[90]

*** Questions With Calculation.[2 Marks Each]**

1. Simplify: $\left\{\left(\frac{1}{3}\right)^{-2}-\left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-2}$

   Ans. : $\left\{\left(\frac{1}{3}\right)^{-2}-\left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-2}$

   $=\left\{\frac{1^{-2}}{3^{-2}}-\frac{1^{-3}}{2^{-3}}\right\}\div\frac{1^{-2}}{4^{-2}}$

   $=\left\{\frac{3^2}{1^2}-\frac{2^3}{1^3}\right\}\div\frac{4^2}{1^2}=\{9-8\}\div 16=\frac{1}{16}$

2. Simplify: $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-5}$

   Ans. : $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-5}=\frac{5^{-7}}{8^{-7}}\times\frac{8^{-5}}{5^{-5}}=\frac{5^{-7}}{5^{-5}}\times\frac{8^{-5}}{8^{-7}}$

   $=5^{(-7)-(-5)}\times 8^{(-5)-(-7)}=5^{-2}\times 8^2=\frac{8^2}{5^2}=\frac{64}{25}$

3. Express 0.00000000000942 in standard form.

   Ans. : $0.00000000000942=\frac{942}{10^{14}}$

   $=\frac{9.42}{10^{12}}$

   $= 9.42\times 10^{-12}$

4. Express the 6020000000000000 in standard form.

   Ans. : 6020000000000000

   $=6020000000000000\times\frac{10^{15}}{10^{15}}=6.02\times 10^{15}$

5. Express 3×10^{-8} in usual form.

   Ans. : We have, 3×10^{-8}

   = 0.00000003

6. Express 1.0001×10^9 in usual form.

   Ans. : 1.0001×10^9

   $= 1.0001\times 1000000000$

   = 1000100000

7. Express the number appearing in the statement in standard form :Charge of an electron is 0.00000000000000000016 coulomb.

   Ans. : 0.00000000000000000016

   $\frac{16}{1000000000000000000000}$

   $=\frac{16}{10^{20}}$

   $=\frac{1.6}{10^{19}}$

   $= 1.6\times$ 10−19

8. Evaluate :$\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

   Ans. : $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

   $=\left(\frac{1^{-1}}{3^{-1}}-\frac{1^{-1}}{4^{-1}}\right)^{-1}=\left(\frac{3^1}{1^1}-\frac{4^1}{1^1}\right)^{-1}$

   $=\left(\frac{3}{1}-\frac{4}{1}\right)^{-1}$ $= (3 - 4)^{-1}$

   $=(-1)^{-1}=\frac{1}{(-1)^1}$

   $=\frac{1}{(-1)}=-1$

9. Find the value of m for which $5^m \div 5^{-3} = 5^5$

Ans. : $5^m \div 5^{-3} = 5^5$

$\therefore \frac{5^m}{5^{-3}} = 5^5$

$\therefore 5^{m-(-3)} = 5^5$

$\therefore 5^{m+3} = 5^5$

$\therefore m + 3 = 5$ [When the bases are same , powers are equal]

$\therefore m = 5 - 3$

$\therefore m = 2$

10. Simplify and express the result in power notation with positive exponent: $(3^{-7} \div 3^{-10}) \times 3^{-5}$

Ans. : $(3^{-7} \div 3^{-10}) \times 3^{-5}$

$= \left(\frac{3^{-7}}{3^{-10}}\right) \times \frac{1}{3^5}$

$= 3^{(-7)-(-10)} \times 3^{\frac{1}{5}}$

$= 3^{-7+10} \times 3^{\frac{1}{5}}$

$= \frac{3^3}{3^5}$

$= \frac{1}{3^{5-3}}$

$= \frac{1}{(3)^2}$

11. Simplify and express the result in power notation with positive exponent.$(-4)^5 \div (-4)^8$

Ans. : $(-4)^5 \therefore (-4)^8$

$= \frac{(-4)^5}{(-4)^8}$

$= \frac{1}{(-4)^{8-5}}$

$= \frac{1}{(-4)^3}$

12. Express $\frac{27}{64}$ and $\frac{-27}{64}$ as powers of a rational number.

Ans. : $\because 27 = 3 \times 3 \times 3 = 3^3$

$(-27) = (-3) \times (-3) \times (-3) = (-3)^3$

and $64 = 4 \times 4 \times 4 = 4^3$

$\therefore \frac{27}{64} = \frac{3^3}{4^3} = \left(\frac{3}{4}\right)^3$

and $\frac{-27}{64} = \frac{(-3)^3}{(4)^3} = \left(\frac{-3}{4}\right)^3$

13. Simplify:

$\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^{3^{-1}} \times 2^{-3}$

Ans. : Given, $\left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^3\right]^{-1} \times 2^{-3} = \left(\frac{1}{4} - \frac{1}{64}\right)^{-1} \times 2^{-3}$

