[10]

*** Choose the right answer from the given options. [1 Marks Each]**

1. Which of the following is a rational number:

(A) $\sqrt{180}$ (B) $\sqrt{31}$ (C) $\sqrt{196}$ (D) 0.323223222322223

Ans. :

c. $\sqrt{196}$

Solution:

Because it is the square of 14 and can be written in the form of $\frac{p}{q}$.

2. Simplified value of $(25)^{\frac{1}{3}} \times 5^{\frac{1}{3}}$ is:

(A) 25 (B) 3 (C) 1 (D) 5

Ans. :

d. 5

Solution:

$(25)^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5^{2\times\frac{1}{3}} \times 5^{\frac{1}{3}}$

$= 5^{\frac{2}{3}} \times 5^{\frac{1}{3}} = 5^{\frac{2}{3}+\frac{1}{3}} = 5^{\frac{3}{3}} = 5$

3. Which one of the following is a correct statement?

(A) Decimal expansion of a rational number is terminating.

(B) Decimal expansion of a rational number is non-terminating.

(C) Decimal expansion of an irrational number is terminating.

(D) Decimal expansion of an irrational number is non-terminating and non-repeating.

Ans. :

d. Decimal expansion of an irrational number is non-terminating and non-repeating.

Solution:

Decimal Expansion of a Rational number is not only terminating,

It can be either terminating like $\frac{1}{2} = 0.5$ or non-terminating Repeating like $\frac{1}{3} = 0.3333333.....$ So option (a) is not true alone.

Now we know that Non-Terminating numbers are of two types:

One is Non-Terminating Repeating and other is Non-Terminating Non-Repeating.

The Decimal expansion of a Rational number matches one of it's kind i.e Non-Terminating Repeating of Non-Terminating numbers.

So Rational number does not consist both the kinds of Non-Terminating numbers.

Hence, they are not Non-Terminating numbers.

An irrational number is always Non-Terminating in nature, but again not of both of it's kinds.

The decimal Expansion of an irrational number is Non-Terminating Non-Repeating in Nature.

So from all above points and theory we can conclude an Irrational number is Non-Terminating but Non-Repeating in nature

i.e. $\sqrt{2} = 1.4142135623730...$

So, option (d) is correct.

4. The sum of two irrational numbers is.

(A) Always an integer. (B) Always irrational. (C) Always rational. (D) Either irrational or rational.

Ans. :

d. Either irrational or rational.

Solution:

The sum of two irrational numbers, in some cases, will be irrational. However, if the irrational parts of the numbers have a zero sum (cancel each other out), the sum will be rational.

5. The value of $64^{-\frac{1}{3}}\left(64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right)$ is:

(A) 1 (B) 13 (C) -3 (D) -2

Ans. :

c. -3

Solution:

Find the value of $64^{\frac{1}{3}}\left(64^{\frac{1}{3}}-64^{\frac{2}{3}}\right)$

So,

$\Rightarrow 64^{\frac{1}{3}}\left(64^{\frac{1}{3}}-64^{\frac{2}{3}}\right)=2^{6\times\frac{1}{3}}\left(2^{6\times\frac{1}{3}}-2^{6\times\frac{2}{3}}\right)$

$=2^{-2}(2^2-2^4)$

$=2^2(4-16)$

$\Rightarrow 64^{\frac{1}{3}}\left(64^{\frac{1}{3}}-64^{\frac{2}{3}}\right)=\frac{1}{2^2}\times -12$

$=\frac{1}{4}\times -12$

$=-3$

Hence the correct statement is c.

6. The simplest form of $25^{\frac{1}{3}}\times 5^{\frac{1}{3}}$ is:

(A) 5 (B) 25 (C) None of these. (D) 125

Ans. :

a. 5

Solution:

$25^{\frac{1}{3}}\times 5^{\frac{1}{3}}$

$=5^{\frac{2}{3}}\times 5^{\frac{1}{3}}$

$=(5)^{\frac{2+1}{3}}\Leftrightarrow 5$

7. The $\frac{p}{q}$ form of the number 0.8 is:

(A) 1 (B) $\frac{1}{8}$ (C) $\frac{8}{10}$ (D) $\frac{8}{100}$

Ans. :

c. $\frac{8}{10}$

Solution:

$\frac{8}{10}$ or $\frac{4}{5}$

8. An irrational number between 5 and 6 is

(A) $\frac{1}{2}(5+6)$ (B) $\sqrt{5+6}$ (C) $\sqrt{5\times 6}$ (D) none of these

Ans. : (c)

We observe that $\frac{1}{2}(5+6)$ is a rational number between 5 and 6. So, option (a) is in correct.

$\sqrt{5+6}=\sqrt{11}=3.3166247\ldots$ is an irrational number not lying between 5 and 6.

$\sqrt{5\times 6}$ is an irrational number lying between $\sqrt{5}$ and $\sqrt{6}$.Hence, option (c) is correct.

