and can be expressed as $(x + 5) + \frac{19x - 30}{x^2 - 5x + 6}$ which is the sum of a polynomial $(x + 5)$ and a proper rational function $\frac{19x - 30}{x^2 - 5x + 6}$.

Example: 9 If $\frac{x^3 - 6x^2 + 10x - 2}{x^2 - 5x + 6} = f(x) + \frac{A}{(x-2)} + \frac{B}{(x-3)}$, then $f(x) =$

- (a) $x - 1$ (b) $x + 1$ (c) x (d) None of these

Solution: (a)
$$\begin{array}{r} x^2 - 5x + 6 \overline{) \begin{array}{r} x^3 - 6x^2 + 10x - 2 \\ x^3 - 5x^2 + 6x \\ \hline -x^2 + 4x - 2 \\ -x^2 + 5x - 6 \\ \hline + \quad - \quad + \\ \hline -x + 4 \end{array}} \end{array}$$

$\therefore f(x) = x - 1.$

1.3.4. GENERAL METHOD OF FINDING OUT THE CONSTANTS

- (1) Express the given fraction into its partial fractions in accordance with the rules written above.
- (2) Then multiply both sides by the denominator of the given fraction and you will get an identity which will hold for all values of x .
- (3) Equate the coefficients of like powers of x in the resulting identity and solve the equations so obtained simultaneously to find the various constant is short method. Sometimes, we substitute particular values of the variable x in the identity obtained after clearing of fractions to find some or all the constants. For non-repeated linear factors, the values of x used as those for which the denominator of the corresponding partial fractions become zero.

Note : $\square$ If the given fraction is improper, then before finding partial fractions, the given fraction must be expressed as sum of a polynomial and a proper fraction by division.

Important Tips

$\Rightarrow$ Some times a suitable substitution transforms the given function to a rational fraction which can be integrated by breaking it into partial fractions.

Example: 10 The coefficient of x^n in the expression $\frac{5x+6}{(2+x)(1-x)}$ when expanded in ascending order is

- (a) $\frac{-2}{3} \frac{(-1)^n}{2^n} + \frac{11}{3}$ (b) $\frac{2}{3} + \frac{(-1)^n}{2^n} - \frac{11}{3}$ (c) $-\frac{2}{3} + \frac{(-1)^n}{3} - \frac{11}{2^n}$ (d) None of these

Solution: (a)
$$\frac{5x+6}{(2+x)(1+x)} = \frac{-4}{2+x} + \frac{11}{1-x}$$

Rewriting the denominators for expressions, we get

$$\begin{aligned}
&= \frac{\frac{-4}{3}}{2\left(1+\frac{x}{2}\right)} + \frac{\frac{11}{3}}{1-x} = \frac{-2}{3}\left(1+\frac{x}{2}\right)^{-1} + \frac{11}{3}(1-x)^{-1} \\
&= \frac{-2}{3}\left[1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}+\dots+(-1)^n\frac{x^n}{2^n}+\dots\right] + \frac{11}{3}[1+x+x^2+\dots+x^n+\dots]
\end{aligned}$$

The coefficient of x^n in the given expression is $\frac{-2}{3}(-1)^n\frac{1}{2^n} + \frac{11}{3}$.

