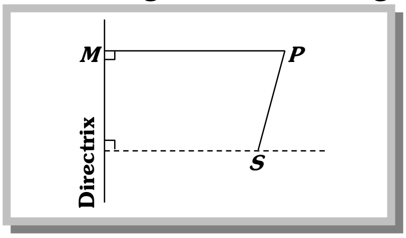
CONIC SECTION : GENERAL

5.0.1. Introduction

The curves obtained by intersection of a plane and a double cone in different orientation are called conic section.

In other words "Graph of a quadratic equation (in two variables) is a "Conic section".

A conic section or conic is the locus of a point P, which moves in such a way that its distance from a fixed point S always bears a constant ratio to its distance from a fixed straight line, all being in the same plane.



$$\frac{SP}{PM} = \text{constant} = e \text{ (eccentricity)}$$

or $\quad SP = e.\,PM$

5.0.2. Definitions of Various important Terms

(1) **Focus :** The fixed point is called the focus of the conic-section.

(2) **Directrix :** The fixed straight line is called the directrix of the conic section.

In general, every central conic has four foci, two of which are real and the other two are imaginary. Due to two real foci, every conic has two directrices corresponding to each real focus.

(3) **Eccentricity :** The constant ratio is called the eccentricity of the conic section and is denoted by e.

If $e = 1$, the conic is called **Parabola**.

If $e < 1$, the conic is called **Ellipse**.

If $e > 1$, the conic is called **Hyperbola**.

If $e = 0$, the conic is called **Circle**.

If $e = \infty$, the conic is called **Pair of the straight lines**.

(4) **Axis:** The straight line passing through the focus and perpendicular to the directrix is called the axis of the conic section. A conic is always symmetric about its axis.

(5) **Vertex:** The points of intersection of the conic section and the axis are called vertices of conic section.

(6) **Centre:** The point which bisects every chord of the conic passing through it, is called the centre of conic.

(7) **Latus-rectum:** The latus-rectum of a conic is the chord passing through the focus and perpendicular to the axis.

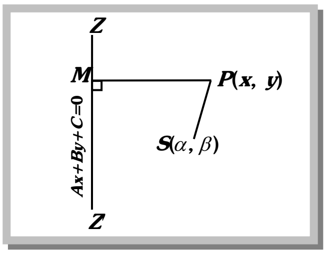
(8) **Double ordinate:** The double ordinate of a conic is a chord perpendicular to the axis.

(9) **Focal chord:** A chord passing through the focus of the conic is called a focal chord.

(10) **Focal distance:** The distance of any point on the conic from the focus is called the focal distance of the point.

5.0.3. General equation of a Conic section when its Focus, Directrix and Eccentricity are given

Let $S(\alpha, \beta)$ be the focus, $Ax + By + C = 0$ be the directrix and e be the eccentricity of a conic. Let $P(h, k)$ be any point on the conic. Let PM be the perpendicular from P, on the directrix. Then by definition



$$SP = ePM \Rightarrow SP^2 = e^2 PM^2$$

$$\Rightarrow (h-\alpha)^2 + (k-\beta)^2 = e^2\left(\frac{Ah + Bk + C}{\sqrt{A^2 + B^2}}\right)^2$$

Thus the locus of (h, k) is $(x-\alpha)^2 + (y-\beta)^2 = e^2 \frac{(Ax + By + C)^2}{(A^2 + B^2)}$ this is the cartesian equation of the conic section which, when simplified, can be written in the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, which is general equation of second degree.

5.0.4. Recognisation of Conics

The equation of conics is represented by the general equation of second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad ......(i)$$

and discriminant of above equation is represented by Δ, where

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

Case I: When $\Delta = 0$

In this case equation (i) represents the degenerate conic whose nature is given in the following table.

S. No.	Condition	Nature of conic
1.	$\Delta = 0$ and $ab - h^2 = 0$	A pair of coincident straight lines
2.	$\Delta = 0$ and $ab - h^2 < 0$	A pair of intersecting straight lines
3.	$\Delta = 0$ and $ab - h^2 > 0$	A point

Case II: When $\Delta \neq 0$

In this case equation (i) represents the non-degenerate conic whose nature is given in the following table.

S. No.	Condition	Nature of conic
1.	$\Delta \neq 0,\ h = 0,\ a = b$	A circle
2.	$\Delta \neq 0,\ ab - h^2 = 0$	A parabola
3.	$\Delta \neq 0,\ ab - h^2 > 0$	An ellipse
4.	$\Delta \neq 0,\ ab - h^2 < 0$	A hyperbola
5.	$\Delta \neq 0,\ ab - h^2 < 0$ and $a + b = 0$	A rectangular hyperbola

5.0.5. Method to find centre of a Conic

Let $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ be the given conic. Find $\frac{\partial S}{\partial x}; \frac{\partial S}{\partial y}$

Solve $\frac{\partial S}{\partial x} = 0, \frac{\partial S}{\partial y} = 0$ for x, y we shall get the required centre (x, y)

$$(x, y) = \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$$

Example: 1 The equation $x^2 - 2xy + y^2 + 3x + 2 = 0$ represents

(a) A parabola (b) An ellipse (c) A hyperbola (d) A circle

Solution: (a) Comparing the given equation with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Here, $a = 1, b = 1, h = -1, g = \frac{3}{2}, f = 0, c = 2$

Now $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$\Rightarrow \Delta = (1)(1)(2) + 2\left(\frac{3}{2}\right)(0)(-1) - (1)(0)^2 - 1\left(\frac{3}{2}\right)^2 - 2(-1)^2 \Rightarrow \Delta = \frac{-9}{4}$ *i.e.*, $\Delta \neq 0$ and $h^2 - ab = 1 - 1 = 0$ *i.e.*, $h^2 = ab$

So given equation represents a parabola.

Example: 2 The centre of $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$ is

(a) (2, 3) (b) (2, –3) (c) (–2, 3) (d) (–2, –3)

Solution: (a) Centre of conic is $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$

Here, $a = 14, h = -2, b = 11, g = -22, f = -29, c = 71$

Centre $\equiv \left(\frac{(-2)(-29) - (11)(-22)}{(14)(11) - (-2)^2}, \frac{(-22)(-2) - (14)(-29)}{(14)(11) - (-2)^2}\right)$

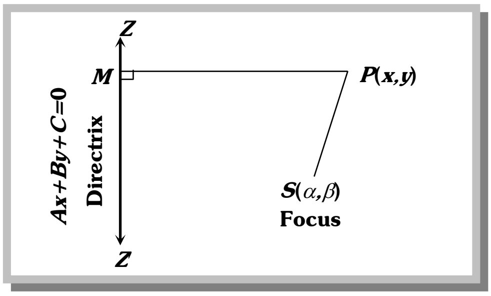
Centre $\equiv (2, 3)$.

PARABOLA

5.1.1 Definition

A parabola is the locus of a point which moves in a plane such that its distance from a fixed point (*i.e.*, focus) in the plane is always equal to its distance from a fixed straight line (*i.e.*, directrix) in the same plane.

General equation of a parabola : Let *S* be the focus, ZZ' be the directrix and let *P* be any point on the parabola. Then by definition,



$$SP = PM \qquad (\because e = 1)$$

$$\sqrt{(x-\alpha)^2 + (y-\beta)^2} = \frac{Ax + By + C}{\sqrt{A^2 + B^2}}$$

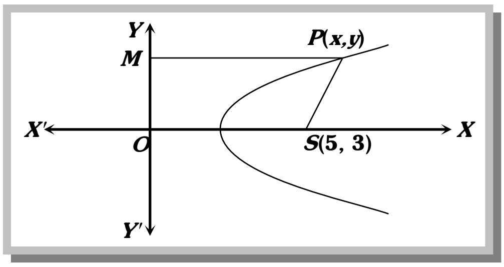
Or $\quad (A^2 + B^2)\{(x-\alpha)^2 + (y-\beta)^2\} = (Ax + By + C)^2$

Example: 1 The equation of parabola whose focus is (5, 3) and directrix is $3x - 4y + 1 = 0$, is

(a) $(4x+3y)^2 - 256x - 142y + 849 = 0$ (b) $(4x-3y)^2 - 256x - 142y + 849 = 0$

(c) $(3x+4y)^2 - 142x - 256y + 849 = 0$ (d) $(3x-4y)^2 - 256x - 142y + 849 = 0$

Solution: (a) $PM^2 = PS^2 \Rightarrow (x-5)^2 + (y-3)^2 = \left(\frac{3x-4y+1}{\sqrt{9+16}}\right)^2$



$\Rightarrow 25(x^2 + 25 - 10x + y^2 + 9 - 6y)$

$= 9x^2 + 16y^2 + 1 - 12xy + 6x - 8y - 12xy$

$\Rightarrow 16x^2 + 9y^2 - 256x - 142y + 24xy + 849 = 0$

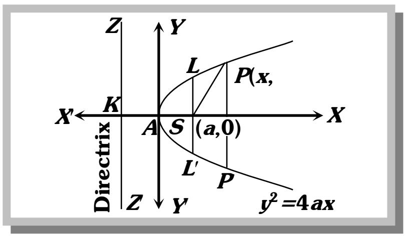
$\Rightarrow (4x+3y)^2 - 256x - 142y + 849 = 0$

5.1.2 Standard equation of the Parabola

Let *S* be the focus ZZ' be the directrix of the parabola and (x, y) be any point on parabola.

Let $AS = AK = a (> 0)$ then coordinate of *S* is (*a*, 0) and the equation of *KZ* is $x = -a$ or $x + a = 0$

Now $SP = PM \Rightarrow (SP)^2 = (PM)^2$

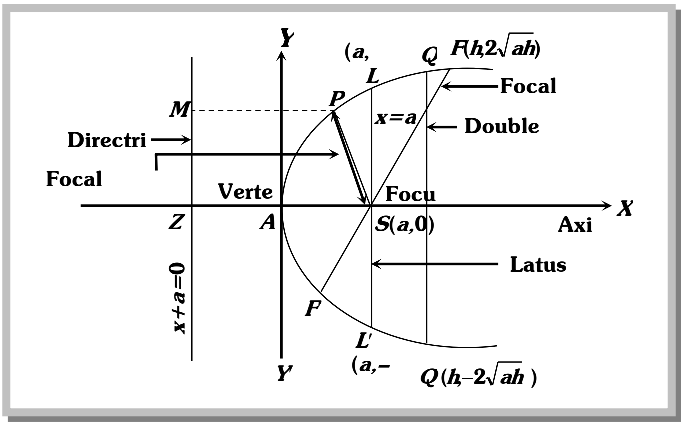


$\Rightarrow (x-a)^2 + (y-0)^2 = (a+x)^2$

$\therefore \boxed{y^2 = 4ax}$

which is the equation of the parabola in its standard form.

Some terms related to parabola



For the parabola $y^2 = 4ax$,

(1) **Axis :** A straight line passes through the focus and perpendicular to the directrix is called the axis of parabola.

For the parabola $y^2 = 4ax$, x-axis is the axis. Here all powers of y are even in $y^2 = 4ax$. Hence parabola $y^2 = 4ax$ is symmetrical about x-axis.

(2) **Vertex :** The point of intersection of a parabola and its axis is called the vertex of the parabola. The vertex is the middle point of the focus and the point of intersection of axis and the directrix.

For the parabola $y^2 = 4ax$, $A(0,0)$ i.e., the origin is the vertex.

(3) **Double-ordinate :** The chord which is perpendicular to the axis of parabola or parallel to directrix is called double ordinate of the parabola.

Let QQ' be the double-ordinate. If abscissa of Q is h then ordinate of Q, $y^2 = 4ah$ or $y = 2\sqrt{ah}$ (for I^{st} Quadrant) and ordinate of Q' is $y = -2\sqrt{ah}$ (for IV^{th} Quadrant). Hence coordinates of Q and Q' are $(h, 2\sqrt{ah})$ and $(h, -2\sqrt{ah})$ respectively.

(4) **Latus-rectum :** If the double-ordinate passes through the focus of the parabola, then it is called latus-rectum of the parabola.

Coordinates of the extremeties of the latus rectum are $L(a, 2a)$ and $L'(a, -2a)$ respectively.

Since $LS = L'S = 2a$ $\therefore$ Length of latus rectum $LL' = 2(LS) = 2(L'S) = 4a$.

(5) **Focal Chord :** A chord of a parabola which is passing through the focus is called a focal chord of the parabola. Here *PP'* and *LL'* are the focal chords.

(6) **Focal distance (Focal length) :** The focal distance of any point P on the parabola is its distance from the focus S *i.e.*, SP.

Here, Focal distance $SP = PM = x + a$

Note : ❑ If length of any double ordinate of parabola $y^2 = 4ax$ is $2l$, then coordinates of end points of this double ordinate are $\left(\frac{l^2}{4a}, l\right)$ and $\left(\frac{l^2}{4a}, -l\right)$.

Important Tips

- The area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8a}(y_1 \sim y_2)(y_2 \sim y_3)(y_3 \sim y_1)$, where $y_1, y_2 y_3$ are the ordinate of the vertices
- The length of the side of an equilateral triangle inscribed in the parabola $y^2 = 4ax$ is $8a\sqrt{3}$ (one angular point is at the vertex).

Example: 2 The point on the parabola $y^2 = 18x$, for which the ordinate is three times the abscissa, is

(a) (6, 2) (b) (–2, –6) (c) (3, 18) (d) (2, 6)

Solution: (d) Given $y = 3x$, then $(3x)^2 = 18x \Rightarrow 9x^2 = 18x \Rightarrow x = 2$ and $y = 6$.

Example: 3 The equation of the directrix of parabola $5y^2 = 4x$ is

(a) $4x - 1 = 0$ (b) $4x + 1 = 0$ (c) $5x + 1 = 0$ (d) $5x - 1 = 0$

Solution: (c) The given parabola is $y^2 = \frac{4}{5}x$. Here $a = \frac{1}{5}$. Directrix is $x = -a = \frac{-1}{5} \Rightarrow 5x + 1 = 0$

Example: 4 The point on the parabola $y^2 = 8x$. Whose distance from the focus is 8, has *x*-coordinate as

(a) 0 (b) 2 (c) 4 (d) 6

Solution: (d) If $P(x_1, y_1)$ is a point on the parabola $y^2 = 4ax$ and S is its focus, then $SP = x_1 + a$

Here $4a = 8 \Rightarrow a = 2;\ SP = 8$

$\therefore\ 8 = x_1 + 2 \Rightarrow x_1 = 6$

Example: 5 If the parabola $y^2 = 4ax$ passes through (–3, 2), then length of its latus rectum is

(a) 2/3 (b) 1/3 (c) 4/3 (d) 4

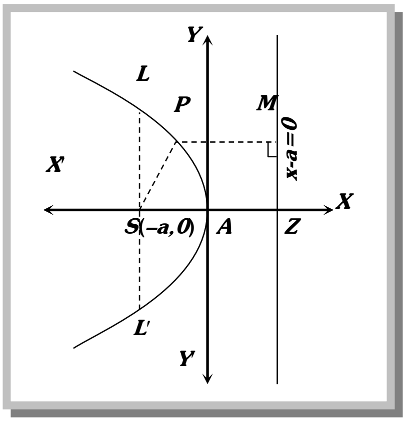
Solution: (c) The point $(-3, 2)$ will satisfy the equation $y^2 = 4ax \Rightarrow 4 = -12a \Rightarrow$ Latus rectum $= 4|a| = 4 \times |-\frac{1}{3}| = \frac{4}{3}$

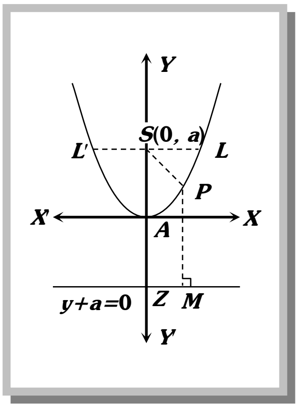
5.1.3 Some other standard forms of Parabola

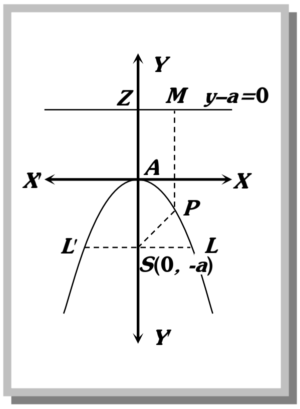
(1) Parabola opening to left (*i.e.* $y^2 = -4ax$); (a > 0)

(2) Parabola opening upwards (*i.e.* $x^2 = 4ay$); (a>0)

(3) Parabola opening down wards (*i.e.* $x^2 = -4ay$); (a > 0)







Important terms	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Coordinates of vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Coordinates of focus	(a, 0)	(–a, 0)	(0, a)	(0, –a)
Equation of the directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Equation of the axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Length of the latusrectum	$4a$	$4a$	$4a$	$4a$
Focal distance of a point $P(x, y)$	$x + a$	$a - x$	$y + a$	$a - y$

Example: 6 Focus and directrix of the parabola $x^2 = -8ay$ are

(a) $(0, -2a)$ and $y = 2a$ (b) $(0, 2a)$ and $y = -2a$ (c) $(2a, 0)$ and $x = -2a$ (d) $(-2a, 0)$ and $x = 2a$

Solution: (a) Given equation is $x^2 = -8ay$

Comparing the given equation with $x^2 = -4AY$, $A = 2a$

Focus of parabola (0,–A) i.e. (0, –2a)

Directrix $y = A$, *i.e.* $y = 2a$

Example: 7 The equation of the parabola with its vertex at the origin, axis on the y-axis and passing through the point (6, –3) is

(a) $y^2 = 12x + 6$ (b) $x^2 = 12y$ (c) $x^2 = -12y$ (d) $y^2 = -12x + 6$

Solution: (c) Since the axis of parabola is y-axis with its vertex at origin.