$= \left(\frac{16-1}{64}\right)^{-1} \times 2^{-3}$

$= \left(\frac{15}{64}\right)^{-1} \times 2^{-3}$

$= \frac{64}{15} \times \frac{1}{8}$ $\left[\because a^{-m} = \frac{1}{a^m}\right]$

$= \frac{8}{15}$

14. Express the following in standard form:
Mass of a molecule of hydrogen gas is about 0.000000000000000000000334 tons.

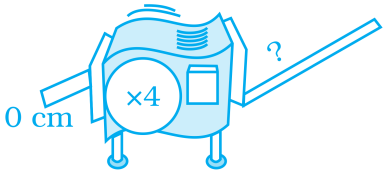
Ans. :
Mass of a molecule of hydrogen gas is about 0.000000000000000000000334 tonnes

Standard form = 0.334×10^{-20}

$= 3.34 \times 10^{-20} \times 10^{-1}$

$= 3.34 \times 10^{-21}$ $[\because a^m \times a^n = (a)^{m+n}]$

15. **Stretching Machine:**
Suppose you have a stretching machine which could stretch almost anything. For example, if you put a 5 metre stick into a (×4) stretching machine (as shown below), you get a 20 metre stick. Now if you put 10cm carrot into a (×4) machine, how long will it be when it comes out?



Ans. : According to the question, if we put a 5m stick into a (×4) stretching machine, then machine produces 20m stick. Similarly, if we put 10cm carrot into a (×4) stretching machine, then machine produce 10 × 4= 40cm stick.

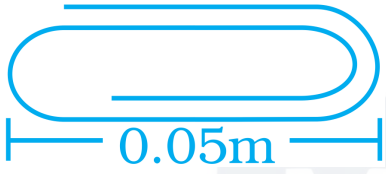
16. Some migratory birds travel as much as 15,000km to escape the extreme climatic conditions at home. Write the distance in metres using scientific notation.

Ans. : Total distance travelled by migratory bird = 15000km
= 15000 × 1000m [∵ 1km = 1000m]
= 15000000m
= 15 × 10^6m
Scientiic notation of 15 × 10^6 = 1.5 × 10^7m

17. A particular star is at a distance of about 8.1 × 10^{13}km from the Earth. Assuring that light travels at 3 × 10^8m per second, find how long does light takes from that star to reach the Earth.

Ans. : The distance between star and Earth = 8.1 × 10^{13}km = 8.1 × 10^{13} × 10^{13}m.
Since, light travels at 3 × 10^8m per second. $[\because 1\text{km} = 1000\text{m}]$
So, time taken by light to reach the Earth
$= \frac{8.1\times10^{13}\times10^3}{3\times10^8} == \frac{8.1\times10^{16}}{3\times10^8} = \frac{8.1}{3} \times 10^8 = 2.7 \times 10^8\text{s}.$

18. The paper clip below has the indicated length. What is the length in standard form.



Length of the paper clip = 0.05m.
In standard form, 0.05m = 0.5 x 10^{-1} = 5.0 x 10^{-2}m.
Hence, the length of the paper clip in standard form is 5.0 x 10^{-2}m.

Ans. : Length of the paper clip = 0.05m.
In standard form, 0.05 m = 0.5 x 10^{-1} = 5.0 x 10^{-2}m.
Hence, the length of the paper clip in standard form is 5.0 x 10^{-2}m.

19. Mass of Mars is 6.42 × 10^{29}kg and mass of the Sun is 1.99 × 10^{30}kg. What is the total mass?

Ans. : Mass of Mars = 6.42 × 10^{29}kg
Mass of the Sun = 1.99 × 10^{30}kg
Total mass of Mars and Sun together = 6.42 × 10^{29} + 1.99 × 10^{30}
= 6.42 × 10^{29} + 19.9 × 10^{29} = 26.32 × 10^{29}kg.