ASSIGNMENT

INTRODUCTION , Non – REPEATED LINEAR FACTORS IN DENOMINATION

Basic Level

1. The remainder obtained when the polynomial $1+x+x^3+x^9+x^{27}+x^{81}+x^{243}$ is divided by $x-1$ is
 (a) 3 (b) 5 (c) 7 (d) 11
2. If $\frac{1}{x(x+1)(x+2)\dots(x+n)} = \frac{A_0}{x} + \frac{A_1}{x+1} + \frac{A_2}{x+2} + \dots + \frac{A_n}{x+n}$ then $A_1 =$
 (a) $\frac{n!(-1)^r}{(n-r)!}$ (b) $\frac{(-1)^r}{n!(n-r)!}$ (c) $\frac{1}{n!(n-r)!}$ (d) None of these
3. $\frac{x+1}{(x-1)(x-2)(x-3)} =$
 (a) $\frac{1}{x-1} + \frac{3}{x-2} + \frac{1}{x-3}$ (b) $-\frac{3}{x-1} + \frac{1}{x-2} + \frac{2}{x-3}$
 (c) $\frac{1}{x-1} - \frac{3}{x-2} + \frac{2}{x-3}$ (d) None of these
4. If $\frac{ax^2+bx+c}{(x-1)(x+2)(2x+3)} = \frac{3}{x-1} + \frac{2}{x+2} - \frac{5}{2x+3}$, then
 (a) $a=5$ (b) $b=-18$ (c) $c=22$ (d) None of these
5. If $\frac{(e^x+2)}{(e^x-1)(2e^x-3)} = -\frac{3}{e^x-1} + \frac{B}{2e^x-3}$, then $B =$
 (a) 1 (b) 3 (c) 5 (d) 7
6. If $\frac{3x+4}{x^2-3x+2} = \frac{A}{x-2} - \frac{B}{x-1}$, then $(A, B) =$
 (a) (7, 10) (b) (10, 7) (c) (10, -7) (d) (-10, 7)

Advance Level

7. If the remainders of the polynomial $f(x)$ when divided by $x+1, x-2, x+2$ are 6, 3, 15 then the remainder of $f(x)$ when divided by $(x+1)(x+2)(x-2)$ is
 (a) $2x^2-3x+1$ (b) $3x^2-2x+1$ (c) $2x^2-x-3$ (d) $3x^2-2x+1$
8. If $\frac{1-\cos x}{\cos x(1+\cos x)} = \frac{\sin \alpha}{\cos x} - \frac{2}{1+\cos x}$, then $\alpha =$
 (a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π
9. If $\frac{x^2}{(x^2+a^2)(x^2+b^2)} = K \left(\frac{a^2}{x^2+a^2} - \frac{b^2}{x^2+b^2} \right)$ then $K =$
 (a) a^2-b^2 (b) $\frac{1}{a+b}$ (c) $\frac{1}{a-b}$ (d) $\frac{1}{a^2-b^2}$

REPEATED LINEAR FACTORS IN DENOMINATOR

Basic Level

10. If $\frac{9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ then $A - B - C =$
- (a) 3 (b) -1 (c) 5 (d) None of these
11. If $\frac{ax+b}{(3x+4)^2} = \frac{1}{3x+4} - \frac{3}{(3x+4)^2}$ then
- (a) $a = 2$ (b) $b = 1$ (c) $a = 3$ (d) $b = 4$
12. $\frac{x^2 + 13x + 15}{(2x+3)(x+3)^2} =$
- (a) $\frac{1}{x+3} - \frac{1}{2x+3} + \frac{5}{(x+3)^2}$ (b) $\frac{1}{2x+3} - \frac{1}{x+3} + \frac{5}{(x+3)^2}$ (c) $\frac{1}{2x+3} + \frac{1}{x+3} - \frac{5}{(x+3)^2}$ (d) $\frac{1}{2x+3} - \frac{1}{x+3} - \frac{5}{(x+3)^2}$
13. The partial fractions of $\frac{3x^3 - 8x^2 + 10}{(x-1)^4}$ is
- (a) $\frac{3}{(x-1)} + \frac{1}{(x-1)^2} + \frac{7}{(x-1)^3} + \frac{5}{(x-1)^4}$ (b) $\frac{3}{(x-1)} + \frac{1}{(x-1)^2} - \frac{7}{(x-1)^3} + \frac{5}{(x-1)^4}$
- (c) $\frac{3}{(x-1)} + \frac{1}{(x-1)^2} - \frac{7}{(x-1)^3} + \frac{5}{(x-1)^4}$ (d) None of these