$\therefore$ Equation of parabola $x^2 = 4ay$. Since it passes through (6, –3) ; $\therefore$ 36= –12a

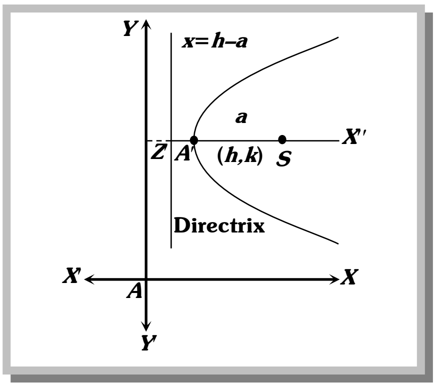
$\Rightarrow a = -3$

$\therefore$ Equation of parabola is $x^2 = -12y$.

5.1.4 Special form of Parabola $(y - k)^2 = 4a(x - h)$

The equation of a parabola with its vertex at (h, k) and axis as parallel to x-axis is $(y - k)^2 = 4a(x - h)$

If the vertex of the parabola is (p, q) and its axis is parallel to y-axis, then the equation of the parabola is $(x - p)^2 = 4b(y - q)$



When origin is shifted at $A(h, k)$ without changing the direction of axes, its equation becomes $(y - k)^2 = 4a(x - h)$ or $(x - p)^2 = 4b(y - q)$

Equation of Parabola	Vertex	Axis	Focus	Directrix	Equation of L.R.	Length of L.R.
$(y-K)^2 = 4a(x-h)$	(h,k)	$y=k$	$(h+a,k)$	$x+a-h=0$	$x=a+h$	$4a$
$(x-p)^2 = 4b(y-q)$	(p,q)	$x=p$	$(p,b+q)$	$y+b-q=0$	$y=b+q$	$4b$

Important Tips

- $y^2 = 4a(x+a)$ is the equation of the parabola whose focus is the origin and the axis is x-axis.
- $y^2 = 4a(x-a)$ is the equation of parabola whose axis is x-axis and y-axis is directrix.
- $x^2 = 4a(y+a)$ is the equation of parabola whose focus is the origin and the axis is y-axis.
- $x^2 = 4a(y-a)$ is the equation of parabola whose axis is y-axis and the directrix is x-axis.
- The equation to the parabola whose vertex and focus are on x-axis at a distance a and a' respectively from the origin is $y^2 = 4(a'-a)(x-a)$.
- The equation of parabola whose axis is parallel to x-axis is $x = Ay^2 + By + C$ and $y = Ax^2 + Bx + C$ is a parabola with its axis parallel to y-axis.

Example: 8 Vertex of the parabola $x^2 + 4x + 2y - 7 = 0$ is

(a) $(-2, 11/2)$ (b) $(-2, 2)$ (c) $(-2, 11)$ (d) $(2, 11)$

Solution: (a) Equation of the parabola is $(x+2)^2 = -2y + 7 + 4 \Rightarrow (x+2)^2 = -2\left(y - \frac{11}{2}\right)$.

Hence vertex is $\left(-2, \frac{11}{2}\right)$.

Example: 9 The focus of the parabola $4y^2 - 6x - 4y = 5$ is

(a) $(-8/5, 2)$ (b) $(-5/8, 1/2)$ (c) $(1/2, 5/8)$ (d) $(6/8, -1/2)$

Solution: (b) Given equation of parabola when written in standard form, we get

$4\left(y - \frac{1}{2}\right)^2 = 6(x+1) \Rightarrow \left(y - \frac{1}{2}\right)^2 = \frac{3}{2}(x+1) \Rightarrow Y^2 = \frac{3}{2}X$ where, $Y = y - \frac{1}{2}, X = x+1$

$\therefore y = Y + \frac{1}{2}, x = X - 1$ ....(i)

Focus $\Rightarrow X = a, \ Y = 0$; $\therefore 4a = \frac{3}{2} \Rightarrow a = \frac{3}{8} \Rightarrow x = \frac{3}{8} - 1 = -\frac{5}{8}$; $y = 0 + \frac{1}{2} = \frac{1}{2} \Rightarrow$ Focus = $\left(-\frac{5}{8}, \frac{1}{2}\right)$

Example: 10 The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is

(a) $x = -1$ (b) $x = 1$ (c) $x = \frac{-3}{2}$ (d) $x = \frac{3}{2}$

Solution: (d) Here, $y^2 + 4y + 4 + 4x - 2 = 0$ or $(y+2)^2 = -4\left(x - \frac{1}{2}\right)$

Let $y + 2 = Y$, $\frac{1}{2} - x = X$. Then the parabola is $Y^2 = 4X$.

$\therefore$ The directrix is $X + 1 = 0$ or $\frac{1}{2} - x + 1 = 0$, $\therefore x = \frac{3}{2}$

Example: 11 The line $x-1=0$ is the directrix of the parabola $y^2-kx+8=0$. Then one of the values of *k* is

(a) $\frac{1}{8}$ (b) 8 (c) 4 (d) $\frac{1}{4}$

Solution: (c) The parabola is $y^2=4\frac{k}{4}\left(x-\frac{8}{k}\right)$. Putting $y=Y,\ x-\frac{8}{k}=X$. The equation is $Y^2=4.\frac{k}{4}.X$

$\therefore$ The directrix is $X+\frac{k}{4}=0$ *i.e.*, $x-\frac{8}{k}+\frac{k}{4}=0$. But $x-1=0$ is the directrix.

So $\frac{8}{k}-\frac{k}{4}=1 \Rightarrow k=-8,4$.

Example: 12 Equation of the parabola with its vertex at (1, 1)and focus (3, 1)is

(a) $(x-1)^2=8(y-1)$ (b) $(y-1)^2=8(x-3)$ (c) $(y-1)^2=8(x-1)$ (d) $(x-3)^2=8(y-1)$

Solution: (c) Given vertex of parabola $(h,k)\equiv(1,1)$ and its focus $(a+h,k)\equiv(3,1)$ or $a+h=3$ or $a=2$. We know that as the *y*-coordinates of vertex and focus are same, therefore axis of parabola is parallel to *x*-axis. Thus equation of the parabola is $(y-k)^2=4a(x-h)$ or $(y-1)^2=4\times2(x-1)$ or $(y-1)^2=8(x-1)$.

5.1.5 Parametric equations of a Parabola

The simplest and the best form of representing the coordinates of a point on the parabola $y^2=4ax$ is $(at^2,2at)$ because these coordinates satisfy the equation $y^2=4ax$ for all values of *t*. The equations $x=at^2, y=2at$ taken together are called the parametric equations of the parabola $y^2=4ax$, *t* being the parameter.

The following table gives the parametric coordinates of a point on four standard forms of the parabola and their parametric equation.

Parabola	$y^2=4ax$	$y^2=-4ax$	$x^2=4ay$	$x^2=-4ay$
Parametric Coordinates	$(at^2,2at)$	$(-at^2,2at)$	$(2at,at^2)$	$(2at,-at^2)$
Parametric Equations	$x=at^2$ $y=2at$	$x=-at^2$ $y=2at$	$x=2at$ $y=at^2$	$x=2at$, $y=-at^2$

Note : ❑ The parametric equation of parabola $(y-k)^2=4a(x-h)$ are $x=h+at^2$ and $y=k+2at$

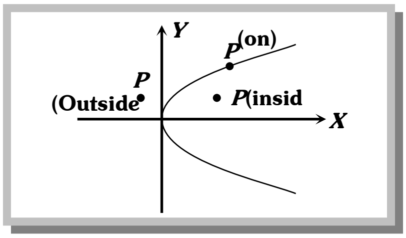
Example: 13 $x-2=t^2, y=2t$ are the parametric equations of the parabola

(a) $y^2=4x$ (b) $y^2=-4x$ (c) $x^2=-4y$ (d) $y^2=4(x-2)$

Solution: (d) Here $\frac{y}{2}=t$ and $x-2=t^2 \Rightarrow (x-2)=\left(\frac{y}{2}\right)^2 \Rightarrow y^2=4(x-2)$

5.1.6 Position of a point and a Line with respect to a Parabola

(1) **Position of a point with respect to a parabola**: The point $P(x_1,y_1)$ lies outside on or inside the parabola $y^2=4ax$ according as $y_1^2-4ax_1 >,=,\text{or} < 0$



(2) **Intersection of a line and a parabola:** Let the parabola be $y^2=4ax$ .....(i)

And the given line be $y=mx+c$ .....(ii)

Eliminating y from (i) and (ii) then $(mx+c)^2 = 4ax$ or $m^2x^2 + 2x(mc-2a) + c^2 = 0$ .....(iii)

This equation being quadratic in *x*, gives two values of *x*. It shows that every straight line will cut the parabola in two points, may be real, coincident or imaginary, according as discriminate of (iii) >, = or < 0

$\therefore$ The line $y = mx + c$ does not intersect, touches or intersect a parabola $y^2 = 4ax$, according as $c >, =, < \frac{a}{m}$

Condition of tangency : The line $y = mx + c$ touches the parabola, if $c = \frac{a}{m}$

Example: 14 The equation of a parabola is $y^2 = 4x$. $P(1,3)$ and $Q(1,1)$ are two points in the *xy*-plane. Then, for the parabola

(a) *P* and *Q* are exterior points (b)P is an interior point while *Q* is an exterior point

(c) *P* and *Q* are interior points (d)*P* is an exterior point while *Q* is an interior point

Solution: (d) Here, $S \equiv y^2 - 4x = 0$; $S(1,3) = 3^2 - 4.1 > 0 \Rightarrow P(1,3)$ is an exterior point.

$S(1,1) = 1^2 - 4.1 < 0 \Rightarrow Q(1,1)$ is an interior point.

Example: 15 The ends of a line segment are $P(1,3)$ and $Q(1,1)$. *R* is a point on the line segment *PQ* such that $PQ : QR = 1 : \lambda$. If *R* is an interior point of the parabola $y^2 = 4x$, then

(a) $\lambda \in (0,1)$ (b) $\lambda \in \left(-\frac{3}{5}, 1\right)$ (c) $\lambda \in \left(\frac{1}{2}, \frac{3}{5}\right)$ (d)None of these

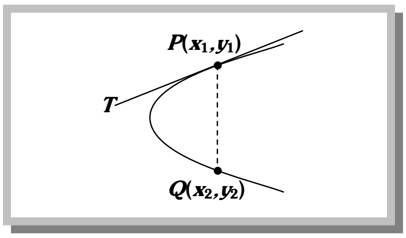
Solution: (a) $R = \left(1, \frac{1+3\lambda}{1+\lambda}\right)$ It is an interior point of $y^2 - 4x = 0$ *iff* $\left(\frac{1+3\lambda}{1+\lambda}\right)^2 - 4 < 0$

$\Rightarrow \left(\frac{1+3\lambda}{1+\lambda} - 2\right)\left(\frac{1+3\lambda}{1+\lambda} + 2\right) < 0 \Rightarrow \left(\frac{\lambda - 1}{1+\lambda}\right)\left(\frac{5\lambda + 3}{1+\lambda}\right) < 0 \Rightarrow (\lambda - 1)\left(\lambda + \frac{3}{5}\right) < 0$

Therefore, $-\frac{3}{5} < \lambda < 1$. But $\lambda > 0$ $\therefore 0 < \lambda < 1 \Rightarrow \lambda \in (0,1)$.

5.1.7 Equation of Tangent in Different forms

(1) **Point Form:** The equation of the tangent to the parabola $y^2 = 4ax$ at a point (x_1, y_1) is $yy_1 = 2a(x + x_1)$



Equation of tangent of all other standard parabolas at (x_1, y_1)	
Equation of parabolas	**Tangent at (x_1, y_1)**
$y^2 = -4ax$	$yy_1 = -2a(x + x_1)$
$x^2 = 4ay$	$xx_1 = 2a(y + y_1)$
$x^2 = -4ay$	$xx_1 = -2a(y + y_1)$

Note : ❑ The equation of tangent at (x_1, y_1) to a curve can also be obtained by replacing x^2 by xx_1, y^2 by yy_1, x by $\frac{x+x_1}{2}$, y by $\frac{y+y_1}{2}$ and xy by $\frac{xy_1+x_1y}{2}$ provided the equation of curve is a polynomial of second degree in x and y.

(2) **Parametric form :** The equation of the tangent to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ is $ty = x + at^2$

Equations of tangent of all other standard parabolas at '*t*'		
Equations of parabolas	**Parametric co-ordinates '*t*'**	**Tangent at '*t*'**
$y^2 = -4ax$	$(-at^2, 2at)$	$ty = -x + at^2$
$x^2 = 4ay$	$(2at, at^2)$	$tx = y + at^2$
$x^2 = -4ay$	$(2at, -at^2)$	$tx = -y + at^2$

(3) **Slope Form:** The equation of a tangent of slope m to the parabola $y^2 = 4ax$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ is $y = mx + \frac{a}{m}$

Equation of parabolas	**Point of contact in terms of slope (*m*)**	**Equation of tangent in terms of slope (*m*)**	**Condition of Tangency**
$y^2 = 4ax$	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y = mx + \frac{a}{m}$	$c = \frac{a}{m}$
$y^2 = -4ax$	$\left(-\frac{a}{m^2}, -\frac{2a}{m}\right)$	$y = mx - \frac{a}{m}$	$c = -\frac{a}{m}$
$x^2 = 4ay$	$(2am, am^2)$	$y = mx - am^2$	$c = -am^2$
$x^2 = -4ay$	$(-2am, -am^2)$	$y = mx + am^2$	$c = am^2$

Important Tips

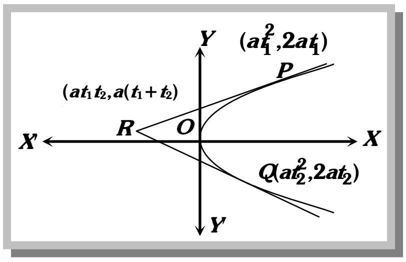
☞ If the straight line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ then $ln = am^2$.

☞ If the line $x\cos\alpha + y\sin\alpha = p$ touches the parabola $y^2 = 4ax$, then $P\cos\alpha + a\sin^2\alpha = 0$ and point of contact is $(a\tan^2\alpha, -2a\tan\alpha)$

☞ If the line $\frac{x}{l} + \frac{y}{m} = 1$ touches the parabola $y^2 = 4a(x+b)$, then $m^2(l+b) + al^2 = 0$

5.1.8 Point of intersection of Tangents at any two points on the Parabola

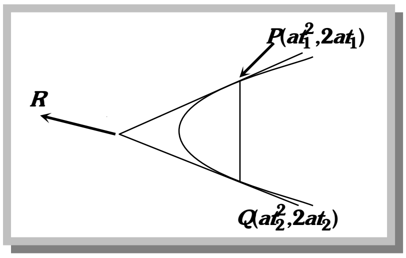
The point of intersection of tangents at two points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ is $(at_1t_2, a(t_1 + t_2))$.



The locus of the point of intersection of tangents to the parabola $y^2 = 4ax$ which meet at an angle α is $(x + a)^2 \tan^2 \alpha = y^2 - 4ax$.

Director circle: The locus of the point of intersection of perpendicular tangents to a conic is known as its director circle. The director circle of a parabola is its directrix.

Note : ❑ Clearly, *x*-coordinates of the point of intersection of tangents at *P* and *Q* on the parabola is the G.M of the *x*-coordinate of *P* and *Q* and y-coordinate is the A.M. of *y*-coordinate of *P* and *Q*.

