Introduction to Trigonometry
CLASS 10 - MATHEMATICS

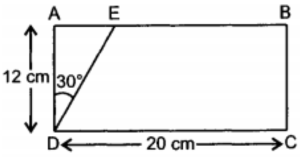
Time Allowed: 2 hours **Maximum Marks :** 235

Section A

1) Prove the trigonometric identity: $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$ [2]
2) Evaluate : $\frac{\cot(90^\circ-\theta)\sin(90^\circ-\theta)}{\sin\theta} + \frac{\cot 40^\circ}{\tan 50^\circ} - (\cos^2 20^\circ + \cos^2 70^\circ)$. [2]
3) If $\sin\alpha = \frac{1}{\sqrt{2}}$ and $\cot\beta = \sqrt{3}$, then find the value of cosec α + cosec β . [2]
4) Prove the trigonometric identity: $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \left(\cos^2\theta - \sin^2\theta\right)$ [2]
5) Prove the identity: $\frac{\sin^2\theta}{1-\cos\theta} = \frac{1+\sec\theta}{\sec\theta}$ [2]
6) Prove that: $\cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A = \cot^2 A - \cot^2 B$ [2]
7) Prove that: $(\operatorname{cosec} A - \cot A)^2 = \frac{1-\cos A}{1+\cos A}$ [2]
8) Prove (cosec A - sin A) (sec A - cos A) $= \frac{1}{\tan A + \cot A}$, where the angles involved are acute angles for which the expressions are defined. [2]
9) If A = B = 60°, verify that $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$ [2]
10) If tan A= $\frac{1}{2}$ and tan B = $\frac{1}{3}$, using tan (A + B) $= \frac{\tan A + \tan B}{1-\tan A\tan B}$, prove that A + B = 45°. [2]
11) Prove that: $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{cosecA - 1}{cosecA+1}$ [2]
12) Evaluate $2\sin^2 30^\circ \tan 60^\circ - 3\cos^2 60^\circ \sec^2 30^\circ$. [2]
13) Express the trigonometric ratio of sec A and tan A in terms of sin A. [2]
14) Prove that: $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} = \left(\frac{1+\sin\theta}{\cos\theta}\right)^2$ [2]
15) Prove that : $\frac{\tan A}{\sec A-1} + \frac{\tan A}{\sec A+1}$ = 2 cosec A. [2]
16) Prove that: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$ [2]
17) If $\cos\theta = \frac{3}{5}$, find the value of $\cot\theta + cosec\theta$ [2]
18) Prove the trigonometric identity: $\frac{\sin A+\cos A}{\sin A-\cos A} + \frac{\sin A-\cos A}{\sin A+\cos A} = \frac{2}{\sin^2 A-\cos^2 A} = \frac{2}{2\sin^2 A-1} = \frac{2}{1-2\cos^2 A}$ [2]
19) If $\tan\theta = \frac{20}{21}$, show that $\frac{(1-\sin\theta+\cos\theta)}{(1+\sin\theta+\cos\theta)} = \frac{3}{7}$. [2]
20) Prove the trigonometric identity: $\frac{1-\cos\theta}{1+\cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$ [2]
21) If $a\cos\theta - b\sin\theta = c$, prove that $(a\sin\theta + b\cos\theta) = \pm\sqrt{a^2+b^2-c^2}$ [2]
22) Prove the trigonometric identity: $\frac{\tan^2 A}{1+\tan^2 A} + \frac{\cot^2 A}{1+\cot^2 A} = 1$ [2]
23) If $\sin\theta + \cos\theta = \sqrt{2}$,then prove that $\tan\theta + \cot\theta = 2$. [2]
24) If 3 cot A = 4, find the value of $\frac{cosec^2 A+1}{cosec^2 A-1}$. [2]
25) If sin A $= \frac{\sqrt{3}}{2}$, find the value of $2\cot^2 A - 1$. [2]
26) Prove $\frac{\cos A - \sin A+1}{\cos A+\sin A-1}$ = cosec A + cot A, using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$. where the angles involved are acute angles for which the expressions are defined. [2]
27) If $\sin A = \frac{3}{4}$, calculate cosA and tanA [2]
28) If $\sin\theta + \cos\theta = \sqrt{3}$, then find the value of $\sin\theta \cdot \cos\theta$. [2]
29) If tan (A + B) $=\sqrt{3}$ and tan (A - B) = $\frac{1}{\sqrt{3}}$; 0° < A + B ≤ 90°; A > B, then find A and B. [2]
30) Prove that: $\sec^4 A(1 - \sin^4 A) - 2\tan^2 A = 1$ [2]

