[147]

*** Questions With Calculation.[3 Marks Each]**

1. Find the value of :$(2^{-1} \times 4^{-1}) \div 2^{-2}$

Ans. : $(2^{-1} \times 4^{-1}) \div 2^{-2}$

$= \{2^{-1} \times (2^2)^{-1}\} \div 2^{-2}$

$= \{2^{-1} \times 2^{2 \times (-1)}\} \div 2^{-2}$

$= (2^{-1} \times 2^{-2}) \div 2^{-2}$

$= 2^{(-1)} \times 2^{(-2)} \div 2^{-2}$

$= 2^{-3} \div 2^{-2}$

$= \frac{2^{-3}}{2^{-2}}$

$= \frac{1}{2^{(-2)-(-3)}}$

$= \frac{1}{2^{-2+3}}$

$= \frac{1}{2^1}$

$= \frac{1}{2}$

2. Evaluate :$\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Ans. : $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

$= \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}}$

$= \frac{5^{-7}}{5^{-4}} \times \frac{8^{-4}}{8^{-7}}$

$= 5^{(-7)-(-4)} \times 8^{(-4)-(-7)}$

$= 5^{-7+4} \times 8^{-4+7}$

$= 5^{-3} \times 8^3$

$\frac{1}{5^3} \times 8^3$

$= \frac{8^3}{5^3}$

$= \frac{512}{125}$

$= 4\frac{12}{125}$

3. Find the value of :$\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Ans. : $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

$= \left(\frac{-2}{3}\right)^{(-2)\times 2}$

$= \left(\frac{-2}{3}\right)^{-4}$

$= \frac{(-2)^{-4}}{(3)^{-4}}$

$= \frac{3^4}{(-2)^4}$

$= \frac{3\times3\times3\times3}{(-2)\times(-2)\times(-2)\times(-2)}$

$= \frac{81}{16}$

$= 5\frac{1}{16}$

4. Find the value of :$\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

Ans. : $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^2$

$= \frac{1^{-2}}{2^{-2}} + \frac{1^{-2}}{3^2} + \frac{1^2}{4}$

$= \frac{2^2}{1^2} + \frac{3^2}{1^2} + \frac{4^2}{1^2}$

$\frac{4}{1}+\frac{9}{1}+\frac{16}{1}$
= 4 + 9 + 16
= 29

5. In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Ans. : Total thickness of books = 5 × 20 mm = 100 mm
Total thickness of paper sheets = 5 × 0.016 mm = 0.080 mm
∴ Total thickness of the stack
= Total thickness of books + Total thickness of paper sheets
= 100 mm + 0.080 mm
= (100 + 0.080) mm
= 100.080 mm
= 1.0008×10^2 mm.

6. $\frac{125\times x^{-3}}{5^{-3}\times 25\times x^{-6}}$

Ans. : Using laws of exponents, $a^m \div a^n = (a)^{n-m}$ and $a^{-m}=\frac{1}{a^m}$
∴ $\frac{125\times x^{-3}}{5^{-3}\times 25\times x^{-6}}$
$=(5)^3\times 5^3\times 5^{-2}\times x^{-3}\times x^6$ [∵ 125 = 5 × 5 × 5 and 25 = 5 × 5]
$=5^4\times x^3$
$=5\times 5\times 5\times 5\times x^3$
$=625x^3$ [∵ $a^m \times a^n = a^{m+n}$]

7. $5^x + 5^{x-1} = 750$

Ans. : $5x+5x-1=750$
$\Rightarrow 5^x+\frac{5x}{5}=750\ \left[\because a^{-m}=\frac{1}{a^m}\right]$
$\Rightarrow 5^x\left(1+\frac{1}{5}\right)=750$
$\Rightarrow 5^x\left(\frac{6}{5}\right)=750$
$\Rightarrow 5^x=750\times\frac{5}{6}$
$\Rightarrow 5^x=125\times 5$
$\Rightarrow 5^x=625$
$\Rightarrow 5^x=5^4$
On compairing the powers of 5, we get
$x=4$

8. About 230 billion litres of water flows through a river each day. How many litres of water flows through that river in a week? How many litres of water flows through the river in an year? Write your answer in standard notation.

Ans. : Water flows through a river in each day = 230000000000 or 230 billion.
Water flows through the river in a week = 7 × 230000000000 ∵ $[1\text{week}=7\text{days}]$
= 1610000000000
= 1610 billion
= 1.61×10^{12} L
Water flows through the river in an year = 230000000000 × 365.∵ $[1\text{year}=365\text{days}]$
= 83950000000000
= 8.395×10^{13} L

9. Express the following in standard form:
Express 2 years in seconds.

Ans. :
Given,
2 year = 2 × 365 days [∵ 1 yr = 365 days approx.)
= 2 × 365 × 24h [∵ 1 day = 24h]
= 2 × 365 × 24 × 60 min [∵ 1 h = 60 min]
= 2 × 365 × 24 × 60 × 60s [∵ 1 min = 60 s)
= 63072000s
Standard form of 63072000
= 63072 × 10 × 10 × 10
= 63072×10^3 [∵ $a^m \times a^n = (a)^{m+n}$]