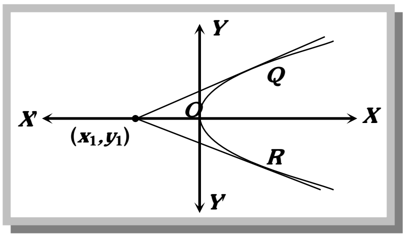


❑ The equation of the common tangents to the parabola $y^2 = 4ax$ and $x^2 = 4by$ is $a^{\frac{1}{3}} x + b^{\frac{1}{3}} y + a^{\frac{2}{3}} b^{\frac{2}{3}} = 0$

❑ The tangents to the parabola $y^2 = 4ax$ at $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ intersect at *R*. Then the area of triangle *PQR* is $\frac{1}{2} a^2 (t_1 - t_2)^3$

5.1.9 Equation of Pair of Tangents from a point to a Parabola

If $y_1^2 - 4ax_1 > 0$, then any point $P(x_1, y_1)$ lies out side the parabola and a pair of tangents *PQ*, *PR* can be drawn to it from *P*



The combined equation of the pair of the tangents drawn from a point to a parabola is $SS' = T^2$ where $S = y^2 - 4ax$, $S' = y_1^2 - 4ax_1$ and $T = yy_1 - 2a(x + x_1)$

Note : ❑ The two tangents can be drawn from a point to a parabola. The two tangent are real and distinct or coincident or imaginary according as the given point lies outside, on or inside the parabola.

Important Tips

☞ Tangents at the extremities of any focal chord of a parabola meet at right angles on the directrix.

☞ Area of the triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points.

☞ If the tangents at the points P and Q on a parabola meet in T, then ST is the geometric mean between SP and SQ, i.e. $ST^2 = SP.SQ$

☞ Tangent at one extremity of the focal chord of a parabola is parallel to the normal at the other extremity.

☞ The angle of intersection of two parabolas $y^2 = 4ax$ and $x^2 = 4by$ is given by $\tan^{-1} \frac{3a^{1/3} b^{1/3}}{2(a^{2/3} + b^{2/3})}$

Example: 16 The straight line $y=2x+\lambda$ does not meet the parabola $y^2=2x$, if

(a) $\lambda<\frac{1}{4}$ (b) $\lambda>\frac{1}{4}$ (c) $\lambda=4$ (d) $\lambda=1$

Solution: (b) $y=2x+\lambda$ does not meet the parabola $y^2=2x$, If $\lambda>\frac{a}{m}=\frac{1}{2.2}=\frac{1}{4}\Rightarrow\lambda>\frac{1}{4}$

Example: 17 If the parabola $y^2=4ax$ passes through the point $(1,-2)$, then the tangent at this point is

(a) $x+y-1=0$ (b) $x-y-1=0$ (c) $x+y+1=0$ (d) $x-y+1=0$

Solution: (c) $\because$ Parabola passes through the point (1, –2), then $4=4a\Rightarrow a=1$.

From $yy_1=2a(x+x_1)\Rightarrow -2y=2(x+1)$

$\therefore$ Required tangent is $x+y+1=0$

Example: 18 The equation of the tangent to the parabola $y^2=16x$, which is perpendicular to the line $y=3x+7$ is

(a) $y-3x+4=0$ (b) $3y-x+36=0$ (c) $3y+x-36=0$ (d) $3y+x+36=0$

Solution: (a) A line perpendicular to the given line is $3y+x=\lambda\Rightarrow y=-\frac{1}{3}x+\frac{\lambda}{3}$

Here $m=-\frac{1}{3}$, $c=\frac{\lambda}{3}$. If we compare $y^2=16x$ with $y^2=4ax$, then $a=4$

Condition for tangency is $c=\frac{a}{m}\Rightarrow\frac{\lambda}{3}=\frac{4}{(-1/3)}\Rightarrow\lambda=-36$.

$\therefore$ Required equation is $x+3y+36=0$.

Example: 19 If the tangent to the parabola $y^2=ax$ makes an angle of 45^o with x-axis, then the point of contact is

(a) $\left(\frac{a}{2},\frac{a}{2}\right)$ (b) $\left(\frac{a}{4},\frac{a}{4}\right)$ (c) $\left(\frac{a}{2},\frac{a}{4}\right)$ (d) $\left(\frac{a}{4},\frac{a}{2}\right)$

Solution: (d) Parabola is $y^2=ax$ *i.e.* $y^2=4\left(\frac{a}{4}\right)x$ .....(i)

Let point of contact is (x_1, y_1).

$\therefore$ Equation of tangent is $y-y_1=\frac{2(a/4)}{y_1}(x-x_1)\Rightarrow y=\frac{a}{2y_1}(x)-\frac{ax_1}{2y_1}+y_1$

Here, $m=\frac{a}{2y_1}=\tan 45^o\Rightarrow\frac{a}{2y_1}=1\Rightarrow y_1=\frac{a}{2}$. From (i), $x_1=\frac{a}{4}$, So point is $\left(\frac{a}{4},\frac{a}{2}\right)$.

Example: 20 The line $x-y+2=0$ touches the parabola $y^2=8x$ at the point

(a) (2, –4) (b) $(1,2\sqrt{2})$ (c) $(4,-4\sqrt{2})$ (d) (2, 4)

Solution: (d) The line $x-y+2=0$ i.e. $x=y-2$ meets parabola $y^2=8x$, if

$\Rightarrow y^2=8(y-2)=8y-16\Rightarrow y^2-8y+16=0$ $\Rightarrow (y-4)^2=0\Rightarrow y=4,4$

$\because$ Roots are equal, $\therefore$ Given line touches the given parabola.

$\therefore$ $x=4-2=2$, Thus the required point is (2, 4).

Example: 21 The equation of the tangent to the parabola at point $(a/t^2, 2a/t)$ is

(a) $ty=xt^2+a$ (b) $ty=x+at^2$ (c) $y=tx+at^2$ (d) $y=tx+(a/t^2)$

Solution: (a) Equation of the tangent to the parabola, $y^2=4ax$ is $yy_1=2a(x+x_1)$

$$\Rightarrow y.\frac{2a}{t} = 2a\left(x + \frac{a}{t^2}\right) \Rightarrow \frac{y}{t} = \left(x + \frac{a}{t^2}\right) \Rightarrow \frac{y}{t} = \frac{t^2 x + a}{t^2} \Rightarrow ty = t^2 x + a$$

Example: 22 Two tangents are drawn from the point (–2, –1) to the parabola $y^2 = 4x$. If α is the angle between these tangents, then $\tan\alpha$ =

(a) 3 (b) 1/3 (c) 2 (d) 1/2

Solution: (a) Equation of pair of tangent from $(-2,-1)$ to the parabola is given by $SS_1 = T^2$ *i.e.* $(y^2 - 4x)(1+8) = [y(-1) - 2(x-2)]^2$

$$\Rightarrow 9y^2 - 36x = [-y - 2x + 4]^2 \Rightarrow 9y^2 - 36x = y^2 + 4x^2 + 16 + 4xy - 16x - 8y$$

$$\Rightarrow \quad 4x^2 - 8y^2 + 4xy + 20x - 8y + 16 = 0$$

$$\therefore \tan\alpha = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\sqrt{4 - 4(-8)}}{4-8}\right| = \left|\frac{12}{-4}\right| = 3$$

Example: 23 If $\left(\frac{a}{b}\right)^{1/3} + \left(\frac{b}{a}\right)^{1/3} = \frac{\sqrt{3}}{2}$, then the angle of intersection of the parabola $y^2 = 4ax$ and $x^2 = 4by$ at a point other than the origin is

(a) $\pi/4$ (b) $\pi/3$ (c) $\pi/2$ (d) None of these

Solution: (b) Given parabolas are $y^2 = 4ax$ ....(i) and $x^2 = 4by$ .....(ii)

These meet at the points (0, 0), $(4a^{1/3}b^{2/3}, 4a^{2/3}b^{1/3})$

Tangent to (i) at $(4a^{1/3}b^{2/3}, 4a^{2/3}b^{1/3})$ is $y.4a^{2/3}b^{1/3} = 2a(x + 4a^{2/3}b^{1/3})$

Slope of the tangent $(m_1) = \frac{2a}{4a^{2/3}b^{1/3}} = \frac{a^{1/3}}{2b^{1/3}}$

Tangent to (ii) at $(4a^{1/3}b^{2/3}, 4a^{2/3}b^{1/3})$ is $x.4a^{1/3}b^{2/3} = 2b(y + 4a^{2/3}b^{1/3})$

Slope of the tangent $(m_2) = \frac{2a^{1/3}}{b^{1/3}}$

If θ is the angle between the two tangents, then $\Rightarrow \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{a^{1/3}}{2b^{1/3}} - \frac{2a^{1/3}}{b^{1/3}}}{1 + \frac{a^{1/3}}{2b^{1/3}} \cdot \frac{2a^{1/3}}{b^{1/3}}}\right|$

$$= \frac{3}{2} \cdot \frac{1}{\left(\frac{a}{b}\right)^{1/3} + \left(\frac{b}{a}\right)^{1/3}} = \frac{3}{2} \cdot \frac{1}{\frac{\sqrt{3}}{2}} = \sqrt{3};$$

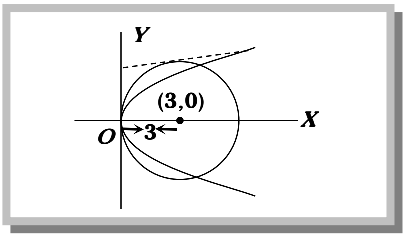
$$\therefore \theta = 60^o = \frac{\pi}{3}$$

Example: 24 The equation of the common tangent touching the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the *x*-axis , is

(a) $\sqrt{3}y = 3x + 1$ (b) $\sqrt{3}y = -(x+3)$ (c) $\sqrt{3}y = x + 3$ (d) $\sqrt{3}y = -(3x+1)$

Solution: (c) Any tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$. It touches the circle if $3 = \left|\frac{3m + \frac{1}{m}}{\sqrt{1+m^2}}\right|$

or $9(1+m^2) = \left(3m + \frac{1}{m}\right)^2$ or $\frac{1}{m^2} = 3$, $\therefore m = \pm\frac{1}{\sqrt{3}}$



For the common tangent to be above the x-axis, $m=\frac{1}{\sqrt{3}}$

$\therefore$ Common tangent is $y=\frac{1}{\sqrt{3}}x+\sqrt{3} \Rightarrow \sqrt{3}y=x+3$

Example: 25 If $a\neq 0$ and the line $2bx+3cy+4d=0$ passes through the points of intersection of the parabolas $y^2=4ax$ and $x^2=4ay$ then

(a) $d^2+(3b-2c)^2=0$ (b) $d^2+(3b+2c)^2=0$ (c) $d^2+(2b-3c)^2=0$ (d) $d^2+(2b+3c)^2=0$

Solution: (d) Given parabolas are $y^2=4ax$ .....(i) and $x^2=4ay$ .....(ii)

from (i) and (ii) $\left(\frac{x^2}{4a}\right)^2=4ax \Rightarrow x^4-64a^3x=0 \Rightarrow x=0,\ 4a \ \therefore y=0,\ 4a$

So points of intersection are $(0,0)$ and $(4a,4a)$

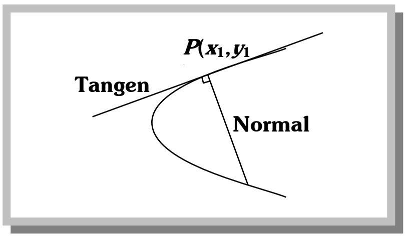
Given, the line $2bx+3cy+4d=0$ passes through (0,0) and $(4a,4a)$

$\therefore d=0 \Rightarrow d^2=0$ and $(2b+3c)^2=0$ $(\because a\neq 0)$

Therefore $d^2+(2b+3c)^2=0$

5.1.10 Equations of Normal in Different forms

(1) **Point form :** The equation of the normal to the parabola $y^2=4ax$ at a point (x_1,y_1) is $y-y_1=-\frac{y_1}{2a}(x-x_1)$



Equation of normals of all other standard parabolas at (x_1, y_1)	
Equation of parabolas	**Normal at (x_1, y_1)**
$y^2=-4ax$	$y-y_1=\frac{y_1}{2a}(x-x_1)$
$x^2=4ay$	$y-y_1=-\frac{2a}{x_1}(x-x_1)$
$x^2=-4ay$	$y-y_1=\frac{2a}{x_1}(x-x_1)$

(2) **Parametric form**: The equation of the normal to the parabola $y^2=4ax$ at $(at^2,2at)$ is $y+tx=2at+at^3$

Equations of normal of all other standard parabola at 't'		
Equations of parabolas	**Parametricco-ordinates**	**Normals at 't'**
$y^2=-4ax$	$(-at^2,2at)$	$y-tx=2at+at^3$

$x^2 = 4ay$	$(2at, at^2)$	$x + ty = 2at + at^3$
$x^2 = -4ay$	$(2at, -at^2)$	$x - ty = 2at + at^3$

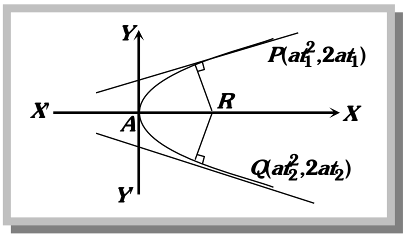
(3) **Slope form:** The equation of normal of slope m to the parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$ at the point $(am^2, -2am)$.

Equations of normal, point of contact, and condition of normality in terms of slope (*m*)			
Equations of parabola	**Point of contact in terms of slope (*m*)**	**Equations of normal in terms of slope (*m*)**	**Condition of normality**
$y^2 = 4ax$	$(am^2, -2am)$	$y = mx - 2am - am^3$	$c = -2am - am^3$
$y^2 = -4ax$	$(-am^2, 2am)$	$y = mx + 2am + am^3$	$c = 2am + am^3$
$x^2 = 4ay$	$\left(-\frac{2a}{m}, \frac{a}{m^2}\right)$	$y = mx + 2a + \frac{a}{m^2}$	$c = 2a + \frac{a}{m^2}$
$x^2 = -4ay$	$\left(\frac{2a}{m}, -\frac{a}{m^2}\right)$	$y = mx - 2a - \frac{a}{m^2}$	$c = -2a - \frac{a}{m^2}$

Note : ❑ The line $lx + my + n = 0$ is a normal to the parabola $y^2 = 4ax$ if $al(l^2 + 2m^2) + m^2 n = 0$

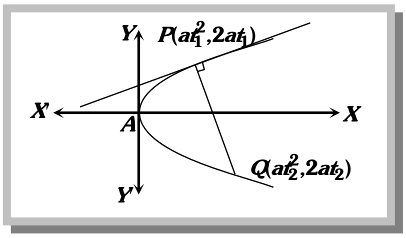
5.1.11 Point of intersection of normals at any two points on the Parabola

If R is the point of intersection then point of intersection of normals at any two points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ is $R[2a + a(t_1^2 + t_2^2 + t_1 t_2), -at_1 t_2(t_1 + t_2)]$



5.1.12 Relation between 't_1' and 't_2' if Normal at 't_1' meets the Parabola again at 't_2'

If the normal at the point $P(at_1^2, 2at_1)$ meets the parabola $y^2 = 4ax$ again at $Q(at_2^2, 2at_2)$, then $t_2 = -t_1 - \frac{2}{t_1}$



Important Tips

- If the normals at points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$ meet on the parabola then $t_1 t_2 = 2$
- If the normal at a point $P(at^2, 2at)$ to the parabola $y^2 = 4ax$ subtends a right angle at the vertex of the parabola then $t^2 = 2$.
- If the normal to a parabola $y^2 = 4ax$, makes an angle ϕ with the axis, then it will cut the curve again at an angle $\tan^{-1}\left(\frac{1}{2}\tan\phi\right)$.
- The normal chord to a parabola $y^2 = 4ax$ at the point whose ordinate is equal to abscissa subtends a right angle at the focus.
- If the normal at two points P and Q of a parabola $y^2 = 4ax$ intersect at a third point R on the curve. Then the product of the ordinate of P and Q is $8a^2$.

