NCERT - MCQ MATHS

CLASS 11 & 12

Contents

CHAPTER 1

SETS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The set of intelligent students in a class is :
(a) a null set (b) a singleton set
(c) a finite set (d) not a well defined collection

2. If the sets A and B are given by $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8, 10\}$ and the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then
(a) $(A \cup B)' = \{5, 7, 9\}$
(b) $(A \cap B)' = \{1, 3, 5, 6, 7\}$
(c) $(A \cap B)' = \{1, 3, 5, 6, 7, 8\}$
(d) None of these

3. If $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6\}$ and $C = \{3, 4, 6, 7\}$, then
(a) $A - (B \cap C) = \{1, 3, 4\}$
(b) $A - (B \cap C) = \{1, 2, 4\}$
(c) $A - (B \cup C) = \{2, 3\}$
(d) $A - (B \cup C) = \{\phi\}$

4. Which of the following is correct?
(a) $A \cup B \neq A \cup A'$
(b) $(A \cap B)' = A' \cup B'$
(c) $(A' \cup B') \neq A' \cup A$
(d) $(A \cap B)' = A' \cap B'$

5. The number of the proper subset of {a, b, c} is:
(a) 3 (b) 8
(c) 6 (d) 7

6. Which one is different from the others ?
(i) empty set (ii) void set (iii) zero set (iv) null set :
(a) (i) (b) (ii)
(c) (iii) (d) (iv)

7. If the sets A and B are as follows :
$A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$, then
(a) $A - B = \{1, 2\}$
(b) $B - A = \{5\}$
(c) $[(A - B) - (B - A)] \cap A = \{1, 2\}$
(d) $[(A - B) - (B - A)] \cup A = \{3, 4\}$

8. If $A = \{x, y\}$ then the power set of A is :
(a) $\{x^x, y^y\}$ (b) $\{\phi, x, y\}$
(c) $\{\phi, \{x\}, \{2y\}\}$ (d) $\{\phi, \{x\}, \{y\}, \{x, y\}\}$

9. The set {x : x is an even prime number} can be written as
(a) {2} (b) {2,4}
(c) {2,14} (d) {2, 4, 14}

10. Given the sets
$A = \{1, 3, 5\}, B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$. Which of the following may be considered as universal set for all the three sets *A*, *B* and *C*?
(a) {0, 1, 2, 3, 4, 5, 6}
(b) ϕ
(c) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
(d) {1, 2, 3, 4, 5, 6, 7, 8}

11. If $A \cup B \neq \phi$, then $n(A \cup B) = ?$
(a) $n(A) + n(B) - n(A \cap B)$
(b) $n(A) - n(B) + n(A \cap B)$
(c) $n(A) - n(B) - n(A \cap B)$
(d) $n(A) + n(B) + n(A \cap B)$

12. Which of the following collections are sets ?
(a) The collection of all the days of a week
(b) A collection of 11 best hockey player of India.
(c) The collection of all rich person of Delhi
(d) A collection of most dangerous animals of India.

13. Which of the following properties are associative law ?
(a) $A \cup B = B \cup A$
(b) $A \cup C = C \cup A$
(c) $A \cup D = D \cup A$
(d) $(A \cup B) \cup C = A \cup (B \cup C)$

14. Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$. Value of $V - B$ and $B - V$ are respectively
(a) {*e*, o} and {*k*} (b) {*e*} and {*k*}
(c) {o} and {*k*} (d) {*e*, o} and {*k*, *i*}

15. Let $A = \{a, b\}, B = \{a, b, c\}$. What is $A \cup B$?
(a) {*a*, *b*} (b) {*a*, *c*}
(c) {*a*, *b*, *c*} (d) {*b*, *c*}

16. If A and B are finite sets, then which one of the following is the correct equation?
(a) $n(A-B) = n(A) - n(B)$
(b) $n(A-B) = n(B-A)$
(c) $n(A-B) = n(A) - n(A \cap B)$
(d) $n(A-B) = n(B) - n(A \cap B)$
[n (A) denotes the number of elements in A]

17. If ϕ denotes the empty set, then which one of the following is correct ?
(a) $\phi \in \phi$ (b) $\phi \in \{\phi\}$
(c) $\{\phi\} \in \{\phi\}$ (d) $0 \in \phi$

18. Which one of the following is an infinite set ?
(a) The set of human beings on the earth
(b) The set of water drops in a glass of water
(c) The set of trees in a forest
(d) The set of all primes

19. Let $A = \{x : x$ is a multiple of $3\}$ and
$B = \{x : x$ is a multiple of $5\}$. Then A Ç B is given by:
(a) $\{15, 30, 45, ...\}$
(b) $\{3, 6, 9, ...\}$
(c) $\{15, 10, 15, 20...\}$
(d) $\{5, 10, 20, ...\}$

20. The set $A = \{x : x \in R, x^2 = 16$ and $2x = 6\}$ equals
(a) ϕ (b) $\{14, 3, 4\}$
(c) $\{3\}$ (d) $\{4\}$

21. $A = \{x : x \neq x\}$ represents
(a) $\{x\}$ (b) $\{1\}$
(c) $\{ \}$ (d) $\{0\}$

22. Which of the following is a null set ?
(a) $\{0\}$
(b) $\{x : x > 0$ or $x < 0\}$
(c) $\{x : x^2 = 4$ or $x = 3\}$
(d) $\{x : x^2 + 1 = 0, x \in \mathbf{R}\}$

23. In a group of 52 persons, 16 drink tea but not coffee, while 33 drink tea. How many persons drink coffee but not tea ?
(a) 17 (b) 36
(c) 23 (d) 19

24. There are 600 student in a school. If 400 of them can speak Telugu, 300 can speak Hindi, then the number of students who can speak both Telugu and Hindi is:
(a) 100 (b) 200
(c) 300 (d) 400

25. In a group of 500 students, there are 475 students who can speak Hindi and 200 can speak Bengali. What is the number of students who can speak Hindi only ?
(a) 275 (b) 300
(c) 325 (d) 350

26. The set builder form of given set $A = \{3, 6, 9, 12\}$ and $B = \{1, 4, 9, ..., 100\}$ is
(a) $A = \{x : x = 3n, n \in N$ and $1 \le n \le 5\}$,
$B = \{x : x = n^2, n \in N$ and $1 \le n \le 10\}$
(b) $A = \{x : x = 3n, n \in N$ and $1 \le n \le 4\}$,
$B = \{x : x = n^2, n \in N$ and $1 \le n \le 10\}$
(c) $A = \{x : x = 3n, n \in N$ and $1 \le n \le 4\}$,
$B = \{x : x = n^2, n \in N$ and $1 < n < 10\}$
(d) None of these

27. Which of the following sets is a finite set?
(a) $A = \{x : x \in Z$ and $x^2 - 5x + 6 = 0\}$
(b) $B = \{x : x \in Z$ and x^2 is even$\}$
(c) $D = \{x : x \in Z$ and $x > -10\}$
(d) All of these

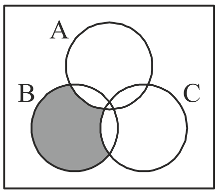
28. Which of the following is a singleton set?
(a) $\{x : |x| = 5, x \in N\}$
(b) $\{x : |x| = 6, x \in Z\}$
(c) $\{x : x^2 + 2x + 1 = 0, x \in N\}$
(d) $\{x : x^2 = 7, x \in N\}$

29. Which of the following is not a null set?
(a) Set of odd natural numbers divisible by 2
(b) Set of even prime numbers
(c) $\{x : x$ is a natural number, $x < 5$ and $x > 7\}$
(d) $\{y : y$ is a point common to any two parallel lines$\}$

30. If $A = \{x : x = n^2, n = 1, 2, 3\}$, then number of proper subsets is
(a) 3 (b) 8
(c) 7 (d) 4

31. Which of the following has only one subset?
(a) $\{ \}$ (b) $\{4\}$
(c) $\{4, 5\}$ (d) $\{0\}$

32. The shaded region in the given figure is



(a) $B \cap (A \cup C)$ (b) $B \cup (A \cap C)$
(c) $B \cap (A - C)$ (d) $B - (A \cup C)$

33. If $A = \{x : x$ is a multiple of $3\}$ and
$B = \{x : x$ is a multiple of $5\}$, then $A - B$ is equal to
(a) $\overline{A} \cap B$ (b) $A \cap \overline{B}$
(c) $\overline{A} \cap \overline{B}$ (d) $\overline{A \cap B}$

34. If A and B be any two sets, then $A \cap (A \cup B)'$ is equal to
(a) A (b) B
(c) ϕ (d) None of these

35. A survey shows that 63% of the people watch a news channel whereas 76% watch another channel. If x% of the people watch both channel, then
(a) $x = 35$ (b) $x = 63$
(c) $39 \le x \le 63$ (d) $x = 39$

36. The set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set-builder form is

(a) $\left\{x : x = \frac{n}{n+1}, \text{ where } n \in N \text{ and } 1 < n < 6\right\}$

(b) $\left\{x : x = \frac{n}{n+1}, \text{ where } n \in N \text{ and } 1 \le n < 6\right\}$

(c) $\left\{x : x = \frac{n}{n+1}, \text{ where } n \in N \text{ and } 1 \le n \le 6\right\}$

(d) None of the above

37. The set {x : x is a positive integer less than 6 and $3^x - 1$ is an even number} in roster form is
(a) {1, 2, 3, 4, 5} (b) {1, 2, 3, 4, 5, 6}
(c) {2, 4, 6} (d) {1, 3, 5}

38. If B = {x : x is a student presently studying in both classes X and XI}. Then, the number of elements in set B are
(a) finite (b) infinite
(c) zero (d) None of these

39. Consider:
X = Set of all students in your school.
Y = Set of all students in your class.
Then, which of the following is true?
(a) Every element of Y is also an element of X
(b) Every element of X is also an element of Y
(c) Every element of Y is not an element of X
(d) Every element of X is not an element of Y

40. If $A \subset B$ and $A \neq B$, then
(a) A is called a proper subset of B
(b) A is called a super set of B
(c) A is not a subset of B
(d) B is a subset of A

41. The set of real numbers {x : a < x < b} is called
(a) open interval (b) closed interval
(c) semi-open interval (d) semi-closed interval

42. Which of the following is true?
(a) $a \in \{\{a\}, b\}$ (b) $\{b, c\} \subset \{a, \{b, c\}\}$
(c) $\{a, b\} \subset \{a, \{b, c\}\}$ (d) None of these

43. The interval [a, b) is represented on the number line as

(a) 

(b) 

(c) 

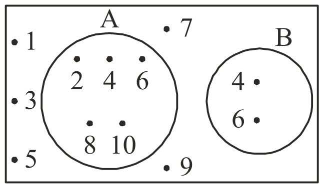
(d) 

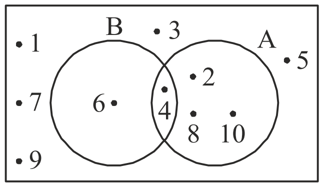
44. The interval represented by

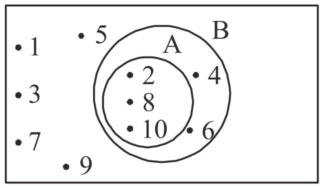
(a) (a, b) (b) [a, b]
(c) [a, b) (d) (a, b]

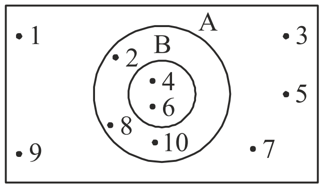
45. The number of elements in $P[P(P(\phi))]$ is
(a) 2 (b) 3
(c) 4 (d) 5

46. If U = {1, 2, 3, 4, ..., 10} is the universal set of A, B and A = {2, 4, 6, 8, 10}, B = {4, 6} are subsets of U, then given sets can be represented by Venn diagram as

(a) 

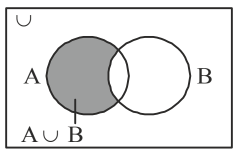
(b) 

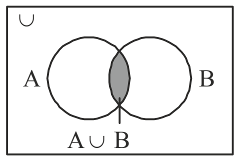
(c) 

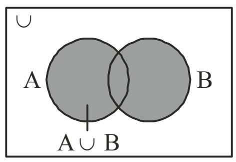
(d) 

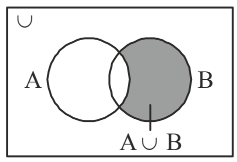
47. Most of the relationships between sets can be represented by means of diagrams which are known as
(a) rectangles (b) circles
(c) Venn diagrams (d) triangles

48. Which of the following represent the union of two sets A and B?

(a) 

(b) 

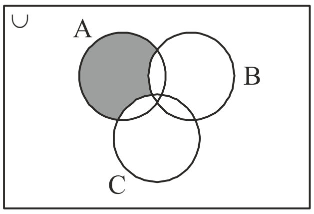
(c) 

(d) 

49. Let X = {Ram, Geeta, Akbar} be the set of students of Class XI, who are in school hockey team and Y = {Geeta, David, Ashok} be the set of students from Class XI, who are in the school football team. Then, $X \cap Y$ is
(a) {Ram, Geeta} (b) {Ram}
(c) {Geeta} (d) None of these

50. Which of the following represent A – B?
(a) $\{x : x \in A \text{ and } x \in B\}$
(b) $\{x : x \in A \text{ and } x \notin B\}$
(c) $\{x : x \in A \text{ or } x \in B\}$
(d) $\{x : x \in A \text{ or } x \notin B\}$

51. The shaded region in the given figure is

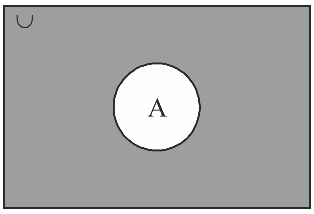


(a) $A \cap (B \cup C)$ (b) $A \cup (B \cap C)$
(c) $A \cap (B - C)$ (d) $A - (B \cup C)$

52. If A and B are non-empty subsets of a set, then $(A - B) \cup (B - A)$ equals to
(a) $(A \cap B) \cup (A \cup B)$ (b) $(A \cup B) - (A - B)$
(c) $(A \cup B) - (A \cap B)$ (d) $(A \cup B) - B$

53. Let A, B, C are three non-empty sets. If $A \subset B$ and $B \subset C$, then which of the following is true?
(a) $B - A = C - B$ (b) $A \cap B \cap C = B$
(c) $A \cup B = B \cap C$ (d) $A \cup B \cup C = A$

54. In the Venn diagram, the shaded portion represents



(a) complement of set A (b) universal set
(c) set A (d) None of these

55. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 3, 5}, B = {2, 4, 6, 7} and C = {2, 3, 4, 8}, then which of the following is true?

(a) $(B \cup C)' = \{1, 5, 9, 10\}$

(b) $(C - A)' = \{1, 2, 3, 5, 6, 7, 9, 10\}$

(c) Both (a) and (b)
(d) None of the above

56. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to
(a) A (b) B
(c) ϕ (d) $A \cap B^c$

57. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to
(a) A (b) B
(c) A^c (d) B^c

58. The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is
(a) {2, 3, 5} (b) {3, 5, 9}
(c) {1, 2, 5, 9} (d) None of these

59. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to
(a) A (b) B
(c) ϕ (d) None of these

60. If A and B are sets, then $A \cap (B - A)$ is
(a) ϕ (b) A
(c) B (d) None of these

61. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5}, then $(A - B) \times (B - C)$ is
(a) {(1, 2), (1, 5), (2, 5)} (b) {(1, 4)}
(c) (1, 4) (d) None of these

62. If n(A) = 3, n(B) = 6 and $A \subseteq B$. Then, the number of elements in $A \cup B$ is equal to
(a) 3 (b) 9
(c) 6 (d) None of these

63. In a battle 70% of the combatants lost one eye, 80% an ear, 75% an arm, 85% a leg, x% lost all the four limbs. The minimum value of x is
(a) 10 (b) 12
(c) 15 (d) None of these

64. If A = {x : x is a multiple of 4} and B = {x : x is a multiple of 6}, then $A \cap B$ consists of all multiples of
(a) 16 (b) 12
(c) 8 (d) 4

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

65. Let P be a set of squares, Q be set of parallelograms, R be a set of quadrilaterals and S be a set of rectangles. Consider the following :
I. $P \subset Q$ II. $R \subset P$
III. $P \subset S$ IV. $S \subset R$
Which of the above are correct?
(a) I, II and III (b) I, III and IV
(c) I, II and IV (d) III and IV

66. Consider the following statements
I. $\phi \in \{\phi\}$ II. $\{\phi\} \subseteq \phi$
Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

67. Consider the following sets.
I. A = {1, 2, 3}
II. $B = \{x \in R : x^2 - 2x + 1 = 0\}$
III. C = {1, 2, 2, 3}
IV. $D = \{x \in R : x^3 - 6x^2 + 11x - 6 = 0\}$
Which of the following are equal?
(a) A = B = C (b) A = C = D
(c) A = B = D (d) B = C = D

68. Consider the following relations:
I. $A - B = A - (A \cap B)$
II. $A = (A \cap B) \cup (A - B)$
III. $A - (B \cup C) = (A - B) \cup (A - C)$
Which of these is/are correct?
(a) Both I and III (b) Only II
(c) Both II and III (d) Both I and II

69. Consider the following statements
I. The vowels in the English alphabet.
II. The collection of books.
III. The rivers of India.
IV. The collection of most talented batsmen of India.
Which of the following is/are well-defined collections?
(a) I and II (b) Only I
(c) I and III (d) I and IV

70. The set of all letters of the word 'SCHOOL' is represented by
I. {S, C, H, O, O, L}
II. {S, C, H, O, L}
III. {C, H, L, O, S}
IV. {S, C, H, L}
The correct code is
(a) I and II (b) I, II and III
(c) II and III (d) I, II, III and IV

71. I. The collection of all months of a year beginning with the letter J.
II. The collection of ten most talented writers of India.
III. A team of eleven best cricket batsmen of the world.
IV. The collection of all boys in your class.
Which of the above are the sets?
(a) I and II (b) I and III
(c) I and IV (d) I, II and III

72. **Statement - I :** The set D = {x : x is a prime number which is a divisor of 60} in roster form is {1, 2, 3, 4, 5}.
Statement - II : The set E = the set of all letters in the word 'TRIGONOMETRY', in the roster form is {T, R, I, G, O, N, M, E, Y}.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

73. The empty set is represented by
I. ϕ II. $\{\phi\}$
III. { } IV. {{ }}
(a) I and II (b) I and III
(c) II and III (d) I and IV

74. **Statement - I :** The set $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$ is the empty set.
Statement - II : The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is an empty set.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

75. State which of the following is/are true?
I. The set of animals living on the Earth is finite.
II. The set of circles passing through the origin (0, 0) is infinite.
(a) Only I (b) Only II
(c) I and II (d) None of these

76. **Statement - I :** The set of positive integers greater than 100 is infinite.
Statement - II : The set of prime numbers less than 99 is finite.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

77. Select the infinite set from the following:
I. The set of lines which are parallel to the X-axis.
II. The set of numbers which are multiples of 5.
III. The set of letters in the English alphabet.
(a) I and II (b) II and III
(c) I and III (d) None of these

78. Consider the following sets.
A = {0},
B = $\{x : x > 15 \text{ and } x < 5\}$,
C = $\{x : x - 5 = 0\}$,
D = $\{x : x^2 = 25\}$,
E = {x : x is an integral positive root of the equation $x^2 - 2x - 15 = 0$}
Choose the pair of equal sets
(a) A and B (b) C and D
(c) C and E (d) B and C

79. **Statement - I :** The set of concentric circles in a plane is infinite.
Statement - II : The set $\{x : x^2 - 3 = 0 \text{ and } x \text{ is rational}\}$ is finite.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

80. Which of the following is/are true?
I. Every set A is a subset of itself.
II. Empty set is a subset of every set.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) None of these

81. Let A = {1, 3, 5} and B = {x : x is an odd natural number less than 6}. Then, which of the following are true?
I. $A \subset B$ II. $B \subset A$
III. $A = B$ IV. $A \not\subset B$
(a) I and II are true (b) I and III are true
(c) I, II and III are true (d) I, II and IV are true

82. Given the sets A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 6, 8}. Then, which of the following may be considered as universal set(s) for all the three sets A, B and C?
I. {0, 1, 2, 3, 4, 5, 6}
II. ϕ
III. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
IV. {1, 2, 3, 4, 5, 6, 7, 8}
(a) Only I (b) Only III
(c) I and III (d) III and IV

83. Which of the following is/are the universal set(s) for the set of isosceles triangles?
I. Set of right angled triangles.
II. Set of scalene triangles.
III. Set of all triangles in a plane.
(a) Only I (b) Only III
(c) II and III (d) None of these

84. **Statement - I :** In the union of two sets A and B, the common elements being taken only once.
Statement - II : The symbol '$\cap$' is used to denote the union.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

85. **Statement - I :** Let A = {a, b} and B = {a, b, c}. Then, $A \not\subset B$.
Statement - II : If $A \subset B$, then $A \cup B = B$.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

86. Which of the following are correct?
I. $A - B = A - (A \cap B)$.
II. $A = (A \cap B) \cup (A - B)$.
III. $A - (B \cup C) = (A - B) \cup (A - C)$.
(a) I and II (b) II and III
(c) I, II and III (d) None of these

87. Which of the following is/are true?
I. If A is a subset of the universal set U, then its complement A' is also a subset of U.
II. If U = {1, 2, 3, ..., 10} and A = {1, 3, 5, 7, 9}, then $(A')' = A$.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) None of these

88. **Statement-I :** Let U be the universal set and A be the subset of U. Then, complement of A is the set of element of A.
Statement-II : The complement of a set A can be represented by A'.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

89. **Statement-I :** The Venn diagram of $(A \cup B)'$ and $A' \cap B'$ are same.
Statement-II : The Venn diagram of $(A \cap B)'$ and $A' \cup B'$ are different.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

90. **Statement-I :** If A, B and C are finite sets, then $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.
Statement-II : If A, B and C are mutually pairwise disjoint, then $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C)$.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

91. In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Then, which of the following is/are true?
I. 150 students were taking at least one juice.
II. 225 students were taking neither apple juice nor orange juice.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) None of these

92. Suppose A be a non-empty set, then the collection of all possible subsets of set A is a power set P(A).
Which of the following is correct?
I. $P(A) \cap P(B) = P(A \cap B)$
II. $P(A) \cup P(B) = P(A \cup B)$
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Both I and II are false

93. Which of the following is correct?
I. Number of subsets of a set A having n elements is equal to 2^n.
II. The power set of a set A contains 128 elements then number of elements in set A is 7.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Both I and II are false

94. Which of the following is correct?
I. Number of non-empty subsets of a set having n elements are $2^n - 1$.
II. The number of non-empty subsets of the set {a, b, c, d} are 15.
(a) Only I is false (b) Only II is false
(c) Both I and II are false (d) Both I and II are true

95. **Statement-I :** If A = {1, 2, 3, 4, 5}, B = {2, 4, 6}, C = {3, 4, 6}, then $(A \cup B) \cap C = \{3, 4, 6\}$
Statement-II : $(A \cup B)' = A' \cap B'$
(a) Only I is true
(b) Only II is true
(c) Both I and II are true.
(d) Both I and II are false.

96. Let A = {3, 6, 9, 12, 15, 18, 21}
B = {4, 8, 12, 16, 20}
C = {2, 4, 6, 8, 10, 12, 14, 16}
and D = {5, 10, 15, 20}
Which of the following is incorrect?
I. $A - B = \{4, 8, 16, 20\}$
II. $(C - B) \cap (D - B) = \phi$
III. $B - C \neq B - D$
(a) Only I & II (b) Only II & III
(c) Only III & I (d) None of these

97. Which of the following is correct?

I. $n(S \cup T)$ is maximum when $n(S \cap T)$ is least.

II. If $n(U) = 1000$, $n(S) = 720$, $n(T) = 450$, then least value of $n(S \cap T) = 170$.

(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Both I and II are false

98. Which of the following is correct?

I. Three sets A, B, C are such that $A = B \cap C$ and $B = C \cap A$, then $A = B$.

II. If $A = \{a, b\}$, then $A \cap P(A) = A$

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

99. Consider the following relations :

I. $A = (A \cap B) \cup (A - B)$

II. $A - B = A - (A \cap B)$

III. $A - (B \cup C) = (A - B) \cup (A - C)$

Which of these is correct?

(a) I and III (b) I and II
(c) Only II (d) II and III

100. Consider the following statements.

I. If A_n is the set of first n prime numbers, then $\overset{10}{\underset{n=2}{U}} A_n$ is equal to {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

II. If A and B are two sets such that $n(A \cup B) = 50$, $n(A) = 28$, $n(B) = 32$, then $n(A \cap B) = 10$.

Which of these is correct?

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

101. Consider the following statements.

I. Let A and B be any two sets. The union of A and B is the set containing the elements of A and B both.

II. The intersection of two sets A and B is the set which consists of common elements of A and B.

Which of the statement is correct?

(a) Only statement-I is true.
(b) Only statement-II is true.
(c) Both statements are true.
(d) Neither I nor II are true.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

102. Match the following statements in column-I with their symbolic forms in column-II.

Column – I	Column – II
A. A is a subset of B	1. if and only if
B. If $A \subset B$ and $B \subset A$, then	2. $A \subset B$
C. A is not a subset of B	3. $A = B$
D. If $a \in A \Rightarrow a \in B$, then	4. $A \not\subset B$
E. The symbol "$\Leftrightarrow$" means	

Codes:

	A	B	C	D	E
(a)	4	3	1	2	3
(b)	2	3	4	2	1
(c)	1	2	3	4	3
(d)	4	3	2	1	4

103. Match the following sets in column -I with the intervals in column -II.

Column – I	Column – II
A. $\{x : x \in R,\ a < x < b\}$	1. $(a, b]$
B. $\{x \in R : a \le x \le b\}$	2. $[a, b)$
C. The set of real numbers x such that $a \le x < b$	3. (a, b)
D. $\{x : x \in R \text{ and } a < x \le b\}$	4. $[a, b]$

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	2	3	4	1
(c)	1	2	3	4
(d)	3	4	2	1

104. Match the following sets in column -I with the equal sets in column-II.

Column – I	Column – II
A. $A \cap B$	1. $(A \cap B) \cup (A \cap C)$
B. $(A \cap B) \cap C$	2. A
C. $\phi \cap A$	3. $A \cap (B \cap C)$
D. $U \cap A$	4. $B \cap A$
E. $A \cap A$	5. ϕ
F. $A \cap (B \cup C)$	

Codes:

	A	B	C	D	E	F
(a)	5	1	4	3	1	2
(b)	3	4	2	1	5	4
(c)	4	3	5	2	2	1
(d)	1	2	3	4	5	2

105. Match the following sets in column -I equal with the sets in column-II.

Column – I	Column – II
A. $A \cup A'$	1. $A' \cap B'$
B. $A \cap A'$	2. $A' \cup B'$
C. $(A \cup B)'$	3. U
D. $(A \cap B)'$	4. ϕ
E. ϕ'	5. A
F. U'	
G. $(A')'$	

Codes:

	A	B	C	D	E	F	G
(a)	1	2	3	4	5	3	2
(b)	3	4	1	2	3	4	5
(c)	4	3	2	1	4	5	3
(d)	5	4	3	2	1	4	1

106.

Column - I (Set)	**Column - II (Roster-form)**
(A) $\{x \in N : x^2 < 25\}$	1. {1, 2, 3, 4, 5 }
(B) Set of integers between – 5 and 5	2. {2, 3, 5}
(C) {x : x is a natural number less than 6}	3. {–4, –3, –2, –1, 0, 1, 2, 3, 4}
(D) {x : x is a prime number which is a divisor of 60}	4. {1, 2, 3, 4}

Codes :

	A	B	C	D
(a)	4	2	1	3
(b)	1	3	4	2
(c)	1	2	3	4
(d)	4	3	1	2

107.

Column - I	**Column - II**
(A) If $A \cup B = A \cap B$, then	1. $A = B$
(B) Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then	2. $A \cup B$
(C) If P(A) = P(B), then	3. $A \subset B$
(D) $A \cup (B - A)$ is equal to	4. $(A \cap B \cap C)'$
(E) Let U be the universal set and $A \cup B \cup C = U$. Then, $\{(A - B) \cup (B - C) \cup (C - A)\}$ is equal to	5. $B = C$
(F) The set $(A \cap B')' \cup (B \cup C)$ is equal to	6. $A' \cup B$

Codes :

	A	B	C	D	E	F
(a)	1	2	3	4	5	6
(b)	3	2	1	5	6	4
(c)	2	1	5	4	6	2
(d)	3	5	1	2	4	6

108. If U = {1, 2, 3, 4, 5, 6, 7}, A = {2, 4, 6}, B = {3, 5} and C = {1, 2, 4, 7}, then match the columns.

Column-I	**Column-II**
(A) $A \cup (B \cap C)$	1. {1, 2, 4, 7}
(B) $(A \cap B) \cup C$	2. {6}
(C) $A \cap (B \cup C)'$	3. {1, 3, 5, 7}
(D) $A' \cup (B \cap C')$	4. {1, 7}
(E) $A' \cap B'$	5. {2, 4, 6}

Codes :

	A	B	C	D	E
(a)	1	5	3	2	4
(b)	5	1	2	3	4
(c)	5	1	3	4	2
(d)	3	4	5	1	2

109. Match the complement of sets of the following sets in column-I with the sets in column-II.

Column - I	**Column - II**
(A) {x: x is a prime number}	1. {x : x is not divisible by 15}
(B) {x : x is a multiple of 3}	2. {x : x is an odd natural number}
(C) {x : x is a natural number divisible by 3 and 5}	3. {x : x is not a prime number}
(D) {x : x is an even natural number}	4. {x : x is not a multiple of 3}

Codes :

	A	B	C	D
(a)	3	4	2	1
(b)	1	2	3	4
(c)	3	4	1	2
(d)	4	3	2	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

110. If X = {1, 2, 3, …, 10} and 'a' represents any element of X, then the set containing all the elements satisfy $a + 2 = 6$, $a \in X$ is

(a) {4} (b) {3}
(c) {2} (d) {5}

111. If a set is denoted as $B = \phi$, then the number of element in B is

(a) 3 (b) 2
(c) 1 (d) 0

112. Let X = {1, 2, 3, 4, 5}. Then, the number of elements in X are

(a) 3 (b) 2
(c) 1 (d) 5

113. If X = {1, 2, 3}, then the number of proper subsets is

(a) 5 (b) 6
(c) 7 (d) 8

114. The number of non-empty subsets of the set {1, 2, 3, 4} is $3 \times a$. The value of 'a' is

(a) 3 (b) 4
(c) 5 (d) 6

115. If $A = \phi$, then the number of elements in P(A) is

(a) 3 (b) 2
(c) 1 (d) 0

116. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + 9y^2 = 144\}$ then the number of points, $A \cap B$ contains is

(a) 1 (b) 2
(c) 3 (d) 4

117. The cardinality of the set $P\{P[P(\phi)]\}$ is
(a) 0 (b) 1
(c) 2 (d) 4

118. If $n(A) = 8$ and $n(A \cap B) = 2$, then $n[(A \cap B)' \cap A]$ is equal to
(a) 8 (b) 6
(c) 4 (d) 2

119. In a school, there are 20 teachers who teach Mathematics or Physics of these, 12 teach Mathematics and 4 teach both Maths and Physics. Then the number of teachers teaching only Physics are
(a) 4 (b) 8
(c) 12 (d) 16

ASSERTION-REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

120. **Assertion :** The number of non-empty subsets of the set {a, b, c, d} are 15.
Reason : Number of non-empty subsets of a set having n elements are $2^n - 1$.

121. Suppose A, B and C are three arbitrary sets and U is a universal set.
Assertion : If $B = U - A$, then $n(B) = n(U) - n(A)$.
Reason : If $C = A - B$, then $n(C) = n(A) - n(B)$.

122. **Assertion :** Let $A = \{1, \{2, 3\}\}$, then
$P(A) = \{\{1\}, \{2, 3\}, \phi, \{1, \{2, 3\}\}\}$.
Reason : Power set is set of all subsets of A.

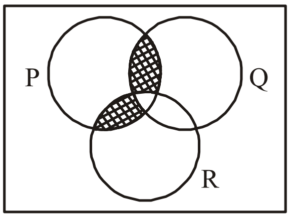
123. **Assertion :** The subsets of the set {1, {2}} are { }, {1}, {{2}} and {1, {2}}.
Reason : The total number of proper subsets of a set containing n elements is $2^n - 1$.

124. **Assertion :** For any two sets A and B, $A - B \subset B'$.
Reason : If A be any set, then $A \cap A' = \phi$.

CRITICAL THINKING TYPE QUESTIONS

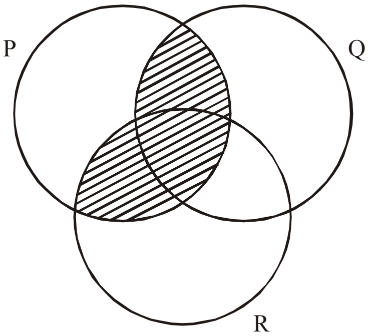
Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

125. What does the shaded portion of the Venn diagram given below represent?



(a) $(P \cap Q) \cap (P \cap R)$
(b) $((P \cap Q) - R) \cup ((P \cap R) - Q)$
(c) $((P \cup Q) - R) \cap ((P \cap R) - Q)$
(d) $((P \cap Q) \cup R) \cap ((P \cup Q) - R)$

126. What does the shaded region represent in the figure given below ?



(a) $(P \cup Q) - (P \cap Q)$
(b) $P \cap (Q \cap R)$
(c) $(P \cap Q) \cap (P \cap R)$
(d) $(P \cap Q) \cup (P \cap R)$

127. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are:
(a) 7, 6 (b) 6, 3
(c) 5, 1 (d) 8, 7

128. If A is the set of the divisors of the number 15, B is the set of prime numbers smaller than 10 and C is the set of even numbers smaller than 9, then $(A \cup C) \cap B$ is the set
(a) {1, 3, 5} (b) {1, 2, 3}
(c) {2, 3, 5} (d) {2, 5}

129. Let S = the set of all triangles, P = the set of all isosceles triangles, Q = the set of all equilateral triangles, R = the set of all right-angled triangles. What do the sets $P \cap Q$ and $R - P$ represents respectively ?
(a) The set of isosceles triangles; the set of non- isosceles right angled triangles
(b) The set of isosceles triangles; the set of right angled triangles
(c) The set of equilateral triangles; the set of right angled triangles
(d) The set of isosceles triangles; the set of equilateral triangles

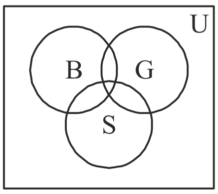
130. If A and B are non-empty sets, then $P(A) \cup P(B)$ is equal to

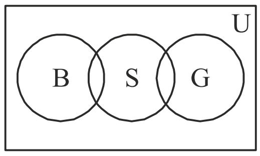
(a) $P(A \cup B)$ (b) $P(A \cap B)$
(c) $P(A) = P(B)$ (d) None of these

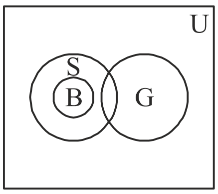
131. Let $A = \{(1, 2), (3, 4), 5\}$, then which of the following is incorrect?

(a) $\{3, 4\} \not\subset A$ as $(3, 4)$ is an element of A
(b) $\{5\}$, $\{(3, 4)\}$ are subsets of A but not elements of A
(c) $\{1, 2\}$, $\{5\}$ are subsets of A
(d) $\{(1, 2), (3, 4), 5\}$ are subset of A

132. Let U be the set of all boys and girls in school. G be the set of all girls in the school. B be the set of all boys in the school and S be the set of all students in the school who take swimming. Some but not all students in the school take swimming.

(a) 

(b) 

(c) 

(d) None of these

133. If $A = \{a, \{b\}\}$, then P(A) equals.

(a) $\{\phi, \{a\}, \{\{b\}\}, \{a, \{b\}\}\}$
(b) $\{\phi, \{a\}\}$
(c) $\{\{a\}, \{b\}, \phi\}$
(d) None of these

134. If A and B are two sets, then $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to

(a) Only A (b) $A \cup B$
(c) $(A \cup B)'$ (d) None of these

135. A market research group conducted a survey of 2000 consumers and reported that 1720 consumers like product P_1 and 1450 consumers like product P_2. What is the least number that must have liked both the products?

(a) 1150 (b) 2000
(c) 1170 (d) 2500

136. In a town of 10000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all of three newspapers, then the number of families which buy A only, is

(a) 4400 (b) 3300
(c) 2000 (d) 500

137. A class has 175 students. The following data shows the number of students opting one or more subjects. Maths–100, Physics–70, Chemistry–40, Maths and Physics–30, Maths and Chemistry–28, Physics and Chemistry–23, Maths, Physics and Chemistry–18. How many have offered Maths alone?

(a) 35 (b) 48
(c) 60 (d) 22

138. If $aN = \{ax : x \in \mathbf{N}\}$, then the set $3N \cap 7N$ is

(a) 21N (b) 10N (c) 4N (d) None

139. If $A = \{x \in \mathbf{R} : 0 < x < 3\}$ and $B = \{x \in \mathbf{R} : 1 \le x \le 5\}$ then $A \Delta B$ is

(a) $\{x \in \mathbf{R} : 0 < x < 1\}$ (b) $\{x \in \mathbf{R} : 3 \le x \le 5\}$
(c) $\{x \in \mathbf{R} : 0 < x < 1 \text{ or } 3 \le x \le 5\}$ (d) ϕ

140. Let A, B, C be finite sets. Suppose that n (A) = 10, n (B) = 15, n (C) = 20, n $(A \cap B)$ = 8 and n $(B \cap C)$ = 9. Then the possible value of n $(A \cup B \cup C)$ is

(a) 26
(b) 27
(c) 28
(d) Any of the three values 26, 27, 28 is possible

141. A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products ?

(a) 170 (b) 280
(c) 220 (d) None

142. Each student in a class of 40, studies at least one of the subjects English, Mathematics and Economics. 16 study English, 22 Economics and 26 Mathematics, 5 study English and Economics, 14 Mathematics and Economics and 2 study all the three subjects. The number of students who study English and Mathematics but not Economics is

(a) 7 (b) 5
(c) 10 (d) 4

143. A survey of 500 television viewers produced the following information, 285 watch football, 195 watch hockey, 115 watch basket-ball, 45 watch football and basket ball, 70 watch football and hockey, 50 watch hockey and basket ball, 50 do not watch any of the three games. The number of viewers, who watch exactly one of the three games are

(a) 325 (b) 310
(c) 405 (d) 372

144. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total 64 played both basketball and hockey, 80 played cricket and basketball and 40 played cricket and hockey, 24 played all the three games. The number of boys who did not play any game is:

(a) 128 (b) 216
(c) 240 (d) 160

145. Let A, B, C be three sets. If $A \in B$ and $B \subset C$, then

(a) $A \subset C$ (b) $A \not\subset C$
(c) $A \in C$ (d) $A \notin C$

146. Let $V = \{a, e, i, o, u\}$, $V - B = \{e, o\}$ and $B - V = \{k\}$. Then, the set B is

(a) $\{a, i, u\}$ (b) $\{a, e, k, u\}$
(c) $\{a, i, k, u\}$ (d) $\{a, e, i, k, u\}$

147. From 50 students taking examination in Mathematics, Physics and Chemistry, each of the students has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. Atmost 19 passed Mathematics and Physics, atmost 29 Mathematics and Chemistry and atmost 20 Physics and Chemistry. Then, the largest numbers that could have passed all three examinations, are

(a) 12 (b) 14
(c) 15 (d) 16

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(d)** **2.** **(a)** **3.** **(b)**

4. **(b)** **Note:** $(A \cup B)' = A' \cap B'$ (By De-morgan's law) and $(A \cap B)' = A' \cup B'$

5. **(d)** The number of proper subsets of $\{1, 2, 3, ... n\}$ is $2^n - 1$.
Hence the number of proper subset of $\{a, b, c\}$ is $2^3 - 1 = 7$

6. **(c)** A set which does not contain any element is called an empty or void or null set.
But zero set contain 0.

7. **(a)** Given $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$
$\therefore A - B = \{1, 2\}$

8. **(d)** Let $A = \{x, y\}$
Power set = Set of all possible subsets of A
$\therefore \; P(A) = \{\phi, \{x\}, \{y\}, \{x, y\}\}$

9. **(a)** **10.** **(c)** **11.** **(a)**

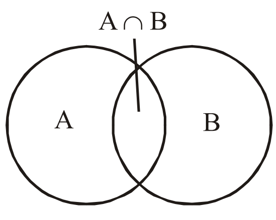
12. **(a)** The days of a week are well defined.
Hence, the collection of all the days of a week, is a set.

13. **(d)**

14. **(a)** We have, $V = \{a, e, i, o, u\}$ and
$B = \{a, i, k, u\}$
$\therefore \; V - B = \{e, o\}$
$\because$ the element e, o belong to V but not to B
$\therefore \; B - V = \{k\}$
$\because$ the element k belong to $B - V$ but not to $V - B$.

15. **(c)** $A \cup B = \{a, b\} \cup \{a, b, c\} = \{a, b, c\}$

16. **(c)** If A and B are finite sets, then



$A - B = A - (A \cap B)$

From the Venn diagram
$\Rightarrow \; n(A - B) = n(A) - n(A \cap B)$

17. **(b)** Since, ϕ is an empty set, $\phi \in \{\phi\}$

18. **(d)** In the given sets, the set of all primes is an infinite set.

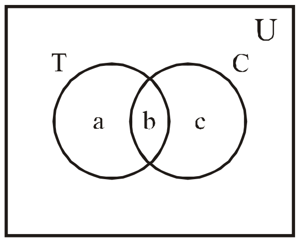
19. **(a)** Given : $A = \{3, 6, 9, 15....\}$ and
$B = \{5, 10, 15, 20,.....\}$
$A \cap B = \{x : x \text{ is multiple of 3 and 5}\}$
$\Rightarrow \quad A \cap B = \{x : x \text{ is multiple of } 15\}$
$\Rightarrow \quad A \cap B = \{15, 30, 45,....\}$

20. **(a)** We have $x^2 = 16 \Rightarrow x = \pm 4$
Also, $2x = 6 \Rightarrow x = 3$
There is no value of x which satisfies both the above equations. Thus the set A contains no elements
$\therefore \; A = \phi$

21. **(c)** Clearly $A = \phi = \{\}$

22. **(d)** $x^2 + 1 = 0$ has no solution in R

23. **(d)** Let T denotes tea drinkers and C denotes coffee drinkers in universal set U.



From the diagram, we get
$a + b + c = 52$ ...(i)
$a = 16$ ...(ii)
$a + b = 33$ ...(iii)
Put a = 16 in equation (iii), we have
$16 + b = 33 \Rightarrow b = 17$
Now, substitute the values of a and b in equation (i), we get
$16 + 17 + c = 52$
$c = 52 - 33 = 19$

24. **(a)** Let $A \equiv$ Set of Tamil speaking students and
$B \equiv$ Hindi speaking students
$n(A) = 400, n(B) = 300$ and $n(A \cup B) = 600$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$\Rightarrow \; n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$= 400 + 300 - 600 = 100$

25. (b) Total number of students = 500

Let H be the set showing number of students who can speak Hindi = 475 and B be the set showing number of students who can speak Bengali = 200

So, n (H) = 475 and n (B) = 200 and given that

$n(B \cup H) = 500$

we have

$n(B \cup H) = n(B) + n(H) - n(B \cap H)$

$\Rightarrow 500 = 200 + 475 - n(B \cap H)$

so, $n(B \cap H) = 175$

Hence, persons who speak Hindi only $= n(H) - n(B \cap H)$

$= 475 - 175 = 300$

26. (b) Given, A = {3, 6, 9, 12}

$= \{x : x = 3n, n \in N \text{ and } 1 \le n \le 4\}$

and $B = \{1, 4, 9, ..., 100\}$

$= \{x : x = n^2, n \in N \text{ and } 1 \le n \le 10\}$

27. (a) (a) $A = \{x : x \in Z \text{ and } x^2 - 5x + 6 = 0\} = \{2, 3\}$

So, A is a finite set

(b) $B = \{x : x \in Z \text{ and } x^2 \text{ is even}\}$

$= \{....., -6, -4, -2, 0, 2, 4, 6, ...\}$

Clearly, B is an infinite set.

(c) $D = \{x : x \in Z \text{ and } x > -10\}$

$= \{-9, -8, -7, ...\}$

Clearly, D is an infinite set.

28. (a)

(a) $|x| = 5 \Rightarrow x = 5$ $[\because x \in N]$

$\therefore$ Given set is singleton.

(b) $|x| = 6 \Rightarrow x = -6, 6$ $[\because x \in Z]$

$\therefore$ Given set is not singleton.

(c) $x^2 + 2x + 1 = 0 \Rightarrow (x + 1)^2 = 0$

$\Rightarrow x = -1, -1$

Since, $-1 \notin N$, $\therefore$ given set $= \phi$

(d) $x^2 = 7 \Rightarrow x = \pm\sqrt{7}$.

29. (b)

(a) There is no odd natural number divisible by 2, so there will be no element in this set, hence it is a null set.

(b) There is only one even prime number which is 2, i.e. there is an element, so it is not a null set.

(c) There is no natural number which is less than 5 and greater than 7, i.e. there is no element, so it is a null set.

(d) Since, parallel lines never intersect each other, so they have no common point, i.e. no element, so it is a

null set.

30. (c) Given that $A = \{x : x = n^2, n = 1, 2, 3\} = \{1, 4, 9\}$

$\therefore$ Number of elements in A is 3.

So, number of proper subsets $= 2^3 - 1 = 7$.

31. (a) Subset of { } i.e., ϕ is ϕ.

Subsets of {4} are ϕ, {4}.

Subsets of {4, 5} are ϕ, {4}, {5}, {4, 5}.

Subsets of {0} are ϕ, {0}.

32. (d) It is clear from the figure that set $A \cup C$ is not shaded and set B is shaded other than $A \cup C$, i.e., $B - (A \cup C)$.

33. (b)

34. (c) $A \cap (A \cup B)' = A \cap (A' \cap B') = (A \cap A') \cap B'$

$= \phi \cap B' = \phi$.

35. (c) Let A and B be the two sets of news channel such that n(A) = 63, n(B) = 76, $n(A \cup B) = 100$

Also, $n(A \cap B) = x$

Using, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 100 = 63 + 76 - x$

$\Rightarrow x = 139 - 100 = 39$

Again, $n(A \cap B) \le n(A)$

$\Rightarrow x \le 63$

$\therefore 39 \le x \le 63$.

36. (c) We see that each member in the given set has the numerator one less than the denominator. Also, the numerator begins from 1 and do not exceed 6. Hence, in the set-builder form, the given set is

$$\left\{x : x = \frac{n}{n+1}, \text{ where } n \in N \text{ and } 1 \le n \le 6\right\}.$$

37. (a) Since, $3^x - 1$ is an even number for all $x \in Z^+$. So, the given set in roster form is {1, 2, 3, 4, 5}.

38. (c) A student cannot study simultaneously in both classes X and XI. Thus, the set B contains no element at all.

39. (a) We note that every element of Y is also an element of X, as if a student is in your class, then he is also in your school.

40. (a) If $A \subset B$ and $A \ne B$, then A is called a proper subset of B and B is called a super set of A.

41. (a) Let $a, b \in R$ and $a < b$. Then, the set of real numbers $\{x : a < x < b\}$ is called an open interval. And a, b do not belong to this interval.

42. (d) a is not an element of {{a}, b}

$\therefore a \notin \{\{a\}, b\}$

{b, c} is the element of {a, {b, c}}

$\therefore \{b, c\} \in \{a, \{b, c\}\}$

$b \in \{a, b\}$ but $b \notin \{a, \{b, c\}\}$

$\therefore \{a, b\} \not\subset \{a, \{b, c\}\}$.

43. (b)  represents [a, b).

 represents (a, b].

 represents (a, b).

 represents [a, b].

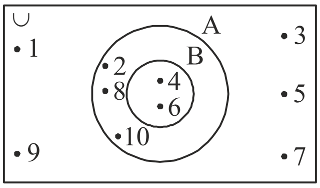
44. (b) The interval in the figure is [a, b].

45. (c) $n[P(\phi)] = 2^0 = 1$ $[\because n(\phi) = 0]$

$n[P(P(\phi))] = 2^1 = 2$

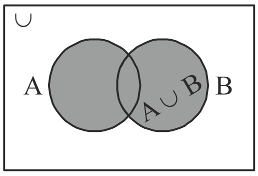
$n[P\{P(P(\phi))\}] = 2^2 = 4$.

46. **(d)** $U = \{1, 2, 3, 4, ..., 10\}$
$A = \{2, 4, 6, 8, 10\}$
$B = \{4, 6\}$
$\because$ All the elements of B are also in A.
$\therefore$ $B \subset A$
$\Rightarrow$ Set B lies inside A in the Venn diagram.



47. **(c)** Most of the relationships between sets can be represented by Venn diagrams.

48. **(c)** The union of two sets A and B can be represented by a Venn diagram as

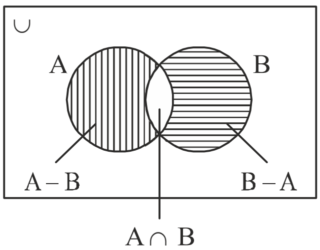


49. **(c)** Here, X = {Ram, Geeta, Akbar}
and Y = {Geeta, David, Ashok}
Then, $X \cap Y$ = {Geeta}

50. **(b)** Using the set-builder form, we can write the definition of difference as
$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

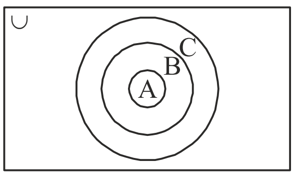
51. **(d)** The shaded region in the figure is $A - (B \cup C)$.

52. **(c)**



Clearly, $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

53. **(c)** If $A \subset B$ and $B \subset C$, then these sets is represented in Venn diagram as



Clearly, $A \cup B = B$
and $B \cap C = B$
Hence, $A \cup B = B \cap C$.

54. **(a)** In the given figure, the shaded portion represents complement of set A.

55. **(c)** $B \cup C = \{2, 3, 4, 6, 7, 8\}$
$(B \cup C)' = U - (B \cup C) = \{1, 5, 9, 10\}$
$C - A = \{4, 8\}$
$(C - A)' = \{1, 2, 3, 5, 6, 7, 9, 10\}$.

56. **(d)** $A \cap (A \cap B)^c = A \cap (A^c \cup B^c)$
$= (A \cap A^c) \cup (A \cap B^c) = \phi \cup (A \cap B^c) = A \cap B^c$.

57. **(a)** $A \cap B \subseteq A$. Hence, $A \cup (A \cap B) = A$.

58. **(b)** Given $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$.
Hence, $A = \{3, 5, 9\}$

59. **(c)** $A \cap (A \cup B)' = A \cap (A' \cap B')$,
$[\because (A \cup B)' = A' \cap B']$
$= (A \cap A') \cap B'$, [By associative law]
$= \phi \cap B'$, $[\because A \cap A' = \phi]$
$= \phi$.

60. **(a)** $A \cap (B - A) = \phi$ $[\because x \in B - A \Rightarrow x \notin A]$

61. **(b)** $A - B = \{1\}$ and $B - C = \{4\}$
$(A - B) \times (B - C) = \{(1, 4)\}$.

62. **(c)** Since $A \subseteq B$,
$\therefore$ $A \cup B = B$
So, $n(A \cup B) = n(B) = 6$.

63. **(a)** Minimum value of $x = 100 - (30 + 20 + 25 + 15)$
$= 100 - 90 = 10$.

64. **(b)** $A = \{4, 8, 12, 16, 20, 24, ...\}$
$B = \{6, 12, 18, 24, 30, ...\}$
$\therefore$ $A \subset B = \{12, 24, ...\} = \{x : x \text{ is a multiple of } 12\}$.

STATEMENT TYPE QUESTIONS

65. **(b)** As given, $P \equiv$ set of square, $Q \equiv$ set of parallelogram, $R \equiv$ set of quadrilaterals and $S \equiv$ set of rectangles.
Since all squares are parallelogram
$\Rightarrow$ $P \subset Q$
Since, all squares are rectangles, $\therefore$ $P \subset S$ and also all rectangles are quadrilateral, $\therefore$ $S \subset R$
$\Rightarrow$ 1, 3 and 4 are correct

66. **(d)** Both statements are incorrect.

67. **(b)**

68. **(d)** Let us consider the sets
$A = \{1, 2, 4\}$, $B = \{2, 5, 6\}$ and $C = \{1, 5, 7\}$
I. $A - B = \{1, 4\}$ and $A - (A \cap B)$
$= \{1, 2, 4\} - \{2\} = \{1, 4\}$
$\therefore$ $A - B = A - (A \cap B)$
II. $(A \cap B) \cup (A - B)$
$= \{2\} \cup \{1, 4\} = \{1, 2, 4\} = A$

III. $A - (B \cup C) = \{1, 2, 4\} - \{1, 2, 5, 6, 7\} = \{4\}$ and $(A - B) \cup (A - C) = \{1, 4\} \cup \{2, 4\} = \{1, 2, 4\}$
$\therefore$ $A - (B \cup C) \neq (A - B) \cup (A - C)$.

69. **(c)** In (i) and (iii), we can definitely decide whether a given particular object belongs to a given collection or not. For example, we can say that the river Nile does not belong to the collection of rivers of India. On the other hand, the river Ganga belongs to this collection.
Again, the collection of most talented batsmen of India and the collection of books is not well-defined, because the criterion for determining most talented batsman and collection of particular kind of books may vary from person-to-person.

70. **(c)** While writing the set in roster form, an element is not generally repeated, i.e. all elements are taken as distinct. The set of letters forming the word 'SCHOOL' is {S, C, H, O, L} or {H, O, L, C, S}. Here, the order of listing elements has no relevance. We can also express it as {S, C, H, O, L}.

71. **(c)** The collection of all months of a year beginning with the letter J and the collection of all boys in your class are well-defined. But the collection of ten most talented writers of India and a team of eleven best cricket batsmen of the world may vary from person-to-person, so these are not well defined. Hence, I and IV represent the sets.

72. **(b)** We can write $60 = 2 \times 2 \times 3 \times 5$
$\therefore$ Prime factors of 60 are 2, 3 and 5.
Hence, the set D in roster form is {2, 3, 5}.
There are 12 letters in the word 'TRIGONOMETRY' out of which three letters T, R and O are repeated. Hence, set E in the roster form is
{T, R, I, G, O, N, M, E, Y}.

73. **(b)** The empty set is denoted by the symbol ϕ or { }.

74. **(b)** The set of real numbers which satisfy $x^2 - 1 = 0$ is $\{-1, 1\}$.
So, Statement I is false.
Given, $x^2 = 16$ and $2x = 6$
$x = 4, -4$ and $x = 3$
$\therefore$ There is no real x which simultaneously satisfied $x^2 = 16$ and $2x = 6$.
So, Statement II is true.

75. **(c)** We do not know the number of animals living on the Earth, but it is some natural number. So, the set of animals living on the Earth is finite. There are infinite circles passing through the origin (0, 0). So, the set of circles passing through the origin (0, 0) is infinite.

76. **(c)** There are infinite positive integer greater than 100. So, the set of positive integers greater than 100 is infinite.
There are 25 prime numbers less than 99.
So, the set of prime numbers less than 99 is finite.

77. **(a)** There are infinite lines parallel to X-axis. So, the set of lines parallel to X-axis is infinite.
There are infinite numbers which are multiple of 5. So, the set of numbers, which are multiple of 5, is infinite.
There are 26 letters in the English alphabet. So, the set of letters in the English alphabet is finite.

78. **(c)** Since, $0 \in A$ and 0 does not belong to any of the sets B, C, D and E, it follows that $A \neq B$, $A \neq C$, $A \neq D$, $A \neq E$.
Since, $B = \phi$, but none of the other sets are empty. Therefore $B \neq C$, $B \neq D$ and $B \neq E$. Also, $C = \{5\}$ but $-5 \in D$, hence $C \neq D$.
Since, $E = \{5\}$, $C = E$. Further, $D = \{-5, 5\}$ and $E = \{5\}$, we find that $D \neq E$. Thus, the only one pair of equal sets is C and E.

79. **(c)** There are infinite concentric circles in a plane. So, the given set is infinite.
Now, $x^2 - 3 = 0$
or $x^2 = 3$
or $x = \pm\sqrt{3}$
Thus, there is no rational number satisfied $x^2 - 3 = 0$. So, given set is null set.

80. **(c)** From the definition of subset, it follows that every set is a subset of itself. Since, the empty set ϕ has no element, we agree to say that ϕ is a subset of every set.

81. **(c)** $A = \{1, 3, 5\}$
$B = \{x : x \text{ is an odd natural number less than } 6\}$
$= \{1, 3, 5\}$
Since, every element of A is in B, so $A \subset B$.
Every element of B is in A, so $B \subset A$.
Then, $A = B$.

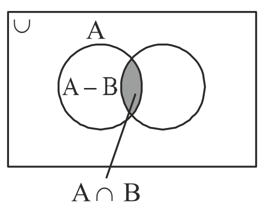
82. **(b)** The universal set must contain the elements 0, 1, 2, 3, 4, 5, 6 and 8.

83. **(b)** From all the three sets, set of all triangles in a plane is the universal set for set of isosceles triangle.

84. **(a)** Let A and B be two sets. Symbolically, the union of A and B write as $A \cup B$ and the common elements of A and B being taken only once.

85. **(b)** $A = \{a, b\}$, $B = \{a, b, c\}$
Since, all the elements of A are in B.
So, $A \subset B$.
Hence, Statement I is false.
$\because$ $A \subset B$
$\Rightarrow$ $A \cup B = B$
Therefore, Statement II is true.

86. **(a)** I.

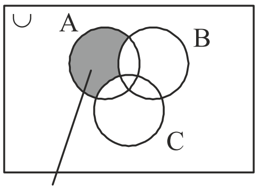


It is clear from the Venn diagram

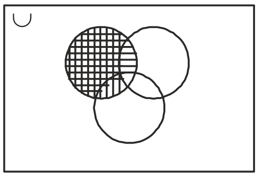
$A - B = A - (A \cap B)$

II. Also, it is clear from above diagram

$A = (A \cap B) \cup (A - B)$

III. 

$A - (B \cup C)$



≡ (horizontal) lines : A – C

||| (vertical) lines : A – B

It is clear from the diagrams

$A - (B \cup C) = (A - B) \cap (A - C)$

87. (c) If A is a subset of the universal set U, then its complement A' is also a subset of U.

We have, $A' = \{2, 4, 6, 8, 10\}$

Hence, $(A')' = \{x : x \in U \text{ and } x \notin A'\}$

$= \{1, 3, 5, 7, 9\} = A$

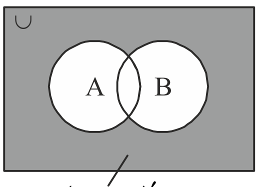
It is clear from the definition of the complement that for any subset of the universal set U, we have

$$(A')' = A$$

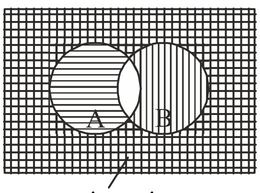
88. (b) Let U be the uni versal set and A is a subset of U. Then, the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A' to denote the complement of A with respect to U. Thus,

$A' = \{x : x \in U \text{ and } x \notin A\}$

Obviously, $A' = U - A$

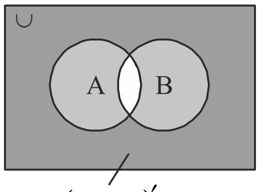
89. (c) 

$(A \cup B)'$

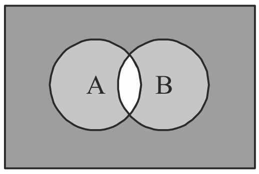


$A' \cap B'$

Clearly, $(A \cup B)'$ and $A' \cap B'$ are same.



$(A \cap B)'$



The shaded region is $A' \cup B'$

Clearly, $(A \cap B)'$ and $A' \cup B'$ are same.

90. (a) If A, B and C are finite sets, then

$n(A \cup B \cup C) = n(A) + n(B \cup C) - n[A \cap (B \cup C)]$

$[\because n(A \cup B) = n(A) + n(B) - n(A \cap B)]$

$= n(A) + n(B) + n(C) - n(B \cap C) - n[A \cap (B \cup C)]$ ... (i)

Since, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, we get

$n[A \cap (B \cup C)] = n(A \cap B) + n(A \cap C) - n[(A \cap B) \cap (A \cap C)]$

$= n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)$

Therefore, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Now, if A, B and C are mutually pairwise disjoint, then

$A \cap B = \phi = B \cap C = A \cap C = A \cap B \cap C$

$\therefore$ $n(A \cup B \cup C) = n(A) + n(B) + n(C)$.

91. (b) Let U denote the set of surveyed students and X denote the set of students taking apple juice and Y denote the set of students taking orange juice. Then,

$n(U) = 400$, $n(X) = 100$, $n(Y) = 150$

and $n(X \cap Y) = 75$

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$= 100 + 150 - 75$

$= 175$

$\therefore$ 175 students were taking at least one juice.

$n(X' \cap Y') = n(X \cup Y)'$

$= n(U) - n(X \cup Y)$

$= 400 - 175$

$= 225$

Hence, 225 students were taking neither apple juice nor orange juice.

92. (a) Let $X \in P(A \cap B)$ …(i)

$\Leftrightarrow$ $x \subset A$ and $x \subset B$

$\Leftrightarrow$ $x \in P(A)$ and $x \in P(B)$

$\Leftrightarrow$ $x \in [P(A) \cap P(B)]$ …(ii)

Hence, from (i) and (ii)

$P(A) \cap P(B) = P(A \cap B)$

Now, $P(A) \cup P(B) \neq P(A \cup B)$, we can prove it by an example.

93. (c) Let $A = \{1, 2, 3, \ldots, n\}$

No. of subsets of $A = 2^n$

$\therefore$ $2^n = 128 \Rightarrow 2^n = 2^7 \Rightarrow n = 7$

$\therefore$ Number of elements in set A = 7

94. (d) Let X = {a, b, c, d}

n(X) = 4

No. of subsets of X = $2^4 = 16$

No. of non-empty subsets of A = 16 – 1 = 15

($\because$ Only one set is empty set)

95. (c) I. $A \cup B = \{1, 2, 3, 4, 5, 6\}$

$(A \cup B) \cap C = \{3, 4, 6\}$

II. De-Morgan's law.

96. (a) Only I and II statements are incorrect.

I. $A - B = \{3, 6, 9, 15, 18, 21\}$

II. $C - B = \{2, 6, 10, 14, 20\}$

$D - B = \{5, 10, 15\}$

$(C - B) \cap (D - B) = \{10\}$

97. (c) Both the statements are true.

II. $n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$= 720 + 450 - n(S \cap T)$

$= 1170 - n(S \cap T)$

$1170 - n(S \cap T) \le n(U)$

$1170 - n(S \cap T) \le 1000$

$\Rightarrow n(S \cap T) \ge 170$

98. (a) Only statement-I is true.

I. Consider $A = B \cap C$

$= (C \cap A) \cap C \Rightarrow A = C \cap A \Rightarrow A = B$

II. A = {a, b}

P(A) = {ϕ, {a}, {b}, {a, b}}

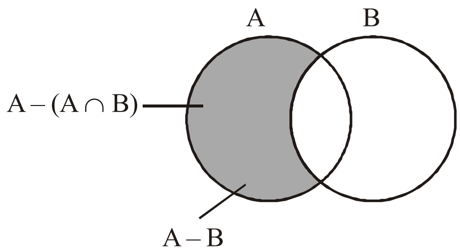
$A \cap P(A) = \phi$

99. (b) I and II are the correct statements.

$A - B = A - (A \cap B)$ is correct.

$A = (A \cap B) \cup (A - B)$ is correct.

Statement-III is false.



100. (c) I. $\bigcup_{n=2}^{10} A_n$ is the set of first 10 prime numbers

$= \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

II. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$50 = 28 + 32 - n(A \cap B)$

$n(A \cap B) = 60 - 50 = 10$

101. (c) By definition of union and intersection of two sets, both the statements are true.

MATCHING TYPE QUESTIONS

102. (b) If A is a subset of B, we write $A \subset B$ and if A is not a subset of B, then we write $A \not\subset B$.

In other words, $A \subset B$ if $a \in A$, then $a \in B$.

Now, if $A \subset B \Rightarrow$ Every element of A is in B and $B \subset A \Rightarrow$ Every element of B is in A, then we can say A and B are the same set, so that we have $A \subset B$ and $B \subset A \Leftrightarrow A = B$,

where '$\Leftrightarrow$' is a symbol for two way implications and usually read as if and only if (briefly written as "iff").

103. (d) The open interval $a < x < b$ is represented by (a, b) or]a, b[. The interval $a \le x \le b$ contain end points also is called closed interval and is denoted by [a, b]. The interval $a \le x < b$ closed at the end a and open at the end b, i.e. [a, b). Similarly, the interval $a < x \le b$ is represented by (a, b].

104. (c) Some properties of operation of intersection are as follows:

A. $A \cap B = B \cap A$ [commutative law]

B. $(A \cap B) \cap C = A \cap (B \cap C)$ [associative law]

C. $\phi \cap A = \phi$ [law of ϕ]

D. $U \cap A = A$ [law of U]

E. $A \cap A = A$ [idempotent law]

F. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [distributive law]

105. (b) By properties of complement of a set,

A. $A \cup A' = U$

B. $A \cap A' = \phi$

By De-Morgan's laws,

C. $(A \cup B)' = A' \cap B'$

D. $(A \cap B)' = A' \cup B'$

By laws of empty set and universal set,

E. $\phi' = U$ and

F. $U' = \phi$

By law of double complementation,

G $(A')' = A$.

106. (d)

107. (d)

(A) Let $x \in A$, then $x \in A \cup B$

$\Rightarrow x \in A \cap B$ ($\because A \cup B = A \cap B$)

$\Rightarrow x \in B$

$\therefore A \subset B$ …(i)

Similarly, if $y \in B$, then $y \in A \cap B$

$\Rightarrow y \in A$

$\therefore B \subset A$ …(ii)

From (i) & (ii), A = B

(C) Let $a \in A$, then there exists $x \in P(A)$ such that $a \in X$.

$\Rightarrow x \in P(B)$ $(\because P(A) = P(B))$

$\Rightarrow a \in B$

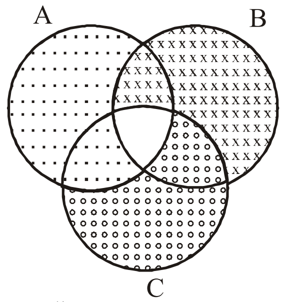
$\Rightarrow A \subset B$ ...(i)

Similarly, we can prove $B \subset A$ ...(ii)

from (i) and (ii), we have $A = B$

(D) $A \cup (B - A) = A \cup (B \cap A') = A \cup B$

(E)



From Venn - diagram

$(A - B) \cup (B - C) \cup (C - A) = (A \cap B \cap C)$.

108. (b) **109. (c)**

INTEGER TYPE QUESTIONS

110. (a) Since, $a + 2 = 6 \Rightarrow a = 4$

$\therefore$ the given set is $\{4\}$.

111. (d) An empty set does not contain any element.

112. (d) Number of elements in $X = 5$

113. (c) $n(X) = 3$

Number of proper subset $= 2^{n(x)} - 1$

$= 2^3 - 1 = 8 - 1 = 7$

114. (c) Total number of subset of given set $\{1, 2, 3, 4\} = 2^4 = 16$

Since, ϕ is the subset of every set.

$\therefore$ Number of non-empty subsets $= 16 - 1 = 15 = 3 \times 5$

115. (c) $n(A) = 0$

$n[P(A)] = 2^0 = 1$

116. (d) A is the set of points on circle.

B is the set of points on ellipse. These two intersects at four points.

$\therefore$ $A \cap B$ contains four points.

117. (d) $P(\phi)$ is the power set of the set ϕ.

$\therefore$ Cardinality $= P\{P[P(\phi)]\} = 4$

118. (b) $n[(A \cap B)' \cap A] = n[(A' \cup B') \cap A]$

$= n[(A' \cap A) \cup (B' \cap A)]$ (Distributive Law)

$= n[\phi \cup (B' \cap A)] = n(A \cap B') = n(A) - n(A \cap B)$

119. (b) Let M = set of Mathematics teachers

P = set of Physics teachers

n(only Maths teacher) $= n(M) - n(M \cap P) = 12 - 4 = 8$

Also, $n(M \cup P) =$ n (only Math teachers)

+ n(Only Physics teachers) + $n(M \cap P)$

$20 = 8 + 4 +$ n (only Physics teachers)

$\Rightarrow n = 8$.

ASSERTION-REASON TYPE QUESTIONS

120. (a) $A = \{a, b, c, d\}$

$\therefore$ $n(A) = 4$

$\therefore$ Number of subsets of $A = 2^4 = 16$, out of which only one set is empty set because empty set is subset of every set.

$\therefore$ Number of non-empty subsets of $A = 2^4 - 1 = 15$.

121. (c) If U is a universal set, then $B = U - A = A'$, for which $n(B) = n(A') = n(U) - n(A)$.

But for any three arbitrary sets A, B and C, we cannot always have $n(C) = n(A) - n(B)$, if $C = A - B$ as it is not specified here whether A is universal set or not. In case if A is not universal set, then we cannot conclude.

$n(C) = n(A) - n(B)$.

Hence, Assertion is true but Reason is false.

122. (d) As $A = \{1, \{2, 3\}\}$

$\therefore$ Subsets of $A = \phi, \{1\}, \{\{2, 3\}\}, \{1, \{2, 3\}\}$

Now, $\{\{2, 3\}\} \subset A$

$\therefore$ $\{\{2, 3\}\} \in P(A)$

$\therefore$ Assertion is false but Reason is obviously true.

123. (b) $\{1\}$ and $\{2\}$ are the element of $\{1, \{2\}\}$.

So, the subsets of the set $\{1, \{2\}\}$ are $\phi, \{1\}, \{\{2\}\}$ and $\{1, \{2\}\}$.

Hence, Assertion is true.

We know, total number of proper subsets of a set containing n elements is $2^n - 1$.

Hence, Reason is true. But Reason is not the correct explanation of Assertion.

124. (b) Let $x \in A - B$

$\Rightarrow$ $x \in A$ and $x \notin B$

$\Rightarrow$ $x \in A$ and $x \in B'$

$\Rightarrow$ $x \in B'$

$\therefore$ $A - B \subset B'$

It is true $A \cap A' = \phi$ [by complement laws]

Hence, both Assertion and Reason are correct but Reason is not a correct explanation of Assertion.

CRITICAL THINKING TYPE QUESTIONS

125. (b) In the given Venn diagram, shaded area between sets P an Q is $(P \cap Q) - R$ and shaded area between P and R is $(P \cap R) - Q$. So, both the shaded area is union of these two area and is represented by

$((P \cap Q) - R) \cup ((P \cap R) - Q)$.

126. (d) The shaded region represents $(P \cap Q) \cup (P \cap R)$.

127. (b) Given : Two finite sets have m and n elements

$\therefore$ $2^m - 2^n = 56$

$\Rightarrow \quad 2^m - 2^n = 64 - 8$

$\Rightarrow \quad 2^m - 2^n = 2^6 - 2^3$

$\Rightarrow \quad m = 6, n = 3$

128. (c) $A = \{1, 3, 5, 15\}, B = \{2, 3, 5, 7\}, C = \{2, 4, 6, 8\}$

$\therefore A \cup C = \{1, 2, 3, 4, 5, 6, 8, 15\}$

$(A \cup C) \cap B = \{2, 3, 5\}$

129. (a) As given :

S = the set of all triangles

P = the set of all isosceles triangles

Q = the set of all equilateral triangles

R = the set of all right angled triangles

$\therefore$ $P \cap Q$ represents the set of isosceles triangles and $R - P$ represents the set of non-isosceles right angled triangles.

130. (d) Let A = {1}, B = {2, 3}, then

$A \cup B = \{1, 2, 3\}$ and $A \cap B = \phi$

Now, $P(A) = \{\phi, \{1\}\}$, $P(B) = \{\phi, \{2\}, \{3\}, \{2, 3\}\}$

$\therefore$ $P(A) \cup P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{2, 3\}\}$

$P(A \cup B) = \{\phi, \{1\}, \{2\}, \{3\}, \{2, 3\}, \{1, 2\}, \{3, 1\}, \{1, 2, 3\}\}$

and $P(A \cap B) = \{\phi\}$.

131. (c) {5} is a subset of A as $5 \in A$

But, {1, 2} is not a subset of A as elements $1, 2 \notin A$.

132. (b)

133. (a) Let B = {b}. Then, A = {a, B}.

$\therefore$ $P(A) = \{\phi, \{a\}, \{B\}, \{a, B\}\}$

$= \{\phi, \{a\}, \{\{b\}\}, \{a, \{b\}\}\}$.

134. (b) $(A - B) \cup (B - A) \cup (A \cap B)$

= only A $\cup$ only B $\cup$ Both A and B

$= A \cup B$.

135. (c) Let U be the set of all consumers who were questioned, A be the set of consumers who liked product P_1 and B be the set of consumers who liked product P_2.

It is given that n(U) = 2000, n(A) = 1720, n(B) = 1450,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 1720 + 1450 - n(A \cap B)$

$= 3170 - n(A \cap B)$

Since, $A \cup B \subseteq U$

$\therefore$ $n(A \cup B) \le n(U)$

$\Rightarrow$ $3170 - n(A \cap B) \le 2000$

$\Rightarrow$ $3170 - 2000 \le n(A \cap B)$

$\Rightarrow$ $n(A \cap B) \ge 1170$

Thus, the least value of $n(A \cap B)$ is 1170.

Hence, the least number of consumers who liked both the products is 1170.

136. (b) n(A) = 40% of 10000 = 4000, n(B) = 2000,

$n(C) = 1000$, $n(A \cap B) = 500$, $n(B \cap C) = 300$,

$n(C \cap A) = 400$, $n(A \cap B \cap C) = 200$

$\therefore \quad n\left(A \cap \bar{B} \cap \bar{C}\right) = n\left\{A \cap (B \cup C)'\right\}$

$= n(A) - n\{A \cap (B \cup C)\}$

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

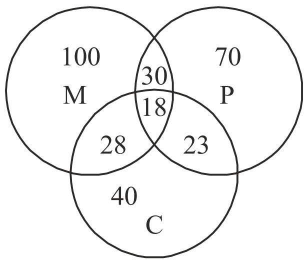
$= 4000 - 500 - 400 + 200 = 3300.$

137. (c) $n(M \cap P' \cap C')$

$= n(M) - [n(M \cap P) + n(M \cap C) - n(M \cap C \cap P)]$

$= 100 - 30 - 28 + 18 = 60$

[This can be solved directly by seeing the Venn Diagram]



138. (a) We have,

$3N = \{3x : x \in \mathbf{N}\} = \{3, 6, 9, 12, 15, 18, 21, 24.....\}$

$= \{x \in \mathbf{N} : x \text{ is a multiple of } 3\}$

and $7N = \{7x : x \in \mathbf{N}\} = \{7, 14, 21, 28.....\}$

$= \{x \in \mathbf{N} : x \text{ is a multiple of } 7\}$

$\therefore 3N \cap 7N = \{x \in \mathbf{N} : x \text{ is a multiple of } 3 \text{ and } 7\}$

$= \{x \in N : x \text{ is a multiple of } 21\} = \{21, 42, ...\}$

$= 21N$

139. (c) From the given we have in interval notation A = (0, 3) and B = [1, 5]

Clearly $A - B = (0, 1) = \{x \in \mathbf{R} : 0 < x < 1\}$

and $B - A = [3, 5] = \{x \in \mathbf{R} : 3 \le x \le 5\}$

$\therefore A \Delta B = (A - B) \cup (B - A) = (0, 1) \cup [3, 5]$

$= \{x \in \mathbf{R} : 0 < x < 1 \text{ or } 3 \le x \le 5\}$

140. (d) We have

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 10 + 15 + 20 - 8 - 9 - n(C \cap A) + n(A \cap B \cap C)$

$= 28 - \{n(C \cap A) - n(A \cap B \cap C)\}$ ...(i)

Since $n(C \cap A) \ge n(A \cap B \cap C)$

We have $n(C \cap A) - n(A \cap B \cap C) \ge 0$ ...(ii)

From (i) and (ii)

$n(A \cup B \cup C) \le 28$ ...(iii)

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 10 + 15 - 8 = 17$

and $n(B \cup C) = n(B) + n(C) - n(B \cap C)$

$= 15 + 20 - 9 = 26$

Since, $n(A \cup B \cup C) \ge n(A \cup C)$ and

$n(A \cup B \cup C) \geq n(B \cup C)$, we have

$n(A \cup B \cup C) \geq 17$ and $n(A \cup B \cup C) \geq 26$

Hence $n(A \cup B \cup C) \geq 26$ ...(iv)

From (iii) and (iv) we obtain

$26 \leq n(A \cup B \cup C) \leq 28$

Also $n(A \cup B \cup C)$ is a positive integer

$\therefore n(A \cup B \cup C) = 26$ or 27 or 28

141. (a) Let U be the set of consumers questioned X, the set of consumers who liked the product A and Y, the set of consumers who liked the product B. Then $n(U) = 1000$, $n(X) = 720$, $n(Y) = 450$

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y) = 1170 - n(X \cap Y)$

$\therefore n(X \cap Y) = 1170 - n(X \cup Y)$

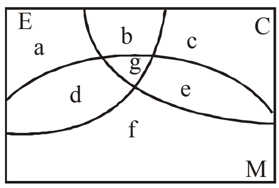
Clearly, $n(X \cap Y)$ is least when $n(X \cup Y)$ is maximum.

Now, $X \cup Y \subset U$

$\therefore n(X \cup Y) \leq n(U) = 1000$

$\therefore$ the maximum value of $n(X \cup Y)$ is 1000.

142. (b) C stands for set of students taking economics



$a + b + c + d + e + f + g = 40$; $a + b + d + g = 16$

$b + c + e + g = 22$; $d + e + f + g = 26$

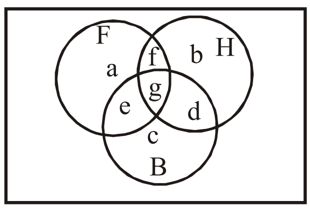
$b + g = 5$; $e + g = 14$; $g = 2$

Go by backward substitution

$e = 12, b = 3, d + f = 12, c + e = 17 \Rightarrow c = 5$; $a + d = 11$

$a + d + f = 18 \Rightarrow f = 7 \quad \therefore d = 12 - 7 = 5$

143. (a)



$a + e + f + g = 285$, $b + d + f + g = 195$

$c + d + e + f = 115$, $e + g = 45$, $f + g = 70$, $d + g = 50$

$a + b + c + d + e + f + g = 500 - 50 = 450$

As in previous question, we obtain

$a + f = 240$, $b + d = 125$, $c + e = 65$

$a + e = 215$, $b + f = 145$; $b + c + d = 165$

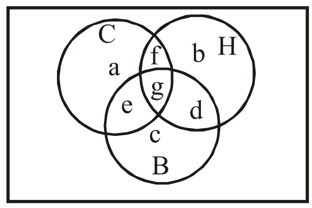
$a + c + e = 255$; $a + b + f = 335$

Solving we get

$b = 95, c = 40, a = 190, d = 30, e = 25, f = 50$ and $g = 20$

Desired quantity $= a + b + c = 325$

144. (d) $a + e + f + g = 224$



$b + d + f + g = 240$

$c + d + e + g = 336$

$d + g = 64$, $e + g = 80$

$f + g = 40$, $g = 24$

$\Rightarrow d = 40$

$e = 56, f = 16$

$a = 128, b = 160, c = 216$

$\therefore$ Boys who did not play any game

$= 800 - (a + b + c + d + e + f + g) = 160$

145. (b) Let $A = \{1\}$, $B = \{\{1\}, 2\}$ and $C = \{\{1\}, 2, 3\}$.

Here, $A \in B$ as $A = \{1\}$ and $B \subset C$ but $A \not\subset C$ as $1 \in A$ but $1 \notin C$.

146. (c) $V = \{a, e, i, o, u\}$

$V - B = \{e, o\}$

i.e., e and o are the elements belong to V but not to B

$B - V = \{k\}$

i.e., k is the element belongs to B but not to V.

$\therefore$ $B = \{a, i, u, k\}$

147. (b) Let M be the set of students passing in Mathematics, P be the set of students passing in Physics and C be the set of students passing in Chemistry.

Now, $n(M \cup P \cup C) = 50$, $n(M) = 37$, $n(P) = 24$, $n(C) = 43$

$n(M \cap P) \leq 19$, $n(M \cap C) \leq 29$, $n(P \cap C) \leq 20$ [given]

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C) \leq 50$

$\Rightarrow 37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C) \leq 50$

$\Rightarrow n(M \cap P \cap C) \leq 50 - 36$

$\Rightarrow n(M \cap P \cap C) \leq 14$

Thus, the largest possible number that could have passed all the three examinations, is 14.