$= 63072 \times 10^3 \times 10^4$

$= 63072 \times 10^7$s

10. Find the value of x, so that:

$(-2)^3 \times (-2)^{-6} = (-2)^{2x-1}$

Ans. : We have, $(-2)^3 \times (-2)^{-6} = (-2)^{2x-1}$

Using law of exponents, $a^m \times a^n = (a)^{m+n}$ [$\because$ a is non-zero integer]

Then, $(-2)^3 \times (-2)^{-6} = (-2)^{2x-1}$

$\Rightarrow (-2)^{3-6} = (-2)^{2x-1}$

$\Rightarrow (-2)^{-3} = (-2)^{2x-1}$

On camparing both side, we get

$-3 = 2x - 1$

$\Rightarrow 2x = -2$

$\Rightarrow x = -1$

11. Find three repeater machines that will do the same work as a (x64) machine. Draw them, or describe them using exponents.

Ans. :

We know that, the possible factors of 64 are 2, 4, 8:

If $2^6 = 64$, $4^3 = 64$ and $8^2 = 64$.

Hence, three repeater machines that would work as a (x64) will be ($x2^6$), ($x4^3$) and ($x8^2$). The diagram of ($x2^6$), ($x4^3$) and ($x8^2$) is given below.

12. Find the value of x, so that:

$(2^{-1} + 4^{-1} + 6^{-1} + 8^{-1})^x = 1$

Ans. : We have, $(2^{-1} + 4^{-1} + 6^{-1} + 8^{-1})^x = 1$

Using law of exponents, $a^{-m} = \frac{1}{a^m}$ [$\because$ a is non-zero integer]

Then, $\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right)^x = 1$

$\Rightarrow \left(\frac{12+6+4+3}{24}\right)^x = 1$ [$\because$ LCM of 2, 4, 6 and 8 = 24]

$\Rightarrow \left(\frac{25}{24}\right)^x = 1$

This can be possible only if x = 0.

Since $a^0 = 1$. [law of exponents]

13. Simplify:

$\frac{(3^{-2})^2 \times (5^2)^{-3} \times (t^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3}$

Ans. : $\frac{(3^{-2})^2 \times (5^2)^{-3} \times (t^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3} = \frac{(3)^{-4} \times (5)^{-6} \times t^{-6}}{(3)^{-10} \times (5)^{-6} \times (t)^{-12}}$ $[\because (a^m)^n = (a)^{mn}]$

$= (3)^{-4} \times (3)^{10} \times (5)^{-6} \times (5)^6 \times (t)^{-6} \times (t)^{12}$

$= (3)^{-4+12} \times (5)^{-6+6} \times (t)^{-6+12}$ $\left[\because a^{-m} = \frac{1}{a^m}\right]$

$= (3)^6 \times 5^0 \times (t)^6 = (3t)^6$ $[\because a^0 = 1]$

14. $\frac{16 \times 10^2 \times 64}{2^4 \times 4^2}$

Ans. : Using laws of exponents, $a^m \div a^n = (a)^{m-n}$ and $a^m \times a^n = a^{m+n}$ [$\because$ a is non-zero integer]

$\therefore \frac{16 \times 10^2 \times 64}{2^4 \times 4^2}$

$= (4)^2 \times 10^2 \times 2^{-4} \times (4)^3 \times 4^{-2}$ [$\because$ 64 = 4 × 4 × 4 and 16 = 4 × 4]

$= (2^2)^3 \times 10^2 \times 2^{-4}$

$= 2^6 \times 10^2 \times 2^{-4}$

$= 2^2 \times 10^2$

$= 4 \times 100$ [$\because 2^2 = 4$]

$= 400$

15. Simplify:

$\left(\frac{4}{13}\right)^4 \times \left(\frac{13}{7}\right)^2 \times \left(\frac{7}{4}\right)^3$

Ans. : $\left(\frac{4}{13}\right)^4 \times \left(\frac{13}{7}\right)^2 \times \left(\frac{7}{4}\right)^3 = \frac{(4)^4}{(13)^4} \times \frac{(13)^2}{(7)^2} \times \frac{(7)^3}{(4)^3}$

$= (4)^4 \times (4)^{-3} \times (13)^2 \times (13)^{-4} \times (7)^3 \times (7)^{-2} \quad \left[\because a^{-m} = \frac{1}{a^m}\right]$

$= (4)^{4-3} \times (13)^{2-4} \times (7)^{3-2} \quad [\because a^m \times a^n = a^{m-n}]$

$= (4)^1 \times (13)^{-2} \times (7)^1$

$= 4 \times \frac{1}{169} \times 7 \quad \left[\because a^{-m} = \frac{1}{a^m}\right]$

$= \frac{28}{169}$

16. Find the value of x, so that:

$\left(\frac{5}{3}\right)^{-2} \times \left(\frac{5}{3}\right)^{-14} = \left(\frac{5}{3}\right)^{8x}$

Ans. : We have, $\left(\frac{5}{3}\right)^{-2} \times \left(\frac{5}{3}\right)^{-14} = \left(\frac{5}{3}\right)^{8x}$

Using law of exponents, $a^m \times a^n = (a)^{m+n}$ [$\because$ a is non-zero integer]