Advance Level

14. The partial fractions of $\frac{x^4 + 24x^2 + 28}{(x^2 + 1)^3}$ are
- (a) $\frac{1}{(x^2 + 1)} + \frac{22}{(x^2 + 1)^2} + \frac{5}{(x^2 + 1)^3}$ (b) $\frac{1}{(x^2 + 1)} + \frac{22}{(x^2 + 1)^2} - \frac{5}{(x^2 + 1)^3}$
- (c) $\frac{1}{(x^2 + 1)} - \frac{22}{(x^2 + 1)^2} - \frac{5}{(x^2 + 1)^3}$ (d) None of these

NON — REPEATED QUADRATIC FACTORS IN DENOMINATOR

Basic Level

15. If $\frac{(x-1)^2}{x^3 + x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$, then
- (a) $A = 1, B = 0, C = 2$ (b) $A = 1, B = 0, C = -2$
- (c) $A = -1, B = 0, C = -2$ (d) None of these
16. If $\frac{2x}{x^3 - 1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$, then
- (a) $A = B = C$ (b) $A = B \neq C$ (c) $A \neq B = C$ (d) $A \neq B \neq C$

17. $\frac{x^2+1}{(2x-1)(x^2-1)} =$

(a) $\frac{-5}{3(2x-1)} + \frac{3}{(x+1)} + \frac{1}{(x-1)}$

(b) $\frac{-5}{3(2x-1)} + \frac{1}{3(x+1)} + \frac{1}{(x-1)}$

(c) $\frac{1}{2x-1} + \frac{5}{(x+1)} - \frac{3}{(x-1)}$

(d) None of these

18. If $\frac{ax-1}{(1-x+x^2)(2+x)} = \frac{x}{1-x+x^2} - \frac{1}{2+x}$, then $a =$

(a) 2

(b) 3

(c) 4

(d) 5

19. $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$, then $(A, B, C) =$

(a) (1, -1, 0)

(b) (-1, 0, -1)

(c) (0, 1, 1)

(d) None of these

Advance Level

20. $\frac{2x}{x^4+x^2+1} =$

(a) $\frac{x+1}{x^2-x+1} + \frac{x-1}{x^2+x-1}$

(b) $\frac{x-1}{x^2-x+1} - \frac{x+1}{x^2+x-1}$

(c) $\frac{x}{x^2-x+1} + \frac{x+1}{x^2+x-1}$

(d) $\frac{1}{x^2-x+1} - \frac{1}{x^2+x+1}$

REPEATED QUADRATIC FACTORS IN DENOMINATOR

Basic Level

21. $\frac{3x^2+5}{(x^2+1)^2} = \frac{a}{x^2+1} + \frac{b}{(x^2+1)^2}$, then $(a, b) =$

(a) (2, 3)

(b) (3, 2)

(c) (-2, 3)

(d) (-3, 2)

IMPROPER FRACTIONS

Basic Level

22. $\frac{(x-a)(x-b)}{(x-c)(x-d)} = \frac{A}{x-c} - \frac{B}{(x-d)} + C$, then $C =$