Example: 26 If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then k is

(a) 3 (b) 9 (c) –9 (d) –3

Solution: (b) Any normal to the parabola $y^2 = 12x$ is $y + tx = 6t + 3t^3$. It is identical with $x + y = k$ if $\frac{t}{1} = \frac{1}{1} = \frac{6t + 3t^3}{k}$

$\therefore t = 1$ and $1 = \frac{6+3}{k} \Rightarrow k = 9$

Example: 27 The equation of normal at the point $\left(\frac{a}{4}, a\right)$ to the parabola $y^2 = 4ax$, is

(a) $4x + 8y + 9a = 0$ (b) $4x + 8y - 9a = 0$ (c) $4x + y - a = 0$ (d) $4x - y + a = 0$

Solution: (b) From $y - y_1 = -\frac{-y_1}{2a}(x - x_1)$

$\Rightarrow y - a = \frac{-a}{2a}\left(x - \frac{a}{4}\right) \Rightarrow 2y + x = 2a + \frac{a}{4} = \frac{9a}{4} \Rightarrow 2y + x - \frac{9a}{4} = 0 \Rightarrow 4x + 8y - 9a = 0$

Example: 28 The point on the parabola $y^2 = 8x$ at which the normal is parallel to the line $x - 2y + 5 = 0$ is

(a) $(-1/2, 2)$ (b) $(1/2, -2)$ (c) $(2, -1/2)$ (d) $(-2, 1/2)$

Solution: (b) Let point be (h, k). Normal is $y - k = \frac{-k}{4}(x - h)$ or $-kx - 4y + kh + 4k = 0$

Gradient $= \frac{-K}{4} = \frac{1}{2} \Rightarrow k = -2$. Substituting (h, k) and $k = -2$ in $y^2 = 8x$, we get $h = \frac{1}{2}$. Hence point is $\left(\frac{1}{2}, -2\right)$

Trick: Here only point $\left(\frac{1}{2}, -2\right)$ will satisfy the parabola $y^2 = 8x$.

Example: 29 The equations of the normal at the ends of the latus rectum of the parabola $y^2 = 4ax$ are given by

(a) $x^2 - y^2 - 6ax + 9a^2 = 0$ (b) $x^2 - y^2 - 6ax - 6ay + 9a^2 = 0$

(c) $x^2 - y^2 - 6ay + 9a^2 = 0$ (d) None of these

Solution: (a) The coordinates of the ends of the latus rectum of the parabola $y^2 = 4ax$ are $(a, 2a)$ and $(a, -2a)$ respectively.

The equation of the normal at $(a, 2a)$ to $y^2 = 4ax$ is $y - 2a = \frac{-2a}{2a}(x - a)$ $\left\{\text{using } y - y_1 = \frac{-y_1}{2a}(x - x_1)\right\}$

Or $x + y - 3a = 0$ .....(i)

Similarly the equation of the normal at $(a, -2a)$ is $x - y - 3a = 0$ .....(ii)

The combined equation of (i) and (ii) is $x^2 - y^2 - 6ax + 9a^2 = 0$.

Example: 30 The locus of the point of intersection of two normals to the parabola $x^2 = 8y$, which are at right angles to each other, is

(a) $x^2 = 2(y - 6)$ (b) $x^2 = 2(y + 6)$ (c) $x^2 = -2(y - 6)$ (d) None of these

Solution: (a) Given parabola is $x^2 = 8y$ .....(i)

Let $(4t_1, 2t_1^2)$ and $Q(4t_2, 2t_2^2)$ be two points on the parabola (i)

Normal at P, Q are $y - 2t_1^2 = -\frac{1}{t_1}(x - 4t_1)$ ......(ii) and $y - 2t_2^2 = -\frac{1}{t_2}(x - 4t_2)$ ......(iii)

(ii)–(iii) gives $2(t_2^2 - t_1^2) = x\left(\frac{1}{t_2} - \frac{1}{t_1}\right) = x\frac{t_1 - t_2}{t_1 t_2}$, $\therefore\ x = -2t_1 t_2(t_2 + t_1)$ .....(iv)

From (ii), $y = 2t_1^2 - \frac{1}{t_1}(-2t_1 t_2(t_2 + t_1) - 4t_1) = 2t_1^2 + 2t_2(t_1 + t_2) + 4 \Rightarrow y = 2t_1^2 + 2t_1 t_2 + 2t_2^2 + 4$ .....(v)

Since normals (ii) and (iii) are at right angles, $\therefore$ $\left(-\frac{1}{t_1}\right)\left(-\frac{1}{t_2}\right) = -1 \Rightarrow t_1 t_2 = -1$

$\therefore$ From (iv), $x = 2(t_1 + t_2)$ and from (v) $y = 2t_1^2 - 2 + 2t_2^2 + 4$

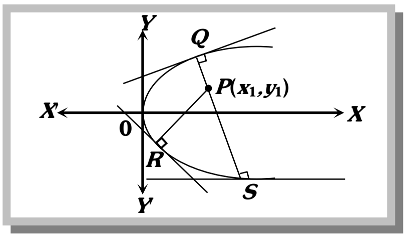
$\Rightarrow y = 2[t_1^2 + t_2^2 + 1] = 2[(t_1 + t_2)^2 - 2t_1 t_2 + 1]$

$\Rightarrow y = 2[(t_1 + t_2)^2 + 2 + 1] = 2[(t_1 + t_2)^2 + 3 \Rightarrow y = 2\left[\frac{x^2}{4} + 3\right] = \frac{x^2}{2} + 6$

$\Rightarrow x^2 = 2(y - 6)$, which is the required locus.

5.1.13 Co-normal Points

The points on the curve at which the normals pass through a common point are called co-normal points.



Q, R, S are co-normal points. The co- normal points are also called the feet of the normals.

If the normal passes through point $P(x_1, y_1)$ which is not on parabola, then $y_1 = mx_1 - 2am - am^3 \Rightarrow am^3 + (2a - x_1)m + y_1 = 0$ ......(i)

Which gives three values of m. Let three values of m are m_1, m_2 and m_3, which are the slopes of the normals at Q, R and S respectively, then the coordinates of Q, R and S are $(am_1^2, -2am_1), (am_2^2, -2am_2)$ and $(am_3^2, -2am_3)$ respectively. These three points are called the feet of the normals.

Now $m_1 + m_2 + m_3 = 0$, $m_1 m_2 + m_2 m_3 + m_3 m_1 = \frac{(2a - x_1)}{a}$ and $m_1 m_2 m_3 = \frac{-y_1}{a}$

In general, three normals can be drawn from a point to a parabola.

(1) The algebraic sum of the slopes of three concurrent normals is zero.

(2) The sum of the ordinates of the co-normal points is zero.

(3) The centroid of the triangle formed by the co-normal points lies on the axis of the parabola.

(4) The centroid of a triangle formed by joining the foots of the normal of the parabola lies on its axis and is given by $\left(\frac{am_1^2+am_2^2+am_3^2}{3}, \frac{2am_1+2am_2+2am_3}{3}\right)=\left(\frac{am_1^2+am_2^2+am_3^2}{3}, 0\right)$

(5) If three normals drawn to any parabola $y^2=4ax$ from a given point (h, k) be real, then $h>2a$ for $a=1$, normals drawn to the parabola $y^2=4x$ from any point (h, k) are real, if $h>2$.

(6) Out of these three at least one is real, as imaginary normals will always occur in pairs.

5.1.14 Circle through Co-normal points

Equation of the circle passing through the three (co-normal) points on the parabola $y^2=4ax$, normal at which pass through a given point (α, β); is $x^2+y^2-(2a+\alpha)x-\frac{\beta}{2}y=0$

(1) The algebraic sum of the ordinates of the four points of intersection of a circle and a parabola is zero.

(2) The common chords of a circle and a parabola are in pairs, equally inclined to the axis of parabola.

(3) The circle through co-normal points passes through the vertex of the parabola.

(4) The centroid of four points; in which a circle intersects a parabola, lies on the axis of the parabola.

Example: 31 The normals at three points P, Q, R of the parabola $y^2=4ax$ meet in (h, k), the centroid of triangle PQR lies on

(a) $x=0$ (b) $y = 0$ (c) $x=-a$ (d) $y=a$

Solution: (b) Since the centroid of the triangle formed by the co-normal points lies on the axis of the parabola.

Example: 32 If two of the three feet of normals drawn from a point to the parabola $y^2=4x$ be (1, 2) and (1, – 2) then the third foot is

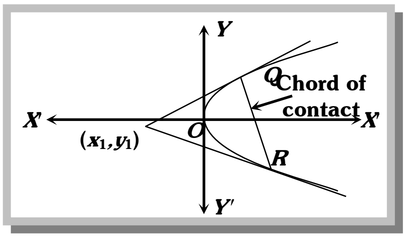
(a) $(2, 2\sqrt{2})$ (b) $(2, -2\sqrt{2})$ (c) $(0, 0)$ (d) None of these

Solution: (c) The sum of the ordinates of the foot = $y_1+y_2+y_3=0$

$\therefore 2+(-2)+y_3=0 \Rightarrow y_3=0$

5.1.15 Equation of the Chord of contact of Tangents to a Parabola

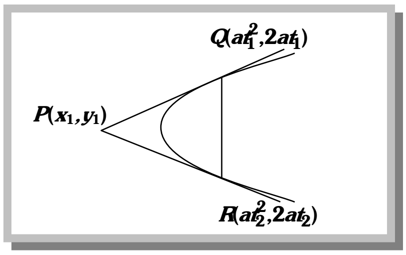
Let *PQ* and *PR* be tangents to the parabola $y^2 = 4ax$ drawn from any external point $P(x_1, y_1)$ then *QR* is called the 'Chord of contact' of the parabola $y^2 = 4ax$.



The chord of contact of tangents drawn from a point (x_1, y_1) to the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$

The equation is same as equation of the tangents at the point (x_1, y_1).

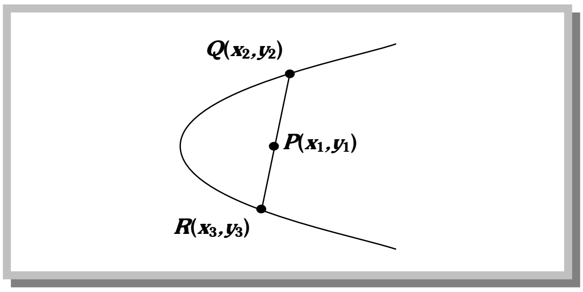
Note :
- ❑ The chord of contact joining the point of contact of two perpendicular tangents always passes through focus.
- ❑ If tangents are drawn from the point (x_1, y_1) to the parabola $y^2 = 4ax$, then the length of their chord of contact is $\frac{1}{|a|}\sqrt{(y_1^2 - 4ax_1)(y_1^2 + 4a^2)}$



- ❑ The area of the triangle formed by the tangents drawn from (x_1, y_1) to $y^2 = 4ax$ and their chord of contact is $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$.

5.1.16 Equation of the Chord of the Parabola which is bisected at a given point

The equation of the chord at the parabola $y^2 = 4ax$ bisected at the point (x_1, y_1) is given by $T = S_1$, where $T = yy_1 - 2a(x + x_1)$ and $S_1 = y_1^2 - 4ax_1$. *i.e.*, $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$



5.1.17 Equation of the Chord joining any two points on the Parabola

Let $P(at_1^2, 2at_1), Q(at_2^2, 2at_2)$ be any two points on the parabola $y^2 = 4ax$. Then, the equation of the chord joining these points is, $y - 2at_1 = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}(x - at_1^2)$ or $y - 2at_1 = \frac{2}{t_1 + t_2}\left(x - at_1^2\right)$ or $y(t_1 + t_2) = 2x + 2at_1t_2$

(1) **Condition for the chord joining points having parameters t_1 and t_2 to be a focal chord:** If the chord joining points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ on the parabola passes through its focus, then $(a, 0)$ satisfies the equation $y(t_1 + t_2) = 2x + 2at_1t_2 \Rightarrow 0 = 2a + 2at_1t_2 \Rightarrow t_1t_2 = -1$ or $t_2 = -\frac{1}{t_1}$

(2) **Length of the focal chord:** The length of a focal chord having parameters t_1 and t_2 for its end points is $a(t_2 - t_1)^2$.

Note: ❑ If one extremity of a focal chord is $(at_1^2, 2at_1)$, then the other extremity $(at_2^2, 2at_2)$ becomes $\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$ by virtue of relation $t_1 t_2 = -1$.

❑ If one end of the focal chord of parabola is $(at^2, 2at)$, then other end will be $\left(\frac{a}{t^2}, -2at\right)$ and length of chord $= a\left(t + \frac{1}{t}\right)^2$.

❑ The length of the chord joining two point 't_1' and 't_2' on the parabola $y^2 = 4ax$ is $a(t_1 - t_2)\sqrt{(t_1 + t_2)^2 + 4}$

❑ The length of intercept made by line $y = mx + c$ between the parabola $y^2 = 4ax$ is $\frac{4}{m^2}\sqrt{a(1+m^2)(a - mc)}$.

Important Tips

☞ The focal chord of parabola $y^2 = 4ax$ making an angle α with the x-axis is of length $4a\operatorname{cosec}^2\alpha$.

☞ The length of a focal chord of a parabola varies inversely as the square of its distance from the vertex.

☞ If l_1 and l_2 are the length of segments of a focal chord of a parabola, then its latus-rectum is $\frac{4l_1 l_2}{l_1 + l_2}$

☞ The semi latus rectum of the parabola $y^2 = 4ax$ is the harmonic mean between the segments of any focal chord of the parabola.

Example: 33 If the points $(au^2, 2au)$ and $(av^2, 2av)$ are the extremities of a focal chord of the parabola $y^2 = 4ax$, then

(a) $uv - 1 = 0$ (b) $uv + 1 = 0$ (c) $u + v = 0$ (d) $u - v = 0$

Solution: (b) Equation of focal chord for the parabola $y^2 = 4ax$ passes through the point $(au^2, 2au)$ and $(av^2, 2av)$

$\Rightarrow$ $y - 2au = \frac{2av - 2au}{av^2 - au^2}(x - au^2) \Rightarrow y - 2au = \frac{2a(v-u)}{a(v-u)(v+u)}(x - au^2) \Rightarrow y - 2au = \frac{2}{v+u}(x - au^2)$

It this is focal chord, so it would passes through focus $(a, 0)$

$\Rightarrow$ $0 - 2au = \frac{2}{v+u}(a - au^2) \Rightarrow -uv - u^2 = 1 - u^2$, $\therefore$ $uv + 1 = 0$

Trick : Given points $(au^2, 2au)$ and $(av^2, 2av)$, then $t_1 = u$ and $t_2 = v$, we know that $t_1 t_2 = -1$. Hence $uv + 1 = 0$.

Example: 34 The locus of the midpoint of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with the directrix

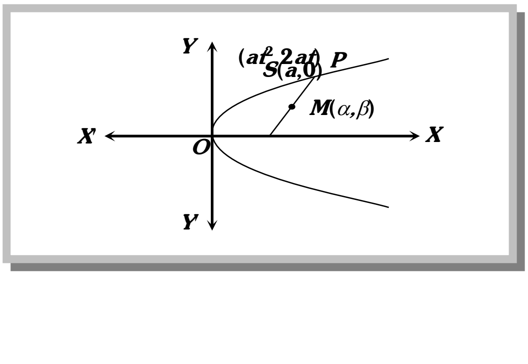
(a) $x = -a$ (b) $x = -\frac{a}{2}$ (c) $x = 0$ (d) $x = \frac{a}{2}$

Solution: (c) Let $M(\alpha, \beta)$ be the mid point of *PS*.

$\alpha = \frac{at^2 + a}{2}, \beta = \frac{2at + 0}{2} \Rightarrow 2\alpha = at^2 + a, \; at = \beta$

$\therefore$ $2\alpha = a.\frac{\beta^2}{a^2} + a$ or $2a\alpha = \beta^2 + a^2$

$\therefore$ The locus is $y^2 = \frac{4a}{2}\left(x - \frac{a}{2}\right) = 4b(x - b), \left\{b = \frac{a}{2}\right\}$



$\therefore$ Directrix is $(x-b)+b=0$ or $x=0$.

Example: 35 The length of chord of contact of the tangents drawn from the point (2, 5) to the parabola $y^2=8x$, is

(a) $\frac{1}{2}\sqrt{41}$ (b) $\sqrt{41}$ (c) $\frac{3}{2}\sqrt{41}$ (d) $2\sqrt{41}$

Solution: (c) Equation of chord of contact of tangents drawn from a point (x_1, y_1) to parabola $y^2=4ax$ is $yy_1=2a(x+x_1)$. So that $5y=2\times 2(x+2) \Rightarrow 5y=4x+8$.

Point of intersection of chord of contact with parabola $y^2=8x$ are $\left(\frac{1}{2}, 2\right)$,(8, 8), So the length of chord is $\frac{3}{2}\sqrt{41}$.