CHAPTER

2

RELATIONS AND FUNCTIONS-I

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If $A \times B = \{(5,5), (5,6), (5,7), (8,6), (8,7), (8,5)\}$, then the value A is
(a) $\{5\}$ (b) $\{8\}$ (c) $\{5, 8\}$ (d) $\{5, 6, 7, 8\}$

2. Which of the following relation is a function ?
(a) $\{(a, b)(b, e)(c, e)(b, x)\}$
(b) $\{(a, d)(a, m)(b, e)(a, b)\}$
(c) $\{(a, d)(b, e)(c, d)(e, x)\}$
(d) $\{(a, d)(b, m)(b, y)(d, x)\}$

3. The relation R defined on the set of natural numbers as $\{(a, b) : \text{a differs from b by 3}\}$ is given
(a) $\{(1,4), (2,5), (3,6), ...\}$ (b) $\{(4,1), (5,2), (6,3), ...\}$
(c) $\{(1,3), (2,6), (3,9), ...\}$ (d) None of these

4. If $f(x) = \dfrac{x}{x-1}$, then $\dfrac{f(a)}{f(a+1)}$ is equal to:
(a) $f(a^2)$ (b) $f\left(\frac{1}{a}\right)$ (c) $f(-a)$ (d) $f\left(\frac{-a}{a-1}\right)$

5. If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function described by the formula, $g(x) = \alpha x + \beta$ then what values should be assigned to α and β?
(a) $\alpha = 1, \beta = 1$ (b) $\alpha = 2, \beta = -1$
(c) $\alpha = 1, \beta = -2$ (d) $\alpha = -2, \beta = -1$

6. If $f : R \to R$ is defined by $f(x) = 3x + |x|$, then $f(2x) - f(-x) - 6x =$
(a) $f(x)$ (b) $2f(x)$
(c) $-f(x)$ (d) $f(-x)$

7. If $f(x) = x^3 - \dfrac{1}{x^3}$, then $f(x) + f\left(\dfrac{1}{x}\right)$ is equal to
(a) $2x^3$ (b) $2\dfrac{1}{x^3}$
(c) 0 (d) 1

8. The domain of $f(x) = \dfrac{1}{\sqrt{2x-1}} - \sqrt{1-x^2}$ is:
(a) $\left]\dfrac{1}{2}, 1\right]$ (b) $[-1, \infty[$
(c) $[1, \infty[$ (d) None of these

9. If $f(x+1) = x^2 - 3x + 2$, then $f(x)$ is equal to:
(a) $x^2 - 5x - 6$ (b) $x^2 + 5x - 6$
(c) $x^2 + 5x + 6$ (d) $x^2 - 5x + 6$

10. If $f(x) = \dfrac{1-x}{1+x}$, then $f\left(\dfrac{1-x}{1+x}\right)$ is equal to:
(a) x (b) $\dfrac{1-x}{1+x}$
(c) $\dfrac{1+x}{1-x}$ (d) $1/x$

11. The Cartesian product of two sets P and Q, i.e., $P \times Q = \phi$, if
(a) either P or Q is the null set
(b) neither P nor Q is the null set
(c) Both (a) and (b)
(d) None of the above

12. A relation is represented by
(a) Roster method (b) Set-builder method
(c) Both (a) and (b) (d) None of these

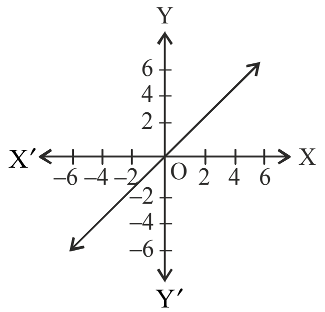
13. Let A = {x, y, z} and B = {a, b, c, d}. Then, which one of the following is not a relation from A to B?
(a) {(x, a), (x, c)} (b) {(y, c), (y, d)}
(c) {(z, a), (z, d)} (d) {(z, b), (y, b), (a, d)}

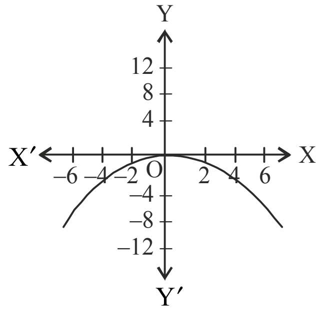
14. Let R be the relation on Z defined by $R = \{(a, b) : a, b \in Z, a - b \text{ is an integer}\}$. Then
(a) domain of R is {2, 3, 4, 5, ...}
(b) range of R is Z
(c) Both (a) and (b)
(d) None of the above

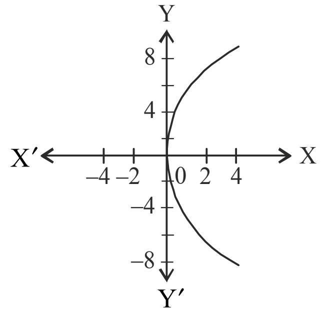
15. There are three relations R_1, R_2 and R_3 such that
$R_1 = \{(2, 1), (3, 1), (4, 2)\}$,
$R_2 = \{(2, 2), (2, 4), (3, 3), (4, 4)\}$ and
$R_3 = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$
Then,
(a) R_1 and R_2 are functions
(b) R_2 and R_3 are functions
(c) R_1 and R_3 are functions
(d) Only R_1 is a function

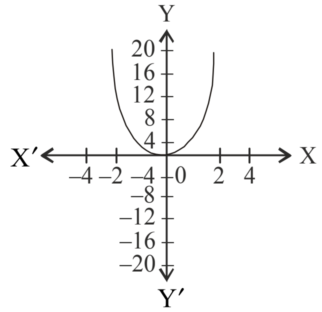
16. Let N be the set of natural numbers and the relation R be defined such that $\{R = (x, y) : y = 2x, x, y \in N\}$. Then,
(a) R is a function
(b) R is not a function
(c) domain, range and co-domain is N
(d) None of the above

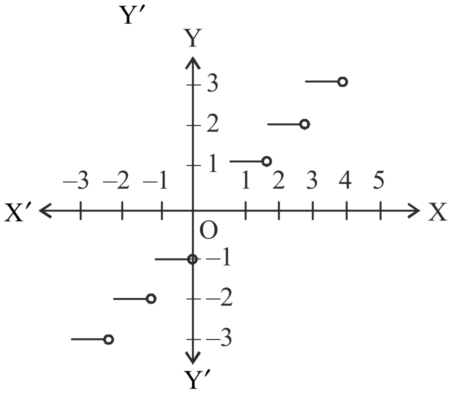
17. The graph of the function $f : R \to R$ and $f(x) = x^2, x \in R$, is

(a) 

(b) 

(c) 

(d) 

18. 

$f(x) = [x]$

Which of the following options identify the above graph?
(a) Modulus function
(b) Greatest integer function
(c) Signum function
(d) None of these

19. The domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ is equal, i.e. $f(x) = g(x)$, is
(a) $\{0, 2\}$ (b) $\left\{\frac{1}{2}, -2\right\}$
(c) $\left\{-\frac{1}{2}, 2\right\}$ (d) $\left\{\frac{1}{2}, 2\right\}$

20. If $g(x) = 1 + \sqrt{x}$ and $f[g(x)] = 3 + 2\sqrt{x} + x$, then $f(x) =$
(a) $1 + 2x^2$ (b) $2 + x^2$
(c) $1 + x$ (d) $2 + x$

21. Let $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$. Then,
(a) f is a relation from A to B
(b) f is a function from A to B
(c) Both (a) and (b)
(d) None of these

22. If $A = \{2, 3, 4, 5\}$ and $B = \{3, 6, 7, 10\}$. R is a relation defined by $R = \{(a, b) : a$ is relatively prime to b, $a \in A$ and $b \in B\}$, then domain of R is
(a) $\{2, 3, 5\}$ (b) $\{3, 5\}$
(c) $\{2, 3, 4\}$ (d) $\{2, 3, 4, 5\}$

23. The domain of relation
$R = \{(x, y) : x^2 + y^2 = 16, x, y \in Z\}$ is
(a) $\{0, 1, 2, 3, 4\}$
(b) $\{-4, -3, -2, -1\}$
(c) $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$
(d) None of the above

24. If $A = \{1, 2\}$, $B = \{1, 3\}$, then $(A \times B) \cup (B \times A)$ is equal to
(a) $\{(1, 3), (2, 3), (3, 1), (3, 2), (1, 1), (2, 1), (1, 2)\}$
(b) $\{(1, 3), (3, 1), (3, 2), (2, 3)\}$
(c) $\{(1, 3), (2, 3), (3, 1), (3, 2), (1, 1)\}$
(d) None of these

25. If $A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$, $C = \{2, 5\}$, then $(A - C) \times (B - C)$ is equal to
(a) $\{(1, 4)\}$ (b) $\{(1, 4), (4, 4)\}$
(c) $\{(4, 1), (4, 4)\}$ (d) $\{(1, 2), (2, 5)\}$

26. Let set $X = \{a, b, c\}$ and $Y = \phi$. The number of ordered pairs in $X \times Y$ are
(a) 0 (b) 1
(c) 2 (d) 3

27. Let $A = \{x, y, z\}$ and $B = \{a, b, c, d\}$. Which one of the following is not a relation from A to B?
(a) $\{(x, a), (x, c)\}$ (b) $\{(y, c), (y, d)\}$
(c) $\{(z, a), (z, d)\}$ (d) $\{(z, b), (y, b), (a, d)\}$

28. A relation R is defined in the set Z of integers as follows $(x, y) \in R$ iff $x^2 + y^2 = 9$. Which of the following is false?
(a) $R = \{(0, 3), (0, -3), (3, 0), (-3, 0)\}$
(b) Domain of $R = \{-3, 0, 3\}$
(c) Range of $R = \{-3, 0, 3\}$
(d) None of these

29. The domain and range of the relation R given by $R = \{(x, y) : y = x + \frac{6}{x};$ where $x, y \in N$ and $x < 6\}$ is
(a) $\{1, 2, 3\}, \{7, 5\}$ (b) $\{1, 2\}, \{7, 5\}$
(c) $\{2, 3\}, \{5\}$ (d) None of these

30. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, then $f\left(\frac{1}{2}\right) \times g(14)$ is
(a) $\frac{1336}{5}$ (b) $\frac{1363}{4}$
(c) 1251 (d) 1608

31. Let $f(x) = 1 + x$, $g(x) = x^2 + x + 1$, then $(f + g)(x)$ at $x = 0$ is
(a) 2 (b) 5
(c) 6 (d) 9

32. If $\phi(x) = a^x$, then $[\phi(p)]^3$ is equal to
(a) $\phi(3p)$ (b) $3\phi(p)$
(c) $6\phi(p)$ (d) $2\phi(p)$

33. Domain of $\sqrt{a^2 - x^2}$, $(a > 0)$ is
(a) $(-a, a)$ (b) $[-a, a]$
(c) $[0, a]$ (d) $(-a, 0]$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

34. Consider the following statements :
I. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$
II. $A \times \phi = \phi$
III. In general, $A \times B \neq B \times A$
Which of the above statements are true ?
(a) Only I (b) Only II
(c) Only III (d) All of the above

35. Consider the following statements:
Statement-I: The Cartesian product of two non-empty sets P and Q is denoted as P × Q and $P \times Q = \{(p, q) : p \in P, q \in Q\}$.
Statement-II: If A = {red, blue} and B = {b, c, s}, then A × B = {(red, b), (red, c), (red, s), (blue, b), (blue, c), (blue, s)}.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

36. Which of the following is/are true?
I. If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (n, m)}.
II. If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y), such that $x \in A$ and $y \in B$.
III. If A = {1, 2} and B = {3, 4}, then $A \times (B \cap \phi) = \phi$.
(a) I and II are true (b) II and III are true
(c) I and III are true (d) All are true

37. Consider the following statements:
Statement-I: If R is a relation from A to B, then domain of R is the set A.
Statement-II: The set of all second elements in a relation R from a set A to a set B is called co-domain of R.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

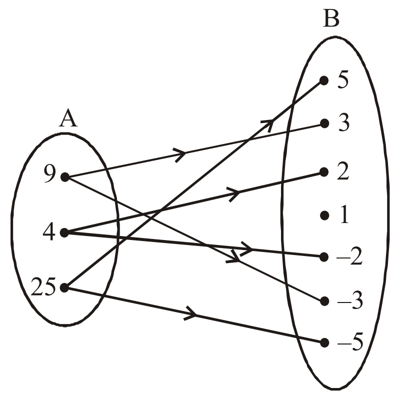
38. Let R be a relation from N to N defined by $R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$. Then, which of the following is/are true?
I. $(a, a) \in R$ for all $a \in N$.
II. $(a, b) \in R$ implies $(b, a) \in R$.
III. $(a, b) \in R$, $(b, c) \in R$ implies $(a, c) \in R$.
(a) I and II are true (b) II and III are true
(c) All are true (d) None of these

39. Consider the following statements:
Statement-I: The domain of the relation $R = \{(a, b) : a \in N, a < 5, b = 4\}$ is {1, 2, 3, 4}.
Statement-II: The range of the relation $S = \{(a, b) : b = |a - 1|, a \in Z \text{ and } |a| \leq 3\}$ is {1, 2, 3, 4}.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

40. Consider the following statements.
Let A = {1, 2, 3, 4} and B = {5, 7, 9}
I. A × B = B × A
II. n (A × B) = n (B × A)
Choose the correct option.
(a) Statement-I is true. (b) Statement-II is true.
(c) Both are true. (d) Both are false.

41. Consider the following statements.
I. Let A and B are non-empty sets such that $A \subseteq B$. Then, $A \times C \subseteq B \times C$.
II. For any two sets A and B, A × B = B × A
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

42. The figure given below shows a relation R between the sets A and B.



Then which of the following is correct?
I. The relation R in set-builder form is {(x,y) : x is the square of y, $x \in A$, $y \in B$}
II. The domain of the relation R is {4, 9, 25}
III. The range of the relation R is {–5, –3, –2, 2, 3, 5}
(a) Only I and II are true. (b) Only II and III are true.
(c) I, II and III are true (d) Neither I, II nor III are true.

43. Consider the following statements.
I. If (a, 1), (b, 2) and (c, 1) are in A × B and n(A) = 3, n (B) = 2, then A = {a, b, c} and B = {1, 2}
II. If A = {1, 2} and B = {3, 4}, then $A \times (B \cap \phi)$ is equal to A × B.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Neither I nor II is true

44. Consider the following statements.
I. Relation R = {(2, 0), (4, 8), (2, 1), (3, 6)} is not a function.
II. If first element of each ordered pair is different with other, then the given relation is a function.

Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Neither I nor II is true.

45. Consider the following statements.
I. If the set A has 3 elements and set B = {3, 4, 5}, then the number of elements in $A \times B = 9$.
II. The domain of the relation R defined by $R = \{(x, x+5) : x \in (0, 1, 2, 3, 4, 5)\}$ is {5, 6, 7, 8, 9, 10}.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Both I and II are false.

46. Consider the following statements.
I. If X = {p, q, r, s} and Y = {1,2, 3, 4, 5}, then {(p, 1), (q, 1), (r, 3), (s, 4)} is a function.
II. Let A = {1, 2, 3, 4, 6}. If R is the relation on A defined by $\{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$.
The relation R in Roster form is {(6, 3), (6, 2), (4, 2)}
Choose the correct option.
(a) Only I is false. (b) Only II is false.
(c) Both I and II are false. (d) Neither I nor II is false.

47. Consider the following statements.
I. Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a linear function from Z to Z. Then, f(x) is 2x – 1.
II. If $f(x) = x^3 - \frac{1}{x^3}$, then $f(x) + f\left(\frac{1}{x}\right)$ is equal to 0.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

48. Consider the following statements.
I. The relation $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$ in Roster form is {(3, 27), (5, 125), (7, 343)}
II. The range of the relation $R = \{(x+2, x+4) : x \in N, x < 8\}$ is {1, 2, 3, 4, 5, 6, 7}.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

49. Consider the following statements
I. Let n(A) = m and n(B) = n. Then the total number of non-empty relations that can be defined from A to B is $2^{mn} - 1$
II. If A = {1, 2, 3}, B = {3, 8}, then $(A \cup B) \times (A \cap B)$ is equal to {(1, 3), (2, 3), (3, 3), (8, 3)}.
III. If $\left(\frac{x}{2} - 1, \frac{y}{9} + 1\right) = (2, 1)$, then the values of x and y respectively are 6 and 0.
Choose the correct option.
(a) Only I and II are false.
(b) Only II and III are true.
(c) Only I and III are true.
(d) All the three statements are true

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

50. Let A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}. Then, match the following in column-I with the sets of ordered pairs in column-II.

Column -I	Column -II
A. $A \times (B \cap C)$	1. $\{(1, 4), (2, 4), (3, 4)\}$
B. $(A \times B) \cap (A \times C)$	2. $\{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$
C. $A \times (B \cup C)$	3. $\{(3, 4)\}$
D. $(A \times B) \cup (A \times C)$	
E. $(A \cap B) \times (B \cap C)$	

Codes:

	A	B	C	D	E
(a)	1	2	1	2	3
(b)	2	2	1	1	3
(c)	1	1	2	2	3
(d)	2	1	2	3	2

51. Let A = {1, 2, 3, 4, ..., 14}. A relation R from A to A is defined by $R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in A\}$.

Column- I	Column-II
A. In Roster form, the relation R is	1. {1, 2, 3, 4}
B. The domain of R is	2. {3, 6, 9, 12}
C. The range of R is	3. {1, 2, 3, 4, ..., 14}
D. The co-domain of R is	4. {(1, 3), (2, 6), (3, 9), (4, 12)}

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	4	1	2	3
(c)	4	2	1	3
(d)	4	3	1	2

52. Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Then, match the functions given in column-I with the expressions in column-II.

Column- I	Column -II
A. $(f + g)(x)$	1. $x^2 - 2x - 1$
B. $(f - g)(x)$	2. $x^2 + 2x + 1$
C. $(fg)(x)$	3. $\frac{x^2}{2x+1}, x \neq -\frac{1}{2}$
D. $\left(\frac{f}{g}\right)(x)$	4. $2x^3 + x^2$

Codes:

	A	B	C	D
(a)	2	1	4	3
(b)	4	1	2	3
(c)	2	1	3	4
(d)	2	4	1	3

53. Let $f(x) = 2x + 5$ and $g(x) = x^2 + x$. Then, match the functions given in column-I with the expressions in column-II.

Column-I	Column-II
A. $(f+g)x$	1. $2x^3 + 7x^2 + 5x$
B. $(f-g)x$	2. $x^2 + 3x + 5$
C. $(fg)(x)$	3. $\frac{2x+5}{x^2+x}$, $x \neq 0, -1$
D. $\left(\frac{f}{g}\right)(x)$	4. $5 + x - x^2$

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	4	1	2	3
(c)	2	1	4	3
(d)	1	4	2	3

54.

Column - I	Column - II
(A) The range of the function f(x) = x, x is a real number, is	1. $[0, \infty)$
(B) The domain of the real function $f(x) = \frac{1}{\sqrt{4-x^2}}$ is	2. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(C) The range of the function $f(x) = \frac{x}{1+x^2}$ is	3. R (Real numbers)
(D) The range of the function defined by $f(x) = \sqrt{x-1}$ is	4. $(-2, 2)$

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	3	4	2	1
(c)	3	2	4	1
(d)	1	4	2	3

55.

Column - I	Column - II
(A) Every function is a	1. real function.
(B) If f is a function from A to B and (a, b) ∈ f, then b is called	2. linear function
(C) If the domain of a function is either R or a subset of R, then it is called a	3. relation
(D) The function f defined by f(x) = mx + c, where m and c are constants, x ∈ R is called	4. the image of 'a' under f.

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	3	4	1	2
(c)	3	1	4	2
(d)	2	1	4	3

56. If f is the identity function and g is the modulus function, then match the column-I with column-II.

Column - I	Column - II
(A) $(f+g)(x) =$	1. $\begin{cases} 0, & x \geq 0 \\ 2x, & x < 0 \end{cases}$
(B) $(f-g)(x) =$	2. $\begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$
(C) $(fg)(x) =$	3. $\begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$
(D) $\left(\frac{f}{g}\right)(x) =$	4. $\begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$

Codes:

(a)	1	4	3	2
(b)	4	1	2	3
(c)	4	1	3	2
(d)	2	1	3	4

57. If A = {3, 4} and B = {5, 6, 7}. Then match the column-I with the column-II.

Column-I	Column-II
(A) Number of relations from A to A is	1. 2^6
(B) Number of relations from B to B is	2. 2^9
(C) Number of relations from A to B is	3. 2^4

Codes:

	A	B	C
(a)	1	2	3
(b)	1	3	2
(c)	2	3	1
(d)	3	2	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

58. If $(4x + 3, y) = (3x + 5, -2)$, then the sum of the values of x and y is
(a) 0 (b) 2 (c) –2 (d) 1

59. If $(x + 3, 4 - y) = (1, 7)$, then the value of $4 + y$ is
(a) 3 (b) 4 (c) 5 (d) 1

60. The number of elements in the set $\{(x, y) : 2x^2 + 3y^2 = 35, x, y \in Z\}$, where Z is the set of all integers,
(a) 8 (b) 2 (c) 4 (d) 6

61. If the set A has 3 elements and the set B = {3, 4}, then the number of elements in A × B is
(a) 6 (b) 9 (c) 8 (d) 2

62. If n (X) = 5 and n (Y) = 7, then the number of relations on X × Y is 2^{5m}. The value of 'm' is
(a) 5 (b) 7
(c) 6 (d) 8

63. If $f(x) = 4x - x^2$, $x \in R$, then $f(b+1) - f(b-1)$ is equal to $m(2-b)$. The value of 'm' is
(a) 2 (b) 3
(c) 4 (d) 5

64. If $f(y) = 2y^2 + by + c$ and $f(0) = 3$ and $f(2) = 1$, then the value of $f(1)$ is
(a) 0 (b) 1
(c) 2 (d) 3

65. Let $X = \{1, 2, 3\}$. The total number of distinct relations that can be defined over X is 2^n. The value of 'n' is
(a) 9 (b) 6
(c) 8 (d) 2

66. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then the value of 'a' is
(a) 3 (b) 0
(c) 2 (d) 1

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

67. Let $A = \{1, 2, 3, 4, 6\}$. If R is the relation on A defined by $\{(a, b) : a, b \in A$, b is exactly divisible by a$\}$.
Assertion : The relation R in Roster form is $\{(6, 3), (6, 2), (4, 2)\}$.
Reason : The domain and range of R is $\{1, 2, 3, 4, 6\}$.

68. Assertion : If $(x + 1, y - 2) = (3, 1)$, then $x = 2$ and $y = 3$.
Reason : Two ordered pairs are equal, if their corresponding elements are equal.

69. Assertion : Let f and g be two real functions given by
$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\}$ and
$g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}$
Then, domain of $f \cdot g$ is given by $\{2, 3, 4, 5\}$.
Reason : Let f and g be two real functions. Then, $(f \cdot g)(x) = f\{g(x)\}$.

70. Assertion : If $f(x) = \frac{1}{x-2}$, $x \neq 2$ and $g(x) = (x-2)^2$, then
$(f+g)(x) = \frac{1+(x-2)^3}{x-2}$, $x \neq 2$.
Reason : If f and g are two functions, then their sum is defined by $(f + g)(x) = f(x) + g(x)\ \forall\ x \in D_1 \cap D_2$, where D_1 and D_2 are domains of f and g, respectively.

71. Assertion : If $A = \{x, y, z\}$ and $B = \{3, 4\}$, then number of relations from A to B is 2^5.
Reason : Number of relations from A to B is $2^{n(A) \times n(B)}$.

72. Let $A = \{a, b, c, d, e, f, g, h\}$ and $R = \{(a, a), (b, b), (a, g), (b, a), (b, g), (g, a), (g, b), (g, g), (b, b)\}$
Consider the following statements:
Assertion : $R \subset A \times A$.
Reason : R is not a relation on A.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

73. Let $n(A) = m$, and $n(B) = n$. Then the total number of non-empty relations that can be defined from A to B is
(a) m^n (b) $n^m - 1$
(c) $mn - 1$ (d) $2^{mn} - 1$

74. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is
(a) 2^9 (b) 9^2
(c) 3^2 (d) $2^9 - 1$

75. If $f(x) = \frac{2^x + 2^{-x}}{2}$, then $f(x + y).\ f(x - y) =$
(a) $\frac{1}{2}[f(2x) + f(2y)]$ (b) $\frac{1}{4}[f(2x) + f(2y)]$
(c) $\frac{1}{2}[f(2x) - f(2y)]$ (d) $\frac{1}{4}[f(2x) - f(2y)]$

76. $f(x) = \frac{x(x-p)}{q-p} + \frac{x(x-q)}{p-q}$, $p \neq q$. What is the value of $f(p) + f(q)$?
(a) $f(p - q)$ (b) $f(p + q)$
(c) $f(p(p + q))$ (d) $f(q(p - q))$

77. If $f(x) = x$ and $g(x) = |x|$, then $(f + g)(x)$ is equal to
(a) 0 for all $x \in R$ (b) 2x for all $x \in R$
(c) $\begin{cases} 2x, & \text{for } x \geq 0 \\ 0, & \text{for } x < 0 \end{cases}$ (d) $\begin{cases} 0, & \text{for } x \geq 0 \\ 2x, & \text{for } x < 0 \end{cases}$

78. Let $A = \{1, 2\}$, $B = \{3, 4\}$. Then, number of subsets of $A \times B$ is
(a) 4 (b) 8
(c) 18 (d) 16

79. If A, B and C are three sets, then
(a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(b) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$
(c) Both (a) and (b)
(d) None of the above

80. If $A = \{8, 9, 10\}$ and $B = \{1, 2, 3, 4, 5\}$, then the number of elements in $A \times A \times B$ are
(a) 15 (b) 30
(c) 45 (d) 75

81. If A, B and C are any three sets, then $A \times (B \cup C)$ is equal to
(a) $(A \times B) \cup (A \times C)$ (b) $(A \cup B) \times (A \cup C)$
(c) $(A \times B) \cap (A \times C)$ (d) None of these

82. If the set A has p elements, B has q elements, then the number of elements in $A \times B$ is
(a) $p + q$ (b) $p + q + 1$
(c) pq (d) p^2

83. If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, c\}$, then $(A - B) \times (B \cap C) =$
(a) $\{(a, c), (a, d)\}$ (b) $\{(a, b), (c, d)\}$
(c) $\{(c, a), (a, d)\}$ (d) $\{(a, c), (a, d), (b, d)\}$

84. If $A = \{a, b\}$, $B = \{c, d\}$, $C = \{d, e\}$, then $\{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$ is equal to
(a) $A \cap (B \cup C)$ (b) $A \cup (B \cap C)$
(c) $A \times (B \cup C)$ (d) $A \times (B \cap C)$

85. Suppose that the number of elements in set A is p, the number of elements in set B is q and the number of elements in $A \times B$ is 7. Then $p^2 + q^2 =$
(a) 42 (b) 49
(c) 50 (d) 51

86. The cartesian product of $A \times A$ has 9 elements, two of which are $(-1, 0)$ and $(0, 1)$, the remaining elements of $A \times A$ is given by
(a) $\{(-1, 1), (0, 0), (-1, -1), (1, -1), (0, -1)\}$
(b) $\{(-1, -1), (0, 0), (-1, 1), (1, -1), (1, 0), (1, 1), (0, -1)\}$
(c) $\{(1, 0), (0, -1), (0, 0), (-1, -1), (1, -1), (1, 1)\}$
(d) None of these

87. Let $A = \{1, 2, 3\}$. The total number of distinct relations that can be defined over A, is
(a) 2^9 (b) 6
(c) 8 (d) 2^6

88. The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(x, y) : |x^2 - y^2| < 16\}$ is given by
(a) $\{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$
(b) $\{(2, 2), (3, 2), (4, 2), (2, 4)\}$
(c) $\{(3, 3), (4, 3), (5, 4), (3, 4)\}$
(d) None of these

89. The relation R defined on set $A = \{x : |x| < 3, x \in I\}$ by $R = \{(x, y) : y = |x|\}$ is
(a) $\{(-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2)\}$
(b) $\{(-2, -2), (-2, 2), (-1, 1), (0, 0), (1, -2), (1, 2), (2, -1), (2, -2)\}$
(c) $\{(0, 0), (1, 1), (2, 2)\}$
(d) None of these

90. The domain of the function $f(x) = \dfrac{|x + 3|}{x + 3}$ is
(a) $\{-3\}$ (b) $R - \{-3\}$
(c) $R - \{3\}$ (d) R

91. Let $n(A) = 8$ and $n(B) = p$. Then, the total number of non-empty relations that can be defined from A to B is
(a) 8^p (b) $n^p - 1$
(c) $8p - 1$ (d) $2^{8p} - 1$

92. The domain of the real valued function
$f(x) = \sqrt{5 - 4x - x^2} + x^2 \log(x + 4)$ is
(a) $(-5, 1)$ (b) $-5 \le x$ and $x \ge 1$
(c) $(-4, 1]$ (d) ϕ

93. The domain of the function
$f(x) = \dfrac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is
(a) $(-1, 0) \cup (1, 2)$
(b) $(1, 2) \cup (2, \infty)$
(c) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
(d) $(1, 2)$

94. The domain of the function $f(x) = \dfrac{1}{\sqrt{9 - x^2}}$ is
(a) $-3 \le x \le 3$ (b) $-3 < x < 3$
(c) $-9 \le x \le 9$ (d) $-9 < x < 9$

95. The domain and range of the real function f defined by $f(x) = |x - 1|$ is
(a) $R, [0, \infty)$ (b) $R, (-\infty, 0)$
(c) R, R (d) $(-\infty, 0), R$

96. Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a linear function from Z into Z, then $f(x) =$
(a) $2x - 1$ (b) $2x$
(c) $2x + 1$ (d) $-2x + 1$

97. The domain of the function f defined by $f(x) = \dfrac{1}{\sqrt{x - |x|}}$ is
(a) R (b) R^+
(c) R^- (d) $\{\phi\}$

98. The domain of the function $f(x) = \dfrac{x^2 + 3x + 5}{x^2 - 5x + 4}$ is
(a) R (b) $R - \{1, 4\}$
(c) $R - \{1\}$ (d) $(1, 4)$

99. The domain and range of the function f given by $f(x) = 2 - |x - 5|$ is
(a) Domain = R^+, Range = $(-\infty, 1]$
(b) Domain = R, Range = $(-\infty, 2]$
(c) Domain = R, Range = $(-\infty, 2)$
(d) Domain = R^+, Range = $(-\infty, 2]$

100. If $P = \{a, b, c\}$ and $Q = \{r\}$, then
(a) $P \times Q = Q \times P$ (b) $P \times Q \ne Q \times P$
(c) $P \times Q \subset Q \times P$ (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (c) $\{5, 8\}$
2. (c) Since in (c) each element is associated with unique element. While in (a) element *b* is associated with two elements, in (b) element *a* is associated with three elements and in (d) element *b* is associated with two elements so relation given in option (c) is function.
3. (b) The set is $\{(a, b): a - b = 3, a, b \in N\}$

Here $a = b + 3$

For $b = 1, a = 4$

For $b = 2, a = 5$

For $b = 3, a = 6$.

and so, on

Hence the given set is

$\{(4, 1), (5, 2), (6, 3)...\}$

4. (a) Given $f(x) = \frac{x}{x-1}$

Then, $f(a) = \frac{a}{a-1}$

and $f(a+1) = \frac{a+1}{a}$

So, $\frac{f(a)}{f(a+1)} = \frac{a}{a-1} \div \frac{a}{a+1} = \frac{a^2}{a^2-1} = f(a^2)$

5. (b) (1, 1) satisfies $g(x) = \alpha x + \beta \therefore \alpha + \beta = 1$

(2, 3) satisfies $g(x) = \alpha x + \beta \therefore 2\alpha + \beta = 3$

Solving the two equation, we get $\alpha = 2, \beta = -1$

It can be checked that other ordered pairs satisfy $g(x) = 2x - 1$

6. (a) $f(x) = 3x + |x|$

$\therefore f(2x) - f(-x) - 6x$

$= 6x + |2x| - 3(-x) - |-x| - cx$

$= 3x + 2|x| - |x| \quad (\because |x| = |-x|)$

$= 3x + |x| = f(x)$

7. (c) Since $f(x) = x^3 - \frac{1}{x^3}$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}} = \frac{1}{x^3} - x^3$$

Hence,

$$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0$$

8. (a) Given, $f(x) = \frac{1}{\sqrt{2x-1}} - \sqrt{1-x^2} = p(x) - q(x)$

where $p(x) = \frac{1}{\sqrt{2x-1}}$ and $q(x) = \sqrt{1-x^2}$

Now, Domain of p(x) exist when $2x - 1 \neq 0$

$\Rightarrow \quad x = \frac{1}{2}$ and $2x - 1 > 0$

$\Rightarrow \quad x = \frac{1}{2}$ and $x > \frac{1}{2}$

$\therefore \quad x \in \left(\frac{1}{2}, \infty\right)$

and domain of q(x) exists when

$1 - x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow |x| \leq 1$

$\therefore \quad -1 \leq x \leq 1$

$\therefore$ Common domain is $\left]\frac{1}{2}, 1\right[$

9. (d) Given function is :

$f(x+1) = x^2 - 3x + 2$

This function is valid for all real values of x. So, putting $x - 1$ in place of x, we get

$f(x) = f(x - 1 + 1)$

$\Rightarrow \quad f(x) = (x-1)^2 - 3(x-1) + 2$

$\Rightarrow \quad f(x) = x^2 - 2x + 1 - 3x + 3 + 2$

$f(x) = x^2 - 5x + 6$

10. (a) Given function is :

$f(x) = \frac{1-x}{1+x}$

Putting $\frac{1-x}{1+x}$ in place of x,

$$\Rightarrow f\left(\frac{1-x}{1+x}\right) = \frac{1 - \left(\frac{1-x}{1+x}\right)}{1 + \left(\frac{1-x}{1+x}\right)} = \frac{1+x-1+x}{1+x+1-x} = \frac{2x}{2}$$

So, $f\left(\frac{1-x}{1+x}\right) = x$

11. **(a)** If either P or Q is the null set, then $P \times Q$ will be an empty set, i.e. $P \times Q = \phi$.

12. **(c)** A relation may be represented algebraically either by the Roster method or by the Set-builder method.

An arrow diagram is a visual representation of a relation.

13. **(d)** In option (d), $a \notin A$

$\therefore$ It it not a relation.

14. **(d)** The difference of two integers is also an integer.

$\therefore$ Domain of R = Z

Range of R = Z

15. **(c)** Since 2, 3, 4 are the elements of domain of R_1 having their unique images, this relation R_1 is a function.

Since, the same first element 2 corresponds to two different images 2 and 4, this relation R_2 is not a function.

Since, every element has one and only one image, this relation R_3 is a function.

16. **(a)** R = {(1, 2), (2, 4), (3, 6), (4, 8), ...}

Since, every natural number N has one and only one image, this relation R is a function.

The domain of R is the set of natural number, i.e. N. The co-domain is also N, and the range is the set of even natural numbers.

17. **(d)** (2,4) is an order pair of the function $f(x) = x^2, x \in R$

But point (2, 4) only lies on the graph given in option (d).

18. **(b)** **Greatest Integer Function:** The function $f : R \to R$ defined by $f(x) = [x], x \in R$ assumes the value of the greatest integer, less than or equal to x. Such a function is called the greatest integer function.

From the definition of [x], we can see that

$[x] = -1$ for $-1 \le x < 0$

$[x] = 0$ for $0 \le x < 1$

$[x] = 1$ for $1 \le x < 2$

$[x] = 2$ for $2 \le x < 3$ and so on.

The graph of the function is given in the question.

19. **(b)** For f(x) = g(x)

$\Rightarrow 2x^2 - 1 = 1 - 3x$

$\Rightarrow 2x^2 + 3x - 2 = 0$

$\Rightarrow 2x^2 + 4x - x - 2 = 0$

$\Rightarrow 2x(x + 2) - 1(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 1) = 0$

$\Rightarrow x = -2, \frac{1}{2}$

$\therefore$ The domain for which the function f(x) = g(x) is $\left\{-2, \frac{1}{2}\right\}$.

20. **(b)** We have, $g(x) = 1 + \sqrt{x}$ and

$f[g(x)] = 3 + 2\sqrt{x} + x$ ...(i)

Also, $f[g(x)] = f(1 + \sqrt{x})$ ...(ii)

By (i) and (ii), we get

$f(1 + \sqrt{x}) = 3 + 2\sqrt{x} + x$

Let $1 + \sqrt{x} = y$ or $x = (y - 1)^2$.

$\therefore$ $f(y) = 3 + 2(y - 1) + (y - 1)^2$

$= 3 + 2y - 2 + y^2 - 2y + 1 = 2 + y^2$

$\therefore$ $f(x) = 2 + x^2$

21. **(a)** Since, first elements of the ordered pairs in f belongs to A and second elements of the ordered pairs belongs to B. So, f is a relation from A to B.

Now, 2 has two different images 9 and 11.

So, f is not a function.

22. **(d)** In Roster form relation R is,

R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)}

$\therefore$ Domain of R = {2, 3, 4, 5}.

23. **(d)** We have, $(x, y) \in R$, if $x^2 + y^2 = 16$

i.e. $y = \pm\sqrt{16 - x^2}$

For, x = 0, y = ± 4

For, x = ± 4, y = 0

We observe that no other values of $x, y \in Z$, which satisfy $x^2 + y^2 = 16$

R = {(0, 4), (0, – 4), (4, 0), (– 4, 0)}

$\therefore$ Domain of R = {0, 4, – 4}.

24. **(a)** $A \times B = \{1, 2\} \times \{1, 3\} = \{(1, 1), (1, 3), (2, 1), (2, 3)\}$

$B \times A = \{1, 3\} \times \{1, 2\} = \{(1, 1), (1, 2), (3, 1), (3, 2)\}$

$\therefore$ $(A \times B) \cup (B \times A)$

$= \{(1, 1), (1, 3), (2, 1), (2, 3), (1, 2), (3, 1), (3, 2)\}$

25. **(b)** A – C = {1, 4} and B – C = {4}

$\therefore$ $(A - C) \times (B - C) = \{1, 4\} \times \{4\} = \{(1, 4), (4, 4)\}$.

26. **(a)** $X \times Y = \{a, b, c\} \times \{\ \} = \phi$

Hence, there are no ordered pairs formed in $X \times Y$.

27. **(d)** $R \subseteq A \times B$

For given A = {x, y, z} and B = {a, b, c, d}

$$A \times B = \left\{\begin{matrix}(x, a), (x, b), (x, c), (x, d), (y, a), (y, b), \\ (y, c), (y, d), (z, a), (z, b), (z, c), (z, d)\end{matrix}\right\}$$

Clearly, {(z, b), (y, b), (a, d)} is not the subset of $A \times B$.

$\therefore$ It is not a relation.

28. **(d)** $x^2 + y^2 = 9 \Rightarrow y^2 = 9 - x^2 \Rightarrow y = \pm\sqrt{9 - x^2}$

$x = 0 \Rightarrow y = \pm\sqrt{9 - 0} = \pm 3 \in Z$

$x = \pm 1 \Rightarrow y = \pm\sqrt{9 - 1} = \pm\sqrt{8} \notin Z$

$x = \pm 2 \Rightarrow y = \pm\sqrt{9-4} = \pm\sqrt{5} \notin Z$

$x = \pm 3 \Rightarrow y = \pm\sqrt{9-9} = 0 \in Z$

$x = \pm 4 \Rightarrow y = \pm\sqrt{9-16} = \pm\sqrt{-7} \notin Z$ and so on.

$\therefore$ R = {(0, 3), (0, –3), (3, 0), (–3, 0)}

Domain of R = {x : (x, y) ∈ R} = {0, 3, –3}

Range of R = {y : (x, y) ∈ R} = {3, –3, 0}.

29. (a) When x = 1, y = 7 ∈ N, so (1, 7) ∈ R

When x = 2, y = 2 + 3 = 5 ∈ N, so (2, 5) ∈ R

Again for x = 3, y = 3 + 2 = 5 ∈ N, (3, 5) ∈ R

Similarly for x = 4, $y = 4 + \frac{6}{4} \notin N$ and for x = 5, $y = 5 + \frac{6}{5} \notin N$.

Thus, R = {(1, 7), (2, 5), (3, 5)}

$\therefore$ Domain of R = {1, 2, 3}

and Range of R = {7, 5}.

30. (b) We have $f(x) = x^2 + 7$ and $g(x) = 3x + 5$

$\therefore f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 7 = \frac{1}{4} + 7 = \frac{29}{4}$

g (14) = 14 × 3 + 5 = 42 + 5 = 47

So, $f\left(\frac{1}{2}\right) \times g(14) = \frac{29}{4} \times 47 = \frac{1363}{4}$.

31. (a) We have $f(x) = 1 + x$, $g(x) = x^2 + x + 1$

$\therefore (f+g)(x) = f(x) + g(x)$

$= 1 + x + x^2 + x + 1 = x^2 + 2x + 2$

$\therefore (f+g)(0) = (0)^2 + 2(0) + 2 = 2.$

32. (a) $[\phi(p)]^3 = (a^p)^3 = a^{3p} = \phi(3p)$.

33. (b) Let $f(x) = \sqrt{a^2 - x^2}$

For f(x) to be defined $a^2 - x^2 \geq 0$

$\Rightarrow x^2 \leq a^2$

$\Rightarrow x \in [-a, a]$.

STATEMENT TYPE QUESTIONS

34. (d)

35. (c) P and Q are two non-empty sets. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, i.e. P × Q = {(p, q) : p ∈ P and q ∈ Q}.

Now, A = {red, blue}, B = {b, c, s}

A × B = Set of all ordered pairs

= {(red, b), (red, c), (red, s), (blue, b), (blue, c), (blue, s)}.

36. (b) I. P = {m, n} and Q = {n, m}

P × Q = {(m, n), (m, m), (n, n), (n, m)}

II. True

III. A = {1, 2}, B = {3, 4}

$B \cap \phi = \phi$

$\therefore A \times (B \cap \phi) = A \times \phi = \phi$

37. (d) The set of first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R ⇒ Domain = A.

The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the co-domain of the relation R.

Note that range ⊆ co-domain.

38. (d) $\therefore 2 \neq 2^2 \Rightarrow (2, 2) \notin R$

Hence, (a, a) ∉ R for all a ∈ N

Let (a, b) ∈ R for all a, b ∈ N

$\Rightarrow a = b^2$ [by definition]

$\Rightarrow b \neq a^2$

⇒ (b, a) ∉ R for all a, b ∈ N

Let (a, b) ∈ R and (b, c) ∈ R for all a, b, c ∈ N

$\Rightarrow a = b^2$ and $b = c^2$

$\Rightarrow a = c^4$

⇒ (a, c) ∉ R, for all a, b, c ∈ N.

39. (a) I. In Roster form,

R = {(1, 4), (2, 4), (3, 4), (4, 4)}

$\therefore$ Domain of R = {1, 2, 3, 4}

II. Here, |a| ≤ 3

⇒ –3 ≤ a ≤ 3

Hence, a = –3, –2, –1, 0, 1, 2, 3

In Roster form,

S = {(–3, 4), (–2, 3), (–1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}

$\therefore$ Range of S = {0, 1, 2, 3, 4}

40. (b) A × B = {(1, 5), (1, 7), (1, 9), (2, 5), (2, 7), (2, 9), (3, 5), (3, 7), (3, 9), (4, 5), (4, 7), (4, 9)}

$$B \times A = \begin{Bmatrix} (5,1), (5,2), (5,3), (5,4) \\ (7,1), (7,2), (7,3), (7,4) \\ (9,1)\ (9,2), (9,3), (9,4) \end{Bmatrix}$$

A × B ≠ B × A but n(A × B) = n(B × A) = 12

41. (a) Only I is true.

Let (x, y) ∈ A × C

⇒ x ∈ A and y ∈ C

⇒ x ∈ B and y ∈ C (∵ A ⊆ B)

⇒ (x, y) ∈ B × C

⇒ A × C ⊆ B × C

42. (c) R = {(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}

Domain = Set of first elements of ordered pairs in R.

Range = Set of second elements of ordered pairs in R.

43. (a) I. A = Set of first elements = {a, b, c}

B = Set of second elements = {1, 2}

II. $B \cap \phi = \phi \quad \therefore A \times \phi = \phi$

44. (c) Both the given statements are true.

I. R is not a function as 2 has two images 0 and 1.

45. **(a)** Only statement-I is true.

II. $R = \{(x, x+5) : x \in (0,1,2,3,4,5)\}$

$R = \{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

$\therefore$ Domain of $R = \{0,1,2,3,4,5\}$

46. **(b)** Only statement-II is false.

II. In Roster form,

$R = \{(1,1),(1,2),(1,3),(1,4),(1,6),(2,4),(2,6),(2,2),(4,4),(6,6),(3,3),(3,6)\}$

47. **(c)** Both the statements are true.

I. Since f(x) is a linear function

$\therefore$ $f(x) = mx + c$

$(1, 1)$ and $(0, -1) \in R$.

$f(1) = m + c, f(0) = c$

$1 = m + c, -1 = c$

$\Rightarrow$ $m = 2$ and $c = -1$

Thus, $f(x) = 2x - 1$

II. $f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$

$\therefore$ $f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0.$

48. **(d)** Both the given statements are false.

I. Correct Roster form is $\{(2,8),(3,27),(5,125),(7,343)\}$

II. Given relation in Roster form is,

$R = \{(3,5),(4,6),(5,7),(6,8),(7,9),(8,10),(9,11)\}$

Range $= \{5,6,7,8,9,10,11\}$.

49. **(d)** II. $A \cup B = \{1,2,3,8\}$

$A \cap B = \{3\}$

$(A \cup B) \times (A \cap B) = \{(1,3),(2,3),(3,3),(8,3)\}$

III. $\frac{x}{2} - 1 = 2 \Rightarrow x = 6$ and $\frac{y}{9} + 1 = 1 \Rightarrow y = 0$

MATCHING TYPE QUESTIONS

50. **(c)** Given, $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$

A. $B \cap C = \{4\}$

$\therefore$ $A \times (B \cap C) = \{(1,4),(2,4),(3,4)\}$

B. $A \times B = \{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$

$A \times C = \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$

$\therefore$ $(A \times B) \cap (A \times C) = \{(1,4),(2,4),(3,4)\}$

C. $B \cup C = \{3,4,5,6\}$

$\therefore$ $A \times (B \cup C) = \{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6)\}$

D. $(A \times B) \cup (A \times C) = \{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6)\}$

E. $A \cap B = \{3\}$, $B \cap C = \{4\}$

$\therefore$ $(A \cap B) \times (B \cap C) = \{3, 4\}$

51. **(b)** We have $3x - y = 0 \Rightarrow y = 3x$

For,

$x = 1, y = 3 \in A$

$x = 2, y = 6 \in A$

$x = 3, y = 9 \in A$

$x = 4, y = 12 \in A$

$x = 5, y = 15 \notin A$

A. In Roster form,

$R = \{(1,3),(2,6),(3,9),(4,12)\}$

B. Domain of R = Set of first element of ordered pairs in R

$= \{1, 2, 3, 4\}$

C. Range of R = Set of second element of ordered pairs in R

$= \{3, 6, 9, 12\}$

D. Co-domain of R is the set A.

52. **(a)** Since, domain of f = domain of g

We have,

$(f + g)(x) = x^2 + 2x + 1$

$(f - g)(x) = x^2 - 2x - 1$

$(fg)(x) = x^2(2x + 1) = 2x^3 + x^2$

$\left(\frac{f}{g}\right)(x) = \frac{x^2}{2x+1}, \ x \neq -\frac{1}{2}.$

53. **(a)** Domain of f = R

Domain of g = R

Domain of f $\cap$ Domain of g = R

A. $f + g : R \to R$ is given by

$(f + g)(x) = f(x) + g(x)$

$= 2x + 5 + x^2 + x$

$= x^2 + 3x + 5$

B. $f - g : R \to R$ is defined as

$(f - g)(x) = f(x) - g(x)$

$= 2x + 5 - x^2 - x$

$= 5 + x - x^2$

C. $(fg)(x) = f(x) \cdot g(x)$

$= (2x + 5)(x^2 + x)$

$= 2x^3 + 2x^2 + 5x^2 + 5x$

$= 2x^3 + 7x^2 + 5x$

D. $g(x) = 0$

$\therefore$ $x^2 + x = 0$

$\Rightarrow$ $x(x + 1) = 0$

$\Rightarrow$ $x = 0, -1$

Domain of $\left(\frac{f}{g}\right)$ = Domain of f $\cap$ Domain of g $- \{0, -1\}$

$= R - \{0, -1\}$

Thus, $\frac{f}{g} : R - \{0, -1\} \to R$ is given by

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+5}{x^2+x}.$

54. **(b)** **(B)** f(x) assumes real values if $4 - x^2 > 0$

$\Rightarrow\ x^2 - 4 < 0 \Rightarrow (x+2)(x-2) < 0$

$\Rightarrow\ x \in (-2, 2)$

$\Rightarrow$ Domain of f = (–2, 2)

(C) $f(x) = \dfrac{x}{1+x^2} = y$ (say)

$\Rightarrow\ y = \dfrac{x}{1+x^2} \quad \Rightarrow\ yx^2 - x + y = 0$

x assumes real values if

$(-1)^2 - 4(y^2) \geq 0 \Rightarrow 4y^2 - 1 \leq 0$

$\Rightarrow\ (2y+1)(2y-1) \leq 0$

$\Rightarrow\ y \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

$\therefore$ Range of f = $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

(D) Let f(x) = y $\Rightarrow y = \sqrt{x-1} \Rightarrow y^2 = x - 1$

$\Rightarrow\ x = y^2 + 1$

Since, $y \geq 0$ and $x \in [1, \infty) \Rightarrow$ Range of f = $[0, \infty)$

55. **(b)**

56. **(b)** (A) $f(x) + g(x) = x + |x| = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$

(B) $f(x) - g(x) = x - |x| = \begin{cases} 0, & x \geq 0 \\ 2x, & x < 0 \end{cases}$

(C) $f(x) . g(x) = x . |x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

(D) $\dfrac{f(x)}{g(x)} = \dfrac{x}{|x|} = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$

57. **(d)** (A) Number of relations from A to A = $2^{n(A) \times n(A)}$

(B) Number of relations from B to B = $2^{n(B) \times n(B)}$

(C) Number of relations from A to B = $2^{n(A) \times n(B)}$

INTEGER TYPE QUESTIONS

58. **(a)** $4x + 3 = 3x + 5 \Rightarrow x = 5 - 3 = 2$ and $y = -2$

$\therefore\ x + y = 2 - 2 = 0$

59. **(d)** $4 - y = 7 \Rightarrow y = -3$

$\therefore\ 4 + y = 4 - 3 = 1$

60. **(a)** Elements in the given set are (2, 3), (–2, –3), (4, 1), (–4, –1), (2, –3), (–2, 3), (–4, 1) and (4, –1). So, number of elements in the set is 8.

61. **(a)** n(A) = 3, n(B) = 2

$n(A \times B) = n(A) \times n(B) = 3 \times 2 = 6$

62. **(b)** Total number of relations from X to Y is 2^{mn}

$\Rightarrow$ No. of relations = $2^{5 \times 7}$

63. **(c)** $f(b+1) = 4(b+1) - (b+1)^2$

$= 4b + 4 - b^2 - 1 - 2b$

$= 2b - b^2 + 3$

$f(b-1) = 4(b-1) - (b-1)^2$

$= 4b - 4 - b^2 - 1 + 2b$

$= 6b - b^2 - 5$

$f(b+1) - f(b-1) = -4b + 8$

$= 4(2 - b) \equiv m(2 - b)$

64. **(a)** $f(y) = 2y^2 + by + c$

$f(0) = c$ | $f(2) = 2(2)^2 + b(2) + c$

$3 = c$ | $1 = 8 + 2b + c$

$2b + c = -7$

$2b + 3 = -7$

$2b = -10$

$b = -5$

Now, $f(1) = 2(1)^2 + b(1) + c$

$= 2 + b + c = 2 - 5 + 3 = 0$

65. **(a)** $n(X \times X) = n(X) . n(X) = 3^2 = 9$

So, the total number of subsets of X × X is 2^9 and a subset of X × X is a relation over the set X.

66. **(c)** $f(x) = ax + b$

$f(-1) = -a + b$ | $f(3) = 3a + b$

$-5 = -a + b$ | $3 = 3a + b$

On solving both the equations, we get a = 2

ASSERTION - REASON TYPE QUESTIONS

67. **(d)** In Roster form R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 4), (2, 6), (2, 2), (4, 4), (6, 6), (3, 3), (3, 6)}

Domain of R = Set of first element of ordered pairs in R = {1, 2, 3, 4, 6}

Range of R = {1, 2, 3, 4, 6}.

68. **(a)** Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

Given, (x + 1, y – 2) = (3, 1)

Then, by the definition

x + 1 = 3 and y – 2 = 1

$\Rightarrow$ x = 2 and y = 3.

69. **(c)** Domain of f = {0, 2, 3, 4, 5}

Domain of g = {1, 2, 3, 4, 5}

Domain of f · g = Domain of f $\cap$ Domain of g = {2, 3, 4, 5}

Hence, Assertion is true.

If f and g be two real functions.

Then, (f · g) (x) = f(x) · g(x)

Hence, Reason is false.

70. **(a)** Given functions are $f(x) = \dfrac{1}{x-2}$, $x \neq 2$ and

$g(x) = (x-2)^2$

$\therefore\ (f+g)(x) = f(x) + g(x) = \dfrac{1}{x-2} + (x-2)^2,\ x \neq 2$

$= \dfrac{1+(x-2)^3}{(x-2)},\ x \neq 2.$

71. (d) We have

$A = \{x, y, z\}, B = \{3, 4\} \Rightarrow n(A) = 3, n(B) = 2$

$\therefore\ n(A \times B) = n(A) \times n(B) = 6$

Therefore, the number of subsets of $A \times B$ is 2^6.

So, the number of relations from A to B is 2^6.

72. (c) We know that every subset of $A \times A$ is a relation on A.

So, Assertion is true but Reason is false.

CRITICALTHINKING TYPE QUESTIONS

73. (d)

74. (a) $A = \{2, 4, 6\}, B = \{2, 3, 5\}$

No. of relations from A to B $= 2^{3 \times 3} = 2^9$

75. (a) We have, $f(x) = \dfrac{2^x + 2^{-x}}{2}$

$\therefore f(x+y).\ f(x-y)$

$= \dfrac{1}{2}(2^{x+y} + 2^{-x-y}).\dfrac{1}{2}(2^{x-y} + 2^{-x+y})$

$= \dfrac{1}{4}\left[(2^{2x} + 2^{-2x}) + (2^{2y} + 2^{-2y})\right]$

$= \dfrac{1}{2}[f(2x) + f(2y)]$

76. (b) In the definition of function

$$f(x) = \frac{x(x-p)}{q-p} + \frac{x(p-q)}{(p-q)} = p$$

Putting p and q in place of x, we get

$$f(p) = \frac{p(p-p)}{q-p} + \frac{p(p-q)}{(p-q)} = p$$

$\Rightarrow\quad f(p) = p$

and $f(q) = \dfrac{q(q-p)}{q-p} + \dfrac{q(p-q)}{(p-q)} = q$

$\Rightarrow\quad f(q) = q$

Putting $x = (p+q)$

$$f(p+q) = \frac{(p+q)(p+q-p)}{(q-p)} + \frac{(p+q)(p+q-q)}{(p-q)}$$

$$= \frac{(p+q)q}{(q-p)} + \frac{(p+q)(p)}{(p-q)} = \frac{pq + q^2 - p^2 - pq}{(q-p)}$$

$$= \frac{q^2 - p^2}{q-p} = \frac{(q-p)(q+p)}{(q-p)}$$

$= p + q = f(q) + f(p)$

So, $f(p) + f(q) = f(p+q)$

77. (c) Given functions are : $f(x) = x$ and $g(x) = |x|$

$\therefore\ (f+g)(x) = f(x) + g(x) = x + |x|$

According to definition of modulus function,

$$(f+g)(x) = \begin{cases} x+x, & x \geq 0 \\ x-x, & x < 0 \end{cases} = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

78. (d) $n(A) = 2$ and $n(B) = 2$

$n(A \times B) = n(A) \times n(B) = 2 \times 2 = 4$

$\therefore$ Number of subset of $A \times B = 2^{n(A \times B)} = 2^4 = 16$

79. (c) Let $(a, b) \in A \times (B \cap C)$

$\Rightarrow\ a \in A$ and $b \in (B \cap C)$

$\Rightarrow\ a \in A$ and ($b \in B$ and $b \in C$)

$\Rightarrow\ (a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

$\Rightarrow\ (a, b) \in A \times B$ and $(a, b) \in (A \times C)$

$(a, b) \in (A \times B) \cap (A \times C)$

$\Rightarrow\ A \times (B \cap C) \subset (A \times B) \cap (A \times C)$ ... (i)

Again, let $(x, y) \in (A \times B) \cap (A \times C)$

$(x, y) \in A \times B$ and $(x, y) \in A \times C$

$\Rightarrow\ (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow\ x \in A$ and ($y \in B$ and $y \in C$)

$\Rightarrow\ x \in A$ and $y \in (B \cap C)$

$\Rightarrow\ (x, y) \in A \times (B \cap C)$

$\Rightarrow\ (A \times B) \cap (A \times C) \subset A \times (B \cap C)$ ... (ii)

From equations (i) and (ii), we get

$A \times (B \cap C) = (A \times B) \cap (A \times C)$ ... (iii)

Now, $A \times (B' \cup C')' = A \times \left[(B')' \cap (C')'\right]$

[by De-Morgan's law]

$= A \times (B \cap C)$ $\quad [\because (A')' = A]$

$= (A \times B) \cap (A \times C)$ [by equation (iii)]

80. (c) $n(A \times B \times C \times .....) = n(A) \times n(B) \times n(C) \times .....$

$\therefore\ n(A \times A \times B) = n(A) \times n(A) \times n(B)$

$[\because n(A) = 3, n(B) = 5]$

$= 3 \times 3 \times 5 = 45$

81. (a) It is distributive law.

82. (c) $n(A \times B) = pq$.

83. (a) If $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, c\}$

$A - B = \{a\}, B \cap C = \{c, d\}$

Then, $(A - B) \times (B \cap C) = \{a\} \times \{c, d\}$

$= \{(a, c), (a, d)\}$

84. (c) $B \cup C = \{c, d\} \cup \{d, e\} = \{c, d, e\}$

$\therefore A \times (B \cup C) = \{a, b\} \times \{c, d, e\}$

$= \{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$

85. (c) $n(A) = p, n(B) = q$

$n(A \times B) = pq = 7$

So, possible values of p, q are 7, 1

$\Rightarrow\ p^2 + q^2 = (7)^2 + (1)^2 = 50.$

86. (b) $(-1, 0) \in A \times A$ and $(0, 1) \in A \times A$

$\therefore$ $(-1, 0) \in A \times A \Rightarrow -1, 0 \in A$

and $(0, 1) \in A \times A \Rightarrow 0, 1 \in A$

$\therefore$ $A = \{-1, 0, 1\}$

$\therefore$ $A \times A = \{-1, 0, 1\} \times \{-1, 0, 1\}$

$= \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)\}$

Since, $(-1, 0)$, $(0, 1)$ already exist.

$\therefore$ Remaining 7 ordered pairs are

$\{(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)\}$

87. (a) $\because n(A \times A) = n(A) \times n(A) = 3^2 = 9$. So, the total number of subsets of $A \times A$ is 2^9.

88. (d) We have $R = \{(x, y) : |x^2 - y^2| < 16\}$

Let $x = 1$,

$|x^2 - y^2| < 16 \Rightarrow |1 - y^2| < 16$

$\Rightarrow$ $|y^2 - 1| < 16 \Rightarrow y = 1, 2, 3, 4$

Let $x = 2$,

$|x^2 - y^2| < 16 \Rightarrow |4 - y^2| < 16$

$\Rightarrow$ $|y^2 - 4| < 16 \Rightarrow y = 1, 2, 3, 4$

Let $x = 3$,

$|x^2 - y^2| < 16 \Rightarrow |9 - y^2| < 16$

$\Rightarrow$ $|y^2 - 9| < 16 \Rightarrow y = 1, 2, 3, 4$

Let $x = 4$,

$|x^2 - y^2| < 16 \Rightarrow |16 - y^2| < 16$

$\Rightarrow$ $|y^2 - 16| < 16 \Rightarrow y = 1, 2, 3, 4, 5$

Let $x = 5$,

$|x^2 - y^2| < 16 \Rightarrow |25 - y^2| < 16$

$\Rightarrow$ $|y^2 - 25| < 16 \Rightarrow y = 4, 5$

$\therefore$ $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (5, 4), (5, 5)\}$.

89. (a) Given, $A = \{x : |x| < 3, x \in I\}$

$A = \{x : -3 < x < 3, x \in I\} = \{-2, -1, 0, 1, 2\}$

Also, $R = \{(x, y) : y = |x|\}$

$\therefore$ $R = \{(-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2)\}$.

90. (b) Here, f(x) is defined only when $x + 3 \neq 0$, i.e. when $x \neq -3$

$\therefore$ $D(f) = R - \{-3\}$.

91. (d) Given $n(A) = 8$ and $n(B) = p$

$\therefore$ Total number of relations from A to B $= 2^{8p}$

$\therefore$ Total number of non-empty relations from A to B $= 2^{8p} - 1$.

92. (c) $f(x) = \sqrt{5 - 4x - x^2} + x^2 \log(x + 4)$

$\Rightarrow$ $5 - 4x - x^2 \geq 0, x + 4 > 0$

$\Rightarrow$ $(x + 5)(x - 1) \leq 0, x > -4$

$\Rightarrow$ $-5 \leq x \leq 1, x > -4$

$\Rightarrow$ $-4 < x \leq 1$.

93. (c) Let $g(x) = \dfrac{3}{4 - x^2}$ $\quad\therefore\quad x \neq \pm 2$

$\therefore$ $D(g(x)) = R - \{-2, 2\}$

$h(x) = \log_{10}(x^3 - x)$

$\therefore$ $x^3 - x > 0$

$\Rightarrow$ $x(x + 1)(x - 1) > 0$

– + – +
–1 0 1

$\therefore$ $x \in (-1, 0) \cup (1, \infty)$

$\therefore$ Domain of f(x) is $(-1, 0) \cup (1, 2) \cup (2, \infty)$.

94. (b) $f(x) = \dfrac{1}{\sqrt{9 - x^2}}$

Clearly, $9 - x^2 > 0 \Rightarrow x^2 - 9 < 0$

$\Rightarrow$ $(x + 3)(x - 3) < 0$

Thus, domain of f(x) is $x \in (-3, 3)$.

95. (a) We have $f(x) = |x - 1|$

Here, f(x) is a modulus function and since modulus of a real number is uniquely defined $\forall$ real positive number.

$\therefore$ The domain of f(x) is R

We see that $f(x) = |x - 1|$

$$f(x) = \begin{cases} x - 1 & , \text{ if } x \geq 1 \\ -(x - 1) & , \text{ if } x < 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x - 1 & , \text{ if } x \geq 1 \\ 1 - x & , \text{ if } x < 1 \end{cases}$$

From above, we observe that in both cases $f(x) \geq 0$.

Hence, range of f(x) is $[0, \infty)$.

96. (a) Since f is a linear function, $f(x) = mx + c$.

Also, since $(1, 1), (0, -1) \in R$,

$f(1) = m + c = 1$ and $f(0) = c = -1$

This gives $m = 2$

$\therefore$ $f(x) = 2x - 1$.

97. (d) Given that $f(x) = \dfrac{1}{\sqrt{x - |x|}}$,

where $x - |x| = \begin{cases} x - x = 0 & , \text{ if } x \geq 0 \\ x - (-x) = 2x & , \text{ if } x < 0 \end{cases}$.

Thus, $\dfrac{1}{\sqrt{x - |x|}}$ is not defined for any $x \in R$.

Hence, f is not defined for any $x \in R$, i.e. domain of $f = \{\phi\}$.

98. (b) Since $x^2 - 5x + 4 = (x - 4)(x - 1)$, the function f is defined for all real numbers except $x = 4$ and $x = 1$.

Hence, the domain of f is $R - \{1, 4\}$.

99. (b) Given $f(x) = 2 - |x - 5|$

Domain of f(x) is defined for all real values of x.

Since, $|x - 5| \geq 0 \Rightarrow -|x - 5| \leq 0$

$\Rightarrow$ $2 - |x - 5| \leq 2 \Rightarrow f(x) \leq 2$

Hence, range of f(x) is $(-\infty, 2]$.

100. (b) Given $P = \{a, b, c\}$ and $Q = \{r\}$

$P \times Q = \{(a, r), (b, r), (c, r)\}$

$Q \times P = \{(r, a), (r, b), (r, c)\}$

Since, by the definition of equality of ordered pairs, the pair (a, r) is not equal to the pair (r, a), we conclude that

$$P \times Q \neq Q \times P$$

However, the number of elements in each set will be the same.

CHAPTER 3

TRIGONOMETRIC FUNCTIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The value of $\tan^2\theta \sec^2\theta (\cot^2\theta - \cos^2\theta)$ is

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

2. Value of cot 5° cot 10° ..... cot 85° is

(a) 0 (b) –1 (c) 1 (d) 2

3. Value of sin 10° + sin 20° + sin 30° +......+sin360° is

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

4. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then value of A + B is

(a) π (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

5. If $\sin 2\theta + \sin 2\phi = 1/2$, $\cos 2\theta + \cos 2\phi = 3/2$, then value of $\cos^2(\theta - \phi)$ is

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $-\frac{5}{8}$ (d) $\frac{3}{5}$

6. If $0 < \theta < 360°$, then solutions of $\cos\theta = -1/2$ are

(a) 120º, 360º (b) 240º, 90º
(c) 60º, 270º (d) 120º, 240º

7. If $\tan\theta = -\frac{1}{\sqrt{3}}$, then general solution of the equation is

(a) $2n\pi + \frac{\pi}{6}, n \in I$ (b) $n\pi + \frac{\pi}{6}, n \in I$
(c) $2n\pi - \frac{\pi}{6}, n \in I$ (d) $n\pi - \frac{\pi}{6}, n \in I$

8. If $2\tan^2\theta = \sec^2\theta$, then general value of θ are

(a) $n\pi \pm \frac{\pi}{4}, n \in I$ (b) $n\pi \pm \frac{\pi}{6}, n \in I$
(c) $2n\pi + \frac{\pi}{4}, n \in I$ (d) $2n\pi \pm \frac{\pi}{6}, n \in I$

9. If $\sin 5x + \sin 3x + \sin x = 0$ and $0 \le x \le \pi/2$, then value of x is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

10. If $y = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$, then value of $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is

(a) $\frac{y}{3}$ (b) y (c) 2y (d) $\frac{3}{2}y$

11. The number of solution of $\tan x + \sec x = 2\cos x$ in $(0, 2\pi)$ is

(a) 2 (b) 3 (c) 0 (d) 1

12. If $\sin A = \frac{3}{5}, 0 < A < \frac{\pi}{2}$ and $\cos B = \frac{-12}{13}, \pi < B < \frac{3\pi}{2}$, then value of sin (A – B) is

(a) $-\frac{13}{82}$ (b) $-\frac{15}{65}$ (c) $-\frac{13}{75}$ (d) $-\frac{16}{65}$

13. Value of tan15°. tan45° tan75° is

(a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) –1

14. Value of

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is

(a) $\frac{1}{8}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

15. The large hand of a clock is 42 cm long. How much distance does its extremity move in 20 minutes?

(a) 88 cm (b) 80 cm (c) 75 cm (d) 77 cm

16. The angle in radian through which a pendulum swings and its length is 75 cm and tip describes an arc of length 21 cm, is

(a) $\frac{7}{25}$ (b) $\frac{6}{25}$ (c) $\frac{8}{25}$ (d) $\frac{3}{25}$

17. The length of an arc of a circle of radius 3 cm, if the angle subtended at the centre is 30° is (π = 3.14)

(a) 1.50 cm (b) 1.35 cm (c) 1.57 cm (d) 1.20 cm

18. A circular wire of radius 7 cm is cut and bent again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is

(a) 50° (b) 210° (c) 100° (d) 60°

19. A circular wire of radius 3 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 48 cm. The angle in degrees which is subtended at the centre of hoop is

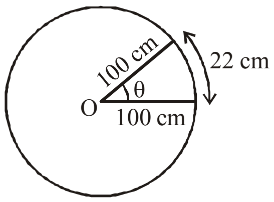
(a) 21.5° (b) 23.5° (c) 22.5° (d) 24.5°

20. The radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm is $\left(\text{Use } \pi = \frac{22}{7}\right)$

(a) 37.5 cm (b) 32.8 cm (c) 35.7 cm (d) 34.5 cm

21. The degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm as shown in figure, is $\left[\text{Use } \pi = \frac{22}{7}\right]$

(a) $12°\,30'$
(b) $12°\,36'$
(c) $11°\,36'$
(d) $11°\,12'$



22. If $\tan\theta = 3$ and θ lies in III[rd] quadrant, then the value of $\sin\theta$ is

(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{2}{\sqrt{10}}$ (c) $\frac{-3}{\sqrt{10}}$ (d) $\frac{-5}{\sqrt{10}}$

23. If $\frac{\sin x}{\cos x} \times \frac{\sec x}{\operatorname{cosec} x} \times \frac{\tan x}{\cot x} = 9$, where $x \in \left(0, \frac{\pi}{2}\right)$, then the value of x is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

24. Find x from the equation:
$\operatorname{cosec}(90° + \theta) + x\cos\theta\cot(90° + \theta) = \sin(90° + \theta)$.
(a) $\cot\theta$ (b) $\tan\theta$ (c) $-\tan\theta$ (d) $-\cot\theta$

25. If A + B = 45°, then (cot A – 1) (cot B – 1) is equal to

(a) 1 (b) $\frac{1}{2}$ (c) –1 (d) 2

26. If $\sin A = \frac{3}{5}$ and A is in first quadrant, then the values of sin 2A, cos 2A and tan 2A are

(a) $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$ (b) $\frac{1}{25}, \frac{7}{25}, \frac{1}{7}$

(c) $\frac{24}{25}, \frac{1}{25}, \frac{24}{7}$ (d) $\frac{1}{25}, \frac{24}{25}, \frac{1}{24}$

27. The value of $\tan(\alpha + \beta)$, given that $\cot\alpha = \frac{1}{2}$, $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and $\sec\beta = \frac{-5}{3}$, $\beta \in \left(\frac{\pi}{2}, \pi\right)$ is

(a) $\frac{1}{11}$ (b) $\frac{2}{11}$ (c) $\frac{5}{11}$ (d) $\frac{3}{11}$

28. The value of tan 75° – cot 75° is equal to

(a) $2\sqrt{3}$ (b) $2+\sqrt{3}$

(c) $2-\sqrt{3}$ (d) 1

29. The value of tan 3A – tan 2A – tan A is equal to
(a) tan 3A tan 2A tan A
(b) –tan 3A tan 2A tan A
(c) tan A tan 2A – tan 2A tan 3A – tan 3A tan A
(d) None of these

30. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then tan(2A + B) is equal to

(a) 1 (b) 2 (c) 3 (d) 4

31. If $\tan\theta = \frac{a}{b}$, then $b\cos 2\theta + a\sin 2\theta$ is equal to

(a) a (b) b (c) $\frac{a}{b}$ (d) None of these

32. Number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, \pi]$ is
(a) 0 (b) 1 (c) 2 (d) 3

33. If $\cos A = \frac{4}{5}$, $\cos B = \frac{12}{13}$, $\frac{3\pi}{2} < A, B < 2\pi$, the value of the $\cos(A + B)$ is

(a) $\frac{65}{33}$ (b) $\frac{33}{65}$ (c) $\frac{30}{65}$ (d) $\frac{65}{30}$

34. What is the value of radian measures corresponding to the 25° measures ?

(a) $\frac{5\pi}{36}$ (b) $\frac{2\pi}{36}$ (c) $\frac{3\pi}{36}$ (d) $\frac{4\pi}{36}$

35. If $\tan\theta = \frac{-4}{3}$, then $\sin\theta$ is

(a) $\frac{-4}{5}$ but not $\frac{4}{5}$ (b) $\frac{-4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) None of these

36. cos (A + B). cos (A – B) is given by:
(a) $\cos^2 A - \cos^2 B$ (b) $\cos(A^2 - B^2)$
(c) $\cos^2 A - \sin^2 B$ (d) $\sin^2 A - \cos^2 B$

37. If $\sin\theta = \frac{24}{25}$ and 0° < θ < 90° then what is the value of $\sin\left(\frac{\theta}{2}\right)$?

(a) $\frac{12}{25}$ (b) $\frac{7}{25}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

38. What is the value of $\sin\left(\frac{5\pi}{12}\right)$?

(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(c) $\frac{\sqrt{3}+\sqrt{2}}{4}$ (d) $\frac{\sqrt{6}+1}{2}$

39. If $x + \frac{1}{x} = 2\cos\theta$, then $x^3 + \frac{1}{x^3}$ is:

(a) $\frac{1}{2}\cos 3\theta$ (b) $2\cos 3\theta$

(c) $\cos 3\theta$ (d) $\frac{1}{3}\cos 3\theta$

40. If $1 + \cot\theta = \operatorname{cosec}\theta$, then the general value of θ is

(a) $n\pi + \frac{\pi}{2}$ (b) $2n\pi - \frac{\pi}{2}$

(c) $2n\pi + \frac{\pi}{2}$ (d) $2n\pi \pm \frac{\pi}{2}$

41. If $\sin 3\alpha = 4\sin\alpha\sin(x + \alpha)\sin(x - \alpha)$, then $x =$

(a) $n\pi \pm \frac{\pi}{6}$ (b) $n\pi \pm \frac{\pi}{3}$

(c) $n\pi \pm \frac{\pi}{4}$ (d) $n\pi \pm \frac{\pi}{2}$

42. The general value of θ satisfying the equation $\tan\theta + \tan\left(\frac{\pi}{2} - \theta\right) = 2$, is

(a) $n\pi \pm \frac{\pi}{4}$ (b) $n\pi + \frac{\pi}{4}$

(c) $2n\pi \pm \frac{\pi}{4}$ (d) $n\pi + (-1)^n \frac{\pi}{4}$

43. The general solution of $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$ is

(a) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3},\ \theta = n\pi;\ n \in I$

(b) $\theta = n\pi;\ n \in I$

(c) $\theta = \frac{n\pi}{2},\ n \in I$

(d) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{2},\ \theta = n\pi;\ n \in I$

44. If $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$, then

(a) $\theta = (6n+1)\frac{\pi}{18},\ \forall\ n \in I$

(b) $\theta = (6n+1)\frac{\pi}{9},\ \forall\ n \in I$

(c) $\theta = (3n+1)\frac{\pi}{9},\ \forall\ n \in I$

(d) $\theta = (3n+1)\frac{\pi}{18}$

45. The most general value of θ satisfying the equations $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha$ is

(a) $2n\pi + \alpha$ (b) $2n\pi - \alpha$
(c) $n\pi + \alpha$ (d) $n\pi - \alpha$

46. If $\sec 4\theta - \sec 2\theta = 2$, then the general value of θ is

(a) $(2n+1)\frac{\pi}{4}$ (b) $(2n+1)\frac{\pi}{10}$

(c) $n\pi + \frac{\pi}{2}$ or $\frac{n\pi}{5} + \frac{\pi}{10}$ (d) $(2n+1)\frac{\pi}{2}$

47. General solution of the equation $\tan\theta\tan 2\theta = 1$ is given by

(a) $(2n+1)\frac{\pi}{4},\ n \in I$ (b) $n\pi + \frac{\pi}{6},\ n \in I$

(c) $n\pi - \frac{\pi}{6},\ n \in I$ (d) $n\pi \pm \frac{\pi}{6},\ n \in I$

48. If $\cot\theta + \cot\left(\frac{\pi}{4} + \theta\right) = 2$, then the general value of θ is

(a) $2n\pi \pm \frac{\pi}{6}$ (b) $2n\pi \pm \frac{\pi}{3}$

(c) $n\pi \pm \frac{\pi}{3}$ (d) $n\pi \pm \frac{\pi}{6}$

49. If $2\cos^2 x + 3\sin x - 3 = 0,\ 0 \le x \le 180°$, then $x =$

(a) 30°, 90°, 150° (b) 60°, 120°, 180°
(c) 0°, 30°, 150° (d) 45°, 90°, 135°

50. If $\cot\theta + \tan\theta = 2\,\text{cosec}\,\theta$, the general value of θ is

(a) $n\pi \pm \frac{\pi}{3}$ (b) $n\pi \pm \frac{\pi}{6}$

(c) $2n\pi \pm \frac{\pi}{3}$ (d) $2n\pi \pm \frac{\pi}{6}$

51. If $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta\tan 3\theta = 1$, then the general value of θ is

(a) $n\pi \pm \frac{\pi}{5}$ (b) $\left(n + \frac{1}{6}\right)\frac{\pi}{5}$

(c) $\left(2n \pm \frac{1}{6}\right)\frac{\pi}{5}$ (d) $\left(n + \frac{1}{3}\right)\frac{\pi}{5}$

52. If $\cos 7\theta = \cos\theta - \sin 4\theta$, then the general value of θ is

(a) $\frac{n\pi}{4}, \frac{n\pi}{3} + \frac{\pi}{18}$ (b) $\frac{n\pi}{3}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$

(c) $\frac{n\pi}{4}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$ (d) $\frac{n\pi}{6}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$

53. Which among the following is/are correct?

(a) The angle is called negative, if the rotation is clockwise
(b) The angle is called positive, if the rotation is anti-clockwise
(c) The amount of rotation performed to get the terminal side from the initial side is called the measure of an angle
(d) All the above are correct

54. Angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of

(a) 1 degree (b) 1 grade (c) 1 radian (d) 1 arc

55. Radian measure of $40°\,20'$ is equal to

(a) $\frac{120\pi}{504}$ radian (b) $\frac{121\pi}{540}$ radian

(c) $\frac{121\pi}{3}$ radian (d) None of these

56. π radian in degree measure is equal to

(a) 18° (b) 180° (c) 200° (d) 360°

57. The value of $\sin\frac{31\pi}{3}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

58. The value of $\cot\left(\frac{-15\pi}{4}\right)$ is

(a) $\frac{-1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

59. If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then the value of $\left(\frac{\text{cosec}\,\theta + \cot\theta}{\sec\theta - \tan\theta}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{13}{2}$ (d) None of these

60. $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$ is equal to

(a) $\sqrt{2}\sin x$ (b) $-2\sin x$

(c) $-\sqrt{2}\sin x$ (d) None of these

61. $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$ is equal to

(a) $\sin 2x$ (b) $\cos 2x$ (c) $\tan 2x$ (d) None of these

62. The solution of $\sin x = -\frac{\sqrt{3}}{2}$ is

(a) $x = n\pi + (-1)^n\frac{4\pi}{3}$, where $n \in Z$

(b) $x = n\pi + (-1)^n\frac{2\pi}{3}$, where $n \in Z$

(c) $x = n\pi + (-1)^n\frac{3\pi}{3}$, where $n \in Z$

(d) None of the above

63. If $x = \sec\theta + \tan\theta$, then $x + \frac{1}{x} =$

(a) 1 (b) $2\sec\theta$ (c) 2 (d) $2\tan\theta$

64. The value of $\frac{\tan 70° - \tan 20°}{\tan 50°} =$

(a) 1 (b) 2 (c) 3 (d) 0

65. $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} =$

(a) 0 (b) 1 (c) 2 (d) 4

66. The value of $\cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \cos^2\frac{5\pi}{12}$ is

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3+\sqrt{3}}{2}$ (d) $\frac{2}{3+\sqrt{3}}$

67. $1 + \cos 2x + \cos 4x + \cos 6x =$

(a) $2\cos x \cos 2x \cos 3x$
(b) $4\sin x \cos 2x \cos 3x$
(c) $4\cos x \cos 2x \cos 3x$
(d) None of these

68. cosec A – 2 cot 2A cos A =

(a) 2 sin A (b) sec A
(c) 2 cos A cot A (d) None of these

69. If $\sin x + \cos x = \frac{1}{5}$, then $\tan 2x$ is

(a) $\frac{25}{17}$ (b) $\frac{7}{25}$ (c) $\frac{25}{7}$ (d) $\frac{24}{7}$

70. If $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta \tan 3\theta = 1$, then the general value of θ is

(a) $n\pi + \frac{\pi}{5}$ (b) $\left(n + \frac{1}{6}\right)\frac{\pi}{5}$

(c) $\left(2n \pm \frac{1}{6}\right)\frac{\pi}{5}$ (d) $\left(n + \frac{1}{3}\right)\frac{\pi}{5}$

71. If $\tan\theta - \sqrt{2}\sec\theta = \sqrt{3}$, then the general value of θ is

(a) $n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{3}$ (b) $n\pi + (-1)^n\frac{\pi}{3} - \frac{\pi}{4}$

(c) $n\pi + (-1)^n\frac{\pi}{3} + \frac{\pi}{4}$ (d) $n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{3}$

72. The general solution of $\sin^2\theta\sec\theta + \sqrt{3}\tan\theta = 0$ is

(a) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}$, $\theta = n\pi$, $n \in Z$
(b) $\theta = n\pi$, $n \in Z$
(c) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}$, $n \in Z$
(d) $\theta = \frac{n\pi}{2}$, $n \in Z$

73. The value of $\frac{\cot 54°}{\tan 36°} + \frac{\tan 20°}{\cot 70°}$ is

(a) 2 (b) 3 (c) 1 (d) 0

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

74. I : $\cos\alpha + \cos\beta + \cos\gamma = 0$
II : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then

(a) I is false and II is true (b) I and II both are true
(c) I and II both are false (d) I is true and II is false

75. Consider the statements given below:

I. $\sin x$ is positive in first and second quadrants.
II. cosec x is negative in third and fourth quadrants.
III. tan x and cot x are negative in second and fourth quadrants.
IV. cos x and sec x are positive in first and fourth quadrants.

Choose the correct option.

(a) All are correct
(b) Only I and IV are correct
(c) Only III and IV are correct
(d) None is correct

76. Which among the following is/are true?

I. The values of cosec x repeat after an interval of 2π.
II. The values of sec x repeat after an interval of 2π.
III. The values of cot x repeat after an interval of π.

(a) I is true (b) II is true
(c) III is true (d) All are true

77. Consider the following statements.

I. cot x decreases from 0 to $-\infty$ in first quadrant and increases from 0 to ∞ in third quadrant.
II. sec x increases from $-\infty$ to -1 in second quadrant and decreases from ∞ to 1 in fourth quadrant.
III. cosec x increases from 1 to ∞ in second quadrant and decreases from -1 to $-\infty$ in fourth quadrant.

Choose the correct option.

(a) I is incorrect (b) II is incorrect
(c) III is incorrect (d) IV is incorrect

78. Consider the statements given below:

I. $2\cos x \cdot \cos y = \cos(x + y) - \cos(x - y)$.
II. $-2\sin x \cdot \sin y = \cos(x + y) - \cos(x - y)$.
III. $2\sin x \cdot \cos y = \sin(x + y) - \sin(x - y)$.
IV. $2\cos x \cdot \sin y = \sin(x + y) + \sin(x - y)$.

Choose the correct statements.

(a) I is correct
(b) II is correct
(c) Both I and II are correct
(d) III is correct

79. If $\sin 2x + \cos x = 0$, then which among the following is/are true?

I. $\cos x = 0$
II. $\sin x = -\frac{1}{2}$
III. $x = (2n + 1)\frac{\pi}{2}$, $n \in Z$
IV. $x = n\pi + (-1)^n\frac{7\pi}{6}$, $n \in Z$

(a) I is true (b) I and II are true
(c) I, II and III are true (d) All are true

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

80.

Column-I (Degree Measure)	Column-II (Radian Measure)
A. 25°	1. $\frac{26\pi}{9}$
B. – 47° 30′	2. $\frac{4\pi}{3}$
C. 240°	3. $\frac{-19\pi}{72}$
D. 520°	4. $\frac{5\pi}{36}$

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	4	3	2	1
(c)	1	3	2	4
(d)	1	4	3	2

81. $\left[\text{Use } \pi = \frac{22}{7}\right]$.

Column-I (Radian Measure)	Column-II (Degree Measure)
A. $\frac{11}{16}$	1. 300°
B. -4	2. 210°
C. $\frac{5\pi}{3}$	3. 39° 22′ 30″
D. $\frac{7\pi}{6}$	4. −229° 5′ 27″

Codes:

	A	B	C	D
(a)	3	4	2	1
(b)	1	4	2	3
(c)	3	4	1	2
(d)	2	4	1	3

82.

Column-I	Column-II
A. $\sin\frac{25\pi}{3}$	1. $-\sqrt{3}$
B. $\cos\frac{41\pi}{4}$	2. $\frac{\sqrt{3}}{2}$
C. $\tan\left(\frac{-16\pi}{3}\right)$	3. 1
D. $\cot\frac{29\pi}{4}$	4. $\frac{1}{\sqrt{2}}$

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	2	1	4	3
(c)	3	1	4	2
(d)	3	4	1	2

83.

Column-I	Column-II
A. $\cos(\pi - x)$	1. $-\cos x$
B. $\sin(\pi - x)$	2. $-\sin x$
C. $\sin(\pi + x)$	3. $\cos x$
D. $\cos(\pi + x)$	4. $\sin x$
E. $\cos(2\pi - x)$	
F. $\sin(2\pi - x)$	

Codes:

	A	B	C	D	E	F
(a)	1	4	2	1	3	2
(b)	1	2	4	1	3	1
(c)	2	4	1	2	3	2
(d)	1	2	2	4	3	1

84.

Column-I	Column-II
A. 1 radian is equal to	1. 0.01746 radian
B. 1° is equal to	2. 57° 16′ (approx.)
C. 3° 45′ is equal to	3. $\frac{9\pi}{32}$ radian
D. 50° 37′ 30″ is equal to	4. $\frac{\pi}{48}$ radian

Codes:

	A	B	C	D
(a)	1	4	3	2
(b)	2	4	1	3
(c)	2	1	4	3
(d)	3	1	4	2

85.

Column-I (Degree measure)	Column-II (Radian measure)
(A) 25°	1. $\frac{-19\pi}{72}$
(B) −47° 30′	2. $\frac{4\pi}{3}$
(C) 240°	3. $\frac{26\pi}{9}$
(D) 520°	4. $\frac{5\pi}{36}$

Codes:

	A	B	C	D
(a)	4	2	1	3
(b)	3	1	2	4
(c)	4	1	2	3
(d)	3	2	1	4

86.