Then, $\left(\frac{5}{3}\right)^{-2} \times \left(\frac{5}{3}\right)^{-14} = \left(\frac{5}{3}\right)^{8x}$

$\Rightarrow \left(\frac{5}{3}\right)^{-2-14} = \left(\frac{5}{3}\right)^{8x}$

$\Rightarrow \left(\frac{5}{3}\right)^{-16} = \left(\frac{5}{3}\right)^{8x}$

On comparing both side, we get

$16 = 8x$

$\Rightarrow x = -2$

17. If $5^{3x-1} \div 25 = 125$, find the value of x.

Ans. : Given,

$5^{3x-1} \div 25 = 125$

$\because 25 = 5 \times 5 = 5^2$

and $125 = 5 \times 5 \times 5 = 5^3$

$\therefore 5^{3x-1} \div (5)^2 = (5)^3$

$\Rightarrow (5)^{3x-1-2} = 5^3$ ($\because a^m \div a^n = (a)^{m-n}$)

$\Rightarrow (5)^{3x-3} = (5)^3$

On camparing both sides

18. Find the value of x^{-3} if $x = (100)^{1-4} \div (100)^0$.

Ans. : Given,

$x = (100)^{1-4} \div (100)^0$

$\Rightarrow x = (100)^{-3} \div (100)^0$

$\Rightarrow x = (100)^{-3-0}$ [$\because a^m \div a^n = (a)^{m-n}$]

$\Rightarrow x = (100)^{-3}$

So, $x^{-3} = [(100)^{-3}]^{-3}$

$= (100)^9$ [$\because (a^m)^n = (a)^{mn}$]

19. By what number should $\left(\frac{-3}{2}\right)^{-3}$ be divided so that the quotient may be $\left(\frac{4}{27}\right)^{-2}$?

Ans. : Let $\left(\frac{-3}{2}\right)^{-3}$ be divided by x to get $\left(\frac{4}{27}\right)^{-2}$ as quotient.

Then, $\left(\frac{-3}{2}\right)^{-3} \div x = \left(\frac{4}{27}\right)^{-2}$

$\Rightarrow x = \left(\frac{-3}{2}\right)^{-3} \div \left(\frac{2^2}{3^3}\right)^{-2}$

$= \left(\frac{-3}{2}\right)^{-3} \div \frac{(2)^{-4}}{(3)^{-6}}$

$= \left(\frac{-3}{2}\right)^{-3} \times \frac{(3)^{-6}}{(2)^{-4}}$

$= \frac{(-3)^{-3} \times (3)^{-6}}{2^{-3} \times 2^{-4}} = \frac{3^{-9}}{2^{-7}}$ [$\because a^m \times a^n = a^{m+n}$]

$= \frac{2^7}{3^9} \left[\because a^{-m} = \frac{1}{a^m} \text{ and } (a^m)^n = (a)^{mn}\right]$

20. Find x.

$-\frac{2}{5}^{2x+6} \times \frac{2}{5}^{3} = \frac{2}{5}^{x+2}$

Ans. : We have, $\frac{2}{5}^{2x+6} \times \frac{2}{5}^{3} = \frac{2}{5}^{x+2}$

Using law of exponents, $a^m \times a^n = (a)^{m+n}$ [$\because$ a is non-zero integer]

Then, $\left(\frac{2}{5}\right)^{2x+6+3} = \left(\frac{2}{5}\right)^{x+2}$

On compairing powers of $\left(\frac{2}{5}\right)$, we get.

2x + 6 + 3 = x + 2

$\Rightarrow$ 2x + 9 = x + 2

$\Rightarrow$ x = -7

21. Find x.

$\frac{-6^{x-7}}{7} = 1$

Ans. : We have, $\left(-\frac{6}{7}\right)^{x-7} = 1$

Using law of exponents, $x^0 = 1$[$\because$ a is non-zero integer]

Then, $\left(-\frac{6}{7}\right)^{x-7} = 1$

it is possible only, if x = 7

So, $\left(\frac{-6}{7}\right)^{7-7} = 1$

$\Rightarrow \left(-\frac{6}{7}\right)^{0} = 1$ [$\because a^0 = 1$]

Hence, $x = 7$

22. Find x.

$-\frac{1}{7}^{-5} + -\frac{1}{7}^{-7} = (-7)^x$

Ans. : Using law of exponents, $a^m + a^n = (a)^{m-n}$ [$\because$ a is non-zero integer]

Then, $\left(-\frac{1}{7}\right)^{-5+7} = (-7)^x$

$\Rightarrow \left(\frac{-1}{7}\right)^2 = (-7)^x$

$\Rightarrow (-7)^2 = (-7)^x$

On comparing powers of (-7), we get x = -2

23. If $\frac{5^m \times 5^3 \times 5^{-2}}{5^{-5}} = 5^{12}$, then find m.

Ans. : Given,

$\frac{5^m \times 5^3 \times 5^{-2}}{5^{-5}} = 5^{12}$

Using laws of exponents, $a^m \div a^n = (a)^{m-n}$ and $a^{-m} = \frac{1}{a^n}$ [$\because$ a is non-zero integer]

Then,

$5^m \times 5^3 \times 5^{-2} \times 5^5 = 5^{12}$

$\Rightarrow 5^m \times 5^8 \times 5^{-2} = 5^{12}$

$\Rightarrow 5^m \times 5^6 = 5^{12}$

$\Rightarrow 5^{m+6} = 5^{12}$ [$\because a^m \times a^n = a^{m+n}$]

On camparing both sides, we get

m + 6 = 12

$\Rightarrow$ m = 6

24. find the value of n.