Example: 36 If *b*, *k* are the intercept of a focal chord of the parabola $y^2=4ax$, then *K* is equal to

(a) $\frac{ab}{b-a}$ (b) $\frac{b}{b-a}$ (c) $\frac{a}{b-a}$ (d) $\frac{ab}{a-b}$

Solution: (a) Let $'t_1'$ $'t_2'$ be the ends of focal chords

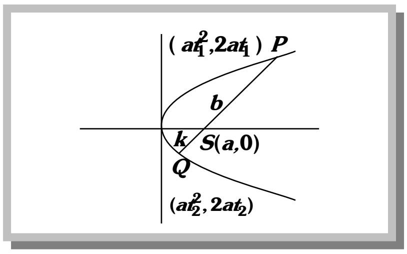
$\therefore$. $t_1t_2=-1$. If S is the focus and *P*, *Q* are the ends of the focal chord, then

$SP=\sqrt{(at_1^2-a)^2+(2at_1-0)^2}=a(t_1^2+1)=b$ (Given).... (i)

$\therefore$ $SQ=a(t_2^2+1)=a\left(\frac{1}{t_1^2}+1\right)$ (Given) $\left[\because t_2=-\frac{1}{t_1} \Rightarrow t_2^2=\frac{1}{t_1^2}\right]$

$=\frac{a(t_1^2+1)}{t_1^2}=k$ ....(ii), $\therefore$ $\frac{b}{k}=t_1^2$ [Divide (i) by (ii)]

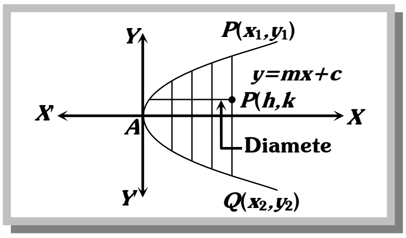
Putting in (1), we get $a\left(\frac{b}{k}+1\right)=b \Rightarrow \frac{ab}{k}+a=b \Rightarrow k=\frac{ab}{b-a}$



5.1.18 Diameter of a Parabola

The locus of the middle points of a system of parallel chords is called a diameter and in case of a parabola this diameter is shown to be a straight line which is parallel to the axis of the parabola.

The equation of the diameter bisecting chords of the parabola $y^2=4ax$ of slope *m* is $y=\frac{2a}{m}$



Note : ❑ Every diameter of a parabola is parallel to its axis.

❑ The tangent at the end point of a diameter is parallel to corresponding system of parallel chords.

❑ The tangents at the ends of any chord meet on the diameter which bisects the chord.

Example: 37 Equation of diameter of parabola $y^2=x$ corresponding to the chord $x-y+1=0$ is

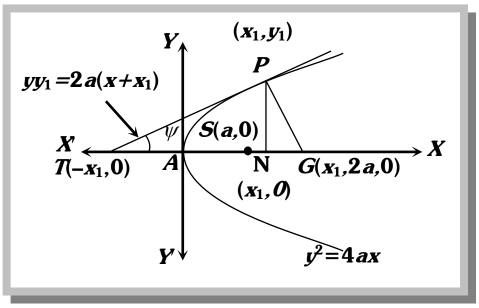
(a) $2y=3$ (b) $2y=1$ (c) $2y=5$ (d) $y=1$

Solution: (b) Equation of diameter of parabola is $y=\frac{2a}{m}$, Here $a=\frac{1}{4}, m=1 \Rightarrow y=\frac{2.\frac{1}{4}}{1} \Rightarrow 2y=1$

5.1.19 Length of Tangent, Subtangent, Normal and Subnormal

Let the parabola $y^2 = 4ax$. Let the tangent and normal at $P(x_1, y_1)$ meet the axis of parabola at T and G respectively, and tangent at $P(x_1, y_1)$ makes angle ψ with the positive direction of x-axis.

$A(0,0)$ is the vertex of the parabola and $PN = y$. Then,



(1) Length of tangent $= PT = PN\operatorname{cosec}\psi = y_1 \operatorname{cosec}\psi$

(2) Length of normal $= PG = PN\operatorname{cosec}(90^o - \psi) = y_1 \sec\psi$

(3) Length of subtangent $= TN = PN\cot\psi = y_1 \cot\psi$

(4) Length of subnormal $= NG = PN\cot(90^o - \psi) = y_1 \tan\psi$

where , $\tan\psi = \frac{2a}{y_1} = m$, [slope of tangent at $P(x, y)$]

Length of tangent, subtangent, normal and subnormal to $y^2 = 4ax$ at $(at^2, 2at)$

(1) Length of tangent at $(at^2, 2at) = 2at\operatorname{cosec}\psi = 2at\sqrt{(1+\cot^2\psi)} = 2at\sqrt{1+t^2}$

(2) Length of normal at $(at^2, 2at) = 2at\sec\psi = 2at\sqrt{(1+\tan^2\psi)} = 2a\sqrt{t^2 + t^2\tan^2\psi} = 2a\sqrt{(t^2+1)}$

(3) Length of subtangent at $(at^2, 2at) = 2at\cot\psi = 2at^2$

(4) Length of subnormal at $(at^2, 2at) = 2at\tan\psi = 2a$

Example: 38 The length of the subtangent to the parabola $y^2 = 16x$ at the point whose abscissa is 4, is

(a) 2 (b) 4 (c) 8 (d) None of these

Solution: (c) Since the length of the subtangent at a point to the parabola is twice the abscissa of the point. Therefore, the required length is 8.

Example: 39 If P is a point on the parabola $y^2 = 4ax$ such that the subtangent and subnormal at P are equal, then the coordinates of P are

(a) $(a, 2a)$ or $(a, -2a)$ (b) $(2a, 2\sqrt{2}a)$ or $(2a, -2\sqrt{2}a)$

(c) $(4a, -4a)$ or $(4a, 4a)$ (d) None of these

Solution: (a) Since the length of the subtangent at a point on the parabola is twice the abscissa of the point and the length of the subnormal is equal to semi-latus-rectum. Therefore if $P(x, y)$ is the required point, then $2x = 2a \Rightarrow x = a$

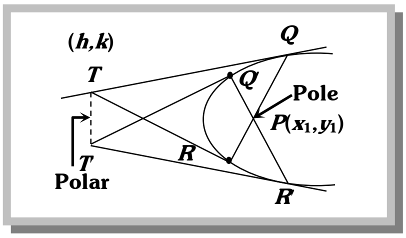
Now (x, y) lies on the parabola $y^2 = 4ax \Rightarrow 4a^2 = y^2 \Rightarrow y = \pm 2a$

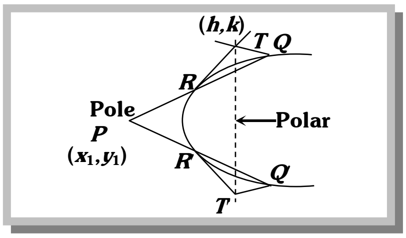
Thus the required points are $(a, 2a)$ and $(a, -2a)$.

5.1.20 Pole and Polar

The locus of the point of intersection of the tangents to the parabola at the ends of a chord drawn from a fixed point P is called the polar of point P and the point P is called the pole of the polar.

Equation of polar: Equation of polar of the point (x_1, y_1) with respect to parabola $y^2 = 4ax$ is same as chord of contact and is given by $yy_1 = 2a(x + x_1)$

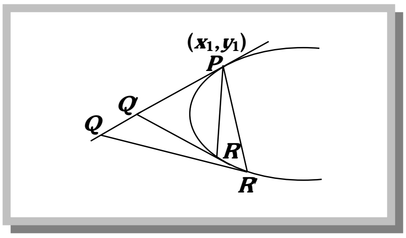




(1) **Polar of the focus is directrix:** Since the focus is $(a, 0)$

$\therefore$ Equation of polar of $y^2 = 4ax$ is $y.0 = 2a(x + a) \Rightarrow x + a = 0$, which is the directrix of the parabola $y^2 = 4ax$.

(2) **Any tangent is the polar of its point of contact:** If the point $P(x_1, y_1)$ be on the parabola. Its polar and tangent at *P* are identical. Hence the tangent is the polar of its own point of contact.



Coordinates of pole: The pole of the line $lx + my + n = 0$ with respect to the parabola $y^2 = 4ax$ is $\left(\frac{n}{l}, \frac{-2am}{l}\right)$.

(i) Pole of the chord joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{y_1 y_2}{4a}, \frac{y_1 + y_2}{2}\right)$ which is the same as the point of intersection of tangents at (x_1, y_1) and (x_2, y_2).

(ii) The point of intersection of the polar of two points *Q* and *R* is the pole of *QR*.

5.1.21 Characterstics of Pole and Polar

(1) **Conjugate points:** If the polar of $P(x_1, y_1)$ passes through $Q(x_2, y_2)$, then the polar of $Q(x_2, y_2)$ goes through $P(x_1, y_1)$ and such points are said to be conjugate points.

Two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are conjugate points with respect to the parabola $y^2 = 4ax$, if $y_1 y_2 = 2a(x_1 + x_2)$.

(2) **Conjugate lines:** If the pole of a line $ax + by + c = 0$ lies on the another line $a_1 x + b_1 y + c_1 = 0$, then the pole of the second line will lie on the first and such lines are said to be conjugate lines.

Two lines $l_1 x + m_1 y + n_1 = 0$ and $l_2 x + m_2 y + n_2 = 0$ are conjugate lines with respect to parabola $y^2 = 4ax$, if $(l_1 n_2 + l_2 n_1) = 2am_1 m_2$

Note :❑The chord of contact and polar of any point on the directrix always passes through focus.

❑The pole of a focal chord lies on directrix and locus of poles of focal chord is the directrix.

❑The polars of all points on directrix always pass through a fixed point and this fixed point is focus.

Example: 40 The pole of the line $2x = y$ with respect to the parabola $y^2 = 2x$ is

(a) $\left(0, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, 0\right)$ (c) $\left(0, -\frac{1}{2}\right)$ (d) None of these

Solution: (a) Let (x_1, y_1) be the pole of line $2x = y$ *w.r.t.* parabola $y^2 = 2x$ its polar is $yy_1 = x + x_1$

Also polar is $y = 2x$, $\therefore \frac{y_1}{1} = \frac{1}{2} = \frac{x_1}{0}$, $\therefore x_1 = 0, y_1 = \frac{1}{2}$. So Pole is $\left(0, \frac{1}{2}\right)$

Example: 41 If the polar of a point with respect to the circle $x^2+y^2=r^2$ touches the parabola $y^2=4ax$, the locus of the pole is

(a) $y^2=-\frac{r^2}{a}x$ (b) $x^2=\frac{-r^2}{a}y$ (c) $y^2=\frac{r^2}{a}x$ (d) $x^2=\frac{r^2}{a}y$

Solution: (a) Polar of a point (x_1,y_1) *w.r.t.* $x^2+y^2=r^2$ is $xx_1+yy_1=r^2$ *i.e.* $yy_1=-xx_1+r^2$

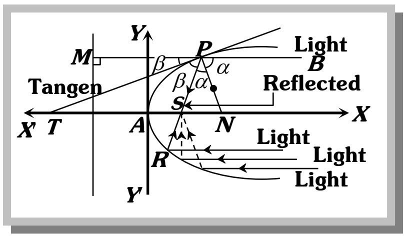
$\Rightarrow y=-\frac{x_1}{y_1}x+\frac{r^2}{y_1} \Rightarrow y=mx+c$, where $m=-\frac{x_1}{y_1}; c=\frac{r^2}{y_1}$

This touches the parabola $y^2=4ax$, If $c=\frac{a}{m} \Rightarrow \frac{r^2}{y_1}=\frac{a}{-x_1/y_1}=-\frac{ay_1}{x_1}$

$\therefore$ Required locus of pole (x_1,y_1) is $\frac{r^2}{y}=\frac{-ay}{x}$ *i.e.*, $y^2=\frac{-r^2}{a}x$

5.1.22 Reflection property of a Parabola

The tangent (PT) and normal (PN) of the parabola $y^2=4ax$ at. P are the internal and external bisectors of $\angle SPM$ and BP is parallel to the axis of the parabola and $\angle BPN=\angle SPN$



Note : ❑ When the incident ray is parallel to the axis of the parabola, the reflected ray will always pass through the focus.

Example: 42 A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $(y-2)^2=4(x+1)$. After reflection, the ray must pass through the point

(a) (0, 2) (b) (2, 0) (c) (0, –2) (d) (–1, 2)

Solution: (a) The equation of the axis of the parabola is $y-2=0$, which is parallel to the x-axis. So, a ray parallel to x-axis is parallel to the axis of the parabola. We know that any ray parallel to the axis of a parabola passes through the focus after reflection. Here (0, 2) is the focus.

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ASSIGNMENT

CONIC SECTION : GENERAL

Basic Level

1. The equation $2x^2+3y^2-8x-18y+35=k$ represents

(a) No locus, if $k>0$ (b) An ellipse, if $k<0$ (c) A point, if $k=0$ (d) A hyperbola, if $k>0$

2. The equation $14x^2-4xy+11y^2-44x-58y+71=0$ represents

(a) A circle (b) An ellipse (c) A hyperbola (d) A rectangular hyperbola

3. Eccentricity of the parabola $x^2-4x-4y+4=0$ is

(a) $e=0$ (b) $e=1$ (c) $e>4$ (d) $e=4$

4. $x^2-4y^2-2x+16y-40=0$ represents

(a) A pair of straight lines (b) An ellipse (c) A hyperbola (d) A parabola

5. The centre of the conic represented by the equation $2x^2-72xy+23y^2-4x-28y-48=0$ is

(a) $\left(\frac{11}{15},\frac{2}{25}\right)$ (b) $\left(\frac{2}{25},\frac{11}{25}\right)$ (c) $\left(\frac{11}{25},-\frac{2}{25}\right)$ (d) $\left(-\frac{11}{25},-\frac{2}{25}\right)$

6. The equation of the parabola with focus (a,b) and directrix $\frac{x}{a}+\frac{y}{b}=1$ is given by

(a) $(ax-by)^2-2a^3x-2b^3y+a^4+a^2b^2+b^4=0$ (b) $(ax+by)^2-2a^3x-2b^3y-a^4+a^2b^2-b^4=0$

(c) $(ax-by)^2+a^4+b^4-2a^3x=0$ (d) $(ax-by)^2-2a^3x=0$

7. The equation of the parabola with focus $(3,0)$ and the directrix $x+3=0$ is

(a) $y^2=3x$ (b) $y^2=2x$ (c) $y^2=12x$ (d) $y^2=6x$

8. The parabola $y^2=x$ is symmetric about

(a) *x*-axis (b) *y*-axis (c) Both *x*-axis and *y*-axis (d) The line $y=x$

9. The focal distance of a point on the parabola $y^2=16x$ whose ordinate is twice the abscissa, is

(a) 6 (b) 8 (c) 10 (d) 12

10. The points on the parabola $y^2=12x$, whose focal distance is 4, are

(a) $(2,\sqrt{3}),(2,-\sqrt{3})$ (b) $(1,2\sqrt{3}),(1,-2\sqrt{3})$ (c) $(1,2)$ (d) None of these

11. The coordinates of the extremities of the latus rectum of the parabola $5y^2=4x$ are

(a) $(1/5,2/5);(-1/5,2/5)$ (b) $(1/5,2/5);(1/5,-2/5)$ (c) $(1/5,4/5);(1/5,-4/5)$ (d) None of these

12. If the vertex of a parabola be at origin and directrix be $x+5=0$, then its latus rectum is

(a) 5 (b) 10 (c) 20 (d) 40

13. The equation of the lines joining the vertex of the parabola $y^2=6x$ to the points on it whose abscissa is 24, is

(a) $y\pm 2x=0$ (b) $2y\pm x=0$ (c) $x\pm 2y=0$ (d) $2x\pm y=0$

14. PQ is a double ordinate of the parabola $y^2=4ax$. The locus of the points of trisection of PQ is

(a) $9y^2=4ax$ (b) $9x^2=4ay$ (c) $9y^2+4ax=0$ (d) $9x^2+4ay=0$

15. The equation of a parabola is $25\{(x-2)^2+(y+5)^2\}=(3x+4y-1)^2$. For this parabola

(a) Vertex $=(2,-5)$ (b) Focus $=(2,-5)$

(c) Directrix has the equation $3x+4y-1=0$ (d) Axis has the equation $3x+4y-1=0$

16. The co-ordinates of a point on the parabola $y^2=8x$, whose focal distance is 4, is

(a) $(2,4)$ (b) $(4,2)$ (c) $(2,-4)$ (d) $(4,-2)$

17. The equation of the parabola with $(-3,0)$ as focus and $x+5=0$ as directrix, is

(a) $x^2=4(y+4)$ (b) $x^2=4(y-4)$ (c) $y^2=4(x+4)$ (d) $y^2=4(x-4)$

Advance Level

18. A double ordinate of the parabola $y^2=8px$ is of length $16p$. The angle subtended by it at the vertex of the parabola is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) None of these

19. If $(2,-8)$ is at an end of a focal chord of the parabola $y^2=32x$; then the other end of the chord is

(a) $(32,32)$ (b) $(32,-32)$ (c) $(-2,8)$ (d) None of these

20. A square has one vertex at the vertex of the parabola $y^2=4ax$ and the diagonal through the vertex lies along the axis of the parabola. If the ends of the other diagonal lie on the parabola, the co-ordinates of the vertices of the square are