Column-I	Column-II
(A) $\sin x =$	1. $\frac{1}{\sqrt{3}}$
(B) $\tan x =$	2. -2
(C) $\cot x =$	3. $\frac{-\sqrt{3}}{2}$
(D) $\sec x =$	4. $\frac{-2}{\sqrt{3}}$
(E) $\text{cosec } x =$	5. $\sqrt{3}$

Codes:

	A	B	C	D	E
(a)	3	5	1	2	4
(b)	1	5	3	2	4
(c)	3	5	1	4	2
(d)	3	1	5	4	2

87.

Column-I (Trigonometric Equation)	**Column-II (General Solution)**
(A) $\cos 4x = \cos 2x$	1. $x = n\pi \pm \frac{\pi}{3}$
(B) $\cos 3x + \cos x - \cos 2x = 0$	2. $x = \frac{n\pi}{2} + \frac{3\pi}{8}$
(C) $\sin 2x + \cos x = 0$	3. $x = 2n\pi \pm \frac{\pi}{3}$
(D) $\sec^2 2x = 1 - \tan 2x$	4. $x = \frac{n\pi}{3}$ or $x = n\pi,\ n \in Z$
(E) $\sin x + \sin 3x + \sin 5x = 0$	5. $x = (2n+1)\frac{\pi}{2}$ or $x = n\pi + (-1)^n.\frac{7\pi}{6}$, $n \in Z$

Codes:

	A	B	C	D	E
(a)	4	3	5	1	2
(b)	4	3	5	2	1
(c)	3	4	5	1	2
(d)	1	3	5	2	4

88.

Column-I	**Column-II**
(A) $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} =$	1. $\tan 2x$
(B) $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} =$	2. $\tan \frac{x-y}{2}$
(C) $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} =$	3. $\frac{-\sin 2x}{\cos 10x}$
(D) $\frac{\sin x - \sin y}{\cos x + \cos y} =$	4. $2 \sin x$
(E) $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} =$	5. $\tan 4x$

Codes:

	A	B	C	D	E
(a)	3	5	2	1	4
(b)	3	5	1	4	2
(c)	3	1	2	5	4
(d)	3	5	1	2	4

89. Let $\sin x = \frac{3}{5}$, x lies in second quadrant.

Column-I (Trigonometric Function)	**Column-II (Value)**
(A) $\cos x =$	1. $-4/3$
(B) $\sec x =$	2. $-3/4$
(C) $\tan x =$	3. $-4/5$
(D) $\text{cosec } x =$	4. $-5/4$
(E) $\cot x =$	5. $5/3$

Codes:

	A	B	C	D	E
(a)	3	4	2	5	1
(b)	3	4	1	5	2
(c)	3	2	4	5	1
(d)	1	2	5	4	3

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

90. The value of cosec $(-1410)°$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) None of these

91. The expression $\cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$ is equal to

(a) -1 (b) 0 (c) 1 (d) None of these

92. If $\sin\theta + \cos\theta = 1$, then $\sin\theta\cos\theta =$

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

93. If $\frac{\cos A}{3} = \frac{\cos B}{4} = \frac{1}{5}$, $\frac{-\pi}{2} < A < 0$ $\frac{-\pi}{2} < B < 0$, then value of $2 \sin A + 4 \sin B$ is $-$ a. The value of 'a' is

(a) 4 (b) 2 (c) 3 (d) 0

94. The value of $\sin 765°$ is $\frac{1}{\sqrt{n}}$. Value of n is

(a) 2 (b) 3 (c) 4 (d) 0

95. The value of cosec $(-1410)°$ is equal to

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) None of these

96. The value of $\tan\frac{19\pi}{3}$ is $\sqrt{n}$. Value of 'n' is

(a) 1 (b) 2 (c) 3 (d) 5

97. The value of $\sin\left(\frac{-11\pi}{3}\right)$ is $\frac{\sqrt{3}}{m}$. Value of 'm' is

(a) 1 (b) 2 (c) 3 (d) 5

98. The value of $\left(1+\cos\frac{\pi}{6}\right)\left(1+\cos\frac{\pi}{3}\right)\left(1+\cos\frac{2\pi}{3}\right)\left(1+\cos\frac{7\pi}{6}\right)$ is $\frac{m}{16}$. Value of m is

(a) 1 (b) 2 (c) 3 (d) 8

99. If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\left(\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}\right)$ is equal to $\frac{m}{m+1}$. The value of m is

(a) 1 (b) 2 (c) 3 (d) 4

100. If $\sin x = \frac{-2\sqrt{6}}{5}$ and x lies in III quadrant, then the value of $\cot x$ is $\frac{1}{m\sqrt{6}}$. Value of m is

(a) 1 (b) 2 (c) 3 (d) 5

101. If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then the value of $\left(\frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}\right)$ is equal to $\frac{1}{m}$. Value of m is

(a) 2 (b) 4 (c) 5 (d) 6

102. The value of

$3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$ is equal to

(a) 2 (b) 1 (c) 3 (d) 4

103. Value of $2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cdot\cos^2\frac{\pi}{3}$ is $\frac{m}{m-1}$. The value of 'm' is

(a) 3 (b) 2 (c) 4 (d) None of these

104. $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$ is equal to

(a) 1 (b) 5 (c) 3 (d) 6

105. Value of

$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$ is

(a) 0 (b) 1 (c) 2 (d) 3

106. $\cot x\cot 2x - \cot 2x\cot 3x - \cot 3x\cot x$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

107. Assertion : The ratio of the radii of two circles at the centres of which two equal arcs subtend angles of 30° and 70° is 21 : 10.

Reason : Number of radians in an angle subtended at the centre of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle.

108. Assertion : If $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$, then $\sin\theta + \cos\theta = \pm\sqrt{2}$.

Reason : $-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$.

109. Assertion : The solution of the equation

$\tan\theta + \tan\left(\theta+\frac{\pi}{3}\right) + \tan\left(\theta+\frac{2\pi}{3}\right) = 3$

is $\theta = \frac{n\pi}{3} + \frac{\pi}{12}$, $n \in I$.

Reason : If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha$, $n \in I$.

110. Assertion : The degree measure corresponding to (–2) radian is –114° 19 min.

Reason : The degree measure of a given radian measure $= \frac{180}{\pi} \times$ Radian measure.

111. Assertion : $\frac{\cos(\pi+x)\cdot\cos(-x)}{\sin(\pi-x)\cdot\cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$

Reason : $\cos(\pi+\theta) = -\cos\theta$ and $\cos(-\theta) = \cos\theta$. Also, $\sin(\pi-\theta) = \sin\theta$ and $\sin(-\theta) = -\sin\theta$.

112. Assertion : If $\tan 2x = -\cot\left(x+\frac{\pi}{3}\right)$, then $x = n\pi + \frac{5\pi}{6}$, $n \in Z$.

Reason : $\tan x = \tan y \Rightarrow x = n\pi + y$, where $n \in Z$.

113. Assertion : The measure of rotation of a given ray about its initial point is called an angle.

Reason : The point of rotation is called a vertex.

114. Assertion : In a unit circle, radius of circle is 1 unit.

Reason : 1 min (or 1′) is divided into 60s.

115. Assertion : Area of unit circle is π unit^2.

Reason : Radian measure of 40° 20′ is equal to $\frac{|2|\pi}{540}$ radian.

116. Assertion : The second hand rotates through an angle of 180° in a minute.

Reason : The unit of measurement is degree in sexagesimal system.

117. Assertion: cosec x is negative in third and fourth quadrants.

Reason : $\cot x$ decreases from 0 to $-\infty$ in first quadrant and increases from 0 to ∞ in third quadrant.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

118. The value of $\tan 20° + 2\tan 50° - \tan 70°$ is equal to

(a) 1 (b) 0

(c) tan 50° (d) None of these

119. If α and β lies between 0 and $\frac{\pi}{2}$ and if $\cos(\alpha+\beta)=\frac{12}{13}$ and $\sin(\alpha-\beta)=\frac{3}{5}$, then value of $\sin 2\alpha$ is

(a) $\frac{55}{56}$ (b) $\frac{13}{58}$ (c) 0 (d) $\frac{56}{65}$

120. The most general value of θ satisfying the equation $\cos\theta=\frac{1}{\sqrt{2}}$ and $\tan\theta=-1$ is

(a) $2n\pi-7\frac{\pi}{4}$ (b) $n\pi-\frac{\pi}{4}$

(c) $n\pi+\frac{\pi}{2}$ (d) $2n\pi+\frac{7\pi}{4}$

121. Value of $\sqrt{3}$ cosec 20° – sec 20° is

(a) 3 (b) $\frac{3}{2}$ (c) 1 (d) 4

122. The solution of the equation $\cos^2\theta+\sin\theta+1=0$, lies in the interval

(a) $\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$ (b) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

(c) $\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$ (d) $\left(\frac{5\pi}{4},\frac{7\pi}{4}\right)$

123. The number of values of x in the interval $[0,3\pi]$ satisfying the equation $2\sin^2 x+5\sin x-3=0$ is

(a) 4 (b) 6 (c) 1 (d) 2

124. Value of $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$ is

(a) $-\frac{1}{2}$ (b) 0 (c) 1 (d) $\frac{\sqrt{3}}{2}$

125. Value of sin 47° + sin 61° – sin 11° – sin 25° is

(a) cos 7º (b) sin 7º (c) sin 61º (d) –sin 25º

126. The value of expression $\sin\theta+\cos\theta$ lies between

(a) –2 and 2 both inclusive

(b) 0 and $\sqrt{2}$ both inclusive

(c) $-\sqrt{2}$ and $\sqrt{2}$ both inclusive

(d) 0 and 2 both inclusive

127. The solution of $\tan 2\theta\tan\theta=1$ is

(a) $2n\pi+\frac{\pi}{3}$ (b) $n\pi+\frac{\pi}{4}$

(c) $2n\pi-\frac{\pi}{6}$ (d) $(2n+1)\frac{\pi}{6}$

128. Number of solutions of equation, sin 5x cos 3x = sin 6x cos 2x, in the interval $[0,\pi]$ is

(a) 4 (b) 5 (c) 3 (d) 2

129. If tan (cot x) = cot (tan x), then

(a) $\sin 2x=\frac{2}{(2n+1)\pi}$ (b) $\sin x=\frac{4}{(2n+1)\pi}$

(c) $\sin 2x=\frac{4}{(2n+1)\pi}$ (d) None of these

130. Find the distance from the eye at which a coin of a diameter 1 cm be placed so as to hide the full moon, it is being given that the diameter of the moon subtends an angle of 31′ at the eye of the observer.

(a) 110 cm (b) 108 cm

(c) 110.9 cm (d) 112 cm

131. A wheel rotates making 20 revolutions per second. If the radius of the wheel is 35 cm, what linear distance does a point of its rim travel in three minutes? $\left(\text{Take } \pi=\frac{22}{7}\right)$

(a) 7.92 km (b) 7.70 km

(c) 7.80 km (d) 7.85 km

132. The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes? (Use π = 3.14)

(a) 2.68 cm (b) 6.28 cm

(c) 6.82 cm (d) 7.42 cm

133. If the arcs of the same lengths in two circles subtend angles 65° and 110° at the centre, the ratio of their radii is

(a) 12 : 13 (b) 22 : 31 (c) 22 : 13 (d) 21 : 13

134. If tan A + cot A = 4, then $\tan^4 A+\cot^4 A$ is equal to

(a) 110 (b) 191 (c) 80 (d) 194

135. If $\frac{\sin A}{\sin B}=m$ and $\frac{\cos A}{\cos B}=n$, then the value of tan B; $n^2<1<m^2$, is

(a) n^2 (b) $\pm\sqrt{\frac{1-n^2}{m^2-1}}$

(c) $\frac{n^2}{(m^2-1)}$ (d) m^2

136. If $\tan(A-B)=1$, $\sec(A+B)=\frac{2}{\sqrt{3}}$, the smallest positive value of B is

(a) $\frac{25\pi}{24}$ (b) $\frac{19\pi}{24}$ (c) $\frac{13\pi}{24}$ (d) $\frac{7\pi}{24}$

137. The value of $4 \sin \alpha \sin\left(\alpha + \frac{\pi}{3}\right) \sin\left(\alpha + \frac{2\pi}{3}\right) =$

(a) $\sin 3\alpha$ (b) $\sin 2\alpha$ (c) $\sin \alpha$ (d) $\sin^2 \alpha$

138. The solution of the equation

$[\sin x + \cos x]^{1 + \sin 2x} = 2, -\pi \le x \le \pi$ is

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

139. If $\tan\theta + \sec\theta = p$, then what is the value of $\sec\theta$?

(a) $\frac{p^2+1}{p^2}$ (b) $\frac{p^2+1}{\sqrt{p}}$

(c) $\frac{p^2+1}{2p}$ (d) $\frac{p+1}{2p}$

140. The number of solutions of the given equation

$\tan\theta + \sec\theta = \sqrt{3}$, where $0 \le \theta \le 2\pi$ is

(a) 0 (b) 1 (c) 2 (d) 3

141. If n is any integer, then the general solution of the equation $\cos x - \sin x = \frac{1}{\sqrt{2}}$ is

(a) $x = 2n\pi - \frac{\pi}{12}$ or $x = 2n\pi + \frac{7\pi}{12}$

(b) $x = n\pi \pm \frac{\pi}{12}$

(c) $x = 2n\pi + \frac{\pi}{12}$ or $x = 2n\pi - \frac{7\pi}{12}$

(d) $x = n\pi + \frac{\pi}{12}$ or $x = n\pi - \frac{7\pi}{12}$

142. If $4\sin^2\theta + 2\left(\sqrt{3}+1\right)\cos\theta = 4 + \sqrt{3}$, then the general value of θ is

(a) $2n\pi \pm \frac{\pi}{3}$ (b) $2n\pi + \frac{\pi}{4}$

(c) $n\pi \pm \frac{\pi}{3}$ (d) $n\pi - \frac{\pi}{3}$

143. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation

$2\sin^2 x + 5\sin x - 3 = 0$ is

(a) 4 (b) 6

(c) 1 (d) 2

144. If $\sin\theta + \cos\theta = 1$, then the general value of θ is

(a) $2n\pi$ (b) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

(c) $2n\pi + \frac{\pi}{2}$ (d) $(2n - 1) + \frac{\pi}{4}$

145. $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then $\theta =$

(a) $\frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{3}$ (b) $\frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{6}$

(c) $\frac{n\pi}{4}$ or $2n\pi \pm \frac{\pi}{6}$ (d) None of these

146. If $\sqrt{2}\sec\theta + \tan\theta = 1$, then the general value of θ is

(a) $n\pi + \frac{3\pi}{4}$ (b) $2n\pi + \frac{\pi}{4}$

(c) $2n\pi - \frac{\pi}{4}$ (d) $2n\pi \pm \frac{\pi}{4}$

147. If $12\cot^2\theta - 31\,\text{cosec}\,\theta + 32 = 0$, then the value of $\sin\theta$ is

(a) $\frac{3}{5}$ or 1 (b) $\frac{2}{3}$ or $\frac{-2}{3}$

(c) $\frac{4}{5}$ or $\frac{3}{4}$ (d) $\pm\frac{1}{2}$

148. If $\sec^2\theta = \frac{4}{3}$, then the general value of θ is

(a) $2n\pi \pm \frac{\pi}{6}$ (b) $n\pi \pm \frac{\pi}{6}$

(c) $2n\pi \pm \frac{\pi}{3}$ (d) $n\pi \pm \frac{\pi}{3}$

149. General solution of $\tan 5\theta = \cot 2\theta$ is

(a) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$ (b) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$

(c) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$ (d) $\theta = \frac{n\pi}{7} + \frac{\pi}{3}$

150. If none of the angles x, y and $(x + y)$ is a multiple of π, then

(a) $\cot(x + y) = \dfrac{\cot x \cdot \cot y - 1}{\cot y + \cot x}$

(b) $\cot(x - y) = \dfrac{\cot x \cdot \cot y + 1}{\cot y - \cot x}$

(c) (a) and (b) are true

(d) (a) and (b) are not true

151. Solution of the equation $3\tan(\theta - 15) = \tan(\theta + 15)$ is

(a) $\theta = \dfrac{n\pi}{2} + (-1)^n \dfrac{\pi}{4}$ (b) $\theta = n\pi + (-1)^n \dfrac{\pi}{3}$

(c) $\theta = n\pi - \dfrac{\pi}{3}$ (d) $\theta = n\pi - \dfrac{\pi}{4}$

152. If angle θ is divided into two parts such that the tangent of one part is K times the tangent to other and ϕ is their difference, then $\sin\theta$ is equal to

(a) $\dfrac{K+1}{K-1}\sin\dfrac{\theta}{2}$ (b) $\dfrac{K+1}{K-1}\sin\dfrac{\phi}{2}$

(c) $\dfrac{K+1}{K-1}\sin\phi$ (d) $\dfrac{K-1}{K+1}\sin\phi$

153. If $m\sin\theta = n\sin(\theta + 2\alpha)$, then $\tan(\theta + \alpha)\cdot\cot\alpha$ is equal to

(a) $\dfrac{m+n}{m-n}$ (b) $\dfrac{m-n}{m+n}$

(c) $\dfrac{m+n}{mn}$ (d) $\dfrac{m-n}{mn}$

154. If $5\tan\theta = 4$, then $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} =$

(a) 0 (b) 1 (c) $\dfrac{1}{6}$ (d) 6

155. $\dfrac{1 + \sin A - \cos A}{1 + \sin A + \cos A} =$

(a) $\sin\dfrac{A}{2}$ (b) $\cos\dfrac{A}{2}$ (c) $\tan\dfrac{A}{2}$ (d) $\cot\dfrac{A}{2}$

156. $\dfrac{1}{4}\left[\sqrt{3}\cos 23° - \sin 23°\right] =$

(a) $\cos 43°$ (b) $\cos 7°$ (c) $\cos 53°$ (d) None of these

157. If $\cos x + \cos y + \cos\alpha = 0$ and $\sin x + \sin y + \sin\alpha = 0$, then $\cot\left(\dfrac{x+y}{2}\right) =$

(a) $\sin\alpha$ (b) $\cos\alpha$ (c) $\cot\alpha$ (d) $\sin\left(\dfrac{x+y}{2}\right)$

158. $\sin 12° \sin 24° \sin 48° \sin 84° =$

(a) $\cos 20° \cos 40° \cos 60° \cos 80°$

(b) $\sin 20° \sin 40° \sin 60° \sin 80°$

(c) $\dfrac{3}{15}$

(d) None of these

159. If $\dfrac{2\sin\alpha}{\{1 + \cos\alpha + \sin\alpha\}} = y$, then $\dfrac{\{1 - \cos\alpha + \sin\alpha\}}{1 + \sin\alpha} =$

(a) $\dfrac{1}{y}$ (b) y (c) $1 - y$ (d) $1 + y$

160. If $\sin 2\theta + \sin 2\phi = \dfrac{1}{2}$ and $\cos 2\theta + \cos 2\phi = \dfrac{3}{2}$, then $\cos^2(\theta - \phi) =$

(a) $\dfrac{3}{8}$ (b) $\dfrac{5}{8}$ (c) $\dfrac{3}{4}$ (d) $\dfrac{5}{4}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (b) $\tan^2\theta \sec^2\theta\,(\cot^2\theta - \cos^2\theta)$

$= \sec^2\theta\,(\tan^2\theta \cot^2\theta - \tan^2\theta \cos^2\theta)$

$= \sec^2\theta\left(1 - \dfrac{\sin^2\theta}{\cos^2\theta}\cos^2\theta\right) = \sec^2\theta(1-\sin^2\theta)$

$= \sec^2\theta.\ \cos^2\theta = 1$

2. (c) $\cot 5° \cot 10°\ ....\ \cot 85°$

$= \cot 5° \cot 10° ....\cot(90° - 10°)\cot(90° - 5°)$

$= \cot 5° \cot 10° ......\ \tan 10° \tan 5°$

$= (\tan 5° \cot 5°)(\tan 10° \cot 10°)........$

$= (1)(1)(1).......= 1$

3. (b) $\because \sin 190° = \sin(180° + 10°) = -\sin 10°$

$\sin 200° = -\sin 20°$

$\sin 210° = -\sin 30°$

....

$\sin 360° = \sin 180° = 0$

$\therefore$ given expression = 0

4. (d) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}} = \dfrac{5/6}{5/6} = 1$

$\therefore A + B = 45° = \dfrac{\pi}{4}$

5. (a) Using cosine formula

$\sin 2\theta + \sin 2\phi = 2\sin(\theta+\phi)\cos(\theta-\phi) = 1/2$ .....(i)

$\cos 2\theta + \cos 2\phi = 2\cos(\theta+\phi)\cos(\theta-\phi) = 3/2$ .....(ii)

Squaring (i) and (ii) and then adding

$4\cos^2(\theta-\phi) = \dfrac{1}{4} + \dfrac{9}{4} = \dfrac{5}{2}$

$\Rightarrow \cos^2(\theta-\phi) = \dfrac{5}{8}$

6. (d) $\cos\theta = -1/2 = \cos 120°$ or $\cos 240°$ $[0 < \theta < 360°]$

$\therefore \theta = 120°, 240°$

7. (d) $\tan\theta = -\dfrac{1}{\sqrt{3}} = \tan\left(-\dfrac{\pi}{6}\right)$

$\therefore \theta = n\pi - \dfrac{\pi}{6}$

8. (a) $2\tan^2\theta = \sec^2\theta = 1 + \tan^2\theta$

$\tan^2\theta = 1 = (1)^2 = \tan^2\dfrac{\pi}{4}$

$\theta = n\pi \pm \dfrac{\pi}{4}, n \in I$

9. (c) $\sin 5x + \sin x = -\sin 3x$

$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0, \cos 2x = -1/2$

$\Rightarrow x = n\pi, x = n\pi \pm (\pi/3)$

So, $x = \pi/3$

10. (b) $\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}.\dfrac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$= \dfrac{(1+\sin\alpha)^2 - \cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \dfrac{(1+\sin^2\alpha + 2\sin\alpha) - (1-\sin^2\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \dfrac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)} = \dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

11. (b) The given equation is $\tan x + \sec x = 2\cos x$;

$\Rightarrow \sin x + 1 = 2\cos^2 x \Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$;

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$;

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = \dfrac{1}{2}, -1$

$\Rightarrow x = 30°, 150°, 270°$.

12. (d) We have : $\sin A = \dfrac{3}{5}$, where $0 < A < \dfrac{\pi}{2}$

$\therefore \quad \cos A = \pm\sqrt{1-\sin^2 A}$

$\Rightarrow \quad \cos A = +\sqrt{1-\sin^2 A} = \sqrt{1-\dfrac{9}{25}} = \dfrac{4}{5}$

[$\because$ cos is positive in first quadrant]

It is given that : $\cos B = \dfrac{-12}{13}$ and $\pi < B < \dfrac{3\pi}{2}$

$\therefore \quad \sin B = \pm\sqrt{1-\cos^2 B}$

$\Rightarrow \quad \sin B = -\sqrt{1-\cos^2 B}$

[$\because$ Sine is negative in the third quadrant]

$\Rightarrow \quad \sin B = -\sqrt{1-\left(\dfrac{-12}{13}\right)^2} = -\dfrac{5}{13}$

Now, $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$= \dfrac{3}{5} \times \dfrac{-12}{13} - \dfrac{4}{5} \times \dfrac{-5}{13} = -\dfrac{16}{65}$

13. (b) $\tan 15°.\ \tan 45° \tan 75°$

$= \tan 15°.\ \tan(60° - 15°).\tan(60° + 15°)$

$= \tan(3 \times 15°) = \tan 45° = 1$

14. (a) $\left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1+\cos\left(\pi-\dfrac{3\pi}{8}\right)\right)\left(1+\cos\left(\pi-\dfrac{\pi}{8}\right)\right)$

$= \left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1-\cos\dfrac{3\pi}{8}\right)\left(1-\cos\dfrac{\pi}{8}\right)$

$= \left(1-\cos^2\dfrac{\pi}{8}\right)\left(1-\cos^2\dfrac{3\pi}{8}\right)$

$= \frac{1}{4}\left(2-1-\cos\frac{\pi}{4}\right)\left(2-1-\cos\frac{3\pi}{4}\right)$

$= \frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)$

$= \frac{1}{4}\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right). = \frac{1}{4}\left(1-\frac{1}{2}\right) = \frac{1}{8}$

15. (a) The large hand of the clock makes a complete revolution in 60 minutes.

$\therefore$ Angle traced out by the large hand in 20 minutes (of time)

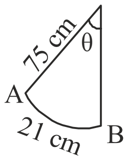
$= \frac{360° \times 20}{60} = 120° = \frac{120\pi}{180}$ radian $= \frac{2\pi}{3}$ radian

Hence, the distance moved by the extremity of the large hand = (42) × $\frac{2\pi}{3}$ = 88 cm. ($\because$ $l = r\theta$)

16. (a) Given, length of pendulum = 75 cm

Radius (r) = length of pendulum = 75 cm

Length of arc (l) = 21 cm



Now, $\theta = \frac{l}{r} = \frac{21}{75} = \frac{7}{25}$ radian.

17. (c) Let l be the length of the arc. We know that,

Angle $\theta = \frac{l}{r}$, where θ is in radian.

Given, r = 3 cm

$\theta = 30° = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$ rad

On putting the values of r and θ, we get

$\frac{\pi}{6} = \frac{l}{3} \Rightarrow l = \frac{\pi}{2} = \frac{3.14}{2} = 1.57$ cm.

18. (b) Circumference of a circular wire of radius 7 cm is $= 2\pi \times 7 = 14\pi$

As we know, $\theta = \frac{l}{r}$

$\Rightarrow \theta = \frac{14\pi}{12} = \frac{7\pi \times 180°}{6\pi} = 210°.$

19. (c) Length of wire = 2π × 3 = 6π cm and r = 48 cm is the radius of the circle. Therefore, the angle θ (in radian) subtended at the centre of the circle is given by

$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{6\pi}{48} = \frac{\pi}{8} = 22.5°.$

20. (c) Here, l = 37.4 cm and $\theta = 60° = \frac{60\pi}{180}$ radian $= \frac{\pi}{3}$

Hence, by $r = \frac{l}{\theta}$, we have

$r = \frac{37.4 \times 3}{\pi} = \frac{37.4 \times 3 \times 7}{22} = 35.7$ cm.

21. (b) Given radius, r = 100 cm and arc length, l = 22 cm

We know that, $l = r\theta$

$\theta = \frac{l}{r} = \frac{\text{Arc length}}{\text{Radius}}$

$= \frac{22}{100} = 0.22$ rad $= 0.22 \times \frac{180}{\pi}$ degree

$= 0.22 \times \frac{180 \times 7}{22} = \frac{22}{100} \times \frac{180 \times 7}{22}$

$= \frac{126}{10} = 12\frac{6°}{10} = 12° + \frac{6}{10} \times 60'$ $[\because 1° = 60']$

$= 12° + 36' = 12°\ 36'$

Hence, the degree measure of the required angle is 12°36′.

22. (c) Given, $\tan\theta = \frac{3}{1}$ and θ lies in III quadrant.

We know that $\sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{3}{1}\right)^2 = 10$

$\Rightarrow \sec\theta = \pm\sqrt{10}$

Since, θ lies in III quadrant, so $\sec\theta = -\sqrt{10}$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\sqrt{10}}$

Also,

$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\frac{1}{\sqrt{10}}\right)^2$

$= 1 - \frac{1}{10} = \frac{9}{10}$

$\Rightarrow \sin\theta = \pm\sqrt{\frac{9}{10}}$

Since, θ lies in III quadrant so $\sin\theta = -\sqrt{\frac{9}{10}} = \frac{-3}{\sqrt{10}}.$

23. (b) $\frac{\sin x}{\cos x} \times \frac{\sec x}{\text{cosec } x} \times \frac{\tan x}{\cot x} = 9$

$\Rightarrow \tan x \times \tan x \times \frac{\tan x}{\cot x} = 9$

$\Rightarrow \tan^4 x = 9$

$\Rightarrow \tan x = \pm\sqrt{3}$

$\Rightarrow x = \frac{\pi}{3} \in \left(0, \frac{\pi}{2}\right).$

24. (b) The given equation is

$\text{cosec}(90° + \theta) + x\cos\theta\cot(90° + \theta) = \sin(90° + \theta)$

$\Rightarrow \sec\theta + x\cos\theta(-\tan\theta) = \cos\theta$

$\Rightarrow \sec\theta - x\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) = \cos\theta$

$\Rightarrow \sec\theta - x\sin\theta = \cos\theta$

$\Rightarrow x\sin\theta = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$

$\Rightarrow x\sin\theta = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$

$\Rightarrow x = \tan\theta.$

25. (d) We have $A + B = \frac{\pi}{4}$

$\Rightarrow \cot(A+B) = \cot\frac{\pi}{4} \Rightarrow \frac{\cot A\cot B - 1}{\cot A + \cot B} = 1$

$\Rightarrow \cot A\cot B - 1 = \cot A + \cot B$

$\Rightarrow$ cot A cot B – cot A – cot B – 1 = 0
$\Rightarrow$ cot A cot B – cot A – cot B + 1 = 2
$\Rightarrow$ cot A(cot B – 1) – 1(cot B – 1) = 2
$\Rightarrow$ (cot A – 1) (cot B – 1) = 2.

26. (a) We have, $\sin A = \frac{3}{5}$

$\Rightarrow \cos A = \sqrt{1-\sin^2 A}$

$= \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

and $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

Now, $\sin 2A = 2\sin A \cdot \cos A = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

$\cos 2A = 1 - 2\sin^2 A = 1 - 2 \times \frac{9}{25} = 1 - \frac{18}{25} = \frac{7}{25}$

and $\tan 2A = \frac{24}{7}$.

27. (b) Given, $\cot \alpha = \frac{1}{2} \Rightarrow \tan \alpha = 2$ and $\sec \beta = \frac{-5}{3}$

Then, $\tan \beta = \sqrt{\sec^2 \beta - 1}$

$\Rightarrow \tan \beta = \pm\sqrt{\frac{25}{9} - 1} = \pm\sqrt{\frac{16}{9}}$

$\Rightarrow \tan \beta = \pm\frac{4}{3}$

But, $\tan \beta = \frac{-4}{3}$

[$\because$ tan β is negative in IInd quadrant]

$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = \frac{2 + \left(-\frac{4}{3}\right)}{1 - (2)\left(\frac{-4}{3}\right)}$

$= \frac{\left(2 - \frac{4}{3}\right)}{\left(1 + \frac{8}{3}\right)} = \frac{2}{11}$.

28. (a) tan 75° – cot 75° = tan(45° + 30°) – cot(45° + 30°)

$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°} - \frac{\cot 45° \cot 30° - 1}{\cot 45° + \cot 30°}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} - \frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} - \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{(3+1+2\sqrt{3})}{3-1} - \frac{(3+1-2\sqrt{3})}{3-1} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}.$

29. (a) tan 3A = tan(2A + A)

$\Rightarrow \tan 3A = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$

$\Rightarrow$ tan 3A – tan 3A tan 2A tan A = tan 2A + tan A
$\Rightarrow$ tan 3A – tan 2A – tan A = tan 3A tan 2A tan A

30. (c) Given, $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ … (i)

Now, tan(2A + B)

$= \frac{\tan 2A + \tan B}{1 - \tan 2A \tan B} = \frac{\frac{2\tan A}{1-\tan^2 A} + \tan B}{1 - \frac{2\tan A}{1-\tan^2 A} \times \tan B}$

$= \frac{\left(\frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}}\right) + \frac{1}{3}}{1 - \left(\frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}}\right) \times \frac{1}{3}} = \frac{\frac{4}{3} + \frac{1}{3}}{1 - \frac{4}{3} \times \frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{9}} = 3.$

31. (b) We have, $\tan\theta = \frac{a}{b}$

Now, b cos 2θ + a sin 2θ

$= b\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + a\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= b\left(\frac{1 - \frac{a^2}{b^2}}{1 + \frac{a^2}{b^2}}\right) + a\left(\frac{2 \times \frac{a}{b}}{1 + \frac{a^2}{b^2}}\right)$

$= b\left(\frac{b^2 - a^2}{b^2 + a^2}\right) + \left(\frac{2\frac{a^2}{b} \times b^2}{b^2 + a^2}\right)$

$= \frac{1}{b^2 + a^2}[b^3 - a^2 b + 2a^2 b]$

$= \frac{1}{(b^2 + a^2)} \times b(a^2 + b^2) = b.$

32. (c) Given, equation is $\tan x + \sec x = 2\cos x$

$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$\Rightarrow 1 + \sin x = 2\cos^2 x$

$\Rightarrow 1 + \sin x = 2(1 - \sin^2 x)$
$\Rightarrow 2\sin^2 x + 2\sin x - \sin x - 1 = 0$
$\Rightarrow 2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$
$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow$ either $\sin x = \dfrac{1}{2}$ or $\sin x = -1$

$\Rightarrow$ either $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6} \in [0, \pi]$ or $x = \dfrac{3\pi}{2}$

But, $x = \dfrac{3\pi}{2}$ can not be possible.

$\therefore$ Number of solutions are 2.

33. (b) Since A and B both lie in the IV quadrant, it follows that $\sin A$ and $\sin B$ are negative. Therefore,

$\sin A = -\sqrt{1-\cos^2 A}$

$\Rightarrow \sin A = -\sqrt{1-\dfrac{16}{25}} = -\dfrac{3}{5}$

and, $\sin B = -\sqrt{1-\cos^2 B}$

$\Rightarrow \sin B = -\sqrt{1-\dfrac{144}{169}} = -\dfrac{5}{13}$

Now, $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \dfrac{4}{5}\times\dfrac{12}{13} - \left(\dfrac{-3}{5}\right)\left(\dfrac{-5}{13}\right) = \dfrac{33}{65}$

34. (a) π radians $= 180°$

$1° = \dfrac{\pi}{180}$ radians

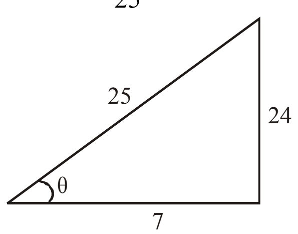
$\therefore \quad 25° = 25\times\dfrac{\pi}{180} = \dfrac{5\pi}{36}$

35. (b) Since $\tan\theta = -\dfrac{4}{3}$ is negative, θ lies either in second quadrant or in fourth quadrant. Thus $\sin\theta = \dfrac{4}{5}$ if θ lies in the second quadrant

or $\sin\theta = -\dfrac{4}{5}$, if θ lies in the fourth quadrant.

36. (c) $\cos(A+B).\cos(A-B) = (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$
$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B.$
$= \cos^2 A(1-\sin^2 B) - \sin^2 A\sin^2 B$
$= \cos^2 A - \cos^2 A\sin^2 B - \sin^2 A\sin^2 B$
$= \cos^2 A - \sin^2 B(\cos^2 A + \sin^2 A)$
$= \cos^2 A - \sin^2 B$

37. (c) We have, $\sin\theta = \dfrac{24}{25}, 0° < \theta < 90°$



$\cos^2\theta = 1-\sin^2\theta = 1-\left(\dfrac{24}{25}\right)^2$

Since θ lies in first quadrant $\Rightarrow \cos\theta = \dfrac{7}{25}$

$\cos\theta = 1-2\sin^2\dfrac{\theta}{2}$

$2\sin^2\dfrac{\theta}{2} = 1-\cos\theta = 1-\dfrac{7}{25}$

$2\sin^2\dfrac{\theta}{2} = \dfrac{18}{25}$

$\sin^2\dfrac{\theta}{2} = \dfrac{9}{25} \Rightarrow \sin\dfrac{\theta}{2} = \pm\dfrac{3}{5}$

$\Rightarrow \sin\dfrac{\theta}{2} = \dfrac{3}{5}$

[Negative sign discarded since θ is in first quadrant]

38. (b) $\sin\dfrac{5\pi}{12} = \sin 75°$

$= \sin(45°+30°) = \sin 45°\cos 30° + \cos 45°\sin 35°$

$= \dfrac{1}{\sqrt{2}}.\dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}}.\dfrac{1}{2} = \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{3}+1}{2}\right)$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

39. (b) Given : $x + \dfrac{1}{x} = 2\cos\theta$ ...(i)

Cubic both sides in eq^n (i) we get

$x^3 + \dfrac{1}{x^3} + 3\left(x + \dfrac{1}{x}\right) = 8\cos^3\theta$

$\Rightarrow \quad x^3 + \dfrac{1}{x^3} + 3(2\cos\theta) = 8\cos^3\theta$

$\Rightarrow \quad x^3 + \dfrac{1}{x^3} = 8\cos^3\theta - 6\cos\theta$

$\Rightarrow \quad x^3 + \dfrac{1}{x^3} = 2(4\cos^3\theta - 3\cos\theta) = 2\cos 3\theta$

40. (c) $1 + \cot\theta = \operatorname{cosec}\theta$

$\Rightarrow \dfrac{1}{\sin\theta} = 1 + \dfrac{\cos\theta}{\sin\theta} \Rightarrow \sin\theta + \cos\theta = 1$

$\sin\theta\sin\dfrac{\pi}{4} + \cos\theta\cos\dfrac{\pi}{4} = \cos\dfrac{\pi}{4}$

$\Rightarrow \cos\left(\theta - \dfrac{\pi}{4}\right) = \cos\dfrac{\pi}{4} \Rightarrow \theta - \dfrac{\pi}{4} = 2n\pi \pm \dfrac{\pi}{4}$

Hence, $\theta = 2n\pi$ or $\theta = 2n\pi + \dfrac{\pi}{2}$

But, $\theta = 2n\pi$ is ruled out

41. (b) $\sin 3\alpha = 4\sin\alpha\sin(x+\alpha)\sin(x-\alpha)$

$\therefore \quad \sin 3\alpha = 4\sin\alpha(\sin^2 x\cos^2\alpha - \cos^2 x\sin^2\alpha)$

$\therefore \quad 3\sin\alpha - 4\sin^3\alpha = 4\sin\alpha(\sin^2 x - \sin^2\alpha)$

$\therefore \quad \sin^2 x = \left(\dfrac{3}{4}\right) \Rightarrow \sin^2 x = \sin^2\dfrac{\pi}{3}$

$\therefore \quad x = n\pi \pm \dfrac{\pi}{3}.$

42. (b) $\tan\theta + \tan\left(\dfrac{\pi}{2} - \theta\right) = 2$

$\Rightarrow \quad \tan\theta + \dfrac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4} \Rightarrow \theta = n\pi + \frac{\pi}{4}.$

43. (b) $\sin^2\theta\sec\theta + \sqrt{3}\tan\theta = 0$

$\therefore \left(\sin^2\theta + \sqrt{3}\sin\theta\right)\sec\theta = 0$

$\therefore \sin\theta\left(\sin\theta + \sqrt{3}\right)\sec\theta = 0$

$\Rightarrow \sin\theta = 0$

$\therefore \theta = n\pi,\ n \in I \quad \left[\because \sin\theta \neq -\sqrt{3},\ \sec\theta \neq 0\right]$

44. (c) $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

$\therefore \tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

$\therefore \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3} \Rightarrow \tan 3\theta = \tan\frac{\pi}{3}$

$\therefore 3\theta = n\pi + \frac{\pi}{3} \Rightarrow \theta = (3n+1)\frac{\pi}{9}.$

45. (a)

46. (c) $\sec 4\theta - \sec 2\theta = 2$

$\therefore \cos 2\theta - \cos 4\theta = 2\cos 4\theta\cos 2\theta$

$\therefore -\cos 4\theta = \cos 6\theta \Rightarrow 2\cos 5\theta\cos\theta = 0$

$\therefore \theta = n\pi + \frac{\pi}{2}$ or $\frac{n\pi}{5} + \frac{\pi}{10}.$

47. (d) $\tan\theta\tan 2\theta = 1$

$\therefore \tan\theta\,\frac{2\tan\theta}{1-\tan^2\theta} = 1$

$\therefore 2\tan^2\theta = 1 - \tan^2\theta$

$\therefore 3\tan^2\theta = 1$

$\therefore \tan^2\theta = \frac{1}{3} = \tan^2\left(\frac{\pi}{6}\right)$

$\therefore \theta = n\pi \pm \frac{\pi}{6}.$

48. (d) $\cot\theta + \cot\left(\frac{\pi}{4}+\theta\right) = 2$

$\therefore \frac{\cos\theta}{\sin\theta} + \frac{\cos\left(\frac{\pi}{4}+\theta\right)}{\sin\left(\frac{\pi}{4}+\theta\right)} = 2$

$\therefore \sin\left(\frac{\pi}{4}+2\theta\right) = 2\sin\theta\sin\left(\frac{\pi}{4}+\theta\right)$

$= \cos\left(\theta - \frac{\pi}{4} - \theta\right) - \cos\left(\theta + \frac{\pi}{4} + \theta\right)$

$\therefore \sin\left(\frac{\pi}{4}+2\theta\right) = \cos\left(\frac{-\pi}{4}\right) - \cos\left(2\theta + \frac{\pi}{4}\right)$

$\Rightarrow \sin\left(\frac{\pi}{4}+2\theta\right) + \cos\left(\frac{\pi}{4}+2\theta\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\cos 2\theta + \frac{1}{\sqrt{2}}\sin 2\theta\right)$

$+ \left(\frac{1}{\sqrt{2}}\cos 2\theta - \frac{1}{\sqrt{2}}\sin 2\theta\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{2}{\sqrt{2}}\cos 2\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 2\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}.$

49. (a) $2\cos^2 x + 3\sin x - 3 = 0$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ or $\sin x = 1$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$, i.e. 30°, 150°, 90°.

50. (c) $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$

$\Rightarrow \frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}.$

51. (b) $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta\tan 3\theta = 1$

$\Rightarrow \frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta\tan 3\theta} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan 5\theta = \tan\frac{\pi}{6}$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{6} \Rightarrow \theta = \left(n + \frac{1}{6}\right)\frac{\pi}{5}.$

52. (c) $\cos 7\theta = \cos\theta - \sin 4\theta$

$\sin 4\theta = \cos\theta - \cos 7\theta$

$\Rightarrow \sin 4\theta = 2\sin(4\theta)\sin(3\theta)$

$\Rightarrow \sin 4\theta = 0 \Rightarrow 4\theta = n\pi$ or

$\sin 3\theta = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right) \Rightarrow 3\theta = n\pi + (-1)^n\frac{\pi}{6}$

$\therefore \theta = \frac{n\pi}{4} = \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}.$

53. (d) In anti-clockwise rotation, the angle is said to be positive.
In clockwise rotation, the angle is said to be negative.
The measure of an angle is the amount of rotation performed to get the terminal side from the initial side.
So, all the statements are correct.

54. (c) Angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of 1 radian.

55. (b) We know that, 180° = π radian

Hence, $40°\,20' = 40\frac{1}{3}$ degree $\quad [\because\ 1° = 60']$

$= \frac{\pi}{180} \times \frac{121}{3}$ radian $= \frac{121\pi}{540}$ radian

Therefore, $40°\,20' = \frac{121\pi}{540}$ radian.

56. (b) π radian = 180°.

57. (a) We know that, values of $\sin x$ repeats after an interval of 2π. Therefore,

$\sin\frac{31\pi}{3} = \sin\left(10\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

58. (b) $\cot\left(-\frac{15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right)$ [$\because$ cot(–θ) = –cot θ]

$= -\cot\left(4\pi - \frac{\pi}{4}\right) = -\cot\left(2\pi \times 2 - \frac{\pi}{4}\right)$

$= -\left(-\cot\frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1$

[$\because$ cot(2nπ – θ) = –cot θ]

59. (d) In III quadrant, only tan θ and cot θ are positive.

$\sin^2\theta = (1 - \cos^2\theta) = \left(1 - \frac{9}{25}\right) = \frac{16}{25}$

$\Rightarrow \sin\theta = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$

$\Rightarrow \sin\theta = \frac{-4}{5}$ (as sin θ is negative in 3rd quadrant)

$\therefore \tan\theta = \left(\frac{-4}{5} \times \frac{5}{-3}\right) = \frac{4}{3}$

and $\cot\theta = \frac{3}{4} \Rightarrow \operatorname{cosec}\theta = \frac{-5}{4}$

and $\sec\theta = \frac{-5}{3}$

$\therefore \frac{(\operatorname{cosec}\theta + \cot\theta)}{(\sec\theta - \tan\theta)} = \frac{\left(\frac{-5}{4} + \frac{3}{4}\right)}{\left(\frac{-5}{3} - \frac{4}{3}\right)} = \frac{\left(\frac{-2}{4}\right)}{\left(\frac{-9}{3}\right)}$

$= \frac{-2}{4} \times \frac{3}{-9} = \frac{1}{6}.$

60. (c) $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$

$= -2\sin\frac{\frac{3\pi}{4} + x + \frac{3\pi}{4} - x}{2}\sin\frac{\frac{3\pi}{4} + x - \frac{3\pi}{4} + x}{2}$

$= -2\sin\frac{3\pi}{4}\sin x = -2\sin\left(\pi - \frac{\pi}{4}\right)\sin x$

$= -2\sin\frac{\pi}{4}\sin x = -2 \times \frac{1}{\sqrt{2}}\sin x = -\sqrt{2}\sin x$

61. (d) $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x}$

$= \frac{2\cos\frac{3x + x}{2} \cdot \sin\frac{3x - x}{2}}{\cos 2x} = \frac{2\cos 2x \cdot \sin x}{\cos 2x}$

$= 2\sin x$

62. (a) We have,

$\sin x = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3} = \sin\left(\pi + \frac{\pi}{3}\right) = \sin\frac{4\pi}{3}$

Hence, $\sin x = \sin\frac{4\pi}{3}$, which gives

$x = n\pi + (-1)^n\frac{4\pi}{3}$, where $n \in Z$.

Note: $\frac{4\pi}{3}$ is one such value of x for which $\sin x = -\frac{\sqrt{3}}{2}$. One may take any other value of x for which $\sin x = -\frac{\sqrt{3}}{2}$. The solutions obtained will be the same although these may apparently look different.

63. (b) Given that, $x = \sec\theta + \tan\theta$

$\Rightarrow x + \frac{1}{x} = \sec\theta + \tan\theta + \frac{1}{\sec\theta + \tan\theta}$

$= \sec\theta + \tan\theta + \sec\theta - \tan\theta = 2\sec\theta$

Aliter: $x = \frac{1 + \sin\theta}{\cos\theta}$

$\therefore x + \frac{1}{x} = \frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta}$

$\Rightarrow \frac{2(1 + \sin\theta)}{\cos\theta(1 + \sin\theta)} = 2\sec\theta.$

64. (b) $\frac{\tan 70° - \tan 20°}{\tan 50°} = \frac{\frac{\sin 70°}{\cos 70°} - \frac{\sin 20°}{\cos 20°}}{\frac{\sin 50°}{\cos 50°}}$

$= \frac{\frac{\sin 70°\cos 20° - \cos 70°\sin 20°}{\cos 70°\cos 20°}}{\frac{\sin 50°}{\cos 50°}}$

$= \frac{2}{2} \times \frac{\sin(70° - 20°)\cos 50°}{\cos 70°\cos 20°\sin 50°}$

$= \frac{2\sin 50°\cos 50°}{2\cos 70°\cos 20°\sin 50°}$

$= \frac{2\cos 50°}{\cos 90° + \cos 50°} = \frac{2\cos 50°}{0 + \cos 50°} = 2.$

65. (d) $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = \frac{\left[\cos 10° - \sqrt{3}\sin 10°\right]}{\sin 10°\cos 10°}$

$= \frac{2\left[\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°\right]}{\sin 10°\cos 10°}$

$= \frac{2[\sin 30°\cos 10° - \cos 30°\sin 10°]}{\sin 10°\cos 10°}$

$= \dfrac{2[\sin(30° - 10°)]}{\sin 10° \cos 10°}$

$= \dfrac{2 \cdot 2\sin(30° - 10°)}{2\sin 10° \cos 10°} = \dfrac{4\sin 20°}{\sin 20°} = 4.$

66. **(a)** $\cos^2\dfrac{\pi}{12} + \cos^2\dfrac{\pi}{4} + \cos^2\dfrac{5\pi}{12}$

$= 1 - \sin^2\left(\dfrac{\pi}{12}\right) + \left(\dfrac{1}{\sqrt{2}}\right)^2 + \cos^2\left(\dfrac{5\pi}{12}\right)$

$= 1 + \dfrac{1}{2} + \left(\cos^2\dfrac{5\pi}{12} - \sin^2\dfrac{\pi}{12}\right)$

$= \dfrac{3}{2} + \cos\left(\dfrac{5\pi}{12} + \dfrac{\pi}{12}\right)\cos\left(\dfrac{5\pi}{12} - \dfrac{\pi}{12}\right)$

$= \dfrac{3}{2} + \cos\dfrac{\pi}{2}\cos\dfrac{\pi}{3} = \dfrac{3}{2} + 0 \cdot \dfrac{1}{2} = \dfrac{3}{2}.$

67. **(c)** $1 + \cos 2x + \cos 4x + \cos 6x$
$= (1 + \cos 6x) + (\cos 2x + \cos 4x)$
$= 2\cos^2 3x + 2\cos 3x \cos x$
$= 2\cos 3x(\cos 3x + \cos x)$
$= 4\cos x \cos 2x \cos 3x.$

68. **(a)** $\text{cosec A} - 2\cot 2A \cos A = \dfrac{1}{\sin A} - \dfrac{2\cos A \cos 2A}{\sin 2A}$

$= \dfrac{1}{\sin A} - \dfrac{2\cos A\cos 2A}{2\sin A\cos A} = \dfrac{1 - \cos 2A}{\sin A} = \dfrac{2\sin^2 A}{\sin A}$
$= 2\sin A.$

69. **(d)** $\sin x + \cos x = \dfrac{1}{5}$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x\cos x = \dfrac{1}{25}$

$\sin 2x = \dfrac{-24}{25} \Rightarrow \cos 2x = \dfrac{-7}{25} \Rightarrow \tan 2x = \dfrac{24}{7}.$

70. **(b)** $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta\tan 3\theta = 1$

$\Rightarrow \sqrt{3}(\tan 2\theta + \tan 3\theta) = 1 - \tan 2\theta\tan 3\theta$

$\Rightarrow \dfrac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta\tan 3\theta} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \tan 5\theta = \tan\dfrac{\pi}{6}$

$\Rightarrow 5\theta = n\pi + \dfrac{\pi}{6} \Rightarrow \theta = \left(n + \dfrac{1}{6}\right)\dfrac{\pi}{5}.$

71. **(d)** $\tan\theta - \sqrt{2}\sec\theta = \sqrt{3}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} - \dfrac{\sqrt{2}}{\cos\theta} = \sqrt{3}$

$\Rightarrow \sin\theta - \sqrt{2} = \sqrt{3}\cos\theta$

$\Rightarrow \sin\theta - \sqrt{3}\cos\theta = \sqrt{2}$

$\Rightarrow \sin\left(\theta - \dfrac{\pi}{3}\right) = \sin\dfrac{\pi}{4}$

$\Rightarrow \theta = n\pi + (-1)^n\dfrac{\pi}{4} + \dfrac{\pi}{3}.$

72. **(b)** The given equation can be written as

$\dfrac{\sin^2\theta}{\cos\theta} + \sqrt{3}\tan\theta = 0$

$\Rightarrow \tan\theta\sin\theta + \sqrt{3}\tan\theta = 0$

$\tan\theta(\sin\theta + \sqrt{3}) = 0$ as $\sin\theta \neq -\sqrt{3}$

Hence, $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in Z.$

73. **(a)** $\tan(90° - \theta) = \cot\theta, \cot(90° - \theta) = \tan\theta$

Therefore, $\dfrac{\cot 54°}{\tan 36°} + \dfrac{\tan 20°}{\cot 70°}$

$= \dfrac{\cot 54°}{\tan(90° - 54°)} + \dfrac{\tan 20°}{\cot(90° - 20°)}$

$= \dfrac{\cot 54°}{\cot 54°} + \dfrac{\tan 20°}{\tan 20°} = 1 + 1 = 2.$

STATEMENT TYPE QUESTIONS

74. **(b)** We have

$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\dfrac{3}{2}$

$\Rightarrow 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)] + 3 = 0$

$\Rightarrow 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)]$
$+ \sin^2\alpha + \cos^2\alpha + \sin^2\beta + \cos^2\beta + \sin^2\gamma + \cos^2\alpha = 0$

$\Rightarrow [\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2\sin\alpha\sin\beta + 2\sin\beta\sin\gamma$
$+ 2\sin\gamma\sin\alpha] + [\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta$
$+ 2\cos\beta\cos\gamma + 2\cos\gamma\cos\alpha] = 0$

$\Rightarrow [\sin\alpha + \sin\beta + \sin\gamma]^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$

$\Rightarrow \sin\alpha + \sin\beta + \sin\gamma = 0$ and $\cos\alpha + \cos\beta + \cos\gamma = 0$

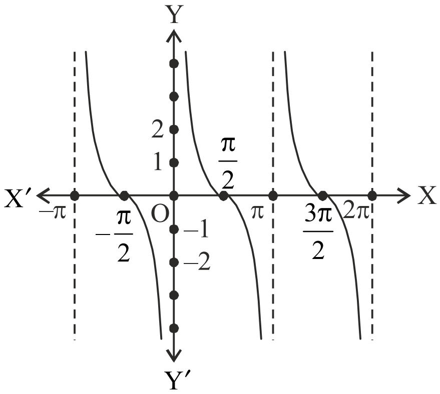
$\therefore$ I and II both are true.

75. **(a)** The signs of trigonometric functions in different quadrants are shown below

	I	II	III	IV
$\sin x$	+	+	–	–
$\cos x$	+	–	–	+
$\tan x$	+	–	+	–
$\text{cosec } x$	+	+	–	–
$\sec x$	+	–	–	+
$\cot x$	+	–	+	–

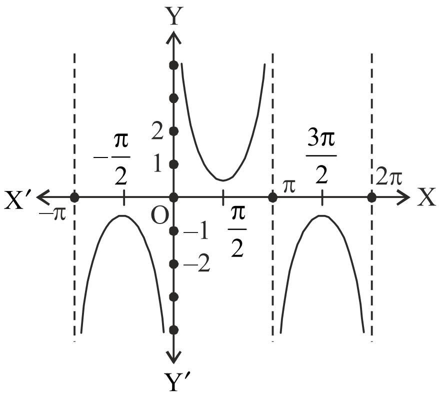
According to the above table, option (a) is correct.

76. **(d)** Using behaviour of trigonometric functions we can draw the graphs of $y = \cot x$, $y = \text{cosec } x$ and $y = \sec x$ as shown below.



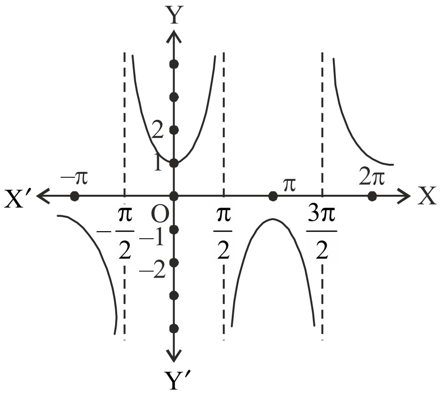
y = cot x

So, we see that the values of cot x repeat after an interval of π.



y = cosec x

Also, we can see that the values of cosec x repeat after an interval of 2π by using above graph of y = cosec x. Similarly, we can say that the values of sec x repeat after an interval of 2π by using the graph of y = sec x as shown below.



y = sec x

Hence, it is concluded that all the given statements are true.

77. (a) Only option (a) is incorrect.

78. (b) As a part of identities from above, we can also show that

I. $2\cos x\cos y = \cos(x+y) + \cos(x-y)$

II. $-2\sin x\sin y = \cos(x+y) - \cos(x-y)$

III. $2\sin x\cos y = \sin(x+y) + \sin(x-y)$

IV. $2\cos x\sin y = \sin(x+y) - \sin(x-y)$

Hence, option (b) is correct.

79. (d) $\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x\cos x + \cos x = 0$ $\quad [\because \sin 2x = 2\sin x\cos x]$

$\Rightarrow \cos x(2\sin x + 1) = 0$

$\Rightarrow \cos x = 0$ or $\sin x = -\frac{1}{2}$

When $\cos x = 0$,

Then, $x = (2n+1)\frac{\pi}{2}$

When $\sin x = -\frac{1}{2}$,

Then, $\sin x = -\sin\frac{\pi}{6}$

$\sin x = \sin\left(\pi + \frac{\pi}{6}\right)$ $\quad [\because \sin(\pi+\theta) = -\sin\theta]$

$\sin x = \sin\frac{7\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n\frac{7\pi}{6}.$ $\quad [n \in Z]$

MATCHING TYPE QUESTIONS

80. (b) We know that,

$$\text{Radian measure} = \frac{\pi}{180} \times \text{Degree measure}$$

A. Radian measure of $25° = \frac{\pi}{180} \times 25° = \frac{5\pi}{36}.$

B. We know that, $30' = \left(\frac{1}{2}\right)^{o}$ $\quad [\because 60' = 1°]$

$\therefore\ -47°\,30' = -\left(47\frac{1}{2}\right)^{o} = \left(\frac{-95}{2}\right)^{o}$

$\therefore$ Radian measure of $(-47°\,30') = \frac{\pi}{180} \times \left(\frac{-95}{2}\right)$

$= \frac{-19\pi}{72}.$

C. Radian measure of $240° = \frac{\pi}{180} \times 240 = \frac{4\pi}{3}.$

D. Radian measure of $520° = \frac{\pi}{180} \times 520 = \frac{26\pi}{9}$

81. (c) We know that

$$\text{Degree measure} = \frac{180}{\pi} \times \text{Radian measure}$$

A. Degree measure of $\frac{11}{16} = \left(\frac{180}{\pi} \times \frac{11}{16}\right)^{o}$

$= \left(\frac{180}{22} \times \frac{11}{16} \times 7\right)^{o}$ $\quad \left[\because \pi = \frac{22}{7}\right]$

$= \left(\frac{90 \times 7}{16}\right)^{o} = \left(\frac{315}{8}\right)^{o}$

$= \left(39\frac{3}{8}\right)^o = 39^\circ\left(\frac{3}{8}\times 60\right)'$ $[\because 1^\circ = 60']$

$= 39^\circ\left(22\frac{1}{2}\right)' = 39^\circ\, 22'\left(\frac{1}{2}\times 60\right)''$ $[\because 1' = 60'']$

$= 39^\circ\, 22'\, 30''$.

B. Degree measure of $-4 = \left(\frac{180}{\pi}\times -4\right)^o$

$= \left(\frac{180}{22}\times -4\times 7\right)^o$ $\left[\because \pi = \frac{22}{7}\right]$

$= \left(\frac{90\times(-28)}{11}\right)^o = -\left(\frac{2520}{11}\right)^o = -\left(229\frac{1}{11}\right)^o$

$= -229^\circ\left(\frac{1}{11}\times 60'\right)$ $[\because 1^\circ = 60']$

$= -229^\circ\left(5\frac{5}{11}\right)'$

$= -229^\circ\, 5'\left(\frac{5}{11}\times 60\right)''$ $[\because 1' = 60'']$

$\approx -229^\circ\, 5'\, 27.3'' \approx -229^\circ\, 5'\, 27''$ (approx.)

C. Degree measure of $\frac{5\pi}{3} = \left(\frac{180}{\pi}\times\frac{5\pi}{3}\right)^o = 300^\circ$.

D. Degree measure of $\frac{7\pi}{6} = \left(\frac{180}{\pi}\times\frac{7\pi}{6}\right)^o = 210^\circ$

82. **(a)** A. $\sin\frac{25\pi}{3} = \sin\left(8\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$[\because \sin(2n\pi + \theta) = \sin\theta]$

B. $\cos\frac{41\pi}{4} = \cos\left(10\pi + \frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$[\because \cos(2n\pi + \theta) = \cos\theta]$

C. $\tan\left(\frac{-16\pi}{3}\right) = -\tan\frac{16\pi}{3}$ $[\because \tan(-\theta) = -\tan\theta]$

$= -\tan\left(5\pi + \frac{\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3}$

$[\because \tan(n\pi + \theta) = \tan\theta]$

D. $\cot\frac{29\pi}{4} = \cot\left(7\pi + \frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1$

$[\because \cot(n\pi + \theta) = \cot\theta]$

83. **(a)** By taking suitable values of x and y in the identities, we get the following results:

$\cos(\pi - x) = -\cos x$; $\sin(\pi - x) = \sin x$

$\cos(\pi + x) = -\cos x$; $\sin(\pi + x) = -\sin x$

$\cos(2\pi - x) = \cos x$; $\sin(2\pi - x) = -\sin x$

84. **(c)** A. 1 radian $= \frac{180^\circ}{\pi} = 57^\circ\, 16'$ (approx.)

B. $1^\circ = \left(\frac{\pi}{180}\right)^c = 0.01746$ radian (approx.)

C. $3^\circ\, 45' = \left(3\frac{45}{60}\right)^o = \left(3\frac{3}{4}\right)^o = \left(\frac{15}{4}\right)^o$

Also, $180^\circ = \pi$ radian

$\Rightarrow 1^\circ = \frac{\pi}{180}$ radian

$\Rightarrow \left(\frac{15}{4}\right)^o = \frac{\pi}{180}\times\frac{15}{4} = \frac{\pi}{48}$ radian

D. $50^\circ\, 37'\, 30'' = 50^\circ + \left(37\frac{30}{60}\right)'$

$= 50^\circ + \left(\frac{75}{2}\right)' = 50^\circ + \left(\frac{75}{2\times 60}\right)^o$

$= \left(\frac{405}{8}\right)^o = \left(\frac{\pi}{180}\times\frac{405}{8}\right)^c = \frac{9\pi}{32}$ radian

85. **(c)** π radians $= 180^\circ$

$\Rightarrow 1^\circ = \frac{\pi}{180}$ radians

(A) $25^\circ = 25\times\frac{\pi}{180} = \frac{5\pi}{36}$

(B) $60' = 1^\circ$ $\therefore 30' = \frac{30^\circ}{60} = \frac{1}{2}^\circ$

$\therefore$ $47^\circ 30' = \left(47 + \frac{1}{2}\right)^o = \left(\frac{95}{2}\right)^o$

$\therefore$ $180^\circ = \pi$ radian

$-\frac{95^\circ}{2} = \frac{-\pi}{180}\times\frac{95}{2}$ radians $= \frac{-19\pi}{72}$ radians

(C) $240^\circ = 240\times\frac{\pi}{180} = \frac{4\pi}{3}$ radians.

(D) $180^\circ = \pi$ radians

$520^\circ = \frac{\pi}{180}\times 520$ radians $= \frac{26\pi}{9}$ radians

86. **(a)** Since x lies in the 3rd quadrant

$\cos x = -\frac{1}{2}$

$\therefore$ $\sin x = -\sqrt{1-\cos^2 x}$ ($\because x$ lies in III rd quadrant)

$= -\sqrt{1-\frac{1}{4}} = -\sqrt{3}/2$

$\tan x = \sqrt{3}$, $\cot x = \frac{1}{\sqrt{3}}$

$\sec x = \left(\frac{1}{\cos x}\right) = -2$, $\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{2}{\sqrt{3}}$

87. **(b)** A. $\cos 4x = \cos 2x$

$\Rightarrow$ $4x = 2n\pi \pm 2x$

Taking + ve sign, we get

$4x = 2n\pi + 2x$

$\Rightarrow$ $4x - 2x = 2n\pi$

$\Rightarrow \quad x = n\pi, n \in Z$

Taking – ve sign

$4x = 2n\pi - 2x$

$\Rightarrow \quad 4x + 2x = 2n\pi$

$\Rightarrow \quad 6x = 2n\pi$

$\Rightarrow \quad x = \frac{n\pi}{3}, n \in Z$

$\therefore$ General solution is $x = \frac{n\pi}{3}$ or $x = n\pi, n \in Z$

B. $\cos 3x + \cos x - \cos 2x = 0$

or $\quad 2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2} - \cos 2x = 0$

or $\quad 2\cos 2x \cos x - \cos 2x = 0$

or $\quad \cos 2x\,(2\cos x - 1) = 0$

If $\cos 2x = 0,\ 2x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{4}$

If $2\cos x - 1 = 0$, $\cos x = \frac{1}{2} \Rightarrow \cos x = \cos\frac{\pi}{3}$

$\Rightarrow \quad x = 2n\pi \pm \frac{\pi}{3}$

C. $\sin 2x + \cos x = 0$

$\Rightarrow \quad 2\cos x \cos x \cos x = 0$

$\Rightarrow \quad \cos x\,(2\sin x + 1) = 0$

$\Rightarrow \quad \cos x = 0$

or $\quad 2\sin x + 1 = 0$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = -\frac{1}{2}$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = \sin\left(\pi + \frac{\pi}{6}\right)$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = \sin\frac{7\pi}{6}$

$\Rightarrow \quad x = (2n+1)\frac{\pi}{2}$

or $\quad x = n\pi + (-1)^n\frac{7\pi}{6}, n \in Z$

Hence, general solution is

$x = (2n+1)\frac{\pi}{2}$ or $x = n\pi + (-1)^n\frac{7\pi}{6}$

where $n \in Z$

D. $\sec^2 2x = 1 - \tan 2x$

$\Rightarrow \quad 1 + \tan^2 2x = 1 - \tan 2x = 0$

$\Rightarrow \quad \tan^2 2x + \tan 2x = 0$

$\Rightarrow \quad \tan 2x(\tan 2x + 1) = 0$

If $\tan 2x = 0 \Rightarrow 2x = n\pi$ or $x = \frac{n\pi}{2}$

If $\tan 2x + 1 = 0 \Rightarrow \tan 2x = -1$

$= \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

$\Rightarrow \quad 2x = n\pi + \frac{3\pi}{4}$ or $x = \frac{n\pi}{2} + \frac{3\pi}{8}$

E. We have, $(\sin 5x + \sin x) + \sin 3x = 0$

$\Rightarrow \quad 2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2} + \sin 3x = 0$

or $\quad 2\sin 3x\cos 2x + \sin 3x = 0$

or $\quad \sin 3x\,(2\cos 2x + 1) = 0$

If $\sin 3x = 0 \Rightarrow 3x = n\pi$ or $x = \frac{n\pi}{3}$

If $2\cos 2x + 1 = 0$, $\cos 2x = \frac{-1}{2}$

$= \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

$\therefore\ 2x = 2n\pi \pm \frac{2\pi}{3}$ or $x = n\pi \pm \frac{\pi}{3}$

88. (d) A. LHS $= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$= \frac{-2\sin\frac{9x+5x}{2}\sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}$$

$$= \frac{-\sin 7x.\sin 2x}{\cos 10x\,.\,\sin 7x} = -\frac{\sin 2x}{\cos 10x} = \text{RHS}$$

B. L.H.S. $= \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$$

$$= \frac{\sin 4x}{\cos 4x} = \tan 4x = \text{R.H.S.}$$

C. L.H.S. $= \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \frac{\sin 3x + \sin x}{\cos 3x + \cos x}$

$$= \frac{2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}}{2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}} = \frac{\sin 2x\cos x}{\cos 2x\cos x}$$

$$= \frac{\sin 2x}{\cos 2x} = \tan 2x$$

D. L.H.S. $= \frac{\sin x - \sin y}{\cos x + \cos y} = \frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$

$$= \frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}} = \tan\frac{x-y}{2} = \text{R.H.S.}$$

E. L.H.S. $= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \frac{-(\sin 3x - \sin x)}{-(\cos^2 x - \sin^2 x)}$

$$= \frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x} = \frac{2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2}}{\cos 2x}$$

$= \dfrac{2\cos 2x \times \sin x}{\cos 2x}$ [$\because \cos 2x = \cos^2 x - \sin^2 x$]

$= 2\sin x$

89. (a) Since x lies in the second quadrant

$\sin x = 3/5$ given

$\cos x = -\sqrt{1-\sin^2 x}$ ($\because x$ lies in II quadrant)

$= -\sqrt{1-\dfrac{9}{25}} = -\dfrac{4}{5}$

$\sec x = -\dfrac{5}{4}$, $\tan x = -\dfrac{3}{4}$

$\operatorname{cosec} x = \dfrac{5}{3}$, $\cot x = -\dfrac{4}{3}$

INTEGER TYPE QUESTIONS

90. (c) As, we know that

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\therefore$ $\operatorname{cosec}(-1410°) = -\operatorname{cosec}(360 \times 4 - 30)°$

$= -(-\operatorname{cosec} 30°)$

$= \operatorname{cosec} 30°$ [$\because \operatorname{cosec}(2n\pi - \theta) = -\operatorname{cosec}\theta$]

$= 2.$

91. (b) Given expression

$= \cos\dfrac{10\pi}{13} + \cos\dfrac{8\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \left(\cos\dfrac{10\pi}{13} + \cos\dfrac{3\pi}{13}\right) + \left(\cos\dfrac{8\pi}{13} + \cos\dfrac{5\pi}{13}\right)$

$= 2\cos\left(\dfrac{13\pi}{2\times 13}\right)\cdot\cos\left(\dfrac{7\pi}{2\times 13}\right)$

$+ 2\cos\left(\dfrac{13\pi}{2\times 13}\right)\cos\left(\dfrac{3\pi}{2\times 13}\right)$

$= 2\cos\dfrac{\pi}{2}\left(\cos\dfrac{7\pi}{26} + \cos\dfrac{3\pi}{26}\right)$ $\left[\because \cos\dfrac{\pi}{2} = 0\right]$

$= 0.$

92. (a) $\sin\theta + \cos\theta = 1$

Squaring on both sides, we get

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\therefore$ $\sin\theta\cos\theta = 0.$

93. (a) $\cos A = \dfrac{3}{5}$, $\cos B = \dfrac{4}{5}$

$\sin A = -\sqrt{1-\dfrac{9}{25}} = \dfrac{-4}{5}$

$\sin B = -\sqrt{1-\dfrac{16}{25}} = \dfrac{-3}{5}$

($\because \angle A$ and $\angle B$ in the 4^{th} quad.)

$\therefore 2\sin A + 4\sin B = 2\left(\dfrac{-4}{5}\right) + 4\left(\dfrac{-3}{5}\right) = -4 = -a$

94. (a) $\sin 765° = \sin(360 \times 2 + 45)° = \sin 45° = \dfrac{1}{\sqrt{2}}$

95. (b) $\operatorname{cosec}(-1410°) = -\operatorname{cosec}(360 \times 4 - 30)°$

$= -(-\operatorname{cosec} 30°) = \operatorname{cosec} 30° = 2$

96. (c) $\tan\dfrac{19\pi}{3} = \tan\left(6\pi + \dfrac{\pi}{3}\right)$

$= \tan\left[2\pi \times 3 + \dfrac{\pi}{3}\right] = \tan\dfrac{\pi}{3} = \sqrt{3}$

97. (b) $\sin\left(\dfrac{-11\pi}{3}\right) = -\sin\left(\dfrac{11\pi}{3}\right) = -\sin\left(4\pi - \dfrac{\pi}{3}\right)$

$= -\sin\left(2\pi \times 2 - \dfrac{\pi}{3}\right) = -\left(-\sin\dfrac{\pi}{3}\right) = \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

98. (c) $\left(1+\cos\dfrac{\pi}{6}\right)\left(1+\cos\dfrac{\pi}{3}\right)\left(1+\cos\dfrac{2\pi}{3}\right)$

$\left(1+\cos\dfrac{7\pi}{6}\right) = \left(1+\dfrac{\sqrt{3}}{2}\right)\left(1+\dfrac{1}{2}\right)\left(1-\dfrac{1}{2}\right)\left(1-\dfrac{\sqrt{3}}{2}\right)$

$= \left(1-\dfrac{3}{4}\right)\left(1-\dfrac{1}{4}\right) = \dfrac{1}{4}\times\dfrac{3}{4} = \dfrac{3}{16}$

99. (c) $\tan\theta = \dfrac{1}{\sqrt{7}} \Rightarrow \cot\theta = \sqrt{7}$

Given expression $= \dfrac{1+\cot^2\theta - 1 - \tan^2\theta}{1+\cot^2\theta + 1 + \tan^2\theta}$

$= \dfrac{\cot^2\theta - \tan^2\theta}{2+\cot^2\theta + \tan^2\theta} = \dfrac{(\sqrt{7})^2 - \left(\dfrac{1}{\sqrt{7}}\right)^2}{2+(\sqrt{7})^2 + \left(\dfrac{1}{\sqrt{7}}\right)^2}$

$= \dfrac{48}{64} = \dfrac{3}{4} = \dfrac{m}{m+1} \Rightarrow m = 3$

100. (b) $\cos^2 x = 1 - \sin^2 x = 1 - \dfrac{24}{25} = \dfrac{1}{25}$

$\Rightarrow \cos x = \dfrac{-1}{5}$

($\because$ $\sin x$ and $\cos x$ are negative in III quad)

$\therefore$ $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{1}{2\sqrt{6}}$

101. (d) $\sin^2\theta = 1 - \cos^2\theta = \left(1-\dfrac{9}{25}\right) = \dfrac{16}{25}$

$\Rightarrow \sin\theta = \dfrac{-4}{5}$

$\therefore$ $\tan\theta = \left(\dfrac{-4}{5}\times\dfrac{5}{-3}\right) = \dfrac{4}{3}$

$\cot\theta = \dfrac{3}{4}$

$\Rightarrow \operatorname{cosec}\theta = \dfrac{-5}{4}$ and $\sec\theta = \dfrac{-5}{3}$

$\therefore$ $\left(\dfrac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}\right) = \dfrac{-2}{4}\times\dfrac{3}{-9} = \dfrac{1}{6}$

102. (b) $3\sin\dfrac{\pi}{6}\sec\dfrac{\pi}{3} - 4\sin\dfrac{5\pi}{6}\cot\dfrac{\pi}{4}$

$= 3\times\dfrac{1}{2}\times 2 - 4\sin\left(\pi - \dfrac{\pi}{6}\right)\times 1$

$= 3 - 4\sin\dfrac{\pi}{6} = 3 - 4\times\dfrac{1}{2} = 1$

103. (a) $2\sin^2\frac{\pi}{6} + \text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\times\left(\frac{1}{2}\right)^2 + \text{cosec}^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$

$= \frac{2}{4} + \text{cosec}^2\frac{\pi}{6}\cos^2\frac{\pi}{3}$

$= \frac{1}{2}+(2)^2\left(\frac{1}{2}\right)^2 = \frac{3}{2} = \frac{m}{m-1} \quad \therefore\ m=3$

104. (d) $\cot^2\frac{\pi}{6} + \text{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$= (\sqrt{3})^2 + \text{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + \text{cosec}\frac{\pi}{6} + 1 = 3 + 2 + 1 = 6$

105. (b) $\text{LHS} = \cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

Now, $\cos\left(\frac{3\pi}{2}+x\right)=\sin x, \cos(2\pi+x)=\cos x$ and

$\cot\left(\frac{3\pi}{2}-x\right)=\tan x, \cot(2\pi+x)=\cot x$

$\text{L.H.S.} = \sin x.\cos x[\tan x+\cot x]$

$= \sin x.\cos x\left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right]$

$= \sin x.\cos x\left[\frac{\sin^2 x+\cos^2 x}{\cos x\sin x}\right]$

$= (\sin x.\cos x)\frac{1}{\cos x\sin x} = 1 \quad [\because \sin^2 x+\cos^2 x=1]$

106. (b) L.H.S. $\cot x\cot 2x - \cot 2x\cot 3x - \cot 3x\cot x$

We have $3x = x + 2x$

$\cot 3x = \cot(x+2x) = \frac{\cot x\cot 2x - 1}{\cot x+\cot 2x}$

By cross multiplication

$\cot 3x(\cot x + \cot 2x) = \cot x\cot 2x - 1$

$\cot x\cot 3x + \cot 2x\cot 3x = \cos x\cot 2x - 1$

$\therefore\ \cos x\cot 2x - \cot 2x\cot 3x - \cot 3x\cot x = 1$

ASSERTION - REASON TYPE QUESTIONS

107. (d) If the radii of the two circles are r_1 and r_2 and l is the length of arc in either case, then

$l = r_1$ (circular measure of 30°) $= r_1\left(\frac{30\pi}{180}\right)$

and also $l = r_2$ (circular measure of 70°) $= r_2\left(\frac{70\pi}{180}\right)$.

So, we must have $\frac{r_1\pi}{6} = \frac{7r_2\pi}{18} \Rightarrow \frac{r_1}{r_2} = \frac{7}{3}$.

108. (d) $\because \tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$

$= \tan\left(\frac{\pi}{2}-\frac{\pi}{2}\cos\theta\right)$

$\therefore\ \frac{\pi}{2}\sin\theta = n\pi + \frac{\pi}{2}-\frac{\pi}{2}\cos\theta$

$\Rightarrow \sin\theta+\cos\theta = 2n+1,\ n\in I$

$\because\ -\sqrt{2} \le \sin\theta+\cos\theta \le \sqrt{2}$

$\therefore\ n = 0, -1$

Then, $\sin\theta + \cos\theta = 1, -1$.

109. (a) Given equation is

$\tan\theta + \tan\left(\theta+\frac{\pi}{3}\right)+\tan\left(\theta+\frac{2\pi}{3}\right) = 3$

$\Rightarrow \tan\theta + \frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}+\frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta} = 3$

$\Rightarrow \tan\theta + \frac{\left(\tan\theta+\sqrt{3}\right)\left(1+\sqrt{3}\tan\theta\right)+\left(\tan\theta-\sqrt{3}\right)\times\left(1-\sqrt{3}\tan\theta\right)}{\left(1-\sqrt{3}\tan\theta\right)\left(1+\sqrt{3}\tan\theta\right)} = 3$

$\Rightarrow \tan\theta + \frac{8\tan\theta}{1-3\tan^2\theta} = 3$

$\Rightarrow \frac{\tan\theta-3\tan^3\theta+8\tan\theta}{1-3\tan^2\theta} = 3$

$\Rightarrow \frac{3\left(3\tan\theta-\tan^3\theta\right)}{1-3\tan^2\theta} = 3$

$\Rightarrow 3\tan 3\theta = 3 \Rightarrow \tan 3\theta = 1$

$\Rightarrow \tan 3\theta = \tan\frac{\pi}{4} \Rightarrow 3\theta = n\pi+\frac{\pi}{4},\ n\in I$

$\Rightarrow \theta = \frac{n\pi}{3}+\frac{\pi}{12},\ n\in I.$

110. (d) Reason is true.

$\therefore$ Degree measure of (–2) radian $= \frac{180}{\pi}\times -2$

$= \frac{180}{22}\times -2\times 7 \quad \left[\because \pi = \frac{22}{7}\right]$

$= \left(-\frac{1260}{11}\right)^{o} = -\left(114\frac{6}{11}\right)^{o} = -114^\circ\left(\frac{6}{11}\times 60\right)'$

$= -114^\circ 32'\left(\frac{8}{11}\times 60\right)'' = -114^\circ 32' 43.6''$

$= -114^\circ 32' 44''$ (approx.)

So, Assertion is false.

111. (a) I. $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)} = \frac{(-\cos x)(\cos x)}{(\sin x)(-\sin x)}$

$\left[\begin{aligned}\because \cos(\pi+\theta) &= -\cos\theta\\ \cos(-\theta) &= \cos\theta\\ \sin(\pi-\theta) &= \sin\theta\\ \sin(-\theta) &= -\sin\theta\end{aligned}\right]$

$= \dfrac{\cos^2 x}{\sin^2 x} = \cot^2 x$

So, both the Assertion and Reason are true and Reason is the correct explanation of Assertion.

112. (a) We have,

$\tan 2x = -\cot\left(x+\dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{2}+x+\dfrac{\pi}{3}\right)$

$\Rightarrow \quad \tan 2x = \tan\left(x+\dfrac{5\pi}{6}\right)$

Therefore, $2x = n\pi + \left(x+\dfrac{5\pi}{6}\right)$, where $n \in Z$

$(\because \tan x = \tan y \Rightarrow x = n\pi + y,\ n \in Z)$

$\Rightarrow \quad x = n\pi + \dfrac{5\pi}{6}$, where $n \in Z$.

113. (b) Both are correct statements. Reason is not the correct explanation for the Assertion.

114. (b) Both Assertion and Reason is correct but Reason is not correct explanation.

115. (b) Both Assertion and Reason is correct. Reason is not the correct explanation for Assertion.

Reason : $40°20' = 40\dfrac{1}{3}$ degree

$= \dfrac{\pi}{180} \times \dfrac{|2|}{3}$ radian $= \dfrac{|2|\pi}{540}$ radian.

116. (d) Assertion is incorrect. The second hand rotates through 360° in a minute.

117. (c) Assertion is correct and Reason is incorrect.

CRITICAL THINKING TYPE QUESTIONS

118. (b) $\tan 20° + 2\tan 50° - \tan 70°$

$= \dfrac{\sin 20°}{\cos 20°} - \dfrac{\sin 70°}{\cos 70°} + 2\tan 50°$

$= \dfrac{\sin 20° \cos 70° - \cos 20° \sin 70°}{\cos 20° \cos 70°} + 2\tan 50°$

$= \dfrac{\sin(20° - 70°)}{\frac{1}{2}[\cos(70°+20°) + \cos(70°-20°)]} + 2\tan 50°$

$= \dfrac{2\sin(-50°)}{\cos 90° + \cos 50°} + 2\tan 50°$

$= \dfrac{-2\sin 50°}{0 + \cos 50°} + 2\tan 50°$

$= -2\tan 50° + 2\tan 50° = 0.$

119. (d) $\sin(2\alpha) = \sin(\alpha+\beta+\alpha-\beta)$

$= \sin(\alpha+\beta)\cos(\alpha-\beta) + \cos(\alpha+\beta)\sin(\alpha-\beta)$

$= \dfrac{5}{13}.\dfrac{4}{5} + \dfrac{12}{13}.\dfrac{3}{5} = \dfrac{56}{65}$

120. (d) $\cos\theta = \dfrac{1}{\sqrt{2}} = \cos\left(\dfrac{\pi}{4}\right)$

$\theta = 2n\pi \pm \dfrac{\pi}{4}$; $n \in I$

Put $n = 1$, $\theta = \dfrac{9\pi}{4}, \dfrac{7\pi}{4}$

$\tan\theta = -1 = \tan\left(\dfrac{-\pi}{4}\right) \Rightarrow \theta = n\pi - \pi/4,\ n \in I$

Put $n = 1$, $\theta = \dfrac{3\pi}{4}$

Put $n = 2$, $\theta = \dfrac{7\pi}{4}$

The common value which satisfies both these equation is $\left(\dfrac{7\pi}{4}\right)$. Hence the general value is $2n\pi + \dfrac{7\pi}{4}$

121. (d) The given expression

$= \dfrac{\sqrt{3}}{\sin 20°} - \dfrac{1}{\cos 20°} = \dfrac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}$

$= \dfrac{2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{\sin 20° \cos 20°}$

$= \dfrac{2(\sin 60° \cos 20° - \cos 60° \sin 20°)}{\sin 20° \cos 20°}$

$= \dfrac{2\sin(60° - 20°)}{\sin 20° \cos 20°} = \dfrac{2\sin 40°}{\sin 20° \cos 20°}$

$= \dfrac{4\sin 40°}{2\sin 20° \cos 20°} = \dfrac{4\sin 40°}{\sin 40°} = 4$

122. (d) We have $\sin^2\theta - \sin\theta - 2 = 0$

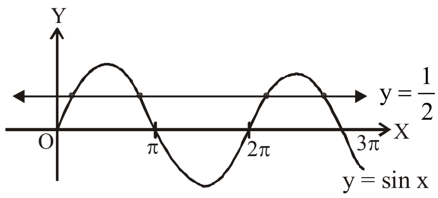
$\Rightarrow (\sin\theta + 1)(\sin\theta - 2) = 0$

As $\sin\theta \neq 2$

$\therefore \sin\theta = -1 = \sin\dfrac{3\pi}{2}$

$\therefore \theta = \dfrac{3\pi}{2} = \dfrac{6\pi}{4} \in \left(\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right)$

123. (a)



$2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow \quad (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \quad \sin x = \dfrac{1}{2}$ and $\sin x \neq -3$

$\therefore$ In $[0, 3\pi]$, x has 4 values.