20. Express $\frac{1.5\times10^6}{2.5\times10^{-4}}$ in the standard form.

Ans. : Given,
$\frac{1.5\times10^6}{2.5\times10^{-4}} = \frac{15}{25} \times 10^{6+4}$ [∵ $a^m \div a^n = (a)^{m-n}$]
$= \frac{3}{5} \times 10^{10}$
$= 0.6 \times 10^{10}$
$= 0.6 \times 10^{10} \times 10^{-1}$
$= 6 \times 10^9$ [∵ $a^m \times a^n = (a)^{m-n}$]

21. By what number should $(-8)^{-3}$ be multiplied so that that the product may be equal to $(-6)^{-3}$?

Ans. : Let x be the number multiplied with $(-8)^{-3} = (-6)^{-3}$
$\Rightarrow \text{x} \times (-8)^{-3} = (-6)^{-3}$
$\Rightarrow \text{x} = \frac{(-6)^{-3}}{(-8)^{-3}} = \frac{(-8)^{-3}}{(-6)^{-3}} = \frac{512}{216} = \frac{64}{27}$

22. The left column of the chart lists the lengths of input pieces of ribbon. Stretching machines are listed across the top. The other entries are the outputs for sending the input ribbon from that row through the machine from that column. Copy

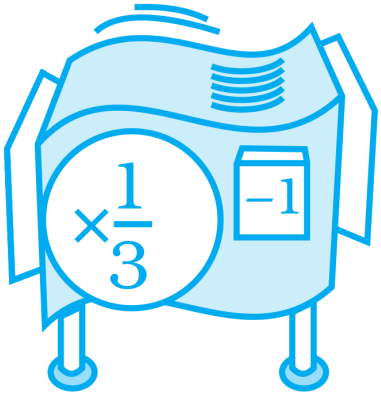
and complete the chart.

Input Length	Machine			
	x2			
	1	5		
3				15
	14		7	

Ans. : In the given table, the left column of chart list is the length of input piece of ribbon. Thus, the outputs for sending the input ribbon are given in the following table.

Input Length	Machine		
	x2	x10	x5
5	1	5	2.5
3	6	30	15
7	14	70	35

23. **Shrinking Machine:** In a shrinking machine, a piece of stick is compressed to reduce its length. If 9cm long sandwich is put into the shrinking machine below, how many cm long will it be when it emerges?



Ans. : According to the question, in a shrinking machine, a piece of stick is compressed to reduce its length. If 9cm long sandwich is put into the shrinking machine,
then the length of sandwich will be $9 \times \frac{1}{3^{-1}} = 9 \times 3 = 27\text{cm}$.

24. Planet A is at a distance of 9.35 × 10^6km from Earth and planet B is 6.27 × 10^7km from Earth. Which planet is nearer to Earth?

Ans. : Distance between planet A and Earth = 9.35 × 10^6km
Distance between planet B and Earth = 6.27 × 10^7km
For finding difference between above two distances,
we have to change both in same exponent of 10, i.e. 9.35 × 10^6
= 0.935 × 10^7
clearly 6.27 × 10^7 is greater.
So, planet A is nearer to Earth.

25. Simplify:
$\left(\frac{1}{5}\right)^{45} \times \left(\frac{1}{5}\right)^{-60} - \left(\frac{1}{5}\right)^{+28} \times \left(\frac{1}{5}\right)^{-43}$

Ans. : $\left(\frac{1}{5}\right)^{45} \times \left(\frac{1}{5}\right)^{-60} - \left(\frac{1}{5}\right)^{+28} \times \left(\frac{1}{5}\right)^{-43} = \frac{1}{(5)^{45}} \times \frac{1}{(5)^{-60}} - \frac{1}{(5)^{28}} \times \frac{1}{(5)^{-43}}$

$= \frac{1}{(5)^{45-60}} - \frac{1}{(5)^{28-43}} \quad [\because a^m \times a^n = a^{m+n}]$

$= \frac{1}{(5)^{-15}} - \frac{1}{(5)^{-15}} = (5)^{15} - (5)^{15} \quad [\because a^{-m} = \frac{1}{a^m}]$

$= 0$

26. At the end of the 20th century, the world population was approximately 6.1 × 10^9 people. Express this population in usual form. How would you say this number in words?

Ans. : Given, at the end of the 20th century, the world population was 6.1 x 10^9 (approx). People population in usual form = 6.1 x 10^9 = 6100000000.
Hence, population in usual form was six thousand one hundred million.