(a) $(4a,4a)$ (b) $(4a,-4a)$ (c) $(0,0)$ (d) $(8a,0)$

OTHER STANDARD FORMS OF PARABOLA

Basic Level

21. A parabola passing through the point (–4,–2) has its vertex at the origin and y-axis as its axis. The latus rectum of the parabola is

(a) 6 (b) 8 (c) 10 (d) 12

22. The focus of the parabola $x^2=-16y$ is

(a) (4, 0) (b) (0, 4) (c) (– 4, 0) (d) (0, – 4)

23. The end points of latus rectum of the parabola $x^2=4ay$ are

(a) $(a,2a),(2a,-a)$ (b) $(-a,2a),(2a,a)$ (c) $(a,-2a),(2a,a)$ (d) $(-2a,a),(2a,a)$

24. The ends of latus rectum of parabola $x^2+8y=0$ are

(a) (–4, –2) and (4, 2) (b) (4, –2) and (–4, 2) (c) (–4, –2) and (4, –2) (d) (4, 2) and (–4, 2)

25. Given the two ends of the latus rectum, the maximum number of parabolas that can be drawn is

(a) 1 (b) 2 (c) 0 (d) Infinite

26. The length of the latus rectum of the parabola $9x^2-6x+36y+19=0$ is

(a) 36 (b) 9 (c) 6 (d) 4

SPECIAL FORMS OF PARABOLA

Basic Level

27. Vertex of the parabola $y^2+2y+x=0$ lies in the quadrant

(a) First (b) Second (c) Third (d) Fourth

28. The vertex of the parabola $3x-2y^2-4y+7=0$ is

(a) (3, 1) (b) (–3, –1) (c) (–3, 1) (d) None of these

29. The vertex of parabola $(y-2)^2=16(x-1)$ is

(a) (2, 1) (b) (1, –2) (c) (–1, 2) (d) (1, 2)

30. The vertex of the parabola $x^2+8x+12y+4=0$ is

(a) (–4, 1) (b) (4, –1) (c) (–4, –1) (d) (4, 1)

31. The axis of the parabola $9y^2-16x-12y-57=0$ is

(a) $3y=2$ (b) $x+3y=3$ (c) $2x=3$ (d) $y=3$

32. The directrix of the parabola $x^2-4x-8y+12=0$ is

(a) $x=1$ (b) $y=0$ (c) $x=-1$ (d) $y=-1$

33. The length of the latus rectum of the parabola $x^2-4x-8y+12=0$ is

(a) 4 (b) 6 (c) 8 (d) 10

34. The latus rectum of the parabola $y^2=5x+4y+1$ is

(a) $\frac{5}{4}$ (b) 10 (c) 5 (d) $\frac{5}{2}$

35. If (2, 0) is the vertex and y-axis the directrix of a parabola then its focus is

(a) (2, 0) (b) (–2, 0) (c) (4, 0) (d) (–4, 0)

36. The length of latus rectum of the parabola $4y^2+2x-20y+17=0$ is

(a) 3 (b) 6 (c) $\frac{1}{2}$ (d) 9

37. The focus of the parabola $y^2=4y-4x$ is

(a) (0, 2) (b) (1, 2) (c) (2, 0) (d) (2, 1)

38. Focus of the parabola $(y-2)^2=20(x+3)$ is

(a) (3, –2) (b) (2, –3) (c) (2, 2) (d) (3, 3)

39. The focus of the parabola $y^2-x-2y+2=0$ is

(a) (1/4, 0) (b) (1, 2) (c) (3/4, 1) (d) (5/4, 1)

40. The focus of the parabola $y=2x^2+x$ is

(a) (0, 0) (b) $\left(\frac{1}{2},\frac{1}{4}\right)$ (c) $\left(-\frac{1}{4},0\right)$ (d) $\left(-\frac{1}{4},\frac{1}{8}\right)$

41. The vertex of a parabola is the point (*a*, *b*) and latus rectum is of length *l*. If the axis of the parabola is along the positive direction of y-axis, then its equation is

(a) $(x+a)^2=\frac{l}{2}(2y-2b)$ (b) $(x-a)^2=\frac{l}{2}(2y-2b)$ (c) $(x+a)^2=\frac{l}{4}(2y-2b)$ (d) $(x-a)^2=\frac{l}{8}(2y-2b)$

42. $y^2-2x-2y+5=0$ represents

(a) A circle whose centre is (1, 1) (b) A parabola whose focus is (1, 2)

(c) A parabola whose directrix is $x=\frac{3}{2}$ (d) A parabola whose directrix is $x=-\frac{1}{2}$

43. The length of the latus rectum of the parabola whose focus is (3, 3) and directrix is $3x-4y-2=0$ is

(a) 2 (b) 1 (c) 4 (d) None of these

44. The equation of the parabola whose vertex is at (2, –1) and focus at (2, – 3) is

(a) $x^2+4x-8y-12=0$ (b) $x^2-4x+8y+12=0$ (c) $x^2+8y=12$ (d) $x^2-4x+12=0$

45. The equation of the parabola with focus (0, 0) and directrix $x+y=4$ is

(a) $x^2+y^2-2xy+8x+8y-16=0$ (b) $x^2+y^2-2xy+8x+8y=0$

(c) $x^2+y^2+8x+8y-16=0$ (d) $x^2-y^2+8x+8y-16=0$

46. The equation of the parabola whose vertex and focus lies on the *x*-axis at distance *a* and a' from the origin, is

(a) $y^2=4(a'-a)(x-a)$ (b) $y^2=4(a'-a)(x+a)$ (c) $y^2=4(a'+a)(x-a)$ (d) $y^2=4(a'+a)(x+a)$

47. The equation of parabola whose vertex and focus are (0, 4) and (0, 2) respectively, is

(a) $y^2-8x=32$ (b) $y^2+8x=32$ (c) $x^2+8y=32$ (d) $x^2-8y=32$

48. The equation of the parabola, whose vertex is (–1, –2) axis is vertical and which passes through the point (3, 6) is

(a) $x^2+2x-2y-3=0$ (b) $2x^2=3y$ (c) $x^2-2x-y+3=0$ (d) None of these

49. The length of the latus rectum of the parabola whose focus is $\left(\frac{u^2}{2g}\sin 2\alpha, -\frac{u^2}{2g}\cos 2\alpha\right)$ and directrix is $y=\frac{u^2}{2g}$, is

(a) $\frac{u^2}{g}\cos^2\alpha$ (b) $\frac{u^2}{g}\cos 2\alpha$ (c) $\frac{2u^2}{g}\cos 2\alpha$ (d) $\frac{2u^2}{g}\cos^2\alpha$

50. The equation of the parabola whose axis is vertical and passes through the points (0, 0), (3, 0) and (–1, 4), is

(a) $x^2-3x-y=0$ (b) $x^2+3x+y=0$ (c) $x^2-4x+2y=0$ (d) $x^2-4x-2y=0$

51. If the vertex and the focus of a parabola are (–1, 1) and (2, 3) respectively, then the equation of the directrix is

(a) $3x+2y+14=0$ (b) $3x+2y-25=0$ (c) $2x-3y+10=0$ (d) None of these

52. If the focus of a parabola is (–2, 1) and the directrix has the equation $x+y=3$, then the vertex is

(a) (0, 3) (b) (–1, 1/2) (c) (–1, 2) (d) (2, –1)

53. The vertex of a parabola is (*a*, 0) and the directrix is $x+y=3a$. The equation of the parabola is

(a) $x^2+2xy+y^2+6ax+10ay+7a^2=0$ (b) $x^2-2xy+y^2+6ax+10ay=7a^2$

(c) $x^2-2xy+y^2-6ax+10ay=7a^2$ (d) None of these

54. The equation of a locus is $y^2+2ax+2by+c=0$, then

(a) It is an ellipse (b) It is a parabola (c) Its latus rectum = *a* (d) Its latus rectum = 2*a*

55. If the vertex of the parabola $y = x^2 - 8x + c$ lies on *x*-axis, then the value of *c* is

(a) –16 (b) –4 (c) 4 (d) 16

56. If the vertex of a parabola is the point (–3, 0) and the directrix is the line $x + 5 = 0$ then its equation is

(a) $y^2 = 8(x+3)$ (b) $x^2 = 8(y+3)$ (c) $y^2 = -8(x+3)$ (d) $y^2 = 8(x+5)$

57. If the parabola $y^2 = 4ax$ passes through (3, 2), then the length of its latusrectum is

(a) 2/3 (b) 4/3 (c) 1/3 (d) 4

58. The extremities of latus rectum of the parabola $(y-1)^2 = 2(x+2)$ are

(a) $\left(-\frac{3}{2}, 2\right)$ (b) $(-2, 1)$ (c) $\left(-\frac{3}{2}, 0\right)$ (d) $\left(-\frac{3}{2}, 1\right)$

59. The equation of parabola is given by $y^2 + 8x - 12y + 20 = 0$. Tick the correct options given below

(a) Vertex (2, 6) (b) Focus (0, 6) (c) Latus rectum = 4 (d) axis *y* = 6

Advance Level

60. The length of the latus rectum of the parabola $169\{(x-1)^2 + (y-3)^2\} = (5x - 12y + 17)^2$ is

(a) $\frac{14}{13}$ (b) $\frac{28}{13}$ (c) $\frac{12}{13}$ (d) None of these

61. The length of the latus rectum of the parabola $x = ay^2 + by + c$ is

(a) $\frac{a}{4}$ (b) $\frac{a}{3}$ (c) $\frac{1}{a}$ (d) $\frac{1}{4a}$

62. If the vertex = (2, 0) and the extremities of the latus rectum are (3, 2) and (3, –2), then the equation of the parabola is

(a) $y^2 = 2x - 4$ (b) $x^2 = 4y - 8$ (c) $y^2 = 4x - 8$ (d) None of these

63. Let there be two parabolas with the same axis, focus of each being exterior to the other and the latus recta being 4*a* and 4*b*. The locus of the middle points of the intercepts between the parabolas made on the lines parallel to the common axis is a

(a) Straight line if $a = b$ (b) Parabola if $a \neq b$ (c) Parabola for all *a*, *b* (d)

64. A line *L* passing through the focus of the parabola $y^2 = 4(x-1)$ intersects the parabola in two distinct points. If '*m*' be the slope of the line *L*, then

(a) –1< *m*< 1 (b) *m*<–1 or *m* > 1 (c) $m \in R$ (d) None of these

PARAMETRIC EQUATIONS OF PARABOLA

Basic Level

65. Which of the following points lie on the parabola $x^2 = 4ay$

(a) $x = at^2, y = 2at$ (b) $x = 2at, y = at$ (c) $x = 2at^2, y = at$ (d) $x = 2at, y = at^2$

66. The parametric equation of a parabola is $x = t^2 + 1, y = 2t + 1$. The cartesian equation of its directrix is

(a) $x = 0$ (b) $x + 1 = 0$ (c) $y = 0$ (d) None of these

67. The parametric representation $(2 + t^2, 2t + 1)$ represents

(a) A parabola with focus at (2, 1) (b) A parabola with vertex at (2, 1)

(c) An ellipse with centre at (2, 1) (d) None of these

68. The graph represented by the equations $x = \sin^2 t,\ y = 2\cos t$ is

(a) A portion of a parabola (b) A parabola

(c) A part of a sine graph (d) A Part of a hyperbola

69. The curve described parametrically by $x = t^2 + t + 1, y = t^2 - t + 1$ represents

(a) A pair of straight lines (b) An ellipse (c) A parabola (d) A hyperbola

POSITION OF A POINT , INTERSECTION OF LINES & PARABOLA , TANGENTS & PAIR OF TANGENTS

Basic Level

70. The equation of the tangent at a point $P(t)$ where '*t*' is any parameter to the parabola $y^2 = 4ax$, is

(a) $yt = x + at^2$ (b) $y = xt + at^2$ (c) $y = xt + \frac{a}{t}$ (d) $y = tx$

71. The condition for which the straight line $y = mx + c$ touches the parabola $y^2 = 4ax$ is

(a) $a = c$ (b) $\frac{a}{c} = m$ (c) $m = a^2c$ (d) $m = ac^2$

72. The line $y = mx + c$ touches the parabola $x^2 = 4ay$, if

(a) $c = -am$ (b) $c = -a/m$ (c) $c = -am^2$ (d) $c = a/m^2$

73. The line $y = 2x + c$ is tangent to the parabola $y^2 = 16x$, if *c* equals

(a) –2 (b) –1 (c) 0 (d) 2

74. The line $y = 2x + c$ is tangent to the parabola $y^2 = 4x$, then *c* =

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 4

75. If line $x = my + k$ touches the parabola $x^2 = 4ay$, then *k* =

(a) $\frac{a}{m}$ (b) am (c) am^2 (d) $-am^2$

76. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$, if

(a) $m = 1$ (b) $m = 2$ (c) $m = 4$ (d) $m = 3$

77. The line $lx + my + n = 0$ will touch the parabola $y^2 = 4ax$, if

(a) $mn = al^2$ (b) $lm = an^2$ (c) $ln = am^2$ (d) $mn = al$

78. The equation of the tangent to the parabola $y^2 = 4x + 5$ parallel to the line $y = 2x + 7$ is

(a) $2x-y-3=0$ (b) $2x-y+3=0$ (c) $2x+y+3=0$ (d) None of these

79. If $lx+my+n=0$ is tangent to the parabola $x^2=y$, then condition of tangency is

(a) $l^2=2mn$ (b) $l=4m^2n^2$ (c) $m^2=4ln$ (d) $l^2=4mn$

80. The point at which the line $y=mx+c$ touches the parabola $y^2=4ax$ is

(a) $\left(\frac{a}{m^2},\frac{2a}{m}\right)$ (b) $\left(\frac{a}{m^2},\frac{-2a}{m}\right)$ (c) $\left(-\frac{a}{m^2},\frac{2a}{m}\right)$ (d) $\left(-\frac{a}{m^2},-\frac{2a}{m}\right)$

81. The locus of a foot of perpendicular drawn to the tangent of parabola $y^2=4ax$ from focus, is

(a) $x=0$ (b) $y=0$ (c) $y^2=2a(x+a)$ (d) $x^2+y^2(x+a)=0$

82. The equation of tangent at the point (1, 2) to the parabola $y^2=4x$, is

(a) $x-y+1=0$ (b) $x+y+1=0$ (c) $x+y-1=0$ (d) $x-y-1=0$

83. The tangent to the parabola $y^2=4ax$ at the point $(a,2a)$ makes with *x*-axis an angle equal to

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

84. A tangents to the parabola $y^2=8x$ makes an angle of 45^o with the straight line $y=3x+5$; then the equation of tangent is

(a) $2x+y-1=0$ (b) $x+2y-1=0$ (c) $2x+y+1=0$ (d) None of these

85. The equation of the tangent to the parabola $y^2=9x$ which goes through the point $(4,10)$ is

(a) $x+4y+1=0$ (b) $9x+4y+4=0$ (c) $x-4y+36=0$ (d) $9x-4y+4=0$

86. The angle of intersection between the curves $y^2=4x$ and $x^2=32y$ at point $(16,8)$ is

(a) $\tan^{-1}\left(\frac{3}{5}\right)$ (b) $\tan^{-1}\left(\frac{4}{5}\right)$ (c) π (d) $\frac{\pi}{2}$

87. The equation of the tangent to the parabola $y=x^2-x$ at the point where $x=1$, is

(a) $y=-x-1$ (b) $y=-x+1$ (c) $y=x+1$ (d) $y=x-1$

88. The point of intersection of the tangents to the parabola $y^2=4ax$ at the points t_1 and t_2 is

(a) $(at_1t_2,a(t_1+t_2))$ (b) $(2at_1t_2,a(t_1+t_2))$ (c) $(2at_1t_2,2a(t_1+t_2))$ (d) None of these

89. The tangents drawn from the ends of latus rectum of $y^2=12x$ meets at

(a) Directrix (b) Vertex (c) Focus (d) None of these

90. Two perpendicular tangents to $y^2=4ax$ always intersect on the line

(a) $x=a$ (b) $x+a=0$ (c) $x+2a=0$ (d) $x+4a=0$

91. The locus of the point of intersection of the perpendicular tangents to the parabola $x^2=4ay$ is

(a) Axis of the parabola (b) Directrix of the parabola

(c) Focal chord of the parabola (d) Tangent at vertex to the parabola

92. The angle between the tangents drawn from the origin to the parabola $y^2=4a(x-a)$ is

(a) 90^o (b) 30^o (c) $\tan^{-1}\frac{1}{2}$ (d) 45^o

93. The angle between tangents to the parabola $y^2=4ax$ at the points where it intersects with the line $x-y-a=0$, is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