$\frac{2^n \times 2^6}{2^{-3}} = 2^{18}$

Ans. : Given,

$\frac{2^n \times 2^6}{2^{-3}} = 2^{18}$

Using law of exponents, $a^{-m} = \frac{1}{a^m}$ [$\because$ a is non-zero integer]

$\Rightarrow 2^n \times 2^6 \times 2^3 = 2^{18}$

$\Rightarrow 2^{n+9} = 2^{18}$ [$\because a^m \times a^n = a^{m+n}$]

On camparing both side, we get

n + 9 = 18 [$\because a^m \div a^n = (a)^{m-n}$]

$\Rightarrow$ n = 9

25. Simplify:

$\left(\left(\frac{-2}{3}\right)^{-2}\right)^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

Ans. : Using law of exponents, $(a^m)^n = (a)^{m\times n}$, $a^{-m} = \frac{1}{a^m}$,$a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n}$ [$\because$ a is non-zero integer]

$\therefore \left(\left(\frac{-2}{3}\right)^{-2}\right)^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

$= \left(\frac{-2}{3}\right)^{(-2)\times 3} \times (3)^4 \times \frac{1}{3} \times \frac{1}{6}$

$= \left(\frac{-2}{3}\right)^{-6} \times 3^4 \times \frac{1}{3} \times \frac{1}{2\times 3}$ [$\because$ 6 = 2 × 3]

$= \left(\frac{3}{-2}\right)^6 \times 3^4 \times \frac{1}{3\times 2\times 3}$

$= \frac{(3)^6}{(-2)^6} \times 3^4 \times \frac{1}{2^1\times 3^2}$

$= \frac{(3)^{6+4}}{(2)^{6+1}\times 3^2}$ [$(-a^m) = a^m$, if m is even number]

$= \frac{(3)^{10}}{2^7\times 3^2} = \frac{3^{10-2}}{2^7}$

$= \frac{3^8}{2^7}$

26. Find × so that $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2x-1}$

Ans. : Given,

$\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2x-1}$

Using law of exponent, $a^m \times a^n = (a)^{m+n}$

Then, $\left(\frac{2}{9}\right)^{3-6} = \left(\frac{2}{9}\right)^{2x-1}$

$\Rightarrow \left(\frac{2}{9}\right)^{-3} = \left(\frac{2}{9}\right)^{2x-1}$

On camparing, we get

-3 = 2× - 1

$\Rightarrow$ -2 = 2×

$\Rightarrow$ × = -1

27. Find x.

$2^x + 2^x + 2^x = 192$

Ans. : $\Rightarrow 2^x(1+1+1) = 192$

$\Rightarrow 3 \times (2^x) = 192$

$\Rightarrow 2x = \frac{192}{3} = 64$

$\Rightarrow 2x = 2\times 2\times 2\times 2\times 2\times 2$

$\Rightarrow 2x = 2^6$

On compairing the powers of 2, we get x = 6

28. By what number should $(-15)^{-1}$ be divided so that quotient may be equal to $(-5)^{-1}$?

Ans. : Let $(-15)^{-1}$ be divided by x to get quotient $(-15)^{-1}$.

So, $\frac{(-15)^{-1}}{x} = (-15)^{-1}$

$\Rightarrow \frac{(-15)^{-1}}{(-15)^{-1}} = x$

$\Rightarrow x = (-15)^{-1+1}$ [$\because a^m + a^n = (a)^{m-n}$]

$\Rightarrow x = (-15)^0 = 1$ [$\because a^0 = 1$]

29. The number of red blood cells per cubic millimetre of blood is approximately 5.5 million. If the average body contains 5 litres of blood, what is the total number of red cells in the body? Write the standard form. (1 litre = 1,00,000mm^3)

Ans. : The average body contain 5L of blood.

Also, the number of red blood cells per cubic millimetre of blood is approximately 5.5 million.

Blood contained by body = 5L = 5 × 100000mm^3

Red blood cells = $5 \times 100000 mm^3$

Blood = $5.5 \times 1000000 \times 5 \times 100000$

$= 55 \times 5 \times 10^{5+5}$

$= 275 \times 10^{10}$

$= 2.75 \times 10^{10} \times 10^2$

$= 2.75 \times 10^{12}$

30. By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplied so that the product may be equal to $\left(\frac{-4}{7}\right)^{-1}$?