124. (b) $\text{LHS} = 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \cos\left(\dfrac{9\pi}{13}+\dfrac{\pi}{13}\right) + \cos\left(\dfrac{9\pi}{13}-\dfrac{\pi}{13}\right) + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \cos\dfrac{10\pi}{13} + \cos\dfrac{8\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \cos\left(\pi - \dfrac{3\pi}{13}\right) + \cos\left(\pi - \dfrac{5\pi}{13}\right) + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= -\cos\dfrac{3\pi}{13} - \cos\dfrac{5\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= 0 = \text{RHS} \qquad [\because \cos(\pi - \theta) = -\cos\theta]$

125. (a) Given value

$= (\sin 47° + \sin 61°) - (\sin 11° + \sin 25°)$

$= 2 \sin 54° \cos 7° - 2 \sin 18° \cos 7°$

$= 2 \cos 7° (\sin 54° - \sin 18°)$

$= 2 \cos 7° \, 2 \cos 36° \sin 18°$

$= 2\cos 7° \dfrac{2\sin 18° \cos 18°}{\cos 18°} \times \cos 36°$

$= \cos 7° \dfrac{2\sin 36° \cos 36°}{\cos 18°}$

$= \cos 7° \dfrac{\sin 72°}{\cos 18°} = \cos 7° \quad [\because \sin 72° = \cos 18°]$

126. (c) Since $\sin\theta + \cos\theta = \sqrt{2}\left[\dfrac{1}{\sqrt{2}}\sin\theta + \dfrac{1}{\sqrt{2}}\cos\theta\right]$

$= \sqrt{2}\left[\sin\theta\cos\dfrac{\pi}{4} + \cos\theta\sin\dfrac{\pi}{4}\right] = \sqrt{2}\sin\left(\theta + \dfrac{\pi}{4}\right)$

which lies between $-\sqrt{2}$ and $\sqrt{2}$

$[\because \sin\left(\theta + \dfrac{\pi}{4}\right)$ lies between -1 and $1]$

127. (d) $\tan 2\theta \tan\theta = 1 \Rightarrow \dfrac{2\tan\theta}{1-\tan^2\theta} . \tan\theta = 1$

$\Rightarrow 2\tan^2\theta = 1 - \tan^2\theta \Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm\dfrac{1}{\sqrt{3}} = \tan\left(\pm\dfrac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi \pm \dfrac{\pi}{6} \; (n \in Z) = (6n \pm 1)\dfrac{\pi}{6}$

or $\tan 2\theta = \cot\theta = \tan\left(\dfrac{\pi}{2} - \theta\right)$

$\Rightarrow 2\theta = n\pi + \dfrac{\pi}{2} - \theta \Rightarrow 3\theta = n\pi + \dfrac{\pi}{2}$

$\Rightarrow \theta = \dfrac{n\pi}{3} + \dfrac{\pi}{6} = (2n+1)\dfrac{\pi}{6}$

128. (b) The given equation can be written as

$\dfrac{1}{2}(\sin 8x + \sin 2x) = \dfrac{1}{2}(\sin 8x + \sin 4x)$

or $\sin 2x - \sin 4x \Rightarrow -2\sin x \cos 3x = 0$

Hence $\sin x = 0$ or $\cos 3x = 0$.

That is, $x = n\pi \, (n \in I)$, or $3x = k\pi + \dfrac{\pi}{2} \; (k \in I)$.

Therefore, since $x \in [0, \pi]$, the given equation is satisfied if $x = 0, \pi, \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}$

129. (c) $\tan(\cot x) = \cot(\tan x) = \tan\left(\dfrac{\pi}{2} - \tan x\right)$

$\Rightarrow \cot x = n\pi + \dfrac{\pi}{2} - \tan x$

$[\because \tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha]$

$\Rightarrow \cot x + \tan x = n\pi + \dfrac{\pi}{2}$

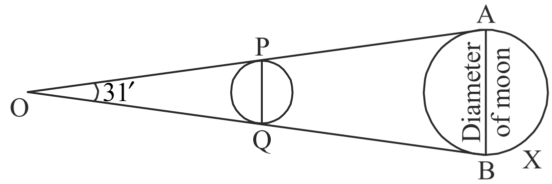
$\Rightarrow \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x} = (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \dfrac{1}{\sin x \cos x} = (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \dfrac{1}{\sin 2x} = \dfrac{(2n+1)\pi}{4}$

$\therefore \sin 2x = \dfrac{4}{(2n+1)\pi}$

130. (c) The coin will just hide the full moon if the lines joining the observer's eye O to the ends A and B of moon's diameter touch the coin at the ends P and Q of its diameter.



Here, $\angle POQ = \angle AOB = 31'$

$= \left(\dfrac{31}{60}\right)^{o} = \dfrac{31}{60} \times \dfrac{\pi}{180}$ radian.

Since, this angle is very small, the diameter PQ of the coin can be regarded as an arc of a circle whose centre is O and radius equal to the distance of the coin from O.

$\therefore \quad \dfrac{31\pi}{60 \times 180} = \dfrac{1}{r} \quad \left(\because \quad \theta = \dfrac{\ell}{r}\right)$

$\Rightarrow \quad r = \dfrac{60 \times 180}{31\pi}$

$\Rightarrow \quad r = \dfrac{60 \times 180 \times 7}{31 \times 22} = 110.9$ cm.

131. (a) Radius of the wheel = 35 cm

$\therefore$ Circumference of the wheel $= 2\pi \times 35$ cm

$= 2 \times \dfrac{22}{7} \times 35$ cm $= 220$ cm.

Hence, the linear distance travelled by a point of the rim in one revolution = 220 cm.

Number of revolutions made by the wheel in 3 minutes

$= 20 \times 3 \times 60 = 3600$

$\therefore$ The linear distance travelled by a point of the rim in 3 minutes $= 220 \times 3600 = 792000$ cm

$= \dfrac{792000}{100000}$ km $= 7.92$ km.

132. (b) In 60 minutes, the minute hand of a watch completes one revolution. Therefore, in 40 minutes, the minute hand turns through $\dfrac{2}{3}$ of a revolution. Therefore, $\theta = \dfrac{2}{3} \times 360°$ or $\dfrac{4\pi}{3}$ radian. Hence, the required distance travelled is given by

$l = r\theta = 1.5 \times \dfrac{4\pi}{3} = 2\pi = 2 \times 3.14 = 6.28$ cm.

133. (c) Let r_1 and r_2 be the radii of the two circles. Given that

$\theta_1 = 65° = \frac{\pi}{180} \times 65 = \frac{13\pi}{36}$ radian

and $\theta_2 = 110° = \frac{\pi}{180} \times 110 = \frac{22\pi}{36}$ radian

Let l be the length of each of the arc.

Then, $l = r_1 \theta_1 = r_2 \theta_2$, which gives

$\frac{13\pi}{36} \times r_1 = \frac{22\pi}{36} \times r_2$, i.e. $\frac{r_1}{r_2} = \frac{22}{13}$

Hence, $r_1 : r_2 = 22 : 13$.

134. (d) $\tan A + \cot A = 4$ … (i)

Squaring (i) both sides, we get

$\tan^2 A + \cot^2 A + 2 = 16$

$\Rightarrow \tan^2 A + \cot^2 A = 14$ … (ii)

Squaring (ii) both sides, we get

$(\tan^2 A + \cot^2 A)^2 = 196$

$\Rightarrow \tan^4 A + \cot^4 A = 196 - 2$

$\Rightarrow \tan^4 A + \cot^4 A = 194$.

135. (b) Given, $\frac{\sin A}{\sin B} = m$

$\Rightarrow \sin A = m \sin B$ ... (i)

and $\frac{\cos A}{\cos B} = n$

$\Rightarrow \cos A = n \cos B$ ... (ii)

Squaring (i) and (ii) and then adding, we get

$1 = m^2 \sin^2 B + n^2 \cos^2 B$

$\Rightarrow \frac{1}{\cos^2 B} = m^2 \frac{\sin^2 B}{\cos^2 B} + n^2$ [Dividing by $\cos^2 B$]

$\Rightarrow \sec^2 B = m^2 \tan^2 B + n^2$

$\Rightarrow 1 + \tan^2 B = m^2 \tan^2 B + n^2$

$\Rightarrow 1 - n^2 = (m^2 - 1) \tan^2 B$

$\Rightarrow \tan^2 B = \frac{1 - n^2}{m^2 - 1}$

$\Rightarrow \tan B = \pm\sqrt{\frac{1 - n^2}{m^2 - 1}}$.

136. (d) $\tan(A - B) = 1 \Rightarrow A - B = 45°$ or $225°$

$\sec(A + B) = \frac{2}{\sqrt{3}} \Rightarrow A + B = 30°$ or $330°$

$A + B = 330° = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ … (i)

and $A - B = 225° = \frac{5\pi}{4}$ … (ii)

Solving (i) and (ii), we get

$2B = \frac{11\pi}{6} - \frac{5\pi}{4} \Rightarrow 2B = \frac{7\pi}{12} \Rightarrow B = \frac{7\pi}{24}$.

137. (a) $4 \sin \alpha \sin\left(\alpha + \frac{\pi}{3}\right) \sin\left(\alpha + \frac{2\pi}{3}\right)$

$= 2 \sin \alpha \left\{2 \sin\left(\alpha + \frac{2\pi}{3}\right) \sin\left(\alpha + \frac{\pi}{3}\right)\right\}$

$= 2 \sin \alpha\, [2 \sin(\alpha + 120°) \sin(\alpha + 60°)]$

$= 2 \sin \alpha\, [\cos(\alpha + 120° - \alpha - 60°) - \cos(\alpha + 120° + \alpha + 60°)]$

$= 2 \sin \alpha\, [\cos 60° - \cos(180° + 2\alpha)]$

$= 2 \sin \alpha \cdot \frac{1}{2} - 2 \sin \alpha\, (-\cos 2\alpha)$

$= \sin \alpha + 2 \cos 2\alpha \sin \alpha$

$= \sin \alpha + \sin(2\alpha + \alpha) - \sin(2\alpha - \alpha)$

$= \sin \alpha + \sin 3\alpha - \sin \alpha = \sin 3\alpha$.

138. (c) $[\sin x + \cos x]^{1 + 2 \sin x \cos x} = 2$

$\Rightarrow (\sin x + \cos x)^{(\sin x + \cos x)^2} = 2$

$\Rightarrow (\sin x + \cos x)^{(\sin x + \cos x)^2} = (\sqrt{2})^{(\sqrt{2})^2}$ … (i)

Comparing (i) both sides, we get

$\sin x + \cos x = \sqrt{2}$

$\Rightarrow \sin\left(x + \frac{\pi}{4}\right) = 1 = \sin\frac{\pi}{2}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{2} - \frac{\pi}{4}$

So, $x = \frac{\pi}{4}$, when $n = 0$.

139. (c) Given that

$\tan\theta + \sec\theta = p$ …(i)

and we know that

$\Rightarrow \sec^2\theta - \tan^2\theta = (\sec\theta - \tan\theta)p$

(multiplying both the sides by $(\sec\theta - \tan\theta)$)

$\Rightarrow (\sec\theta - \tan\theta)\, p = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ …(ii)

On solving equations (i) and (ii), we get

$2\sec\theta = \frac{p^2 + 1}{p} \Rightarrow \sec\theta = \frac{p^2 + 1}{2p}$

140. (c) We have,

$\sec\theta + \tan\theta = \sqrt{3}$ … (i)

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$ … (ii)

$[\because \sec^2\theta - \tan^2\theta = 1]$

By solving (i) and (ii), we get

$\tan\theta = \frac{1}{2}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$

$\therefore \tan\theta = \tan\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi + \frac{\pi}{6}$

$\therefore$ Solutions for $0 \le \theta \le 2\pi$ are $\frac{\pi}{6}$ and $\frac{7\pi}{6}$.

Hence, there are two solutions.

141. (c) Given equation is $\cos x - \sin x = \frac{1}{\sqrt{2}}$

Dividing equation by $\sqrt{2}$,

$\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{1}{2}$

$\Rightarrow \cos\left(\frac{\pi}{4} + x\right) = \cos\frac{\pi}{3}$

$\Rightarrow \frac{\pi}{4} + x = 2n\pi \pm \frac{\pi}{3}$

$x = 2n\pi + \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi + \frac{\pi}{12}$

or $x = 2n\pi - \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi - \frac{7\pi}{12}.$

142. (a) $4\sin^2\theta + 2(\sqrt{3}+1)\cos\theta = 4 + \sqrt{3}$

$\Rightarrow 4 - 4\cos^2\theta + 2(\sqrt{3}+1)\cos\theta = 4 + \sqrt{3}$

$\Rightarrow 4\cos^2\theta - 2(\sqrt{3}+1)\cos\theta + \sqrt{3} = 0$

$\Rightarrow \cos\theta = \frac{2(\sqrt{3}+1) \pm \sqrt{4(\sqrt{3}+1)^2 - 16\sqrt{3}}}{8}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ or $\frac{1}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$ or $2n\pi \pm \frac{\pi}{3}.$

143. (a) $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow \sin x = \frac{-5 \pm \sqrt{25+24}}{4} = \frac{-5 \pm 7}{4} = -3, \frac{1}{2}$

But $\sin x \neq -3$

$\therefore \sin x = \frac{1}{2}$

$\therefore$ Number of solution in $[0, 3\pi]$ will be equal to 4.

144. (b) $\sin\theta + \cos\theta = 1$

Dividing by $\sqrt{1^2+1^2} = \sqrt{2}$,

$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\Rightarrow \theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}.$

145. (a) $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\Rightarrow 2\sin 4\theta\cos 2\theta + \sin 4\theta = 0$

$\Rightarrow \sin 4\theta(2\cos 2\theta + 1) = 0$

$\Rightarrow 2\cos 2\theta = -1 \Rightarrow \cos 2\theta = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow 2\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

and $\sin 4\theta = 0 \Rightarrow 4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$

$\theta = \frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{3}.$

146. (c) $\sqrt{2}\sec\theta + \tan\theta = 1$

$\Rightarrow \frac{\sqrt{2}}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \sin\theta - \cos\theta = -\sqrt{2}$

Dividing by $\sqrt{2}$ on both sides, we get

$\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta = -1$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = 1$

$\Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \cos(0)$

$\Rightarrow \theta + \frac{\pi}{4} = 2n\pi \pm 0 \Rightarrow \theta = 2n\pi - \frac{\pi}{4}.$

147. (c) $12\cot^2\theta - 31\operatorname{cosec}\theta + 32 = 0$

$\Rightarrow 12(\operatorname{cosec}^2\theta - 1) - 31\operatorname{cosec}\theta + 32 = 0$

$\Rightarrow 12\operatorname{cosec}^2\theta - 31\operatorname{cosec}\theta + 20 = 0$

$\Rightarrow 12\operatorname{cosec}^2\theta - 16\operatorname{cosec}\theta - 15\operatorname{cosec}\theta + 20 = 0$

$\Rightarrow (4\operatorname{cosec}\theta - 5)(3\operatorname{cosec}\theta - 4) = 0$

$\Rightarrow \operatorname{cosec}\theta = \frac{5}{4}, \frac{4}{3}$

$\therefore \sin\theta = \frac{4}{5}, \frac{3}{4}.$

148. (b) We have $\sec^2\theta = \frac{4}{3}$

$\Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos^2\theta = \cos^2\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$ .... $\left[\begin{array}{l}\text{If } \cos^2\theta = \cos^2\alpha \\ \Rightarrow \theta = n\pi \pm \alpha\end{array}\right]$

149. (a) We have $\tan 5\theta = \cot 2\theta$

$\Rightarrow \tan 5\theta = \tan\left(\frac{\pi}{2} - 2\theta\right) \ldots \left[\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right]$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{2} - 2\theta \Rightarrow 7\theta = n\pi + \frac{\pi}{2}$

$\Rightarrow \theta = \frac{n\pi}{7} + \frac{\pi}{14}.$

150. (c) Since, none of the x, y and $(x + y)$ is multiple of π, we find that $\sin x$, $\sin y$ and $\sin(x + y)$ are non-zero. Now,

$\cot(x + y) = \frac{\cos(x+y)}{\sin(x+y)} = \frac{\cos x\cos y - \sin x\sin y}{\sin x\cos y + \cos x\sin y}$

On dividing numerator and denominator by $\sin x \sin y$, we have

$\cot(x + y) = \frac{\cot x\cot y - 1}{\cot y + \cot x}$

On replacing y by (–y) in above identity, we get

$\cot(x - y) = \frac{\cot x \cdot \cot y + 1}{\cot y - \cot x}.$

151. (a) Given, $3\tan(\theta - 15) = \tan(\theta + 15)$

$\frac{\tan A}{\tan B} = \frac{3}{1},$

where $A = \theta + 15°$, $B = \theta - 15°$

On applying componendo and dividendo, we get

$$\Rightarrow \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{3+1}{3-1} \Rightarrow \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}} = 2$$

$$\Rightarrow \frac{\sin(A+B)}{\sin(A-B)} = 2$$

$$\Rightarrow \sin 2\theta = 2 \sin 30°$$

$$\Rightarrow \sin 2\theta = 2 \cdot \frac{1}{2} = 1 = \sin\frac{\pi}{2}$$

$$\Rightarrow 2\theta = n\pi + (-1)^n \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{4}.$$

152. (c) Let $\theta = \alpha + \beta$. Then, $\tan \alpha = K \tan \beta$

or $\frac{\tan\alpha}{\tan\beta} = \frac{K}{1}$

Applying componendo and dividendo, we have

$$\frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \frac{K+1}{K-1}$$

or $\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta} = \frac{K+1}{K-1}$

i.e., $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{K+1}{K-1}$

Given that, $\alpha - \beta = \phi$ and $\alpha + \beta = \theta$. Therefore,

$$\frac{\sin\theta}{\sin\phi} = \frac{K+1}{K-1} \quad \text{or} \quad \sin\theta = \frac{K+1}{K-1}\sin\phi.$$

153. (a) We have, $m \sin \theta = n \sin(\theta + 2\alpha)$

$$\Rightarrow \frac{\sin(\theta+2\alpha)}{\sin\theta} = \frac{m}{n}$$

Using componendo and dividendo, we get

$$\frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{2\sin\left(\frac{\theta+2\alpha+\theta}{2}\right)\cdot\cos\left(\frac{\theta+2\alpha-\theta}{2}\right)}{2\cos\left(\frac{\theta+2\alpha+\theta}{2}\right)\cdot\sin\left(\frac{\theta+2\alpha-\theta}{2}\right)} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{2\sin(\theta+\alpha)\cdot\cos\alpha}{2\cos(\theta+\alpha)\cdot\sin\alpha} = \frac{m+n}{m-n}$$

$$\Rightarrow \tan(\theta+\alpha)\cdot\cot\alpha = \frac{m+n}{m-n}$$

154. (c) $5\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{5}$

$$\therefore \sin\theta = \frac{4}{\sqrt{41}} \text{ and } \cos\theta = \frac{5}{\sqrt{41}}$$

$$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5\times\frac{4}{\sqrt{41}} - 3\times\frac{5}{\sqrt{41}}}{5\times\frac{4}{\sqrt{41}} + 2\times\frac{5}{\sqrt{41}}}$$

$$= \frac{20-15}{20+10} = \frac{5}{30} = \frac{1}{6}.$$

155. (c) $\frac{1+\sin A - \cos A}{1+\sin A + \cos A}$

$$= \frac{2\sin^2\frac{A}{2} + 2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2} + 2\sin\frac{A}{2}\cos\frac{A}{2}} \quad \left(\begin{array}{l}\text{as} \\ \sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} \\ \cos A = 2\cos^2\frac{A}{2} - 1 \\ \cos A = 1 - 2\sin^2\frac{A}{2}\end{array}\right)$$

$$= \frac{2\sin\frac{A}{2}\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)}{2\cos\frac{A}{2}\left(\cos\frac{A}{2} + \sin\frac{A}{2}\right)} = \tan\frac{A}{2}.$$

Trick: Put A = 60°

Then, $\frac{1+\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{2}\right)}{1+\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)} = \frac{1+\sqrt{3}}{3+\sqrt{3}} = \frac{1}{\sqrt{3}},$

which is given by option (c), i.e. $\tan\frac{60°}{2} = \frac{1}{\sqrt{3}}.$

156. (d) $\frac{1}{4}\left\{\sqrt{3}\cos 23° - \sin 23°\right\}$

$$= \frac{1}{2}\{\cos 30°\cos 23° - \sin 30°\sin 23°\}$$

$$= \frac{1}{2}\cos\{30° + 23°\} = \frac{1}{2}\cos 53°.$$

157. (c) Given equation $\cos x + \cos y + \cos\alpha = 0$ and $\sin x + \sin y + \sin\alpha = 0$.

The given equation may be written as

$\cos x + \cos y = -\cos\alpha$ and $\sin x + \sin y = -\sin\alpha$.

Therefore,

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = -\cos\alpha \quad \text{... (i)}$$

$$2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = -\sin\alpha \quad \text{... (ii)}$$

Divide (i) by (ii), we get

$$\frac{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{\cos\alpha}{\sin\alpha}$$

$$\Rightarrow \cot\left(\frac{x+y}{2}\right) = \cot\alpha.$$

158. (a) $\sin 12° \sin 24° \sin 48° \sin 84°$

$$= \frac{1}{4}(2\sin 12°\sin 48°)(2\sin 24°\sin 84°)$$

$$= \frac{1}{2}(\cos 36° - \cos 60°)(\cos 60° - \cos 108°)$$

$$= \frac{1}{4}\left(\cos 36° - \frac{1}{2}\right)\left(\frac{1}{2} + \sin 18°\right)$$

$$= \frac{1}{4}\left\{\frac{1}{4}\left(\sqrt{5}+1\right) - \frac{1}{2}\right\}\left\{\frac{1}{2} + \frac{1}{4}\left(\sqrt{5}-1\right)\right\} = \frac{1}{16}$$

and $\cos 20° \cos 40° \cos 60° \cos 80°$

$= \frac{1}{2}[\cos(60° - 20°) \cos 20° \cos(60° + 20°)]$

$= \frac{1}{2}\left[\frac{1}{4}\cos 3(20°)\right] = \frac{1}{8}\cos 60° = \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}.$

159. (b) We have, $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

Then, $\frac{4\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = y$

$\Rightarrow \frac{2\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}} \times \frac{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)}{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)} = y$

$\Rightarrow \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = y.$

Trick: Put value of $\theta = 30°$ and check.

160. (b) Given, $\sin 2\theta + \sin 2\phi = \frac{1}{2}$ ... (i)

and $\cos 2\theta + \cos 2\phi = \frac{3}{2}$ ... (ii)

Square and adding,

$\therefore$ $(\sin^2 2\theta + \cos^2 2\theta) + (\sin^2 2\phi + \cos^2 2\phi)$

$+ 2[\sin 2\theta \sin 2\phi + \cos 2\theta \cos 2\phi] = \frac{1}{4} + \frac{9}{4}$

$\Rightarrow \cos 2\theta \cos 2\phi + \sin 2\theta \sin 2\phi = \frac{1}{4}$

$\Rightarrow \cos(2\theta - 2\phi) = \frac{1}{4} \Rightarrow \cos 2(\theta - \phi) = \frac{1}{4}$

$\Rightarrow 2\cos^2(\theta - \phi) - 1 = \frac{1}{4} \Rightarrow \cos^2(\theta - \phi) = \frac{5}{8}$

CHAPTER 4

PRINCIPLE OF MATHEMATICAL INDUCTION

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Let P(n) be statement $2^n < n!$. Where n is a natural number, then P(n) is true for:

(a) all n (b) all $n > 2$
(c) all $n > 3$ (d) None of these

2. If $P(n) = 2 + 4 + 6 + ... + 2n, n \in \mathbf{N}$, then $P(k) = k(k+1) + 2$ $\Rightarrow P(k+1) = (k+1)(k+2) + 2$ for all $k \in \mathbf{N}$. So we can conclude that $P(n) = n(n+1) + 2$ for

(a) all $n \in \mathbf{N}$ (b) $n > 1$
(c) $n > 2$ (d) nothing can be said

3. Let $T(k)$ be the statement $1 + 3 + 5 + ... + (2k - 1) = k^2 + 10$ Which of the following is correct?

(a) $T(1)$ is true
(b) $T(k)$ is true $\Rightarrow$ $T(k + 1)$ is true
(c) $T(n)$ is true for all $n \in \mathbf{N}$
(d) All above are correct

4. Let $S(K) = 1 + 3 + 5 ... + (2K - 1) = 3 + K^2$, then which of the following is true?

(a) Principle of mathematical induction can be used to prove the formula
(b) $S(K) \Rightarrow S(K+1)$
(c) $S(K) \not\Rightarrow S(K+1)$
(d) $S(1)$ is correct

5. Let P(n) : "$2^n < (1 \times 2 \times 3 \times ... \times n)$". Then the smallest positive integer for which P(n) is true is

(a) 1 (b) 2
(c) 3 (d) 4

6. A student was asked to prove a statement $P(n)$ by induction. He proved that $P(k + 1)$ is true whenever $P(k)$ is true for all $k > 5 \in$ N and also that $P(5)$ is true. On the basis of this he could conclude that P(n) is true

(a) for all $n \in$ N
(b) for all $n > 5$
(c) for all $n \geq 5$
(d) for all $n < 5$

7. If $P(n) : 2 + 4 + 6 + ... + (2n), n \in N$, then $P(k) = k(k + 1) + 2$ implies $P(k + 1) = (k + 1)(k + 2) + 2$ is true for all $k \in N$. So statement $P(n) = n(n + 1) + 2$ is true for:

(a) $n \geq 1$
(b) $n \geq 2$
(c) $n \geq 3$
(d) None of these

8. If $P(n)$: "$46^n + 16^n + k$ is divisible by 64 for $n \in$ N" is true, then the least negative integral value of k is.

(a) -1 (b) 1
(c) 2 (d) -2

9. Use principle of mathematical induction to find the value of k, where $(10^{2n-1} + 1)$ is divisible by k.

(a) 11 (b) 12
(c) 13 (d) 9

10. For all $n \geq 1$, $1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 =$

(a) $\dfrac{n(n+1)}{6}$
(b) $n(n + 1)(2n - 1)$
(c) $\dfrac{n(n-1)(2n+1)}{2}$
(d) $\dfrac{n(n+1)(2n+1)}{6}$

11. P(n) : $2.7^n + 3.5^n - 5$ is divisible by

(a) 24, $\forall$ n $\in$ N
(b) 21, $\forall$ n $\in$ N
(c) 35, $\forall$ n $\in$ N
(d) 50, $\forall$ n $\in$ N

12. For all $n \geq 1$, $\dfrac{1}{1.2} + \dfrac{1}{2.3} + \dfrac{1}{3.4} + ... + \dfrac{1}{n(n+1)} =$

(a) n (b) $\dfrac{n}{n+1}$
(c) $\dfrac{(n+1)}{n}$ (d) $\dfrac{4n+3}{2n}$

13. For every positive integer n, $7^n - 3^n$ is divisible by

(a) 7 (b) 3
(c) 4 (d) 5

14. By mathematical induction,

$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+.....+\frac{1}{n(n+1)(n+2)}$ is equal to

(a) $\frac{n(n+1)}{4(n+2)(n+3)}$

(b) $\frac{n(n+3)}{4(n+1)(n+2)}$

(c) $\frac{n(n+2)}{4(n+1)(n+3)}$

(d) None of these

15. By using principle of mathematical induction for every natural number, $(ab)^n$ =

(a) $a^n b^n$ (b) $a^n b$

(c) ab^n (d) 1

16. If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by

(a) 113 (b) 123

(c) 133 (d) None of these

17. For all $n \in N$, $41^n - 14^n$ is a multiple of

(a) 26 (b) 27

(c) 25 (d) None of these

18. The remainder when 5^{4n} is divided by 13, is

(a) 1 (b) 8

(c) 9 (d) 10

19. If m, n are any two odd positive integers with n < m, then the largest positive integer which divides all the numbers of the type $m^2 - n^2$ is

(a) 4 (b) 6

(c) 8 (d) 9

20. For natural number n, $2^n (n-1)! < n^n$, if

(a) $n < 2$ (b) $n > 2$ (c) $n \geq 2$ (d) $n > 3$

21. For all $n \in N$, $3.5^{2n+1} + 2^{3n+1}$ is divisible by

(a) 19 (b) 17

(c) 23 (d) 25

22. Principle of mathematical induction is used

(a) to prove any statement

(b) to prove results which are true for all real numbers

(c) to prove that statements which are formulated in terms of n, where n is positive integer

(d) in deductive reasoning

23. For all $n \in N$, $1.3 + 2.3^2 + 3.3^3 + ... + n.3^n$ is equal to

(a) $\frac{(2n+1)3^{n+1}+3}{4}$

(b) $\frac{(2n-1)3^{n+1}+3}{4}$

(c) $\frac{(2n+1)3^{n}+3}{4}$

(d) $\frac{(2n-1)3^{n+1}+1}{4}$

24. For all $n \in N$,

$1+\frac{1}{1+2}+\frac{1}{1+2+3}+.....+\frac{1}{1+2+3+.....+n}$

is equal to

(a) $\frac{3n}{n+1}$ (b) $\frac{n}{n+1}$

(c) $\frac{2n}{n-1}$ (d) $\frac{2n}{n+1}$

25. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

(a) 7 (b) 5

(c) 9 (d) 17

26. The statement P(n)

"$1 \times 1! + 2 \times 2! + 3 \times 3! + ... + n \times n! = (n+1)! - 1$" is

(a) True for all n > 1 (b) Not true for any n

(c) True for all $n \in N$ (d) None of these

27. If n is a natural number, then $\left(\frac{n+1}{2}\right)^n \geq n!$ is true when

(a) $n > 1$ (b) $n \geq 1$

(c) $n > 2$ (d) $n \geq 2$

28. For natural number n, $(n!)^2 > n^n$, if

(a) $n > 3$ (b) $n > 4$

(c) $n \geq 4$ (d) $n \geq 3$

STATEMENT TYPE QUESTIONS

Directions : Read the following statement and choose the correct option from the given below four options.

29. **Statement-I :** $1 + 2 + 3 + ... + n < \frac{1}{8}(2n+1)^2$, $n \in N$.

Statement-II : $n(n+1)(n+5)$ is a multiple of 3, $n \in N$.

(a) Only Statement I is true

(b) Only Statement II is true

(c) Both Statements are true

(d) Both Statements are false

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, Reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, Reason is incorrect

(d) Assertion is incorrect, Reason is correct.

30. **Assertion :** For every natural number $n \geq 2$,

$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+.....+\frac{1}{\sqrt{n}}>\sqrt{n}$

Reason : For every natural number $n \geq 2$,

$\sqrt{n(n+1)} < n + 1$.

31. **Assertion :** $11^{m+2} + 12^{2m+1}$ is divisible by 133 for all $m \in N$.

Reason : $x^n - y^n$ is divisible by $x + y$, $\forall\ n \in N, x \neq y$.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

32. The greatest positive integer, which divides $n(n+1)(n+2)(n+3)$ for all $n \in \mathbf{N}$, is

(a) 2 (b) 6
(c) 24 (d) 120

33. Let $P(n) : n^2 + n + 1$ is an even integer. If $P(k)$ is assumed true then $P(k+1)$ is true. Therefore $P(n)$ is true.

(a) for $n > 1$ (b) for all $n \in N$
(c) for $n > 2$ (d) None of these

34. By the principle of induction $\forall\ n \in N$, 3^{2n} when divided by 8, leaves remainder

(a) 2 (b) 3
(c) 7 (d) 1

35. If n is a positive integer, then $5^{2n+2} - 24n - 25$ is divisible by

(a) 574 (b) 575
(c) 674 (d) 576

36. The greatest positive integer, which divides $(n + 1)(n + 2)(n + 3) \ldots\ldots (n + r)$ for all $n \in W$, is

(a) r (b) r!
(c) n + r (d) (r + 1)!

37. If $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$, then P(n) is true for

(a) $n \geq 1$ (b) $n > 0$
(c) $n < 0$ (d) $n \geq 2$

38. For all $n \in N$,

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\ldots\left(1+\frac{(2n+1)}{n^2}\right)$$

is equal to

(a) $\frac{(n+1)^2}{2}$ (b) $\frac{(n+1)^3}{3}$
(c) $(n+1)^2$ (d) None of these

39. For all $n \in N$, the sum of $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is

(a) a negative integer (b) a whole number
(c) a real number (d) a natural number

40. For given series:

$1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 + \ldots\ldots$,

if S_n is the sum of n terms, then

(a) $S_n = \frac{n(n+1)^2}{2}$, if n is even

(b) $S_n = \frac{n^2(n+1)}{2}$, if n is odd

(c) Both (a) and (b) are true
(d) Both (a) and (b) are false

41. When 2^{301} is divided by 5, the least positive remainder is

(a) 4 (b) 8
(c) 2 (d) 6

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(c)** Let $P(n): 2^n < n!$

Then $P(1): 2^1 < 1!$, which is true

Now $P(2): 2^2 < 2!$, which is not true

Also $P(3): 2^3 < 3!$, which is not true

$P(4): 2^4 < 4!$, which is true

Let $P(k)$ is true if $k \geq 4$

That is $2^k < k!$, $k \geq 4$

$\Rightarrow 2.2^k < 2(k!) \Rightarrow 2^{k+1} < k(k!)$ $[\because k \geq 4 > 2]$

$\Rightarrow 2^{k+1} < (k+1)! \Rightarrow P(k+1)$ is true.

Hence, we conclude that $P(n)$ is not true for $n = 2, 3$ but holds true for $n \geq 4$.

2. **(d)** We note that $P(1) = 2$ and hence, $P(n) = n(n+1) + 2$ is not true for $n = 1$.
So the principle of mathematical induction is not applicable and nothing can be said about the validity of the statement $P(n) = n(n+1) + 2$.

3. **(b)** When $k = 1$, LHS = 1 but RHS = 1 + 10 = 11

$\therefore$ $T(1)$ is not true

Let $T(k)$ is true.

i.e., $1 + 3 + 5 + ... + (2k-1) = k^2 + 10$

Now, $1 + 3 + 5 + ... + (2k-1) + (2k+1)$

$= k^2 + 10 + 2k + 1 = (k+1)^2 + 10$

$\therefore T(k+1)$ is true.

i.e., $T(k)$ is true $\Rightarrow T(k+1)$ is true.

But $T(n)$ is not true for all $n \in \mathbf{N}$, as $T(1)$ is not true.

4. **(b)** $S(K) = 1+3+5+...+(2K-1) = 3 + K^2$

$S(1) = 1 = 3 + 1$, which is not true

$\because$ $S(1)$ is not true.

$\therefore$ P.M.I cannot be applied

Let $S(K)$ is true, i.e. $1 + 3 + 5 + ... + (2K-1) = 3 + K^2$

$\Rightarrow 1 + 3 + 5 + ... + (2K-1) + 2K + 1$

$= 3 + K^2 + 2K + 1 = 3 + (K+1)^2$

$\therefore S(K) \Rightarrow S(K+1)$

5. **(d)** Since $P(1): 2 < 1$ is false

$P(2): 2^2 < 1 \times 2$ is false

$P(3): 2^3 < 1 \times 2 \times 3$ is false

$P(4): 2^4 < 1 \times 2 \times 3 \times 4$ is true

6. **(c)** Since $P(5)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

7. **(d)** $P(1) = 2$ and $k(k+1) + 2 = 4$, So $P(1)$ is not true.
Mathematical Induction is not applicable.

8. **(a)** For $n = 1$, $P(1)$: $65 + k$ is divisible by 64.

Thus k, should be –1

Since 65 – 1 = 64 is divisible by 64.

9. **(a)** Let P(n) be the statement given by

$P(n): 10^{2n-1} + 1$ is divisible by 11

For n = 1, $P(1): 10^{(2 \times 1)-1} + 1 = 11$,

which is divisible by 11.

So, P(1) is true.

Let P(k) be true, i.e. $10^{2k-1} + 1$ is divisible by 11

$\Rightarrow 10^{2k-1} + 1 = 11\lambda$, for some $\lambda \in N$ ... (i)

We shall now show that P(k + 1) is true. For this, we have to show that $10^{2(k+1)-1} + 1$ is divisible by 11.

Now, $10^{2(k+1)-1} + 1 = 10^{2k-1}\,10^2 + 1$

$= (11\lambda - 1)100 + 1$ [Using (i)]

$= 1100\lambda - 99 = 11(100\lambda - 9) = 11\mu$,

where $\mu = 100\lambda - 9 \in N$

$\Rightarrow 10^{2(k+1)-1} + 1$ is divisible by 11

$\Rightarrow$ P(k + 1) is true.

Thus, P(k + 1) is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, P(k) is true for all n ∈ N, i.e. $10^{2n-1} + 1$ is divisible by 11 for all n ∈ N.

10. **(d)** Let the given statement be P(n), i.e.

$$P(n): 1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

For n = 1,

$$P(1): 1 = \frac{1(1+1)((2\times 1)+1)}{6} = \frac{1\times 2\times 3}{6} = 1,$$

which is true.

Assume that P(k) is true for some positive integer k,

$$\text{i.e. } 1^2 + 2^2 + 3^2 + 4^2 + ... + k^2 = \frac{k(k+1)(2k+1)}{6} \quad ... (i)$$

We shall now prove that P(k + 1) is also true,
i.e. $1^2 + 2^2 + 3^2 + 4^2 + ... + k^2 + (k+1)^2$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

Now, L.H.S. $= (1^2 + 2^2 + 3^2 + 4^2 + ... + k^2) + (k+1)^2$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \quad \text{[Using (i)]}$$

$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6} = \text{R.H.S.}$$

Thus, P(k + 1) is true, whenever P(k) is true. Hence, from the principle of mathematical induction, the statement P(n) is true for all natural numbers n.

11. **(a)** P(n) : $2.7^n + 3.5^n - 5$ is divisible by 24.

For n = 1,

P(1) : 2.7 + 3.5 – 5 = 24, which is divisible by 24.

Assume that P(k) is true,

i.e. $2.7^k + 3.5^k - 5 = 24q$, where $q \in N$ ... (i)

Now, we wish to prove that P(k + 1) is true whenever P(k) is true, i.e. $2.7^{k+1} + 3.5^{k+1} - 5$ is divisible by 24.

We have,

$2.7^{k+1} + 3.5^{k+1} - 5 = 2.7^k \cdot 7^1 + 3.5^k \cdot 5^1 - 5$

$= 7[2.7^k + 3.5^k - 5 - 3.5^k + 5] + 3.5^k \cdot 5 - 5$

$= 7[24q - 3.5^k + 5] + 15.5^k - 5$

$= (7 \times 24q) - 21.5^k + 35 + 15.5^k - 5$

$= (7 \times 24q) - 6.5^k + 30 = (7\times 24q) - 6(5^k - 5)$

$= (7 \times 24q) - 6(4p)$ [$\because$ $(5^k - 5)$ is a multiple of 4]

$= (7 \times 24q) - 24p = 24(7q - p)$

$= 24 \times r$; $r = 7q - p$, is some natural number ... (ii)

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

12. **(b)** Let P(n) : $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

For n = 1,

P(1) : $\frac{1}{1.2} = \frac{1}{2} = \frac{1}{1+1}$, which is true.

Assume that P(k) is true for some natural number k,

i.e. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{k(k+1)} = \frac{k}{k+1}$ ... (i)

We shall now prove that P(k + 1) is true, i.e.

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

$$\text{L.H.S.} = \left[\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{k(k+1)}\right] + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} \quad \text{[Using (i)]}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{(k^2+2k+1)}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2} = \text{R.H.S.}$$

Thus, P(k + 1) is true whenever P(k) is true. Hence, by the principle of mathematical induction, P(n) is true for all natural numbers.

13. **(c)** Let P(n) : $7^n - 3^n$ is divisible by 4.

For n = 1,

P(1) : $7^1 - 3^1 = 4$, which is divisible by 4. Thus, P(n) is true for n = 1.

Let P(k) be true for some natural number k,

i.e. P(k) : $7^k - 3^k$ is divisible by 4.

We can write $7^k - 3^k = 4d$, where $d \in N$ ... (i)

Now, we wish to prove that P(k + 1) is true whenever P(k) is true, i.e. $7^{k+1} - 3^{k+1}$ is divisible by 4.

Now, $7^{(k+1)} - 3^{(k+1)} = 7^{(k+1)} - 7.3^k + 7.3^k - 3^{(k+1)}$

$= 7(7^k - 3^k) + (7 - 3)3^k = 7(4d) + 4.3^k$ [using (i)]

$= 4(7d + 3^k)$, which is divisible by 4.

Thus, P(k + 1) is true whenever P(k) is true. Therefore, by the principle of mathematical induction the statement is true for every positive integer n.

14. **(b)** Let P(n) : $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + ... + \frac{1}{n(n+1)(n+2)}$

$$= \frac{n(n+3)}{4(n+1)(n+2)}$$

For n = 1,

$$\text{L.H.S.} = \frac{1}{1\cdot2\cdot3} = \frac{1}{6}$$

$$\text{and} \quad \text{R.H.S.} = \frac{1(1+3)}{4(1+1)(1+2)} = \frac{1}{6}$$

$\therefore$ P(1) is true.

Let P(k) is true, then

$$\text{P(k)} : \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + ... + \frac{1}{k(k+1)(k+2)}$$

$$= \frac{k(k+3)}{4(k+1)(k+2)} \quad ... (i)$$

For n = k + 1,

$$\text{P(k + 1)} : \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + ...$$

$$+ \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

$$\text{L.H.S.} = \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + ...$$

$$+ \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

[from (i)]

$$= \frac{(k+1)^2(k+4)}{4(k+1)(k+2)(k+3)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)} = \text{R.H.S.}$$

Hence, P(k + 1) is true.

Hence, by principle of mathematical induction for all $n \in N$, P(n) is true.

15. (a) Let P(n) be the given statement,

i.e. $P(n) : (ab)^n = a^n b^n$

We note that P(n) is true for n = 1, since $(ab)^1 = a^1 b^1$

Let P(k) be true,

i.e. $(ab)^k = a^k b^k$ ... (i)

We shall now prove that P(k + 1) is true whenever P(k) is true.

Now, we have $(ab)^{k+1} = (ab)^k (ab)$

$= (a^k b^k)(ab)$ [by using (i)]

$= (a^k \cdot a^1)(b^k \cdot b^1) = a^{k+1} \cdot b^{k+1}$

Therefore, P(k + 1) is also true whenever P(k) is true. Hence, by principle of mathematical induction, P(n) is true for all $n \in N$.

16. (c) On putting n = 1 in $11^{n+2} + 12^{2n+1}$, we get

$11^{1+2} + 12^{(2\times 1)+1} = 11^3 + 12^3 = 3059$,

which is divisible by 133 only.

17. (b) Let P(n) be the statement given by

$P(n) : 41^n - 14^n$ is a multiple of 27

For n = 1,

i.e. $P(1) = 41^1 - 14^1 = 27 = 1 \times 27$,

which is a multiple of 27.

$\therefore$ P(1) is true.

Let P(k) be true, i.e. $41^k - 14^k = 27\lambda$ ... (i)

For n = k + 1,

$41^{k+1} - 14^{k+1} = 41^k\, 41 - 14^k\, 14$

$= (27\lambda + 14^k)\, 41 - 14^k\, 14$ [using (i)]

$= (27\lambda \times 41) + (14^k \times 41) - (14^k \times 14)$

$= (27\lambda \times 41) + 14^k (41 - 14)$

$= (27\lambda \times 41) + (14^k \times 27)$

$= 27(41\lambda + 14^k)$,

which is a multiple of 27.

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

18. (a) For n = 1,

$5^4 = 625 = (624 + 1) = (48 \times 13) + 1$,

i.e 5^4 leaves 1 as remainder when divided by 13.

19. (c) Let m = 2k + 1, n = 2k – 1 ($k \in N$)

$\therefore\ m^2 - n^2 = 4k^2 + 1 + 4k - 4k^2 + 4k - 1 = 8k$

Hence, all the numbers of the form $m^2 - n^2$ are always divisible by 8.

20. (b) The condition $2^n (n - 1)! < n^n$ is satisfied for n > 2.

21. (b) $3.5^{2n+1} + 2^{3n+1}$

Put n = 1, we get

$(3 \times 5^3) + 2^4 = 391$, which is divisible by 17.

Put n = 2, we get

$(3 \times 5^5) + 2^7 = 9503$, which is divisible by 17 only.

22. (c) In algebra or in other discipline of Mathematics, there are certain results or statements that are formulated in terms of n, where n is a positive integer. To prove such statement, the well-suited principle, i.e. used-based on the specific technique is known as the principle of mathematical induction.

23. (b) Let the statement P(n) be defined as

$P(n) = 1.3 + 2.3^2 + 3.3^3 + \ldots.. + n.3^n$

$$= \frac{(2n-1)3^{n+1} + 3}{4}$$

Step I : For n = 1,

$$P(1) : 1.3 = \frac{(2.1-1)3^{1+1} + 3}{4} = \frac{3^2 + 3}{4}$$

$$= \frac{9+3}{4} = \frac{12}{4} = 3 = 1.3, \text{ which is true.}$$

Step II : Let it is true for n = k,

i.e. $1.3 + 2.3^2 + 3.3^3 + \ldots.. + k.3^k$

$$= \frac{(2k-1)3^{k+1} + 3}{4} \quad \text{... (i)}$$

Step III : For n = k + 1,

$(1.3 + 2.3^2 + 3.3^3 + \ldots.. + k.3^k) + (k + 1)3^{k+1}$

$$= \frac{(2k-1)3^{k+1} + 3}{4} + (k+1)3^{k+1}$$

[Using equation (i)]

$$= \frac{(2k-1)3^{k+1} + 3 + 4(k+1)3^{k+1}}{4}$$

$$= \frac{3^{k+1}(2k - 1 + 4k + 4) + 3}{4}$$

[taking 3^{k+1} common in first and last term of numerator part]

$$= \frac{3^{k+1}(6k+3) + 3}{4} = \frac{3^{k+1} \cdot 3(2k+1) + 3}{4}$$

[taking 3 common in first term of numerator part]

$$= \frac{3^{(k+1)+1}[2k + 2 - 1] + 3}{4}$$

$$= \frac{[2(k+1) - 1]3^{(k+1)+1} + 3}{4}$$

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

24. (d) Let the statement P(n) be defined as

$$P(n): 1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots.. + \frac{1}{1+2+3+\ldots..+n}=\frac{2n}{n+1}$$

i.e. $P(n): 1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots..+\frac{2}{n(n+1)}=\frac{2n}{n+1}$

$\left[\because \text{ sum of natural numbers} = \frac{n(n+1)}{2}\right]$

Step I : For n = 1,

$P(1): 1=\frac{2\times 1}{1+1}=\frac{2}{2}=1$, which is true.

Step II : Let it is true for n = k,

i.e. $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots..+\frac{2}{k(k+1)}=\frac{2k}{k+1}$ ... (i)

Step III : For n = k + 1,

$$\left(1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots..+\frac{2}{k(k+1)}\right)+\frac{2}{(k+1)(k+2)}$$

$=\frac{2k}{k+1}+\frac{2}{(k+1)(k+2)}$ [using equation (i)]

$=\frac{2k(k+2)+2}{(k+1)(k+2)}=\frac{2\left[k^2+2k+1\right]}{(k+1)(k+2)}$

[taking 2 common in numerator part]

$=\frac{2(k+1)^2}{(k+1)(k+2)}$ $[\because (a+b)^2=a^2+2ab+b^2]$

$=\frac{2(k+1)}{k+2}=\frac{2(k+1)}{(k+1)+1}$

Therefore, P(k + 1) is true, when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

25. (c) $10^n + 3(4^{n+2}) + 5$

Taking n = 2;

$10^2 + 3\times 4^4 + 5 = 100 + 768 + 5 = 873$

Therefore, this is divisible by 9.

26. (c) Check for n = 1, 2, 3, ..., it is true for all $n \in N$.

27. (b) Check through option, the condition $\left(\frac{n+1}{2}\right)^n \geq n!$ is true for $n \geq 1$.

28. (d) Check through option, condition $(n!)^2 > n^n$ is true when $n \geq 3$.

STATEMENT TYPE QUESTIONS

29. (c) I. Let the statement P(n) be defined as

$P(n): 1+2+3+\ldots..+n<\frac{1}{8}(2n+1)^2$

Step I : For n = 1,

$P(1): 1<\frac{1}{8}(2.1+1)^2 \Rightarrow 1<\frac{1}{8}\times 3^2$

$\Rightarrow 1<\frac{9}{8}$, which is true.

Step II : Let it is true for n = k.

$1+2+3+\ldots..+k<\frac{1}{8}(2k+1)^2$ ... (i)

Step III : For n = k + 1,

$(1+2+3+\ldots..+k)+(k+1)<\frac{1}{8}(2k+1)^2+(k+1)$

[using equation (i)]

$=\frac{(2k+1)^2}{8}+\frac{k+1}{1}=\frac{(2k+1)^2+8k+8}{8}$

$=\frac{4k^2+1+4k+8k+8}{8}$

$=\frac{4k^2+12k+9}{8}=\frac{(2k+3)^2}{8}$

$=\frac{(2k+2+1)^2}{8}=\frac{[2(k+1)+1]^2}{8}$

$\Rightarrow 1+2+3+\ldots..+k+(k+1)<\frac{[2(k+1)+1]^2}{8}$

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

II. Let the statement P(n) be defined as

P(n) : n(n + 1) (n + 5) is a multiple of 3.

Step I : For n = 1,

$P(1): 1(1+1)(1+5) = 1\times 2\times 6 = 12 = 3\times 4$, which is a multiple of 3, that is true.

Step II : Let it is true for n = k,

i.e. $k(k+1)(k+5) = 3\lambda$

$\Rightarrow k(k^2+5k+k+5) = 3\lambda$

$\Rightarrow k^3+6k^2+5k = 3\lambda$ ... (i)

Step III : For n = k + 1, $(k+1)(k+1+1)(k+1+5)$

$= (k+1)(k+2)(k+6) = (k^2+2k+k+2)(k+6)$

$= (k^2+3k+2)(k+6)$

$= k^3+6k^2+3k^2+18k+2k+12$

$= k^3 + 9k^2 + 20k + 12$
$= (3\lambda - 6k^2 - 5k) + 9k^2 + 20k + 12$ [using equation (i)]
$= 3\lambda + 3k^2 + 15k + 12$
$= 3(\lambda + k^2 + 5k + 4)$, which is a multiple of 3.
Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.
Hence, both the statements are true.

ASSERTION - REASON TYPE QUESTIONS

30. (a) **Assertion :** Let P(n) : $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}$

For n = 2,

P(2) : $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} > \sqrt{2}$, which is true.

Assume P(k) is true,

i.e. $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{k}} > \sqrt{k}$ ... (i)

For n = k + 1, we have to show that

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ ... (ii)

L.H.S. $= \left(\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}}\right) + \frac{1}{\sqrt{k+1}}$ ... (iii)

Reason : For n = k,

$\sqrt{k(k+1)} < k + 1$

$\Rightarrow \sqrt{k}\sqrt{k+1} < \sqrt{k+1}\sqrt{k+1}$

$\Rightarrow \sqrt{k} < \sqrt{k+1}$

$\because \sqrt{k+1} > \sqrt{k}$ for $k \geq 2$

$\Rightarrow 1 > \frac{\sqrt{k}}{\sqrt{k+1}}$

$\Rightarrow \sqrt{k} > \frac{k}{\sqrt{k+1}}$, (Multiplying by $\sqrt{k}$)

$\Rightarrow \sqrt{k} > \frac{(k+1)-1}{\sqrt{k+1}} \Rightarrow \sqrt{k} > \sqrt{k+1} - \frac{1}{\sqrt{k+1}}$

$\Rightarrow \sqrt{k} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ ... (iv)

From (iii) and (iv),

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k}$

$+ \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ [Using (i)]

Hence, (ii) is true for n = k + 1
Hence, P(n) is true for $n \geq 2$
So, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

31. (c) If $11^{m+2} + 12^{2m+1}$ is divisible by 133, then
$11^{m+2} + 12^{2m+1} = 133\lambda, \lambda \in N$ ... (i)
Hence, $11^{(m+1)+2} + 12^{2(m+1)+1}$
$= (11^{m+2} \times 11) + (12^{2m+1} \times 12^2)$
$= (133\lambda - 12^{2m+1}) \times 11 + (144 \times 12^{2m+1})$ [using (i)]
$= (11 \times 133\lambda) - (11 \times 12^{2m+1}) + (144 \times 12^{2m+1})$
$= (11 \times 133\lambda) + (133 \times 12^{2m+1})$

CRITICALTHINKING TYPE QUESTIONS

32. (c) The product of r consecutive integers is divisible by r ! . Thus $n(n+1)(n+2)(n+3)$ is divisible by 4 ! = 24.

33. (d) $P(1)$ is not true (Principle of induction is not applicable). Also $n(n+1)+1$ is always an odd integer.

34. (d) Let P(n) be the statement given by
P(n) : 3^{2n} when divided by 8, the remainder is 1.
or P(n) : $3^{2n} = 8\lambda + 1$ for some $\lambda \in N$
For n = 1,
P(1) : $3^2 = (8 \times 1) + 1 = 8\lambda + 1$, where $\lambda = 1$
$\therefore$ P(1) is true.
Let P(k) be true.
Then, $3^{2k} = 8\lambda + 1$ for some $\lambda \in N$ ... (i)
We shall now show that P(k + 1) is true, for which we have to show that $3^{2(k+1)}$ when divided by 8, the remainder is 1.
Now, $3^{2(k+1)} = 3^{2k} \cdot 3^2 = (8\lambda + 1) \times 9$ [Using (i)]
$= 72\lambda + 9 = 72\lambda + 8 + 1 = 8(9\lambda + 1) + 1$
$= 8\mu + 1$, where $\mu = 9\lambda + 1 \in N$
$\Rightarrow$ P(k + 1) is true.
Thus, P(k + 1) is true, whenever P(k) is true.
Hence, by the principle of mathematical induction P(n) is true for all $n \in N$.

35. (d) Let P(n) be the statement given by
P(n) : $5^{2n+2} - 24n - 25$ is divisible by 576.
For n = 1,
P(1) : $5^{2+2} - 24 - 25 = 625 - 49 = 576$,
which is divisible by 576.
$\therefore$ P(1) is true.
Let P(k) be true,
i.e. P(k) : $5^{2k+2} - 24k - 25$ is divisible by 576.
$\Rightarrow 5^{2k+2} - 24k - 25 = 576\lambda$ ... (i)
We have to show that P(k + 1) is true,
i.e. $5^{2k+4} - 24k - 49$ is divisible by 576

Now, $5^{2k+4} - 24k - 49$

$= 5^{2k+2+2} - 24k - 49 = 5^{2k+2} \cdot 5^2 - 24k - 49$

$= (576\lambda + 24k + 25) \cdot 25 - 24k - 49$ [from (i)]

$= 576.25\lambda + 600k + 625 - 24k - 49$

$= 576.25\lambda + 576k + 576$

$= 576\{25\lambda + k + 1\}$, which is divisible by 576.

$\therefore$ P(k + 1) is true whenever P(k) is true.

So, P(n) is true for all $n \in N$.

36. **(b)** The product of r consecutive natural numbers is divisible by r! and not by (r + 1)!

37. **(d)** Let $P(n) : \frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$

For n = 1,
P(n) is not true.
For n = 2,

$$P(2) : \frac{4^2}{2+1} < \frac{4!}{(2)^2} \Rightarrow \frac{16}{3} < \frac{24}{4} \text{ which is true.}$$

Let for n = m > 2, P(n) is true, i.e.

$$\frac{4^m}{m+1} < \frac{(2m)!}{(m!)^2}$$

$$\text{Now, } \frac{4^{m+1}}{m+2} = \frac{4^m}{m+2} \cdot \frac{4(m+1)}{m+2} < \frac{(2m)!}{(m!)^2} \cdot \frac{4(m+1)}{(m+2)}$$

$$= \frac{(2m)!(2m+1)(2m+2)4(m+1)(m+1)^2}{(2m+1)(2m+2)(m!)^2(m+1)^2(m+2)}$$

$$= \frac{[2(m+1)]!}{[(m+1)!]^2} \cdot \frac{2(m+1)^2}{(2m+1)(m+2)} < \frac{[2(m+1)]!}{[(m+1)!]^2}$$

$$\left[\because \frac{2(m+1)^2}{(2m+1)(m+2)} < 1 \ \forall\ m > 2\right]$$

Hence, for $n \geq 2$, P(n) is true.

38. **(c)** Let the statement P(n) be defined as

$$P(n) : \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2n+1)}{n^2}\right) = (n+1)^2$$

Step I : For n = 1,

$$\text{i.e. } P(1) : \left(1+\frac{3}{1}\right) = (1+1)^2 = 2^2 = 4 = \left(1+\frac{3}{1}\right),$$

which is true.

Step II : Let it is true for n = k,

$$\text{i.e. } \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{2k+1}{k^2}\right) = (k+1)^2 \ \ldots \text{(i)}$$

Step III : For n = k + 1,

$$\left\{\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{2k+1}{k^2}\right)\right\}\left(1+\frac{2k+1+2}{(k+1)^2}\right)$$

$$= (k+1)^2\left(1+\frac{2k+3}{(k+1)^2}\right) \quad \text{[using equation (i)]}$$

$$= (k+1)^2\left[\frac{(k+1)^2+2k+3}{(k+1)^2}\right]$$

$= k^2 + 2k + 1 + 2k + 3$

$= (k+2)^2 = [(k+1)+1]^2$ $[\because (a+b)^2 = a^2 + 2ab + b^2]$

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

39. **(d)** Let the statement P(n) be defined as

$$P(n): \frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15} \text{ is a natural number for all } n \in N.$$

Step I : For n = 1,

$$P(1) : \frac{1}{5} + \frac{1}{3} + \frac{7}{15} = 1 \in N$$

Hence, it is true for n = 1.

Step II : Let it is true for n = k,

$$\text{i.e. } \frac{k^5}{5} + \frac{k^3}{3} + \frac{7k}{15} = \lambda \in N \quad \ldots \text{(i)}$$

Step III : For n = k + 1,

$$\frac{(k+1)^5}{5} + \frac{(k+1)^3}{3} + \frac{7(k+1)}{15}$$

$$= \frac{1}{5}(k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1) + \frac{1}{3}(k^3 + 3k^2 + 3k + 1) + \frac{7}{15}k + \frac{7}{15}$$

$$= \left(\frac{k^5}{5} + \frac{k^3}{3} + \frac{7}{15}k\right) + (k^4 + 2k^3 + 3k^2 + 2k) + \frac{1}{5} + \frac{1}{3} + \frac{7}{15}$$

$= \lambda + k^4 + 2k^3 + 3k^2 + 2k + 1$

[using equation (i)]

which is a natural number, since $\lambda\ k \in N$.

Therefore, P(k + 1) is true, when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

40. **(c)** Let $P(n) : S_n = \begin{cases} \dfrac{n(n+1)^2}{2}, & \text{when n is even} \\ \dfrac{n^2(n+1)}{2}, & \text{when n is odd} \end{cases}$

Also, note that any term T_n of the series is given by

$$T_n = \begin{cases} n^2, & \text{if n is odd} \\ 2n^2, & \text{if n is even} \end{cases}$$

We observe that P(1) is true, since

$$P(1) : S_1 = 1^2 = 1 = \frac{1 \cdot 2}{2} = \frac{1^2 \cdot (1+1)}{2}$$

Assume that P(k) is true for some natural number k, i.e

Case I : When k is odd, then k + 1 is even. We have,

$$P(k+1) : S_{k+1} = 1^2 + 2 \times 2^2 + ... + k^2 + 2 \times (k+1)^2$$

$$= \frac{k^2(k+1)}{2} + 2 \times (k+1)^2$$

$$\left[\text{as k is odd, } 1^2 + 2 \times 2^2 + ... + k^2 = k^2 \frac{(k+1)}{2}\right]$$

$$= \frac{(k+1)}{2}[k^2 + 4(k+1)]$$

$$= \frac{k+1}{2}[k^2 + 4k + 4]$$

$$= \frac{k+1}{2}(k+2)^2$$

$$= (k+1)\frac{[(k+1)+1]^2}{2}$$

So, P(k + 1) is true, whenever P(k) is true, in the case when k is odd.

Case II : When k is even, then k + 1 is odd.

$$\text{Now, } P(k+1) : S_{k+1} = 1^2 + 2 \times 2^2 + ... + 2 \cdot k^2 + (k+1)^2$$

$$= \frac{k(k+1)^2}{2} + (k+1)^2$$

$$\left[\text{as k is even, } 1^2 + 2 \times 2^2 + ... + 2k^2 = k\frac{(k+1)^2}{2}\right]$$

$$= \frac{(k+1)^2(k+2)}{2} = \frac{(k+1)^2((k+1)+1)}{2}$$

Therefore, P(k + 1) is true, whenever P(k) is true for the case when k is even.

Thus, P(k + 1) is true whenever P(k) is true for any natural number k. Hence, P(n) true for all natural numbers n.

41. **(c)** $2^4 \equiv 1 \pmod 5 \Rightarrow (2^4)^{75} \equiv (1)^{75} \pmod 5$

i.e. $2^{300} \equiv 1 \pmod 5 \Rightarrow 2^{300} \times 2 \equiv (1.2) \pmod 5$

$\Rightarrow 2^{301} \equiv 2 \pmod 5$

$\therefore$ Least positive remainder is 2.

CHAPTER

5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Value of $\left(\frac{2i}{1+i}\right)^2$ is

(a) i (b) $2i$
(c) $1-i$ (d) $1-2i$

2. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$ then

(a) $a=2, b=-1$ (b) $a=1, b=0$
(c) $a=0, b=1$ (d) $a=-1, b=2$

3. $1 + i^2 + i^4 + i^6 + ... + i^{2n}$ is

(a) positive (b) negative
(c) 0 (d) cannot be determined

4. If $(x+iy)(2-3i) = 4+i$, then

(a) $x=-14/13, y=5/13$ (b) $x=5/13, y=14/13$
(c) $x=14/13, y=5/13$ (d) $x=5/13, y=-14/13$

5. If $4x + i(3x-y) = 3 + i(-6)$, where x and y are real numbers, then the values of x and y are

(a) $x=\frac{3}{5}$ and $y=\frac{33}{4}$ (b) $x=\frac{3}{4}$ and $y=\frac{22}{3}$
(c) $x=\frac{3}{4}$ and $y=\frac{33}{4}$ (d) $x=\frac{3}{4}$ and $y=\frac{33}{5}$

6. If $z = x - i\,y$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)\Big/(p^2+q^2)$ is equal to

(a) –2 (b) –1 (c) 2 (d) 1

7. The polar form of the complex number $(i^{25})^3$ is

(a) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (b) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$
(c) $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ (d) $\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

8. If $z_1 = \sqrt{3} + i\sqrt{3}$ and $z_2 = \sqrt{3} + i$, then in which quadrant $\left(\frac{z_1}{z_2}\right)$ lies?

(a) I (b) II (c) III (d) IV

9. The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in R$, $a \neq 0$, $b^2 - 4ac < 0$, are given by $x = ?$

(a) $\frac{b \pm \sqrt{4ac - b^2 i}}{2a}$ (b) $\frac{-b \pm \sqrt{4ac + b^2 i}}{2a}$
(c) $\frac{-b \pm \sqrt{4ac - b^2 i}}{2a}$ (d) $\frac{-b \pm \sqrt{4ab - c^2 i}}{2a}$

10. If $x^2 + x + 1 = 0$, then what is the value of x ?

(a) $\frac{1+\sqrt{3}i}{2}$ (b) $\frac{-1\pm\sqrt{3}i}{2}$
(c) $\frac{-1\pm\sqrt{3}i}{3}$ (d) $\frac{-1\pm\sqrt{2}i}{2}$

11. The solution of $\sqrt{3x^2-2} = 2x-1$ are :

(a) (2, 4) (b) (1, 4) (c) (3, 4) (d) (1, 3)

12. If α, β are roots of the equation $x^2 - 5x + 6 = 0$, then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is

(a) $2x^2 - 11x + 30 = 0$ (b) $-x^2 + 11x = 0$
(c) $x^2 - 11x + 30 = 0$ (d) $2x^2 - 5x + 30 = 0$

13. Value of k such that equations $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have one common root, is

(a) –1, –2 (b) $-3, -\frac{27}{4}$
(c) $3, \frac{4}{27}$ (d) –2, – 3

14. If a < b < c < d, then the nature of roots of $(x-a)(x-c) + 2(x-b)(x-d) = 0$ is

(a) real and equal (b) complex
(c) real and unequal (d) None of these

15. For the equation $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$, if the product of roots is zero, then sum of roots is

(a) $-\frac{2bc}{b+c}$ (b) $\frac{2ca}{c+a}$
(c) $\frac{bc}{c+a}$ (d) $\frac{-bc}{b+c}$

16. Product of real roots of the equation $t^2x^2+|x|+9 = 0$

(a) is always positive (b) is always negative
(c) does not exist (d) None of these

17. If p and q are the roots of the equation $x^2+px+q=0$, then
(a) $p=1, q=-2$ (b) $p=0, q=1$
(c) $p=-2, q=0$ (d) $p=-2, q=1$

18. The roots of the given equation
$(p-q)x^2+(q-r)x+(r-p)=0$ are :
(a) $\frac{p-q}{r-p}, 1$ (b) $\frac{q-r}{p-q}, 1$
(c) $\frac{r-p}{p-q}, 1$ (d) None of these

19. If α, β are the roots of $ax^2+bx+c=0$, then $\alpha\beta^2+\alpha^2\beta+\alpha\beta$ equals
(a) $\frac{c(a-b)}{a^2}$ (b) 0
(c) $\frac{-bc}{a^2}$ (d) abc

20. The roots of equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ is
(a) one (b) two
(c) infinite (d) None of these

21. If $z_1=3+i$ and $z_2=i-1$, then
(a) $|z_1+z_2|>|z_1|+|z_2|$ (b) $|z_1+z_2|<|z_1|-|z_2|$
(c) $|z_1+z_2|\le|z_1|+|z_2|$ (d) $|z_1+z_2|<|z_1|+|z_2|$

22. Let z be any complex number such that | z | = 4 and $\arg(z)=\frac{5\pi}{6}$, then value of z is
(a) $-2\sqrt{3}-2i$ (b) $2\sqrt{3}-i$
(c) $\sqrt{2}+3i$ (d) $-2\sqrt{3}+2i$

23. If $z=\frac{3-i}{2+i}+\frac{3+i}{2-i}$, then value of arg (zi) is
(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

24. If the complex numbers z_1, z_2, z_3 represents the vertices of an equilateral triangle such that $|z_1|=|z_2|=|z_3|$, then value of $z_1+z_2+z_3$ is
(a) 0 (b) 1 (c) 2 (d) $\frac{3}{2}$

25. $(1+i)^8+(1-i)^8$ equal to
(a) 2^8 (b) 2^5 (c) $2^4\cos\frac{\pi}{4}$ (d) $2^8\cos\frac{\pi}{8}$

26. The conjugate of the complex number $\frac{2+5i}{4-3i}$ is equal to :
(a) $\frac{7-26i}{25}$ (b) $\frac{-7-26i}{25}$
(c) $\frac{-7+26i}{25}$ (d) $\frac{7+26i}{25}$

27. If z = 1 + i, then the multiplicative inverse of z^2 is (where, $i=\sqrt{-1}$)
(a) 2i (b) 1 – i
(c) $-\frac{i}{2}$ (d) $\frac{i}{2}$

28. $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$ is equal to :
(a) $\frac{1}{2}+\frac{9}{2}i$ (b) $\frac{1}{2}-\frac{9}{2}i$
(c) $\frac{1}{4}-\frac{9}{4}i$ (d) $\frac{1}{4}+\frac{9}{4}i$

29. The complex number $\left(\frac{1+2i}{1-i}\right)$ lies in:
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

30. Amplitude of $\frac{1+\sqrt{3}i}{\sqrt{3}+1}$ is :
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

31. The value of $(1+i)^4\left(1+\frac{1}{i}\right)^4$ is
(a) 12 (b) 2
(c) 8 (d) 16

32. Evaluate: $(1+i)^6+(1-i)^3$.
(a) –2 – 10i (b) 2 – 10i
(c) –2 + 10i (d) 2 + 10i

33. If $(x+iy)^{\frac{1}{3}}=a+ib$, where x, y, a, b ∈ R, then $\frac{x}{a}-\frac{y}{b}=$
(a) a^2-b^2 (b) $-2(a^2+b^2)$
(c) $2(a^2-b^2)$ (d) a^2+b^2

34. The value of $\frac{i^{4n+1}-i^{4n-1}}{2}$. is
(a) i (b) 2i (c) –i (d) –2i

35. $\sqrt{-3}\sqrt{-6}$ is equal to
(a) $3\sqrt{2}$ (b) $-3\sqrt{2}$ (c) $3\sqrt{2}i$ (d) $-3\sqrt{2}i$

36. If z(2 – i) = (3 + i), then z^{20} is equal to
(a) 2^{10} (b) -2^{10}
(c) 2^{20} (d) -2^{20}

37. The real part of $\frac{(1+i)^2}{(3-i)}$ is
(a) $\frac{1}{3}$ (b) $\frac{1}{5}$
(c) $-\frac{1}{3}$ (d) None of these

38. The multiplicative inverse of $\frac{3+4i}{4-5i}$ is
(a) $\frac{8}{25}-\frac{31}{25}i$ (b) $-\frac{8}{25}-\frac{31}{25}i$
(c) $-\frac{8}{25}+\frac{31}{25}i$ (d) None of these

39. What is the conjugate of $\dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$?

(a) $-3i$ (b) $3i$ (c) $\frac{3}{2}i$ (d) $-\frac{3}{2}i$

40. If $z = \dfrac{7-i}{3-4i}$, then $|z|^{14} =$

(a) 2^7 (b) $2^7 i$ (c) -2^7 (d) $-2^7 i$

41. Represent $z = 1+i\sqrt{3}$ in the polar form.

(a) $\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$ (b) $\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}$

(c) $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ (d) $4\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

42. The modulus of $\dfrac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$ is

(a) 2 (b) 4 (c) $3\sqrt{2}$ (d) $2\sqrt{2}$

43. The argument of the complex number $\left(\frac{i}{2}-\frac{2}{i}\right)$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{2}$

44. The square root of $(7-24i)$ is

(a) $\pm(3-5i)$ (b) $\pm(3+4i)$
(c) $\pm(3-4i)$ (d) $\pm(4-3i)$

45. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$.

(a) $\pm\frac{\sqrt{19}}{5}i$ (b) $\pm\frac{\sqrt{19}i}{2}$

(c) $\dfrac{-1\pm\sqrt{19}i}{2\sqrt{5}}$ (d) $\dfrac{-1\pm\sqrt{19}i}{\sqrt{5}}$

46. If α and β are the roots of the equation $x^2 + 2x + 4 = 0$, then $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$ is equal to

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 32 (d) $\frac{1}{4}$

47. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$ equals

(a) $\frac{ac}{b}$ (b) 1 (c) $\frac{ab}{c}$ (d) $\frac{b}{ac}$

48. If 1 – i, is a root of the equation $x^2 + ax + b = 0$, where a, b ∈ R, then the values of a and b are,
(a) 2, 2 (b) –2, 2 (c) –2, –2 (d) 1, 2

49. Which of the following is correct for any two complex numbers z_1 and z_2?
(a) $|z_1 z_2| = |z_1||z_2|$
(b) $\arg(z_1 z_2) = \arg(z_1)\cdot\arg(z_2)$
(c) $|z_1 + z_2| = |z_1| + |z_2|$
(d) $|z_1 + z_2| \ge |z_1| - |z_2|$

50. A number z = a + ib, where a and b are real numbers, is called
(a) complex number (b) real number
(c) natural number (d) integer

51. If $ax^2 + bx + c = 0$ is a quadratic equation, then equation has no real roots, if
(a) D > 0 (b) D = 0
(c) D < 0 (d) None of these

52. If z = a + ib, then real and imaginary part of z are
(a) Re(z) = a, Im(z) = b (b) Re(z) = b, Im(z) = a
(c) Re(z) = a, Im(z) = ib (d) None of these

53. Which of the following options defined 'imaginary number'?
(a) Square root of any number
(b) Square root of positive number
(c) Square root of negative number
(d) Cube root of number

54. If $x = \sqrt{-16}$, then
(a) x = 4i (b) x = 4
(c) x = – 4 (d) All of these

55. If $z_1 = 6 + 3i$ and $z_2 = 2 - i$, then $\frac{z_1}{z_2}$ is equal to

(a) $\frac{1}{5}(9+12i)$ (b) $9+12i$

(c) $3+2i$ (d) $\frac{1}{5}(12+9i)$

56. The value of $(1+i)^5 \times (1-i)^5$ is
(a) –8 (b) 8i (c) 8 (d) 32

57. If $z_1 = 2 - i$ and $z_2 = 1 + i$, then value of $\left|\dfrac{z_1+z_2+1}{z_1-z_2+1}\right|$ is

(a) 2 (b) 2i (c) $\sqrt{2}$ (d) $\sqrt{2}i$

58. If $\dfrac{(1+i)^3}{(1-i)^3}-\dfrac{(1-i)^3}{(1+i)^3} = x+iy$

(a) x = 0, y = –2 (b) x = –2, y = 0
(c) x = 1, y = 1 (d) x = –1, y = 1

59. Additive inverse of 1 – i is
(a) 0 + 0i (b) –1 – i
(c) –1 + i (d) None of these

60. If z is a complex number such that $z^2 = (\bar{z})^2$, then
(a) z is purely real
(b) z is purely imaginary
(c) either z is purely real or purely imaginary
(d) None of these

61. If $|z| = 1$, $(z \neq -1)$ and $z = x + iy$, then $\left(\frac{z-1}{z+1}\right)$ is

(a) purely real (b) purely imaginary
(c) zero (d) undefined

62. If $\bar{z}$ be the conjugate of the complex number z, then which of the following relations is false?

(a) $|z| = |\bar{z}|$ (b) $z \cdot \bar{z} = |\bar{z}|^2$
(c) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ (d) $\arg z = \arg \bar{z}$

63. If $\sqrt{a+ib} = x + iy$, then possible value of $\sqrt{a-ib}$ is

(a) $x^2 + y^2$ (b) $\sqrt{x^2 + y^2}$
(c) $x + iy$ (d) $x - iy$

64. A value of k for which the quadratic equation $x^2 - 2x(1 + 3k) + 7(2k + 3) = 0$ has equal roots is

(a) 1 (b) 2 (c) 3 (d) 4

65. The roots of the equation $3^{2x} - 10.3^x + 9 = 0$ are

(a) 1, 2 (b) 0, 2 (c) 0, 1 (d) 1, 3

66. If $x^2 + y^2 = 25$, $xy = 12$, then x =

(a) {3, 4} (b) {3, –3}
(c) {3, 4, –3, –4} (d) {–3, –3}

67. If the roots of the equations $px^2 + 2qx + r = 0$ and $qx^2 - 2(\sqrt{pr})x + q = 0$ be real, then

(a) $p = q$ (b) $q^2 = pr$
(c) $p^2 = qr$ (d) $r^2 = pq$

68. If $a > 0$, $b > 0$, $c > 0$, then both the roots of the equation $ax^2 + bx + c = 0$.

(a) Are real and negative (b) Have negative real parts
(c) Are rational numbers (d) None of these

69. If a and b are the odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0$, $a \neq 0$, will be

(a) rational (b) irrational
(c) non-real (d) equal

70. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then (p, q) =

(a) (–4, 7) (b) (4, –7) (c) (4, 7) (d) (–4, –7)

71. If the sum of the roots of the equation $x^2 + px + q = 0$ is equal to the sum of their squares, then

(a) $p^2 - q^2 = 0$ (b) $p^2 + q^2 = 2q$
(c) $p^2 + p = 2q$ (d) None of these

72. If a root of the equations $x^2 + px + q = 0$ and $x^2 + \alpha x + \beta = 0$ is common, then its value will be (where $p \neq \alpha$ and $q \neq \beta$)

(a) $\frac{q-\beta}{\alpha-p}$ (b) $\frac{p\beta-\alpha q}{q-\beta}$

(c) $\frac{q-\beta}{\alpha-p}$ or $\frac{p\beta-\alpha q}{q-\beta}$ (d) None of these

73. If $x^2 + ax + 10 = 0$ and $x^2 + bx - 10 = 0$ have a common root, then $a^2 - b^2$ is equal to

(a) 10 (b) 20 (c) 30 (d) 40

74. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then

(a) $a < 2$ (b) $2 \le a \le 3$
(c) $3 < a \le 4$ (d) $a > 4$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

75. **Statement - I :** Roots of quadratic equation $x^2 + 3x + 5 = 0$ is $x = \frac{-3 \pm i\sqrt{11}}{2}$.

Statement - II : If $x^2 - x + 2 = 0$ is a quadratic equation, then its roots are $\frac{1 \pm i\sqrt{7}}{2}$.

(a) Statement I is correct (b) Statement II is correct
(c) Both are correct (d) Both are incorrect

76. **Statement - I :** Let z_1 and z_2 be two complex numbers such that $\overline{z_1} + i\,\overline{z_2} = 0$ and $\arg(z_1 \cdot z_2) = \pi$, then $\arg(z_1)$ is $\frac{3\pi}{4}$.

Statement - II : $\arg(z_1 \cdot z_2) = \arg z_1 + \arg z_2$.

(a) Statement I is correct (b) Statement II is correct
(c) Both are correct (d) Neither I nor II is correct

77. Which of the following are correct?

I. Modulus of $\frac{1+i}{1-i}$ is 1.

II. Argument of $\frac{1+i}{1-i}$ is $\frac{\pi}{2}$.

III. Modulus of $\frac{1}{1+i}$ is $\sqrt{2}$.

IV. Argument of $\frac{1}{1+i}$ is $\frac{\pi}{4}$.

(a) I and II are correct (b) III and IV are correct
(c) I, II and III are correct (d) All are correct

78. **Statement - I :** If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$.

Statement II : If $z = x + iy$, then $|z| = \sqrt{x^2 + y^2}$.

(a) Statement I is correct (b) Statement II is correct
(c) Both are correct (d) Neither I nor II is correct

79. Consider the following statements

I. Additive inverse of $(1 - i)$ is equal to $-1 + i$.

II. If z_1 and z_2 are two complex numbers, then $z_1 - z_2$ represents a complex number which is sum of z_1 and additive inverse of z_2.

III. Simplest form of $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ is $1 + 2\sqrt{2}$ i.

Choose the correct option.
(a) Only I and II are correct.
(b) Only II and III are correct.
(c) I, II and III are correct.
(d) I, II and III are incorrect.

80. Consider the following statements.
I. Representation of $z = x + iy$ in terms of r and θ is called polar form of the complex number.
II. $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

Choose the correct option.
(a) Only I is incorrect.
(b) Only II is correct.
(c) Both I and II are incorrect.
(d) Both I and II are correct.

81. Consider the following statements.
I. Let z_1 and z_2 be two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $\arg(z_1) - \arg(z_2) = 0$
II. Roots of quadratic equation $x^2 + 3x + 5 = 0$ is $x = \frac{-3 \pm i\sqrt{11}}{2}$

Choose the correct option.
(a) Only I is correct. (b) Only II is correct.
(c) Both are correct. (d) Neither I nor II is correct.

82. Consider the following statements.
I. The value of $x^3 + 7x^2 - x + 16$, when $x = 1 + 2i$ is $-17 + 24i$.
II. If $iz^3 + z^2 - z + i = 0$ then $|z| = 1$

Choose the correct option.
(a) Only I is correct. (b) Only II is correct.
(c) Both are correct. (d) Both are incorrect.

83. Consider the following statements.
I. If z, z_1, z_2 be three complex numbers then $z\bar{z} = |z|^2$
II. The modulus of a complex number $z = a + ib$ is defined as $|z| = \sqrt{a^2 + b^2}$.
III. Multiplicative inverse of $z = 3 - 2i$ is $\frac{3}{13} + \frac{2}{13}i$

Choose the correct option.
(a) Only I and II are correct.
(b) Only II and III are correct.
(c) Only I and III are correct.
(d) All I, II and III are correct.

84. Consider the following statements.
I. Modulus of $\frac{1+i}{1-i}$ is 1.
II. Argument of $\frac{1+i}{1-i}$ is $\frac{\pi}{2}$.

Choose the correct option.
(a) Only I is correct. (b) Only II is correct.
(c) Both are correct. (d) Both are incorrect.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

85.

Column - I (Complex Nos.)	Column - II (Multiplicative inverse)
(A) $4 - 3i$	(1) $\frac{\sqrt{5}}{14} - i\frac{3}{14}$
(B) $\sqrt{5} + 3i$	(2) $\frac{4}{25} + i\frac{3}{25}$
(C) $-i$	(3) $0 + i$

Codes:

	A	B	C
(a)	1	2	3
(b)	2	1	3
(c)	1	3	2
(d)	2	3	1

86. Simplify the complex numbers given in column-I and match with column-II.

Column - I	Column - II
(A) $(1-i)^4$	(1) $-\left(\frac{22}{3} + i\frac{107}{27}\right)$
(B) $\left(\frac{1}{3} + 3i\right)^3$	(2) $-4 + 0i$
(C) $\left(-2 - \frac{1}{3}i\right)^3$	(3) $-\frac{242}{27} - 26i$

Codes:

	A	B	C
(a)	1	2	3
(b)	2	1	3
(c)	3	1	2
(d)	2	3	1

87.

Column - I	Column -II
(A) i^{-1}	(1) -1
(B) i^{-2}	(2) $-i$
(C) i^{-3}	(3) i
(D) i^4	(4) 1

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	2	1	3	4
(c)	2	3	4	1
(d)	1	4	3	2

88.

Column - I (Complex Number)	Column - II (a + ib form)
(A) $(1-i)-(-1+6i)$	(1) $-\frac{21}{5}-\frac{21}{10}i$
(B) $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$	(2) –4
(C) $\left(\frac{1}{3}+3i\right)^3$	(3) $-\frac{22}{3}-\frac{107}{27}i$
(D) $(1-i)^4$	(4) $2-7i$
(E) $\left(-2-\frac{1}{3}i\right)^3$	(5) $\frac{-242}{27}-26i$

Codes:

	A	B	C	D	E
(a)	5	4	3	2	1
(b)	4	1	5	2	3
(c)	4	2	5	1	3
(d)	3	1	2	5	4

89.

Column – I (Complex Number)	Column - II (Multiplicative Inverse)
(A) $4-3i$	(1) $\frac{\sqrt{5}}{14}-\frac{3}{14}i$
(B) $\sqrt{5}+3i$	(2) $\frac{1}{49}-\frac{4\sqrt{3}\,i}{49}$
(C) $-i$	(3) $0+i.1$
(D) $\left(2+\sqrt{3}\,i\right)^2$	(4) $\frac{4}{25}+i\frac{3}{25}$

Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	4	3	1	2
(c)	2	1	3	4
(d)	4	1	3	2

90.

Column - I (Quadratic Equation)	Column - II (Roots)
(A) $2x^2+x+1=0$	(1) $\frac{1\pm\sqrt{7}\,i}{2}$
(B) $x^2+3x+9=0$	(2) $\frac{-1\pm\sqrt{7}\,i}{4}$
(C) $-x^2+x-2=0$	(3) $\frac{-3\pm\sqrt{11}\,i}{2}$
(D) $x^2+3x+5=0$	(4) $\frac{-3\pm\left(3\sqrt{3}\right)i}{2}$

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	2	4	1	3
(d)	2	1	4	3

91.

Column - I (Complex Number)	Column - II (Polar form)
(A) $(1-i)$	(1) $\sqrt{2}\left[\cos\left(\frac{-3\pi}{4}\right)+i\sin\left(\frac{-3\pi}{4}\right)\right]$
(B) $(-1+i)$	(2) $2\left[\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right]$
(C) $(-1-i)$	(3) $\sqrt{2}\left[\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right]$
(D) $\left(\sqrt{3}+i\right)$	(4) $\sqrt{2}\left[\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right]$

Codes:

	A	B	C	D
(a)	3	4	1	2
(b)	3	1	4	2
(c)	2	4	1	3
(d)	2	1	4	3

92.