27. Express the following in standard form:
Express 56km in m.

Ans. :

Given,

56km = 56 × 1000m [$\because$ 1km= 1000m]

= 56000m

Standard form of 56000m = 56×10^3

= $5.6 \times 10^3 \times 10^1$

= 5.6×10^4m [$\because a^m \times a^n = (a)^{m+n}$]

28. Simplify:

$\frac{(9)^3 \times 27 \times t^4}{(3)^{-2} \times (3)^4 \times t^2}$

Ans. : $\frac{(9)^3 \times 27 \times t^4}{(3)^{-2} \times (3)^4 \times t^2} = \frac{(3^2)^3 \times (3)^3 \times t^4}{(3)^{-2} \times (3)^4 \times t^2}$ $[\because 3 \times 3 = 3^2 = 9]$

$\frac{(3)^6 \times (3)^3 \times t^4}{(3)^{-2} \times (3)^4 \times t^2} = (3)^6 \times (3)^3 \times (3)^2 \times (3)^{-4} \times t^4 \times t^{-2}$ $\left[\because a^{-m} = \frac{1}{a^m}\right]$

$= (3)^{11-4} \times t^{4-2} = (3)^7 \times t^2$ $[\because a^m \times a^n = a^{m+n}]$

29. Simplify:

$\left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-3}$

Ans. : $\left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-3}$

$= (3^3 - 2^3) \div (4)^3$

$= (27 - 8) \div 64 = \frac{19}{64}$

30. Find x, if

$\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$

Ans. : $\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$

$\Rightarrow \left(\frac{1}{4}\right)^{-12} = \left(\frac{1}{4}\right)^{-4x}$

$\Rightarrow \left(\frac{1}{4}\right)^{-12} = \left(\frac{1}{4}\right)^{-4x}$

Comparing, we get:

$-4x = -12 \Rightarrow x = \frac{-12}{-4} = 3$

$\therefore x = 3$

31. Find x, if

$\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$

Ans. : $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$

$\Rightarrow \left(\frac{2}{5}\right)^{-3+15} = \left(\frac{2}{5}\right)^{2+3x}$

$\Rightarrow \left(\frac{2}{5}\right)^{12} = \left(\frac{2}{5}\right)^{2+3x}$

Comaring, we get:

$2 + 3x = 12$

$3x = 12 - 2 = 10$

$x = \frac{10}{3}$

32. Simplify:

$(2^2 + 3^2 - 4^2) \div \left(\frac{3}{2}\right)^2$

Ans. : $(2^2 + 3^2 - 4^2) \div \left(\frac{3}{2}\right)^2$

$= (4 \div 9 - 16) \div \frac{9}{4} = -3 \times \frac{4}{9}$

$= \frac{-1 \times 4}{3} = \frac{-4}{3}$

33. Write the following numbers in the usual form:

1.0001×10^9

Ans. : 1.0001×10^9

$= 1.0001 \times 1000000000$

$= 1000100000$

34. Write the following numbers in the usual form:

3.61492×10^6

Ans. : 3.61492×10^6

$= 3.61492 \times 1000000$

$= 3614920$

35. Simplify:

$\left[\left\{\left(\frac{-1}{4}\right)^2\right\}\right]^{-1}$

Ans. : $\left[\left\{\left(\frac{-1}{4}\right)^2\right\}\right]^{-1} = \left(\frac{-1}{4}\right)^{2\times(-2)(-2)}$

$= \left(\frac{-1}{4}\right)^4 = \left(\frac{-1}{4}\right) \times \left(\frac{-1}{4}\right) \times \left(\frac{-1}{4}\right)$

$\times \left(\frac{-1}{4}\right) = \frac{1}{256}$

36. Evaluate:

$\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3}$

Ans. : $\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3}$

$= \left\{(3)^3 - (2)^3\right\} \div (4)^3$

$= (27 - 8) \div 64$

$= \frac{19}{64}$

37. Evaluate:

$\left[\left\{\left(\frac{-1}{3}\right)^2\right\}^{-2}\right]^{-1}$

Ans. : $\left[\left\{\left(\frac{-1}{3}\right)^2\right\}^{-2}\right]^{-1}$

$= \left(\frac{-1}{3}\right)^{2\times(-2)(-1)}$

$= \left(\frac{-1}{3}\right)^4$

$= \left(\frac{-1}{3}\right) \times \left(\frac{-1}{3}\right) \times \left(\frac{-1}{3}\right) \times \left(\frac{-1}{3}\right)$