Ans. : Let x be multiplied, the

$x \times \left(\frac{1}{2}\right)^{-1} = \left(\frac{-4}{7}\right)^{-1}$

$x = \left(\frac{-4}{7}\right)^{-1} \div \left(\frac{1}{2}\right)^{-1}$

$= \left(\frac{7}{-4}\right)^{1} \div \left(\frac{2}{1}\right)^{1} = \frac{7}{-4} \div 2$

$= \frac{7}{-4} \times \frac{1}{2} = \frac{7}{-8} = \frac{-7}{8}$

$\therefore$ Required number $= \frac{-7}{8}$

31. Find x, if

$\left(\frac{8}{3}\right)^{2x+3} \times \left(\frac{8}{3}\right)^{5} = \left(\frac{8}{3}\right)^{x+2}$

Ans. : $\left(\frac{8}{3}\right)^{2x+3} \times \left(\frac{8}{3}\right)^{5} = \left(\frac{8}{3}\right)^{x+2}$

$\Rightarrow \left(\frac{8}{3}\right)^{2x+1+5} = \left(\frac{2}{5}\right)^{2+3x}$

Comaring, we get:

$2x + 1 + 5 = x+2$

$\Rightarrow 2x - x = 2 - 1 - 5 \Rightarrow x = -4$

$\therefore x = -4$

32. Find the value of x for which $5^{2x} \div 5^3 = 5^5$.

Ans. : $5^{2x} \div 5^3 = 5^5$

$\Rightarrow 5^{2x} \div 5^5 = 5^{-3}$

$\Rightarrow 5^{2x} = 5^{5-3} = 5^2$

Comparing, we get:

$2x = 2 \Rightarrow x = \frac{2}{2} = 1$

$\therefore x = 1$

33. By what number should $(-15)^{-1}$ be ddivded so that the quotient may be equal to $(-15)^{-1}$?

Ans. : Let $(-15)^{-1}$ be divided by x

$\therefore (-15)^{-1} \div x = (-5)^{-1}$

$\Rightarrow \left(\frac{1}{-15}\right) \div x = \left(\frac{1}{-5}\right)^{1}$

$\Rightarrow \frac{1}{-15} \times \frac{1}{x} = \frac{1}{-5}$

$\Rightarrow \frac{1}{-15x} = \frac{1}{-5}$

$\Rightarrow x = \frac{-5}{-15} = \frac{1}{3}$

$\therefore$ Required number $= \frac{1}{3}$

34. If $x = \left(\frac{3}{2}\right)^2 \times \left(\frac{2}{3}\right)^{-4}$, find the value of x^{-2}

Ans. : $x = \left(\frac{3}{2}\right)^2 \times \left(\frac{2}{3}\right)^{-4}$

$\left(\frac{3}{2}\right)^2 \times \left(\frac{2}{3}\right)^{-4}$

$= \left(\frac{3}{2}\right)^{2+4} = \left(\frac{3}{2}\right)^6$

$x^2 = \left[\left(\frac{3}{2}\right)^6\right] = \left(\frac{3}{2}\right)^{-12}$

$= \left(\frac{2}{3}\right)^{12}$

35. If $x = \left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$, find the value of x^1

Ans. : $= \left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$

$\left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$

$= \left(\frac{5}{4} \div \frac{1}{4}\right)^{2}$

$= \left(\frac{5}{4} \times \frac{4}{1}\right)^{2} = (5)^2 = 25$

$\therefore x^1 = (25)^{-1} = \frac{1}{25}$

36. Simplify:

$\left\{\left(\frac{2}{3}\right)\right\} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1}$

Ans. : $\left\{\left(\frac{2}{3}\right)\right\} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1}$

$= \left(\frac{2}{3}\right)^{2\times3} \times \left(\frac{3}{1}\right)^{4} \times \frac{1}{3^1} \times \frac{1}{6^1}$

$= \left(\frac{2}{3}\right)^{6} \times (3)^4 \times \frac{1}{3} \times \frac{1}{6}$

$= \frac{2\times2\times2\times2\times2\times2}{3\times3\times3\times3\times3\times3} \times 3 \times 3 \times 3 \times 3 \times \frac{1}{3} \times \frac{1}{6}$

$= \frac{64}{3\times3\times3\times6} = \frac{64}{81\times2} = \frac{32}{81}$

37. Find x, if

$\left(\frac{1}{2}\right)^{-19} \div \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{-2x+1}$

Ans. : $\left(\frac{1}{2}\right)^{-19} \div \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{-2x+1}$

$\Rightarrow \left(\frac{-1}{2}\right)^{-19-8} = \left(\frac{-1}{2}\right)^{-4x+1}$

$\Rightarrow \left(\frac{1}{4}\right)^{-27} = \left(\frac{1}{4}\right)^{-2x+1}$

Comparing, we get:

$-2x + 1 = -27$

$-2x = -27 - 1 = -28$

$x = \frac{-28}{-2} = 14$

$\therefore x = 14$

38. By what number should $\left(\frac{5}{3}\right)^{-2}$ be multiplied so that the product may be $\left(\frac{7}{3}\right)^{-1}$?