Column - I	Column - II
(A) If z = x + iy, then modulus z is	(1) $a+i0$
(B) The modulus of complex number x + iy is	(2) $0+bi$
(C) Complex numbers which lie on x-axis are in the form of	(3) $\sqrt{x^2+y^2}$
(D) Complex numbers which lie on y-axis are in the form of	(4) distance of the point from the origin

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	4	3	1	2
(c)	4	1	3	2
(d)	2	3	1	4

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

93. $i^{57}+\frac{1}{i^{25}}$, when simplified has the value

(a) 0 (b) $2i$
(c) $-2i$ (d) 2

94. If $z = 2-3i$, then the value of $z^2-4z+13$ is

(a) 1 (b) –1 (c) 0 (d) None of these

95. If $\frac{c+i}{c-i}=a+ib$, where a, b, c are real, then a^2+b^2 is equal to:

(a) 7 (b) 1 (c) c^2 (d) $-c^2$

96. If $x + iy = \frac{a + ib}{a - ib}$, then $x^2 + y^2 =$

(a) 1 (b) 2 (c) 0 (d) 4

97. If $z = x + iy$, $z^{\frac{1}{3}} = a - ib$ and $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$, then value of k equals

(a) 2 (b) 4 (c) 6 (d) 1

98. $2x^2 - (p + 1)x + (p - 1) = 0$. If $\alpha - \beta = \alpha\beta$, then what is the value of p?

(a) 1 (b) 2 (c) 3 (d) –2

99. If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, then $z_1 + z_2$ equals to $a + ai$. Value of 'a' is equal to

(a) 3 (b) 4 (c) 5 (d) 2

100. If $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$, then $z_1 - z_2$ equals to $-1 + bi$. The value of 'b' is

(a) 1 (b) 2 (c) 3 (d) 5

101. If $z = 5i\left(\frac{-3}{5}i\right)$, then z is equal to $3 + bi$. The value of 'b' is

(a) 1 (b) 2 (c) 0 (d) 3

102. If $z_1 = 6 + 3i$ and $z_2 = 2 - i$, then $\frac{z_1}{z_2}$ is equal to $\frac{1}{a}(9 + 12i)$. The value of 'a' is

(a) 1 (b) 2 (c) 4 (d) 5

103. Value of $i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3}$ is

(a) 0 (b) 1 (c) 2 (d) 3

104. If $z = i^9 + i^{19}$, then z is equal to $a + ai$. The value of 'a' is

(a) 0 (b) 1 (c) 2 (d) 3

105. If $z = i^{-39}$, then simplest form of z is equal to $a + i$. The value of 'a' is

(a) 0 (b) 1 (c) 2 (d) 3

106. If $(1 - i)^n = 2^n$, then the value of n is

(a) 1 (b) 2
(c) 0 (d) None of these

107. The value of $(1 + i)^5 (1 - i)^5$ is 2^n. 'n' is equal to

(a) 2 (b) 3 (c) 4 (d) 5

108. The value of $(1 + i)^8 + (1 - i)^8$ is 2^n. Value of n is

(a) 2 (b) 3 (c) 4 (d) 5

109. Roots of $x^2 + 2 = 0$ are $\pm\sqrt{n}\,i$. The value of n is

(a) 1 (b) 2 (c) 3 (d) 4

110. If $z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$ and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$, then $|z_1 z_2|$ is equal to $\sqrt{m}$. Value of m is

(a) 6 (b) 3 (c) 2 (d) 5

111. The modulus of $\sqrt{2i} - \sqrt{-2i}$ is:

(a) 2 (b) $\sqrt{2}$ (c) 0 (d) $2\sqrt{2}$

112. If $z = x + iy$, $z^{1/3} = a - ib$, then $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$ where k is equal to

(a) 1 (b) 2 (c) 3 (d) 4

113. If the equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 - 1) + px + 4x^2 + 2 = 0$ have both roots common, then the value of $(2r - p)$ is :

(a) 0 (b) 1/2
(c) 1 (d) None of these

114. $\arg \bar{z} + \arg z; z \neq 0$ is equal to :

(a) $\frac{\pi}{4}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

115. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to

(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $\frac{-\pi}{2}$

116. If $z = 2 - 3i$, then value of $z^2 - 4z + 13$ is

(a) 0 (b) 1 (c) 2 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

117. Assertion : Let f(x) be a quadratic expression such that $f(0) + f(1) = 0$. If –2 is one of the root of $f(x) = 0$, then other root is $\frac{3}{5}$.

Reason : If α and β are the zeroes of $f(x) = ax^2 + bx + c$, then sum of zeroes $= -\frac{b}{a}$, product of zeroes $= \frac{c}{a}$.

118. Assertion : If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then $\frac{z_1}{z_2}$ is purely imaginary.

Reason : If z is purely imaginary, then $z + \bar{z} = 0$.

119. Assertion : The greatest integral value of λ for which $(2\lambda - 1)x^2 - 4x + (2\lambda - 1) = 0$ has real roots, is 2.

Reason : For real roots of $ax^2 + bx + c = 0$, $D \geq 0$.

120. Assertion : Consider z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| + |z_1 - z_2|$, then $\text{Im}\left(\frac{z_1}{z_2}\right) = 0$.

Reason : $\arg(z) = 0 \Rightarrow z$ is purely real.

121. Assertion : If P and Q are the points in the plane XOY representing the complex numbers z_1 and z_2 respectively, then distance $|PQ| = |z_2 - z_1|$.

Reason : Locus of the point P(z) satisfying $|z - (2 + 3i)| = 4$ is a straight line.

122. Assertion : The equation $ix^2 - 3ix + 2i = 0$ has non-real roots.

Reason : If a, b, c are real and $b^2 - 4ac \geq 0$, then the roots of the equation $ax^2 + bx + c = 0$ are real and if $b^2 - 4ac < 0$, then roots of $ax^2 + bx + c = 0$ are non-real.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

123. If $|z-4|<|z-2|$, its solution is given by
(a) $Re(z)>0$ (b) $Re(z)<0$
(c) $Re(z)>3$ (d) $Re(z)>2$

124. The equation whose roots are twice the roots of the equation, $x^2-3x+3=0$ is:
(a) $4x^2+6x+3=0$ (b) $2x^2-3x+3=0$
(c) $x^2-3x+6=0$ (d) $x^2-6x+12=0$

125. The roots of the equation $4^x - 3 \cdot 2^{x+3} + 128 = 0$ are
(a) 4 and 5 (b) 3 and 4
(c) 2 and 3 (d) 1 and 2

126. If one root of the equation $x^2+px+12=0$ is 4, while the equation $x^2+px+q=0$ has equal roots , then the value of 'q' is
(a) 4 (b) 12
(c) 3 (d) $\frac{49}{4}$

127. For the equation $3x^2+px+3=0, p>0$, if one of the root is square of the other, then p is equal to
(a) 1/3 (b) 1
(c) 3 (d) 2/3

128. Value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}-1$ is
(a) –2 (b) 0 (c) –1 (d) 1

129. Modulus of $z=\frac{(1+i\sqrt{3})(\cos\theta+i\sin\theta)}{2(1-i)(\cos\theta-i\sin\theta)}$ is
(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

130. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are
(a) $\sqrt{2}$ and $\frac{\pi}{6}$ (b) 1 and 0
(c) 1 and $\frac{\pi}{3}$ (d) 1 and $\frac{\pi}{4}$

131. If $|z^2-1|=|z|^2+1$, then z lies on
(a) imaginary axis (b) real axis
(c) origin (d) None of these

132. If $z=2+i$, then $(z-1)(\bar{z}-5)+(\bar{z}-1)(z-5)$ is equal to
(a) 2 (b) 7
(c) –1 (d) –4

133. If $z = r(\cos\theta + i\sin\theta)$, then the value of $\frac{z}{\bar{z}}+\frac{\bar{z}}{z}$ is
(a) $\cos 2\theta$ (b) $2\cos 2\theta$
(c) $2\cos\theta$ (d) $2\sin\theta$

134. The square root of i is
(a) $\pm\frac{1}{\sqrt{2}}(-1+i)$ (b) $\pm\frac{1}{\sqrt{2}}(1+i)$
(c) $\pm\frac{1}{\sqrt{2}}(1-i)$ (d) None of these

135. The number of real roots of $\left(x+\frac{1}{x}\right)^3+\left(x+\frac{1}{x}\right)=0$ is
(a) 0 (b) 2
(c) 4 (d) 6

136. If the roots of the equation $\frac{a}{x-a}+\frac{b}{x-b}=1$ are equal in magnitude and opposite in sign, then
(a) a = b (b) a + b = 1
(c) a – b = 1 (d) a + b = 0

137. Find the value of *a* such that the sum of the squares of the roots of the equation $x^2-(a-2)x-(a+1)=0$ is least.
(a) 4 (b) 2
(c) 1 (d) 3

138. If α, β are the roots of the equation $(x-a)(x-b)=5$, then the roots of the equation $(x-\alpha)(x-\beta)+5=0$ are
(a) a, 5 (b) b, 5
(c) a, α (d) a, b

139. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right|=1$ lies on
(a) circle $x^2+y^2=1$ (b) the x-axis
(c) the y-axis (d) the line x + y = 1

140. The value of $(z+3)(\bar{z}+3)$ is equivalent to
(a) $|z+3|^2$ (b) $|z-3|$
(c) z^2+3 (d) None of these

141. $|z_1+z_2|=|z_1|+|z_2|$ is possible, if
(a) $z_2=\bar{z}_1$ (b) $z_2=\frac{1}{z_1}$
(c) $\arg(z_1)=\arg(z_2)$ (d) $|z_1|=|z_2|$

142. sin x + i cos 2x and cos x – i sin 2x are conjugate to each other for
(a) $x=n\pi$ (b) $x=\left(n+\frac{1}{2}\right)\frac{\pi}{2}$
(c) $x=0$ (d) No value of x

143. The modulus of the complex number z such that $|z+3-i|=1$ and $\arg(z)=\pi$ is equal to
(a) 3 (b) 2
(c) 9 (d) 4

144. If $Z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$, then polar form of Z is
(a) $\sqrt{2}\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$ (b) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$
(c) $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ (d) $\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$

145. $(x - iy)(3 + 5i)$ is the conjugate of $(-6 - 24i)$, then x and y are
(a) $x = 3, y = -3$ (b) $x = -3, y = 3$
(c) $x = -3, y = -3$ (d) $x = 3, y = 3$

146. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then
(a) $|z| = 0$ (b) $|z| = 1$
(c) $|z| > 1$ (d) $|z| < 1$

147. The amplitude of $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$ is
(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{\pi}{15}$

148. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then $(x^2 + y^2)^2 =$
(a) $\frac{a^2+b^2}{c^2+d^2}$ (b) $\frac{a+b}{c+d}$
(c) $\frac{c^2+d^2}{a^2+b^2}$ (d) $\left(\frac{a^2+b^2}{c^2+d^2}\right)^2$

149. If the equation $(m - n)x^2 + (n - 1)x + 1 - m = 0$ has equal roots, then l, m and n satisfy
(a) $2l = m + n$ (b) $2m = n + l$
(c) $m = n + l$ (d) $l = m + n$

150. If the product of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ be 2, then the sum of roots is
(a) 1 (b) –1
(c) 2 (d) –2

151. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} =$
(a) $\frac{2}{a}$ (b) $\frac{2}{b}$ (c) $\frac{2}{c}$ (d) $-\frac{2}{a}$

152. If one root of $ax^2 + bx + c = 0$ be square of the other, then the value of $b^3 + ac^2 + a^2c$ is
(a) 3abc (b) –3abc
(c) 0 (d) None of these

153. If α, β are the roots of $(x - a)(x - b) = c$, $c \neq 0$, then the roots of $(x - \alpha)(x - \beta) + c = 0$ shall be
(a) a, c (b) b, c
(c) a, b (d) a + c, b + c

154. If the roots of the equation $ax^2 + bx + c = 0$ are α, β, then the value of $\alpha\beta^2 + \alpha^2\beta + \alpha\beta$ will be
(a) $\frac{c(a-b)}{a^2}$ (b) 0
(c) $-\frac{bc}{a^2}$ (d) None of these

155. If α, β be the roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 \cdot (2\beta - 35)^3$ is equal to
(a) 1 (b) 64
(c) 8 (d) None of these

156. If the sum of the roots of the equation $x^2 + px + q = 0$ is three times their difference, then which one of the following is true?
(a) $9p^2 = 2q$ (b) $2q^2 = 9p$
(c) $2p^2 = 9q$ (d) $9q^2 = 2p$

157. If the ratio of the roots of $x^2 + bx + c = 0$ and $x^2 + qx + r = 0$ be the same, then
(a) $r^2c = b^2q$ (b) $r^2b = c^2q$
(c) $rb^2 = cq^2$ (d) $rc^2 = bq^2$

158. If the roots of the equation $x^2 - 5x + 16 = 0$ are α, β and the roots of equation $x^2 + px + q = 0$ are $\alpha^2 + \beta^2, \frac{\alpha\beta}{2}$, then
(a) $p = 1, q = -56$ (b) $p = -1, q = -56$
(c) $p = 1, q = 56$ (d) $p = -1, q = 56$

159. If A.M. of the roots of a quadratic equation is $\frac{8}{5}$ and A.M. of their reciprocals is $\frac{8}{7}$, then the equation is
(a) $5x^2 - 16x + 7 = 0$ (b) $7x^2 - 16x + 5 = 0$
(c) $7x^2 - 16x + 8 = 0$ (d) $3x^2 - 12x + 7 = 0$

160. If the roots of $4x^2 + 5k = (5k + 1)x$ differ by unity, then the negative value of k is
(a) –3 (b) –5
(c) $-\frac{1}{5}$ (d) $-\frac{3}{5}$

161. Sum of all real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is
(a) 2 (b) 4 (c) 5 (d) 6

162. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is
(a) 6 (b) 0 (c) 4 (d) 10

163. Value of $\frac{(\cos\theta + i\sin\theta)^4}{(\cos\theta - i\sin\theta)^3}$ is
(a) $\cos 5\theta + i\sin 5\theta$ (b) $\cos 7\theta + i\sin 7\theta$
(c) $\cos 4\theta + i\sin 4\theta$ (d) $\cos\theta + i\sin\theta$

164. The value of $2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots\ldots\ldots\ldots \infty}}$ is
(a) $1 - \sqrt{2}$ (b) $1 + \sqrt{2}$
(c) $1 \pm \sqrt{2}$ (d) None of these

165. If $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots \text{to } \infty}}}$, then
(a) x is an irrational number
(b) $2 < x < 3$
(c) $x = 3$
(d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** $\left(\frac{2i}{1+i}\right)^2 = \frac{4i}{1+i^2+2i} = \frac{-4}{1-1+2i} = \frac{-4}{2i}$

$= \frac{-2}{i} = 2i \left(\because \frac{1}{i} = -i\right)$

2. **(b)** $\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{(1-i)^2}{1-i^2} = \frac{1+i^2-2i}{2} = -i$

$\therefore (-i)^{100} = (i)^{100} = (i^4)^{25} = 1$

$\Rightarrow 1 = a + ib$

$\Rightarrow a = 1, b = 0$

3. **(d)** Given expression $= 1 + i^2 + i^4 + ... + i^{2n}$
$= 1 - 1 + 1 - 1 + ... + (-1)^n$, which cannot be determined unless n is known.

4. **(b)** $x + iy = \frac{4+i}{2-3i} = \frac{(4+i)(2+3i)}{13} = \frac{5+14i}{13}$

$\therefore x = 5/13$, $y = 14/13$

5. **(c)** We have, $4x + i(3x - y) = 3 + i(-6)$
Now, equating the real and the imaginary parts of above equation, we get
$4x = 3$ and $3x - y = -6$

$\Rightarrow x = \frac{3}{4}$ and $3 \times \frac{3}{4} - y = -6$

or $\frac{9}{4} + 6 = y \Rightarrow \frac{9+24}{4} = y$

$\therefore y = \frac{33}{4}$

hence, $x = \frac{3}{4}$ and $y = \frac{33}{4}$

6. **(a)** $z^{\frac{1}{3}} = p + iq$

$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p + iq)$

$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2$

$y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2$

$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2$

$\therefore \left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2) = -2$

7. **(b)** $z = (i^{25})^3 = (i)^{75} = i^{4 \times 18 + 3} = (i^4)^{18}(i)^3$
$= i^3 = -i = 0 - i$
Polar form of $z = r(\cos\theta + i\sin\theta)$

$= 1\left\{\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right\}$

$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

8. **(a)** $\frac{z_1}{z_2} = \frac{\sqrt{3}+i\sqrt{3}}{\sqrt{3}+i} = \left(\frac{3+\sqrt{3}}{4}\right) + \left(\frac{3-\sqrt{3}}{4}\right)i$

which is represented by a point in first quadrant.

9. **(c)** Quadratic equation

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b \pm \sqrt{4ac - b^2 i}}{2a}$

10. **(b)** $b^2 - 4ac = 1^2 - 4 \times 1 \times 1$
$[\because a = 1, b = 1, c = 1]$
$b^2 - 4ac = 1 - 4 = -3$
$\therefore$ the solutions are given by

$x = \frac{-1 \pm \sqrt{-3}}{2 \times 1} = \frac{-1 \pm \sqrt{3}i}{2}$

11. **(d)** Given $\sqrt{3x^2 - 2} = 2x - 1$
squaring both the sides
$\Rightarrow 3x^2 - 2 = 4x^2 + 1 - 4x \Rightarrow x^2 - 4x + 3 = 0$
$\Rightarrow (x - 3)(x - 1) = 0 \Rightarrow x = 1, 3.$

12. **(c)** Let $\alpha + 3 = x$
$\therefore \alpha = x - 3$ (replace x by $x - 3$)
So the required equation
$(x - 3)^2 - 5(x - 3) + 6 = 0$
$\Rightarrow x^2 - 6x + 9 - 5x + 15 + 6 = 0$
$\Rightarrow x^2 - 11x + 30 = 0$

13. **(b)** Let α be the common root
$\therefore \quad 2\alpha^2 + k\alpha - 5 = 0$
$\alpha^2 - 3\alpha - 4 = 0$
Solving both equations

$\frac{\alpha^2}{-4k - 15} = \frac{\alpha}{-5 + 8} = \frac{1}{-6 - k}$

$\Rightarrow \alpha^2 = \frac{4k + 15}{k + 6}$ and $\alpha = \frac{-3}{k + 6}$

$\Rightarrow \left(\frac{-3}{k+6}\right)^2 = \frac{4k+15}{k+6}$

$\Rightarrow (4k+15)(k+6) = 9$

$\Rightarrow 4k^2 + 39k + 81 = 0$

$\Rightarrow k = -3$ or $k = -27/4$

14. (c) Here, $3x^2 - (a+c+2b+2d)x + (ac+2bd) = 0$

$\therefore D = (a+c+2b+2d)^2 - 12(ac+2bd)$

$= [(a+2d)-(c+2b)]^2 + 4(a+2d)(c+2b) - 12(ac+2bd)$

$= [(a+2d)-(c+2b)]^2 + 8(c-b)(d-a) > 0.$

Hence roots are real and unequal.

15. (a) $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$

$\frac{b-a}{x^2+(b+a)x+ab} = \frac{1}{x+c}$

or $x^2 + (a+b)x + ab = (b-a)x + (b-a)c$

or $x^2 + 2ax + ab + ca - bc = 0$

Since product of the roots = 0

$ab + ca - bc = 0$

$a = \frac{bc}{b+c}.$

Thus, sum of roots $= -2a = \frac{-2bc}{b+c}$

16. (a) Product of real roots $= \frac{9}{t^2} > 0, \ \forall\, t \in R$

$\therefore$ Product of real roots is always positive.

17. (a) $p + q = -p$ and $pq = q \Rightarrow q(p-1) = 0$

$\Rightarrow q = 0$ or $p = 1$.

If $q = 0$, then $p = 0$. i.e. $p = q$

$\therefore p = 1$ and $q = -2$.

18. (c) Given equation is

$(p-q)x^2 + (q-r)x + (r-p) = 0$

By using formula for finding the roots

viz: $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$, we get

$x = \frac{(r-q) \pm \sqrt{(q-r)^2 - 4(r-p)(p-q)}}{2(p-q)}$

$\Rightarrow x = \frac{(r-q) \pm (q+r-2p)}{2(p-q)} = \frac{r-p}{p-q}, 1$

19. (a) Given. $ax^2 + bx + c = 0$ and α, β are roots of given equation

$\therefore \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ ...... (i)

Now, $\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\beta+\alpha) + \alpha\beta$

$= \frac{c}{a}\cdot\left(-\frac{b}{a}\right) + \frac{c}{a}$ [Using equation (i)]

$= -\frac{cb}{a^2} + \frac{c}{a}$

$= \frac{-cb+ac}{a^2} = \frac{c(a-b)}{a^2}$

20. (b) Consider the given equation

$x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

By taking L.C.M, we get

$\frac{x(x-1)-2}{x-1} = \frac{x-1-2}{x-1}$

$\Rightarrow x(x-1) - 2 = x - 3$

$\Rightarrow x^2 - x - 2 = x - 3$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1, 1$

Thus, the given equation has two roots.

21. (d) $z_1 + z_2 = 2 + 2i$

$\Rightarrow |z_1 + z_2| = \sqrt{4+4} = \sqrt{8}$

Now $|z_1| = \sqrt{10}$, $|z_2| = \sqrt{2}$.

It is clear that, $|z_1 + z_2| < |z_1| + |z_2|$

22. (d) Let $z = r(\cos\theta + i\sin\theta)$. Then $r = 4$, $\theta = \frac{5\pi}{6}$

$\therefore z = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$= 4\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = -2\sqrt{3} + 2i$

23. (d) $z = \frac{3-i}{2+i} + \frac{3+i}{2-i} = \frac{(3-i)(2-i)+(3+i)(2+i)}{(2+i)(2-i)}$

$\Rightarrow z = 2 \Rightarrow (iz) = 2i$, which is the positive imaginary quantity

$\therefore \arg(iz) = \frac{\pi}{2}$

24. (a) Let the complex number z_1, z_2, z_3 denote the vertices A, B, C of an equilateral triangle ABC. Then, if O be the origin we have $OA = z_1$, $OB = z_2$, $OC = z_3$,

Therefore $|z_1| = |z_2| = |z_3| \Rightarrow OA = OB = OC$

i.e. O is the circumcentre of ΔABC

Hence $z_1 + z_2 + z_3 = 0$.

25. (b) $(1+i)^8 + (1-i)^8$

$= \{(1+i)^2\}^4 + \{(1-i)^2\}^4$

$= \{1+2i+i^2\}^4 + \{1-2i+i^2\}^4$

$= (1+2i-1)^4 + (1-2i-1)^4$

$= 2^4 . i^4 + (-2)^4 . i^4$

$= 2^4 + 2^4$ [Since $i^4 = 1$]

$= 2 \times 2^4$

$= 2^5$

26. (c) Let $z=\dfrac{2+5i}{4-3i}$ Rationalize,

$$=\frac{2+5i}{4-3i}\times\frac{4+3i}{4+3i}$$

$$=\frac{8+26i-15}{(4)^2-(3i)^2}=\frac{8+26i-15}{16+(9)}\quad(\because i^2=-1)$$

$$=\frac{-7+26i}{16+9}=\frac{-7+26i}{25}$$

27. (c) Let $z=1+i$

then $z^2=(1+i)^2$

$=1^2+i^2+2.1.i$

$=1+i^2+2i$

$=1-1+2i$ $\quad(\because i^2=-1)$

$=2i$

Now, $2i\times-\dfrac{i}{2}\Rightarrow -i^2=1$

Hence, $-\dfrac{i}{2}$ is multiplicative inverse of z^2.

28. (d) Let $z=\left(\dfrac{1}{1-2i}+\dfrac{2}{1+i}\right)\left(\dfrac{3+4i}{2-4i}\right)$

$$=\left[\frac{1+i+3-6i}{(1-2i)(1+i)}\right]\left[\frac{3+4i}{2-4i}\right]$$

$$=\left[\frac{4-5i}{3-i}\right]\left[\frac{3+4i}{2-4i}\right]=\left[\frac{32+i}{2-14i}\right]$$

$$=\frac{32+i}{2-14i}\times\frac{2+14i}{2+14i}=\frac{64+448i+2i-14}{4+196}$$

$$=\frac{50+450i}{200}=\frac{1}{4}+\frac{9}{4}i$$

29. (b) Let $z=\dfrac{1+2i}{1-i}$ be the given complex number.

$$\Rightarrow z=\frac{1+2i}{1-i}\times\frac{1+i}{1+i}=\frac{1+i+2i+2i^2}{1-i^2}$$

$$=\frac{-1+3i}{2}=\frac{-1}{2}+\frac{3}{2}i$$

$\Rightarrow (x,y)=\left(\dfrac{-1}{2},\dfrac{3}{2}\right)$ which lies in IInd quadrant.

30. (c) Let $r(\cos\theta+i\sin\theta)=\dfrac{1+i\sqrt{3}}{\sqrt{3}+1}=\dfrac{1}{\sqrt{3}+1}+i\dfrac{\sqrt{3}}{\sqrt{3}+1}$

$\Rightarrow r\cos\theta=\dfrac{1}{\sqrt{3}+1}$; $r\sin\theta=\dfrac{\sqrt{3}}{\sqrt{3}+1}$

$\Rightarrow \tan\theta=\sqrt{3}\Rightarrow\theta=\dfrac{\pi}{3}$.

31. (d) $(1+i)^4\times\left(1+\dfrac{1}{i}\right)^4=(1+i)^4\times(1-i)^4$

$=(1-i^2)^4=(1+1)^4=2^4=16.$

32. (a) $(1+i)^6=\{(1+i)^2\}^3=(1+i^2+2i)^3=(1-1+2i)^3$

$=8i^3=-8i$ and

$(1-i)^3=1-i^3-3i+3i^2$

$=1+i-3i-3=-2-2i$

$\therefore (1+i)^6+(1-i)^3=-8i-2-2i=-2-10i$

33. (b) $(x+iy)^{\frac{1}{3}}=a+ib$

$\Rightarrow x+iy=(a+ib)^3$

$\Rightarrow x+iy=a^3-ib^3+i3a^2b-3ab^2$

$=a^3-3ab^2+i(3a^2b-b^3)$

$\Rightarrow x=a^3-3ab^2$ and $y=3a^2b-b^3$

So, $\dfrac{x}{a}-\dfrac{y}{b}=a^2-3b^2-3a^2+b^2$

$=-2a^2-2b^2=-2(a^2+b^2)$

34. (a) $\dfrac{i^{4n+1}-i^{4n-1}}{2}=\dfrac{i^{4n}i-i^{4n}i^{-1}}{2}$

$$=\frac{i-\frac{1}{i}}{2}=\frac{i^2-1}{2i}=\frac{-2}{2i}=i$$

35. (b) $\sqrt{-3}=i\sqrt{3},\ \sqrt{-6}=i\sqrt{6}$

So, $\sqrt{(-3)}\sqrt{(-6)}=i^2\,3\sqrt{2}=-3\sqrt{2}$

36. (b) We have, $z(2-i)=(3+i)$

$\Rightarrow z=\left(\dfrac{3+i}{2-i}\right)\times\left(\dfrac{2+i}{2+i}\right)=\dfrac{5+5i}{5}$

$\Rightarrow z=1+i$

$\Rightarrow z^2=2i\Rightarrow z^{20}=-2^{10}$

37. (d) $(1+i)^2=1+i^2+2i=2i$

$\therefore \dfrac{(1+i)^2}{3-i}=\dfrac{2i(3+i)}{3^2-i^2}=\dfrac{6i-2}{10}=\dfrac{-1+3i}{5}$

$\therefore$ Real part $=\dfrac{-1}{5}$.

38. (b) Let $z=\dfrac{3+4i}{4-5i}\times\dfrac{4+5i}{4+5i}=-\dfrac{8}{41}+\dfrac{31}{41}i$

Then, $\bar{z}=-\dfrac{8}{41}-\dfrac{31}{41}i$

and $|z|=\sqrt{\left(-\dfrac{8}{41}\right)^2+\left(\dfrac{31}{41}\right)^2}=\dfrac{5}{\sqrt{41}}$

$\therefore$ Multiplicative inverse of z

$$=\frac{\bar{z}}{|z|^2}=\frac{-\frac{8}{41}-\frac{31}{41}i}{\frac{25}{41}}=-\frac{8}{25}-\frac{31}{25}i$$

39. (c) Let $z = \dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}} \times \dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}+\sqrt{5-12i}}$

$= \dfrac{5+12i+5-12i+2\sqrt{25+144}}{5+12i-5+12i}$

$= \dfrac{3}{2i} = \dfrac{3i}{-2} = 0 - \dfrac{3}{2}i$

Therefore, the conjugate of $z = 0 + \dfrac{3}{2}i$

40. (a) $z = \dfrac{7-i}{3-4i} = \dfrac{7-i}{3-4i} \times \dfrac{3+4i}{3+4i} = \dfrac{21+4+i(28-3)}{25}$

$= 1 + i$

$\therefore \ |z| = |1+i| = \sqrt{2}$

$\therefore \ |z|^{14} = \left(\sqrt{2}\right)^{14} = \left[\left(\sqrt{2}\right)^2\right]^7 = 2^7$

41. (c) Let $1 = r\cos\theta$, $\sqrt{3} = r\sin\theta$
By squaring and adding, we get
$r^2(\cos^2\theta + \sin^2\theta) = 4$
i.e, $r = \sqrt{4} = 2$

Therefore, $\cos\theta = \dfrac{1}{2}$, $\sin\theta = \dfrac{\sqrt{3}}{2}$, which gives $\theta = \dfrac{\pi}{3}$

Therefore, required polar form is

$z = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right).$

42. (d) $\dfrac{\left(1+i\sqrt{3}\right)(2+2i)}{\sqrt{3}-i} = \dfrac{2+2\sqrt{3}i+2i-2\sqrt{3}}{\sqrt{3}-i}$

$= \dfrac{\left(2-2\sqrt{3}\right)+\left(2\sqrt{3}+2\right)i}{\sqrt{3}-i} \times \dfrac{\sqrt{3}+i}{\sqrt{3}+i}$

$= \dfrac{2\sqrt{3}-6+2i-2\sqrt{3}i+6i+2\sqrt{3}i-2\sqrt{3}-2}{3+1}$

$= \dfrac{8i-8}{4} = -2+2i$

$\therefore$ Modulus $= \sqrt{(-2)^2+(2)^2} = 2\sqrt{2}.$

43. (d) Since $\left(\dfrac{i}{2} - \dfrac{2}{i}\right) = \dfrac{i}{2} - \dfrac{2i}{i^2} = \dfrac{i}{2} + 2i = \dfrac{5}{2}i$

So, argument is $\tan^{-1}\left(\dfrac{b}{a}\right) = \tan^{-1}\left(\dfrac{\frac{5}{2}}{0}\right) = \dfrac{\pi}{2}.$

44. (d) Let $z = 7 - 24i$
$= 7 - 2\cdot 4\cdot 3i = 16 - 9 - 2\cdot 4\cdot 3i$
$= (4)^2 + (-3i)^2 - 2\cdot 4\cdot 3i$
$= (4-3i)^2$

$\therefore \ \sqrt{7-24i} = \pm(4-3i)$

45. (c) Here, $b^2 - 4ac = 1^2 - 4 \times \sqrt{5} \times \sqrt{5} = 1 - 20 = -19$
Therefore, the solutions are

$\dfrac{-1\pm\sqrt{-19}}{2\sqrt{5}} = \dfrac{-1\pm\sqrt{19}i}{2\sqrt{5}}$

46. (d) Given equation is $x^2 + 2x + 4 = 0$
Since α, β are roots of this equation
$\therefore \ \alpha + \beta = -2$ and $\alpha\beta = 4$

Now, $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3} = \dfrac{\alpha^3+\beta^3}{(\alpha\beta)^3}$

$= \dfrac{(\alpha+\beta)\left(\alpha^2+\beta^2-\alpha\beta\right)}{(\alpha\beta)^3} = \dfrac{(-2)\left((\alpha+\beta)^2-3\alpha\beta\right)}{4\times4\times4}$

$= \dfrac{-2(4-12)}{4\times4\times4} = \dfrac{(-2)\times(-8)}{4\times4\times4} = \dfrac{1}{4}$

47. (d) Since α, β are roots of the equation
$ax^2 + bx + c = 0$

$\therefore \ \alpha + \beta = \dfrac{-b}{a}$, $\alpha\beta = \dfrac{c}{a}$ ... (i)

Now, $\dfrac{1}{a\alpha+b} + \dfrac{1}{a\beta+b} = \dfrac{a(\alpha+\beta)+2b}{(a\alpha+b)(a\beta+b)}$

$= \dfrac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

$= \dfrac{a\left(-\frac{b}{a}\right)+2b}{a^2\cdot\frac{c}{a}+ab\left(-\frac{b}{a}\right)+b^2} = \dfrac{b}{ac}.$ [using (i)]

48. (b) Since complex roots always occur in conjugate pair.
$\therefore$ Other conjugate root is $1 + i$.

Sum of roots $= \dfrac{-a}{1} = (1-i) + (1+i) \Rightarrow a = -2$

Product of roots $= \dfrac{b}{1} = (1-i)(1+i) \Rightarrow b = 2$

49. (a) (a) $|z_1 z_2| = |z_1||z_2|$
(b) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
(c) $|z_1 + z_2| \neq |z_1| + |z_2|$
(d) $|z_1 + z_2| \le |z_1| - |z_2|$

50. (a) A number $z = a + ib$ where $a, b \in R$ is called complex number.

51. (c) For a quadratic equation $ax^2 + bx + c = 0$

$x = \dfrac{-b\pm\sqrt{D}}{2a}$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

For real roots $D \ge 0$. If roots are not real, then $D < 0$.

52. **(a)** Here, $z = a + ib$, then real part of z, i.e., Re(z) = a and imaginary part of z, i.e., Im(z) = b.

53. **(c)** Square root of negative number is imaginary in general $(a)^{\frac{1}{2n}}$, where $a < 0$ and $n \in N$ gives imaginary number.

54. **(a)** Here, $x = \sqrt{-16}$

$$x = \sqrt{-1 \times 16}$$

$$= \sqrt{-1} \times \sqrt{4 \times 4} = 4i$$

55. **(a)** Let $z_1 = 6 + 3i$ and $z_2 = 2 - i$

Then, $\frac{z_1}{z_2} = (6+3i)\frac{1}{2-i} = \frac{(6+3i)(2+i)}{(2-i)(2+i)}$

$$= (6+3i)\left(\frac{2}{2^2+(-1)^2} + i\frac{1}{2^2+(-1)^2}\right)$$

$$= (6+3i)\left(\frac{2}{5} + i\frac{1}{5}\right)$$

$$= (6+3i)\frac{(2+i)}{5}$$

$$= \frac{1}{5}[12 - 3 + i(6 + 6)]$$

$$= \frac{1}{5}(9 + 12i)$$

56. **(d)** $(1 + i)^5 (1 - i)^5 = (1 - i^2)^5$
$= 2^5 = 32$

57. **(c)** $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{2 - i + 1 + i + 1}{2 - i - (1+i) + 1}\right|$

[$\because z_1 = 2 - i$ and $z_2 = 1 + i$]

$$= \left|\frac{4}{2 - i - 1 - i + 1}\right| = \left|\frac{4}{2 - 2i}\right| = \left|\frac{2}{1-i}\right| = \frac{2}{|1-i|}$$

$\left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$

$$= \frac{2}{\sqrt{(1)^2 + (-1)^2}} \qquad \left[\because |z| = \sqrt{a^2 + b^2}\right]$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}.$$

58. **(a)** $\frac{(1+i)^3}{(1-i)^3} - \frac{(1-i)^3}{(1+i)^3} = x + iy$

$\Rightarrow \frac{(1 + i^2 + 2i)^3 - (1 + i^2 - 2i)^3}{(1 - i^2)^3} = x + iy$

$\Rightarrow \frac{8i^3 + 8i^3}{2^3} = x + iy$

$\Rightarrow 2i^3 = x + iy \Rightarrow -2i = x + iy$

$\Rightarrow x = 0, y = -2$

59. **(c)** If $z = x + iy$ is the additive inverse of $1 - i$, then
$(x + iy) + (1 - i) = 0$
$\Rightarrow x + 1 = 0, y - 1 = 0$
$\Rightarrow x = -1, y = 1$
$\therefore$ The additive inverse of $1 - i$ is $z = -1 + i$

Trick: Since $(1 - i) + (-1 + i) = 0$.

60. **(c)** Let $z = x + iy$, then its conjugate $\overline{z} = x - iy$

Given that $z^2 = (\overline{z})^2$

$\Rightarrow x^2 - y^2 + 2ixy = x^2 - y^2 - 2ixy$

$\Rightarrow 4ixy = 0$

If $x \neq 0$, then $y = 0$ and if $y \neq 0$, then $x = 0$.

61. **(b)** $z = x + iy$

$\Rightarrow |z|^2 = x^2 + y^2 = 1$ ... (i)

Now, $\left(\frac{z-1}{z+1}\right) = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$

$$= \frac{(x^2 + y^2 - 1) + 2iy}{(x+1)^2 + y^2} = \frac{2iy}{(x+1)^2 + y^2}$$

[By equation (i)]

Hence, $\left(\frac{z-1}{z+1}\right)$ is purely imaginary.

62. **(d)** Let $z = x + iy$, $\overline{z} = x - iy$

Since $\arg(z) = \theta = \tan^{-1}\frac{y}{x}$

$\arg(\overline{z}) = \theta = \tan^{-1}\left(\frac{-y}{x}\right)$

Thus, $\arg(z) \neq \arg(\overline{z})$.

63. **(d)** $\sqrt{a + ib} = x + yi$

$\Rightarrow \left(\sqrt{a+ib}\right)^2 = (x + yi)^2$

$\Rightarrow a = x^2 - y^2$, $b = 2xy$ and hence

$\sqrt{a - ib} = \sqrt{x^2 - y^2 - 2xyi} = \sqrt{(x - yi)^2} = x - iy$

Note: In the question, it should have been given that $a, b, x, y \in R$.

64. **(b)** Given equation is $x^2 - 2x(1 + 3k) + 7(2k + 3) = 0$

Since, it has equal roots.

$\therefore$ Discriminant D = 0

$\Rightarrow b^2 - 4ac = 0 \Rightarrow 4(1 + 3k)^2 - 4 \times 7(2k + 3) = 0$

$\Rightarrow 1 + 9k^2 + 6k - 14k - 21 = 0$

$\Rightarrow 9k^2 - 8k - 20 = 0$

$\Rightarrow 9k^2 - 18k + 10k - 20 = 0$

$\Rightarrow 9k(k - 2) + 10(k - 2) = 0$

$\Rightarrow k = \frac{-10}{9}, 2$

Only k = 2 satisfy given equation.

65. **(b)** Given equation is $3^{2x} - 10.3^x + 9 = 0$ can be written as $(3^x)^2 - 10(3^x) + 9 = 0$
Let $a = 3^x$, then it reduces to the equation
$a^2 - 10a + 9 = 0 \Rightarrow (a - 9)(a - 1) = 0$
$\Rightarrow a = 9, 1$
Now, $a = 3^x$
$\Rightarrow 9 = 3^x \Rightarrow 3^2 = 3^x \Rightarrow x = 2$
and $1 = 3^x \Rightarrow 3^0 = 3^x \Rightarrow x = 0$
Hence, roots are 0, 2.

66. **(c)** $x^2 + y^2 = 25$ and $xy = 12$

$$\Rightarrow x^2 + \left(\frac{12}{x}\right)^2 = 25$$

$\Rightarrow x^4 + 144 - 25x^2 = 0$
$\Rightarrow (x^2 - 16)(x^2 - 9) = 0$
$\Rightarrow x^2 = 16$ and $x^2 = 9$
$\Rightarrow x = \pm 4$ and $x = \pm 3$.

67. **(b)** Equations $px^2 + 2qx + r = 0$ and
$qx^2 - 2\left(\sqrt{pr}\right)x + q = 0$ have real roots, then
from first
$4q^2 - 4pr \geq 0 \Rightarrow q^2 - pr \geq 0$
$\Rightarrow q^2 \geq pr$ ... (i)
and from second
$4(pr) - 4q^2 \geq 0$ (for real root)
$\Rightarrow pr \geq q^2$ ... (ii)
From (i) and (ii), we get result
$q^2 = pr$.

68. **(b)** The roots of the equations are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) Let $b^2 - 4ac > 0$, $b > 0$
Now, if $a > 0$, $c > 0$, $b^2 - 4ac < b^2$
$\Rightarrow$ the roots are negative.
(ii) Let $b^2 - 4ac < 0$, then the roots are given by

$$x = \frac{-b \pm i\sqrt{(4ac - b^2)}}{2a}, \quad \left(i = \sqrt{-1}\right)$$

which are imaginary and have negative real part. $[\because b > 0]$
$\therefore$ In each case, the roots have negative real part.

69. **(a)** Given equation $2ax^2 + (2a + b)x + b = 0$, $(a \neq 0)$
Now, its discriminant $D = B^2 - 4AC$
$= (2a + b)^2 - 4.2ab = (2a - b)^2$
Hence, D is a perfect square. So, given equation has rational roots.

70. **(a)** Since $2 + i\sqrt{3}$ is a root, therefore, $2 - i\sqrt{3}$ will be other root. Now sum of the roots $= 4 = -p$ and product of roots $= 7 = q$. Hence $(p, q) = (-4, 7)$.

71. **(c)** Let the roots be α and β
$\Rightarrow \alpha + \beta = -p$, $\alpha\beta = q$
Given, $\alpha + \beta = \alpha^2 + \beta^2$
But $\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$
$\Rightarrow -p = (-p)^2 - 2q$
$\Rightarrow p^2 - 2q = -p \Rightarrow p^2 + p = 2q$

72. **(c)** Let the common root be y.
Then, $y^2 + py + q = 0$ and $y^2 + \alpha y + \beta = 0$
On solving by cross multiplication, we have

$$\frac{y^2}{p\beta - q\alpha} = \frac{y}{q - \beta} = \frac{1}{\alpha - p}$$

$$\therefore \quad y = \frac{q - \beta}{\alpha - p} \text{ and } \frac{y^2}{y} = y = \frac{p\beta - q\alpha}{q - \beta}.$$

73. **(d)** Let α be a common root, then
$\alpha^2 + a\alpha + 10 = 0$ ... (i)
and $\alpha^2 + b\alpha - 10 = 0$ ... (ii)
From (i) – (ii),

$$(a - b)\alpha + 20 = 0 \Rightarrow \alpha = -\frac{20}{a - b}$$

Substituting the value of α in (i), we get

$$\left(-\frac{20}{a - b}\right)^2 + a\left(-\frac{20}{a - b}\right) + 10 = 0$$

$\Rightarrow 400 - 20a(a - b) + 10(a - b)^2 = 0$
$\Rightarrow 40 - 2a^2 + 2ab + a^2 + b^2 - 2ab = 0$
$\Rightarrow a^2 - b^2 = 40$.

74. **(a)** Given equation is $x^2 - 2ax + a^2 + a - 3 = 0$
If roots are real, then $D \geq 0$
$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$
$\Rightarrow -a + 3 \geq 0$
$\Rightarrow a - 3 \leq 0 \Rightarrow a \leq 3$
As roots are less than 3, hence $f(3) > 0$.
$9 - 6a + a^2 + a - 3 > 0$
$\Rightarrow a^2 - 5a + 6 > 0$
$\Rightarrow (a - 2)(a - 3) > 0 \Rightarrow$ either $a < 2$ or $a > 3$
Hence, $a < 2$ satisfy all.

STATEMENT TYPE QUESTIONS

75. **(c)** I. Given $x^2 + 3x + 5 = 0$
On comparing the given equation with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = 3$, $c = 5$
Now, $D = b^2 - 4ac$
$= (3)^2 - 4 \times 1 \times 5 = 9 - 20 = -11 < 0$

$$\Rightarrow x = \frac{-3 \pm \sqrt{-11}}{2 \times 1}$$

$$\therefore x = \frac{-3 \pm i\sqrt{11}}{2} \quad \left[\because \sqrt{-1} = i\right]$$

II. Given $x^2 - x + 2 = 0$
On comparing the given equation with $ax^2 + bx + c = 0$, we get

a = 1, b = –1, c = 2

Now, $D = b^2 - 4ac$

$= (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$

$\Rightarrow x = \dfrac{-(-1) \pm \sqrt{-7}}{2 \times 1}$

$= \dfrac{1 \pm i\sqrt{7}}{2}$ $\quad [\because \sqrt{-1} = i]$

76. (c) Given that, $\overline{z_1} + i\,\overline{z_2} = 0$

$\Rightarrow z_1 = iz_2$, i.e. $z_2 = -iz_1$

Thus, $\arg(z_1 z_2) = \arg z_1 + \arg(-iz_1) = \pi$

$\because \arg(z_1 z_2) = \arg z_1 + \arg z_2$

$\Rightarrow \arg(-iz_1^{\,2}) = \pi$

$\Rightarrow \arg(-i) + 2\arg(z_1) = \pi$

$\Rightarrow \dfrac{-\pi}{2} + 2\arg(z_1) = \pi$

$\Rightarrow \arg(z_1) = \dfrac{3\pi}{4}$

77. (a) I. We have,

$\dfrac{1+i}{1-i} = \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i} = \dfrac{1-1+2i}{1+1} = i = 0 + i$

Now, let us put $0 = r\cos\theta$, $1 = r\sin\theta$

Squaring and adding,

$r^2 = 1$, i.e, $r = 1$

So, $\cos\theta = 0$, $\sin\theta = 1$

Therefore, $\theta = \dfrac{\pi}{2}$

Hence, the modulus of $\dfrac{1+i}{1-i}$ is 1 and the argument is $\dfrac{\pi}{2}$.

II. We have :

$\dfrac{1}{1+i} = \dfrac{1-i}{(1+i)(1-i)} = \dfrac{1-i}{1+1} = \dfrac{1}{2} - \dfrac{i}{2}$

Let $\dfrac{1}{2} = r\cos\theta$, $-\dfrac{1}{2} = r\sin\theta$

Proceeding as

$r = \dfrac{1}{\sqrt{2}}$; $\cos\theta = \dfrac{1}{\sqrt{2}}$, $\sin\theta = \dfrac{-1}{\sqrt{2}}$

Therefore, $\theta = \dfrac{-\pi}{4}$

[$\because \cos\theta > 0$ and $\sin\theta < 0$ is in IV quadrant]

Hence, the modulus of $\dfrac{1}{1+i}$ is $\dfrac{1}{\sqrt{2}}$ and the argument is $-\dfrac{\pi}{4}$.

78. (c) $(a + ib)(c + id)(e + if)(g + ih) = A + iB$

Taking modulus on both sides, we get

$|(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|$

$\Rightarrow |a + ib|\,|c + id|\,|e + if|\,|g + ih| = |A + iB|$

$[\because |z_1 z_2 \ldots z_n| = |z_1||z_2||z_3|\ldots|z_n|]$

$\Rightarrow \sqrt{a^2+b^2}\sqrt{c^2+d^2}\sqrt{e^2+f^2}\sqrt{g^2+h^2} = \sqrt{A^2+B^2}$

$\left[\because \text{If } z = a + ib, \text{ then } |z| = \sqrt{a^2+b^2}\right]$

Squaring on both sides, we get

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

79. (c) I. Additive inverse of $(1 - i) = -(1 - i) = -1 + i$

II. Since, difference of two complex numbers is also a complex number and $z_1 - z_2$ can be written as $(z_1) + (-z_2)$ which is sum of z_1 and additive inverse of z_2.

III. $\dfrac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \dfrac{1+\sqrt{2}i}{1+\sqrt{2}i} = \dfrac{5+5\sqrt{2}i+\sqrt{2}i-2}{1+2}$

$= \dfrac{3+6\sqrt{2}i}{3} = 1+2\sqrt{2}i$

80. (d) By definition, both the statements are correct.

81. (c) II. $x^2 + 3x + 5 = 0$

$x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(5)}}{2}$

$= \dfrac{-3 \pm \sqrt{9-20}}{2}$

$= \dfrac{-3 \pm \sqrt{11}i}{2}$

82. (c) I. $x = 1 + 2i$

$\Rightarrow (x - 1) = 2i$

$\Rightarrow (x - 1)^2 = (2i)^2 \Rightarrow x^2 - 2x + 5 = 0$

Consider

$x^3 + 7x^2 - x + 16 = x(x^2 - 2x + 5) + 9(x^2 - 2x + 5) + (12x - 29)$

$= x(0) + 9(0) + 12x - 29$

$= -17 + 24i$

II. $iz^3 + z^2 - z + i = 0$

$z^3 - iz^2 + iz + 1 = 0$ (Dividing both side by i)

$\Rightarrow (z - i)(z^2 + i) = 0$

$\Rightarrow z = i$ or $z^2 = -i$

Now, $z = i \Rightarrow |z| = |i| = 1$

$z^2 = -i \Rightarrow |z^2| = |-i| = 1$

$\Rightarrow |z|^2 = 1$

$\Rightarrow |z| = 1$

83. (d) I. $z\bar{z} = (a + ib)(a - ib)$

$= a^2 - (ib)^2 = a^2 + b^2$

$= |z|^2$

III. $z^{-1} = \frac{3}{(3)^2+(-2)^2} + \frac{i(2)}{3^2+(-2)^2}$

$= \frac{3}{13} + \frac{2}{13}i$

84. (c) $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1-i^2}$

$= \frac{1+i^2+2i}{2} = \frac{2i}{2} = i \equiv 0+i$

$\left|\frac{1+i}{1-i}\right| = |i| = 1$

Now, $r\cos\theta = 0, r\sin\theta = 1$

$r^2 = 1 \Rightarrow r = 1$

$\therefore \cos\theta = 0$ and $\sin\theta = 1$

$\Rightarrow \theta = \frac{\pi}{2}$

MATCHING TYPE QUESTIONS

85. (b) A. Let $z = 4 - 3i$

Then, its multiplicative inverse is

$\frac{1}{z} = \frac{1}{4-3i} = \frac{1}{4-3i} \times \frac{4+3i}{4+3i} = \frac{4+3i}{16-9i^2}$

[use $(a-b)(a+b) = a^2 - b^2$]

$= \frac{4+3i}{16+9}$ [$\because i^2 = -1$]

$= \frac{4+3i}{25} = \frac{4}{25} + \frac{3i}{25}$

B. Let $z = \sqrt{5} + 3i$

Then, its multiplicative inverse is

$\frac{1}{z} = \frac{1}{\sqrt{5}+3i} = \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$

$= \frac{\sqrt{5}-3i}{5-9i^2}$ [use $(a+b)(a-b) = a^2 - b^2$]

$= \frac{\sqrt{5}-3i}{5+9} = \frac{\sqrt{5}-3i}{14}$ [$\because i^2 = -1$]

$= \frac{\sqrt{5}}{14} - \frac{3i}{14}$

C. Let $z = -i$

Then, its multiplicative inverse is

$\frac{1}{z} = -\frac{1}{i} = -\frac{1}{i} \times \frac{i}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i$ [$\because i^2 = -1$]

$= 0 + i.$

86. (d) A. $(1-i)^4 = [(1-i)^2]^2$

$= (1 + i^2 - 2i)^2$ [use $(a-b)^2 = a^2 + b^2 - 2ab$]

$= (1 - 1 - 2i)^2$ [$\because i^2 = -1$]

$= (-2i)^2 = (-2)^2\, i^2$

$= 4(-1) = -4 + 0i$

B. $\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3 \times \frac{1}{3} \times 3i\left(\frac{1}{3}+3i\right)$

$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$

$= \frac{1}{27} - 27i + 3i \times \frac{1}{3} + 3i \times 3i$ [$\because i^3 = -i$]

$= \frac{1}{27} - 27i + i + 9i^2$

$= \frac{1}{27} - 27i + i - 9$ [$\because i^2 = -1$]

$= \left(\frac{1}{27} - \frac{9}{1}\right) - i(27-1)$

$= \left(\frac{1-243}{27}\right) - 26i = -\frac{242}{27} - 26i$

C. $(-1)^3\left(2+\frac{1}{3}i\right)^3$

$= -\left[(2)^3 + \left(\frac{1}{3}i\right)^3 + 3 \times 2 \times \frac{1}{3}i\left(2+\frac{1}{3}i\right)\right]$

$= -\left[8 + \frac{1}{27}i^3 + 2i\left(2+\frac{1}{3}i\right)\right]$

$= -\left[8 - \frac{1}{27}i + 4i + \frac{2}{3}i^2\right]$ [$\because i^3 = -i$]

$= -\left[8 - \frac{1}{27}i + 4i - \frac{2}{3}\right]$ [$\because i^2 = -1$]

$= -\left[\left(\frac{8}{1} - \frac{2}{3}\right) + i\left(\frac{4}{1} - \frac{1}{27}\right)\right]$

$= -\left[\left(\frac{24-2}{3}\right) + i\left(\frac{108-1}{27}\right)\right]$

$= -\left[\frac{22}{3} + i\frac{107}{27}\right]$

87. (b) We know that,

$i = \sqrt{-1},\ i^2 = -1$

$\Rightarrow i^{-1} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{-1} = -i$

$\Rightarrow i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$

$\Rightarrow \ i^{-3} = \dfrac{1}{i^3} = \left\{\dfrac{i}{i^3 \times i}\right\}$

[multiplying numerator and denominator by i]

$\Rightarrow \ \dfrac{i}{i^4} = i$

$\Rightarrow \ i^{-4} = \dfrac{1}{i^4} = \dfrac{1}{1} = 1$

88. (b) (A) $(1-i)-(-1+i.6) = (1-i)+(1-6i)$
$= 1+1-i-6i$
$= 2-7i = (a+ib),$
where $a = 2, b = -7$

(B) $\left(\dfrac{1}{5}+i.\dfrac{2}{5}\right)-\left(4+i.\dfrac{5}{2}\right) = \left(\dfrac{1}{5}+\dfrac{2}{5}i\right)+\left(-4-\dfrac{5}{2}i\right)$

$= \dfrac{1}{5}-4+\dfrac{2}{5}i-\dfrac{5}{2}i = -\dfrac{19}{5}-\left(-\dfrac{2}{5}+\dfrac{5}{2}\right)i$

$= -\dfrac{21}{5}-\dfrac{21}{10}i$

(C) $\left(\dfrac{1}{3}+3i\right)^3 = \left(\dfrac{1}{3}\right)^3+3\left(\dfrac{1}{3}\right)^2(3i)+3.\left(\dfrac{1}{3}\right)(3i)^2+(3i)^3$

$= \dfrac{1}{27}+i+9(-1)+27i^3$

$= \dfrac{1}{27}+i+9(-1)+27i.(i^2)$

$= \dfrac{1}{27}+i+9(-1)+27i.(-1)$

$= \dfrac{1}{27}+i-9-27i = -\dfrac{242}{27}-26i$

(D) $(1-i)^4 = [(1-i)^2]^2$
$= (1+i^2-2i)^2 = (1-1-2i)^2$
$= (-2i)^2 = 4i^2 = 4(-1) = -4$

(E) $\left(-2-\dfrac{1}{3}i\right)^3$

$= (-2)^3-3(-2)^2.\left(\dfrac{1}{3}i\right)+3(-2)\left(-\dfrac{1}{3}i\right)^2-\left(\dfrac{1}{3}i\right)^3$

$= -8-4i-6\times\dfrac{1}{9}(i^2)-\dfrac{1}{27}i^3$

$= -8-4i-\dfrac{2}{3}(-1)-\dfrac{1}{27}i.i^2$

$= -8-4i+\dfrac{2}{3}-\dfrac{1}{27}i.(-1)$

$= -8-4i+\dfrac{2}{3}+\dfrac{1}{27}i$

$= -\dfrac{22}{3}-\dfrac{107}{27}i$

89. (d) (A) We have multiplicative inverse of $4-3i$

$= \dfrac{1}{4-3i}\times\dfrac{4+3i}{4+3i}$

$= \dfrac{4+3i}{4^2-9i^2} = \dfrac{4+3i}{16+9} = \dfrac{4+3i}{25} = \dfrac{4}{25}+i\dfrac{3}{25}$

(B) We have multiplicative inverse of $\sqrt{5}+3i$

$= \dfrac{1}{\sqrt{5}+3i}\times\dfrac{\sqrt{5}-3i}{\sqrt{5}-3i}$ (multiply by conjugate)

$= \dfrac{\sqrt{5}-3i}{5-9i^2} = \dfrac{\sqrt{5}-3i}{5+9} = \dfrac{\sqrt{5}-3i}{14} = \dfrac{\sqrt{5}}{14}-\dfrac{3}{14}i$

$[\because\ (a+ib)(a-ib) = a^2+b^2]$

(C) We have multiplicative inverse of $-i = \dfrac{1}{-i}$.

Multiply by conjugate

$= \dfrac{1}{-i}\times\dfrac{i}{i} = \dfrac{-i}{i^2} = \dfrac{-i}{-1} = i = 0+i\,.\,1$

(D) $z = (2+\sqrt{3}i)^2 = 4+3i^2+4\sqrt{3}i$

$= 1+4\sqrt{3}i$

$\therefore\ \dfrac{1}{z} = \dfrac{1}{4+\sqrt{3}i} = \dfrac{1-4\sqrt{3}i}{1+48}$ (On rationalizing)

90. (c) (A) $2x^2+x+1 = 0$. Comparing with $ax^2+bx+c = 0$
$a = 2, b = 1, c = 1$
$b^2-4ac = 1^2-4.2.1 = 1-8 = -7$

$\therefore\ x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{-1\pm\sqrt{-7}}{2.2}$

$= \dfrac{-1\pm\sqrt{7}i}{4}$

(B) $x^2+3x+9 = 0 \therefore a = 1, b = 3, c = 9$
$b^2-4ac = 3^2-4.1.9 = 9-36 = -27$

$\therefore\ x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{-3\pm\sqrt{-27}}{2\times 1}$

$= \dfrac{-3\pm(3\sqrt{3})i}{2}$

(C) $-x^2+x-2 = 0$ or $x^2-x+2 = 0$
$a = 1, b = -1, c = 2$

Hence, $x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-1)\pm\sqrt{(-1)^2-4\times1\times2}}{2\times1}$

$= \dfrac{1\pm\sqrt{1-8}}{2}$

$= \dfrac{1\pm\sqrt{-7}}{2} = \dfrac{1\pm\sqrt{7}i}{2}$

(D) $x^2+3x+5=0$

$a=1, b=3, c=5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-3\pm\sqrt{(3)^2-4\times1\times5}}{2\times1}$$

$$x=\frac{-3\pm\sqrt{9-20}}{2}$$

$$x=\frac{-3\pm\sqrt{-11}}{2}$$

$$x=\frac{-3\pm\sqrt{11}\,i}{2}$$

91. (a)

(A) We have $1-i=r(\cos\theta+i\sin\theta)$

$\Rightarrow$ $r\cos\theta=1, r\sin\theta=-1$

By squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=1^2+(-1)^2$

$\Rightarrow r^2 . 1=1+1 \Rightarrow r^2=2$

$\therefore$ $r=\sqrt{2}$, By dividing $\frac{r\sin\theta}{r\cos\theta}=\frac{-1}{1}=-1$

$\Rightarrow$ $\tan\theta=-1$ i.e., θ lies in fourth quadrant.

$\Rightarrow$ $\theta=-45$

$\Rightarrow$ $\theta=-\frac{\pi}{4}$

$\therefore$ Polar form of $1-i$

$$=\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$$

(B) We have $-1+i=r(\cos\theta+i\sin\theta)$

$\Rightarrow$ $r\cos\theta=-1$ and $r\sin\theta=1$

By squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=(-1)^2+1^2 \Rightarrow r^2 . 1=1+1$

$\therefore$ $r^2=2$ $\therefore$ $r=\sqrt{2}$

By dividing, $\frac{r\sin\theta}{r\cos\theta}=\frac{1}{-1}=-1 \Rightarrow \tan\theta=-1$

$\therefore$ θ lies in second quadrant ;

$\theta=180°-45°=135°$ i.e $\theta=\frac{3\pi}{4}$

$\therefore$ Polar form of $-1+i$

$$=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$

(C) we have $-1-i=r(\cos\theta+i\sin\theta)$

$\Rightarrow$ $r\cos\theta=-1$ and $r\sin\theta=-1$

By squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=(-1)^2+(-1)^2$

$\Rightarrow$ $r^2 . 1=1+1$

$\Rightarrow$ $r^2=2$

$\therefore$ $r=\sqrt{2}$

By dividing $\frac{r\sin\theta}{r\cos\theta}=\frac{-1}{-1}=1 \Rightarrow \tan\theta=1$

$\therefore$ θ lies in IIIrd quadrant.

$\theta=-180°+45°=-135°$ or $\theta=-\frac{3\pi}{4}$

$\therefore$ Polar form of $-1-i$

$$=\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)$$

(D) $r=\sqrt{3}+i=r(\cos\theta+i\sin\theta)$

$\therefore$ $r\cos\theta=\sqrt{3},\ r\sin\theta=1$

Squaring and adding $r^2=3+1=4, r=2$

Also $\tan\theta=\frac{1}{\sqrt{3}}$, $\sin\theta$ and $\cos\theta$ both are positive.

$\therefore$ θ lies in the I quadrant

$\therefore$ $\theta=30°=\frac{\pi}{6}$

$\therefore$ Polar form of z is $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

92. (b) $z=x+iy \Rightarrow |z|=\sqrt{x^2+y^2}$

INTEGER TYPE QUESTIONS

93. (a) $i^{57}+\frac{1}{i^{25}}=(i^4)^{14}.i+\frac{1}{(i^4)^6.i}$

$=i+\frac{1}{i}$ $(\because i^4=1)$

$=i-i$ $\left(\because\frac{1}{i}=-i\right)$

$=0$

94. (c) $z=2-3i \Rightarrow z-2=-3i$

Squaring, we get

$z^2-4z+4=-9 \Rightarrow z^2-4z+13=0$

95. (b) Given: $\frac{c+i}{c-i}=a+ib$

Then, $a+ib=\frac{c+i}{c-i}\times\frac{c+i}{c+i}=\frac{c^2-1+2ic}{c^2+1}$

$\Rightarrow$ $a=\frac{c^2-1}{c^2+1}$ and $b=\frac{2c}{c^2+1}$

$\Rightarrow$ $(a^2+b^2)=\frac{(c^2-1)^2+4c^2}{(c^2+1)^2}$

$$=\frac{c^4+1-2c^2+4c^2}{(c^2+1)^2}=\frac{(c^2+1)^2}{(c^2+1)^2}=1.$$

96. (a) We have,

$$x + iy = \frac{a + ib}{a - ib} \quad \text{... (i)}$$

Its conjugate,

$$x - iy = \frac{a - ib}{a + ib} \quad \text{... (ii)}$$

Multiply (i) and (ii),

$$(x + iy)(x - iy) = \frac{a + ib}{a - ib} \times \frac{a - ib}{a + ib}$$

$x^2 + y^2 = 1.$

97. (b) $(x + iy)^{\frac{1}{3}} = a - ib$

$x + iy = (a - ib)^3 = (a^3 - 3ab^2) + i(b^3 - 3a^2 b)$

$\Rightarrow x = a^3 - 3ab^2,\ y = b^3 - 3a^2 b$

$\Rightarrow \frac{x}{a} = a^2 - 3b^2,\ \frac{y}{b} = b^2 - 3a^2$

$\therefore \frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - b^2 + 3a^2$

$\frac{x}{a} - \frac{y}{b} = 4(a^2 - b^2) = k(a^2 - b^2)$

$\therefore k = 4.$

98. (b) $2x^2 - (p + 1)x + (p - 1) = 0$

Given $\alpha - \beta = \alpha\beta \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = \alpha^2 \beta^2$

$\Rightarrow \frac{(p-1)^2}{4} = \frac{(p+1)^2}{4} - \frac{4(p-1)}{2}$

$\Rightarrow 2(p - 1) = p \Rightarrow p = 2.$

99. (c) $z_1 = 2 + 3i, z_2 = 3 + 2i$

$z_1 + z_2 = (2 + 3i) + (3 + 2i) = 5 + 5i$

Hence, a = 5

100. (d) $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$

$z_1 - z_2 = (2 + 3i) - (3 - 2i) = -1 + 5i \equiv -1 + bi$

Hence, b = 5.

101. (c) $z = 5i\left(\frac{-3}{5}i\right) = -3i^2 = -3(-1) = 3 \equiv 3 + 0i$

Hence, b = 0

102. (d) $\frac{z_1}{z_2} = \frac{6+3i}{2-i} = \frac{6+3i}{2-i} \times \frac{2+i}{2+i}$

$= \frac{12+6i+6i-3}{4-i^2} = \frac{9+12i}{5}$

$= \frac{1}{5}(9+12i) \equiv \frac{1}{a}(9+12i)$

Hence, a = 5.

103. (a) Consider

$i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3}$

$= (i^4)^k + (i^4)^k . i + (i^4)^k . i^2 + (i^4)^k . i^3$

$= 1 + i + i^2 + i^3 = 1 + i - 1 - i = 0$

104. (a) $z = i^9 + i^{19} = (i^4)^2 . i + (i^4)^4 . i^3$

$= i + i^3 \qquad (\because i^4 = 1)$

$= i - i = 0 \qquad (\because i^3 = 1)$

$\equiv 0 + 0i$

Hence, a = 0

105. (a) $z = i^{-39} = \frac{1}{i^{39}} = \frac{1}{(i^4)^9} \cdot \frac{1}{i^3}$

$= 1 \cdot \frac{1}{i^3} \qquad (\because i^4 = 1)$

$= \frac{1}{i^2 \cdot i} = \frac{-1}{i} \qquad (\because i^2 = -1)$

$= i \qquad (\because \frac{1}{i} = -i)$

$\equiv 0 + i$

Hence, value of a = 0.

106. (c) $(1 - i)^n = 2^n$

Take modulus, both the side $|(1 - i)^n| = |2^n|$

$|1 - i|^n = |2|^n$

$\Rightarrow \left[\sqrt{1^2 + (-1)^2}\right]^n = 2^n$

$\Rightarrow \left(\sqrt{2}\right)^n = 2^n \Rightarrow 2^{\frac{n}{2}} = 2^n$

$\Rightarrow \frac{n}{2} = n \quad \Rightarrow n = 0$

107. (d) $(1 + i)^5 (1 - i)^5 = [(1 + i)(1 - i)]^5$

$= (1 - i^2)^5 = [1 - (-1)]^5$

$= 2^5$

108. (d) $(1 + i)^8 + (1 - i)^8 = [\{(1 + i)^2\}^4 + \{(1 - i)^2\}^4]$

$= [(1 + i^2 + 2i)^4 + (1 + i^2 - 2i)^4]$

$= [(2i)^4 + (-2i)^4] = 16i^4 + 16i^4$

$= 32i^4 = 32 = 2^5$

109. (b) $x^2 + 2 = 0 \Rightarrow x^2 = -2$

$\Rightarrow x = \pm\sqrt{-2} = \pm\sqrt{2}\, i$

110. (a) $z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right]$

$= \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = 1 + i$

$|z_1| = \sqrt{2}$

and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right]$

$= \sqrt{3}\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]$

$|z_2| = \sqrt{\frac{3}{4}+\frac{9}{4}} = \sqrt{3}$

$|z_1 z_2| = |z_1||z_2| = \sqrt{2}\cdot\sqrt{3} = \sqrt{6}$

111. (a) As we know, if z = a + ib, then

$|z| = \sqrt{a^2+b^2}$

Let $z = \sqrt{2i} - \sqrt{-2i}$

$= \sqrt{2i} - i\sqrt{2i}\ \left(\because \sqrt{-1} = i\right)$

$= \sqrt{2i}(1-i)$

Now, $|z| = |\sqrt{2}\sqrt{i}(1-i)|$

$= \sqrt{2}\,|\sqrt{i}|\,|1-i| = \sqrt{2}\times 1\times\sqrt{1^2+(-1)^2}$

$= \sqrt{2}\times\sqrt{2} = 2$

112. (d) $z^{1/3} = a - ib \Rightarrow z = (a-ib)^3$

$\therefore\ x + iy = a^3 + ib^3 - 3ia^2b - 3ab^2$. Then

$x = a^3 - 3ab^2 \Rightarrow \frac{x}{a} = a^2 - 3b^2$

$y = b^3 - 3a^2b \Rightarrow \frac{y}{b} = b^2 - 3a^2$

So, $\frac{x}{a} - \frac{y}{b} = 4(a^2 - b^2)$

113. (a) Given equations are

$k(6x^2+3) + rx + 2x^2 - 1 = 0$ and

$6k(2x^2-1) + px + 4x^2 + 2 = 0$

$\Rightarrow (6k+2)x^2 + rx + 3k - 1 = 0$ ...(i)

$\Rightarrow (12k+4)x^2 + px - 6k + 2 = 0$ ...(ii)

Let α and β be the roots of both equations (i) and (ii).

$\therefore\ \alpha+\beta = \frac{-r}{6k+2}$ (from (i))

and $\alpha+\beta = \frac{-p}{12k+4}$ (from (ii))

$\therefore\ \frac{-r}{2(1+3k)} = \frac{-p}{4(1+3k)} \Rightarrow \frac{-r}{2} = \frac{-p}{4}$

$\Rightarrow -2r = -p \Rightarrow 2r - p = 0.$

114. (c) Let $z = r(\cos\theta + i\sin\theta)$

Then $r = |z|$ and $\theta = \arg(z)$

Now $z = r(\cos\theta + i\sin\theta)$

$\Rightarrow \bar{z} = r(\cos\theta - i\sin\theta)$

$= r[\cos(-\theta) + i\sin(-\theta)]$

$\therefore\ \arg(\bar{z}) = -\theta$

$\Rightarrow \arg(\bar{z}) = -\arg(z)$

$\Rightarrow \arg(\bar{z}) + \arg(z) = 0.$

115. (c) $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow$ z_1 and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.

116. (a) We have, z = 2 – 3i

$\Rightarrow z - 2 = -3i \Rightarrow (z-2)^2 = (-3i)^2$

$\Rightarrow z^2 - 4z + 4 = 9i^2 \Rightarrow z^2 - 4z + 13 = 0$

ASSERTION - REASON TYPE QUESTIONS

117. (a) Since x = –2 is a root of f(x).

$\therefore$ f(x) = (x + 2) (ax + b)

But f(0) + f(1) = 0

$\therefore$ 2b + 3a + 3b = 0

$\Rightarrow -\frac{b}{a} = \frac{3}{5}.$

118. (b) We have,

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$\Rightarrow |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2) = |z_1|^2 + |z_2|^2$

where $\theta_1 = \arg(z_1)$, $\theta_2 = \arg(z_2)$

$\Rightarrow \cos(\theta_1 - \theta_2) = 0$

$\Rightarrow \theta_1 - \theta_2 = \frac{\pi}{2}$

$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$

$\Rightarrow \mathrm{Re}\left(\frac{z_1}{z_2}\right) = 0$

$\therefore\ \frac{z_1}{z_2}$ is purely imaginary.

If z is purely imaginary, then $z + \bar{z} = 0$.

119. (d) For real roots, $D \ge 0$

$\Rightarrow (-4)^2 - 4(2\lambda - 1)(2\lambda - 1) \ge 0 \Rightarrow (2\lambda - 1)^2 \le 4$

$\Rightarrow -2 \le 2\lambda - 1 \le 2$

$\Rightarrow -\frac{1}{2} \le \lambda \le \frac{3}{2}$

$\therefore$ Integral values of λ are 0 and 1

Hence, greatest integral value of λ = 1.

120. (a) We have, arg(z) = 0

$\Rightarrow$ z is purely real

$\therefore$ Reason is true.

Also, $|z_1| = |z_2| + |z_1 - z_2|$

$\Rightarrow |z_1 - z_2|^2 = (|z_1| - |z_2|)^2$

$\Rightarrow |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$= |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \quad \Rightarrow \theta_1 - \theta_2 = 0$

$\Rightarrow \arg(z_1) - \arg(z_2) = 0$

$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = 0$

$\Rightarrow \frac{z_1}{z_2}$ is purely real.

$\Rightarrow \text{Im}\left(\frac{z_1}{z_2}\right) = 0$

121. (c) Assertion is a standard result.

$|z - (2 + 3i)| = 4$

$\Rightarrow$ Distance of P(z) from the point (2, 3) is equal to 4.

$\Rightarrow$ Locus of P is a circle with centre at (2, 3) and radius 4.

122. (d) We have,

$ix^2 - 3ix + 2i = 0$

or $i(x^2 - 3x + 2) = 0$

$\Rightarrow x^2 - 3x + 2 = 0 \qquad [\because i \neq 0]$

$\Rightarrow x = 1, 2$, which are real.

CRITICALTHINKING TYPE QUESTIONS

123. (c) Given $|z-4|<|z-2|$ Let $z = x + iy$

$\Rightarrow |(x-4)+iy)|<|(x-2)+iy|$

$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$

$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$

$\Rightarrow x > 3 \Rightarrow \text{Re}(z) > 3$

124. (d) The given equation is $x^2 - 3x + 3 = 0$

Let a, b be the roots of the given equation then,

$a + b = 3, ab = 3$

We know, $(a - b)^2 = (a + b)^2 - 4ab = 9 - 12$

$\Rightarrow a - b = \sqrt{3}i$

So, $a = \frac{3+\sqrt{3}i}{2}$ and $b = \frac{3-\sqrt{3}i}{2}$

If A and B are the roots of the new equation which are double of the founded roots then

$A = 3 + \sqrt{3}i$ and $B = 3 - \sqrt{3}i$

So, $A + B = 6$ and $AB = 9 + 3 = 12$

Thus the new equation is

$x^2 - 6x + 12 = 0$

125. (b) We have, $4^x - 3 \,.\, 2^{x+3} + 128 = 0$

$\Rightarrow 2^{2x} - 3 \,.\, 2^x \,.\, 2^3 + 128 = 0$

$\Rightarrow 2^{2x} - 24 \,.\, 2^x + 128 = 0$

$\Rightarrow y^2 - 24y + 128 = 0$ where $2^x = y$

$\Rightarrow (y - 16)(y - 8) = 0 \Rightarrow y = 16, 8$

$\Rightarrow 2^x = 16$ or $2^x = 8 \Rightarrow x = 4$ or 3

126. (d) 4 is a root of $x^2 + px + 12 = 0$

$\Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$

Now, the equation $x^2 + px + q = 0$ has equal roots.

$\therefore p^2 - 4q = 0 \Rightarrow q = \frac{p^2}{4} = \frac{49}{4}$

127. (c) Let α, α^2 be the roots of $3x^2 + px + 3$.

$\therefore \quad \alpha + \alpha^2 = -p/3$ and $\alpha^3 = 1$

$\Rightarrow \quad (\alpha - 1)(\alpha^2 + \alpha + 1) = 0$

$\Rightarrow \quad \alpha = 1$ or $\alpha^2 + \alpha = -1$

If $\alpha = 1, p = -6$ which is not possible as $p > 0$

If $\alpha^2 + \alpha = -1 \Rightarrow -p/3 = -1 \Rightarrow p = 3$.

128. (a) Given expression

$= \frac{i^{10}\left(i^{582} + i^{580} + i^{578} + i^{576} + i^{574}\right)}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$

$= i^{10} - 1 = (i^2)^5 - 1 = (-1)^5 - 1$

$= -1 - 1 = -2$

129. (c) $|z| = \frac{|1+i\sqrt{3}|\,|\cos\theta + i\sin\theta|}{2|1-i|\,|\cos\theta - i\sin\theta|} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

130. (b) Suppose, $z = \frac{1+2i}{1-(1-i)^2}$

$= \frac{1+2i}{1-(1^2 + i^2 - 2i)} \qquad [\text{using } (a-b)^2]$

$= \frac{1+2i}{1+2i} \qquad (\because i^2 = -1)$

$= 1 = 1 + 0\,.\,i$

$|z| = \sqrt{(\text{Real part})^2 + (\text{Img. Part})^2}$

and amp $(z) = \tan^{-1}\left[\frac{\text{Img. part}}{\text{Real part}}\right]$

$\therefore \quad |z| = 1$ and amp $(z) = \tan^{-1}\left(\frac{0}{1}\right) = 0$

131. (a) Let $z = x + iy$,

$\because \quad |z^2 - 1| = |z|^2 + 1$

$\Rightarrow \quad |x^2 - y^2 - 1 + i2xy| = |x + iy|^2 + 1$

$\Rightarrow \quad (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$

$\Rightarrow \quad 4x^2 = 0 \Rightarrow x = 0$

Hence, z lies on y-axis or imaginary axis.

132. (d) $(z-1)(\bar{z}-5) + (\bar{z}-1)(z-5)$

$= 2\text{Re}\left[(z-1)(\bar{z}-5)\right]$

$\left[\because z_1\bar{z}_2 + z_2\bar{z}_1 = 2\text{Re}(z_1 z_2)\right]$

$= 2\text{ Re}[(1 + i)(-3 - i)] = 2(-2) = -4$

[Given $z = 2 + i$]

133. (b) Given, $z = r(\cos\theta + i\sin\theta)$;

$\bar{z} = r(\cos\theta - i\sin\theta)$

$\therefore \quad \dfrac{z}{\bar{z}} = \dfrac{r(\cos\theta + i\sin\theta)}{r(\cos\theta - i\sin\theta)}$

$= (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)^{-1}$

$= (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$

$= (\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$

$\therefore \quad \dfrac{\bar{z}}{z} = (\cos\theta - i\sin\theta)(\cos\theta + i\sin\theta)^{-1}$

$= (\cos\theta - i\sin\theta)(\cos\theta - i\sin\theta)$

$= (\cos\theta - i\sin\theta)^2 = (\cos 2\theta - i\sin 2\theta)$

$\therefore \quad \dfrac{z}{\bar{z}} + \dfrac{\bar{z}}{z} = \cos 2\theta + i\sin 2\theta + \cos 2\theta - i\sin 2\theta$

$= 2\cos 2\theta$

134. (b) Let $z = i = \dfrac{1}{2} + \dfrac{1}{2}i^2 + 2\cdot\dfrac{1}{\sqrt{2}}\cdot\dfrac{1}{\sqrt{2}}i$

$= \left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}i\right)^2 + 2\dfrac{1}{\sqrt{2}}\cdot\dfrac{1}{\sqrt{2}}i$

$= \left(\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}i\right)^2$

$\therefore \quad \sqrt{i} = \dfrac{\pm 1}{\sqrt{2}}(1+i).$

135. (a) We have, $\left(x + \dfrac{1}{x}\right)^3 + \left(x + \dfrac{1}{x}\right) = 0$

$\Rightarrow \left(x + \dfrac{1}{x}\right)\left[\left(x + \dfrac{1}{x}\right)^2 + 1\right] = 0$

$\Rightarrow$ either $x + \dfrac{1}{x} = 0$

$\Rightarrow x^2 = -1 \Rightarrow x = \pm i$

or $\left(x + \dfrac{1}{x}\right)^2 + 1 = 0$

$\Rightarrow x^2 + \dfrac{1}{x^2} + 3 = 0$

$\Rightarrow x^4 + 3x^2 + 1 = 0$

$\Rightarrow x^2 = \dfrac{-3 \pm \sqrt{9-4}}{2} = \dfrac{-3 \pm \sqrt{5}}{2} < 0$

$\therefore$ There is no real root.

136. (d) Given equation is $\dfrac{a}{x-a} + \dfrac{b}{x-b} = 1$

$\Rightarrow a(x-b) + b(x-a) = (x-a)(x-b)$

$\Rightarrow x^2 - x(a+b) + ab = ax - ab + bx - ab$

$\Rightarrow x^2 - 2x(a+b) + 3ab = 0$

So, sum of roots $= \alpha + (-\alpha) = 2(a+b)$

or $a + b = 0$.

137. (c) Let α, β be the roots of the equation.

$\therefore \quad \alpha + \beta = a - 2$ and $\alpha\beta = -(a+1)$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (a-2)^2 + 2(a+1)$

$= (a-1)^2 + 5$

$\therefore \quad \alpha^2 + \beta^2$ will be minimum if $(a-1)^2 = 0$, i.e. $a = 1$.

138. (d) Since α, β are roots of the equation

$(x-a)(x-b) = 5$ or $x^2 - (a+b)x + (ab-5) = 0$

$$\left.\begin{aligned}\therefore\ \alpha+\beta &= a+b \text{ or } a+b = \alpha+\beta \\ \text{and } \alpha\beta &= ab-5 \text{ or } ab = \alpha\beta+5\end{aligned}\right\} \quad \text{... (i)}$$

Taking another equation

$(x-\alpha)(x-\beta) + 5 = 0$

or $x^2 - (\alpha+\beta)x + (\alpha\beta+5) = 0$

or $x^2 - (a+b)x + ab = 0$ [using (i)]

$\therefore$ Its roots are a, b.

139. (b) Given, $\left|\dfrac{i+z}{i-z}\right| = 1$

Let $z = x + iy$

$\therefore \quad \left|\dfrac{i + x + iy}{i - (x+iy)}\right| = 1$

$\Rightarrow \left|\dfrac{x + i(1+y)}{-x + i(1-y)}\right| = 1$

$\Rightarrow \sqrt{x^2 + (1+y)^2} = \sqrt{(-x)^2 + (1-y)^2}$

$\Rightarrow x^2 + 1 + y^2 + 2y = x^2 + 1 + y^2 - 2y$

$\Rightarrow 4y = 0 \Rightarrow y = 0$

Hence, z lies on x-axis.

140. (a) $(z+3)(\bar{z}+3) = z\bar{z} + 3z + 3\bar{z} + (3)^2$

$= |z|^2 + 3\left(\dfrac{z+\bar{z}}{2}\right) \times 2 + 3^2 \qquad \left[\because |z|^2 = z\bar{z}\right]$

$= |z|^2 + 2 \times 3 \times (\text{Re}(z)) + 3^2$

$= |z|^2 + 2\text{Re}(3z) + (3)^2 = |z+3|^2$

141. (c) Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$,

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$\because \quad |z_1 + z_2| = |z_1| + |z_2|$ [given]

$\Rightarrow |(r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)|$

$= r_1 + r_2$

$\Rightarrow \sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2)} = r_1 + r_2$

$\Rightarrow r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2) = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1$

$\Rightarrow \theta_1 - \theta_2 = 0 \Rightarrow \theta_1 = \theta_2$

$\Rightarrow \arg(z_1) = \arg(z_2)$.

142. (d) Let $z = \sin x + i\cos 2x$

According to the given condition,

$\bar{z} = \cos x - i\sin 2x$

$\therefore$ $\sin x - i\cos 2x = \cos x - i\sin 2x$

$\Rightarrow$ $(\sin x - \cos x) + i(\sin 2x - \cos 2x) = 0$

On equating real and imaginary parts, we get

$\sin x - \cos x = 0$, $\sin 2x - \cos 2x = 0$

$\Rightarrow$ $\tan x = 1$ and $\tan 2x = 1$

$\Rightarrow$ $x = \frac{\pi}{4}$ and $2x = \frac{\pi}{4}$

$\Rightarrow$ $x = \frac{\pi}{4}$ and $x = \frac{\pi}{8}$

which is not possible.

143. (a) Let $z = x + iy$

$\therefore$ $|z + 3 - i| = |(x + 3) + i(y - 1)| = 1$

$\Rightarrow$ $\sqrt{(x+3)^2 + (y-1)^2} = 1$ ... (i)

$\because$ $\arg z = \pi$

$\Rightarrow$ $\tan^{-1}\frac{y}{x} = \pi$

$\Rightarrow$ $\frac{y}{x} = \tan\pi = 0$

$\Rightarrow$ $y = 0$ ... (ii)

From equations (i) and (ii), we get

$x = -3$, $y = 0$

$\therefore$ $z = -3$

$\Rightarrow$ $|z| = |-3| = 3$

144. (b) Given that :

$$Z = \frac{1-i}{\frac{1}{2} + \frac{\sqrt{3}}{2}i}$$

$$= \frac{2(i-1)}{1 + i\sqrt{(3)}} \times \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}}$$

$$= \frac{2\left(i + \sqrt{3} - 1 + i\sqrt{3}\right)}{1+3}$$

$$= \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$$

Now, put $\frac{\sqrt{3}-1}{2} = r\cos\theta$, $\frac{\sqrt{3}+1}{2} = r\sin\theta$

Squaring and adding, we obtain

$$r^2 = \left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2$$

$$= \frac{2\left(\sqrt{3}\right)^2 + 1}{4} = \frac{2 \times 4}{4} = 2$$

Hence, $r = \sqrt{2}$ which gives :

$$\cos\theta = \frac{\sqrt{3}-1}{2\sqrt{2}}, \ \sin\theta = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Therefore, $\theta = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$

Hence, the polar form is $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$.