$= \frac{1}{81}$

38. Evaluate:

$\left(\frac{-3}{5}\right)^{-4} \times \left(\frac{-2}{5}\right)^2$

Ans. : $\left(\frac{-3}{5}\right)^{-4} \times \left(\frac{-2}{5}\right)^2$

$= \left(\frac{5}{-3}\right)^4 \times \left(\frac{-2}{5}\right)^2$

$= \frac{5^4}{-3^4} \times \frac{-2^2}{5^2}$

$= 5^{(4-2)} \times \frac{-2^2}{-3^4}$

$= 5^2 \times \frac{-2^2}{-3^4}$

$= 25 \times \frac{4}{81}$

$= \frac{100}{81}$

39. Evaluate:

$\left\{\left(\frac{-2}{3}\right)^2\right\}^{-2}$

Ans. : $\left\{\left(\frac{-2}{3}\right)^2\right\}^{-2}$

$= \left(\frac{-2}{3}\right)^{2\times(-2)}$

$= \left(\frac{-2}{3}\right)^{-4}$

$= \left(\frac{-3}{2}\right)^{4}$

$= \frac{(-3)\times(-3)\times(-3)\times(-3)}{2\times2\times2\times2}$

$= \frac{81}{16}$

40. Evaluate

(i) $\frac{8^{-1}\times5^3}{2^{-4}}$

(ii) $\left(5^{-1}\times2^{-1}\right)\times6^{-1}$

Ans. : (i) We have,

$\frac{8^{-1}\times5^3}{2^{-4}}$

$= \frac{2^4\times5^3}{8} \quad \left[\because a^{-m}=\frac{1}{a^m}\right]$

$= \frac{16\times125}{8} = 2\times125 = 250$

(ii) **Ans.** $\frac{1}{60}$

41. Write the value of $(13)^{-13}+(13)^{13}$.

Ans. : $(13)^{-13}+(13)^{13} = (r3)^{-13}\times\frac{1}{(13)^{13}} = \frac{1}{(13)^{26}}$

42. Solve the following

(i) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{2}\right)^{-3}$

(ii) $\left(\frac{2}{3}\right)^{-2}\times\left(\frac{2}{3}\right)^{5}$

Ans. : (i)

$$\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{2}\right)^{-3} = \left(\frac{1}{2}\right)^{-2}\times\left(\frac{1}{2}\right)^{3} = \left(\frac{1}{2}\right)^{-2+3} = \left(\frac{1}{2}\right)^{1} = \frac{1}{2}$$

(ii)

$$\left(\frac{2}{3}\right)^{-2}\times\left(\frac{2}{3}\right)^{5} = \left(\frac{2}{3}\right)^{-2+5} = \left(\frac{2}{3}\right)^{3} = \frac{(2)^3}{(3)^3}$$

$$= \frac{2\times2\times2}{3\times3\times3} = \frac{8}{27}$$

43. Express the product of 3.2×10^6 and 4.1×10^{-1} in the standard form.

Ans. : We have, $\left(3.2\times10^6\right)\times\left(4.1\times10^{-1}\right)$

$= 3.2\times10^6\times4.1\times10^{-1} = 3.2\times4.1\times10^{6-1}$

$= 13.12\times10^5 = 1.312\times10\times10^5 = 1.312\times10^6$

44. Express $\frac{1.5\times10^6}{2.5\times10^4}$ in the standard form.

Ans. : We have, $\frac{1.5\times10^6}{2.5\times10^4}$

$= \frac{1.5\times10^2\times10^4}{2.5\times10^4} = \frac{150}{2.5}\times10^{4-4}$

$= \frac{150}{2.5}\times10^0 = 60\times1 = 60$

$= 6.0\times10^1 = 6.0\times10$

45. If $\frac{5^m\times5^3\times5^{-2}}{5^5} = 5^{12}$. then find the value of m.

Ans. : Given, $\frac{5^m\times5^3\times5^{-2}}{5^{-5}} = 5^{12}$

$\Rightarrow \quad \frac{5^{m+3-2}}{5^{-5}} = 5^{12}$

$\Rightarrow \quad 5^{m+1+5} = 5^{12}$

$\Rightarrow \quad (5)^{m+6} = (5)^{12}$

$\Rightarrow \quad m + 6 = 12 \qquad [\because \text{bases are same}]$

$\Rightarrow \quad m = 12 - 6 = 6$

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