Ans. : Let x be multiplied, then

$x \times \left(\frac{5}{3}\right)^{-2} = \left(\frac{7}{3}\right)^{-1} \Rightarrow x = \left(\frac{7}{3}\right)^{-1} \div \left(\frac{5}{3}\right)^{-2}$

$\Rightarrow x = \left(\frac{3}{7}\right)^{1} \div \left(\frac{3}{5}\right)^{2} = \frac{3}{7} \div \frac{9}{25} = \frac{3}{7} \times \frac{25}{9}$

$= \frac{25}{21}$

$\therefore$ Required number $= \frac{25}{21}$

39. Find x, if

$\left(\frac{2}{5}\right)^{-3} \times \left(\frac{3}{2}\right)^{15} = \left(\frac{2}{5}\right)^{2x+1}$

Ans. : $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{3}{2}\right)^{15} = \left(\frac{2}{5}\right)^{2x+1}$

$\Rightarrow \left(\frac{2}{5}\right)^{-3+15} = \left(\frac{2}{5}\right)^{2+3x}$

$\Rightarrow \left(\frac{2}{5}\right)^{12} = \left(\frac{2}{5}\right)^{2+3x}$

Comparing, we get:

$2 + 3x = 12$

$3x = 12 - 2 = 10$

$x = \frac{10}{3}$

40. Find x, if

$\left(\frac{3}{5}\right)^{-3} \times \left(\frac{3}{2}\right)^{5} = \left(\frac{3}{2}\right)^{2x+1}$

Ans. : $\left(\frac{3}{5}\right)^{-3} \times \left(\frac{3}{2}\right)^{5} = \left(\frac{3}{2}\right)^{2x+1}$

$\Rightarrow \left(\frac{3}{2}\right)^{-3+5} = \left(\frac{3}{2}\right)^{2x+1}$

$\Rightarrow \left(\frac{3}{2}\right)^{2} = \left(\frac{3}{2}\right)^{2x+1}$

Comparing, we get:

$2x + 1 = 2$

$\Rightarrow 2x = 2 - 1 = 1$

$\Rightarrow x = \frac{1}{2}$

41. Simplify: $\left(3^{-1} + 6^{-1}\right) \div \left(\frac{3}{4}\right)^{-1}$

Ans. : $\left(3^{-1} + 6^{-1}\right) \div \left(\frac{3}{4}\right)^{-1}$

$= \left(\frac{1}{3} + \frac{1}{6}\right) \div \left(\frac{4}{3}\right)^{1}$

$= \left(\left[\frac{1\times2}{3\times2}\right] + \left[\frac{1\times1}{6\times1}\right]\right) \div \left(\frac{4}{3}\right)$

$= \left(\frac{2+1}{6}\right) \div \left(\frac{4}{3}\right)$

$= \left(\frac{3}{6}\right) \div \left(\frac{4}{3}\right)$

$= \left(\frac{1}{2}\right) \div \left(\frac{4}{3}\right)$

$= \left(\frac{1}{2}\right) \times \left(\frac{3}{4}\right)$

$= \left(\frac{3}{8}\right)$

$\therefore \left(3^{-1} + 6^{-1}\right) \div \left(\frac{3}{4}\right)^{-1} = \frac{3}{8}$

42. Mass of earth is (5.97×10^{24})kg and mass of moon is (7.35×10^{22})kg. What is the total mass of the two?

Ans. : Mass of the earth = (15.97×10^{24})kg

and mass of the moon = (7.35×10^{22})kg

= Total mass of the earth and moon

$= (5.97 \times 10^{24}) + (7.35 \times 10^{22})\text{kg}$

$= \left(\frac{597}{100} \times 10^{24} + 7.35 \times 10^{22}\right)\text{kg}$

$= \left(\frac{597}{10^{22}} \times 10^{24} + 7.35 \times 10^{22}\right)\text{kg}$

$= (597 \times 10^{22} + 7.35 \times 10^{22})\text{kg}$

$= 10^{22}(597 + 7.35)\text{kg}$

$= (604.35)10^{22}\text{kg}$

43. If $5^{2x+1} \div 25 = 125$, find the value of x.

Ans. : $5^{2x+1} \div 25 = 125$

$\Rightarrow 5^{2x+1} \times \frac{1}{25} = 125$

$\Rightarrow 5^{2x+1} = 125 \times 25$

$\Rightarrow 5^{2x+1} = 5^3 \times 5^2 = 5^{3+2}$

$\Rightarrow 5^{2x+1} = 5^5$

$\therefore 2x + 1 = 5$

$\Rightarrow 2x = 5 - 1 = 4$

$\Rightarrow 2x = 4$

$\Rightarrow x = \frac{4}{2} = 2$

Hence, $x = 2$

44. Find the value of x for which $\left(\frac{4}{9}\right)^{4} \times \left(\frac{4}{9}\right)^{-7} = \left(\frac{4}{9}\right)^{2x-1}$

Ans. : $\left(\frac{4}{9}\right)^{4} \times \left(\frac{4}{9}\right)^{-7} = \left(\frac{4}{9}\right)^{2x-1}$

$\Rightarrow \left(\frac{4}{9}\right)^{4-7} = \left(\frac{4}{9}\right)^{2x-1}$

$\Rightarrow \left(\frac{4}{9}\right)^{-3} = \left(\frac{4}{9}\right)^{2x-1}$

$\therefore 2x - 1 = -3$

$\Rightarrow 2x = -3 + 1$

$\Rightarrow 2x = -2$

$\Rightarrow x = \frac{-2}{2}$

$\Rightarrow x = -1$

Hence $x = -1$

45. Find the value of x for which $\left(\frac{5}{3}\right)^{-4} \times \left(\frac{5}{3}\right)^{-5} = \left(\frac{5}{3}\right)^{3x}$