145. (a) $(x - iy)(3 + 5i)$

$= 3x + 5xi - 3yi - 5yi^2$

$= 3x + (5x - 3y)i + 5y$ $[\because i^2 = -1]$

$= (3x + 5y) + (5x - 3y)i$ ... (i)

Given, $(x - iy)(3 + 5i) = -6 + 24i$

$\left[\begin{array}{l}\text{using equation (i), and } z = (a + ib) \\ \Rightarrow \bar{z} = (a - ib)\end{array}\right]$

On comparing the real and imaginary parts of both sides, we get

$3x + 5y = -6$ and $5x - 3y = 24$

Solving the above equations by substitution or elimination method, we get

$x = 3$, $y = -3$

146. (b) Let $z = x + iy$, then $\frac{z-1}{z+1} = \frac{x-1+iy}{x+1+iy}$

$$= \frac{(x-1+iy)}{(x+1+iy)} \times \frac{(x+1-iy)}{(x+1-iy)}$$

$$= \frac{\left[\left(x^2-1\right) - i(x-1)y + i(x+1)y + y^2\right]}{\left[(x+1)^2 + y^2\right]}$$

For purely imaginary, real (z) = 0

$\Rightarrow$ $x^2 + y^2 = 1$, $|z| = 1$.

147. (c) $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right) = 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + i\,2\sin^2\frac{\pi}{10}$

$$= 2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$$

For amplitude, $\tan\theta = \dfrac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}} = \tan\frac{\pi}{10} \Rightarrow \theta = \frac{\pi}{10}$.

148. (a) $x + iy = \sqrt{\frac{a+ib}{c+id}} \Rightarrow x - iy = \sqrt{\frac{a-ib}{c-id}}$

Also, $x^2 + y^2 = (x + iy)(x - iy) = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$

$\Rightarrow$ $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

149. (b) As sum of coefficients is zero, hence one root is 1 and other root is $\frac{l-m}{m-n}$.

Since roots are equal,

$\therefore$ $\frac{l-m}{m-n} = 1 \Rightarrow 2m = n + l$.

150. (b) It is given that

$$\alpha\beta = 2 \Rightarrow \frac{3a+4}{a+1} = 2$$

$\Rightarrow$ 3a + 4 = 2a + 2 $\Rightarrow$ a = –2

Also, $\alpha + \beta = -\frac{2a+3}{a+1}$

Putting this value of a, we get sum of roots

$$= -\frac{2a+3}{a+1} = -\frac{-4+3}{-2+1} = -1.$$

151. (d) $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$ and $\alpha^2 + \beta^2 = \frac{(b^2 - 2ac)}{a^2}$

Now, $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} = \frac{\alpha(a\alpha + b) + \beta(a\beta + b)}{(a\beta + b)(a\alpha + b)}$

$$= \frac{a(\alpha^2 + \beta^2) + b(\alpha + \beta)}{\alpha\beta a^2 + ab(\alpha + \beta) + b^2} = \frac{a\frac{(b^2 - 2ac)}{a^2} + b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2 + ab\left(-\frac{b}{a}\right) + b^2}$$

$$= \frac{b^2 - 2ac - b^2}{a^2c - ab^2 + ab^2} = \frac{-2ac}{a^2c} = -\frac{2}{a}.$$

152. (a) Let α, α^2 be the two roots. Then,

$$\alpha + \alpha^2 = -\frac{b}{a} \quad ...(i)$$

$$\text{and } \alpha \cdot \alpha^2 = \frac{c}{a} \quad ...(ii)$$

On cubing both sides of (i),

$$\alpha^3 + \alpha^6 + 3\alpha\alpha^2(\alpha + \alpha^2) = -\frac{b^3}{a^3}$$

$$\Rightarrow \frac{c}{a} + \frac{c^2}{a^2} + 3\frac{c}{a}\left(-\frac{b}{a}\right) = -\frac{b^3}{a^3} \quad \text{[by (i) and (ii)]}$$

$\Rightarrow$ $b^3 + ac^2 + a^2c = 3abc$.

153. (c) Given (x – a) (x – b) = c

$\therefore$ $\alpha + \beta = a + b$ and $\alpha\beta = ab - c$

Now, given equation $(x - \alpha)(x - \beta) + c = 0$

$\Rightarrow$ $x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$

If its roots be p and q, then

$p + q = (\alpha + \beta) = a + b$

$pq = \alpha\beta + c = ab - c + c = ab$

So, it can be given by $x^2 - (a + b)x + ab = 0$

So, its roots will be a and b.

154. (a) $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\beta + \alpha) + \alpha\beta$

$$= \alpha\beta(1 + \alpha + \beta) = \frac{c}{a}\left\{1 + \left(-\frac{b}{a}\right)\right\} = \frac{c(a-b)}{a^2}.$$

155. (b) Since α, β are the roots of the equation $2x^2 - 35x + 2 = 0$

Also, $\alpha\beta = 1$

$\therefore$ $2\alpha^2 - 35\alpha = -2$ or $2\alpha - 35 = \frac{-2}{\alpha}$

$2\beta^2 - 35\beta = -2$ or $2\beta - 35 = \frac{-2}{\beta}$

Now, $(2\alpha - 35)^3 (2\beta - 35)^3 = \left(\frac{-2}{\alpha}\right)^3 \left(\frac{-2}{\beta}\right)^3$

$$= \frac{8 \cdot 8}{\alpha^3\beta^3} = \frac{64}{1} = 64.$$

156. (c) Let α, β are roots of $x^2 + px + q = 0$

So, $\alpha + \beta = -p$ and $\alpha\beta = q$

Given that $(\alpha + \beta) = 3(\alpha - \beta) = -p$

$\Rightarrow$ $\alpha - \beta = \frac{-p}{3}$

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow$ $\frac{p^2}{9} = p^2 - 4q$ or $2p^2 = 9q$.

157. (c) Let α, β be the roots of $x^2 + bx + c = 0$ and α', β' be the roots of $x^2 + qx + r = 0$.

Then, $\alpha + \beta = -b$, $\alpha\beta = c$, $\alpha' + \beta' = -q$, $\alpha'\beta' = r$

It is given that $\frac{\alpha}{\beta} = \frac{\alpha'}{\beta'} \Rightarrow \frac{\alpha + \beta}{\alpha - \beta} = \frac{\alpha' + \beta'}{\alpha' - \beta'}$

$$\Rightarrow \frac{(\alpha + \beta)^2}{(\alpha - \beta)^2} = \frac{(\alpha' + \beta')^2}{(\alpha' - \beta')^2} \Rightarrow \frac{b^2}{b^2 - 4c} = \frac{q^2}{q^2 - 4r}$$

$\Rightarrow$ $b^2 r = q^2 c$.

158. (b) Since roots of the equation $x^2 - 5x + 16 = 0$ are α, β.

$\Rightarrow$ $\alpha + \beta = 5$ and $\alpha\beta = 16$ and $\alpha^2 + \beta^2 + \frac{\alpha\beta}{2} = -p$

$\Rightarrow$ $(\alpha + \beta)^2 - 2\alpha\beta + \frac{\alpha\beta}{2} = -p$

$\Rightarrow$ $25 - 32 + 8 = -p$

$\Rightarrow$ $p = -1$ and $(\alpha^2 + \beta^2)\left(\frac{\alpha\beta}{2}\right) = q$

$\Rightarrow$ $[(\alpha + \beta)^2 - 2\alpha\beta]\left[\frac{\alpha\beta}{2}\right] = q$

$\Rightarrow$ $q = [25 - 32]\frac{16}{2} = -56$

So, p = –1, q = –56.

159. (a) Let the roots are α and β.

$\Rightarrow \dfrac{\alpha+\beta}{2} = \dfrac{8}{5}$

$\Rightarrow \alpha + \beta = \dfrac{16}{5}$ ... (i)

and $\dfrac{\frac{1}{\alpha}+\frac{1}{\beta}}{2} = \dfrac{8}{7} \Rightarrow \dfrac{\alpha+\beta}{2\alpha\beta} = \dfrac{8}{7} \Rightarrow \dfrac{\left(\frac{16}{5}\right)}{2\left(\frac{8}{7}\right)} = \alpha\beta$

$\Rightarrow \alpha\beta = \dfrac{7}{5}$ ... (ii)

$\therefore$ Equation is $x^2 - \left(\dfrac{16}{5}\right)x + \dfrac{7}{5} = 0$

$\Rightarrow 5x^2 - 16x + 7 = 0$

160. (c) $4x^2 + 5k = (5k + 1)x$

$\Rightarrow 4x^2 - (5k + 1)x + 5k = 0; (\alpha - \beta) = 1$

$\therefore \alpha + \beta = \dfrac{(5k+1)}{4}$ and $\alpha\beta = \dfrac{5k}{4}$

Now, $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$\Rightarrow \alpha - \beta = \sqrt{\dfrac{(5k+1)^2}{16} - \dfrac{4 \cdot 5k}{4}} = 1$

$\therefore 25k^2 - 70k - 15 = 0$

$\Rightarrow (5k + 1)(k - 3) = 0 \Rightarrow k = -\dfrac{1}{5}, 3.$

161. (b) **Case I :** $x - 2 > 0$, Putting $x - 2 = y$, $y > 0$

$x > 2$

$\therefore y^2 + y - 2 = 0 \Rightarrow y = -2, 1$

$\Rightarrow x = 0, 3$

But $0 < 2$, Hence $x = 3$ is the real root.

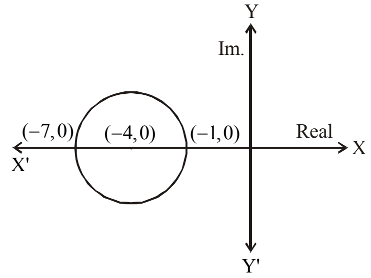
Case II : $x - 2 < 0 \Rightarrow x < 2, y < 0$

$y^2 - y - 2 = 0 \Rightarrow y = 2, -1 \Rightarrow x = 4, x = 1$

Since $4 \not< 2$, only $x = 1$ is the real root.

Hence the sum of the real roots = 3 + 1 = 4

162. (a) z lies on or inside the circle with centre (– 4, 0) and radius 3 units.



From the Argand diagram maximum value of $|z+1|$ is 6.

163. (b) $\dfrac{(\cos\theta + i\sin\theta)^4}{(\cos\theta - i\sin\theta)^3} = (\cos\theta + i\sin\theta)^4 (\cos\theta - i\sin\theta)^{-3}$

$= (\cos 4\theta + i\sin 4\theta)\{\cos(-\theta) + i\sin(-\theta)\}^{-3}$

$= (\cos 4\theta + i\sin 4\theta)\{\cos(-3)(-\theta) + i\sin(-3)(-\theta)\}$

$= (\cos 4\theta + i\sin 4\theta)\{\cos 3\theta + i\sin 3\theta\}$

$= \cos 4\theta\cos 3\theta - \sin 4\theta\sin 3\theta$

$+ i(\sin 4\theta\cos 3\theta + \sin 3\theta\cos 4\theta)$

$= \cos(4\theta + 3\theta) + i\sin(4\theta + 3\theta) = \cos 7\theta + i\sin 7\theta$

164. (b) Let $x = 2 + \dfrac{1}{2 + \dfrac{1}{2 + ...\infty}}$

$\Rightarrow x = 2 + \dfrac{1}{x}$ [On simplification]

$\Rightarrow x = 1 \pm \sqrt{2}$

But the value of the given expression cannot be negative or less than 2, therefore $1 + \sqrt{2}$ is required answer.

165. (c) $x = \sqrt{6+x} \Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0 \Rightarrow (x - 3)(x + 2) = 0 \Rightarrow x = 3, -2$

$x = -2$ will be rejected as $x > 0$. Hence, $x = 3$ is the solution.

CHAPTER 6

LINEAR INEQUALITY

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

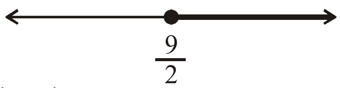
1. If x is real number and $|x| < 3$, then
(a) $x \geq 3$ (b) $-3 < x < 3$
(c) $x \leq -3$ (d) $-3 \leq x \leq 3$

2. Given that x, y and b are real numbers and $x < y$, $b < 0$, then
(a) $\frac{x}{b} < \frac{y}{b}$ (b) $\frac{x}{b} \leq \frac{y}{b}$
(c) $\frac{x}{b} > \frac{y}{b}$ (d) $\frac{x}{b} \geq \frac{y}{b}$

3. Solution of a linear inequality in variable x is represented on number line is

(a) $x \in (-\infty, 5)$ (b) $x \in (-\infty, 5]$
(c) $x \in [5, \infty)$ (d) $x \in (5, \infty)$

4. Solution of linear inequality in variable x is represented on number line is



(a) $x \in \left(\frac{9}{2}, \infty\right)$ (b) $x \in [\frac{9}{2}, \infty)$
(c) $x \in (-\infty, \frac{9}{2})$ (d) $x \in (-\infty, \frac{9}{2}]$

5. If $|x+3| \geq 10$, then
(a) $x \in (-13, 7]$ (b) $x \in (-13, 7)$
(c) $x \in (-\infty, 13] \cup [-7, \infty)$ (d) $x \in (-\infty, -13] \cup [7, \infty)$

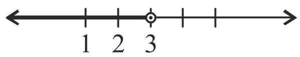
6. Let $\frac{C}{5} = \frac{F-32}{9}$. If C lies between 10 and 20, then :
(a) $50 < F < 78$ (b) $50 < F < 68$
(c) $49 < F < 68$ (d) $49 < F < 78$

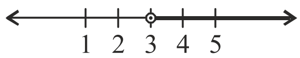
7. The solution set of the inequality $4x + 3 < 6x + 7$ is
(a) $[-2, \infty)$ (b) $(-\infty, -2)$
(c) $(-2, \infty)$ (d) None of these

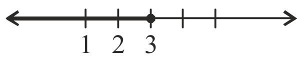
8. Which of the following is the solution set of $3x - 7 > 5x - 1 \ \forall \ x \in R$?
(a) $(-\infty, -3)$ (b) $(-\infty, -3]$
(c) $(-3, \infty)$ (d) $(-3, 3)$

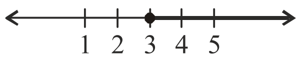
9. The solution set of the inequality $37 - (3x + 5) \geq 9x - 8 (x - 3)$ is
(a) $(-\infty, 2)$ (b) $(-\infty, -2)$
(c) $(-\infty, 2]$ (d) $(-\infty, -2]$

10. The graph of the solution on number line of the inequality $3x - 2 < 2x + 1$ is

(a) 

(b) 

(c) 

(d) 

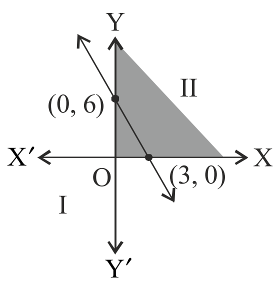
11. The solution set of the inequalities $6 \leq -3(2x - 4) < 12$ is
(a) $(-\infty, 1]$ (b) $(0, 1]$
(c) $(0, 1] \cup [1, \infty)$ (d) $[1, \infty)$

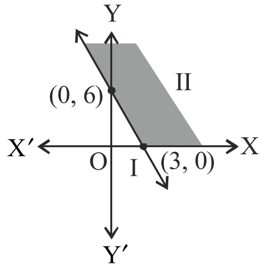
12. Which of the following is the solution set of linear inequalities $2(x - 1) < x + 5$ and $3(x + 2) > 2 - x$?
(a) $(-\infty, -1)$ (b) $(-1, 1)$ (c) $(-1, 7)$ (d) $(1, 7)$

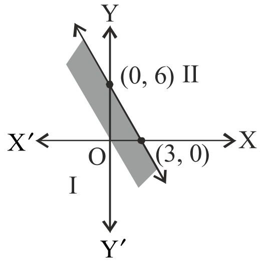
13. x and b are real numbers. If $b > 0$ and $|x| > b$, then
(a) $x \in (-b, \infty)$ (b) $x \in (-\infty, b)$
(c) $x \in (-b, b)$ (d) $x \in (-\infty, -b) \cup (b, \infty)$

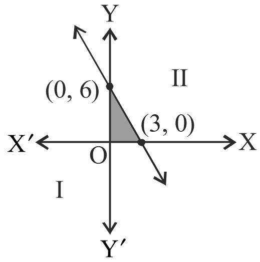
14. If $a < b$ and $c < 0$, then
(a) $\frac{a}{c} = \frac{b}{c}$ (b) $\frac{a}{c} > \frac{b}{c}$
(c) $\frac{a}{c} < \frac{b}{c}$ (d) None of these

15. The graph of the inequality $40x + 20y \leq 120$, $x \geq 0$, $y \geq 0$ is

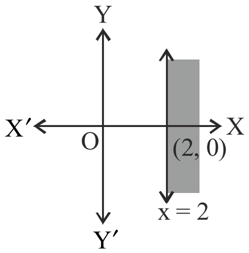
(a) 

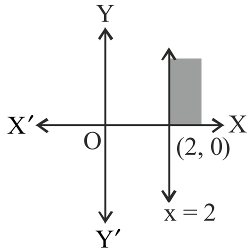
(b) 

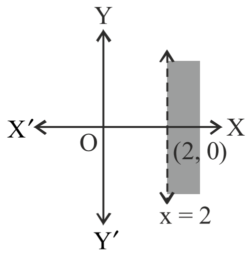
(c) 

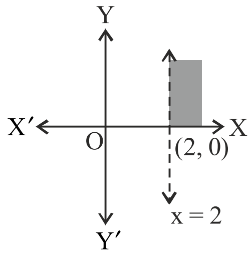
(d) 

16. The graphical solution of $3x - 6 \geq 0$ is

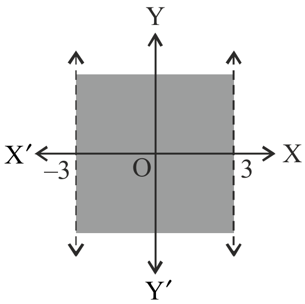
(a) 

(b) 

(c) 

(d) 

17. The inequality representing the following graph is



(a) $|x| < 3$ (b) $|x| \leq 3$ (c) $|x| > 3$ (d) $|x| \geq 3$

18. The solutions of the system of inequalities $3x - 7 < 5 + x$ and $11 - 5x \leq 1$ on the number line is

(a) 

(b) 

(c) 

(d) None of the above

19. The solution set of the inequalities $3x - 7 > 2(x - 6)$ and $6 - x > 11 - 2x$, is

(a) $(-5, \infty)$ (b) $[5, \infty)$ (c) $(5, \infty)$ (d) $[-5, \infty)$

20. If $\frac{5 - 2x}{3} \leq \frac{x}{6} - 5$, then $x \in$

(a) $[2, \infty)$ (b) $[-8, 8]$ (c) $[4, \infty)$ (d) $[8, \infty)$

21. If $\frac{3x - 4}{2} \geq \frac{x + 1}{4} - 1$, then $x \in$

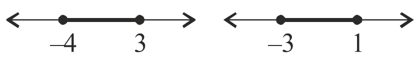
(a) $[1, \infty)$ (b) $(1, \infty)$ (c) $(-5, 5)$ (d) $[-5, 5]$

22. If $-5 \leq \frac{5 - 3x}{2} \leq 8$, then $x \in$

(a) $\left[-\frac{11}{3}, 5\right]$ (b) $[-5, 5]$

(c) $\left[-\frac{11}{3}, \infty\right)$ (d) $(-\infty, \infty)$

23. Solutions of the inequalities comprising a system in variable x are represented on number lines as given below, then



(a) $x \in (-\infty, -4] \cup [3, \infty)$
(b) $x \in [-3, 1]$
(c) $x \in (-\infty, -4) \cup [3, \infty)$
(d) $x \in [-4, 3]$

24. The inequality $\frac{2}{x} < 3$ is true, when x belongs to

(a) $\left[\frac{2}{3}, \infty\right)$ (b) $\left(-\infty, \frac{2}{3}\right]$

(c) $(-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$ (d) None of these

25. Solution of $|3x + 2| < 1$ is

(a) $\left[-1, -\frac{1}{3}\right]$ (b) $\left\{-\frac{1}{3}, -1\right\}$

(c) $\left(-1, -\frac{1}{3}\right)$ (d) None of these

26. Solution of $|x-1| \geq |x-3|$ is
(a) $x \leq 2$ (b) $x \geq 2$ (c) $[1, 3]$ (d) None of these

27. If $-3x + 17 < -13$, then
(a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$
(c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10)$

28. If $|x + 2| \leq 9$, then
(a) $x \in (-7, 11)$ (b) $x \in [-11, 7]$
(c) $x \in (-\infty, -7) \cup (11, \infty)$ (d) $x \in (-\infty, -7) \cup [11, \infty)$

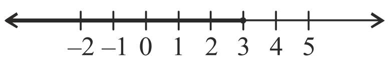
STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

29. Consider the following statements about Linear Inequalities :
I. Two real numbers or two algebraic expressions related by the symbols $<$, $>$, $\leq$ or $\geq$ form an inequality.
II. When equal numbers added to (or subtracted from) both sides of an inequality then the inequality does not changed.
III. When both sides of an inequality multiplied (or divided) by the same positive number then the inequality does not changed.
Which of the above statements are true ?
(a) Only I (b) Only II
(c) Only III (d) All of the above

30. Consider the following statements:
Statement-I : Consider the inequality $30x < 200$ such that x is not a negative integer or fraction. Then, the value of x, which make the inequality a true statement are 1, 2, 3, 4, 5, 6.
Statement-II : The solution of an inequality in one variable is the value of that variable which makes it a true statement.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

31. Consider the following statements:
Statement-I : The solution set of $7x + 3 < 5x + 9$ is $(-\infty, 3)$.
Statement-II : The graph of the solution of above inequality is represented by



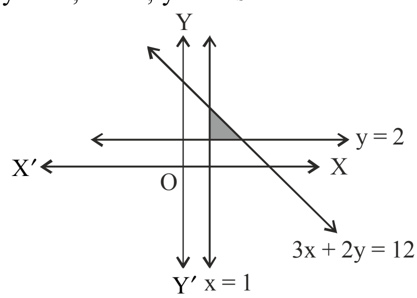
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

32. Consider the following statements:
Statement-I : The solution set of $5x - 3 < 7$, when x is an integer, is $\{....., -3, -2, -1\}$.
Statement-II : The solution of $5x - 3 < 7$, when x is a real number, is $(-\infty, 2)$.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

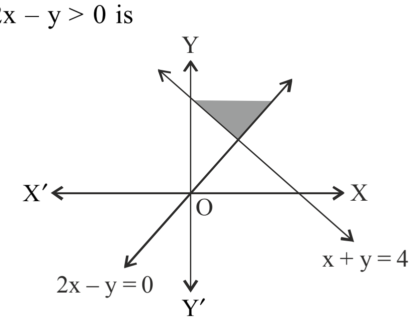
33. Consider the following statements:
Statement-I : The solution set of the inequality
$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3} \text{ is } (-\infty, 2).$$
Statement-II : The solution set of the inequality
$$\frac{1}{2}\left(\frac{3x}{5}+4\right) \geq \frac{1}{3}(x-6) \text{ is } (-\infty, 120].$$
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

34. Consider the following statements:
Statement-I : The region containing all the solutions of an inequality is called the solution region.
Statement-II : The half plane represented by an inequality is checked by taking any point on the line.
Choose the correct option.
(a) Statement I is true (b) Statement II is true
(c) Both are true (d) Both are false

35. Which of the following is/are true?
I. The graphical solution of the system of inequalities $3x + 2y \leq 12$, $x \geq 1$, $y \geq 2$ is

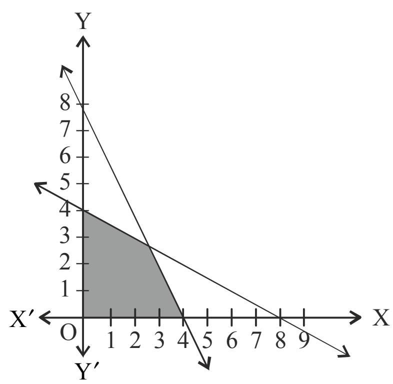


II. The region represented by the solution set of the inequalities $2x + y \geq 6$, $3x + 4y \leq 12$ is bounded.
III. The solution set of the inequalities $x + y \geq 4$, $2x - y > 0$ is



(a) Only I is true (b) I and II are true
(c) I and III are true (d) Only III is true

36. Which of the following linear inequalities satisfy the shaded region of the given figure.
I. $x + 2y \leq 8$ II. $x \geq 0$, $y \geq 0$
III. $x \leq 0$, $y \leq 0$ IV. $2x + y \leq 8$
V. $4x + 5y \leq 40$



(a) I, III and V (b) I, IV and V
(c) I, III and IV (d) I, II, and IV

37. Consider the following statements.

I. Inequalities involving the symbol $\geq$ or $\leq$ are called slack inequalities.

II. Inequalities which do not involve variables are called numerical inequalities.

Choose the correct option.

(a) Only I is true (b) Only II is true.
(c) Both are true. (d) Both are false.

38. Consider the following statements.

I. Solution set of the inequality $-15 < \frac{3(x-2)}{5} \leq 0$ is $(-23, 2]$

II. Solution set of the inequality $7 \leq \frac{3x+11}{2} \leq 11$ is $\left[1, \frac{11}{3}\right]$

III. Solution set of the inequality $-5 \leq \frac{2-3x}{4} \leq 9$ is $[-1, 1] \cup [3, 5]$

Choose the correct option

(a) Only I and II are true. (b) Only II and III are true.
(c) Only I and III are true. (d) All are true.

39. Consider the following statements.

I. Equal numbers may be added to (or subtracted from) both sides of an inequality.

II. When both sides are multiplied (or divided) by a negative number, then the inequality is reversed.

Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Both I and II are false.

40. Consider the following statements.

I. Solution set of $24x < 100$ is $\{1, 2, 3, 4\}$, when x is a natural number.

II. Solution set of $24x < 100$ is $\{\ldots, -3, -2, -1, 0, 1, 2, 3, 4\}$, when x is an integer.

Choose the correct option.

(a) Only I is false. (b) Only II is false.
(c) Both are false. (d) Both are true.

41. I. When x is an integer, the solution set of $3x + 8 > 2$ is $\{-1, 0, 1, 2, 3, \ldots\}$.

II. When x is a real number, the solution set of $3x + 8 > 2$ is $\{-1, 0, 1\}$.

Choose the correct option.

(a) Only I is incorrect.
(b) Only II is incorrect.
(c) Both I and II are incorrect.
(d) Both I and II are correct.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

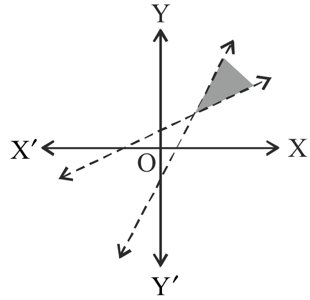
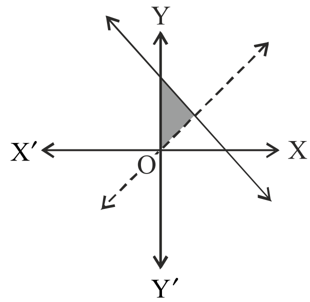
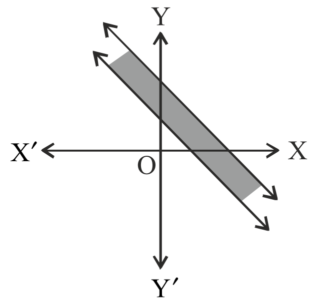
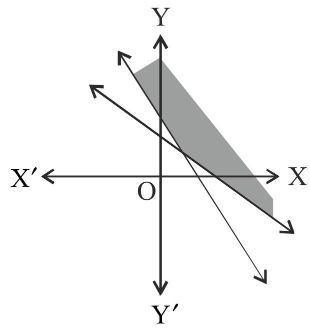
42.

Column - I (Linear inequations)	**Column - II (Solution set)**
(A) $2x - 4 \leq 0$	(1) $[3, \infty)$
(B) $-3x + 12 < 0$	(2) $(3, \infty)$
(C) $4x - 12 \geq 0$	(3) $(-\infty, 2]$
(D) $7x + 9 > 30$	(4) $(4, \infty)$

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	3	1	4	2
(c)	2	4	1	3
(d)	2	1	4	3

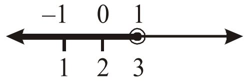
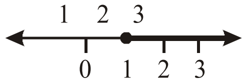
43. Match the linear inequalities given in column-I with solution set representing by graphs in column-II

Column-I	**Column-II**
A. $2x - y > 1$, $x - 2y < -1$	1. 
B. $x + y \leq 6$, $x + y \geq 4$	2. 
C. $2x + y \geq 8$, $x + 2y \geq 10$	3. 
D. $x + y \leq 9$, $y > x$, $x \geq 0$	4. 

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	2	1	4	3
(c)	1	3	4	2
(d)	3	4	2	1

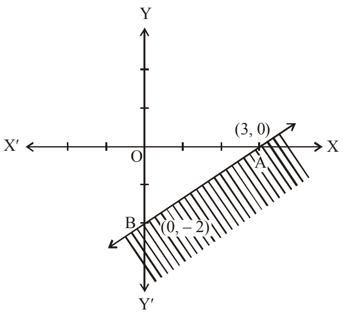
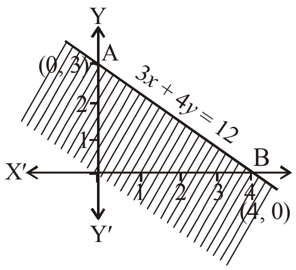
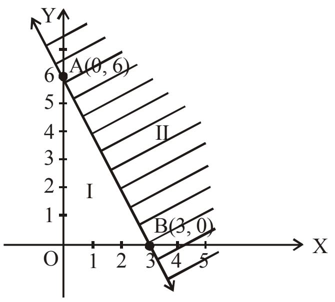
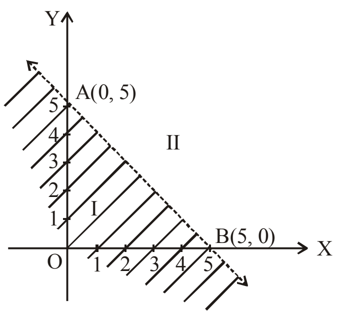
44.

Column - I (Linear inequations)	**Column - II (Solution on number line)**
(A) $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$	(1) 
(B) $3x - 2 < 2x + 1$	(2)
(C) $3(1 - x) < 2(x + 4)$	(3) 
(D) $3x - 7 < 5 + x$ and $11 - 5x \leq 1$	(4) 

Codes

	A	B	C	D
(a)	4	2	3	1
(b)	1	3	2	4
(c)	4	3	2	1
(d)	1	2	3	4

45.

Column - I (Inequality)	**Column - II (Graph)**
(A) $x + y < 5$	(1) 
(B) $2x + y \geq 6$	(2) 
(C) $3x + 4y \leq 12$	(3) 
(D) $2x - 3y > 6$	(4) 

Codes

	A	B	C	D
(a)	4	2	3	1
(b)	4	3	2	1
(c)	1	2	3	4
(d)	1	3	2	4

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

46. The solution set of the inequality $4x + 3 < 6x + 7$ is $(-a, \infty)$. The value of 'a' is
(a) 1 (b) 4
(c) 2 (d) None of these

47. The set of real x satisfying the inequality $\frac{5-2x}{3} \leq \frac{x}{6} - 5$ is $[a, \infty)$. The value of 'a' is
(a) 2 (b) 4 (c) 6 (d) 8

48. The solution set of the inequality $3(2 - x) \geq 2(1 - x)$ is $(-\infty, a]$. The value of 'a' is
(a) 2 (b) 3 (c) 4 (d) 5

49. The solution set of $\frac{2x-1}{3} \geq \left(\frac{3x-2}{4}\right) - \left(\frac{2-x}{5}\right)$ is $(-\infty, a]$. The value of 'a' is
(a) 2 (b) 3 (c) 4 (d) 5

50. If $5x + 1 > -24$ and $5x - 1 < 24$, then $x \in (-a, a)$. The value of 'a' is
(a) 2 (b) 3 (c) 4 (d) 5

51. If x satisfies the inequations $2x - 7 < 11$ and $3x + 4 < -5$, then x lies in the interval $(-\infty, -m)$. The value of 'm' is
(a) 2 (b) 3 (c) 4 (d) 5

52. If $|x| < 3$ and x is a real number, then $-m < x < m$. The value of m is
(a) 3 (b) 4 (c) 2 (d) 1

53. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.
(a) 2 (b) 9 (c) 8 (d) 7

54. The solution of the inequality $-8 \leq 5x - 3 < 7$ is $[-a, b)$. Sum of 'a' and 'b' is
(a) 1 (b) 2 (c) 3 (d) 4

55. The number of pairs of consecutive odd natural numbers both of which are larger than 10, such that their sum is less than 40, is
(a) 4 (b) 6 (c) 3 (d) 8

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

56. **Assertion :** The inequality $ax + by < 0$ is strict inequality.
Reason : The inequality $ax + b \geq 0$ is slack inequality.

57. **Assertion :** If $a < b$, $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.
Reason : If both sides are divided by the same negative quantity, then the inequality is reversed.

58. **Assertion :** $|3x - 5| > 9 \Rightarrow x \in \left(-\infty, \frac{-4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$.
Reason : The region containing all the solutions of an inequality is called the solution region.

59. **Assertion :** A line divides the cartesian plane in two part(s).
Reason : If a point $P(\alpha, \beta)$ on the line $ax + by = c$, then $a\alpha + b\beta = c$.

60. **Assertion :** Each part in which a line divides the cartesian plane, is known as half plane.
Reason : A point in the cartesian plane will either lie on a line or will lie in either of half plane I or II.

61. **Assertion :** Two real numbers or two algebraic expressions related by the symbol $<$, $>$, $\leq$ or $\geq$ forms an inequality.
Reason : The inequality $ax + by < 0$ is strict inequality.

62. **Assertion :** The inequality $3x + 2y \geq 5$ is the linear inequality.
Reason : The solution of $5x - 3 < 7$, when x is a real number, is $(-\infty, 2)$.

63. **Assertion :** If $3x + 8 > 2$, then $x \in \{-1, 0, 1, 2, \ldots\}$, when x is an integer.
Reason : The solution set of the inequality $4x + 3 < 5x + 7$ $\forall$ $x \in R$ is $[4, \infty)$.

64. **Assertion :** Graph of linear inequality in one variable is a visual representation.
Reason : If a point satisfying the line $ax + by = c$, then it will lie in upper half plane.

65. **Assertion :** The region containing all the solutions of an inequality is called the solution region.
Reason : The values of x, which make an inequality a true statement, are called solutions of the inequality.

66. **Assertion :** A non-vertical line will divide the plane into left and right half planes.
Reason : The solution region of a system of inequalities is the region which satisfies all the given inequalities in the system simultaneously.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

67. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then what can you say about breadth?
(a) breadth = 20 (b) breadth ≤ 20
(c) breadth ≥ 20 (d) breadth $\neq 20$

68. The set of real values of x satisfying $|x - 1| \leq 3$ and $|x - 1| \geq 1$ is
(a) $[2, 4]$ (b) $(-\infty, 2] \cup [4, +\infty)$
(c) $[-2, 0] \cup [2, 4]$ (d) None of these

69. The marks obtained by a student of class XI in first and second terminal examinations are 62 and 48, respectively. The minimum marks he should get in the annual examination to have an average of at least 60 marks, are
(a) 70 (b) 50 (c) 74 (d) 48

70. Ravi obtained 70 and 75 marks in first two unit tests. Then, the minimum marks he should get in the third test to have an average of at least 60 marks, are
(a) 45 (b) 35 (c) 25 (d) None of these

71. The pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23, are
(a) (4, 6), (6, 8), (8, 10), (10, 12)
(b) (6, 8), (8, 10), (10, 12)
(c) (6, 8), (8, 10), (10, 12), (12, 14)
(d) (8, 10), (10, 12)

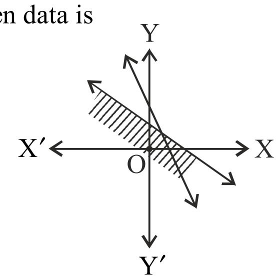
72. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. The possible length of the shortest board, if the third piece is to be at least 5 cm longer than the second, is
(a) less than 8 cm
(b) greater than or equal to 8 cm but less than or equal to 22 cm
(c) less than 22 cm
(d) greater than 22 cm

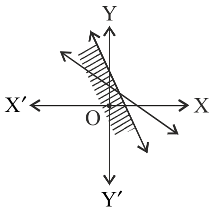
73. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then
(a) breadth > 20 cm (b) length < 20 cm
(c) breadth $\geq$ 20 cm (d) length $\leq$ 20 cm

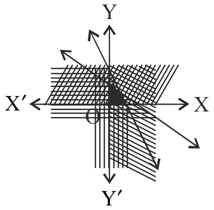
74. The set of values of x satisfying $2 \leq |x - 3| < 4$ is
(a) $(-1, 1] \cup [5, 7)$ (b) $-4 \leq x \leq 2$
(c) $-1 < x < 7$ or $x \geq 5$ (d) $x < 7$ or $x \geq 5$

75. IQ of a person is given by the formula
$$IQ = \frac{MA}{CA} \times 100$$
where, MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12 years children, then the range of their mental age is
(a) $9.8 \leq MA \leq 16.8$ (b) $10 \leq MA \leq 16$
(c) $9.6 \leq MA \leq 16.8$ (d) $9.6 \leq MA \leq 16.6$

76. A furniture dealer deals in only two items — tables and chairs. He has ` 15,000 to invest and a space to store atmost 60 pieces. A table costs him ` 750 and chair ` 150. Suppose he makes x tables and y chairs
The graphical solution of the inequations representing the given data is

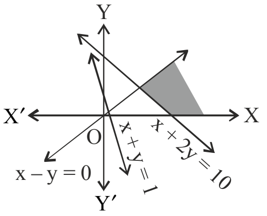
(a) 

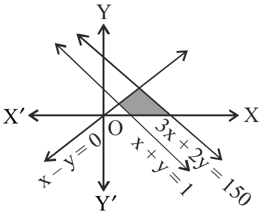
(b) 

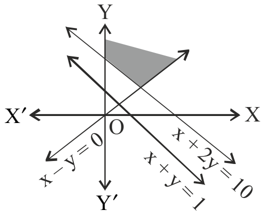
(c) 

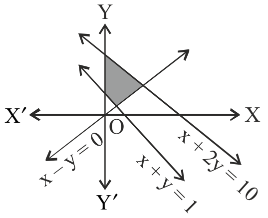
(d) None of these

77. The graphical solution of the inequalities $x + 2y \le 10$, $x + y \ge 1$, $x - y \le 0$, $x \ge 0$, $y \ge 0$ is

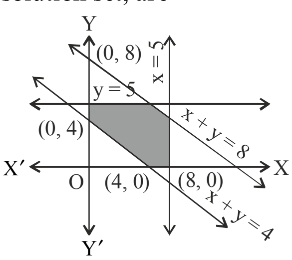
(a) 

(b) 

(c) 

(d) 

78. Linear inequalities for which the shaded region for the given figure is the solution set, are



(a) $x + y \le 8$, $x + y \le 4$, $x \le 5$, $y \le 5$, $x \ge 0$, $y \ge 0$
(b) $x + y \le 8$, $x + y \ge 4$, $x \le 5$, $y \le 5$, $x \ge 0$, $y \ge 0$
(c) $x + y \ge 8$, $x + y \ge 4$, $x \ge 5$, $y \ge 5$, $x \ge 0$, $y \ge 0$
(d) None of the above

79. A solution of 8% boric is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 L of the 8% solution, of the 2% solution will have to be added is
(a) more than 320 and less than 1000
(b) more than 160 and less than 320
(c) more than 320 and less than 1280
(d) more than 320 and less than 640

80. A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$, respectively, where x is the number of cassettes produced and sold in a week.
The number of cassettes must be sold by the company to realise some profit, is
(a) more than 2000 (b) less than 2000
(c) more than 1000 (d) less than 1000

81. A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?
(a) more than 120 litres but less than 300 litres
(b) more than 140 litres but less than 600 litres
(c) more than 100 litres but less than 280 litres
(d) more than 160 litres but less than 500 litres

82. If $\frac{|x+3|+x}{x+2} > 1$, then $x \in$
(a) $(-5, -2)$ (b) $(-1, \infty)$
(c) $(-5, -2) \cup (-1, \infty)$ (d) None of these

83. If $|2x - 3| < |x + 5|$, then x belongs to
(a) $(-3, 5)$ (b) $(5, 9)$ (c) $\left(-\frac{2}{3}, 8\right)$ (d) $\left(-8, \frac{2}{3}\right)$

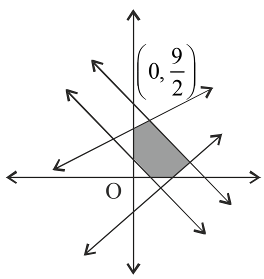
84. Solution of $(x - 1)^2 (x + 4) < 0$ is
(a) $(-\infty, 1)$ (b) $(-\infty, -4)$ (c) $(-1, 4)$ (d) $(1, 4)$

85. Solution of $\left|1 + \frac{3}{x}\right| > 2$ is
(a) $(0, 3]$ (b) $[-1, 0)$
(c) $(-1, 0) \cup (0, 3)$ (d) None of these

86. Solution of $|2x - 3| < |x + 2|$ is
(a) $\left(-\infty, \frac{1}{3}\right)$ (b) $\left(\frac{1}{3}, 5\right)$
(c) $(5, \infty)$ (d) $\left(-\infty, \frac{1}{3}\right) \cup (5, \infty)$

87. Solution of $\left|x + \frac{1}{x}\right| > 2$ is
(a) $R - \{0\}$
(b) $R - \{-1, 0, 1\}$
(c) $R - \{1\}$
(d) $R - \{-1, 1\}$

88. Which of the following linear inequalities satisfy the shaded region of the given figure?



(a) $2x + 3y \ge 3$
(b) $3x + 4y \le 18$
(c) $x - 6y \le 3$
(d) All of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** $|x| < 3 \Rightarrow -3 < x < 3$

2. **(c)** $x < y \Rightarrow \frac{x}{b} > \frac{y}{b}$

3. **(d)** 4. **(b)**

5. **(d)** $|x+3| \geq 10,$

$\Rightarrow x+3 \leq -10$ or $x+3 \geq 10$

$\Rightarrow x \leq -13$ or $x \geq 7$

$\Rightarrow x \in (-\infty, -13] \cup [7, \infty)$

6. **(b)** Given : $\frac{C}{5} = \frac{F-32}{9}$ and $10 < C < 20$.

$\Rightarrow C = \frac{5F-(32)5}{9}$

Since, $10 < C < 20$

$\Rightarrow 10 < \frac{5F-160}{9} < 20$

$\Rightarrow 90 < 5F - 160 < 180$

$\Rightarrow 90 + 160 < 5F < 180 + 160$

$\Rightarrow 250 < 5F < 340$

$\Rightarrow \frac{250}{5} < F < \frac{340}{5}$

$\Rightarrow 50 < F < 68$

7. **(c)** We have, $4x + 3 < 6x + 7$

or $4x - 6x < 6x + 4 - 6x$

or $-2x < 4$ or $x > -2$

i.e. all the real numbers which are greater than –2, are the solutions of the given inequality. Hence, the solution set is $(-2, \infty)$.

8. **(a)** We have, $3x - 7 > 5x - 1$

Transferring the term 5x to L.H.S. and the term –7 to R.H.S.

Dividing both sides by 2,

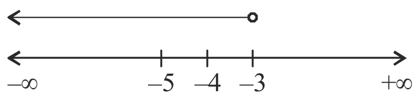
$3x - 5x > -1 + 7$

$\Rightarrow -2x > 6$

$\Rightarrow \frac{2x}{2} < -\frac{6}{2}$

$\Rightarrow x < -3$

With the help of number line, we can easily look for the numbers less than –3.



$\therefore$ Solution set is $(-\infty, -3)$, i.e. all the numbers lying between $-\infty$ and –3 but $-\infty$ and –3 are not included as $x < -3$.

9. **(c)** We have, $37 - (3x + 5) \geq 9x - 8(x - 3)$

$(37 - 3x - 5) \geq 9x - 8x + 24$

$\Rightarrow 32 - 3x \geq x + 24$

Transferring the term 24 to L.H.S. and the term (–3x) to R.H.S.

$32 - 24 \geq x + 3x$

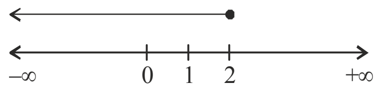
$\Rightarrow 8 \geq 4x$

$\Rightarrow 4x \leq 8$

Dividing both sides by 4,

$\frac{4x}{4} \leq \frac{8}{4}$

$\Rightarrow x \leq 2$

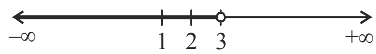


$\therefore$ Solution set is $(-\infty, 2]$.

10. **(a)** We have, $3x - 2 < 2x + 1$

Transferring the term 2x to L.H.S. and the term (–2) to R.H.S.

$3x - 2x < 1 + 2 \Rightarrow x < 3$



All the numbers on the left side of 3 will be less than it.

$\therefore$ Solution set is $(-\infty, 3)$.

11. **(b)** The given inequality $6 \leq -3(2x - 4) < 12$

$6 \leq -6x + 12 < 12$

Adding (–12) to each term,

$6 - 12 \leq -6x + 12 - 12 < 12 - 12$

$\Rightarrow -6 \leq -6x < 0$

Dividing by (–6) to each term,

$\frac{-6}{-6} \geq \frac{-6x}{-6} > \frac{0}{-6}$

$\Rightarrow 1 \geq x > 0 \quad \Rightarrow \quad 0 < x \leq 1$

$\therefore$ Solution set is $(0, 1]$.

12. **(c)** We have the given inequalities as

$2(x - 1) < x + 5$ and $3(x + 2) > 2 - x$

Now, $2x - 2 < x + 5$

Transferring the term x to L.H.S and the term –2 to R.H.S.

$2x - x < 5 + 2$

$\Rightarrow \quad x < 7$ ... (i)

and $3(x + 2) > 2 - x$

$\Rightarrow \quad 3x + 6 > 2 - x$

Transferring the term (–x) to L.H.S. and the term 6 to R.H.S.,

$\Rightarrow 3x + x > 2 - 6$

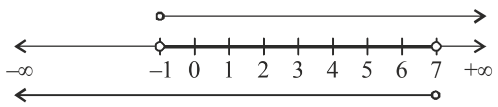
$\Rightarrow 4x > -4$

Dividing both sides by 4,

$x > \frac{-4}{4}$

$\Rightarrow x > -1$ ... (ii)

$\Rightarrow$ Draw the graph of inequalities (i) and (ii) on the number line.



Hence, solution set of the inequalities are real numbers, x lying between –1 and 7 excluding 1 and 7. i.e. $-1 < x < 7$

$\therefore$ Solution set is (–1, 7) or] –1, 7[.

13. **(d)** We have, $|x| > b, b > 0$

$\Rightarrow x < -b$ and $x > b \Rightarrow x \in (-\infty, -b) \cup (b, \infty)$

14. **(b)** We have,

$a < b$ and $c < 0$

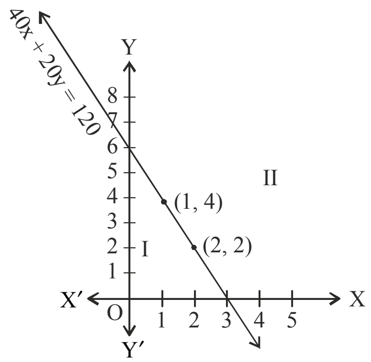
Dividing both sides of $a < b$ by c. Since, c is a negative number, sign at inequality will get reversed.

Hence, $\frac{a}{c} > \frac{b}{c}$.

15. **(d)** We have,

$40x + 20y \le 120, x \ge 0, y \ge 0$ ... (i)

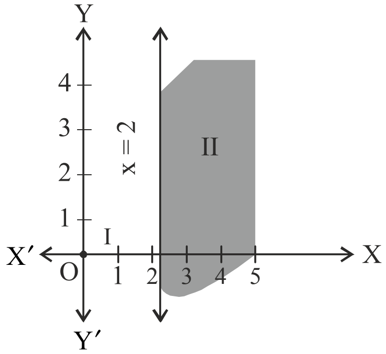
In order to draw the graph of the inequality (i), we take one point say (0, 0), in half plane I and check whether values of x and y satisfy the inequality or not.



We observe that x = 0, y = 0 satisfy the inequality. Thus, we say that the half plane I is the graph of the inequality. Since, the points on the line also satisfy the inequality (i) above, the line is also a part of the graph. Thus, the graph of the given inequality is half plane I including the line itself. Clearly, half plane II is not the part of the graph. Hence, solutions of inequality (i) will consists of all the points of its graph (half plane I including the line).

Also, since it is given $x > 0, y > 0$, x and y can only take positive values in half plane I.

16. **(a)** Graph of $3x - 6 = 0$ is given in the figure,



We select a point say (0, 0) and substituting it in given inequality, we see that

$3(0) - 6 \ge 0$ or $-6 \ge 0$, which is false.

Thus, the solution region is the shaded region on the right hand side of the line x = 2.

Also, all the points on the line $3x - 6 = 0$ will be included in the solution. Hence, a dark line is drawn in the solution region.

17. **(a)** The shaded region in the figure lies between x = –3 and x = 3 not including the line x = –3 and x = 3 (lines are dotted).

Therefore, $-3 < x < 3$

$\Rightarrow |x| < 3$ $\quad [\because |x| < a \Leftrightarrow -a < x < a]$

18. **(b)** Given inequalities are

$3x - 7 < 5 + x$ ... (i)

and $11 - 5x \le 1$ ... (ii)

From inequality (i), we have

$3x - 7 < 5 + x$

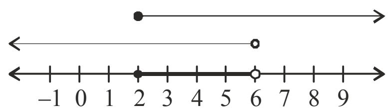
or $x < 6$ ... (iii)

Also, from inequality (ii), we have

$11 - 5x \le 1$

or $-5x \le -10$, i.e. $x \ge 2$ ... (iv)

If we draw the graph of inequalities (iii) and (iv) on the number line, we see that the values of x, which are common to both, are shown by bold line in figure.



19. **(c)** We have $3x - 7 > 2(x - 6)$

$\Rightarrow 3x - 7 > 2x - 12$

Transferring the term 2x to L.H.S. and the term (–7) to R.H.S.,

$3x - 2x > -12 + 7$

$\Rightarrow x > -5$ ... (i)

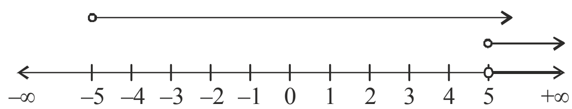
and $6 - x > 11 - 2x$

Transferring the term (–2x) to L.H.S. and the term 6 to R.H.S.,

$-x + 2x > 11 - 6$

$\Rightarrow x > 5$ ... (ii)

Draw the graph of inequations (i) and (ii) on the number line,



Hence, solution set of the equations are real numbers, x lying on greater than 5 excluding 5.

i.e., $x > 5$

$\therefore$ Solution set is $(5, \infty)$ or $]5, \infty[$.

20. **(d)** We have $\frac{5-2x}{3} \le \frac{x}{6} - 5$

or $2(5 - 2x) \le x - 30$ or $10 - 4x \le x - 30$

or $-5x \le -40$ or $x \ge 8$

Thus, all real numbers which are greater than or equal to 8 are the solutions of the given inequality, i.e., $x \in [8, \infty)$.

21. **(a)** We have $\frac{3x-4}{2} \ge \frac{x+1}{4} - 1$

or $\frac{3x-4}{2} \ge \frac{x-3}{4}$

or $2(3x - 4) \ge (x - 3)$

or $6x - 8 \geq x - 3$
or $5x \geq 5$ or $x \geq 1$
Thus, all real numbers which are greater than or equal to 1 is the solution set of the given inequality.
$\therefore \quad x \in [1, \infty)$.

22. **(a)** We have $-5 \leq \frac{5-3x}{2} \leq 8$
or $-10 \leq 5 - 3x \leq 16$ or $-15 \leq -3x \leq 11$
or $5 \geq x \geq -\frac{11}{3}$,
which can be written as $\frac{-11}{3} \leq x \leq 5$
$\therefore \quad x \in \left[\frac{-11}{3}, 5\right]$.

23. **(a)** Common solution of the inequalities is from $-\infty$ to -4 and 3 to ∞.

24. **(c)** **Case I :**
When $x > 0$, $\frac{2}{x} < 3 \Rightarrow 2 < 3x \Rightarrow \frac{2}{3} < x$ or $x > \frac{2}{3}$
Case II :
When $x < 0$, $\frac{2}{x} < 3 \Rightarrow 2 > 3x \Rightarrow \frac{2}{3} > x$ or $x < \frac{2}{3}$,
which is satisfied when $x < 0$.
$\therefore \quad x \in (-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$.

25. **(c)** $|3x + 2| < 1 \Leftrightarrow -1 < 3x + 2 < 1$
$\Leftrightarrow \quad -3 < 3x < -1 \Leftrightarrow -1 < x < -\frac{1}{3}$.

26. **(b)** $|x - 1|$ is the distance of x from 1.
$|x - 3|$ is the distance of x from 3.
The point $x = 2$ is equidistant from 1 and 3. Hence, the solution consists of all $x \geq 2$.

27. **(a)** $-3x < -13 - 17$
$-3x < -30 \Rightarrow x > 10$
$\Rightarrow \quad x \in (10, \infty)$.

28. **(b)** Given, $|x + 2| \leq 9$
$\Rightarrow \quad -9 \leq x + 2 \leq 9$
$\Rightarrow \quad -11 \leq x \leq 7$

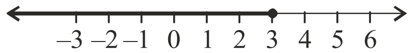
STATEMENT TYPE QUESTIONS

29. **(d)**

30. **(b)** For $x = 0$,
L.H.S. = 30(0) = 0 < 200 (R.H.S.), which is true.
For $x = 1$,
L.H.S. = 30(1) = 30 < 200 (R.H.S.), which is true.
For $x = 2$,
L.H.S. = 30(2) = 60 < 200, which is true.
For $x = 3$,
L.H.S. = 30(3) = 90 < 200, which is true.
For $x = 4$,
L.H.S. = 30(4) = 120 < 200, which is true.
For $x = 5$,
L.H.S. = 30(5) = 150 < 200, which is true.
For $x = 6$,
L.H.S. = 30(6) = 180 < 200, which is true.

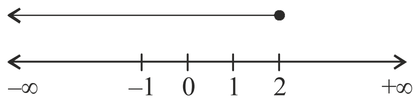
In the above situation, we find that the values of x, which makes the above inequality a true statement are 0, 1, 2, 3, 4, 5, 6. These values of x, which make above inequality a true statement are called solutions of inequality and the set {0, 1, 2, 3, 4, 5, 6} is called its solution set.
Thus, any solution of an inequality in one variable is a value of the variable which makes it a true statement.

31. **(a)** We have, $7x + 3 < 5x + 9$
or $2x < 6$ or $x < 3$
$\Rightarrow \quad x \in (-\infty, 3)$
The graphical representation of the solutions are given in figure.



32. **(b)** We have, $5x - 3 < 7$
Adding 3 on both sides,
$5x - 3 + 3 < 7 + 3$
$\Rightarrow \quad 5x < 10$
Dividing both sides by 5,
$\frac{5x}{5} < \frac{10}{5} \quad \Rightarrow \quad x < 2$
I. When x is an integer, the solution of the given inequality is {....., –1, 0, 1}.
II. When x is a real number, the solution of given inequality is $(-\infty, 2)$, i.e. all the numbers lying between $-\infty$ and 2 but ∞ and 2 are not included as $x < 2$.

33. **(b)** I. We have, $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$
$\Rightarrow \quad \frac{3x-6}{5} \leq \frac{10-5x}{3}$
$\Rightarrow \quad 9x - 18 \leq 50 - 25x$
Transferring the terms (–25x) to L.H.S. and the term (–18) to R.H.S.
$9x + 25x \leq 50 + 18$
$\Rightarrow \quad 34x \leq 68$
$\Rightarrow \quad x \leq \frac{68}{34} \Rightarrow x \leq 2$



$\therefore$ Solution set is $(-\infty, 2]$

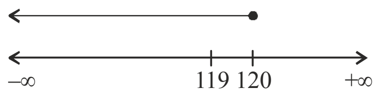
II. We have, $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6)$
$\Rightarrow \quad \frac{1}{2}\left(\frac{3x}{5} + \frac{4}{1}\right) \geq \frac{1}{3}(x - 6)$
Taking L.C.M. in L.H.S.,
$\frac{1}{2}\left(\frac{3x+20}{5}\right) \geq \frac{1}{3}(x - 6)$
$\Rightarrow \quad \frac{3x+20}{10} \geq \frac{x-6}{3}$
$\Rightarrow \quad 3(3x + 20) \geq 10(x - 6)$
$\Rightarrow \quad 9x + 60 \geq 10x - 60$

Transferring the term 10x to L.H.S. and the term 60 to R.H.S.

$9x - 10x \geq -60 - 60 \Rightarrow -x \geq -120$

Multiplying both sides by –1,

$x \leq 120$



∴ Solution set is $(-\infty, 120]$.

34. **(a)** I. The region containing all the solutions of an inequality is called the solution region.

II. In order to identify the half plane represented by an inequality, it is just sufficient to take any point (a, b) (not on line) and check whether it satisfies the inequality or not. If it satisfies, then the inequality represents the half plane and shade the region, which contains the point, otherwise the inequality represents that half plane which does not contains the point within it. For convenience, the point (0, 0) is preferred.

35. **(a)** I. The given system of inequalities

$3x + 2y \leq 12$ ... (i)

$x \geq 1$ ... (ii)

$y \geq 2$ ... (iii)

Step I: Consider the given inequations as strict equations

i.e. $3x + 2y = 12$, $x = 1$, $y = 2$

Step II: Draw the table for $3x + 2y = 12$

x	0	4
y	6	0

(i.e., Find the points on x-axis and y-axis)

Step III: Plot the points and draw the graph For $3x + 2y = 12$, and

Graph of $x = 1$ will be a line parallel to y-axis cutting x-axis at 1.

and Graph of $y = 2$ will be a line parallel to x-axis cutting y-axis at 2.

Step IV: Take a point (0, 0) and put it in the given inequations (i), (ii) and (iii).

i.e., $0 + 0 \leq 12$, $0 \leq 12$ [true]

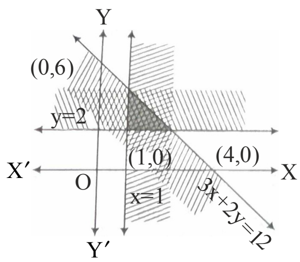
So, the shaded region will be towards the origin

$0 \geq 1$ [false]

So, the shaded region will be away from the origin

$0 \geq 2$ [false]

So, the shaded region will be away from the origin.



Thus, common shaded region shown the solution of the inequalities.

II. The given system of inequalities

$2x + y \geq 6$ ... (i)

$3x + 4y \leq 12$ ... (ii)

Step I: Consider the given inequations as strict equations

i.e., $2x + y = 6$

$3x + 4y = 12$

Step II: Find the points on the x-axis and y-axis for

$2x + y = 6$

x	0	3
y	6	0

and $3x + 4y = 12$

x	0	4
y	3	0

Step III: Plot the points and draw the graph using the above tables.

Step IV: Take a point (0, 0) and putting in the given inequations (i) and (ii),

i.e. $0 + 0 \geq 6$

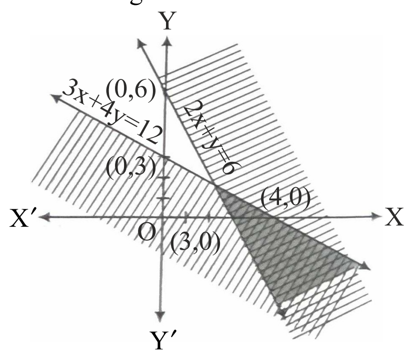
$0 \geq 6$ [false]

So, the shaded region will be away from the origin.

and $0 + 0 \leq 12$

$0 \leq 12$ (True)

So, the shaded region will be towards the origin.



Thus, common shaded region shows the solution of the inequality.

Since, common shaded region is not enclosed. So, it is not bounded.

III. The given system of inequalities

$x + y \geq 4$ ... (i)

$2x - y > 0$ ... (ii)

Step I: Consider the given inequations as strict equations

i.e., $x + y = 4$, $2x - y = 0$

Step II: Find the points on the x-axis and y-axis for

$x + y = 4$

x	0	4
y	4	0

and $2x - y = 0$

x	0	1
y	0	2

Step III: Plot the points to draw the graph using the above tables.

Step IV: Take a point (0, 0) and put it in the inequation (i)

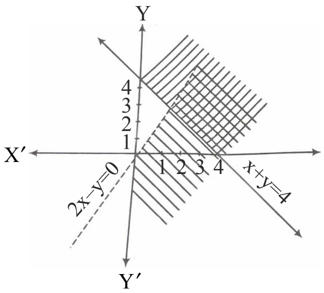
$0 + 0 \geq 4$ [false]

So, the shaded region will be away from the origin.

Take a point (1, 0) and put it in the inequation (ii)

$2 - 0 > 0$ [true]

So, the shaded region will be towards the point (1, 0)



Thus, the common shaded region shows the solution of the inequalities.

36. (d) 37. (c)

38. (a) I. $-75 < 3x - 6 \Rightarrow -23 < x$

$3x - 6 \leq 0 \Rightarrow x \leq 2$

II. $14 \leq 3x + 11 \Rightarrow 3 \leq 3x \Rightarrow 1 \leq x$

$3x + 11 \leq 22 \Rightarrow 3x \leq 11 \Rightarrow x \leq \frac{11}{3}$

III. $-20 \leq 2 - 3x \Rightarrow x \leq \frac{22}{3}$

$2 - 3x \leq 36 \Rightarrow -34 \leq 3x \Rightarrow x \geq \frac{-34}{3}$

39. (c) Both the statements are correct.

40. (d) We are given :

$24x < 100$

or $\frac{24x}{24} < \frac{100}{24}$

or $x < \frac{100}{24}$

(I) When x is natural number, the following values of x make the statement true

$x = 1, 2, 3, 4.$

The solution set = {1, 2, 3, 4}

(II) When x is an integer, in this case the solutions of the given inequality are ......, –3, –2, –1, 0, 1, 2, 3, 4.

∴ The solution set of the inequality is {...., –3, –2, –1, 0, 1, 2, 3, 4}.

41. (b) Inequality is $3x + 8 > 2$

Transposing 8 to RHS $3x > 2 - 8 = -6$

Dividing by 3, $x > -2$

(I) When x is an integer the solution is {–1, 0, 1, 2, 3, ...}

(II) When x is real, the solution is $(-2, \infty)$.

MATCHING TYPE QUESTIONS

42. (a) (A) $2x - 4 \leq 0 \Rightarrow x \leq 2$

(B) $-3x + 12 < 0 \Rightarrow x > 4$

(C) $4x - 12 \geq 0 \Rightarrow x \geq 3$

(D) $7x + 9 > 30 \Rightarrow 7x > 21 \Rightarrow x > 3$

43. (c) A. The given system of inequalities

$2x - y > 1$ ... (i)

$x - 2y < -1$ ... (ii)

Step I : Consider the inequations as strict equations i.e. $2x - y = 1$ and $x - 2y = -1$

Step II: Find the points on the x-axis and y-axis for $2x - y = 1$.

x	0	$\frac{1}{2}$
y	–1	0

and $x - 2y = -1$

x	0	–1
y	$\frac{1}{2}$	0

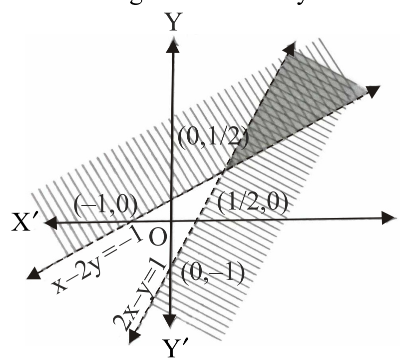
Step III: Plot the graph using the above tables.

Step IV: Take a point (0, 0) and put it in the inequations (i) and (ii).

$0 - 0 > 1$, i.e., $0 > 1$ [false]

So, the shaded region will be away from the origin and $0 - 0 < -1$, i.e., $0 < -1$ [false]

So, the shaded region will be away from the origin.



Thus, common shaded region shows the solution of the inequalities.

B. The given system of inequalities

$x + y \leq 6$ ... (i)

$x + y \geq 4$ ... (ii)

Step I: Consider the inequations as strict equations i.e. $x + y = 6$ and $x + y = 4$

Step II: Find the points on the x-axis and y-axis for

$x + y = 6.$

x	0	6
y	6	0

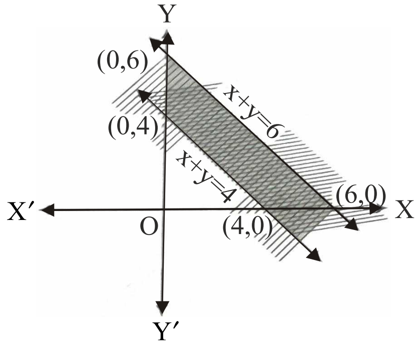
and $x + y = 4$

x	0	4
y	4	0

Step III: Plot the graph using the above tables.

Step IV: Take a point (0, 0) and put it in the inequations (i) and (ii),

i.e. $0 + 0 \leq 6$ i.e., $0 \leq 6$ [true]



So, the shaded region will be towards the origin.

and 0 + 0 ≥ 4 ⇒ 0 ≥ 4 [false]

So, the shaded region will be away from the origin.

Thus, common shaded region shows the solution of the inequalities.

C. The given system of inequalities

2x + y ≥ 8 ... (i)

x + 2y ≥ 10 ... (ii)

Step I: Consider the inequations as strict equations i.e. 2x + y = 8 and x + 2y = 10

Step II: Find the points on the x-axis and y-axis for

2x + y = 8

x	0	4
y	8	0

and x + 2y = 10

x	0	10
y	5	0

Step III: Plot the points using the above tables and draw the graph.

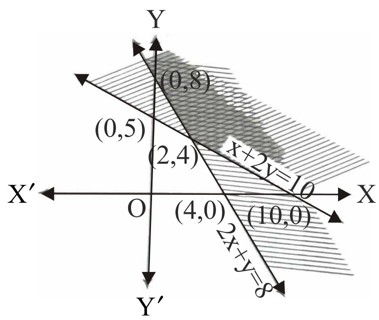
Step IV: Take a point (0, 0) and put it in the given inequations (i) and (ii),

i.e., 0 + 0 ≥ 8 i.e. 0 ≥ 8 [false]

So, the shaded region will be away from the origin.

i.e., 0 + 0 ≥ 10, i.e. 0 ≥ 10 [false]

So, the shaded region will be away from the origin.



Thus, common shaded region shows the solution of the inequalities.

D. The given system of inequalities

x + y ≤ 9 ... (i)

y > x ... (ii)

x ≥ 0 ... (iii)

Step I: Consider the inequations as strict equations i.e. x + y = 9, y = x, x = 0

Step II: Find the points on the x-axis and y-axis for

x + y = 9

x	0	9
y	9	0

and y = x

x	1	2	3
y	1	2	3

Step III: Plot the points using the above tables and draw the graph

For x + y = 9 and

For y = x

Graph of x = 0 will be the y-axis.

Step IV: Take a point (0, 0), put it in the inequations (i), (ii) and (iii), we get

0 + 0 ≤ 9 [true]

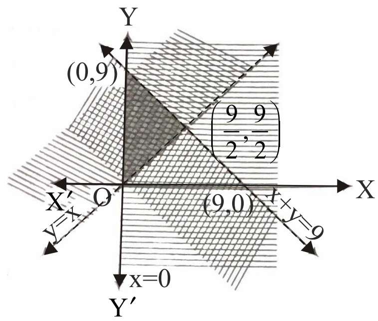
So, the shaded region will be towards the origin.

Take a point (0, 1), put in y > x, 1 > 0 [true]

So, the shaded region will be towards the origin.

Take a point (1, 0), put it in x ≥ 0, 1 ≥ 0 [true]

So, the shaded region will be towards the origin.



Thus, common shaded region shows the solution of the inequalities.

44. **(c)** (A) $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

$\Rightarrow \frac{3x-4}{2} \geq \frac{x+1-4}{4}$

$\Rightarrow 3x - 4 \geq \frac{x-3}{2}$

$\Rightarrow 6x - 8 \geq x - 3$

$\Rightarrow 5x \geq 5 \Rightarrow x \geq 1$

(B) $3x - 2 < 2x + 1 \Rightarrow x < 3$

(C) $3(1-x) < 2(x+4) \Rightarrow 3 - 3x < 2x + 8$

$\Rightarrow -5 < 5x \Rightarrow x > -1$

(D) $3x - 7 < 5 + x \Rightarrow 2x < 12 \Rightarrow x < 6$

$11 - 5x \leq 1 \Rightarrow 10 \leq 5x \Rightarrow 2 \leq x$

45. **(b)** (A) We draw the graph of the equation

$x + y = 5$ ...(i)

Putting $y = 0$, $x = 5$, therefore the point on the x-axis is (5, 0). The point on the y-axis is (0, 5). AB is the graph of (i) (See Fig)

Putting $x = 0, y = 0$ in the given inequality, we have $0 + 0 < 5$ or $5 > 0$ which is true. Hence, origin lies in the half plane region I.
Clearly, any point on the line does not satisfy the given inequality.
Hence, the shaded region I excluding the points on the line is the solution region of the inequality.

(B) We draw the graph of the equation
$$2x + y = 6 \quad ...(i)$$
Putting $x = 0, y = 6$, therefore the point on y-axis is (0, 6) and the point on x-axis is (3, 0). AB is the graph of (i).
Putting $x = 0, y = 0$ in the given inequality, we have $2(0) + 0 \geq 6$ or $0 \geq 6$, which is false.
Hence, origin does not lie in the half plane region I.
Clearly, any point on the line satisfy the given inequality.
Hence, the shaded region II including the points on the line is the solution region of the inequality.

(C) We draw the graph of the equation $3x + 4y = 12$.
The line passes through the points (4, 0), (0, 3).
This line is represented by AB.
Now consider the inequality $3x + 4y \leq 12$
Putting $x = 0, y = 0$
$0 + 0 = 0 \leq 12$, which is true
$\therefore$ Origin lies in the region of $3x + 4y \leq 12$
The shaded region represents this inequality.

(D) We draw the graph of $2x - 3y = 6$
The line passes through (3, 0), (0, –2)
AB represents the equation $2x - 3y = 6$
Now consider the inequality $2x - 3y > 6$
Putting $x = 0, y = 0$
$0 = 0 > 6$ is not true.
$\therefore$ Origin does not lie in the region of $2x - 3y > 6$
The graph of $2x - 3y > 6$ is shown as shaded area.

INTEGER TYPE QUESTIONS

46. (c) $4x + 3 < 6x + 7$
$\Rightarrow -2x < 4$
$\Rightarrow -x < 2 \Rightarrow x > -2$
$\Rightarrow x \in (-2, \infty)$

47. (d) $\dfrac{5-2x}{3} \leq \dfrac{x}{6} - 5$
$\Rightarrow \dfrac{5-2x}{3} \leq \dfrac{x-30}{6}$
$\Rightarrow 5 - 2x \leq \dfrac{x-30}{2}$
$\Rightarrow 10 - 4x \leq x - 30 \Rightarrow 40 \leq 5x$
$\Rightarrow 8 \leq x \Rightarrow x \in [8, \infty)$

48. (c) $3(2 - x) \geq 2(1 - x)$
$\Rightarrow 6 - 3x \geq 2 - 2x$
$\Rightarrow -x \geq -4 \Rightarrow x \leq 4$
$\Rightarrow x \in (-\infty, 4]$

49. (a) $\dfrac{2x-1}{3} \geq \dfrac{15x - 10 - 8 + 4x}{20}$
$\Rightarrow \dfrac{2x-1}{3} \geq \dfrac{19x-18}{20}$
$\Rightarrow 40x - 20 \geq 57x - 54$
$\Rightarrow -17x \geq -34 \Rightarrow x \leq 2$
$\Rightarrow x \in (-\infty, 2]$

50. (d) Given inequality is $5x + 1 > -24$
$\Rightarrow 5x > -25 \Rightarrow x > -5$
Also, $5x - 1 < 24$
$\Rightarrow 5x < 25 \Rightarrow x < 5$
Hence, $-5 < x < 5 \Rightarrow x \in (-5, 5)$

51. (b) $2x - 7 < 11 \Rightarrow 2x < 18 \Rightarrow x < 9$
$3x + 4 < -5 \Rightarrow 3x < -9 \Rightarrow x < -3$
Hence, common solution is $x < -3$.
So, $x \in (-\infty, -3)$

52. (a) By definition of $|x|$, we have
$|x| < 3 \Rightarrow -3 < x < 3$
$\Rightarrow m = 3$.

53. (b) Let shortest side measure x cm. Therefore the longest side will be $3x$ cm and third side will be $(3x - 2)$ cm
According to the problem,
$x + 3x + 3x - 2 \geq 61$
$\Rightarrow 7x - 2 \geq 61$ or $7x \geq 63$
$\Rightarrow x \geq 9$cm
Hence, the minimum length of the shortest side is 9 cm and the other sides measure 27 cm and 25 cm.

54. (c) $-8 \leq 5x - 3 \Rightarrow -5 \leq 5x \Rightarrow -1 \leq x$
$5x - 3 < 7 \Rightarrow 5x < 10 \Rightarrow x < 2$
Hence, common sol is $-1 \leq x < 2$
$\Rightarrow x \in [-1, 2)$
$\Rightarrow a = 1, b = 2$ and $a + b = 3$

55. (a) Let x and x + 2 be two odd natural numbers.
we have, $x > 10$ …(i)
and $x + (x + 2) < 40$ …(ii)
On solving (i) and (ii), we get
$10 < x < 19$
So, required pairs are (11, 13), (13, 15), (15, 17) and (17, 19)

ASSERTION - REASON TYPE QUESTIONS

56. (b) Let us consider some inequalities :
$ax + b < 0$ … (i)
$ax + b > 0$ … (ii)
$ax + b \leq 0$ … (iii)
$ax + b \geq 0$ … (iv)
$ax + by > c$ … (v)
$ax + by \leq c$ … (vi)
$ax^2 + bx + c > 0$ … (vii)
$ax^2 + bx + c \leq 0$ … (viii)
Inequalities (i), (ii), (v) and (vii) are strict inequalities, while inequalities (iii), (iv), (vi) and (viii) are slack inequalities.
$\therefore$ Both Assertion and Reason are correct but Reason cannot explain Assertion.

57. **(d)** Assertion is false, Reason is true because if $a < b$, $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

58. **(b)** We have, $|3x - 5| > 9$
$\Rightarrow 3x - 5 < -9$ or $3x - 5 > 9$
$\Rightarrow 3x < -4$ or $3x > 14$
$\Rightarrow x < \frac{-4}{3}$ or $x > \frac{14}{3}$
$\therefore x \in \left(-\infty, \frac{-4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$.

59. **(b)** Both Assertion and Reason are correct but Reason is not correct explanation for the Assertion.

60. **(b)** Both are correct.

61. **(b)** Both are correct; Reason is not the correct explanation of Assertion.

62. **(b)** Both Assertion and Reason are correct but Reason is not the correct explanation.
Reason: $5x - 3 < 7$
$\Rightarrow 5x < 10 \Rightarrow x < 2$
$\Rightarrow x \in (-\infty, 2)$

63. **(c)** Assertion is correct.
$3x + 8 > 2 \Rightarrow 3x > -6$
$\Rightarrow x > -2$
$\Rightarrow x \in \{-1, 0, 1, 2, \ldots\}$
Reason is incorrect.
$4x + 3 < 5x + 7$
$-x < 4 \Rightarrow x > -4$
$\Rightarrow x \in (-4, \infty)$

64. **(c)** Assertion is correct. Reason is incorrect.
If a point satisfying the line $ax + by = c$, then it will lie on the line.

65. **(b)** Both are correct but Reason is not the correct explanation.

66. **(d)** Assertion is incorrect. Reason is correct.

CRITICAL THINKING TYPE QUESTIONS

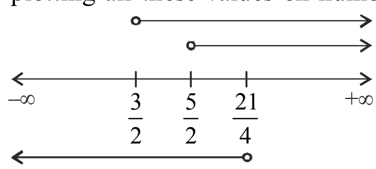
67. **(c)** If x cm is the breadth, then
$2(3x + x) \geq 160 \Rightarrow x \geq 20$

68. **(c)** $|x - 1| \leq 3 \Rightarrow -3 \leq x - 1 \leq 3 \Rightarrow -2 \leq x \leq 4$
and $|x - 1| \geq 1 \Rightarrow x - 1 \leq -1$ or $x - 1 \geq 1$
$\Rightarrow x \leq 0$ or $x \geq 2$
Taking the common values of x, we get
$x \in [-2, 0] \cup [2, 4]$

69. **(a)** Let x be the marks obtained by student in the annual examination. Then,
$\frac{62 + 48 + x}{3} \geq 60$
or $110 + x \geq 180$
or $x \geq 70$
Thus, the student must obtain a minimum of 70 marks to get an average of at least 60 marks.

70. **(b)** Let Ravi got x marks in third unit test.
$\therefore$ Average marks obtained by Ravi
$= \frac{\text{Sum of marks in all tests}}{\text{Number of tests}} = \frac{70 + 75 + x}{3} = \frac{145 + x}{3}$
Now, it is given that he wants to obtain an average of at least 60 marks.
At least 60 marks means that the marks should be greater than or equal to 60.
i.e. $\frac{145 + x}{3} \geq 60$
$\Rightarrow 145 + x \geq 60 \times 3$
$\Rightarrow 145 + x \geq 180$
Now, transferring the term 145 to R.H.S.,
$x \geq 180 - 145 \quad \Rightarrow \quad x \geq 35$
i.e. Ravi should get greater than or equal to 35 marks in third unit test to get an average of at least 60 marks.
$\therefore$ Minimum marks Ravi should get = 35.

71. **(b)** Let numbers are $2x$ and $2x + 2$
Then, according to the question,
$2x > 5 \Rightarrow x > \frac{5}{2}$
and $2x + 2 > 5 \Rightarrow 2x > 5 - 2$
$\Rightarrow 2x > 3 \Rightarrow x > \frac{3}{2}$
and $2x + 2x + 2 < 23 \Rightarrow 4x < 23 - 2$
$\Rightarrow 4x < 21 \quad \Rightarrow \quad x < \frac{21}{4}$
Now, plotting all these values on number line



From above graph, it is clear that $x \in \left(\frac{5}{2}, \frac{21}{4}\right)$ in which integer values are $x = 3, 4, 5$.
When $x = 3$, pair is $(2 \times 3, 2 \times 3 + 2) = (6, 8)$
When $x = 4$, pair is $(2 \times 4, 2 \times 4 + 2) = (8, 10)$
When $x = 5$, pair is $(2 \times 5, 2 \times 5 + 2) = (10, 12)$
$\therefore$ Required pairs are (6, 8), (8, 10), (10, 12).

72. **(b)** Let the shortest side be x cm.
Then, by given condition, second length = x + 3 cm
Third length = 2x cm
Also given, total length = 91
Hence, sum of all the three lengths should be less than or equal to 91
$x + x + 3 + 2x \leq 91$
$\Rightarrow 4x + 3 \leq 91$
Subtracting (–3) to each term,
$-3 + 4x + 3 \leq 91 - 3$
$\Rightarrow 4x \leq 88$
$\Rightarrow \frac{4x}{4} \leq \frac{88}{4} \quad \Rightarrow \quad x \leq \frac{88}{4}$
$\Rightarrow x \leq 22$ cm ... (i)
Again, given that
Third length $\geq$ second length + 5

$\Rightarrow\ 2x \ge (x + 3) + 5$

$\Rightarrow\ 2x \ge x + (3 + 5)$

Transferring the term x to L.H.S.,

$2x - x \ge 8$

$\Rightarrow\ x \ge 8$ ... (ii)

From equations (i) and (ii), length of shortest board should be greater than or equal to 8 but less than or equal to 22, i.e., $8 \le x \le 22$.

73. (c) Let breadth of rectangle be x cm.

$\therefore$ Length of rectangle = 3x

Perimeter of rectangle = 2(Length + Breadth)

$= 2(x + 3x) = 8x$

Given, Perimeter $\ge$ 160 cm

$8x \ge 160$

Dividing both sides by 8,

$x \ge 20$ cm

74. (a) We have, $2 \le |x - 3| < 4$

Case I : If $x < 3$, then

$2 \le |x - 3| < 4$

$\Rightarrow\ 2 \le -(x - 3) < 4$

$\Rightarrow\ 2 \le -x + 3 < 4$

Subtracting 3 from both sides,

$-1 \le -x < 1$

Multiplying (–1) on both sides,

$-1 < x \le 1$

$\Rightarrow\ x \in (-1, 1]$

Case II : If $x > 3$, then

$2 \le |x - 3| < 4$

$\Rightarrow\ 2 \le x - 3 < 4$

Adding 3 on both sides,

$\Rightarrow\ 5 \le x < 7$

Hence, the solution set of given inequality is $x \in (-1, 1] \cup [5, 7)$.

75. (c) We have

$$IQ = \frac{MA}{CA} \times 100$$

$$\Rightarrow\ IQ = \frac{MA}{12} \times 100 \qquad [\because CA = 12 \text{ years}]$$

$$= \frac{25}{3} MA$$

Given, $80 \le IQ \le 140$

$$\Rightarrow\ 80 \le \frac{25}{3} MA \le 140$$

$$\Rightarrow\ 240 \le 25MA \le 420$$

$$\Rightarrow\ \frac{240}{25} \le MA \le \frac{420}{25}$$

$\Rightarrow\ 9.6 \le MA \le 16.8$

76. (c) The inequalities are :

$750x + 150y \le 15000$

i.e. $5x + y \le 100$ … (i)

$x + y \le 60$ … (ii)

$x \ge 0$ … (iii)

$y \ge 0$ … (iv)

The lines corresponding to (i) and (ii) are

$5x + y = 100$ … (v)

$x + y = 60$ … (vi)

Table for $5x + y = 100$

x	0	20
y	100	0

Table for $x + y = 60$

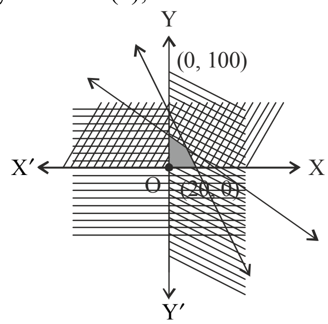
x	0	60
y	60	0

First, draw the lines (v) and (vi)

$\because\ 5(0) + 0 \le 100$

i.e., $0 \le 100$, which is true.

Therefore, inequality (i) represent the half plane made by the line (v), which contains the origin.



Again, $0 + 0 \le 60$

i.e. $0 \le 60$, which is true.

Therefore, inequality (ii) represent the half plane made by the line (vi) which contains origin. Inequality $x \ge 0$ represent the half plane on the right of y-axis. Inequality $y \ge 0$ represent the half plane above x-axis.