(a) 5

(b) 4

(c) 3

(d) 1

23. The partial fractions of $\frac{x^2-5}{x^2-3x+2}$ are

(a) $1 + \frac{1}{(x-1)} - \frac{1}{(x-2)^2}$

(b) $\frac{1}{(x-1)} + \frac{1}{(x-2)^2}$

(c) $\frac{1}{x-1} + \frac{1}{(x-2)^2}$

(d) $1 + \frac{4}{(x-1)} - \frac{1}{(x-2)}$

Advance Level

24. If $\frac{x^3}{(2x-1)(x+2)(x-3)} = p + \frac{q}{2x-1} + \frac{r}{x+2} + \frac{s}{x-3}$, then

(a) $p=1$

(b) $p=2$

(c) $p=\frac{1}{2}$

(d) $6q-3r+2s=3$

25. The partial fraction of $\frac{6x^4 + 5x^3 + x^2 + 5x + 2}{1 + 5x + 6x^2} =$

(a) $x^2 + \frac{1}{1+2x} + \frac{1}{1+3x}$

(b) $x^2 - \frac{1}{1+2x} + \frac{1}{1+3x}$

(c) $x^2 + \frac{1}{1+2x} - \frac{1}{1-3x}$

(d) None of these

26. If $\frac{\sin^2 x + 1}{2\sin^2 x - 5\sin x + 3} = \frac{A}{(2\sin x - 3)} + \frac{B}{(\sin x - 1)} + C$, then

(a) $A = \frac{13}{2}$

(b) $B = 2$

(c) $C = 1$

(d) $A + B + C = 5$

MISCELLANEOUS PROBLEMS

Basic Level

27. The coefficient of x^4 in the expansion of the expression $\frac{3x}{(x-2)(x+1)}$ is

(a) $-\frac{15}{16}$

(b) $\frac{15}{16}$

(c) $-\frac{16}{15}$

(d) $\frac{16}{15}$

28. The coefficient of x^5 in the expansion of $\frac{x^2 + 1}{(x^2 + 4)(x - 2)}$ is

(a) $\frac{1}{256}$

(b) $\frac{1}{562}$

(c) $\frac{1}{265}$

(d) $-\frac{1}{256}$

29. The coefficient of x^n in the expression $\frac{x-4}{x^2-5x+6}$ when expanded in ascending powers of x is

(a) $\frac{-1}{2^n} - \frac{1}{3^{n+1}}$

(b) $\frac{1}{2^n} - \frac{1}{3^{n+1}}$

(c) $\frac{-1}{2^n} + \frac{1}{3^{n+1}}$

(d) $\frac{-1}{2^n} + \frac{1}{3^{n+1}}$

ANSWER

LOGARITHMS

ASSIGNMENT (BASIC & ADVANCE LEVEL)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	c	b	c	d	c	d	b, c	c	c	c	d	b	c	b	b	d	c	d	a
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
d	c	b	a	a	c	c	a,b ,c, d	b	d	b	a,b, c	a	b	a	a	b	b	c	a,b, d
41	42																		
c	a																		

INDICES AND SURDS

ASSIGNMENT (BASIC & ADVANCE LEVEL)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	c	d	b	d	a	a	d	b	a,d	b	c	b	a,b	b,c	a	a,c, d	c	b	b
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	b	c	a	a	b	a	d	c	b	b	b	d	b	c	d	b	b	a	d
41	42	43	44	45															
d	b	a,b ,c	a	a,d															

PARTIAL FRACTIONS

ASSIGNMENT (BASIC & ADVANCE LEVEL)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	b	c	a,c	d	b	a	c	d	c	b,c	a	c	a	b	d	b	b	a	d
21	22	23	24	25	26	27	28	29											
b	d	d	c,d	a	a,d	b	d	c											

ANSWER

LOGARITHMS

ASSIGNMENT (BASIC & ADVANCE LEVEL)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	b	c	b	c	d	c	d	b, c	c	c	c	d	b	c	b	b	d	c	d	a
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	d	c	b	a	a	c	c	a,b ,c, d	b	d	b	a,b, c	a	b	a	a	b	b	c	a,b, d
	41	42																		
	c	a																		

INDICES AND SURDS

ASSIGNMENT (BASIC & ADVANCE LEVEL)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	c	c	d	b	d	a	a	d	b	a,d	b	c	b	a,b	b,c	a	a,c, d	c	b	b
	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	b	b	c	a	a	b	a	d	c	b	b	b	d	b	c	d	b	b	a	d
	41	42	43	44	45															
	d	b	a,b ,c	a	a,d															

PARTIAL FRACTIONS

ASSIGNMENT (BASIC & ADVANCE LEVEL)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	b	c	a,c	d	b	a	c	d	c	b,c	a	c	a	b	d	b	b	a	d
21	22	23	24	25	26	27	28	29											
b	d	d	c,d	a	a,d	b	d	c											