9. The value of $\sqrt[4]{\sqrt[3]{2^2}}$ is:

a. $2^{-\frac{1}{6}}$

b. 2^{-6}

c. $2^{\frac{1}{6}}$

d. 2^6

Ans. :

c. $2^{\frac{1}{6}}$

Solution:

$\sqrt[4]{\sqrt[3]{2^2}}=\sqrt[4]{\sqrt[3]{4}}=\sqrt[4]{4^{\frac{1}{3}}}=4^{\frac{1}{3}\times\frac{1}{4}}=4^{\frac{1}{12}}=2^{2\times\frac{1}{12}}=2^{\frac{1}{6}}$

Hence, the correct answer is option (c).

10. The simplest for of $0.\overline{32}$ is:

a. $\frac{16}{45}$

b. $\frac{32}{99}$

c. $\frac{29}{90}$

d. None of these.

Ans. :

c. $\frac{29}{90}$

Solutions:

Let $x = 0.\overline{32}$
Then, $x = 0.3222 \ldots (i)$
$\therefore 10x = 3.222 \ldots (ii)$
and $100x = 32.222 \ldots (iii)$
On subtracting (ii) from (iii), we get
$90x = 29$
$\Rightarrow x = \frac{29}{90}$
Hence, the correct option is (c).

*** Answer the following short questions. [2 Marks Each]** **[10]**

11. Rationalise the denominator of the following:
$\frac{\sqrt{40}}{\sqrt{3}}$

Ans. : Let $E = \frac{\sqrt{40}}{\sqrt{3}}$
For rationalising the denominator, multiplying numerator and denominator by $\sqrt{3}$
$E = \frac{\sqrt{40}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{40\times3}}{(\sqrt{3})^2} = \frac{\sqrt{120}}{3}$
$= \frac{\sqrt{2\times2\times2\times5\times3}}{3} = \frac{2}{3}\sqrt{30}$

12. Find two rational and two irrational number between 0.5 and 0.55.

Ans. : The two rational numbers between 0.5 and 0.55 are: 0.51 and 0.52
The two irrational numbers between 0.5 and 0.55 are: 0.505005000... and 0.5101100111000...
Disclaimer: There are infinite number of rational and irrational numbers between 0.5 and 0.55.

13. Simplify:
$3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

Ans. : $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$
$3\sqrt{9\times5} - \sqrt{25\times5} + \sqrt{100\times2} - \sqrt{25\times2}$
$= 3\times3\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2}$
$= 9\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2}$
$= 4\sqrt{5} + 5\sqrt{2}$

14. Solve for $x\left(\frac{2}{5}\right)^{2x-2} = \frac{32}{3125}$.

Ans. : $x\left(\frac{2}{5}\right)^{2x-2} = \frac{32}{3125}$
$\Rightarrow \left(\frac{2}{5}\right)^{2x-2} = \frac{2^5}{5^5}$
$\Rightarrow \left(\frac{2}{5}\right)^{2x-2} = \left(\frac{2}{5}\right)^5$
$\Rightarrow 2x - 2 = 5$
$\Rightarrow 2x = 7$
$\Rightarrow x = \frac{7}{2}$

15. Examine whether the following numbers are rational or irrational.
$\sqrt[3]{5} \times \sqrt[3]{25}$

Ans. : $\sqrt[3]{5} \times \sqrt[3]{25}$
$= \sqrt[3]{5\times25}$
$= \sqrt[3]{125}$
$= 5$, which is an integer
Hence, $\sqrt[3]{5} \times \sqrt[3]{25}$ is rational.

*** Answer the following questions. [3 Marks Each]** **[6]**

16. Find the values of a and b in the following:
$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 2 - b\sqrt{6}$

Ans. : $LHS = \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$
$= \frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$
$= \frac{6+2\sqrt{6}+3\sqrt{6}+6}{18-12}$

$= \frac{12+5\sqrt{6}}{6} = 2\frac{5\sqrt{6}}{6}$

Now, $2 - b\sqrt{6} = 2 + \frac{5}{6}\sqrt{6} \Rightarrow b = -\frac{5}{6}$

17. Find two irrational numbers between 0.5 and 0.55.

Ans. : Let a = 0.5 = 0.50 and b = 0.55
We observe that in the second decimal place a has digit 0 and b has digit 5, therefore a < 0 so, if we consider irrational numbers
x = 0.51051005100051...
y = 0.530535305353530...
We find that a < x < y < b
Hence x and y are required irrational numbers.

*** Questions with calculation. [4 Marks Each]** **[4]**

18. If $x = 9 - 4\sqrt{5}$, find the value of $x^2 - \frac{1}{x^2}$.

Ans. : $x = 9 - 4\sqrt{5}$

$\Rightarrow \frac{1}{x} = \frac{1}{9-4\sqrt{5}} = \frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}}$

$= \frac{9+4\sqrt{5}}{9^2-\left(4\sqrt{5}\right)^2} = \frac{9+4\sqrt{5}}{81-80} = 9 + 4\sqrt{5}$

$\Rightarrow x + \frac{1}{x} = 9 - 4\sqrt{5} + 9 + 4\sqrt{5} = 18$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 18^2 = 324$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 + 2 \times x \times \frac{1}{x} = 324$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) + 2 = 324$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 322$

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