77. (d) The given system of inequalities

$x + 2y \le 10$ ... (i)

$x + y \ge 1$ ... (ii)

$x - y \le 0$ ... (iii)

$x \ge 0,\ y \ge 0$ ... (iv)

Step I: Consider the given inequations as strict equations,

i.e. $x + 2y = 10,\ x + y = 1,\ x - y = 0$

and $x = 0,\ y = 0$

Step II: Find the points on the x-axis and y-axis for

$x + 2y = 10$

x	0	10
y	5	0

and $x + y = 1$

x	0	1
y	1	0

For $x - y = 0$

x	1	2
y	1	2

Step III: Plot the graph of $x + 2y = 10$, $x + y = 1$, $x - y = 0$ using the above tables.

Step IV: Take a point (0, 0) and put it in the inequalities (i) and (ii),

$0 + 0 \le 10$ [true]

So, the shaded region will be towards origin,

and $0 + 0 \ge 1$ [false]

So, the shaded region will be away from the origin.

Again, take a point (2, 2) and put it in the inequation (iv), we get

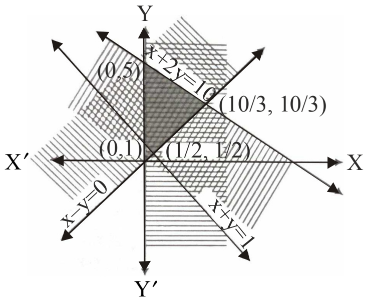
$2 \geq 0, 2 \geq 0$ [true]

So, the shaded region will be towards point (2, 2).

And take a point (0, 1) and put it in the inequation (iii), we get

$0 - 1 \leq 0$ [true]

So, the shaded region will be towards point (0, 1).



Thus, the common shaded region shows the solution of the inequalities.

78. (b) (i) Consider the line $x + y = 8$. We observe that the shaded region and origin lie on the same side of this line and (0, 0) satisfies $x + y \leq 8$. Therefore, $x + y \leq 8$ is the linear inequality corresponding to the line $x + y = 8$.

(ii) Consider $x + y = 4$. We observe that shaded region and origin are on the opposite side of this line and (0, 0) satisfies $x + y \leq 4$. Therefore, we must have $x + y \geq 4$ as linear inequalities corresponding to the line $x + y = 4$.

(iii) Shaded portion lie below the line $y = 5$. So, $y \leq 5$ is the linear inequality corresponding to $y = 5$.

(iv) Shaded portion lie on the left side of the line $x = 5$. So, $x \leq 5$ is the linear inequality corresponding to $x = 5$.

(v) Also, the shaded region lies in the first quadrant only. Therefore, $x \geq 0$, $y \geq 0$.

In view of (i), (ii), (iii), (iv) and (v) above the linear inequalities corresponding to the given solutions are: $x + y \leq 8$, $x + y \geq 4$, $y \leq 5$, $x \leq 5$, $x \geq 0$ and $y \geq 0$.

79. (c) Let the 2% boric acid solution be x L.

$\therefore$ Mixture = (640 + x)L

Now, according to the question, two conditions arise :

I. 2% of x + 8% of 640 > 4% of (640 + x)

II. 2% of x + 8% of 640 < 6% of (640 + x)

From condition I,

$$\frac{2}{100} \times x + \frac{8}{100} \times 640 > \frac{4}{100} \times (640 + x)$$

Multiplying both sides by 100,

$$100 \times \left[\frac{2x}{100} + \frac{8}{100} \times 640\right] > \frac{4}{100} \times (640 + x) \times 100$$

$\Rightarrow 2x + 8 \times 640 > 4 \times 640 + 4x$

Transferring the term 4x to L.H.S. and the term (8 × 640) to R.H.S.

$2x - 4x > 4 \times 640 - 8 \times 640$

$\Rightarrow -2x > 640(4 - 8)$

$\Rightarrow -2x > -4 \times 640$

Dividing both sides by –2,

$$\frac{-2x}{-2} < \frac{-4 \times 640}{-2}$$

$\Rightarrow x < 2 \times 640$

$\Rightarrow x < 1280$ ... (i)

From condition II,

$$\frac{2}{100} \times x + \frac{8}{100} \times 640 < \frac{6}{100} \times (640 + x)$$

$$\Rightarrow 100 \times \left[\frac{2x}{100} + \frac{8}{100} \times 640\right] < [6 \times 640 + 6x] \times \frac{100}{100}$$

$\Rightarrow 2x + 8 \times 640 < 6 \times 640 + 6x$

Transferring the term 6x to L.H.S. and the term (8 × 640) to R.H.S.,

$2x - 6x < 6 \times 640 - 8 \times 640$

$\Rightarrow -4x < 640(6 - 8) \Rightarrow -4x < -2 \times 640$

Dividing both sides by –4,

$$\frac{-4x}{-4} > \frac{-2 \times 640}{-4}$$

$\Rightarrow x > 320$ ... (ii)

Hence, from equations (i) and (ii),

$320 < x < 1280$ i.e., $x \in (320, 1280)$

The number of litres to be added should be greater than 320 L and less than 1280 L.

80. (a) Given, $C(x) = 26000 + 30x$

and $R(x) = 43x$

$\therefore$ Profit = R(x) – C(x)

$= 43x - (26000 + 30x) = 13x - 26000$

For some profit, $13x - 26000 > 0$

$\Rightarrow 13x > 26000$

$\Rightarrow x > 2000$

81. (a) Let x litres of 30% acid solution is required to be added. Then,

Total mixture = (x + 600) litres

$\therefore$ 30% of x + 12% of 600 > 15% of (x + 600)

and 30% of x + 12% of 600 < 18% of (x + 600)

or $\frac{30x}{100} + \frac{12}{100}(600) > \frac{15}{100}(x + 600)$

and $\frac{30x}{100} + \frac{12}{100}(600) < \frac{18}{100}(x + 600)$

or $30x + 7200 > 15x + 9000$

and $30x + 7200 < 18x + 10800$

or $15x > 1800$ and $12x < 3600$

or $x > 120$ and $x < 300$

i.e. $120 < x < 300$

Thus, the number of litres of the 30% solution of acid will have to be more than 120 litres but less than 300 litres.

82. (c) We have $\frac{|x + 3| + x}{x + 2} > 1$

$$\Rightarrow \frac{|x + 3| + x}{x + 2} - 1 > 0 \Rightarrow \frac{|x + 3| - 2}{x + 2} > 0$$

Now, two cases arise :

Case I : When $x + 3 \geq 0$, i.e. $x \geq -3$. Then,

$\frac{|x+3|-2}{x+2} > 0 \Rightarrow \frac{x+3-2}{x+2} > 0$

$\Rightarrow \frac{x+1}{x+2} > 0$

$\Rightarrow$ {(x + 1) > 0 and x + 2 > 0}

or {x + 1 < 0 and x + 2 < 0}

$\Rightarrow$ {x > –1 and x > –2} or {x < –1 and x < –2}

$\Rightarrow$ x > –1 or x < –2

$\Rightarrow$ $x \in (-1, \infty)$ or $x \in (-\infty, -2)$

$\Rightarrow$ $x \in (-3, -2) \cup (-1, \infty)$ [Since $x \geq -3$] ... (i)

Case II : When x + 3 < 0, i.e. x < –3

$\frac{|x+3|-2}{x+2} > 0 \Rightarrow \frac{-x-3-2}{x+2} > 0$

$\Rightarrow \frac{-(x+5)}{x+2} > 0 \Rightarrow \frac{x+5}{x+2} < 0$

$\Rightarrow$ (x + 5 < 0 and x + 2 > 0) or (x + 5 > 0 and x + 2 < 0)

$\Rightarrow$ (x < –5 and x > –2) or (x > –5 and x < –2)

it is not possible.

$\Rightarrow$ $x \in (-5, -2)$ ... (ii)

Combining (i) and (ii), the required solution is $x \in (-5, -2) \cup (-1, \infty)$.

83. (c) We have, $|2x-3| < |x+5|$

$\Rightarrow |2x-3| - |x+5| < 0$

$$\Rightarrow \begin{cases} 3-2x+x+5<0, & x \leq -5 \\ 3-2x-x-5<0, & -5<x\leq \frac{3}{2} \\ 2x-3-x-5<0, & x>\frac{3}{2} \end{cases}$$

$$\Rightarrow \begin{cases} x>8, & x \leq -5 \\ x>-\frac{2}{3}, & -5<x\leq \frac{3}{2} \\ x<8, & x>\frac{3}{2} \end{cases}$$

$\Rightarrow x \in \left(-\frac{2}{3}, \frac{3}{2}\right] \cup \left(\frac{3}{2}, 8\right) \Rightarrow x \in \left(-\frac{2}{3}, 8\right)$

84. (b) $(x-1)^2$ is always positive except when x = 1 (and then it is 0)

$\therefore$ Solution is when x + 4 < 0 and $x \neq 1$

i.e. x < –4, $x \neq 1$

$\therefore$ $x \in (-\infty, -4)$.

85. (c) $\left|1+\frac{3}{x}\right| > 2$

Case I : $1+\frac{3}{x} > 2 \Rightarrow \frac{3}{x} > 1$ (Clearly x > 0)

$\Rightarrow$ 3 > x or x < 3

Case II : $1+\frac{3}{x} < -2 \Rightarrow \frac{3}{x} < -3$ (Clearly x < 0)

$\Rightarrow$ 3 > –3x $\Rightarrow$ –1 < x or x > –1

Hence, either 0 < x < 3 or –1 < x < 0

86. (b) $|2x-3| < |x+2|$

$\Rightarrow -|x+2| < 2x-3 < |x+2|$ ... (i)

Case I : $x + 2 \geq 0$. Then by (i),

$-(x+2) < 2x-3 < x+2$

$\Rightarrow -x-2 < 2x-3 < x+2$

$\Rightarrow$ 1 < 3x and x < 5 $\Rightarrow \frac{1}{3} < x < 5$

Case II : x + 2 < 0. Then by (i),

$(x+2) < 2x-3 < -(x+2)$

$\Rightarrow -(x+2) > 2x-3 > (x+2)$

$\Rightarrow$ 1 > 3x and x > 5 $\Rightarrow \frac{1}{3} \leq x$ and x > 5, Not possible.

87. (b) $\left|x+\frac{1}{x}\right| > 2$ [Clearly $x \neq 0$]

$\Rightarrow \left|\frac{x^2+1}{x}\right| > 2 \Rightarrow \frac{x^2+1}{|x|} > 2$ [$\because x^2+1>0$]

$\Rightarrow x^2+1 > 2|x|$

$\Rightarrow |x|^2 - 2|x| + 1 > 0 \Rightarrow (|x|-1)^2 > 0$

$\Rightarrow |x| \neq 1 \Rightarrow x \neq -1, 1$

$\therefore$ $x \in R - \{-1, 0, 1\}$.

88. (d) From the graph,

$-7x + 4y \leq 14$, $x - 6y \leq 3$

$3x + 4y \leq 18$, $2x + 3y \geq 3$

CHAPTER

7

PERMUTATIONS AND COMBINATIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then the value of r is
(a) 41 (b) 14 (c) 10 (d) 51

2. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then the value of n is:
(a) 20 (b) 19 (c) 18 (d) 17

3. If ${}^{30}C_{r+2} = {}^{30}C_{r-2}$, then r equals:
(a) 8 (b) 15 (c) 30 (d) 32

4. The sum of the digits in the unit place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is
(a) 432 (b) 108 (c) 36 (d) 18

5. In an examination, there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is :
(a) 11 (b) 12 (c) 27 (d) 63

6. How many arrangements can be made out of the letters of the word " MOTHER" taken four at a time so that each arrangement contains the letter ´M´?
(a) 240 (b) 120 (c) 60 (d) 360

7. In how many ways can a bowler take four wickets in a single 6-ball over ?
(a) 6 (b) 15 (c) 20 (d) 30

8. There are four chairs with two chairs in each row. In how many ways can four persons be seated on the chairs, so that no chair remains unoccupied ?
(a) 6 (b) 12 (c) 24 (d) 48

9. If a secretary and a joint secretary are to be selected from a committee of 11 members, then in how many ways can they be selected ?
(a) 110 (b) 55 (c) 22 (d) 11

10. A bag contains 3 black, 4 white and 2 red balls, all the balls being different. Number of selections of atmost 6 balls containing balls of all the colours is
(a) 1008 (b) 1080 (c) 1204 (d) 1130

11. Number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour.
(a) $6 \times (9!)^2$ (b) 12!
(c) $4 \times (8!)^2$ (d) $5 \times (9!)^2$

12. Number of words each containing 3 consonants and 2 vowels that can be formed out of 5 consonants and 4 vowels is
(a) 3600 (b) 7200 (c) 6728 (d) 2703

13. Every body in a room shakes hands with every body else. If total number of hand-shaken is 66, then the number of persons in the room is
(a) 11 (b) 10 (c) 12 (d) 19

14. Number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is
(a) 652 (b) 650 (c) 651 (d) 640

15. Total number of eight digit numbers in which all digits are different is
(a) $\frac{8.7!}{3}$ (b) $\frac{5.8!}{3}$ (c) $\frac{8.9!}{2}$ (d) $\frac{9.9!}{2}$

16. Number of words from the letters of the words BHARAT in which B and H will never come together is
(a) 210 (b) 240 (c) 422 (d) 400

17. Four couples (husband and wife) decide to form a committee of four members, then the number of different committees that can be formed in which no couple finds a place is
(a) 15 (b) 16 (c) 20 (d) 21

18. Number of different ways in which 8 persons can stand in a row so that between two particular person A and B there are always two person is
(a) 11 (b) 13 (c) 15 (d) 16

19. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are
(a) 216 (b) 375 (c) 400 (d) 720

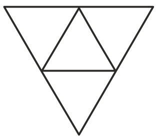
20. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number
(a) 601 (b) 600 (c) 603 (d) 602

21. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
(a) $8.{}^{6}C_4.{}^{7}C_4$ (b) $6.7.{}^{8}C_4$
(c) $6.8.{}^{7}C_4$. (d) $7.{}^{6}C_4.{}^{8}C_4$

22. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ?
(a) 16 (b) 36 (c) 60 (d) 180

23. The number of ways in which four letters of the word MATHEMATICS can be arranged is given by
(a) 136 (b) 192 (c) 1680 (d) 2454

24. In how many ways can this diagram be coloured subject to the following two conditions?
(i) Each of the smaller triangle is to be painted with one of three colours: red, blue or green.
(ii) No two adjacent regions have the same colour.



(a) 20 (b) 24 (c) 28 (d) 30

25. There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip?
(a) 72 (b) 144 (c) 14 (d) 19

26. In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together?
(a) 41472 (b) 41470 (c) 41400 (d) 41274

27. The number of ways of distributing 50 identical things among 8 persons in such a way that three of them get 8 things each, two of them get 7 things each, and remaining 3 get 4 things each, is equal to
(a) $\dfrac{(50!)(8!)}{(8!)^3(3!)^2(7!)^2(4!)^3(2!)}$
(b) $\dfrac{(50!)(8!)}{(8!)^3(7!)^3(4!)^3}$
(c) $\dfrac{(50!)}{(8!)^3(7!)^2(4!)^3}$
(d) $\dfrac{(8!)}{(3!)^2(2!)}$

28. If eleven members of a committee sit at a round table so that the President and Secretary always sit together, then the number of arrangements is
(a) $10! \times 2$ (b) $10!$ (c) $9! \times 2$ (d) $11! \times 2!$

29. ABC is a triangle. 4, 5, 6 points are marked on the sides AB, BC, CA, respectively, the number of triangles on different side is
(a) $(4+5+6)!$ (b) $(4-1)(5-1)(6-1)$
(c) $5!\,4!\,6!$ (d) $4 \times 5 \times 6$

30. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to
(a) 60 (b) 120 (c) 7200 (d) 720

31. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is
(a) 6 (b) 18 (c) 12 (d) 9

32. The number of ways in which 3 prizes can be distributed to 4 children, so that no child gets all the three prizes, are
(a) 64 (b) 62 (c) 60 (d) None of these

33. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then, what is the rank of the word RACHIT?
(a) 479 (b) 480 (c) 481 (d) 482

34. The number of chords that can be drawn through 21 points on a circle, is
(a) 200 (b) 190 (c) 210 (d) None of these

35. The number of ways a student can choose a programme out of 5 courses, if 9 courses are available and 2 specific courses are compulsory for every student is
(a) 35 (b) 40 (c) 24 (d) 120

36. The number of ways in which we can choose a committee from four men and six women so that the committee include at least two men and exactly twice as many women as men is
(a) 94 (b) 126 (c) 128 (d) None of these

37. A father with 8 children takes them 3 at a time to the zoological garden, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is
(a) 56 (b) 100 (c) 112 (d) None of these

38. 4 buses runs between Bhopal and Gwalior. If a man goes from Gwalior to Bhopal by a bus and comes back to Gwalior by another bus, then the total possible ways are
(a) 12 (b) 16 (c) 4 (d) 8

39. Six identical coins are arranged in a row. The number of ways in which the number of tails is equal to the number of heads is
(a) 20 (b) 9 (c) 120 (d) 40

40. The figures 4, 5, 6, 7, 8 are written in every possible order. The number of numbers greater than 56000 is
(a) 72 (b) 96 (c) 90 (d) 98

41. There are 5 roads leading to a town from a village. The number of different ways in which a villager can go to the town and return back, is
(a) 25 (b) 20 (c) 10 (d) 5

42. The number of numbers that can be formed with the help of the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy odd places, is
(a) 24 (b) 18 (c) 12 (d) 30

43. In a circus, there are ten cages for accommodating ten animals. Out of these, four cages are so small that five out of 10 animals cannot enter into them. In how many ways will it be possible to accommodate ten animals in these ten cages?
(a) 66400 (b) 86400 (c) 96400 (d) None of these

44. On the occasion of Deepawali festival, each student of a class sends greeting cards to the others. If there are 20 students in the class, then the total number of greeting cards exchanged by the students is
(a) ${}^{20}C_2$ (b) $2 \cdot {}^{20}C_2$
(c) $2 \cdot {}^{20}P_2$ (d) None of these

45. To fill 12 vacancies, there are 25 candidates of which five are from scheduled caste. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, then the number of ways in which the selection can be made

(a) ${}^5C_3 \times {}^{22}C_9$ (b) ${}^{22}C_9 - {}^5C_3$

(c) ${}^{22}C_3 + {}^5C_3$ (d) None of these

46. 12 persons are to be arranged to a round table. If two particular persons among them are not to be side by side, the total number of arrangements is

(a) 9(10!) (b) 2(10!) (c) 45(8!) (d) 10!

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

47. The number of 3 letters words, with or without meaning which can be formed out of the letters of the word 'NUMBER'.

Statement I : When repetition of letters is not allowed is 120.

Statement II : When repetition of letters is allowed is 216.

Choose the correct option.

(a) Only Statement I is correct

(b) Only Statement II is correct

(c) Both I and II are correct

(d) Both I and II are false

48. The number of 4 letter words that can be formed from letters of the word 'PART', when:

Statement I : Repetition is not allowed is 24.

Statement II : Repetition is allowed is 256.

Which of the above statement(s) is/are true?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

49. Consider the following statements:

Statement I : The number of diagonals of n-sided polygon is ${}^nC_2 - n$.

Statement II : A polygon has 44 diagonals. The number of its sides are 10.

Choose the correct option from the choices given below.

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Both I and II are false

50. A committee of 7 has to be formed from 9 boys and 4 girls.

I. In 504 ways, this can be done, when the committee consists of exactly 3 girls.

II. In 588 ways, this can be done, when the committee consists of at least 3 girls.

Choose the correct option.

(a) Only I is true. (b) Only II is true.

(c) Both are true. (d) Both are false.

51. Consider the following statements.

I. ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2} = {}^{n+2}C_r$

II. ${}^nC_p = {}^nC_q \Rightarrow p = q$ or $p + q = n$

Choose the correct option.

(a) Only I is true. (b) Only II is true.

(c) Both are true. (d) Both are false.

52. Consider the following statements.

I. Value of ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$ is zero.

II. The total number of 9 digit numbers which have all different digits is 9!

Choose the correct option.

(a) Only I is true (b) Only II is true.

(c) Both are true. (d) Both are false.

53. Consider the following statements.

I. Three letters can be posted in five letter boxes in 3^5 ways.

II. In the permutations of n things, r taken together, the number of permutations in which m particular things occur together is ${}^{n-m}P_{r-m} \times {}^rP_m$.

Choose the correct option.

(a) Only I is false. (b) Only II is false.

(c) Both are false. (d) Both are true.

54. Consider the following statements.

I. If some or all n objects are taken at a time, the number of combinations is $2^n - 1$.

II. An arrangement in a definite order which can be made by taking some or all of a number of things is called a permutation.

Choose the correct option.

(a) Only I is true. (b) Only II is true.

(c) Both are true. (d) Both are false.

55. Consider the following statements.

I. If there are n different objects, then ${}^nC_r = {}^nC_{n-r}$, $0 \le r \le n$.

II. If there are n different objects, then ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$, $0 \le r \le n$

Choose the correct option.

(a) Both are false. (b) Both are true.

(c) Only I is true. (d) Only II is true.

56. Consider the following statements.

I. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the values of n and r are 3 and 2 respectively.

II. From a class of 32 students, 4 are to be chosen for a competition. This can be done in ${}^{32}C_2$ ways.

Choose the correct option.

(a) Only I is true. (b) Only II is true.

(c) Both are false. (d) Both are true.

57. Consider the following statements.

I. If n is an even natural number, then the greatest among ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ is ${}^nC_{n/2}$.

II. If n is an odd natural number, then the greatest among ${}^nC_0, {}^nC_1, {}^nC_2 \ldots, {}^nC_n$ is ${}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$.

Choose the correct option.

(a) Only I is false. (b) Only II is true.

(c) Both are true. (d) Both are false.

58. Consider the following statements.
If n is a natural number and r is non-negative integer such that $0 \le r \le n$, then

I. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

II. ${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$

Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

59. Consider the following statements.
I. The continued product of first n natural numbers is called the permutation.
II. L.C.M of 4!, 5! and 6! is 720.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

60.

Column - I	Column - II
A. $\frac{7!}{5!}$ equals	1. 28
B. $\frac{12!}{(10!)(2!)}$ equals	2. 42
C. $\frac{8!}{6! \times 2!}$ equals	3. 66

Codes:

	A	B	C
(a)	1	2	3
(b)	1	3	2
(c)	3	2	1
(d)	2	3	1

61. Using the digits 1, 2, 3, 4, 5, 6, 7, a number of 4 different digits is formed. Find :

Column - I	Column - II
A. How many numbers are formed?	1. 840
B. How many numbers are exactly divisible by 2?	2. 200
C. How many numbers are exactly divisible by 25?	3. 360
D. How many of these are exactly divisible by 4?	4. 40

Match the questions in column-I with column-II and choose the correct option from the codes given below.

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	3	1	4	2
(c)	1	3	4	2
(d)	4	2	3	1

62.

Column - I	Column -II
(A) Value of n, if $(n+2)! = 2550 \times n!$, is	(1) 5
(B) Value of n, if $(n+1)! = 12\,(n-1)!$, is	(2) 121
(C) Value of x, if $\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}$, is	(3) 2730
(D) Value of P(15, 3) is	(4) 49
(E) Value of n, if 2. P(5, 3) = P(n, 4), is	(5) 3

Codes

	A	B	C	D	E
(a)	4	5	2	3	1
(b)	1	3	5	2	4
(c)	4	2	5	3	1
(d)	1	5	3	2	4

63.

Column-I	Column-II
(A) If P(n, 4) = 20 . P(n, 2) then the value of n is	(1) 28
(B) ${}^5P_r = 2\,{}^6P_{r-1}$	(2) 4
(C) ${}^5P_r = {}^6P_{r-1}$	(3) 7
(D) Value of $\frac{8!}{6! \times 2!}$ is	(4) 3

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	3	4	1	2
(c)	4	2	3	1
(d)	3	4	2	1

64.

Column - I	Column - II
(A) If ${}^nC_8 = {}^nC_2$. Find ${}^nC_2$.	(1) 5
(B) Determine n if ${}^{2n}C_3 : {}^nC_2 = 12 : 1$	(2) 91
(C) Determine n if ${}^{2n}C_3 : {}^nC_3 = 11 : 1$	(3) 6
(D) If ${}^nC_8 = {}^nC_6$, then the value of ${}^nC_2$ is	(4) 45

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	1	3	4
(d)	2	3	1	4

65.

Column - I	Column - II
(A) If ${}^nP_r = 720$ and ${}^nC_r = 120$, then the value of 'r' is	(1) 3
(B) If ${}^{2n}C_3 : {}^nC_3 = 11 : 1$, then the value of 'n' is	(2) 4950
(C) If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then the value of 'n' is	(3) 19
(D) Value of ${}^{100}C_{98}$ is	(4) 6

Codes

	A	B	C	D
(a)	1	4	3	2
(b)	1	3	4	2
(c)	2	4	3	1
(d)	2	3	4	1

66. How many words (with or without dictionary meaning) can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

Column - I	Column - II
A. 4 letters are used at a time	1. 720
B. All letters are used at a time	2. 240
C. All letters are used but the first is a vowel	3. 360

Match the statements in column-I with column-II and choose the correct options from the codes given below.

Codes:

	A	B	C
(a)	1	2	3
(b)	3	1	2
(c)	2	1	3
(d)	3	2	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

67. If ${}^nC_9 = {}^nC_8$, what is the value of ${}^nC_{17}$?
(a) 1 (b) 0 (c) 3 (d) 17

68. If ${}^{10}C_x = {}^{10}C_{x+4}$, then the value of x is
(a) 5 (b) 4 (c) 3 (d) 2

69. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals`
(a) 5 (b) 7 (c) 6 (d) 4

70. Total number of ways of selecting five letters from letters of the word INDEPENDENT is
(a) 70 (b) 72 (c) 75 (d) 80

71. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, then the value of $x = 2^m$. The value of m is
(a) 2 (b) 4 (c) 6 (d) 5

72. Value of $\frac{n!}{(n-r)!}$ when n = 6, r = 2 is 5 m. The value of m is
(a) 2 (b) 4 (c) 6 (d) 5

73. Find n if ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$
(a) 2 (b) 6 (c) 8 (d) 9

74. Determine n if ${}^{2n}C_3 : {}^nC_2 = 12 : 1$
(a) 5 (b) 3 (c) 4 (d) 1

75. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides.
If $T_{n+1} - T_n = 21$, then n equals`
(a) 5 (b) 7 (c) 6 (d) 4

76. The number of values of r satisfying the equation ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is
(a) 1 (b) 2 (c) 3 (d) 4

77. What is the value of ${}^nP_0$?
(a) 0 (b) 1 (c) ∞ (d) $\frac{1}{2}$

78. What is the value of ${}^nC_n$?
(a) 0 (b) ∞ (c) r (d) 1

79. What is the value of ${}^nC_0$?
(a) 0 (b) ∞
(c) 1 (d) None of these

80. If ${}^nC_9 = {}^nC_8$, what is the value of ${}^nC_{17}$?
(a) 1 (b) 0 (c) 3 (d) 17

81. If ${}^{10}C_x = {}^{10}C_{x+4}$, then the value of x is
(a) 5 (b) 4 (c) 3 (d) 2

82. If the ratio ${}^{2n}C_3 : {}^nC_3$ is equal to 11 : 1, n equals
(a) 2 (b) 6 (c) 8 (d) 9

83. The number of combinations of 4 different objects A, B, C, D taken 2 at a time is
(a) 4 (b) 6 (c) 8 (d) 7

84. If ${}^{12}P_r = {}^{11}P_6 + 6.\, {}^{11}P_5$, then r is equal to:
(a) 6 (b) 5
(c) 7 (d) None of these

85. $({}^8C_1 - {}^8C_2 + {}^8C_3 - {}^8C_4 + {}^8C_5 - {}^8C_6 + {}^8C_7 - {}^8C_8)$ equals:
(a) 0 (b) 1 (c) 70 (d) 256

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

86. **Assertion :** If the letters W, I, F, E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE occurs in the 24^{th} position.
Reason : The number of ways of arranging four distinct objects taken all at a time is C(4, 4).

87. **Assertion :** A number of four different digits is formed with the help of the digits 1, 2, 3, 4, 5, 6, 7 in all possible ways. Then, number of ways which are exactly divisible by 4 is 200.
Reason : A number divisible by 4, if unit place digit is divisible by 4.

88. **Assertion :** Product of five consecutive natural numbers is divisible by 4!.
Reason : Product of n consecutive natural numbers is divisible by (n + 1)!

89. Assertion : The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^9C_3$.

Reason : The number of ways of choosing any 3 places, from 9 different places is $^9C_3$.

90. Assertion : A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 with repetition. The total number formed are 216.

Reason : If sum of digits of any number is divisible by 3 then the number must be divisible by 3.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

91. $^nC_r + 2\,^nC_{r-1} + {}^nC_{r-2}$ is equal to:

(a) $^{n+2}C_r$ (b) $^nC_{r+1}$
(c) $^{n-1}C_{r+1}$ (d) None of these

92. If ^nC_r denotes the number of combination of n things taken r at a time, then the expression $^nC_{r+1} + {}^nC_{r-1} + 2\times{}^nC_r$ equals

(a) $^{n+1}C_{r+1}$ (b) $^{n+2}C_r$
(c) $^{n+2}C_{r+1}$ (d) $^{n+1}C_r$

93. Given 12 points in a plane, no three of which are collinear. Then number of line segments can be determined, are:

(a) 76 (b) 66 (c) 60 (d) 80

94. There are 10 true-false questions in a examination. Then these questions can be answered in:

(a) 100 ways (b) 20 ways
(c) 512 ways (d) 1024 ways

95. The total number of ways of selecting six coins out of 20 one rupee coins, 10 fifty paise coins and 7 twenty five paise coins is:

(a) $^{37}C_6$ (b) 56 (c) 28 (d) 29

96. In a chess tournament where the participants were to play one game with one another, two players fell ill having played 6 games each, without playing among themselves. If the total number of games is 117, then the number of participants at the beginning was :

(a) 15 (b) 16 (c) 17 (d) 18

97. In how many ways can 10 lion and 6 tigers be arranged in a row so that no two tigers are together?

(a) $10! \times {}^{11}P_6$ (b) $10! \times {}^{10}P_6$
(c) $6! \times {}^{10}P_7$ (d) $6! \times {}^{10}P_6$

98. In how many ways can the letters of the word CORPORATION be arranged so that vowels always occupy even places ?

(a) 120 (b) 2700 (c) 720 (d) 7200

99. What is $\dfrac{(n+2)!+(n+1)!(n-1)!}{(n+1)!(n-1)!}$ equal to ?

(a) 1 (b) Always an odd integer
(c) A perfect square (d) None of these

100. The number of numbers of 9 different non-zero digits such that all the digits in the first four places are less than the digit in the middle and all the digits in the last four places are greater than the digit in the middle is

(a) 2 (4 !) (b) $(4\,!)^2$
(c) 8 ! (d) None of these

101. Number of 6 digit numbers that can be made with the digits 1, 2, 3 and 4 and having exactly two pairs of digits is

(a) 978 (b) 1801 (c) 1080 (d) 789

102. Number of 5 digit numbers that can be made using the digits 1 and 2 and in which at least one digit is different.

(a) 30 (b) 25 (c) 28 (d) 31

103. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls, Number or ways in which we can place the balls in the boxes (order is not considered in the box) so that no box remains empty is

(a) 150 (b) 160 (c) 12 (d) 19

104. In an examination, there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is

(a) 11 (b) 12 (c) 27 (d) 63

105. Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

(a) 12 (b) 13 (c) 14 (d) 15

106. Four writers must write a book containing 17 chapters. The first and third writer must write 5 chapters each, the second writer must write 4 chapters and fourth writer must write three chapters. The number of ways that can be found to divide the book between four writers, is

(a) $\dfrac{17!}{(5!)^2\,4!\,3!\,2!}$ (b) $\dfrac{17!}{5!\,4!\,3!\,2!}$

(c) $\dfrac{17!}{(5!)^2\,4!\,3!}$ (d) $\dfrac{17!}{(5!)^2\times 4\times 3}$

107. A student has to answer 10 questions, choosing at least 4 from each of parts A and B. If there are 6 questions in Part A and 7 in Part B, in how many ways can the student choose 10 questions?

(a) 266 (b) 260 (c) 256 (d) 270

108. In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?

(a) $^{52}C_{18}\,{}^{35}C_2$

(b) $^{52}C_{18}\times{}^{35}C_2 + {}^{52}C_{19}\times{}^{35}C_1 + {}^{52}C_{20}$

(c) $^{52}C_{18} + {}^{35}C_2 + {}^{52}C_{19}$

(d) $^{52}C_{18}\times{}^{35}C_2 + {}^{35}C_1\times{}^{52}C_{19}$

109. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II, unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed?
(a) 40 (b) 45 (c) 42 (d) 41

110. There were two women participants in a chess tournament. The number of games the men played between themselves exceeded by 52 the number of games they played with women. If each player played one game with each other, the number of men in the tournament, was
(a) 10 (b) 11 (c) 12 (d) 13

111. For a game in which two partners play against two other partners, six persons are available. If every possible pair must play with every other possible pair, then the total number of games played is
(a) 90 (b) 45 (c) 30 (d) 60

112. A house master in a vegetarian boarding school takes 3 children from his house to the nearby dhaba for non-vegetarian food at a time as often as he can, but he does not take the same three children more than once. He finds that he goes to the dhaba (road side hotel) 84 times more than a particular child goes with him. Then the number of children taking non-vegetarian food in his hostel, is
(a) 15 (b) 5 (c) 20 (d) 10

113. The number of circles that can be drawn out of 10 points of which 7 are collinear, is
(a) 120 (b) 113
(c) 85 (d) 86

114. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty?
(a) 50 (b) 100
(c) 150 (d) 200

115. The number of ways of dividing 52 cards amongst four players so that three players have 17 cards each and the fourth player have just one card, is
(a) $\dfrac{52!}{(17!)^3}$ (b) 52!
(c) $\dfrac{52!}{17!}$ (d) None of these

116. The number of 3 digit numbers having at least one of their digit as 5 are
(a) 250 (b) 251
(c) 252 (d) 253

117. The number of 4-digit numbers that can be formed with the digits 1, 2, 3, 4 and 5 in which at least 2 digits are identical, is
(a) 505 (b) $4^5 - 5!$
(c) 600 (d) None of these

118. If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is
(a) 324 (b) 341
(c) 359 (d) None of these

119. How many numbers lying between 999 and 10000 can be formed with the help of the digits 0, 2, 3, 6, 7, 8, when the digits are not repeated?
(a) 100 (b) 200
(c) 300 (d) 400

120. Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side of the table. The number of ways in which the seating arrangement can be done equals
(a) ${}^{11}C_4\,(9!)^2$ (b) ${}^{11}C_6\,(9!)^2$
(c) ${}^6P_0 \times {}^5P_0$ (d) None of these

121. At an election, a voter may vote for any number of candidates not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote, is
(a) 6210 (b) 385
(c) 1110 (d) 5040

122. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is
(a) 140 (b) 196
(c) 280 (d) 346

123. Ten persons, amongst whom are A, B and C to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is
(a) $\dfrac{10!}{6}$ (b) 3! 7!
(c) ${}^{10}P_3 \cdot 7!$ (d) None of these

124. A car will hold 2 in the front seat and 1 in the rear seat. If among 6 persons 2 can drive, then the number of ways in which the car can be filled is
(a) 10 (b) 20
(c) 30 (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(a)** $\frac{56!}{(50-r)!} = 30800\left(\frac{54!}{(51-r)!}\right)$

$\Rightarrow 56 \times 55 = \frac{30800}{51-r}$

$\Rightarrow 51-r = \frac{30800}{56 \times 55} \Rightarrow 51-r = 10 \Rightarrow 41 = r$

2. **(b)** Given ${}^{n+2}C_8 : {}^{n-2}P_4 = 57:16$

$\frac{{}^{n+2}C_8}{{}^{n-2}P_4} = \frac{57}{16}$ $\left[\because {}^nC_r = \frac{n!}{r!(n-r)!} \text{ and } {}^nP_r = \frac{n!}{(n-r)!}\right]$

$\Rightarrow \frac{(n+2)!}{8!(n+2-8)!} \times \frac{(n-2-4)!}{(n-2)!} = \frac{57}{16}$

$\Rightarrow \frac{(n+2)(n+1)n.(n-1)}{8.7.6.5.4.3.2.1} = \frac{57}{16}$

$\Rightarrow (n+2)(n+1)n(n-1) = 143640$

$\Rightarrow (n^2+n-2)(n^2+n) = 143640$

$\Rightarrow (n^2+n)^2 - 2(n^2+n) + 1 = 143640 + 1$

$\Rightarrow (n^2+n-1)^2 = (379)^2$

$\Rightarrow n^2+n-1 = 379$ $[\because n^2+n-1 > 0]$

$\Rightarrow n^2+n-1-379 = 0$

$\Rightarrow n^2+n-380 = 0$

$\Rightarrow (n+20)(n-19) = 0$

$\Rightarrow n = -20, n = 19$

$\because$ n is not negative.

$\therefore n = 19$

3. **(b)** Let ${}^{30}C_{r+2} = {}^{30}C_{r-2}$ ...(i)

We know, If ${}^nC_{r_1} = {}^nC_{r_2}$, then $r_1 + r_2 = n$

In above given equation (i), we have

$n = 30, r_1 = r+2, r-2 = r_2$

$\therefore r_1 + r_2 = r+2+r-2 = 2r$

and $n = 30$

$\therefore 2r = 30 \Rightarrow r = 15$

4. **(b)** The total number of numbers that can be formed with the digits 3, 4, 5, 6 taken all at a time = ${}^4P_4 = 4! = 24$. Consider the digits at the unit places in all these number. Each of the digits 3, 4, 5, 6 occurs in 3! = 6 times in unit's place. So, total of the digits at the unit places

$= (3+4+5+6)\,6 = 108$.

[Similary, the sum of the digits in the other places will also be 108]

5. **(d)** Student has 4 choices to answer the question.

$\therefore$ Total no. of ways to answer the question

$= 4 \times 4 \times 4 = 64$ $(\because$ total choices = 4)

But out of these there is only one way such that all answers are correct.

$\therefore$ Required number of ways of (student can fail to get all answers correct = 1 – 64 = 63.

6. **(a)** There are six letters in MOTHER, all different, i.e. arrangement can be made out of the letters of the word MOTHER taken four at a time with M present in every arrangement.

So, rest 3 letters can be arrangement from 5 letters

So, total number of ways = $4 \times {}^5P_3$

$= 4 \times \frac{5!}{(5-3)!} = \frac{4 \times 5 \times 4 \times 3 \times 2}{2} = 240$

7. **(b)** There are 6 balls in one over and 4 wickets are to be taken. So, 4 balls are to succeed. This can be done in ${}^6C_4$ ways.

$\Rightarrow$ Required number of ways = ${}^6C_4$

$= \frac{6!}{4!2!} = \frac{6 \times 5}{2} = 15$

8. **(c)** First chair can be occupied in 4 ways and second chair can be occupied in 3 ways, third chair can be occupied in 2 ways and last chair can be occupied in one way only. So total number of ways = $4 \times 3 \times 2 \times 1 = 24$.

9. **(b)** Selection of 2 members out of 11 has ${}^{11}C_2$ number of ways

So, ${}^{11}C_2 = 55$

10. **(a)** The required number of selections

$= {}^3C_1 \times {}^4C_1 \times {}^2C_1\,({}^6C_3 + {}^6C_2 + {}^6C_0) = 42 \times 4! = 1008$

11. **(d)** Ten pearls of one colour can be arranged in $\frac{1}{2}.(10-1)!$ ways. The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour = 10!

$\therefore$ Required number of ways = $\frac{1}{2} \times 9! \times 10! = 5\,(9!)^2$

12. **(b)** 3 consonants and 2 vowels from 5 consonants and 4 vowels can be selected in ${}^5C_3 \times {}^4C_2 = 60$ ways. But total number of words with 3 + 2 = 5 letters = 5! ways = 120

$\therefore$ The required number of words = $60 \times 120 = 7200$

13. **(c)** If number of persons = n.

Then total number of hand-shaken = ${}^nC_2 = 66$

$\Rightarrow n(n-1) = 132$

$\Rightarrow (n+11)(n-12) = 0$

$\therefore n = 12$ $(\because n \neq -11)$

14. (a) Other than 2 numbers, remaining five places are filled by 1 and 3 and for each place there is two conditions.
No. of ways for five places = $2 \times 2 \times 2 \times 2 \times 2 = 2^5$
For 2 numbers, selecting 2 places out of 7 = ${}^7C_2$
$\therefore$ Required no. of ways = ${}^7C_2 . 2^5 = 652$

15. (d) There are ten digits 0, 1, 2, ..., 9. Permutations of these digits taken eight at a time = ${}^{10}P_8$ which includes permutations having 0 at the first. When 0 is fixed at the first place, then number of such permutations = ${}^9P_7$.
So, required number

$$= {}^{10}P_8 - {}^9P_7 = \frac{10!}{2} - \frac{9!}{2} = \frac{9.\,9!}{2}$$

16. (b) There are 6 letters in the word BHARAT, 2 of them are identical. Hence total number of words = 6!/2! = 360
Number of words in which B and H come together

$$= \frac{5!2!}{2!} = 120$$

$\therefore$ The required number of words = 360 – 120 = 240

17. (b) The number of committees of 4 gentlemen = ${}^4C_4 = 1$
The number of committees of 3 gentlemen, 1 wife
$= {}^4C_3 \times {}^1C_1$
($\because$ after selecting 3 gentlemen only 1 wife is left who can be included)
The number of committees of 2 gentlemen, 2 wives
$= {}^4C_2 \times {}^2C_2$
The number of committees of 1 gentleman, 3 wives
$= {}^4C_1 \times {}^3C_3$
The number of committees of 4 wives = 1
$\therefore$ The required number of committees = 1 + 4 + 6 + 4 + 1
= 16

18. (d) The number of 4 persons including A and B = ${}^6C_2$
Considering these four as a group, number of arrangements with the other four = 5!
But in each group the number of arrangements = 2! × 2!
$\therefore$ Required number of ways = ${}^6C_2 \times 5! \times 2! \times 2! + 1 = 16$

19. (d) Required number of numbers
$= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720.$

20. (a) Alphabetical order is
A, C, H, I, N, S
No. of words starting with A = 5!
No. of words starting with C = 5!
No. of words starting with H = 5!
No. of words starting with I = 5!
No. of words starting with N = 5!
SACHIN – 1
$\therefore$ Sachin appears at serial no. 601

21. (d) First let us arrange M, I, I, I, P, P
Which can be done in $\frac{7!}{4!2!}$ ways
Now 4 S can be kept at any of the ticked places in ${}^8C_4$ ways so that no two S are adjacent.
Total required ways

$$= \frac{7!}{4!2!}\,{}^8C_4 = \frac{7!}{4!2!}\,{}^8C_4 = 7 \times {}^6C_4 \times {}^8C_4$$

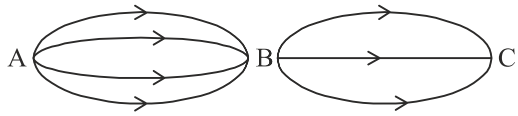
22. (c) X - X - X - X - X. The four digits 3, 3, 5,5 can be arranged at (-) places in $\frac{4!}{2!2!} = 6$ ways.
The five digits 2, 2, 8, 8, 8 can be arranged at (X) places in $\frac{5!}{2!3!}$ ways = 10 ways
Total no. of arrangements = $6 \times 10 = 60$ ways

23. (d) Two pairs of identical letters can be arranged in ${}^3C_2$ $\frac{4!}{2!2!}$ ways. Two identical letters and two different letters can be arranged in ${}^3C_1 \times {}^7C_2 \times \frac{4!}{2!}$ ways. All different letters can be arranged in ${}^8P_4$ ways
$\therefore$ Total no. of arrangements

$$= {}^3C_2\,\frac{4!}{2!2!} + {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} + \frac{8!}{4!} = 2454.$$

24. (b) These conditions are satisfied exactly when we do as follows: First paint the central triangle in any one of the three colours. Next, paint the remaining 3 triangles, with any one of the remaining two colours. By the fundamental principle of counting, this can be done in 3 × 2 × 2 × 2 = 24 ways.

25. (a) In the following figure :



There are 4 bus routes from A to B and 3 routes from B to C. Therefore, there are 4 × 3 = 12 ways to go from A to C. It is round trip so the man will travel back from C to A via B. It is restricted that man cannot use same bus routes from C to B and B to A more than once. Thus, there are 2 × 3 = 6 routes for return journey. Therefore, the required number of ways = 12 × 6 = 72.

26. (a) First, we take books of a particular subject as one unit. Thus, there are 4 units which can be arranged in 4! = 24 ways. Now, in each of the arrangements, mathematics books can be arranged in 3! ways, history books in 4! ways, chemistry books in 3! ways and biology books in 2! ways. Thus, the total number of ways = 4! × 3! × 4! × 3! × 2! = 41472.

27. (d) Number of ways of dividing 8 persons in three groups, first having 3 persons, second having 2 persons and third having 3 persons = $\frac{8!}{3!2!3!}$. Since all the 50 things are identical.

So, required number = $\frac{8!}{(3!)^2 \cdot (2!)}$

28. (c) Since, out of eleven members, two members sit together, then the number of arrangements = 9! × 2 ($\because$ two members can sit in two ways).

29. (d) Required number of such triangles
$= {}^4C_1 \times {}^5C_1 \times {}^6C_1 = 4 \times 5 \times 6$

30. (c) Given 4 vowels and 5 consonants
$\therefore$ Total number of words = $^4C_2 \times {^5C_3} \times 5!$
$= 6 \times 10 \times 120 = 7200$.

31. (b) Total number of parallelograms formed
$= {^4C_2} \times {^3C_2} = 6 \times 3 = 18$

32. (c) Each of the three prizes can be given to any of the four children.
$\therefore$ Total number of ways of distributing prizes
$= 4 \times 4 \times 4 = 64$
Number of ways in which one child gets all prizes = 4
$\therefore$ Number of ways in which no child gets all the three prizes = 64 – 4 = 60

33. (c) In the word 'RACHIT', the number of words beginning with A, C, H, I is 5! and the next word we get RACHIT.
$\therefore$ Required number of words
$= 4 \times 5! + 1 = 4 \times 120 + 1 = 481$

34. (c) Number of chords that can be drawn through 21 points on circle = Number of ways of selecting 2 points from 21 points on circle

$$= {^{21}C_2} = \frac{21 \times 20}{2 \times 1} = 210$$

35. (a) Total number of available courses = 9
Out of these, 5 courses have to be chosen. But it is given that 2 courses are compulsory for every student, i.e. you have to choose only 3 courses, out of 7.

It can be done in $^7C_3$ ways $= \frac{7 \times 6 \times 5}{6} = 35$ ways.

36. (a) There are two possibilities :

	Men	Women
(i)	2	4
(ii)	3	6

(i) Number of ways of choosing a committee of 2 men and 4 women = $^4C_2 \times {^6C_4}$

$$= \frac{4 \times 3}{2 \times 1} \times \frac{6 \times 5}{2 \times 1} = 90$$

(ii) Number of ways of choosing a committee of 3 men and 6 women = $^4C_3 \times {^6C_6}$
$= 4 \times 1 = 4$

$\therefore$ Required number of ways = 94

37. (a) Number of times he will go to garden
= Number of ways of selecting 3 children from 8 children

$$= {^8C_3} = \frac{8 \times 7 \times 6}{3 \times 2} = 56$$

38. (a) Since the man can go in 4 ways and can back in 3 ways.
Therefore, total number of ways are $4 \times 3 = 12$ ways.

39. (a) Required number of ways $= \frac{6!}{3!\,3!} = \frac{720}{6 \times 6} = 20$
[Number of heads = 3, number of tails = 3 and coins are identical]

40. (c) Required number of ways = 5! – 4! – 3!
= 120 – 24 – 6 = 90
[Number will be less than 56000 only if either 4 occurs on the first place or 5, 4 occurs on the first two places].

41. (a) The man can go in 5 ways and he can return in 5 ways. Hence, total number of ways are $5 \times 5 = 25$.

42. (b) The 4 odd digits 1, 3, 3, 1 can be arranged in the 4 odd places in $\frac{4!}{2!\,2!} = 6$ ways and 3 even digits 2, 4, 2 can be arranged in the three even places in $\frac{3!}{2!}$ = 3 ways
Hence, the required number of ways = $6 \times 3 = 18$.

43. (b) At first, we have to accommodate those 5 animals in cages which cannot enter in 4 small cages, therefore number of ways are $^6P_5$. Now, after accommodating 5 animals we left with 5 cages and 5 animals, therefore, number of ways are 5!. Hence, required number of ways = $^6P_5 \times 5! = 86400$.

44. (b) $2 \cdot {^{20}C_2}$ {Since two students can exchange cards each other in two ways}.

45. (a) The selection can be made in $^5C_3 \times {^{22}C_9}$ ways.
{Since 3 vacancies filled from 5 candidates in $^5C_3$ ways and now remaining candidates are 22 and remaining seats are 9}.

46. (a) 12 persons can be seated around a round table in 11! ways. The total number of ways in which 2 particular persons sit side by side is $10! \times 2!$. Hence, the required number of arrangements $= 11! - 10! \times 2! = 9 \times 10!$.

STATEMENT TYPE QUESTIONS

47. (c) I. Number of 3 letter words (repetition not allowed)
$= 6 \times 5 \times 4 = 120$
(as first place can be filled in 6 different ways, second place can be filled in 5 different ways and third place can be filled in 4 different ways)
II. Number of 3 letter words (repetition is allowed)
$= 6 \times 6 \times 6 = 216$
(as each of the place can be filled in 6 different ways)

48. (c) I. Number of 4 letter words that can be formed from alphabets of the word 'PART'
$= {^4P_4} = 4! = 24$
II. Number of 4 letter words that can be formed when repetition is allowed = $4^4 = 256$

49. (a) I. In n-sided polygon, the number of vertices = n
$\therefore$ Number of lines that can be formed using n points = $^nC_2$.
Out of these, $^nC_2$ lines, n lines from the polygon.
$\therefore$ Number of diagonals = $^nC_2 - n$
II. Let the number of sides of a polygon = n
Number of diagonal = Number of line segment joining any two vertices of polygon – Number of sides
$= {^nC_2} - n$

$$= \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$$

Now, $\frac{n(n-3)}{2} = 44$

$\Rightarrow n^2 - 3n - 88 = 0$
$\Rightarrow (n - 11)(n + 8) = 0$
$\Rightarrow n = 11$
or $n = -8$ rejected.

50. (c) (I) A committee consisting of 3 girls and 4 boys can be formed in ${}^4C_3 \times {}^9C_4$ ways

$= {}^4C_1 \times {}^9C_4 = \frac{4}{1} \times \frac{9\times8\times7\times6}{1.2.3.4}$ ways

= 504 ways

(II) A committee having at least 3 girls will consists of (a) 3 girls 4 boys, (b) 4 girls 3 boys

This can be done in ${}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$ ways

$= \frac{4}{1} \times \frac{9\times8\times7\times6}{1\times2\times3\times4} + 1 \times \frac{9\times8\times7}{1\times2\times3}$ ways

= 504 + 84 ways = 588 ways

51. (c) (I) ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$

$= \left[{}^nC_r + {}^nC_{r-1}\right] + \left[{}^nC_{r-1} + {}^nC_{r-2}\right]$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r.$

(II) If ${}^nC_p = {}^nC_q \Rightarrow {}^nC_p = {}^nC_{n-q}$

$\Rightarrow p = q$ or $p = n - q \left[\because {}^nC_r = {}^nC_{n-r}\right]$

52. (a) I. ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$
$= {}^{15}C_7 + {}^{15}C_6 - {}^{15}C_6 - {}^{15}C_7 = 0$

II. Total number $= 10! - 9! = 9 \times 9!$

53. (c) Both are false

I. Correct is 5^3.
($\because$ each one of the three letters can be posted in anyone of the five letter boxes.)

II. Statement will be true if m particular things always occur.

54. (c) Both are true statements.

55. (b) Both are true statements.

I. ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!\,[n-(n-r)]!}$

$= {}^nC_{n-r}$

II. ${}^nC_r + {}^nC_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$

$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$

$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right] = \frac{(n+1)!}{r!(n+1-r)!}$

56. (a) I. ${}^nP_r = {}^nP_{r+1}$

$\Rightarrow n - r = 1$ …(i)

and ${}^nC_r = {}^nC_{r-1}$

$\Rightarrow n - r + 1 = r \Rightarrow n - 2r = -1$ …(ii)

On solving (i) and (ii), we get
n = 3 and r = 2

II. Required no. of ways $= {}^{32}C_4 = \frac{32!}{4!\,28!}$

57. (c) Both are true.

58. (c) Both statements are true.

59. (b) I. The continued product of first n natural numbers is called the 'n factorial'.

II. $5! = 5 \times 4!$
$6! = 6 \times 5 \times 4!$

$\therefore$ L.C.M. of 4!, 5!, 6! = L.C.M. $[4!, 5\times4!, 6\times5\times4!]$
$= 4! \times 5 \times 6 = 6! = 720$

MATCHING TYPE QUESTIONS

60. (d) A. $\frac{7!}{5!} = \frac{7\times6\times5!}{5!} = 42$

B. $\frac{12!}{10!\,2!} = \frac{12\times11\times10!}{10!\times2\times1} = 66$

C. $\frac{8!}{6!\,2!} = \frac{8\times7\times6!}{6!\times2\times1} = 28$

61. (c) A. The number of 4 different digits $= {}^7P_4$

$= \frac{7!}{(7-4)!}$

$= 7 \times 6 \times 5 \times 4 = 840$

B. The numbers exactly divisible by 2
= Number of ways of filling first 3 places
× Number of ways of filling unit's place
$= {}^6P_3 \times 3$

$= \frac{6!}{(6-3)!} \times 3 = \frac{6!}{(3!)} \times 3$

$= 6 \times 5 \times 4 \times 3 = 360$

C. Number of 4-digit numbers divisible by 25
= Numbers ending with 25 or 75

5×4   25 or 75
= ☐☐   ☐☐

$= 5 \times 4 \times 2 = 40$

($\because$ when numbers end with 25 or 75, the other two places can be filled in 5 and 4 ways)

D. Number of 4-digit numbers divisible by 4
= Numbers ending with 12, 16, 24, 32, 36, 64, 72, 76, 52, 56

Now, number ending with 12

= ☐ ☐ ☐ ☐ = 20
4 × 5 × 1 × 1

Similarly, numbers ending with other number (16, 24, ...) = 20 each

$\therefore$ Required numbers $= 10 \times 20 = 200$

62. (a) (A) $(n+2)(n+1)\,n! = 2550 \times n!$

$\Rightarrow n^2 + 3n - 2548 = 0$
$\Rightarrow (n + 52)(n - 49) = 0$
$\Rightarrow n = 49$

(B) $(n+1)\,n\,(n-1)! = 12(n-1)!$

$\Rightarrow n^2 + n - 12 = 0 \Rightarrow (n+4)(n-3) = 0$
$\Rightarrow n = 3$

(C) $\frac{1}{9!}\left[1+\frac{1}{10}\right]=\frac{x}{11\times10}\times\frac{1}{9!}$

$\Rightarrow \frac{11}{10}=\frac{x}{11\times10}\Rightarrow x=11\times11=121$

(D) $P(15,3)=\frac{15!}{12!}=\frac{15\times14\times13\times12!}{12!}=2730$

(E) $P(n,4)=2.\,P(5,3)$

$\Rightarrow \frac{n!}{(n-4)!}=2\cdot\left[\frac{5!}{(5-3)!}\right]$

$\Rightarrow n(n-1)(n-2)(n-3)=\frac{2(5!)}{2!}$

$\Rightarrow n(n-1)(n-2)(n-3)=5\times4\times3\times2\times1$

$\Rightarrow n(n-1)(n-2)(n-3)=5\times(5-1)\times(5-2)\times(5-3)$

$\Rightarrow n=5$

63. **(d)** (A) $\frac{n!}{(n-4)!}=20\cdot\frac{n!}{(n-2)!}$

$\Rightarrow (n-2)!=20(n-4)!$

$\Rightarrow (n-2)(n-3)=5\times4$

$\Rightarrow n-3=4\Rightarrow n=7$

(B) We have,

${}^5P_r=2\,{}^6P_{r-1}$

or $\frac{5!}{(5-r)!}=2\left[\frac{6!}{(6-r+1)!}\right]$

or $\frac{5!}{(5-r)!}=2\left[\frac{6\times5!}{(7-r)!}\right]$

or $\frac{1}{(5-r)!}=\frac{12}{(7-r)(6-r)(5-r)!}$

or $(7-r)(6-r)=12$

or $42-7r-6r+r^2=12$

or $r^2-13r+30=0$

or $r^2-10r-3r+30=0$

or $r(r-10)-3(r-10)=0$

or $(r-10)(r-3)=0$

or $r=10$ or $r=3$

Hence, $r=3$

[$r=10\Rightarrow {}^5P_{10}$ which is meaningless]

(C) We have,

${}^5P_r={}^6P_{r-1}$

or $\frac{5!}{(5-r)!}=2\left[\frac{6!}{[6-(r-1)]!}\right]$

or $\frac{5!}{(5-r)!}=\frac{6\times5!}{(6-r+1)!}$

or $\frac{5!}{(5-r)!}=\frac{6\times5!}{(7-r)!}$

or $\frac{5!}{(5-r)!}=\frac{6\times5!}{(7-r)(6-r)(5-r)!}$

or $(7-r)(6-r)=6$

or $r^2-13r+36=0$

or $r=4,9$

or $r=4$

[$r=9\Rightarrow {}^5P_r$ which is meaningless]

(D) $\frac{8!}{6!\times2!}=\frac{8\times7\times6\times5\times4\times3\times2\times1}{(6\times5\times4\times3\times2\times1)\times(2\times1)}$

$=\frac{8\times7}{2\times1}=28$

64. **(b)** (A) We have

${}^nC_r={}^nC_{n-r}$

${}^nC_2={}^nC_{n-2}$

${}^nC_8={}^nC_{n-2}\Rightarrow n-2=8$ or $n=10$

${}^nC_2={}^{10}C_2=\frac{10\times9}{1\times2}=45$

(B) ${}^{2n}C_3:{}^nC_2=12:1$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{1.2.3}\div\frac{n(n-1)}{1.2}=\frac{12}{1}$

$\left[{}^nC_r=\frac{n(n-1)......(n-r+1)}{1.2.3........n}\right]$

or $\frac{2n(2n-1)2(n-1)}{6}\times\frac{2}{n(n-1)}=\frac{12}{1}$

or $\frac{4n(2n-1)(n-1)}{3}\times\frac{1}{n(n-1)}=12$

$2n-1=9,\ 2n=10$ or $n=5$

(C) ${}^{2n}C_3:{}^nC_3=11:1$

or $\frac{2n(2n-1)(2n-2)}{1.2.3}\div\frac{n(n-1)(n-2)}{1.2.3}=\frac{11}{1}$

or $\frac{4n(n-1)(2n-1)}{6}\times\frac{6}{n(n-1)(n-2)}=\frac{11}{1}$

or $\frac{4(2n-1)}{n-2}=11$

$4(2n-1)=11(n-2)$ or $8n-4=11n-22$

or $3n=18\ \therefore\ n=6$

(D) ${}^nC_8={}^nC_6\Rightarrow n=8+6=14$

$\therefore {}^nC_2={}^{14}C_2=\frac{14}{2}\times\frac{13}{1}\times{}^{12}C_0$

$=\frac{14}{2}\times\frac{13}{1}\times1=91$

65. **(a)** (A) $120=\frac{720}{r!}\Rightarrow r!=6\Rightarrow r!=3!\Rightarrow r=3$

(B) $\frac{(2n)!}{(2n-3)!}\times\frac{(n-3)!}{n!}=\frac{11}{1}$

$\frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)}=\frac{11}{1}$

$\Rightarrow \frac{4(2n-1)}{n-2}=\frac{11}{1}\Rightarrow 3n=18\Rightarrow n=6$

(C) $\frac{(n+2)!}{8!(n-6)!}\times\frac{(n-6)!}{(n-2)!}=\frac{57}{16}$

$\Rightarrow (n+2)(n+1)(n)(n-1)=\frac{57}{16}\times8!$

$\Rightarrow (n-1)n(n+1)(n+2)=18\times19\times20\times21$

$\Rightarrow n-1=18\Rightarrow n=19$

(D) $^{100}C_{98} = {}^{100}C_{100-98} = {}^{100}C_2$

$= \frac{100}{2}\times\frac{99}{1}\times {}^{98}C_0 \left(\because {}^nC_r = \frac{n}{r}\cdot {}^{n-1}C_{r-1}\right)$

$= 4950$

66. (b) A. Number of words using 4 letters out of 6 letters

$= {}^6P_4 = \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360$

B. Number of words using all letters

$= {}^6P_6 = 6! = 720$

C. Number of words starting with vowel

= Number of ways of choosing first letter (out of O and A) × Number of ways of arranging 5 alphabets

$= 2 \times 5! = 2 \times 120 = 240$

INTEGER TYPE QUESTIONS

67. (a) $\frac{n!}{9!(n-9)!} = \frac{n!}{8!(n-8)!}$

$\Rightarrow \frac{1}{9\times 8!(n-9)!} = \frac{1}{8!(n-8)(n-9)!}$

$\Rightarrow \frac{1}{9} = \frac{1}{(n-8)} \Rightarrow 9 = n-8$

$\Rightarrow 9+8 = n \Rightarrow n = 17$

$\therefore {}^nC_{17} = {}^{17}C_{17} = 1 \quad [\because {}^nC_n = 1]$

68. (c) We have $^{10}C_x = {}^{10}C_{x+4}$

$\Rightarrow x + x + 4 = 10 \Rightarrow 2x = 6 \Rightarrow x = 3$

69. (b) $^{n+1}C_3 - {}^nC_3 = 21 \quad \because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^nC_2 = 21 \Rightarrow n = 7$

70. (b) There are 11 letters in the given word which are as follows (NNN) (EEE) (DD)IPT

Five letters can be selected in the following manners :

(i) All letters different : $^6C_5 = 6$

(ii) Two similar and three different : $^3C_1 \cdot {}^5C_3 = 30$

(iii) Three similar and two different : $^2C_1 . {}^5C_2 = 20$

(iv) Three similar and two similar : $^2C_1 . {}^2C_1 = 4$

(v) Two similar, two similar and one different :

$^3C_2 . {}^4C_1 = 12$

$\therefore$ Total selections $= 6 + 30 + 20 + 4 + 12 = 72$

71. (d) $\frac{1}{6!}+\frac{1}{7!} = \frac{1}{6!}+\frac{1}{7.6!} = \frac{x}{8.7.6.!}$

$\Rightarrow \frac{1}{6!}\left(1+\frac{1}{7}\right) = \frac{x}{8.7.6!}$

$\Rightarrow \frac{8}{7} = \frac{x}{8.7} \Rightarrow x = 64$

72. (c) n = 6, r = 2

$\frac{n!}{(n-r)!} = \frac{6!}{(6-2)!} = \frac{6!}{4!} = 6\times 5$

73. (d) $\frac{^{n-1}P_3}{^nP_4} = \frac{1}{9} \Rightarrow \frac{^{n-1}P_3}{n.{}^{n-1}P_3} = \frac{1}{9}$

$\Rightarrow \frac{1}{n} = \frac{1}{9}$ or $n = 9$

74. (a) $^{2n}C_3 : {}^nC_2 = 12 : 1$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{1.2.3} \div \frac{n(n-1)}{1.2} = \frac{12}{1}$

$\left[{}^nC_r = \frac{n(n-1)......(n-r+1)}{1.2.3........n}\right]$

or $\frac{2n(2n-1)2(n-1)}{6}\times\frac{2}{n(n-1)} = \frac{12}{1}$

or $\frac{4n(2n-1)(n-1)}{3}\times\frac{1}{n(n-1)} = 12$

$2n-1 = 9, 2n = 10$ or $n = 5$

75. (b) $^{n+1}C_3 - {}^nC_3 = 21$

$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^nC_2 = 21 \Rightarrow n = 7$

76. (b) $^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$

$\Rightarrow {}^{39}C_{3r-1} + {}^{39}C_{3r} = {}^{39}C_{r^2-1} + {}^{39}C_{r^2}$

$\Rightarrow {}^{40}C_{3r} = {}^{40}C_{r^2}$

$\Rightarrow r^2 = 3r$ or $r^2 = 40 - 3r \Rightarrow r = 0, 3$ or $-8, 5$

3 and 5 are the values as the given equation is not defined by $r = 0$ and $r = -8$. Hence, the number of values of r is 2.

77. (b) $^nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$

78. (d) $^nC_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = \frac{n!}{n!} = 1$

79. (c) $^nC_0 = \frac{n!}{0!(n-0)} = \frac{n!}{0!n!} = \frac{n!}{n!} = 1$

$\therefore {}^nC_{17} = {}^{17}C_{17} = 1 \ [\because {}^nC_n = 1]$

80. (a) $\frac{n!}{9!(n-9)!} = \frac{n!}{8!(n-8)!}$

$\Rightarrow \frac{1!}{9\times 8!(n-9)!} = \frac{1!}{8!(n-8)(n-9)!}$

$\Rightarrow \frac{1}{9} = \frac{1}{(n-8)} \Rightarrow 9 = n-8$

$\Rightarrow 9+8 = n \Rightarrow n = 17$

$\therefore {}^nC_{17} = {}^{17}C_{17} = 1 \quad [\because {}^nC_n = 1]$

81. (c) We have $^{10}C_x = {}^{10}C_{x+4}$

$\Rightarrow x + x + 4 = 10 \Rightarrow 2x = 6 \Rightarrow x = 3$

82. (b) We have,

$^{2n}C_3 : {}^nC_3 = 11 : 1$

$\Rightarrow \frac{^{2n}C_3}{^nC_3} = \frac{11}{1} \Rightarrow \frac{\frac{(2n)!}{(2n-3)!3!}}{\frac{n!}{(n-3)!(3!)}} = \frac{11}{1}$

$\Rightarrow \frac{(2n)!}{(2n-3)!}\times\frac{(n-3)!}{n!} = \frac{11}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!}$

$\times\frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$

$\Rightarrow \dfrac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = \dfrac{11}{1}$

$\Rightarrow \dfrac{4(2n-1)}{n-2} = \dfrac{11}{1}$

$\Rightarrow 8n-4 = 11n-22 \Rightarrow 3n = 18 \Rightarrow n = 6$

83. (b) The combination will be AB, AC, AD, BC, BD and CD.

84. (a) Given: ${}^{12}P_r = {}^{11}P_6 + 6.{}^{11}P_5$

We know that

${}^{n-1}P_r + r\,.\,{}^{n-1}P_{r-1} = r!\,{}^nC_r$

$\therefore\ {}^{11}P_6 + 6.\ {}^{11}P_5 = 6!\ {}^{12}C_6$

$\Rightarrow\ {}^{12}P_6 = 6!\ {}^{12}C_6$

$\therefore\ \dfrac{12!}{6!} = 6!\dfrac{12!}{6!6!}$ which are equal

$\therefore\ r = 6$

85. (b) Let

$A = {}^8C_1 - {}^8C_2 + {}^8C_3 - {}^8C_4 + {}^8C_5 - {}^8C_6 + {}^8C_7 - {}^8C_8$

$= \dfrac{8!}{1!7!} - \dfrac{8!}{2!6!} + \dfrac{8!}{3!5!} - \dfrac{8!}{4!4!} + \dfrac{8!}{5!3!} - \dfrac{8!}{6!2!} + \dfrac{8!}{7!1!} - \dfrac{8!}{0!8!}$

Note: ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

Thus,

$A = 8 - \dfrac{8\times7}{2} + \dfrac{8\times7\times6}{3\times2} - \dfrac{8\times7\times6\times5}{4\times3\times2\times1}$

$+ \dfrac{8\times7\times6}{3\times2} - \dfrac{8\times7}{2} + 8 - 1$

And $A = 8 - 28 + 56 - 70 + 56 - 28 + 8 - 1 = 1$

ASSERTION - REASON TYPE QUESTIONS

86. (c) Number of ways of arranging four distinct objects in a line is ${}^4P_4 = 4! = 24$.

Hence, Statement II is false.

Again, when W, I, F, E are arranged in all possible ways, then number of words formed is 4! = 24 and WIFE occurs last of all as its letters are against alphabetical order.

87. (c) For the number exactly divisible by 4, then last two digits must be divisible by 4, the last two digits are viz.

12, 16, 24, 32, 36, 52, 56, 64, 72, 76

Total 10 ways. Now, the remaining two first places on the left of 4-digit numbers are to be filled from the remaining 5-digits and this can be done in ${}^5P_2$ = 20 ways.

$\therefore$ Required number of ways = 20 × 10 = 200.

88. (c) Product of n consecutive natural numbers

= (m + 1) (m + 2) (m + 3) ..... (m + n), m ∈ whole number

$= \dfrac{(m+n)!}{m!} = n! \times \dfrac{(m+n)!}{m!\,n!}$

$= n! \times {}^{m+n}C_m$

$\Rightarrow$ Product is divisible by n!, then it is always divisible by (n – 1)! but not by (n + 1)!

89. (a) Let the number of ways of distributing n identical objects among r persons such that each person gets at least one object is same as the number of ways of selecting (r – 1) places out of (n – 1) different places, i.e. ${}^{n-1}C_{r-1}$.

90. (d) Number form by using 1, 2, 3, 4, 5 = 5! = 120

Number formed by using 0, 1, 2, 4, 5

4	4	3	2	1

= 4.4.3.2.1 = 96

Total number formed, divisible by 3 (taking numbers without repetition) = 216

Statement 1 is false and statement 2 is true.

CRITICALTHINKING TYPE QUESTIONS

91. (a) Let $A = {}^nC_r + 2{}^nC_{r-1} + {}^nC_{r-2}$

$= \dfrac{n!}{r!(n-r)!} + \dfrac{2n!}{(r-1)!(n-r+1)!} + \dfrac{n!}{(r-2)!(n-r+2)!}$

$= \dfrac{n![(n-r+2.n-r+1)+2(n-r+2)r+r(r-1)]}{r!(n-r+2)!}$

$= \dfrac{\begin{bmatrix} n![(n^2-nr+n-nr+r^2-r+2n-2r+2 \\ +2nr-2r^2+4r+r^2-r] \end{bmatrix}}{r!(n-r+2)!}$

$= \dfrac{(n^2+3n+2)n!}{r!(n-r+2)!} = \dfrac{(n+1)(n+2)n!}{r!(n-r+2)!}$

$= \dfrac{(n+2)!}{r!(n+2-r)!} = {}^{n+2}C_r.$

92. (c) ${}^nC_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r$

$= {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1}$

$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

93. (b) To find number of line segment we will have to draw the line segments joining two points. If n is the number of such lines segments, then

$n = {}^{12}C_2 = \dfrac{12!}{2!(12-2)!} = \dfrac{12\times11\times10!}{2\times10!} = 66.$

94. (d) There are 10 questions with options of false/ true. It means each question has two options. Thus the number of ways that these questions can be answered

$= 2^{10} = 1024$ ways.

95. (c) Since we know that the total number of selections of r things from n things where each thing can be repeated as many times as one can, is ${}^{n+r-1}C_r$

Here r = 6 ($\because$ we have to select 6 coins)

and n = 3 ($\because$ it is repeated 3 times)

$\therefore$ Required number $= {}^{3+6-1}C_6 = 28$

96. (a) Let the no. of participants at the beginning was n.

Now, we have 6 games and each participant will play 2 games.

$\therefore$ Total no. of games played by 2 persons

$= 6 \times 2 = 12$

Since, two players fell ill having played 6 games each, without playing among them selves and total no. of games = 117

$\therefore \frac{n(n-1)}{2} = 117 - 12$

$\Rightarrow n(n-1) = 2(105) = 210$

$\Rightarrow n^2 - n - 210 = 0$

$\Rightarrow n^2 - 15n + 14n - 210 = 0$

$\Rightarrow n(n-15) + 14(n-15) = 0$

$\Rightarrow n = -14, 15$

But no. of participants can not be –ve

$\therefore n = 15$.

97. (a) There are 10 lions and there is no restrictions on arranging lions. They can be arranged in 10! ways. But there is a restriction in arrangements of tigers that no two tigers come together. So two tiger are to be arranged on the either side of a lion. This gives 11 places for tigers and there are 6 tigers. So, tigers can be arranged in $^{11}P_6$ ways.
So, total arrangemns are $10! \times {}^{11}P_6$

98. (d) In the word CORPORATION, there are 11 positions, there are 3 vowels O, A and I and they can occupy even places only (2nd 4th, 6th, 8th and 10th positions), total 5 positions : This can be done in $^5C_3$ ways.
There are remaining 6 positions for odd numbered places (i.e. 1, 3, 5, 7, 9, 11) and these are be occupied by 5 consonants, namely, C, R, P, T, N.
This can be done in $^6C_5$ ways.
Total number of ways $= {}^5C_3 \times {}^6C_5 = 7200$

99. (c) Given expression is :

$$\frac{(n+2)! + (n+1)!(n-1)!}{(n+1)!(n-1)!} = x \text{ (let)}$$

$$\Rightarrow x = \frac{(n+2)(n+1)n(n-1)! + (n+1)(n-1)!}{(n+1)(n-1)!}$$

$= (n+2)n + 1 = n^2 + 2n + 1 = (n+1)^2$

Which is a perfect square.

100. (b) $x_1\, x_2\, x_3\, x_4\, \textcircled{x_5}\, x_6\, x_7\, x_8\, x_9$. Under the given situation x_5 can be 5 only. The selection for x_1, x_2, x_3, x_4 must be from 1, 2, 3, 4, so they can be arranged 4 ! ways. Again the selection of x_6, x_7, x_8, x_9 must be from 6, 7, 8, 9 so they can be arranged in 4! ways.
Desired number of ways $= (4!)\,(4!) = (4!)^2$

101. (c) The number will have 2 pairs and 2 different digits.
The number of selections $= {}^4C_2 \times {}^2C_2$, and for each selection, number of arrangements $= \frac{6!}{2!2!}$.

Thus, the required number $= {}^4C_2 \times {}^2C_2 \times \frac{6!}{2!2!} = 1080$

102. (a) Total number of numbers without restriction $= 2^5$
Two numbers have all the digits equal. So,
The required number $= 2^5 - 2 = 30$

103. (a) One possible arrangement = | 2 | 2 | 1 |
Three such arrangements are possible. Therefore, the number of ways $= ({}^5C_2)({}^3C_2)({}^1C_1)(3) = 90$
The other possible arrangements = | 1 | 1 | 3 |
Three such arrangements are possible.
Thus, the number of ways $= ({}^5C_1)({}^4C_1)({}^3C_3)(3) = 60$
Hence, the total number of ways = 90 + 60 = 150.

104. (d) There are three multiple choice questions, each has four possible answers. Therefore, the total number of possible answers will be $4 \times 4 \times 4 = 64$. Out of these, possible answers only one will be correct and hence the number of ways in which a student can fail to get all correct answers is $64 - 1 = 63$.

105. (a) There will be as many signals as there are ways of filling in 2 vacant places in succession by the 4 flags of different colours. The upper vacant place can be filled in 4 different ways by anyone of the 4 flags; following which, the lower vacant place can be filled in 3 different ways by anyone of the remaining 3 different flags. Hence, by the multiplication principle, the required number of signals $= 4 \times 3 = 12$.

106. (c) Evidently, (c) is correct option because we have to divide 17 into four groups each distinguishable into groups of 5, 5, 4 and 3.

107. (a) The possibilities are:
4 from Part A and 6 from Part B
or 5 from Part A and 5 from Part B
or 6 from Part A and 4 from Part B
Therefore, the required number of ways is
$= {}^6C_4 \times {}^7C_6 + {}^6C_5 \times {}^7C_5 + {}^6C_6 \times {}^7C_4$
$= 105 + 126 + 35 = 266$.

108. (b) The following are the number of possible choices:
$^{52}C_{18} \times {}^{35}C_2$ (18 families having atmost 2 children and 2 selected from other type of families)
$^{52}C_{19} \times {}^{35}C_1$ (19 families having atmost 2 children and 1 selected from other type of families)
$^{52}C_{20}$ (All selected 20 families having atmost 2 children). Hence, the total number of possible choices is : $= {}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$

109. (d) Let us make the following cases :
Case I : Boy borrows Mathematics Part II, then he borrows Mathematics Part I also. So, the number of possible choices is $^6C_1 = 6$.
Case II : Boy does not borrow Mathematics Part II, then the number of possible choices is $^7C_3 = 35$.
Hence, the total number of possible choices is $= 35 + 6 = 41$.

110. (d) Let there were n men playing in the tournament with 2 women. According to the given condition,
$^nC_2 - {}^nC_1 \times {}^2C_1 = 52$

$\Rightarrow \frac{n(n-1)}{2} - 2n = 52$

$\Rightarrow n^2 - n - 4n = 104$

$\Rightarrow n^2 - 5n - 104 = 0$

$\Rightarrow n = 13$.

111. (b) For one game four persons are required.
This can be done in $^6C_4 = 15$ ways.
Once a set of 4 persons are selected, number of games possible will be $\frac{{}^4C_2}{2} = 3$ games.

$\therefore$ Total number of possible games $= 3 \times 15 = 45$.

112. (d) The number of times the house master goes to dhaba is ${}^nC_3$. Let n be the number of children taking non-vegetarian food.

Now, ${}^nC_3 - {}^{n-1}C_2 = 84$

$\Rightarrow \dfrac{n(n-1)(n-2)}{6} - \dfrac{(n-1)(n-2)}{2} = 84$

$\Rightarrow (n-1)(n-2)\left[\dfrac{n}{6} - \dfrac{1}{2}\right] = 84$

$\Rightarrow (n-1)(n-2)(n-3) = 6 \times 6 \times 14$

$\Rightarrow (n-1)(n-2)(n-3) = 3 \times 2 \times 3 \times 2 \times 7 \times 2 = 7 \times 8 \times 9$

$\Rightarrow (n-1) = 9 \Rightarrow n = 10.$

113. (c) Required number

$= {}^3C_3 + {}^3C_2 \times {}^7C_1 + {}^7C_2 \times {}^3C_1$

$= 1 + 3 \times 7 + 21 \times 3 = 1 + 21 + 63 = 85.$

114. (c) Let the boxes be marked as A, B and C. We have to ensure that no box remains empty and all five balls have to put in. There will be two possibilities :

(i) Any two box containing one ball each and 3rd box containing 3 balls. Number of ways

$= A(1)\ B(1)\ C(3)$

$= {}^5C_1 \cdot {}^4C_1 \cdot {}^3C_3 = 5 \cdot 4 \cdot 1 = 20$

(ii) Any two box containing 2 balls each and third containing 1 ball, the number of ways

$= A(2)\ B(2)\ C(1) = {}^5C_2 \cdot {}^3C_2 \cdot {}^1C_1$

$= 10 \times 3 \times 1 = 30$

Since, the box containing 1 ball could be any of the three boxes A, B, C. Hence, the required number of ways = 30 × 3 = 90.

Hence, total number of ways = 60 + 90 = 150.

115. (a) For the first player, distribute the cards in ${}^{52}C_{17}$ ways. Now, out of 35 cards left, 17 cards can be put for second player in ${}^{35}C_{17}$ ways. Similarly, for third player put them in ${}^{18}C_{17}$ ways. One card for the last player can be put in ${}^1C_1$ way. Therefore, the required number of ways for the proper distribution

$= {}^{52}C_{17} \times {}^{35}C_{17} \times {}^{18}C_{17} \times {}^1C_1$

$= \dfrac{52!}{35!17!} \times \dfrac{35!}{18!17!} \times \dfrac{18!}{17!1!} \times 1! = \dfrac{52!}{(17!)^3}.$

116. (c) Total number of 3-digit numbers having at least one of their digits as 5 = Total number of 3-digit numbers – (Total number of 3-digit numbers in which 5 does not appear at all)

$= 9 \times 10 \times 10 - 8 \times 9 \times 9$

$= 900 - 648 = 252$

117. (a) Total number of 4-digit numbers = 5 × 5 × 5 × 5 = 625 (as each place can be filled by anyone of the numbers 1, 2, 3, 4 and 5)

Numbers in which no two digits are identical

= 5 × 4 × 3 × 2 = 120 (i.e. repetition not allowed)

(as 1st place can be filled in 5 different ways, 2nd place can be filled in 4 different ways and so on)

Number of 4-digits numbers in which at least 2 digits are identical = 625 – 120 = 505

118. (a) The number of words starting from A are 5! = 120

The number of words starting from I are 5! = 120

The number of words starting from KA are 4! = 24

The number of words starting from KI are 4! = 24

The number of words starting from KN are 4! = 24

The number of words starting from KRA are 3! = 6

The number of words starting from KRIA are 2! = 2

The number of words starting from KRIN are 2! = 2

The number of words starting from KRISA are 1! = 1

The number of words starting from KRISNA are 1! = 1

Hence, rank of word 'KRISNA

= 2(120) + 3(24) + 6 + 2(2) + 2(1) = 324

119. (c) The numbers between 999 and 10000 are all 4-digit numbers. The number of 4-digit numbers formed by digits 0, 2, 3, 6, 7, 8 is ${}^6P_4 = 360$.

But here those numbers are also involved which begin from 0. So, we take those numbers as three-digit numbers.

Taking initial digit 0, the number of ways to fill remaining 3 places from five digits 2, 3, 6, 7, 8 are ${}^5P_3 = 60$

So, the required numbers = 360 – 60 = 300.

120. (b) After sending 4 to one side and 3 to other side. We have to select 5 for one side and 6 for other side from remaining.

This can be done in ${}^{11}C_5 \times {}^6C_6$ ways $= {}^{11}C_5$

Now, there are 9 on each side of the long table and each can be arranged in 9! ways.

∴ Required number of ways $= {}^{11}C_5 \times 9! \times 9!$

$= {}^{11}C_6 \times (9!)^2$ $\quad [\because {}^nC_r = {}^nC_{n-r}]$

121. (b) Total number of ways

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 10 + 45 + 120 + 210 = 385$

122. (b) The number of choices available to him

$= {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$

$= \dfrac{5!}{4!\,1!} \times \dfrac{8!}{6!\,2!} + \dfrac{5!}{5!\,0!} \times \dfrac{8!}{5!\,3!}$

$= 5 \times \dfrac{8 \times 7}{2} + 1 \times \dfrac{8 \times 7 \times 6}{3 \times 2}$

$= 5 \times 4 \times 7 + 8 \times 7$

$= 140 + 56 = 196$

123. (a) For A, B, C to speak in order of alphabets, 3 places out of 10 may be chosen first in ${}^{10}C_3$ ways.

The remaining 7 persons can speak in 7! ways. Hence, the number of ways in which all the 10 persons can speak is ${}^{10}C_3 \cdot 7! = \dfrac{10!}{3!} = \dfrac{10!}{6}.$

124. (d) Since 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in ${}^2C_1$ ways. Now from the remaining 5 persons we have to select 2 which can be done in ${}^5C_2$ ways. But the front seat and the rear seat person can interchange among themselves. Therefore, the required number of ways in which the car can be filled is ${}^5C_2 \times {}^2C_1 \times 2!$ $= 20 \times 2 = 40.$

CHAPTER 8
BINOMIAL THEOREM

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. How many terms are present in the expansion of $\left(x^2+\frac{2}{x^2}\right)^{11}$?
(a) 11 (b) 12 (c) 10 (d) 11!