Ans. : $\left(\frac{5}{3}\right)^{-4} \times \left(\frac{5}{3}\right)^{-5} = \left(\frac{5}{3}\right)^{3x}$

$\Rightarrow \left(\frac{5}{3}\right)^{-4+(-5)} = \left(\frac{5}{3}\right)^{3x}$

$\Rightarrow \left(\frac{5}{3}\right)^{-9} = \left(\frac{5}{3}\right)^{3x}$

$\therefore 3x = -9$

$\Rightarrow x = \frac{-9}{3}$

$= -3$

Hence $x = -3$

46. In a stack, there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm . What is the total thickness of the stack?

Ans. : Given, thickness of each book $= 20mm$

$\therefore$ Thickness of 5 books $= 20 \times 5 = 100mm$

Thickness of each paper sheet $= 0.016mm$

$\therefore$ Thickness of 5 paper sheets $= 5 \times 0.016 = 0.08mm$

Hence, the total thickness of the stack

$=$ Thickness of 5 books

+Thickness of 5 paper sheets

$= 100 + 0.08 = 100.08$

$= 10008 \times 10^2$

Hence, the total thickness of the stack is $10008 \times 10^2 mm$.

47. The cells of a bacteria double in every 30 min . A scientist begins with a single cell.

(i) How many cells will be there after

(a) 10 h ? (b) 25 h ?

(ii) What type of value is depicted by the cells of bacteria?

Ans. : (i) The cells of a bacteria double in every 30 min i.e. number of cells of a bacteria after $30min = 2$

$\therefore$ Number of cells of a bacteria after 1 h

$= 2 \times 2 = 2^2 = 2^{2\times1}$

Number of cells of a bacteria after $1\frac{1}{2}h$

$= 2 \times 2^2 = 2^3 = 2^{2\times\frac{3}{2}}$

and number of cells of a bacteria after 2 h

$= 2 \times 2^3 = 2^4 = 2^{2\times2}$

(a) Thus, the number of cells after 10 h

$= 2^{2\times10} = 2^{20}$

(b) The number of cells after 25 h

$= 2^{2\times25} = 2^{50}$

(ii) The value depicted by the cells of bacteria here is that it double itself after 30 min or it grows itself in t h by $2^{2\times t}$.

48. Find the value of x^{-3}, if $x = (100)^{1-4} + (100)^0$.

Ans. : Given, $x = (100)^{1-4} + (100)^0$

$x = (100)^{-3} + 1 \quad \left[\because (a)^0 = 1\right]$

$\Rightarrow \quad x = (100)^{-3} \times \frac{1}{1}$

$\Rightarrow \quad x = \frac{1}{(100)^3} \times \frac{1}{1} \quad \left[\because a^{-m} = \frac{1}{a^m}\right]$

$\Rightarrow \quad x = \frac{1}{(100)^3}$

Now, $\quad x^{-3} = \left(\frac{1}{(100)^3}\right)^{-3}$

$\Rightarrow \quad x^{-3} = \frac{1}{(100^3)^{-3}}$

$\Rightarrow \quad x^{-3} = \frac{1}{(100)^{-9}} \Rightarrow x^{-3} = (100)^9$

49. Consider a quantity of a radioactive substance. The fraction of this quantity that remains after t half-lives can be found using the expression 3^{-t}.
(i) What fraction of substance remains after 7 half-lives ?
(ii) After how many half-lives, will the fraction be $\frac{1}{243}$ of the original ?

Ans. : (i) The fraction of substance remains after
7 half-lives $= 3^{-7} = \frac{1}{3^7}$
(ii) Let after x half-lives, the fraction will be $\frac{1}{243}$ of the original.
$\therefore \quad 3^{-x} = \frac{1}{243} = \frac{1}{3\times3\times3\times3\times3} = \frac{1}{3^5}$

$\Rightarrow \quad 3^{-x} = 3^{-5}$
$\Rightarrow \quad (3)^{-x} = (3)^{-5}$
$\Rightarrow \quad -x = -5 \quad [\because \text{ bases are same }]$
$\Rightarrow \quad x = 5$

[25]

*** Questions With Calculation.[5 Marks Each]**

50. Simplify : $\frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}$

Ans. : $\frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}$
$= \frac{3^{-5}\times(2\times5)^{-5}\times(5\times5\times5)}{5^{-7}\times(2\times3)^{-5}}$
$= \frac{3^{-5}\times2^{-5}\times5^{-5}\times5^3}{5^{-7}\times2^{-5}\times3^{-5}}$
$= \frac{5^{-5}\times5^3}{5^{-7}}$
$= \frac{5^{(5)+3}}{5^{-7}}$
$= \frac{5^{-2}}{5^{-7}}$
$= 5^{(-2)-(-7)}$
$= 5^{-2+7}$
$= 5^5$