Section B

31) When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$. [3]
32) If $\sec\theta + \tan\theta = p$, prove that $\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$ [3]
33) In a right triangle ABC, right - angled at B, if tan A = 1, then verify that 2 sin A cos A = 1. [3]
34) Given that 16 cot A = 12; find the value of $\frac{\sin A+\cos A}{\sin A-\cos A}$. [3]
35) Prove the trigonometric identity: $\sec^6\theta = \tan^6\theta + 3\tan^2\theta \sec^2\theta + 1$ [3]
36) If $\sin A = \frac{1}{3}$, evaluate cos A cosec A + tan A sec A. [3]
37) If tan A = n tanB and sin A = m sinB, then prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$ [3]
38) A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732][3]
39) If, $\sin\theta = \frac{a^2-b^2}{a^2+b^2}$ find the values of the other five trigonometric ratios. [3]
40) If $\sin\theta + \cos\theta = \sqrt{2}$, then evaluate $\tan\theta + \cot\theta$. [3]
41) If $\sin\theta = \frac{4}{5}$, what is the value of $\cot\theta + cosec\theta$? [3]
42) Prove: $\frac{1}{(\cot A)(\sec A)-\cot A} - \operatorname{cosec} A = \operatorname{cosec} A - \frac{1}{(\cot A)(\sec A)+\cot A}$ [3]
43) Prove the identity: $\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A - cosec^3 A} = \sin^2 A\cos^2 A$ [3]
44) If $\sec\theta = \frac{5}{4}$, find the value of $\frac{\sin\theta-2\cos\theta}{\tan\theta-\cot\theta}$. [3]
45) If $cosecA = \sqrt{10}$ find other five trigonometric ratios. [3]
46) In $\triangle ABC$, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$. Find the value of cos A cos C - sin A sin C [3]
47) If cosB= $\frac{1}{3}$ find the other five trigonometric ratios. [3]
48) Prove that $\sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta) = 1$ [3]
49) Evaluate: $\tan^2 30^\circ + \tan^2 60^\circ + \tan^2 45^\circ$ [3]
50) If $\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right) = 1$ and $\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right) = 1$, prove that $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 2$ [3]
51) In the given figure, ABCD is a rectangle with AD = 12 cm and DC = 20 cm. Line segment DE is drawn making an angle of 30° with AD, intersecting AB at E. Find the length of DE and AE.



[3]
52) Find the value of the trigonometric ratios if: $cosec\theta = \sqrt{10}$ [3]
53) $\angle A$ and $\angle B$ are acute angles such that cos A = cos B, then show that $\angle A = \angle B$. [3]
54) In△ ABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:
    i. Sin A cos A
    ii. Sin C cos C

    [3]
55) If $\cos\theta = \frac{12}{13}$, show that $\sin\theta(1-\tan\theta) = \frac{35}{156}$ [3]

Section C

56) If cos A - sin A = m and cos A + sin A = n. Show that: $\frac{m^2-n^2}{m^2+n^2}$ = - 2 sin A. cos A $= -\frac{2}{\tan A+\cot A}$ [5]

57) Evaluate: $\frac{\tan^2 60^\circ+4\sin^2 45^\circ+3\sec^2 60^\circ+5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ+\sec 60^\circ-\cot^2 30^\circ}$ [5]

58) Prove that: $\frac{\sec A-\tan A}{\sec A+\tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$ [5]

59) If $\tan\theta = \frac{a}{b}$, find the value of $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$. [5]

60) If secθ = $\frac{5}{4}$, verify that $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\sin\theta}{\sec\theta}$. [5]

61) If $\cot\theta = \frac{1}{\sqrt{3}}$, show that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$ [5]

62) If a cosθ +b sin θ =m and a sin θ - b cos θ =n, prove that m^2 + n^2 = a^2 + b^2. [5]

63) Prove that: $\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}$ = tan θ + cot θ [5]

64) Prove the following identity: $\frac{\sin A}{\sec A+\tan A-1} + \frac{\cos A}{cosecA+\cot A-1}$ = 1 [5]

65) Prove the trigonometric identity:
$2\sec^2\theta - \sec^4\theta - 2\cos ec^2\theta + \cos ec^4\theta = \cot^4\theta - \tan^4\theta$ [5]

66) Prove that: (1 - sinθ + cos θ)^2 = 2(1 + cos θ) (1 - sin θ). [5]

67) If cosec □ + cot □ = p, then prove that cos □ $=\frac{p^2-1}{p^2+1}$. [5]

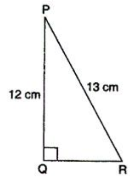
68) Prove that : (sin A + sec A)^2 + (cos A + cosec A)^2 = (1 + sec A cosec A)^2. [5]

69) Evaluate: $4\left(\sin^4 60^\circ+\cos^4 30^\circ\right)-3\left(\tan^2 60^\circ-\tan^2 45^\circ\right)+5\cos^2 45^\circ$ [5]

70) Prove that: $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$ = sec θ cosec θ - 2 sin θ cos θ . [5]

71) If cosecθ - sinθ = l and secθ - cosθ = m, prove that l^{2}m^2(l^2 + m^2 + 3) = 1. [5]

72) In figure, find tan P - cot R.



[5]

73) If $\sec A = \frac{17}{8}$, verify that $\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$. [5]

74) Prove that: $\left(\frac{1}{\sec^2\theta-\cos^2\theta} + \frac{1}{cosec^2\theta-\sin^2\theta}\right)\sin^2\theta\cos^2\theta = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$ [5]

75) In aΔABC right angled at B, $\angle A = \angle C$. find the values of
   i. Sin A cos C + cos A sinC.
   ii. Sin A sin B + cos A cosB.

   [5]