94. The equation of latus rectum of a parabola is $x+y=8$ and the equation of the tangent at the vertex is $x+y=12$, then length of the latus rectum is

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$ (c) 8 (d) $8\sqrt{2}$

95. If the segment intercepted by the parabola $y^2=4ax$ with the line $lx+my+n=0$ subtends a right angle at the vertex, then

(a) $4al+n=0$ (b) $4al+4am+n=0$ (c) $4am+n=0$ (d) $al+n=0$

96. Tangents at the extremities of any focal chord of a parabola intersect

(a) At right angles (b) On the directrix (c) On the tangent at vertex (d) None of these

97. Angle between two curves $y^2=4(x+1)$ and $x^2=4(y+1)$ is

(a) 0^o (b) 90^o (c) 60^o (d) 30^o

98. The angle of intersection between the curves $x^2=4(y+1)$ and $x^2=-4(y+1)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{\pi}{2}$

99. If the tangents drawn from the point (0, 2) to the parabola $y^2=4ax$ are inclined at an angle $\frac{3\pi}{4}$, then the value of *a* is

(a) 2 (b) –2 (c) 1 (d) None of these

100. The point of intersection of the tangents to the parabola $y^2=4x$ at the points, where the parameter '*t*' has the value 1 and 2, is

(a) (3, 8) (b) (1, 5) (c) (2, 3) (d) (4, 6)

101. The tangents from the origin to the parabola $y^2+4=4x$ are inclined at

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

102. The number of distinct real tangents that can be drawn from (0, –2) to the parabola $y^2=4x$ is

(a) One (b) Two (c) Zero (d) None of these

103. If two tangents drawn from the point (α,β) to the parabola $y^2=4x$ be such that the slope of one tangent is double of the other, then

(a) $\beta=\frac{2}{9}\alpha^2$ (b) $\alpha=\frac{2}{9}\beta^2$ (c) $2\alpha=9\beta^2$ (d) None of these

104. If $y+b=m_1(x+a)$ and $y+b=m_2(x+a)$ are two tangents to the parabola $y^2=4ax$, then

(a) $m_1+m_2=0$ (b) $m_1m_2=1$ (c) $m_1m_2=-1$ (d) None of these

105. If $y=mx+c$ touches the parabola $y^2=4a(x+a)$, then

(a) $c=\frac{a}{m}$ (b) $c=am+\frac{a}{m}$ (c) $c=a+\frac{a}{m}$ (d) None of these

106. The angle between the tangents drawn from a point $(-a,2a)$ to $y^2=4ax$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

107. The tangents to the parabola $y^2 = 4ax$ at $(at_1^2, 2at_1)$; (at_2^2, at_2) intersect on its axis, then

(a) $t_2 = t_2$ (b) $t_1 = -t_2$ (c) $t_1 t_2 = 2$ (d) $t_1 t_2 = -1$

108. If perpendiculars are drawn on any tangent to a parabola $y^2 = 4ax$ from the points $(a \pm k, 0)$ on the axis. The difference of their squares is

(a) 4 (b) $4a$ (c) $4k$ (d) $4ak$

109. The straight line $kx + y = 4$ touches the parabola $y = x - x^2$, if

(a) $k = -5$ (b) $k = 0$ (c) $k = 3$ (d) k takes any real value

110. If a tangent to the parabola $y^2 = ax$ makes an angle 45^o with x-axis, its points of contact will be

(a) $(a/2, a/4)$ (b) $(-a/2, a/4)$ (c) $(a/4, a/2)$ (d) $(-a/4, a/2)$

111. The equations of common tangent to the parabola $y^2 = 4ax$ and $x^2 = 4by$ is

(a) $xa^{1/3} + yb^{1/3} + (ab)^{2/3} = 0$ (b) $\frac{x}{a^{1/3}} + \frac{y}{b^{1/3}} + \frac{1}{(ab)^{2/3}} = 0$

(c) $xb^{\frac{1}{3}} + ya^{\frac{1}{3}} - (ab)^{\frac{2}{3}} = 0$ (d) $\frac{x}{b^{1/3}} + \frac{y}{a^{1/3}} - \frac{1}{(ab)^{2/3}} = 0$

112. The range of values of λ for which the point $(\lambda, -1)$ is exterior to both the parabolas $y^2 = |x|$ is

(a) (0, 1) (b) (–1, 1) (c) (–1, 0) (d) None of these

Advance Level

113. The line $x\cos\alpha + y\sin\alpha = p$ will touch the parabola $y^2 = 4a(x + a)$, if

(a) $p\cos\alpha + a = 0$ (b) $p\cos\alpha - a = 0$ (c) $a\cos\alpha + p = 0$ (d) $a\cos\alpha - p = 0$

114. If the straight line $x + y = 1$ touches the parabola $y^2 - y + x = 0$, then the coordinates of the point of contact are

(a) (1, 1) (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (c) (0, 1) (d) (1, 0)

115. The equation of common tangent to the circle $x^2 + y^2 = 2$ and parabola $y^2 = 8x$ is

(a) $y = x + 1$ (b) $y = x + 2$ (c) $y = x - 2$ (d) $y = -x + 2$

116. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(a) $3y = 9x + 2$ (b) $y = 2x + 1$ (c) $2y = x + 8$ (d) $y = x + 2$

117. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are

(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$ (c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$

118. If the line $lx + my + n = 0$ is a tangent to the parabola $y^2 = 4ax$, then locus of its point of contact is

(a) A straight line (b) A circle (c) A parabola (d) Two straight lines

119. The tangent drawn at any point P to the parabola $y^2 = 4ax$ meets the directrix at the point K, then the angle which KP subtends at its focus is

(a) 30^o (b) 45^o (c) 60^o (d) 90^o

120. The point of intersection of tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is

(a) (1, 0) (b) (–1, 0) (c) (0, 1) (d) (0, –1)

121. If y_1, y_2 are the ordinates of two points *P* and *Q* on the parabola and y_3 is the ordinate of the point of intersection of tangents at *P* and *Q*, then

(a) y_1, y_2, y_3 are in A. P. (b) y_1, y_3, y_2 are in A. P. (c) y_1, y_2, y_3 are in G.P. (d) y_1, y_3, y_2 are in G. P.

122. If the tangents at *P* and *Q* on a parabola meet in *T*, then *SP*,*ST* and *SQ* are in

(a) A. P. (b) G. P. (c) H. P. (d) None of these

123. The equation of the parabola whose focus is the point (0, 0) and the tangent at the vertex is $x - y + 1 = 0$ is

(a) $x^2 + y^2 - 2xy - 4x + 4y - 4 = 0$ (b) $x^2 + y^2 - 2xy + 4x - 4y - 4 = 0$

(c) $x^2 + y^2 + 2xy - 4x + 4y - 4 = 0$ (d) $x^2 + y^2 + 2xy - 4x - 4y + 4 = 0$

124. The two parabolas $y^2 = 4x$ and $x^2 = 4y$ intersect at a point *P*, whose abscissae is not zero, such that

(a) They both touch each other at *P*

(b) They cut at right angles at *P*

(c) The tangents to each curve at *P* make complementary angles with the *x*-axis

(d) None of these

125. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then , a point of intersection of the circle and the parabola is

(a) $\left(\frac{p}{2}, p\right)$ (b) $\left(\frac{p}{2}, -p\right)$ (c) $\left(\frac{-p}{2}, p\right)$ (d) $\left(\frac{-p}{2}, -p\right)$

126. The angle of intersection of the curves $y^2 = 2x/\pi$ and $y = \sin x$, is

(a) $\cot^{-1}(-1/\pi)$ (b) $\cot^{-1}\pi$ (c) $\cot^{-1}(-\pi)$ (d) $\cot^{-1}(1/\pi)$

127. *P* is a point. Two tangents are drawn from it to the parabola $y^2 = 4x$ such that the slope of one tangent is three times the slope of the other. The locus of *P* is

(a) A straight line (b) A circle (c) A parabola (d) An ellipse

128. The parabola $y^2 = kx$ makes an intercept of length 4 on the line $x - 2y = 1$.Then *k* is

(a) $\frac{\sqrt{105} - 5}{10}$ (b) $\frac{5 - \sqrt{105}}{10}$ (c) $\frac{5 + \sqrt{105}}{10}$ (d) None of these

129. The triangle formed by the tangents to a parabola $y^2 = 4ax$ at the ends of the latus rectum and the double ordinates through the focus is

(a) Equilateral (b)Isosceles

(c) Right-angled isosceles (d)Dependent on the value of *a* for its classification

130. The equation of the tangent at the vertex of the parabola $x^2 + 4x + 2y = 0$ is

(a) $x = -2$ (b) $x = 2$ (c) $y = 2$ (d) $y = -2$

131. The locus of the point of intersection of the perpendicular tangents to the parabola $x^2 - 8x + 2y + 2 = 0$ is

(a) $2y-15=0$ (b) $2y+15=0$ (c) $2x+9=0$ (d) None of these

132. If P,Q,R are three points on a parabola $y^2=4ax$, whose ordinates are in geometrical progression, then the tangents at P and R meet on

(a) The line through Q parallel to x-axis (b) The line through Q parallel to y-axis

(c) The line joining Q to the vertex (d) The line joining Q to the focus

133. The tangents at three points A, B, C on the parabola $y^2=4x$; taken in pairs intersect at the points P, Q and R. If Δ, Δ' be the areas of the triangles ABC and PQR respectively, then

(a) $\Delta=2\Delta'$ (b) $\Delta'=2\Delta$ (c) $\Delta=\Delta'$ (d) None of these

134. If the line $y=mx+a$ meets the parabola $y^2=4ax$ in two points whose abscissa are x_1 and x_2, then x_1+x_2 is equal to zero if

(a) $m=-1$ (b) $m=1$ (c) $m=2$ (d) $m=-1/2$

135. Two tangents of the parabola $y^2=8x$, meet the tangent at its vertex in the points P and Q. If $PQ=4$, locus of the point of intersection of the two tangents is

(a) $y^2=8(x+2)$ (b) $y^2=8(x-2)$ (c) $x^2=8(y-2)$ (d) $x^2=8(y+2)$

136. If perpendicular be drawn from any two fixed points on the axis of a parabola at a distance d from the focus on any tangent to it, then the difference of their squares is

(a) a^2-d^2 (b) a^2+d^2 (c) $4ad$ (d) $2ad$

137. Two straight lines are perpendicular to each other. One of them touches the parabola $y^2=4a(x+a)$ and the other touches $y^2=4b(x+b)$. Their point of intersection lies on the line

(a) $x-a+b=0$ (b) $x+a-b=0$ (c) $x+a+b=0$ (d) $x-a-b=0$

138. The point $(a, 2a)$ is an interior point of the region bounded by the parabola $y^2=16x$ and the double ordinate through the focus. Then a belongs to the open interval

(a) $a<4$ (b) $0<a<4$ (c) $0<a<2$ (d) $a>4$

139. The number of points with integral coordinates that lie in the interior of the region common to the circle $x^2+y^2=16$ and the parabola $y^2=4x$ is

(a) 8 (b) 10 (c) 16 (d) None of these

NORMAL IN DIFFERENT FORMS , INTERSECTION OF NORMALS

Basic Level

140. The maximum number of normal that can be drawn from a point to a parabola is

141. The centroid of the triangle formed by joining the feet of the normals drawn from any point to the parabola $y^2 = 4ax$, lies on

(a) Axis (b) Directrix (c) Latus rectum (d) Tangent at vertex

142. If the line $2x + y + k = 0$ is normal to the parabola $y^2 = -8x$, then the value of *k* will be

(a) –16 (b) –8 (c) –24 (d) 24

143. The point on the parabola $y^2 = 8x$ at which the normal is inclined at 60^o to the x-axis has the coordinates

(a) $(6, -4\sqrt{3})$ (b) $(6, 4\sqrt{3})$ (c) $(-6, -4\sqrt{3})$ (d) $(-6, 4\sqrt{3})$

144. If the normals at two points *P* and *Q* of a parabola $y^2 = 4ax$ intersect at a third point *R* on the curve, then the product of ordinates of *P* and *Q* is

(a) $4a^2$ (b) $2a^2$ (c) $-4a^2$ (d) $8a^2$

145. The equation of normal to the parabola at the point $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$, is

(a) $y = m^2x - 2mx - am^3$ (b) $m^3y = m^2x - 2am^2 - a$ (c) $m^3y = 2am^2 - m^2x + a$ (d) None of these

146. At what point on the parabola $y^2 = 4x$, the normal makes equal angles with the coordinate axes

(a) $(4, 4)$ (b) $(9, 6)$ (c) $(4, -4)$ (d) $(1, -2)$

147. The slope of the normal at the point $(at^2, 2at)$ of the parabola $y^2 = 4ax$, is

(a) $\frac{1}{t}$ (b) *t* (c) $-t$ (d) $-\frac{1}{t}$

148. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then

(a) $t_2 = -t_1 - \frac{2}{t_1}$ (b) $t_2 = -t_1 + \frac{2}{t_1}$ (c) $t_2 = t_1 - \frac{2}{t_1}$ (d) $t_2 = t_1 + \frac{2}{t_1}$

149. The normal to the parabola $y^2 = 8x$ at the point (2, 4) meets the parabola again at the point

(a) $(-18, -12)$ (b) $(-18, 12)$ (c) $(18, 12)$ (d) $(18, -12)$

150. If a normal drawn to the parabola $y^2 = 4ax$ at the point $(a, 2a)$ meets parabola again on $(at^2, 2at)$, then the value of *t* will be

(a) 1 (b) 3 (c) –1 (d) –3

151. The arithmetic mean of the ordinates of the feet of the normals from (3, 5) to the parabola $y^2 = 8x$ is

(a) 4 (b) 0 (c) 8 (d) None of these

152. If the normal to $y^2 = 12x$ at (3, 6) meets the parabola again in (27, –18) and the circle on the normal chord as diameter is

(a) $x^2 + y^2 + 30x + 12y - 27 = 0$ (b) $x^2 + y^2 + 30x + 12y + 27 = 0$

(c) $x^2 + y^2 - 30x - 12y - 27 = 0$ (d) $x^2 + y^2 - 30x + 12y - 27 = 0$

153. The number of distinct normal that can be drawn from $\left(\frac{11}{4}, \frac{1}{4}\right)$ to the parabola $y^2 = 4x$ is

(a) 3 (b) 2 (c) 1 (d) 4

154. The normal chord of a parabola $y^2 = 4ax$ at (x_1, x_1) subtends a right angle at the

(a) Focus (b) Vertex (c) End of the latus-rectum (d)None of these

155. The normal at $(ap^2, 2ap)$ on $y^2 = 4ax$, meets the curve again at $(aq^2, 2aq)$ then

(a) $p^2 + pq + 2 = 0$ (b) $p^2 - pq + 2 = 0$ (c) $q^2 + pq + 2 = 0$ (d) $p^2 + pq + 1 = 0$

156. The angle between the normals to the parabola $y^2 = 24x$ at points (6, 12) and (6, –12) is

(a) 30^o (b) 45^o (c) 60^o (d) 90^o

Advance Level

157. The centre of a circle passing through the point (0,1) and touching the curve $y = x^2$ at (2, 4) is

(a) $\left(\frac{-16}{5}, \frac{27}{10}\right)$ (b) $\left(\frac{-16}{7}, \frac{5}{10}\right)$ (c) $\left(\frac{-16}{5}, \frac{53}{10}\right)$ (d) None of these

158. The length of the normal chord to the parabola $y^2 = 4x$, which subtends right angle at the vertex is

(a) $6\sqrt{3}$ (b) $3\sqrt{3}$ (c) 2 (d) 1

159. Three normals to the parabola $y^2 = x$ are drawn through a point $(C, 0)$ then

(a) $C = \frac{1}{4}$ (b) $C = \frac{1}{2}$ (c) $C > \frac{1}{2}$ (d) None of these

160. If the tangent and normal at any point *P* of a parabola meet the axes in *T* and *G* respectively, then

(a) $ST \neq SG = SP$ (b) $ST - SG \neq SP$ (c) $ST = SG = SP$ (d) $ST = SG.SP$

161. The number of distinct normals that can be drawn from (–2, 1) to the parabola $y^2 - 4x - 2y - 3 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 0

162. The set of points on the axis of the parabola $y^2 = 4x + 8$ from which the 3 normals to the parabola are all real and different is

(a) $\{(k, 0) \mid k \leq -2\}$ (b) $\{(k, 0) \mid k > -2\}$ (c) $\{(0, k) \mid k > -2\}$ (d) None of these

163. The area of the triangle formed by the tangent and the normal to the parabola $y^2 = 4ax$; both drawn at the same end of the latus rectum, and the axis of the parabola is