51. Simplify : $\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}(t \neq 0)$

Ans. : $\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}(t \neq 0)$
$\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}$
$= \frac{25\times\frac{1}{t^4}}{\frac{1}{5^3}\times10\times\frac{1}{t^8}}$
$= \frac{\frac{25}{t^4}}{\frac{1}{125}\times10\times\frac{1}{t^8}}$
$= \frac{\frac{25}{t^4}}{\frac{25}{25t^8}}$
$= \frac{25}{t^4}\times\frac{25t^8}{2}$
$= \frac{625t^{8-4}}{2}$
$= \frac{625}{2}t^4$

52. Find the value of
(i) $\left(3^0+4^{-1}\right)\times2^2$
(ii) $\left(2^{-1}\times4^{-1}\right)+2^{-2}$
(iii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

(iv) $\left(3^{-1}+4^{-1}+5^{-1}\right)^{0}$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^{2}$

Ans. : (i) We have, $\left(3^{0}+4^{-1}\right)\times 2^{2}$

$=\left(1+\frac{1}{4}\right)\times 4 \quad \left[\because a^{0}=1 \text{ and } a^{-m}=\frac{1}{a^{m}}\right]$

$=\left(\frac{4+1}{4}\right)\times 4=\frac{5}{4}\times 4=5$

(ii) We have, $\left(2^{-1}\times 4^{-1}\right)+2^{-2}$

$=\left(\frac{1}{2}\times\frac{1}{4}\right)+2^{-2} \quad \left[\because a^{-m}=\frac{1}{a^{m}}\right]$

$=\left(\frac{1}{8}\right)+\left(\frac{1}{2^{2}}\right)=\frac{1}{8}+\frac{1}{4}$

$=\frac{1}{8}\times\frac{4}{1}=\frac{1}{2} \quad \left[\because \frac{a}{b}+\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}\right]$

(iii) **Ans.** 29

(iv) **Ans.** 1

(v)**Ans.** $\frac{81}{16}$

53. Express the following numbers in usual form.

(i) 3.02×10^{-6}

(ii) 4.5×10^{4}

(iii) 3×10^{-8}

(iv) 1.0001×10^{9}

(v) 5.8×10^{12}

(vi) 3.61492×10^{6}

Ans. : (i) We have,

$3.02\times 10^{-6} = \frac{3.02}{10^{6}}=\frac{3.02}{1000000}$

$=0.00000302 \quad \left[\because a^{-m}=\frac{1}{a^{m}}\right]$

(ii) We have,

$4.5\times 10^{4}=4.5\times 10000=45000$

(iii) We have,

$3\times 10^{-8}=\frac{3}{10^{8}}=\frac{3}{100000000}$

$=0.00000003 \quad \left[\because a^{-m}=\frac{1}{a^{m}}\right]$

(iv) We have,

$1.0001\times 10^{9}=1.0001\times 1000000000=1000100000$

(v) We have,

$5.8\times 10^{12} = 5.8\times 1000000000000$

$=5800000000000$

(vi) We have,

$3.61492\times 10^{6} = 3.61492\times 1000000$

$=3614920$

54. Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}m$.

(ii) Charge of an electron is 0.00000000000000000016 coulomb.

(iii) Size of a bacteria is 0.0000005 m .

(iv) Size of a plant cell is 0.00001275 m .

(v) Thickness of a thick paper is 0.07 mm .

Ans. : (i) Here, $\frac{1}{1000000}=\frac{1}{10^{6}}$

$=1\times 10^{-6}m \quad \left[\because \frac{1}{a^{m}}=a^{-m}\right]$

Hence, 1 micron $=1\times 10^{-6}m$.

(ii) Here, 0.00000000000000000016 coulomb.

$=\frac{16}{100000000000000000000}$

$=\frac{16}{10^{20}}=\frac{16\times 10}{10^{20}}$

$= 16 \times 10 \times 10^{-20} \quad \left[\because \frac{1}{a^m} = a^{-m}\right]$

$= 16 \times 10^{1-20}$

$= 16 \times 10^{-19}$ coulomb $\left[\because a^m \times a^n = a^{m+n}\right]$

Hence, the charge of an electron is 16×10^{-19} coulomb.

(iii)

$$\text{Here, } 0.0000005 = \frac{5}{10000000}$$

$$= \frac{5}{10^7} = 5 \times 10^{-7} m \quad \left[\because \frac{1}{a^m} = a^{-m}\right]$$

Hence, the size of a bacteria is $5 \times 10^{-7} m$.

(iv)

$$\text{Here, } 0.00001275 = \frac{1275}{100000000}$$

$$= \frac{1275}{10^8} = \frac{1.275 \times 1000}{10^8}$$

$= 1.275 \times 10^3 \times 10^{-6} \quad \left[\because \frac{1}{a^m} = a^{-m}\right]$

$= 1.275 \times 10^{3-8}$

$= 1.275 \times 10^{-5} m \left[\because a^m \times a^n = a^{m+n}\right]$

Hence, the size of a plant cell is $1.275 \times 10^{-5} m$.

(v) Here, $0.07 = \frac{7}{100} = \frac{7}{10^2} = 7 \times 10^{-2} mm$

Hence, the thickness of a thick paper is $7 \times 10^{-2} mm$.

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