(a) $2\sqrt{2}\,a^2$ (b) $2a^2$ (c) $4a^2$ (d) None of these

164. If a chord which is normal to the parabola $y^2 = 4ax$ at one end subtends a right angle at the vertex, then its slope is

(a) 1 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) 2

165. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissae are in A.P., then the slopes of the tangents at the three co-normal points are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

166. If $x = my + c$ is a normal to the parabola $x^2 = 4ay$, then the value of c is

(a) $-2am - am^3$ (b) $2am + am^3$ (c) $-\frac{2a}{m} - \frac{a}{m^3}$ (d) $\frac{2a}{m} + \frac{a}{m^3}$

167. The normal at the point $P(ap^2, 2ap)$ meets the parabola $y^2 = 4ax$ again at $Q(aq^2, 2aq)$ such that the lines joining the origin to P and Q are at right angle. Then

(a) $p^2 = 2$ (b) $q^2 = 2$ (c) $p = 2q$ (d) $q = 2p$

168. If $y = 2x + 3$ is a tangent to the parabola $y^2 = 24x$, then its distance from the parallel normal is

(a) $5\sqrt{5}$ (b) $10\sqrt{5}$ (c) $15\sqrt{5}$ (d) None of these

169. If $P(-3, 2)$ is one end of the focal chord PQ of the parabola $y^2 + 4x + 4y = 0$, then the slope of the normal at Q is

(a) $\frac{-1}{2}$ (b) 2 (c) $\frac{1}{2}$ (d) –2

170. The distance between a tangent to the parabola $y^2 = 4ax$ which is inclined to axis at an angle α and a parallel normal is

(a) $\frac{a\cos\alpha}{\sin^2\alpha}$ (b) $\frac{a\sin\alpha}{\cos^2\alpha}$ (c) $\frac{a}{\sin\alpha\cos^2\alpha}$ (d) $\frac{a}{\cos\alpha\sin^2\alpha}$

171. If the normal to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ cuts the parabola again at $Q(aT^2, 2aT)$, then

(a) $-2 \le T \le 2$ (b) $T \in (-\infty, -8) \cup (8, \infty)$ (c) $T^2 < 8$ (d) $T^2 \ge 8$

CHORDS

Basic Level

172. The locus of the middle points of the chords of the parabola $y^2 = 4ax$ which passes through the origin is

(a) $y^2 = ax$ (b) $y^2 = 2ax$ (c) $y^2 = 4ax$ (d) $x^2 = 4ay$

173. In the parabola $y^2 = 6x$, the equation of the chord through vertex and negative end of latus rectum, is

(a) $y = 2x$ (b) $y + 2x = 0$ (c) $x = 2y$ (d) $x + 2y = 0$

174. From the point $(-1, 2)$ tangent lines are drawn to the parabola $y^2 = 4x$, then the equation of chord of contact is

(a) $y = x + 1$ (b) $y = x - 1$ (c) $y + x = 1$ (d) None of these

175. A set of parallel chords of the parabola $y^2 = 4ax$ have their mid points on

(a) Any straight line through the vertex (b) Any straight line through the focus

(c) A straight line parallel to the axis (d) Another parabola

176. The length of the chord of the parabola $y^2 = 4ax$ which passes through the vertex and makes an angle θ with the axis of the parabola, is

(a) $4a\cos\theta\,\text{cosec}^2\theta$ (b) $4a\cos^2\theta\,\text{cosec}\,\theta$ (c) $a\cos\theta\,\text{cosec}^2\theta$ (d) $a\cos^2\theta\,\text{cosec}\,\theta$

177. If *PSQ* is the focal chord of the parabola $y^2 = 8x$ such that $SP = 6$. Then the length *SQ* is

(a) 6 (b) 4 (c) 3 (d) None of these

178. The locus of the middle points of parallel chords of a parabola $x^2 = 4ay$ is a

(a) Straight line parallel to the axis

(b) Straight line parallel to the *y*-axis

(c) Circle

(d) Straight line parallel to a bisector of the angles between the axes

179. The locus of the middle points of chords of the parabola $y^2 = 8x$ drawn through the vertex is a parabola whose

(a) focus is (2, 0) (b) Latus rectum =8 (c) Focus is (0, 2) (d) Latus rectum =4

180. 't_1' and 't_2' are two points on the parabola $y^2 = 4x$. If the chord joining them is a normal to the parabola at 't_1', then

(a) $t_1 + t_2 = 0$ (b) $t_1(t_1 + t_2) = 0$ (c) $t_1(t_1 + t_2) + 2 = 0$ (d) $t_1 t_2 + 1 = 0$

181. The locus of the middle points of chords of a parabola which subtend a right angle at the vertex of the parabola is

(a) A circle (b) An ellipse (c) A parabola (d) None of these

182. AB is a chord of the parabola $y^2 = 4ax$. If its equation is $y = mx + c$ and it subtends a right angle at the vertex of the parabola then

(a) $c = 4am$ (b) $a = 4mc$ (c) $c = -4am$ (d) $a + 4mc = 0$

183. The length of a focal chord of parabola $y^2 = 4ax$ making an angle θ with the axis of the parabola is

(a) $4a\operatorname{cosec}^2\theta$ (b) $4a\sec^2\theta$ (c) $a\operatorname{cosec}^2\theta$ (d) None of these

184. If (*a*, *b*) is the mid point of a chord passing through the vertex of the parabola $y^2 = 4x$, then

(a) $a = 2b$ (b) $2a = b$ (c) $a^2 = 2b$ (d) $2a = b^2$

185. The mid-point of the chord $2x + y - 4 = 0$ of the parabola $y^2 = 4x$ is

(a) $\left(\frac{5}{2}, -1\right)$ (b) $\left(-1, \frac{5}{2}\right)$ (c) $\left(\frac{3}{2}, -1\right)$ (d) None of these

186. If $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are two variable points on the curve $y^2 = 4ax$ and *PQ* subtends a right angle at the vertex, then $t_1 t_2$ is equal to

(a) –1 (b) – 2 (c) –3 (d) –4

187. If $(at^2, 2at)$ are the coordinates of one end of a focal chord of the parabola $y^2 = 4ax$, then the coordinate of the other end are

(a) $(at^2, -2at)$ (b) $(-at^2, -2at)$ (c) $\left(\frac{a}{t^2}, \frac{2a}{t}\right)$ (d) $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

188. If *b* and *c* are the lengths of the segments of any focal chord of a parabola $y^2 = 4ax$, then the length of the semi- latusrectum is

(a) $\frac{b+c}{2}$ (b) $\frac{bc}{b+c}$ (c) $\frac{2bc}{b+c}$ (d) $\sqrt{bc}$

189. The ratio in which the line segment joining the points $(4,-6)$ and $(3,1)$ is divided by the parabola $y^2=4x$ is

(a) $\frac{-20\pm\sqrt{155}}{11}:1$ (b) $\frac{-2\pm 2\sqrt{155}}{11}:1$ (c) $-20\pm 2\sqrt{155}:11$ (d) $-2\pm\sqrt{155}:11$

190. If the lengths of the two segments of focal chord of the parabola $y^2=4ax$ are 3 and 5, then the value of *a* will be

(a) $\frac{15}{8}$ (b) $\frac{15}{4}$ (c) $\frac{15}{2}$ (d) 15

Advance Level

191. If 'a' and 'c' are the segments of a focal chord of a parabola and *b* the semi-latus rectum, then

(a) a,b,c are in A. P. (b) a,b,c are in G. P. (c) a,b,c are in H. P. (d) None of these

192. The locus of mid point of that chord of parabola which subtends right angle on the vertex will be

(a) $y^2-2ax+8a^2=0$ (b) $y^2=a(x-4a)$ (c) $y^2=4a(x-4a)$ (d) $y^2+3ax+4a^2=0$

193. The HM of the segments of a focal chord of the parabola $y^2=4ax$ is

(a) $4a$ (b) $2a$ (c) a (d) a^2

194. The length of a focal chord of the parabola $y^2=4ax$ at a distance *b* from the vertex is c. Then

(a) $2a^2=bc$ (b) $a^3=b^2c$ (c) $ac=b^2$ (d) $b^2c=4a^3$

195. A chord PP' of a parabola cuts the axis of the parabola at *O*. The feet of the perpendiculars from *P* and *P*' on the axis are *M* and *M*' respectively. If V is the vertex then *VM*, *VO*, *VM*' are in

(a) *A.P.* (b) *G.P.* (c) *H.P.* (d) None of these

196. The chord AB of the parabola $y^2=4ax$ cuts the axis of the parabola at C. If $A=(at_1^2, 2at_1)$; $B=(at_2^2, 2at_2)$ and $AC:AB=1:3$, then

(a) $t_2=2t_1$ (b) $t_2+2t_1=0$ (c) $t_1+2t_2=0$ (d) None of these

197. The locus of the middle points of the focal chord of the parabola $y^2=4ax$ is

(a) $y^2=a(x-a)$ (b) $y^2=2a(x-a)$ (c) $y^2=4a(x-a)$ (d) None of these

198. If $(4,-2)$ is one end of a focal chord of the parabola $y^2=x$, then the slope of the tangent drawn at its other end will be

(a) $-\frac{1}{4}$ (b) -4 (c) 4 (d) $\frac{1}{4}$

199. If (a_1,b_1) and (a_2,b_2) are extremities of a focal chord of the parabola $y^2=4ax$, then $a_1a_2=$

(a) $4a^2$ (b) $-4a^2$ (c) a^2 (d) $-a^2$

200. The length of the chord of the parabola $y^2=4ax$ whose equation is $y-x\sqrt{2}+4a\sqrt{2}=0$ is

(a) $2\sqrt{11}a$ (b) $4\sqrt{2}a$ (c) $8\sqrt{2}a$ (d) $6\sqrt{3}a$

201. If the line $y=x\sqrt{3}-3$ cuts the parabola $y^2=x+2$ at *P* and *Q* and if *A* be the point $(\sqrt{3},0)$, then *AP*. *AQ* is

(a) $\frac{2}{3}(\sqrt{3}+2)$ (b) $\frac{4}{3}(\sqrt{3}+2)$ (c) $\frac{4}{3}(2-\sqrt{3})$ (d) $2\sqrt{3}$

202. A triangle *ABC* of area Δ is inscribed in the parabola $y^2 = 4ax$ such that the vertex *A* lies at the vertex of the parabola and *BC* is a focal chord. The difference of the distances of *B* and *C* from the axis of the parabola is

(a) $\frac{2\Delta}{a}$ (b) $\frac{2\Delta}{a^2}$ (c) $\frac{a}{2\Delta}$ (d) None of these

DIAMETER OF PARABOLA , LENGTH OF TANGENT , NORMAL & SUBNORMAL , POLE & POLAR

Basic Level

203. The length of the subnormal to the parabola $y^2 = 4ax$ at any point is equal to

(a) $\sqrt{2}a$ (b) $2\sqrt{2}$ (c) $a/\sqrt{2}$ (d) $2a$

204. The polar of focus of a parabola is

(a) *x*-axis (b) *y*-axis (c) Directrix (d) Latus rectum

205. Locus of the poles of focal chords of a parabola is .....of parabola

(a) The tangent at the vertex (b)The axis (c) A focal chord (d) The directrix

206. The subtangent, ordinate and subnormal to the parabola $y^2 = 4ax$ at a point (different from the origin) are in

(a) *A.P.* (b) *G.P.* (c) *H.P.* (d) None of these

MISCELLANEOUS PROBLEMS

Basic Level

207. The equation of a circle passing through the vertex and the extremities of the latus rectum of the parabola $y^2 = 8x$ is

(a) $x^2 + y^2 + 10x = 0$ (b) $x^2 + y^2 + 10y = 0$ (c) $x^2 + y^2 - 10x = 0$ (d) $x^2 + y^2 - 5x = 0$

208. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, whose vertices are at the parabola, then the length of its side is equal to

(a) $8a$ (b) $8a\sqrt{3}$ (c) $a\sqrt{2}$ (d) None of these

209. The area of triangle formed inside the parabola $y^2 = 4x$ and whose ordinates of vertices are 1, 2 and 4 will be

(a) $\frac{7}{2}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

210. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum is

(a) 12 sq. units (b) 16 sq. units (c) 18 sq. units (d) 24 sq. units

211. The vertex of the parabola $y^2 = 8x$ is at the centre of a circle and the parabola cuts the circle at the ends of its latus rectum. Then the equation of the circle is

(a) $x^2 + y^2 = 4$ (b) $x^2 + y^2 = 20$ (c) $x^2 + y^2 = 80$ (d) None of these

212. The circle $x^2 + y^2 + 2\lambda x = 0, \lambda \in R$, touches the parabola $y^2 = 4x$ externally. Then

(a) $\lambda > 0$ (b) $\lambda < 0$ (c) $\lambda > 1$ (d) None of these

213. The length of the common chord of the parabola $2y^2 = 3(x+1)$ and the circle $x^2 + y^2 + 2x = 0$ is

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) None of these

214. The circles on focal radii of a parabola as diameter touch

(a) The tangent at the vertex (b) The axis (c) The directrix (d) None of these

Advance Level

215. The ordinates of the triangle inscribed in parabola $y^2 = 4ax$ are y_1, y_2, y_3, then the area of triangle is

(a) $\frac{1}{8a}(y_1 + y_2)(y_2 + y_3)(y_3 + y_1)$ (b) $\frac{1}{4a}(y_1 + y_2)(y_2 + y_3)(y_3 + y_1)$

(c) $\frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$ (d) $\frac{1}{4a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$

216. Which one of the following curves cuts the parabola $y^2 = 4ax$ at right angles

(a) $x^2 + y^2 = a^2$ (b) $y = e^{-x/2a}$ (c) $y = ax$ (d) $x^2 = 4ay$

217. On the parabola $y = x^2$, the point least distant from the straight line $y = 2x - 4$ is

(a) (1, 1) (b) (1, 0) (c) (1, –1) (d) (0, 0)

218. Let the equations of a circle and a parabola be $x^2 + y^2 - 4x - 6 = 0$ and $y^2 = 9x$ respectively. Then

(a) (1, –1) is a point on the common chord of contact

(b) The equation of the common chord is $y + 1 = 0$

(c) The length of the common chord is 6 (d) None of these

219. *P* is a point which moves in the *x*-*y* plane such that the point *P* is nearer to the centre of square than any of the sides. The four vertices of the square are $(\pm a, \pm a)$. The region in which *P* will move is bounded by parts of parabola of which one has the equation

(a) $y^2 = a^2 + 2ax$ (b) $x^2 = a^2 + 2ay$ (c) $y^2 + 2ax = a^2$ (d) None of these

220. The focal chord to $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are

(a) {–1, 1} (b) {–2, 2} (c) {–2, 1/2} (d) {2, –1/2}

221. Let *PQ* be a chord of the parabola $y^2 = 4x$. A circle drawn with PQ as a diameter passes through the vertex *V* of the parabola. If $ar(\Delta PVQ) = 20$ unit ^{2}, then the coordinates of *P* are

(a) (16, 8) (b) (16, –8) (c) (–16, 8) (d) (–16, –8)

222. A normal to the parabola $y^2 = 4ax$ with slope *m* touches the rectangular hyperbola $x^2 - y^2 = a^2$, if

(a) $m^6 + 4m^4 - 3m^2 + 1 = 0$ (b) $m^6 - 4m^4 + 3m^2 - 1 = 0$ (c) $m^6 + 4m^4 + 3m^2 + 1 = 0$ (d) $m^6 - 4m^4 - 3m^2 + 1 = 0$

ANSWER

Basic & Advance Level

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	b	b	c	d	a	c	a	b	b	a	c	c	a	b,c	a,c	c	b	a	a,b,c,d
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
b	d	d	c	b	d	d	b	d	a	a	d	c	c	c	c	a	c	d	c
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
b	c	a	b	a	a	c	a	d	a	a	c	b	b,d	a	a	b	a,c	a,b,d	b
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
c	c	a,b	d	d	a	b	b	c	a	b	c	d	b	a	a	c	b	d	a
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
a	a	b	c	c,d	a	d	a	a	b	b	a	d	d	a	a,b	b	c	a,b	c
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
d	b	b	c	b	b	b	d	a,c	c	a	b	a	c	b	d	b	c	d	b
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
b	b	c	c	a,b	b	c	a	c	c	a	b	a	c	a	c	c	b	a	d
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
a	d	a	d	c	d	c	a	d	d	b	d	a	a	a	d	c	a	c	c
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
a	d	c	c	b	a	a	c	a	c	d	b	b	b	c	a	c	b	d	c
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
c	c	a	d	a	d	d	c	c	a	c	a	b	d	b	b	b	c	c	d
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
b	a	d	c	d	b	c	b	d	c	b	a	a	a	c	b	a	a,c	a,b,c	a
221	**222**																		
a,b	c																		