2. The total number of terms in the expansion of $(x+a)^{51}-(x-a)^{51}$ after simplification is
(a) 102 (b) 25 (c) 26 (d) None of these

3. The term independent of x in the expansion of $\left(2x+\frac{1}{3x^2}\right)^9$ is
(a) 2^{nd} (b) 3^{rd} (c) 4^{th} (d) 5^{th}

4. In the expansion of $\left(\sqrt[3]{\frac{x}{3}}-\sqrt{\frac{3}{x}}\right)^{10}$, $x>0$, the constant term is
(a) –70 (b) 70 (c) 210 (d) –210

5. The coefficient of x^{-12} in the expansion of $\left(x+\frac{y}{x^3}\right)^{20}$ is
(a) $^{20}C_8$ (b) $^{20}C_8\,y^8$ (c) $^{20}C_{12}$ (d) $^{20}C_{12}\,y^{12}$

6. In the binomial expansion of $(a-b)^n$, $n\geq 5$ the sum of the 5th and 6th terms is zero. Then a/b equals :
(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$ (c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

7. If the coefficients of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal, then n is
(a) 56 (b) 55 (c) 45 (d) 15

8. The coefficient of the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is
(a) 5/4 (b) 7/4 (c) 9/4 (d) None of these

9. The coefficient of x^p and x^q (p and q are positive integers) in the expansion of $(1+x)^{p+q}$ are
(a) equal
(b) equal with opposite signs
(c) reciprocal of each other
(d) None of these

10. If t_r is the rth term in the expansion of $(1+x)^{101}$, then the ratio $\frac{t_{20}}{t_{19}}$ equal to
(a) $\frac{20x}{19}$ (b) $83x$ (c) $19x$ (d) $\frac{83x}{19}$

11. r and n are positive integers $r>1$, $n>2$ and coefficient of $(r+2)^{th}$ term and $3r^{th}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals
(a) $3r$ (b) $3r+1$ (c) $2r$ (d) $2r+1$

12. In the expansion of $\left(x+\frac{2}{x^2}\right)^{15}$, the term independent of x is :
(a) $^{15}C_6.26$ (b) $^{15}C_5.2^5$
(c) $^{15}C_4.2^4$ (d) None of these

13. The formula
$$(a+b)^m = a^m + ma^{m-1}b + \frac{m(m-1)}{1.2}a^{m-2}b^2 + \ldots \text{ holds when}$$
(a) $b<a$ (b) $a<b$
(c) $|a|<|b|$ (d) $|b|<|a|$

14. $\frac{1}{\sqrt{5+4x}}$ can be expanded by binomial theorem, if
(a) $x<1$ (b) $|x|<1$
(c) $|x|<\frac{5}{4}$ (d) $|x|<\frac{4}{5}$

15. The expansion of $\frac{1}{(4-3x)^{1/2}}$ by binomial theorem will be valid, if
(a) $x<1$ (b) $|x|<1$
(c) $-\frac{2}{\sqrt{3}}<x<\frac{2}{\sqrt{3}}$ (d) None of these

16. If the coefficients of 2^{nd}, 3^{rd} and the 4^{th} terms in the expansion of $(1+x)^n$ are in A.P., then value of n is
(a) 3 (b) 7 (c) 11 (d) 14

17. If in the binomial expansion of $(1+x)^n$ where n is a natural number, the coefficients of the 5th, 6th and 7th terms are in A.P., then n is equal to:
(a) 7 or 13 (b) 7 or 14 (c) 7 or 15 (d) 7 or 17

18. The coefficient of the middle term in the expansion of $(2+3x)^4$ is :
(a) 6 (b) 5! (c) 8! (d) 216

19. If the r^{th} term in the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4, then r is equal to
(a) 2 (b) 3 (c) 4 (d) 5

20. What is the middle term in the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$?
(a) $C(12, 7)\, x^3 y^{-3}$ (b) $C(12, 6)\, x^{-3} y^3$
(c) $C(12, 7)\, x^{-3} y^3$ (d) $C(12, 6)\, x^3 y^{-3}$

21. If x^4 occurs in the rth term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then what is the value of r ?
(a) 4 (b) 8 (c) 9 (d) 10

22. What is the coefficient of x^3y^4 in $(2x+3y^2)^5$?
(a) 240 (b) 360 (c) 720 (d) 1080

23. If the coefficient of x^7 in $\left[ax^2+\frac{1}{bx}\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation
(a) $a-b=1$ (b) $a+b=1$ (c) $\frac{a}{b}=1$ (d) $ab=1$

24. If A and B are coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ then :
(a) A = B (b) 2A = B (c) A = 2B (d) AB = 2

25. What is the coefficient of x^3 in $\frac{(3-2x)}{(1+3x)^3}$?
(a) –272 (b) –540 (c) –870 (d) –918

26. If 'n' is positive integer and three consecutive coefficient in the expansion of $(1+x)^n$ are in the ratio 6 : 33 : 110, then n is equal to :
(a) 9 (b) 6 (c) 12 (d) 16

27. $\sqrt{5}\,[(\sqrt{5}+1)^{50}-(\sqrt{5}-1)^{50}]$ is
(a) an irrational number (b) 0
(c) a natural number (d) None of these

28. The number of term in the expansion of $[(x+4y)^3\,(x-4y)^3]^2$ is
(a) 6 (b) 7 (c) 8 (d) 32

29. The term independent of x in the expansion of $\left(\sqrt[6]{x}-\frac{1}{\sqrt[3]{x}}\right)^9$ is
(a) $-{}^9C_3$ (b) $-{}^9C_4$ (c) $-{}^9C_5$ (d) $-{}^8C_3$

30. If the coefficients of r^{th} and $(r+1)^{th}$ terms in the expansion of $(3+7x)^{29}$ are equal, then the value of r is
(a) 31 (b) 11 (c) 18 (d) 21

31. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is
(a) 1594 (b) 792 (c) 924 (d) 2924

32. The coefficient of x^{-7} in the expansion of $\left[ax-\frac{1}{bx^2}\right]^{11}$ will be :
(a) $\frac{462}{b^5}a^6$ (b) $\frac{462a^5}{b^6}$ (c) $\frac{-462a^5}{b^6}$ (d) $\frac{-462a^6}{b^5}$

33. The coefficient of x^3 in the expansion of $\left(x-\frac{1}{x}\right)^7$ is :
(a) 14 (b) 21 (c) 28 (d) 35

34. Find the largest coefficient in the expansion of $(4+3x)^{25}$.
(a) $(3)^{25}\times{}^{25}C_{10}\left(\frac{4}{3}\right)^{11}$ (b) $20\times{}^{25}C_{11}\left(\frac{4}{3}\right)^{14}$
(c) $(2)^8\times{}^{25}C_{11}\left(\frac{5}{2}\right)^{11}$ (d) $(4)^{25}\times{}^{25}C_{11}\times\left(\frac{3}{4}\right)^{11}$

35. If $(1+x)^n = C_0+C_1x+C_2x^2+.....+C_nx^n$, then value of $\frac{(C_0+C_1)(C_1+C_2).....(C_{n-1}+C_n)}{C_0C_1C_2.....C_{n-1}}$ is
(a) $\frac{(n+3)^3}{(2n)!}$ (b) $\frac{(n+1)^n}{n!}$ (c) $\frac{(2n)!}{(n+1)!}$ (d) $\frac{(n-1)^n}{n!}$

36. Notation form of $(a+b)^n$ is
(a) $\sum_{k=0}^{n}{}^nC_k a^{n+k} b^k$ (b) $\sum_{k=0}^{n}{}^nC_k a^{n-k} b^k$
(c) $\sum_{k=0}^{n}{}^nC_k b^{n+k} a^k$ (d) None of these

37. In every term, the sum of indices of a and b in the expansion of $(a+b)^n$ is
(a) n (b) n + 1 (c) n + 2 (d) n – 1

38. The approximation of $(0.99)^5$ using the first three terms of its expansion is
(a) 0.851 (b) 0.751 (c) 0.951 (d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

39. The largest term in the expansion of $(3+2x)^{50}$, where $x=\frac{1}{5}$, is
I. 5^{th} II. 3^{rd}
III. 7^{th} IV. 6^{th}
Choose the correct option
(a) Only I (b) Only II
(c) Both I and IV (d) Both III and IV

40. Consider the following statements.
I. Coefficient of x^r in the binomial expansion of $(1+x)^n$ is ${}^nC_r$.
II. Coefficient of $(r+1)^{th}$ term in the binomial expansion of $(1+x)^n$ is ${}^nC_r$.
Choose the correct option.
(a) Only I is correct (b) Only II is correct
(c) Both are correct. (d) Both are incorrect.

41. Consider the following statements.

I. General term of the expansion of $(x+y)^n$ is ${}^nC_r x^{n-r} y^r$.

II. The coefficients ${}^nC_r$ occuring in the binomial theorem are known as binomial coefficients.

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

42. Consider the following statements.

I. General term in the expansion of $(x^2-y)^6$ is $(-1)^r x^{12-2r} \cdot y^r$

II. 4^{th} term in the expansion of $(x-2y)^{12}$ is $-1760x^9y^3$.

Choose the correct option.

(a) Only I is false (b) Only II is false
(c) Both are false (d) Both are true

43. Consider the following statements.

Binomial expansion of $(x+a)^n$ contains $(n+1)$ terms.

I. If n is even, then $\left(\frac{n}{2}+1\right)$th term is the middle term.

II. If n is odd, then $\left(\frac{n+1}{2}\right)$th is the middle term.

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

44. Consider the following statements.

I. The number of terms in the expansion of $(x+a)^n$ is $n+1$.

II. The binomial expansion is briefly written as

$$\sum_{r=0}^{n} {}^nC_r x^{n-r}.a^r$$

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

45.

Column I (Expression)	Column II (Expansion)
A. $(1-2x)^5$	1. $\frac{x^5}{243}+\frac{5}{81}.x^3+\frac{10}{27}.x+\frac{10}{9}.\frac{1}{x}+\frac{5}{3}.\frac{1}{x^3}+\frac{1}{x^5}$
B. $\left(\frac{2}{x}-\frac{x}{2}\right)^5$	2. $1-10x+40x^2-80x^3+80x^4-32x^5$
C. $(2x-3)^6$	3. $32x^{-5}-40x^{-3}+20x^{-1}-5x+\frac{5}{8}x^3-\frac{1}{32}x^5$
D. $\left(\frac{x}{3}+\frac{1}{x}\right)^5$	4. $64x^6-576x^5+2160x^4-4320x^3+4860x^2-2916x+729$

Codes

	A	B	C	D
(a)	2	4	3	1
(b)	2	3	4	1
(c)	1	3	4	2
(d)	1	4	3	2

46. Using Binomial Theorem, evaluate expression given in column-I and match with column-II.

Column I	Column II
A. $(96)^3$	1. 104060401
B. $(102)^5$	2. 9509900499
C. $(101)^4$	3. 11040808032
D. $(99)^5$	4. 884736

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	1	3	4
(d)	2	3	1	4

47.

Column-I	Column-II
A. Coefficient of x^5 in $(x+3)^8$ is	1. 18564
B. Coefficient of a^5b^7 in $(a-2b)^{12}$ is	2. $61236\, x^5y^5$
C. 13^{th} term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$, is	3. 1512
D. Middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$, is	4. -101376

Codes

	A	B	C	D
(a)	3	1	4	2
(b)	2	1	4	3
(c)	2	4	1	3
(d)	3	4	1	2

48.

Column-I	Column-II
A. Term independent of x in the expansion of $\left(x^2+\frac{1}{x}\right)^9$ is	1. 6^{th} term
B. Term independent of x in the expansion of $\left(x^2+\frac{1}{2x}\right)^{12}$ is	2. 10^{th} term
C. Term independent of x in the expansion of $\left(2x-\frac{1}{x}\right)^{10}$ is	3. 9^{th} term
D. Term independent of x in the expansion of $\left(x^3+\frac{3}{x^2}\right)^{15}$ is	4. 7^{th} term

Codes

	A	B	C	D
(a)	2	1	3	4
(b)	4	3	1	2
(c)	4	1	2	3
(d)	3	2	1	4

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

49. If the second, third and fourth terms in the expansion of $(a+b)^n$ are 135, 30 and 10/3 respectively, then the value of n is
(a) 6 (b) 5 (c) 4 (d) None of these

50. Coefficient of x^{13} in the expansion of $(1-x)^5(1+x+x^2+x^3)^4$ is
(a) 4 (b) 6 (c) 32 (d) 5

51. If x^4 occurs in the t^{th} term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then the value of t is equal to :
(a) 7 (b) 8 (c) 9 (d) 10

52. In the expansion of $(1+x)^{18}$, if the coefficients of $(2r+4)^{th}$ and $(r-2)^{th}$ terms are equal, then the value of r is :
(a) 12 (b) 10 (c) 8 (d) 6

53. A positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6, is
(a) 3 (b) 4 (c) 0 (d) None of these

54. If the coefficients of 2^{nd}, 3^{rd} and the 4^{th} terms in the expansion of $(1+x)^n$ are in A.P., then value of n is
(a) 3 (b) 7 (c) 11 (d) 14

55. If the coefficient of x in $(x^2+k/x)^5$ is 270, then the value of k is
(a) 2 (b) 3 (c) 4 (d) 5

56. If the r^{th} term in the expansion of
$$\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$$ contains x^4, then the value of r is
(a) 2 (b) 3 (c) 4 (d) 5

57. The number of zero terms in the expansion of $(1+3\sqrt{2}x)^9+(1-3\sqrt{2}x)^9$ is
(a) 2 (b) 3 (c) 4 (d) 5

58. Number of terms in the expansion of $\left(1+5\sqrt{2}x\right)^9+\left(1-5\sqrt{2}x\right)^9$ is
(a) 2 (b) 3 (c) 4 (d) 5

59. Value of 'a', if 17^{th} and 18^{th} terms in the expansion of $(2+a)^{50}$ are equal, is
(a) 1 (b) 2 (c) 3 (d) 4

60. One value of α for which the coefficients of the middle terms in the expansion of $(1+\alpha x)^4$ and $(1-\alpha x)^6$ are equal, is $\frac{-3}{10}$. Other value of 'α' is
(a) 0 (b) 1 (c) 2 (d) 3

61. Number of terms involving x^6 in the expansion of $\left(2x^2-\frac{3}{x}\right)^{11}$, $r \neq 0$, is
(a) 1 (b) 2 (c) 6 (d) 0

62. If the fourth term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then the value of $a \times n$ is
(a) 2 (b) 6 (c) 3 (d) 4

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

63. **Assertion :** The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^m$ is $\frac{(4m)!}{(2m!)^2}$.
Reason : The coefficient of x^6 in the expansion of $(1+x)^n$ is ${}^nC_6$.

64. **Assertion :** If $(1+ax)^n = 1+8x+24x^2+...$, then the values of a and n are 2 and 4 respectively.
Reason : $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+...$ for all $n \in Z^+$.

65. **Assertion :** If $a_n = \sum_{r=0}^{n}\frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n}\frac{r}{{}^nC_r}$ is equal to $\frac{n}{2}a_n$.
Reason : ${}^nC_r = {}^nC_{n-r}$.

66. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r$, then
Assertion : $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)...\left(1+\frac{C_n}{C_{n-1}}\right)=\frac{(n+1)^n}{n!}$
Reason : ${}^nC_r = \frac{n(n-1)...(n-r+1)}{r(r-1)...1}$

67. **Assertion :** The r^{th} term from the end in the expansion of $(x+a)^n$ is ${}^nC_{n-r+1}\,x^{r-1}a^{n-r+1}$.
Reason : The r^{th} term from the end in the expansion of $(x+a)^n$ is $(n-r+2)^{th}$ term.

68. **Assertion :** In the expansion of $(x+2y)^8$, the middle term is 4^{th} term.
Reason : If n is even in the expansion of $(a+b)^n$, then $\left(\frac{n}{2}+1\right)^{th}$ term is the middle term.

69. **Assertion :** ${}^nC_0+{}^nC_2+{}^nC_4+..... = 2^{n-1}$
Reason : ${}^nC_1+{}^nC_3+{}^nC_5+...... = 2^{n-1}$

70. **Assertion:** Number of terms in the expansion of $[(3x+y)^8-(3x-y)^8]$ is 4.
Reason: If n is even, then $\{(x+a)^n-(x-a)^n\}$ has $\frac{n}{2}$ terms.

71. **Assertion:** Number of terms in the expansion of

$\left(\sqrt{x}+\sqrt{y}\right)^{10}+\left(\sqrt{x}-\sqrt{y}\right)^{10}$ is 6.

Reason: If n is even, then the expansion of

$\{(x+a)^n+(x-a)^n\}$ has $\left(\frac{n}{2}+1\right)$ terms.

72. **Assertion:** General term of the expansion $(x + 2y)^9$ is $^9C_r.2^r.x^{9-r}.y^r$.

Reason: General term of the expansion $(x+a)^n$ is given by $T_{r+1} = {}^nC_r\, x^{n-r}.a^r$

73. **Assertion.** The coefficients of the expansions are arranged in an array. This array is called Pascal's triangle.

Reason: There are 11th terms in the expansion of $(4x+7y)^{10} + (4x-7y)^{10}$.

74. **Assertion.** In the binomial expansion $(a + b)^n$, r^{th} term is $^nC_r.a^{n-r}.b^r$.

Reason. If n is odd, then there are two middle terms.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

75. After simplification, what is the number of terms in the expansion of $[(3x+y)^5]^4-[(3x-y)^4]^5$?

(a) 4 (b) 5 (c) 10 (d) 11

76. The term independent of x in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x>0$, is 'a' times the corresponding binomial coefficient. Then 'a' is

(a) 3 (b) 1/3 (c) –1/3 (d) None of these

77. The term independent of x in the expansion of $\left(\frac{1-x}{1+x}\right)^2$ is

(a) 4 (b) 3 (c) 1 (d) None of these

78. The middle term in the expansion of

$\left(1+\frac{1}{x^2}\right)\left(1+x^2\right)^n$ is

(a) $^{2n}C_n\, x^{2n}$ (b) $^{2n}C_n\, x^{-2n}$
(c) $^{2n}C_n$ (d) $^{2n}C_{n-1}$

79. What are the values of k if the term independent of x in the expansion of $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$ is 405?

(a) ±3 (b) ±6 (c) ±5 (d) ±4

80. If 7^9+9^7 is divided by 64 then the remainder is

(a) 0 (b) 1 (c) 2 (d) 63

81. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is

(a) 6th term (b) 7th term
(c) 5th term (d) 8th term

82. The middle term in the expansion of $\left(1+\frac{1}{x^2}\right)^n\left(1+x^2\right)^n$ is

(a) $^{2n}C_n\, x^{2n}$ (b) $^{2n}C_n\, x^{-2n}$
(c) $^{2n}C_n$ (d) $^{2n}C_{n-1}$

83. The value of $^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3$ is

(a) $^{55}C_4$ (b) $^{55}C_3$ (c) $^{56}C_3$ (d) $^{56}C_4$

84. In the expansion of $(1+x)^{50}$, the sum of the coefficients of odd powers of x is :

(a) 0 (b) 2^{49} (c) 2^{50} (d) 2^{51}

85. Expand by using binomial and find the degree of polynomial

$\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$ is

(a) 7 (b) 6 (c) 5 (d) 4

86. Value of $\sum_{r=1}^{10} r.\frac{^nC_r}{^nC_{r-1}}$ is

(a) $10n-45$ (b) $10n+45$
(c) $10n-35$ (d) $10n^2-35$

87. If $(1+x)^{2n}=a_0+a_1x+a_2x^2+.....+a_{2n}x^{2n}$, then

(a) $a_0+a_2+a_4+....=\frac{1}{2}(a_0+a_1+a_2+a_3+....)$
(b) $a_{n+1}<a_n$
(c) $a_{n-3}=a_{n+3}$
(d) All of these

88. If $(1+ax)^n=1+8x+24x^2+\ldots$ then the values of a and n are

(a) n = 4, a = 2 (b) n = 5, a = 1
(c) n = 8, a = 3 (d) n = 8, a = 2

89. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is

(a) $(-1)^{n-1}n$ (b) $(-1)^n(1-n)$
(c) $(-1)^{n-1}(n-1)^2$ (d) $(n-1)$

90. The sum of the series

$^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....-.....+{}^{20}C_{10}$ is

(a) 0 (b) $^{20}C_{10}$ (c) $-^{20}C_{10}$ (d) $\frac{1}{2}\,^{20}C_{10}$

91. The coefficient of x^{32} in the expansion of :

$\left(x^4-\frac{1}{x^3}\right)^{15}$ is:

(a) $^{-15}C_3$ (b) $^{15}C_4$ (c) $^{-15}C_5$ (d) $^{15}C_2$

92. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

(a) $1-\frac{3}{8}x^2$ (b) $3x+\frac{3}{8}x^2$
(c) $-\frac{3}{8}x^2$ (d) $\frac{x}{2}-\frac{3}{8}x^2$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** 12 terms. [$\because$ No. of terms in $(x+a)^n = n+1$)]

2. **(c)** Since the total number of terms are 52 of which 26 terms get cancelled.

3. **(c)** Suppose $(r+1)^{th}$ term is independent of x. We have

$$T_{r+1} = {}^9C_r\,(2x)^{9-r}\left(\frac{1}{3x^2}\right)^r = {}^9C_r 2^{9-r}\frac{1}{3^r}\,.\,x^{9-3r}$$

This term is independent of x if $9-3r=0$

i.e., $r=3$.

Thus, 4th term is independent of x.

4. **(c)** The constant term

$$= {}^{10}C_6\left(\sqrt[3]{\frac{x}{3}}\right)^6\left(-\sqrt{\frac{3}{x}}\right)^4 = {}^{10}C_4\,\frac{1}{3^2}\,.\,3^2 = 210$$

5. **(b)** Suppose x^{-12} occurs is $(r+1)^{th}$ term.

We have

$$T_{r+1} = {}^{20}C_r\,x^{20-r}\left(\frac{y}{x^3}\right)^r = {}^{20}C_r\,x^{20-4r}\;y^r$$

This term contains x^{-12} if $20-4r=-12$ or $r=8$.

$\therefore$ The coefficient of x^{-12} is ${}^{20}C_8\,y^8$.

6. **(b)** Given,

$T_5 + T_6 = 0$

$\Rightarrow {}^nC_4\,a^{n-4}b^4 - {}^nC_5\,a^{n-5}b^5 = 0$

$\Rightarrow {}^nC_4\,a^{n-4}b^4 = {}^nC_5\,a^{n-5}b^5$

$$\Rightarrow \frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{n-4}{5}$$

7. **(b)** Since $T_{r+1} = {}^nC_r\,a^{n-r}x^r$ in expansion of $(a+x)^n$,

Therefore,

$$T_8 = {}^nC_7(2)^{n-7}\left(\frac{x}{3}\right)^7 = {}^nC_7\frac{2^{n-7}}{3^7}x^7$$

and $$T_9 = {}^nC_8(2)^{n-8}\left(\frac{x}{3}\right)^8 = {}^nC_8\frac{2^{n-8}}{3^8}x^8$$

Therefore, $${}^nC_7\frac{2^{n-7}}{3^7} = {}^nC_8\frac{2^{n-8}}{3^8}$$

(since it is given that coefficient of x^7 = coefficient of x^8)

$$\Rightarrow \frac{n!}{7!\,(n-7)!}\times\frac{8!\,(n-8)!}{n!} = \frac{2^{n-8}}{3^8}\cdot\frac{3^7}{2^{n-7}}$$

$$\Rightarrow \frac{8}{n-7} = \frac{1}{6} \Rightarrow n = 55$$

8. **(a)** The $(r+1)$th term in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is given by

$$T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r = {}^{10}C_r\,\frac{x^{5-(r/2)}}{3^{5-(r/2)}}\cdot\frac{3^r}{2^r\,x^{2r}}$$

$$= {}^{10}C_r\,\frac{3^{(3r/2)-5}}{2^r}\,x^{5-(5r/2)}.$$

For T_{r+1} to be independent of x, we must have

$5-(5r/2)=0$ or $r=2$.

Thus, the 3rd term is independent of x and is equal to

$${}^{10}C_2\,\frac{3^{3-5}}{2^2} = \frac{10\times 9}{2}\times\frac{3^{-2}}{4} = \frac{5}{4}$$

9. **(a)** Coefficient of x^p and x^q in the expansion of $(1+x)^{p+q}$ are ${}^{p+q}C_p$ and ${}^{p+q}C_q$.

and $${}^{p+q}C_p = {}^{p+q}C_q = \frac{(p+q)!}{p!\,q!}$$

10. **(d)** t_r is the rth term in the expansion of $(1+x)^{101}$.

$t_r = {}^{101}C_{r-1}.\,(x)^{(r-1)}$

$$\therefore\ \frac{t_{20}}{t_{19}} = \frac{{}^{101}C_{19}}{{}^{101}C_{18}}.\frac{x^{19}}{x^{18}} = \frac{{}^{101}C_{19}\,x}{{}^{101}C_{18}} = \frac{\frac{101!}{19!82!}}{\frac{101!}{18!83!}}x = \frac{83x}{19}$$

11. **(c)** $t_{r+2} = {}^{2n}C_{r+1}\;x^{r+1};\ t_{3r} = {}^{2n}C_{3r-1}\,x^{3r-1}$

Given ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$;

$\Rightarrow {}^{2n}C_{2n-(r+1)} = {}^{2n}C_{3r-1}$

$\Rightarrow 2n-r-1 = 3r-1 \Rightarrow 2n = 4r \Rightarrow n = 2r$

12. **(b)** On comparing with the expansion of $(x+a)^n$, we get

$$x = x,\ a = \frac{2}{x^2},\ n = 15$$

Now, r^{th} term of $\left(x+\frac{2}{x^2}\right)^{15}$ is given as

$T_{r+1} = {}^nC_r\,x^{n-r}\,a^r$

$$= {}^{15}C_r\,(x)^{15-r}\left(\frac{2}{x^2}\right)^r$$

$= {}^{15}C_r\,x^{15-r}\,2^r\,.\,x^{-2r} = {}^{15}C_r\,x^{15-3r}\,2^r$

Now, in the expansion of $\left(x+\frac{2}{x^2}\right)^{15}$, the term is independent of x if $15-3r=0$

i.e., $r=5$

$\therefore$ Term independent of $x = {}^{15}C_5\,.\,2^5$

13. **(d)** The expression can be written as $a^m\left\{\left(1+\frac{b}{a}\right)^m\right\}$

14. **(c)** The given expression can be written as $5^{-1/2}\left(1+\frac{4}{5}x\right)^{-1/2}$

and it is valid only when $\left|\frac{4}{5}x\right| < 1 \Rightarrow |x| < \frac{5}{4}$

15. (d) The given expression can be written as $4^{-1/2}\left(1-\frac{3}{4}x\right)^{-1/2}$

and it is valid only when $\left|\frac{3}{4}x\right|<1 \Rightarrow -\frac{4}{3}<x<\frac{4}{3}$

16. (b) $2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow n = 2$ or 7

17. (b) In the binomial expansion of $(1+x)^n$,

$T_r = {}^nC_{r-1}.(x)^{r-1}$

For $r = 5, T_5 = {}^nC_4x^4$

$r = 6, T_6 = {}^nC_5x^5$

and $r = 7, T_7 = {}^nC_6x^6$

Since, the coefficients of these terms are in A.P.

$\Rightarrow T_5 + T_7 = 2T_6$

$\Rightarrow {}^nC_4 + {}^nC_6 = 2 \times {}^nC_5$

$\Rightarrow \frac{n!}{(n-4)!4!}+\frac{n!}{(n-6)!6!}=\frac{2\times n!}{(n-5)!5!}$

$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{4!}+\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!}=\frac{2n(n-1)(n-2)(n-3)(n-4)}{5!}$

$\Rightarrow \frac{1}{4!}+\frac{(n-4)(n-5)}{6!}=\frac{2(n-4)}{5!}$

$\Rightarrow \frac{1}{1}+\frac{(n-4)(n-5)}{5\times 6}=\frac{2(n-4)}{5}$

$\Rightarrow \frac{30+n^2-9n+20}{5\times 6}=\frac{2n-8}{5}$

$\Rightarrow n^2 - 9n + 50 = 6(2n - 8)$

$\Rightarrow n^2 - 9n + 50 - 12n + 48 = 0$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow (n - 7)(n - 14) = 0$

$\Rightarrow n = 7$ or $n = 14$.

18. (d) When exponent is n then total number of terms are n + 1. So, total number of terms in $(2 + 3x)^4 = 5$
Middle term is 3rd.

$\Rightarrow T_3 = {}^4C_2(2)^2.(3x)^2$

$=\frac{4\times3\times2\times1}{2\times1\times2}\times4\times9x^2 = 216x^2$

$\therefore$ Coefficient of middle term is 216

19. (b) $T_r = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-r+1}\left(\frac{-2}{x^2}\right)^{r-1}$

$=[{}^{10}C_{r-1}]\,x^{13-r-2r}.(-2)^{r-1}\left(\frac{1}{3}\right)^{10-r}$

r^{th} term contains x^4 when $13 - 3r = 4 \Rightarrow r = 3$

20. (d) In the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$, n = 12 (even)

then middle term is $\frac{12}{2}+1 = 7^{th}$ term.

$(r + 1)^{th}$ term,

$T_{r+1} = {}^{12}C_r\left[\frac{x\sqrt{y}}{3}\right]^{12-r}.\left(-\frac{3}{y\sqrt{x}}\right)^r$

$\therefore T_7 = T_{6+1} = {}^{12}C_6\left(\frac{x\sqrt{y}}{3}\right)^6\left(-\frac{3}{y\sqrt{x}}\right)^6$

$= {}^{12}C_6\frac{x^6y^3}{y^6x^3} = {}^{12}C_6x^3y^{-3} = C(12,6)x^3y^{-3}$

21. (c) In the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, let T_r is the r^{th} term

$T_r = 15_{C_{r-1}}(x^4)^{15-r+1}\left(\frac{1}{x^3}\right)^{r-1}$

$= 15_{C_{r-1}}x^{64-4r-3r+3} = 15_{C_{r-1}}x^{67-7r}$

x^4 occurs in this term

$\Rightarrow 4 = 67 - 7r \Rightarrow 7r = 63 \Rightarrow r = 9$.

22. (c) $T_r = {}^nC_{r-1}(2x)^{r-1}(3y^2)^{n-r+1}$

$T_4 = T_{3+1} = {}^5C_3(2x)^3(3y^2)^2$

$=\frac{5!}{3!2!}2^3.x^3.9y^4 = \frac{5.4}{2.1}\times8\times9\times x^3y^4 = 720\,x^3y^4$

$\therefore$ Coefficient of $x^3y^4 = 720$

23. (d) T_{r+1} in the expansion

$\left[ax^2+\frac{1}{bx}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$

$= {}^{11}C_r(a)^{11-r}(b)^{-r}(x)^{22-2r-r}$

For the coefficient of x^7, we have

$22 - 3r = 7 \Rightarrow r = 5$

$\therefore$ Coefficient of $x^7 = {}^{11}C_5(a)^6\,(b)^{-5}$ ...(i)

Again T_{r+1} in the expansion

$\left[ax-\frac{1}{bx^2}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(-\frac{1}{bx^2}\right)^r$

$= {}^{11}C_r(a)^{11-r}(-1)^r\times(b)^{-r}(x)^{-2r}(x)^{11-r}$

For the coefficient of x^{-7}, we have

$11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

$\therefore$ Coefficient of $x^{-7} = {}^{11}C_6\,a^5\times1\times(b)^{-6}$

$\therefore$ Coefficient of x^7 = Coefficient of x^{-7}

$\Rightarrow {}^{11}C_5(a)^6(b)^{-5} = {}^{11}C_6\,a^5\times(b)^{-6}$

$\Rightarrow ab = 1$.

24. (c) We have

$(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1\,x + {}^{2n}C_2\,x^2 +....+{}^{2n}C_nx^n+....+{}^{2n}C_{2n}x^{2n}$ ...(i)

$(1+x)^{2n-1} = {}^{2n-1}C_0 + {}^{2n-1}C_1 x + {}^{2n-1}C_2 x^2 + \ldots + {}^{2n-1}C_n x^n + \ldots + {}^{2n-1}C_{2n-1} x^{2n-1}$ ...(ii)

According to the given data and equations (i) and (ii), we can claim that

$A = {}^{2n}C_n$ and $B = {}^{2n-1}C_n$

$\Rightarrow \frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{\frac{2n!}{n!n!}}{\frac{(2n-1)!}{n!(n-1)!}}$

$\Rightarrow \frac{A}{B} = \frac{2n(2n-1)!}{n(n-1)!} \times \frac{(n-1)!}{(2n-1)!} = 2$

$\Rightarrow A = 2B$

25. (d) $\frac{(3-2x)}{(1+3x)^3} = (3-2x)(1+3x)^{-3}$

$= (3-2x)\left[1 - 9x + \frac{(-3)(-4)}{2!}.9x^2 + \frac{(-3)(-4)(-5)}{3!}.27x^3 + \ldots\right]$

[Expanding $(1+3x)^{-3}$]

$= (3-2x)(1 - 9x + 54x^2 - 270x^3 + \ldots\ldots)$

$\therefore$ Coefficient of $x^3 = -270 \times 3 - 2 \times 54$

$= -810 - 108 = -918$

26. (c) Let the consecutive coefficient of $(1+x)^n$ are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

From the given condition,

${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6 : 33 : 110$

Now ${}^nC_{r-1} : {}^nC_r = 6 : 33$

$\Rightarrow \frac{n!}{(r-1)!\,(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{6}{33}$

$\Rightarrow \frac{r}{n-r+1} = \frac{2}{11} \Rightarrow 11r = 2n - 2r + 2$

$\Rightarrow 2n - 13r + 2 = 0$ ....(i)

and ${}^nC_r : {}^nC_{r+1} = 33 : 110$

$\Rightarrow \frac{n!}{r!(n-r)!} \times \frac{(r+1)!\,(n-r-1)!}{n!} = \frac{33}{110} = \frac{3}{10}$

$\Rightarrow \frac{(r+1)}{n-r} = \frac{3}{10} \Rightarrow 3n - 13r - 10 = 0$ ...(ii)

Solving (i) & (ii), we get n = 12

27. (c) $\sqrt{5}\left[\left(\sqrt{5}+1\right)^{50} - \left(\sqrt{5}-1\right)^{50}\right]$

$= 2\sqrt{5}\left[{}^{50}C_1\left(\sqrt{5}\right)^{49} + {}^{50}C_3\left(\sqrt{5}\right)^{47} + \ldots\right]$

$= 2\left[{}^{50}C_1(\sqrt{5})^{50} + {}^{50}C_3(\sqrt{5})^{48} + \ldots\right]$

= a natural number

28. (b) $[(x+4y)^3(x-4y)^3]^2 = [\{x^2 - (4y)^2\}]^6$

$= (x^2 - 16y^2)^6$

$\therefore$ No. of terms in the expansion = 7

29. (a) $T_{r+1} = {}^9C_r\left(\sqrt[6]{x}\right)^{9-r}\left(-\frac{1}{\sqrt[3]{x}}\right)^r$

$= {}^9C_r(-1)^r . x^{\frac{9-r}{6}-\frac{r}{3}} = {}^9C_r . x^{\left(\frac{9-3r}{6}\right)}$

Now $\frac{9-3r}{6} = 0 \Rightarrow r = 3$;

Thus, term independent of $x = -{}^9C_3$

30. (d) $T_{r+1} = {}^{29}C_r.3^{29-r}.(7x)^r = ({}^{29}C_r . 3^{29-r} . 7^r)\, x^r$

$\therefore$ a_r = coefficient of (r + 1)[th] term $= {}^{29}C_r . 3^{29-r} . 7^r$

Now, $a_r = a_{r-1}$

$\Rightarrow {}^{29}C_r . 3^{29-r} . 7^r = {}^{29}C_{r-1} . 3^{30-r} . 7^{r-1}$

$\Rightarrow \frac{{}^{29}C_r}{{}^{29}C_{r-1}} = \frac{3}{7} \Rightarrow \frac{30-r}{r} = \frac{3}{7} \Rightarrow r = 21$

31. (c) We have $2^n = 4096 = 2^{12} \Rightarrow n = 12$;

the greatest coeff = coeff of middle term.

So, middle term = t_7

Coeff of $t_7 = {}^{12}C_6 = \frac{12!}{6!6!} = 924$.

32. (b) Suppose x^{-7} occurs in (r + 1)[th] term.

we have $T_{r+1} = {}^nC_r x^{n-r} a^r$ in $(x + a)^n$.

In the given question, n = 1, x = ax, $a = \frac{-1}{bx^2}$

$\therefore$ $T_{r+1} = {}^{11}C_r(ax)^{11-r}\left(\frac{-1}{bx^2}\right)^r$

$= {}^{11}C_r a^{11-r} b^{-r} x^{11-3r}(-1)^r$

This term contains x^{-7} if $11 - 3r = -7$

$\Rightarrow$ r = 6

Therefore, coefficient of x^{-7} is

${}^{11}C_6(a)^5\left(\frac{-1}{b}\right)^6 = \frac{462}{b^6}a^5$

33. (b) Given, $\left(x - \frac{1}{x}\right)^7$ and the (r + 1)[th] term in the expansion of $(x + a)^n$ is $T_{(r+1)} = {}^nC_r(x)^{n-r}a^r$

$\therefore$ (r + 1)[th] term in expansion of

$\left(x - \frac{1}{x}\right)^7 = {}^7C_r(x)^{7-r}\left(-\frac{1}{x}\right)^r$

$= {}^7C_r(x)^{7-2r}(-1)^r$

Since x^3 occurs in T_{r+1}

$\therefore$ $7 - 2r = 3 \Rightarrow r = 2$

thus the coefficient of $x^3 = {}^7C_2(-1)^2 = \frac{7\times 6}{2\times 1} = 21$.

34. (d) $(4+3x)^{25} = 4^{25}\left(1 + \frac{3}{4}x\right)^{25}$

Let (r + 1)[th] term will have largest coefficient

$\Rightarrow \frac{\text{Coefficient of } T_{r+1}}{\text{Coefficient of } T_r} \geq 1$

$$\Rightarrow \frac{{}^{25}C_r\left(\frac{3}{4}\right)^r}{{}^{25}C_{r-1}\left(\frac{3}{4}\right)^{r-1}} \geq 1$$

$$\Rightarrow \left(\frac{25-r+1}{r}\right)\frac{3}{4} \geq 1 \Rightarrow r \leq \frac{78}{7}$$

Largest possible value of r is 11

$\therefore$ Coefficient of $T_{12} = 4^{25} \times {}^{25}C_{11} \times \left(\frac{3}{4}\right)^{11}$

35. (b) The given expression,

$$\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right).....\left(1+\frac{C_n}{C_{n-1}}\right)$$

$$=\left(1+\frac{n}{1}\right)\left(1+\frac{n-1}{2}\right)\left(1+\frac{n-2}{3}\right)...\left(1+\frac{1}{n}\right)$$

$(\because C_0 = C_n = 1)$

$$= \frac{(n+1)^n}{n!}$$

36. (b) The notation $\sum_{k=0}^{n} {}^nC_k a^{n-k} b^k$ stands for

${}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + ... + {}^nC_r a^{n-r} b^r + ... + {}^nC_n a^{n-n} b^n$
where, $b^0 = 1 = a^{n-n}$.
Hence, the notation form of $(a + b)^n$ is

$$(a+b)^n = \sum_{k=0}^{n} {}^nC_k a^{n-k} b^k$$

37. (a) In the expansion of $(a + b)^n$, the sum of the indices of a and b is $n + 0 = n$ in the first term, $(n-1) + 1 = n$ in the second term and so on.
Thus, it can be seen that the sum of the indices of a and b is n in every term of the expansion.

38. (c) Now, $(0.99)^5 = (1 - 0.01)^5$
$= {}^5C_0(1)^5 - {}^5C_1(1)^4(0.01) + {}^5C_2(1)^3(0.01)^2$
(ignore the other terms)

$$= 1 - 5 \times 1 \times 0.01 + \frac{5\times4}{2} \times 1 \times 0.01 \times 0.01$$

$= 1 - 0.05 + 10 \times 0.0001 = 1 - 0.05 + 0.001$
$= 1.001 - 0.05 = 0.951$

STATEMENT TYPE QUESTIONS

39. (d) $\because (3+2x)^{50} = 3^{50}\left(1+\frac{2x}{3}\right)^{50}$

Here, $T_{r+1} = 3^{50}\ {}^{50}C_r\left(\frac{2x}{3}\right)^{r-1}$

and $T_r = 3^{50}\ {}^{50}C_{r-1}\left(\frac{2x}{3}\right)^{r-1}$

But $x = \frac{1}{5}$ [given]

$$\therefore \frac{T_{r+1}}{T_r} \geq 1 \Rightarrow \frac{{}^{50}C_r}{{}^{50}C_{r-1}}\frac{2}{3}\cdot\frac{1}{5} \geq 1$$

$\Rightarrow 102 - 2r \geq 15r \Rightarrow r \leq 6$
$\Rightarrow r = 6$
Therefore, there are two greatest terms T_r and T_{r+1} i.e., T_6 and T_7.

40. (c) Both are correct.

41. (c)

42. (a) I. General term $= T_{r+1} = {}^6C_r (x^2)^{6-r}(-y)^r$

$$= (-1)^r \frac{6!}{r!(6-r)!} .x^{12-2r} .y^r$$

II. 4^{th} term $= T_{3+1}$ in the expansion of $(x + (-2y))^{12}$

$$= {}^{12}C_3 x^{12-3}[-2y]^3$$

$$= \frac{12.11.10}{1.2.3} x^9 (-1)^3 .2^3 .y^3$$

$= -220 \times 8.x^9.y^3 = -1760\,x^9\,y^3.$

43. (a) Statement II is false.

If n is odd, then $\left(\frac{n+1}{2}\right)$th and $\left(\frac{n+3}{2}\right)$th terms are the two middle terms.

44. (c)

MATCHING TYPE QUESTIONS

45. (b) (A) $(1 - 2x)^5$
$= {}^5C_0.1^5 + {}^5C_1.1^4.(-2x) + {}^5C_2.1^3(-2x)^2 + {}^5C_3.1^2(-2x)^3 + {}^5C_4.1^1(-2x)^4 + {}^5C_5 1^0(-2x)^5$

$$= 1.1 + 5.1.(-2x) + \frac{5.4}{1.2}.1.4x^2 + \frac{5.4}{1.2}.1(-8x^3) + \frac{5}{1}.1.16x^4 + (-32x^5)$$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5.$

(B) $\left[\frac{2}{x} + \left(-\frac{x}{2}\right)\right]^5$

$$= C(5,0)\left(\frac{2}{x}\right)^5 + C(5,1)\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right) + C(5,2)\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2 + C(5,3)\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3 + C(5,4)\left(\frac{2}{x}\right)\left(-\frac{x}{2}\right)^4 + C(5,5)\left(-\frac{x}{2}\right)^5$$

$$= 1\left(\frac{2}{x}\right)^5 + 5\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right) + 10\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2 + 10\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3 + 5\left(\frac{2}{x}\right)\left(-\frac{x}{2}\right)^4 + \left(-\frac{x}{2}\right)^5$$

$$= 32x^{-5} - 40x^{-3} + 20x^{-1} - 5x + \frac{5}{8}x^3 - \frac{1}{32}x^5$$

(C) $(2x-3)^6$

$={}^6C_0(2x)^6+{}^6C_1(2x)^5(-3)+{}^6C_2(2x)^4(-3)^2$
$+{}^6C_3(2x)^3(-3)^3+{}^6C_4(2x)^2(-3)^4$
$+{}^6C_5(2x)(-3)^5+{}^6C_6(2x)^0(-3)^6$

$=64x^6+\frac{6}{1}(32x^5)(-3)+\frac{6\,.\,5}{1\,.\,2}(16x^4)9$

$+\frac{6\,.\,5\,.\,4}{1\,.\,2\,.\,3}(8x^3)(-27)+\frac{6\,.\,5}{1\,.\,2}(4x^2)81$

$+\frac{6}{1}(2x)(-243)+729$

$=64x^6-576x^5+2160x^4-4320x^3$
$+4860x^2-2916x+729$

(D) $\left(\frac{x}{3}+\frac{1}{x}\right)^5={}^5C_0\left(\frac{x}{3}\right)^5\left(\frac{1}{x}\right)^0+{}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right)$

$+{}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2+{}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3$

$+{}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4+{}^5C_5\left(\frac{x}{3}\right)^0\left(\frac{1}{x}\right)^5$

$=\frac{x^5}{243}+\frac{5}{1}.\frac{x^4}{81}.\frac{1}{x}+\frac{5.4}{1.2}.\frac{x^3}{27}.\frac{1}{x^2}$

$+\frac{5.4}{1.2}.\frac{x^2}{9}.\frac{1}{x^3}+\frac{5}{1}.\frac{x}{3}.\frac{1}{x^4}+\frac{1}{x^5}$

$=\frac{x^5}{243}+\frac{5}{81}.x^3+\frac{10}{27}.x+\frac{10}{9}.\frac{1}{x}+\frac{5}{3}.\frac{1}{x^3}+\frac{1}{x^5}$

46. **(a)** (A) $(96)^3=(100-4)^3$

$={}^3C_0\,(100)^3-{}^3C_1(100)^2(4)+{}^3C_2(100)\,(4)^2$
$-{}^3C_3(4)^3$

(B) $(102)^5=(100+2)^5$

(C) $(101)^4=(100+1)^4$

(D) $(99)^5=(100-1)^5$

47. **(d)** (A) General term in $(x+3)^8={}^8C_r x^{8-r}.\,3^r$

We have to find the coefficient of x^5

$8-r=5,\ r=8-5=3$

$\therefore$ Coefficient of x^5 (putting $r=3$)

$={}^8C_3\,.\,3^3=\frac{8.7.6}{1.2.3}\,.\,27=56.27=1512$

(B) $(a-2b)^{12}=[\,a+(-2b)]^{12}$

General term $T_{r+1}=C(12,r)\,a^{12-r}(-2b)^r$.

Putting $12-r=5$ or $12-5=r\Rightarrow r=7$

$T_{7+1}=C\,(12,7)\,a^{12-7}\,(-2b)^7$

$=C(12,7)\,a^5\,(-2b)^7=C(12,7)\,(-2)^7\,a^5b^7$

Hence required coefficient is $C(12,7)\,(-2)^7$

$=-\frac{12!}{7!\,5!}.2^7=\frac{-12\times11\times10\times9\times8\times7!}{7!\times5\times4\times3\times2\times1}\times2^7$

$=8\times-11\times9\times2^7$

$=-99\times8\times128=-101376$

(C) 13th term, $T_{13}=T_{12+1}$

$={}^{18}C_{12}\,(9x)^{18-12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$={}^{18}C_6 9^6.x^6(-1)^{12}.\frac{1}{3^{12}}\times\frac{1}{x^6}$

$=18564\times(3^2)^6.\frac{1}{3^{12}}\times\frac{x^6}{x^6}$

$=18564\times\frac{3^{12}}{3^{12}}=18564$

(D) Number of terms in the expansion is $10+1=11$ (odd)

Middle term of the expansion is $\left(\frac{n}{2}+1\right)^{th}$ term

$=(5+1)^{th}$ term $=6^{th}$ term

$T_6=T_{5+1}=C(10,5)\left(\frac{x}{3}\right)^{10-5}(9y)^5$

$=C(10,5)\frac{x^5}{3^5}9^5y^5=C(10,5)3^5x^5y^5$

$=\frac{10!}{5!(10-5)!}3^5x^5y^5=\frac{10!}{5!\,5!}3^5x^5y^5$

$=\frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!}3^5x^5y^5=61236x^5y^5$

48. **(b)** (A) General term of

$\left(x^2+\frac{1}{x}\right)^9$ is $T_{r+1}={}^9C_r\,(x^2)^{9-r}\left(\frac{1}{x}\right)^r$

$=(x^{18-2r}.\,x^{-r}).{}^9C_r=x^{18-3r}.{}^9C_r$

Term independent of x $\Rightarrow 18-3r=0\Rightarrow r=6$ i.e. 7th term.

(B) General term $={}^{12}C_r(x^2)^{12-r}(2x)^{-r}$
$={}^{12}C_r\,x^{24-2r-r}\cdot2^{-r}$

Term independent of x $\Rightarrow 24-3r=0\Rightarrow r=8$ i.e. 9th term.

(C) General term $={}^{10}C_r\,(2x)^{10-r}\left(-\frac{1}{x}\right)^r$
$={}^{10}C_r\,2^{10-r}.x^{10-r}.(-1)^r.x^{-r}$

Term independent of x $\Rightarrow 10-2r=0\Rightarrow r=5$ i.e. 6th term.

(D) General term $={}^{15}C_r\,(x^3)^{15-r}\left(\frac{3}{x^2}\right)^r$
$={}^{15}C_r.x^{45-3r}.3^r.x^{-2r}$
$={}^{15}C_r.x^{45-5r}.3^r$

Term independent of x $\Rightarrow 45-5r=0\Rightarrow r=9$ i.e., 10th term

INTEGER TYPE QUESTIONS

49. (b) $T_2 = {}^nC_1\, ab^{n-1} = 135$ ...(i)

$T_3 = {}^nC_2\, a^2b^{n-2} = 30$ ...(ii)

$T_4 = {}^nC_3\, a^3b^{n-3} = \frac{10}{3}$ ...(iii)

Dividing (i) by (ii)

$$\frac{{}^nC_1 ab^{n-1}}{{}^nC_2 a^2 b^{n-2}} = \frac{135}{30}$$

$$\frac{n}{\frac{n}{2}(n-1)} \cdot \frac{b}{a} = \frac{9}{2} \quad \text{...(iv)}$$

$$\frac{b}{a} = \frac{9}{4}(n-1) \quad \text{...(v)}$$

Dividing (ii) by (iii)

$$\frac{\frac{n(n-1)}{2}}{\frac{n(n-1)(n-2)}{3.2}} \cdot \frac{b}{a} = 9 \quad \text{...(vi)}$$

Eliminating a and b from (v) and (vi), we get

$n = 5$

50. (a) Expression $= (1-x)^5 .(1+x)^4 (1+x^2)^4$

$= (1-x)(1-x^2)^4(1+x^2)^4$

$= (1-x)(1-x^4)^4$

$\therefore$ Coefficient of $x^{13} = -{}^4C_3(-1)^3 = 4$

51. (c) The binomial expansion of $(x+a)^n$ gives $(t+1)^{th}$ term $= T_{t+1} = {}^nC_t x^{n-t} a^t$

We have expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$.

On comparing with $(x+a)^n$, we get

$x = x^4,\ a = \frac{1}{x^3}, n = 15$

$\therefore$ t^{th} term

$= T_t = {}^{15}C_{t-1}(x^4)^{15-(t-1)} . \left(\frac{1}{x^3}\right)^{t-1}$

$= {}^{15}C_{t-1}(x)^{60-4t+4} . (x)^{-3t+3}$

$= {}^{15}C_{t-1}(x)^{67-7t}$

Since, x^4 occurs in the t^{th} term

$\therefore$ $67 - 7t = 4 \Rightarrow 7t = 63 \Rightarrow t = 9$

52. (d) Since the coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^n = {}^nC_r$

$\therefore$ In the expansion of $(1+x)^{18}$

coefficient of $(2r+4)^{th}$ term $= {}^{18}C_{2r+3,}$

Similarly, coefficient of $(r-2)^{th}$ term in the expansion of $(1+x)^{18} = {}^{18}C_{r-3}$

If ${}^nC_r = {}^nC_s$ then $r + s = n$

So, ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$ gives

$2r + 3 + r - 3 = 18$

$\Rightarrow 3r = 18 \Rightarrow r = 6.$

53. (a) Given expansion is $(1+x)^m$. Now,

General term $= T_{r+1} = {}^mC_r\, x^r$

Put r = 2, we have

$T_3 = {}^mC_2 . x^2$

According to the question $C(m, 2) = 6$

or $\frac{m(m-1)}{2!} = 6$

$\Rightarrow m^2 - m = 12$

or $m^2 - m - 12 = 0$

$\Rightarrow m^2 - 4m + 3m - 12 = 0$

or $(m-4)(m+3) = 0$

$\therefore$ $m = 4$, since $m \neq -3$

54. (b) $2\, {}^nC_2 = {}^nC_1 + {}^nC_3 \Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow n = 2$ or 7

55. (b) **Hint :** $T_{r+1} = {}^5C_r(x^2)^{5-r}(k/x)^r = {}^5C_r\, k^r x^{10-3r}$

For coefficient of x, $10 - 3r = 1 \Rightarrow r = 3$

coefficient of $x = {}^5C_3\, k^3 = 270$

$\Rightarrow k^3 = \frac{270}{10} = 27 \therefore k = 3$

56. (b) **Hint :** $T_r = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-(r-1)}\left(-\frac{2}{x^2}\right)^{r-1}$

$= {}^{10}C_{r-1}\left(\frac{1}{3}\right)^{11-r} .(-2)^{r-1} x^{13-3r}$

for coefficient of x^4, $13 - 3r = 4 \Rightarrow r = 3$

57. (d) **Hint :** Given expression

$= 2[1 + {}^9C_2(3\sqrt{2}x)^2 + {}^9C_4(3\sqrt{2}x)^4 + {}^9C_6(3\sqrt{2}x)^6 + {}^9C_8(3\sqrt{2}x)^8]$

$\therefore$ the number of non-zero terms is 5

58. (d) If n is odd, then the expansion of $(x+a)^n + (x-a)^n$ contains $\left(\frac{n+1}{2}\right)$ terms. So, the expansion of $\left(1+5\sqrt{2}x\right)^9 + \left(1-5\sqrt{2}x\right)^9$ has $\left(\frac{9+1}{2}\right) = 5$ terms.

59. (a) $T_{17} = {}^{50}C_{16} \times 2^{34} \times a^{16}$

$T_{18} = {}^{50}C_{17} \times 2^{33} \times a^{17}$

Given $T_{17} = T_{18}$

$\Rightarrow \frac{{}^{50}C_{16}}{{}^{50}C_{17}} \times 2 = \frac{a^{17}}{a^{16}}$

$\Rightarrow a = \frac{50!}{34!16!} \times \frac{33!17! \times 2}{50!} = \frac{17}{34} \times 2 = 1$

60. (a) In the expansion of $(1+\alpha x)^4$

Middle term $= {}^4C_2(\alpha x)^2 = 6\alpha^2x^2$

In the expansion of $(1-\alpha x)^6$,

Middle term $= {}^6C_3(-\alpha x)^3 = -20\alpha^3x^3$

It is given that

Coefficient of the middle term in $(1+\alpha x)^4$ = Coefficient of the middle term in $(1-\alpha x)^6$

$\Rightarrow 6\alpha^2 = -20\alpha^3$

$\Rightarrow \alpha = 0, \alpha = -\frac{3}{10}$

61. **(d)** Suppose x^6 occurs in $(r+1)^{th}$ term in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$

Now, $T_{r+1} = {}^{11}C_r (2x^2)^{11-r}\left(-\frac{3}{x}\right)^r$

$= {}^{11}C_r(-1)^r 2^{11-r} 3^r x^{22-3r}$

For this term to contain x^6, we must have

$22 - 3r = 6 \Rightarrow r = \frac{16}{3}$, which is a fraction.

But, r is a natural number. Hence, there is no term containing x^6.

62. **(c)** $T_{3+1} = \frac{5}{2}$

$\Rightarrow {}^nC_3 (ax)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$

$\Rightarrow {}^nC_3 a^{n-3}.x^{n-6} = \frac{5}{2}$ ...(i)

$\Rightarrow n - 6 = 0 \Rightarrow n = 6$

($\because$ RHS of above equality is independent of x)

Put n = 6 in (i), we get

${}^6C_3 a^3 = \frac{5}{2} \Rightarrow a^3 = \frac{1}{8}$

$\Rightarrow a = \frac{1}{2}$ and $n = 6$

Hence, $a \times \frac{1}{n} = \frac{1}{2} \times 6 = 3$

ASSERTION - REASON TYPE QUESTIONS

63. **(d)** $\left(x + \frac{1}{x} + 2\right)^m = \left(\frac{x^2 + 2x + 1}{x}\right)^m = \frac{(1+x)^{2m}}{x^m}$

Term independent of x is coefficient of x^m in the expansion of $(1+x)^{2m} = {}^{2m}C_m = \frac{(2m)!}{(m!)^2}$

Coefficient of x^6 in the expansion of $(1+x)^n$ is ${}^nC_6$

64. **(a)** Given that, $(1 + ax)^n = 1 + 8x + 24x^2 + ...$

$\Rightarrow 1 + \frac{n}{1}ax + \frac{n(n-1)}{1.2}a^2x^2 + ... = 1 + 8x + 24x^2 + ...$

On comparing the coefficient of x, x^2, we get

$na = 8, \frac{n(n-1)}{2}a^2 = 24$

$\Rightarrow na(na - a) = 48 \Rightarrow 8(8 - a) = 48$

$\Rightarrow 8 - a = 6 \Rightarrow a = 2 \therefore n \times 2 = 8 \Rightarrow n = 4$

65. **(a)** Let $b = \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n-(n-r)}{{}^nC_r}$

$= n\sum_{r=0}^{n} \frac{1}{{}^nC_r} - \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$

$= na_n - \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}}$ $\left(\because {}^nC_r = {}^nC_{n-r}\right)$

$= n\,a_n - b$

$\therefore 2b = na_n \Rightarrow b = \frac{n}{2}a_n$

66. **(b)** We have, $\left(1 + \frac{C_1}{C_0}\right)\left(1 + \frac{C_2}{C_1}\right)...\left(1 + \frac{C_n}{C_{n-1}}\right)$

$= \left(1 + \frac{n}{1}\right)\left[1 + \frac{\frac{n(n-1)}{2!}}{n}\right]...\left(1 + \frac{1}{n}\right)$

$= \frac{(1+n)}{1}.\frac{(1+n)}{2}.\frac{(1+n)}{3}...\frac{(1+n)}{n} = \frac{(1+n)^n}{n!}$

67. **(a)** There are (n + 1) terms in the expansion of $(x + a)^n$. Observing the terms, we can say that the first term from the end is the last term, i.e., $(n + 1)^{th}$ term of the expansion and $n + 1 = (n + 1) - (1 - 1)$. The second term from the end is the n^{th} term of the expansion and $n = (n + 1) - (2 - 1)$.

The third term from the end is the $(n - 1)^{th}$ term of the expansion and $n - 1 = (n + 1) - (3 - 1)$, and so on. Thus, r^{th} term from the end will be term number $(n + 1) - (r - 1) = (n - r + 2)$ of the expansion and the $(n - r + 2)^{th}$ term is ${}^nC_{n-r+1}x^{r-1}a^{n-r+1}$.

68. **(d)** In the expansion of $(x + 2y)^8$, the middle term is $\left(\frac{8}{2}+1\right)^{th}$ i.e., 5th term.

69. **(a)** In the binomial expression, we have

$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + ... + {}^nC_nb^n$ ...(i)

The coefficients ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ..., ${}^nC_n$ are known as binomial or combinatorial coefficients.

Putting a = b = 1 in (i), we get

${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

Thus, the sum of all binomial coefficients is equal to 2^n.

Again, putting a = 1 and b = – 1 in Eq. (i), we get

${}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ...$

Thus, the sum of all the odd binomial coefficients is equal to the sum of all the even binomial coefficients and each is equal to $\frac{2^n}{2} = 2^{n-1}$.

$\Rightarrow {}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}$

70. **(a)** Both Assertion and Reason are correct.

Also, Reason is the correct explanation for the Assertion.

71. **(a)** Both are correct and Reason is the correct explanation.

72. **(a)** **Assertion:** $(x + 2y)^9$

$n = 9, a = 2y$

$\therefore T_{r+1} = {}^9C_r.x^{9-r}.(2y)^r$

$= {}^9C_r.2^r.x^{9-r}.y^r$

73. **(c)** Assertion is correct. Reason is false.

Total number of terms $= \left(\frac{n}{2}+1\right) = 5+1=6$

74. **(d)** Assertion is false and Reason is true.

CRITICALTHINKING TYPE QUESTIONS

75. **(c)** Given expression is :
$[(3x+y)^5]^4 - [(3x-y)^4]^5 = [(3x+y)]^{20} - [(3x-y)]^{20}$
First and second expansion will have 21 terms each but odd terms in second expansion be Ist, 3rd, 5th,.....21st will be equal and opposite to those of first expansion.
Thus, the number of terms in the expansion of above expression is 10.

76. **(d)** $T_{r+1} = {}^{18}C_r(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$

$= (-1)^r\, {}^{18}C_r\, 9^{18-\frac{3r}{2}}\, x^{18-\frac{3r}{2}}$
is independent of x provided $r = 12$ and then $a = 1$.

77. **(c)** $(1-x)^2(1+x)^{-2} = (1-2x+x^2)(1-2x+3x^2+..........)$
The term independent of x is 1.

78. **(c)**

79. **(a)** Given expansion is $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$

$(r+1)_{th}$ term, $T_{r+1} = {}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{k}{x^2}\right)^r$

$\Rightarrow T_{r+1} = {}^{10}C_r x^{5-r/2}.(k)^r.x^{-2r}$

$\therefore T_{r+1} = {}^{10}C_r x^{(10-5r)/2}(k)^r$

Since, T_{r+1} is independent of x

$\therefore \frac{10-5r}{2} = 0 \Rightarrow r = 2$

$\therefore 405 = {}^{10}C_2(k)^2$
$405 = 45 \times k^2$
$\Rightarrow k^2 = 9 \Rightarrow k = \pm 3$

80. **(a)** We have
$7^9 + 9^7 = (8-1)^9 + (8+1)^7 = (1+8)^7 - (1-8)^9$

$= [1 + {}^7C_1 8 + {}^7C_2 8^2 + ... + {}^7C_7 8^7]$

$-[1 - {}^9C_1 8 + {}^9C_2 8^2 - ..... - {}^9C_9 8^9]$

$= {}^7C_1 8 + {}^9C_1 8 + [{}^7C_2 + {}^7C_3.8 + ... - {}^9C_2 + {}^9C_3.8 - ...]8^2$

$= 8(7+9) + 64k = 8..16 + 64k = 64q$,
where $q = k+2$
Thus, $7^9 + 9^7$ is divisible by 64.

81. **(d)** $T_{r+1} = \frac{n(n-1)(n-2).........(n-r+1)}{r!}(x)^r$

For first negative term,

$n-r+1<0 \Rightarrow r > n+1$

$\Rightarrow r > \frac{32}{5}$ $\therefore r = 7.$ $\left(\because n = \frac{27}{5}\right)$

Therefore, first negative term is T_8.

82. **(c)** $\left(1+\frac{1}{x^2}\right)^n (1+x^2)^n = \frac{\left(1+x^2\right)^{2n}}{x^{2n}}$,
numerator has $(2n+1)$ terms.

$\therefore$ The middle terms is $\frac{1}{x^{2n}}$ $[{}^{(2n)}C_n(x^2)^n] = {}^{(2n)}C_n$.

83. **(d)** ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

$= {}^{50}C_4 + \left[{}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3\right]$

We know $\left[{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right]$

$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= ({}^{51}C_4 + {}^{51}C_3) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

Proceeding in the same way, we get

${}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$.

84. **(b)** Binomial expansion of
$(1+x)^{50} = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots + C_{50}x^{50}$
and in given expression
Putting x = 1, we get
$2^{50} = C_0 + C_1 + C_2 + C_3 \ldots + C_{50}$ … (i)
and putting x = – 1
$0 = C_0 - C_1 + C_2 - C_3 \ldots.. + C_{50}$ … (ii)
Subtracting (ii) from (i), we get
$2^{50} = 2(C_1 + C_3 + C_5 + \ldots C_{49})$

$\Rightarrow C_1 + C_3 + C_5 + \ldots C_{49} = \frac{2^{50}}{2} = 2^{49}$

Sum of the coefficient of odd powers of x = 2^{49}

85. **(a)** $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

$= 2[x^5 + {}^5C_2 x^3(x^3-1) + {}^5C_4 x(x^3-1)^2]$
$= 2[x^5 + 10x^3(x^3-1) + 5x(x^6 - 2x^3 + 1]$
$= 10x^7 + 20x^6 + 2x^5 - 20x^4 - 20x^3 + 10x$
$\therefore$ polynomial has degree 7.

86. **(a)** $\frac{r.{}^nC_r}{{}^nC_{r-1}} = \frac{n.{}^{n-1}C_{r-1}}{{}^nC_{r-1}}$

$= n.\frac{(n-1)!}{(r-1)!(n-r)!} \times \frac{(r-1)!(n-r+1)!}{n!}$

$= n - r + 1$

Sum $= n + (n-1) + ... + (n-9) = 10n - 45$

87. **(d)** $a_0 + a_1 + a_2 + ... = 2^{2n}$ and $a_0 + a_2 + a_4 + ... = 2^{2n-1}$
$a_n = {}^{2n}C_n$ = the greatest coefficient, being the middle coefficient
$a_{n-3} = {}^{2n}C_{n-3} = {}^{2n}C_{2n-(n-3)} = {}^{2n}C_{n+3} = a_{n+3}$

88. **(a)** $na = 8 \Rightarrow n^2a^2 = 64, \frac{n(n-1)}{2}a^2 = 24$

since $\frac{2n}{n-1} = \frac{8}{3} \Rightarrow 6n = 8n - 8$

$\Rightarrow n = 4, a = 2$

89. **(b)** Coeff. of x^n in $(1+x)(1-x)^n$ = coeff. of x^n in

$(1+x)(1-{}^nC_1x+{}^nC_2x^2-....+(-1)^n\,{}^nC_nx^n)$

$= (-1)^n\,{}^nC_n + (-1)^{n-1}\,{}^nC_{n-1} = (-1)^n + (-1)^{n-1}.n$

$= (-1)^n(1-n)$

90. **(d)** We know that, $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + ...\ {}^{20}C_{10}x^{10} + ...\ {}^{20}C_{20}x^{20}$

Put $x = -1$, $(0) = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} - {}^{20}C_{11} .... + {}^{20}C_{20}$

$\Rightarrow 0 = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9] + {}^{20}C_{10}$

$\Rightarrow {}^{20}C_{10} = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9 + {}^{20}C_{10}]$

$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$

91. **(b)** We know by Binomial expansion, that $(x+a)^n$

$= {}^nC_0x^na^0 + {}^nC_1x^{n-1}.a + {}^nC_2x^{n-2}a^2 + {}^nC_3x^{n-3}a^3. + {}^nC_4x^{n-4}.a^4 + ... + {}^nC_nx^0a^n$

Given expansion is $\left(x^4 - \frac{1}{x^3}\right)^{15}$

On comparing we get $n = 15, x = x^4, a = \left(-\frac{1}{x^3}\right)$

$\therefore$ We have

$$\left(x^4-\frac{1}{x^3}\right)^{15} = {}^{15}C_0(x^4)^{15}\left(-\frac{1}{x^3}\right)^0 + {}^{15}C_1(x^4)^{14}\left(-\frac{1}{x^3}\right) + {}^{15}C_2(x^4)^{13}\left(-\frac{1}{x^3}\right)^2 + {}^{15}C_3(x^4)^{12}\left(-\frac{1}{x^3}\right)^3 + {}^{15}C_4(x^4)^{11}\left(-\frac{1}{x^3}\right)^4 + ...$$

$T_{r+1} = {}^{15}C_r(x^4)^{15-r}.\left(-\frac{1}{x^3}\right)^r = -{}^{15}C_r\,x^{60-7r}$

$\Rightarrow x^{60-7r} = x^{32} \qquad \Rightarrow 60 - 7r = 32$

$\Rightarrow 7r = 28 \Rightarrow r = 4$

So, 5th term, contains x^{32}

$= {}^{15}C_4(x^4)^{11}\left(-\frac{1}{x^3}\right)^4 = {}^{15}C_4\,x^{44}x^{-12} = {}^{15}C_4x^{32}$.

Thus, coefficient of $x^{32} = {}^{15}C_4$.

92. **(c)** $\because$ x^3 and higher powers of x may be neglected

$$\therefore \frac{(1+x)^{\frac{3}{2}} - \left(1+\frac{x}{2}\right)^3}{(1-x)^{\frac{1}{2}}}$$

$$= (1-x)^{\frac{-1}{2}}\left[\left(1+\frac{3}{2}x+\frac{\frac{3}{2}.\frac{1}{2}}{2!}x^2\right) - \left(1+\frac{3x}{2}+\frac{3.2}{2!}\frac{x^2}{4}\right)\right]$$

$$= \left[1+\frac{x}{2}+\frac{\frac{1}{2}.\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right] = \frac{-3}{8}x^2$$

(as x^3 and higher powers of x can be neglected)

CHAPTER

9

SEQUENCES AND SERIES

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Let a_1, a_2, a_3, ...be the sequence, then the sum expressed as $a_1 + a_2 + a_3 + ... + a_n$ is called ....
(a) Sequence (b) Series
(c) Finite (d) Infinite

2. The third term of a geometric progression is 4. The product of the first five terms is :
(a) 4^3 (b) 4^5 (c) 4^4 (d) 4^7

3. In an AP. the pth term is q and the $(p + q)$th term is 0. Then the qth term is
(a) $-p$ (b) p (c) $p+q$ (d) $p-q$

4. If a, b, c, d, e, f are in A.P., then $e - c$ is equal to:
(a) $2(c-a)$ (b) $2(d-c)$ (c) $2(f-d)$ (d) $(d-c)$

5. The fourth, seventh and tenth terms of a G.P. are p, q, r respectively, then :
(a) $p^2 = q^2 + r^2$ (b) $q^2 = pr$
(c) $p^2 = qr$ (d) $pqr + pq + 1 = 0$

6. If 1, a and P are in A. P. and 1, g and P are in G. P., then
(a) $1 + 2a + g^2 = 0$ (b) $1 + 2a - g^2 = 0$
(c) $1 - 2a - g^2 = 0$ (d) $1 - 2a + g^2 = 0$

7. For a, b, c to be in G.P. What should be the value of $\frac{a-b}{b-c}$?
(a) ab (b) bc
(c) $\frac{a}{b}$ or $\frac{b}{c}$ (d) None of these

8. What is the sum of terms equidistant from the beginning and end in an A.P. ?
(a) First term – Last term (b) First term × Last term
(c) First term + Last term (d) First term ÷ Last term

9. The first and eight terms of a G.P. are x^{-4} and x^{52} respectively. If the second term is x^t, then t is equal to:
(a) –13 (b) 4 (c) $\frac{5}{2}$ (d) 3

10. If the p^{th}, q^{th} and r^{th} terms of a G.P. are again in G.P., then which one of the following is correct?
(a) p, q, r are in A.P.
(b) p, q, r are in G.P.
(c) p, q, r are in H.P.
(d) p, q, r are neither in A.P. nor in G.P. nor in H.P.

11. If $5(3^{a-1} + 1)$, $(6^{2a-3} + 2)$ and $7(5^{a-2} + 5)$ are in AP, then what is the value of a?
(a) 7 (b) 6
(c) 5 (d) None of these

12. If p^{th} term of an AP is q, and its q^{th} term is p, then what is the common difference ?
(a) – 1 (b) 0 (c) 2 (d) 1

13. If a, b, c are in geometric progression and a, 2b, 3c are in arithmetic progression, then what is the common ratio r such that $0 < r < 1$?
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

14. If 1, x, y, z, 16 are in geometric progression, then what is the value of x + y + z ?
(a) 8 (b) 12 (c) 14 (d) 16

15. The product of first nine terms of a GP is, in general, equal to which one of the following?
(a) The 9th power of the 4th term
(b) The 4th power of the 9th term
(c) The 5th power of the 9th term
(d) The 9th power of the 5th term

16. In a G.P. if $(m + n)^{th}$ term is p and $(m - n)^{th}$ term is q, then m^{th} term is:
(a) $\frac{p}{q}$ (b) $\frac{q}{p}$ (c) pq (d) $\sqrt{pq}$

17. The following consecutive terms
$\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ of a series are in:
(a) H.P. (b) G.P.
(c) A.P. (d) A.P., G.P.

18. The series $(\sqrt{2}+1), 1, (\sqrt{2}-1)$ ... is in :
(a) A.P. (b) G.P.
(c) H.P. (d) None of these

19. Three numbers form an increasing G.P. If the middle number is doubled, then the new numbers are in A.P. The common ratio of the G.P. is:
(a) $2 - \sqrt{3}$ (b) $2 + \sqrt{3}$
(c) $\sqrt{3} - 2$ (d) $3 + \sqrt{2}$

20. If the sum of the first $2n$ terms of 2, 5, 8, ... is equal to the sum of the first n terms of 57, 59, 61......., then n is equal to
(a) 10 (b) 12 (c) 11 (d) 13

21. There are four arithmetic means between 2 and –18. The means are
(a) –4, –7, –10, –13 (b) 1, –4, –7, –10
(c) –2, –5, –9, –13 (d) –2, –6, –10, –14

22. The arithmetic mean of three observations is x. If the values of two observations are y, z; then what is the value of the third observation ?
(a) x (b) $2x - y - z$
(c) $3x - y - z$ (d) $y + z - x$

23. What is the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...$?
(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

24. $\frac{1}{q+r}, \frac{1}{r+p}, \frac{1}{p+q}$ are in A.P. then,
(a) p, q, r are in A.P (b) p^2, q^2, r^2 are in A.P
(c) $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in A.P (d) $p + q + r$ are in A.P

25. If G be the geometric mean of x and y, then $\frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} =$
(a) G^2 (b) $\frac{1}{G^2}$ (c) $\frac{2}{G^2}$ (d) $3G^2$

26. In a Geometric Progression with first term a and common ratio r, what is the Arithmetic Mean of the first five terms?
(a) $a + 2r$ (b) $a r^2$
(c) $a (r^5 - 1)/[5(r - 1)]$ (d) $a (r^4 - 1)/[5(r - 1)]$

27. If p, q, r are in A.P., a is G.M. between p & q and b is G.M. between q and r, then a^2, q^2, b^2 are in
(a) G.P. (b) A.P.
(c) H.P (d) None of these

28. Sum of n terms of series 1.3+3.5+5.7+.......... is
(a) $\frac{1}{3}n(n+1)(2n+1) - n$ (b) $\frac{3}{2}n(n+1)(2n+1) - n$
(c) $\frac{4}{5}n(n+1)(2n+1) - n$ (d) $\frac{2}{3}n(n+1)(2n+1) - n$

29. Let a_1, a_2, a_3............ be terms of an A.P. If $\frac{a_1 + a_2 + ...a_p}{a_1 + a_2 + ...+ a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals
(a) $\frac{41}{11}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{11}{41}$

30. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression equals
(a) $\sqrt{5}$ (b) $\frac{1}{2}(\sqrt{5} - 1)$
(c) $\frac{1}{2}(1 - \sqrt{5})$ (d) $\frac{1}{2}\sqrt{5}$.

31. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is
(a) –4 (b) –12 (c) 12 (d) 4

32. The harmonic mean of $\frac{a}{1-ab}$ and $\frac{a}{1+ab}$ is :
(a) a (b) $\frac{a}{1 - a^2b^2}$
(c) $\frac{1}{1 - a^2b^2}$ (d) $\frac{a}{1 + a^2b^2}$

33. If arithmetic mean of a and b is $\frac{(a^{n+1} + b^{n+1})}{a^n + b^n}$, then the value of n is equal to
(a) –1 (b) 0 (c) 1 (d) 2

34. The H. M between roots of the equation $x^2 - 10x + 11 = 0$ is equal to :
(a) $\frac{1}{5}$ (b) $\frac{5}{21}$ (c) $\frac{21}{20}$ (d) $\frac{11}{5}$

35. If m arithmetic means are inserted between 1 and 31 so that the ratio of the 7^{th} and $(m - 1)^{th}$ means 5 : 9, then the value of m is
(a) 10 (b) 11 (c) 12 (d) 14

36. Let S_n denote the sum of first n terms of an A.P. If $S_{2n} = 3 S_n$, then the ratio S_{3n}/ S_n is equal to :
(a) 4 (b) 6 (c) 8 (d) 10

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

37. Consider the following statements
I. If $a_1, a_2, ..., a_n$ ... is a sequence, then the expression $a_1 + a_2 + ... + a_n + ...$ is called a series.
II. Those sequences whose terms follow certain patterns are called progressions.
Choose the correct option.
(a) Only I is false (b) Only II is false
(c) Both are false (d) Both are true

38. Consider the following statements.
I. A sequence is called an arithmetic progression if the difference of a term and the previous term is always same.
II. Arithmetic Mean (A.M.) A of any two numbers a and b is given by $\frac{1}{2}(a + b)$ such that a, A, b are in A.P.
The arithmetic mean for any n positive numbers $a_1, a_2, a_3, ..., a_n$ is given by
$$\text{A.M.} = \frac{a_1 + a_2 + ... + a_n}{n}$$
Choose the correct option.
(a) Only I is true (b) Both are true
(c) Only II is true (d) Both are false

39. **Statement I:** Three numbers a, b, c are in A.P., then b is called the arithmetic mean of a and c.
Statement II: Three numbers a, b, c are in A.P. iff $2b = a + c$.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

40. **Statement I:** If 'a' is the first term and 'd' is the common difference of an A.P., then its n^{th} term is given by
$a_n = a - (n - 1)d$
Statement II: The sum S_n of n terms of an A.P. with first term 'a' and common difference 'd' is given by $S_n = \frac{n}{2}\{2a + (n - 1)d\}$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

41. Consider the following statements.
I. The n^{th} term of a G.P. with first term 'a' and common ratio 'r' is given by $a_n = a.r^{n-1}$.
II. Geometric mean of a and b is given by $(ab)^{1/3}$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

42. I. Three numbers a, b, c are in G.P. iff $b^2 = ac$
II. The reciprocals of the terms of a given G.P. form a G.P.
III. If $a_1, a_2, ..., a_n, ...$ is a G.P., then the expression $a_1 + a_2 + ... + a_n + ...$ is called a geometric series.
Choose the correct option.
(a) Only I and II are true
(b) Only II and III are true
(c) All are true
(d) Only I and III are true

43. I. If each term of a G.P. be raised to the same power, the resulting sequence also forms a G.P.
II. 25^{th} term of the sequence 4, 9, 14, 19, ... is 124.
Choose the correct option.
(a) Both are true (b) Both are false
(c) Only I is true (d) Only II is true

44. I. 18^{th} term of the sequence 72, 70, 68, 66, ... is 40.
II. 4^{th} term of the sequence $8 - 6i, 7 - 4i, 6 - 2i, ...$ is purely real.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

45. I. 37 terms are there in the sequence 3, 6, 9, 12, ..., 111.
II. General term of the sequence 9, 12, 15, 18, ... is $3n + 8$.
Choose the correct option.
(a) Only I is true (b) Only II is true.
(c) Both are true (d) Both are false

46. I. 11^{th} terms of the G.P. 5, 10, 20, 40, ... is 5120
II. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtained quadratic equation is $x^2 - 16x + 25 = 0$
Choose the correct option.
(a) Only I is true (b) Only II is true.
(c) Both are true (d) Both are false.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

47.

Column - I	Column - II
A. Sum of 20 terms of the A.P. 1, 4, 7, 10, ... is	1. 70336
B. Sum of the series 5+13+21+...+181 is	2. 156375
C. The sum of all three digit natural numbers, which are divisible by 7, is	3. 2139
D. The sum of all natural numbers between 250 and 1000 which are exactly divisible by 3, is	4. 590

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	3	1	4
(d)	2	1	3	4

48.

Column - I	Column - II
A. Sum of 7 terms of the G.P. 3, 6, 12, ... is	1. $\frac{10}{9}[10^n - 1] + n^2$
B. Sum of 10 terms of the G.P. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ...$ is	2. $\frac{1023}{512}$
C. Sum of the series $2 + 6 + 18 + ... + 4374$ is	3. 381
D. Sum to n terms of the series $11 + 103 + 1005 + ...$ is	4. 6560

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	1	4	2	3
(c)	3	4	2	1
(d)	3	2	4	1

49.

Column - I	Column - II
A. Sum to infinity of the G.P. $\frac{-5}{4}, \frac{5}{16}, \frac{-5}{64}, ...$ is	1. $\frac{2}{3}$
B. Value of $6^{1/2}.6^{1/4}.6^{1/8}$ .... ∞ is	2. $\frac{1}{3}$
C. If the first term of a G.P. is 2 and the sum to infinity is 6 then the common ratio is	3. -1
D. If each term of an infinite G.P. is twice the sum of the terms following it, then the common ratio of the G.P. is	4. 6

Codes

	A	B	C	D
(a)	2	1	4	3
(b)	2	4	1	3
(c)	3	4	1	2
(d)	3	1	4	2

50. If the sequence is defined by $a_n = n(n + 2)$, then match the columns.

Column - I	Column - II
A. $a_1 =$	1. 35
B. $a_2 =$	2. 24
C. $a_3 =$	3. 8
D. $a_4 =$	4. 3
E. $a_5 =$	5. 15

Codes

	A	B	C	D	E
(a)	4	3	5	2	1
(b)	4	2	5	3	1
(c)	1	3	2	5	4
(d)	3	4	5	1	2

51. If the n^{th} term of the sequence is defined as $a_n = \frac{2n-3}{6}$, then match the columns.

Column - I	Column - II
A. $a_1 =$	1. 1/6
B. $a_2 =$	2. 1/2
C. $a_3 =$	3. 5/6
D. $a_4 =$	4. –1/6
E. $a_5 =$	5. 7/6

Codes

	A	B	C	D	E
(a)	4	1	3	2	5
(b)	5	3	2	1	4
(c)	4	3	3	1	5
(d)	4	1	2	3	5

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

52. If the j^{th} term and k^{th} term of an A.P. are k and j respectively, the $(k+j)$ th term is
(a) 0 (b) 1
(c) $k+j+1$ (d) $k+j-1$

53. Third term of the sequence whose n^{th} term is $a_n = 2^n$, is
(a) 2 (b) 4 (c) 8 (d) 3

54. The Fibonacci sequence is defined by $1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2}$, $n > 2$. Then value of $\frac{a_{n+1}}{a_n}$ for n = 2, is
(a) 1 (b) 2 (c) 3 (d) 4

55. If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116. then the last term is
(a) 0 (b) 2 (c) 4 (d) 6

56. If the sum of first *p* terms of an A.P. is equal to the sum of the first *q* terms then the sum of the first $(p + q)$ terms, is
(a) 0 (b) 1 (c) 2 (d) 3

57. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between *a* and *b*, then the value of *n* is
(a) 1 (b) 2 (c) 3 (d) 4

58. The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°. The number of the sides of the polygon is
(a) 6 (b) 9 (c) 8 (d) 5

59. Which term of the following sequence $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$.......is $\frac{1}{19683}$?
(a) 3 (b) 9
(c) 6 (d) None of these

60. How many terms of G.P. $3, 3^2, 3^3$, ... are needed to give the sum 120?
(a) 3 (b) 4 (c) 5 (d) 6

61. If f is a function satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in N$. such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$, find the value of *n*.
(a) 2 (b) 4 (c) 6 (d) 8

62. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then the common ratio is
(a) 5 (b) 1 (c) 4 (d) 3

63. How many terms of the geometric series 1 + 4 + 16 + 64 + ... will make the sum 5461?
(a) 3 (b) 4 (c) 5 (d) 7

64. Let T_r be the rth term of an A.P. whose first term is a and common difference is *d*. If for some positive integers $m, n,\ m \neq n,\ T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals
(a) $\frac{1}{m} + \frac{1}{n}$ (b) 1 (c) $\frac{1}{mn}$ (d) 0

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

65. **Assertion:** For $x = \pm 1$, the numbers $\frac{-2}{7}$, x, $\frac{-7}{2}$ are in G.P.
Reason: Three numbers a, b, c are in G.P. if $b^2 = ac$.

66. **Assertion:** Sum to n terms of the geometric progression $x^3, x^5, x^7, ... (x \neq \pm 1)$ is $\frac{x^3(1-x^{2n})}{(1-x^2)}$.

Reason: If 'a' is the first term and r is common ratio of a G.P. then sum to n terms is given as

$S_n = \frac{a(r^n-1)}{r-1}$ or $= \frac{a(1-r^n)}{1-r}$ if $r \neq 1$.

67. **Assertion:** Value of a_{17}, whose n^{th} term is $a_n = 4n - 3$, is 65.
Reason: Value of a_9, whose n^{th} term is $a_n = (-1)^{n-1}.n^3$.

68. **Assertion:** If each term of a G.. P. is multiplied or divided by some fixed non-zero number, the resulting sequence is also a G P.
Reason: If $-1 < r < 1$, i.e. $|r| < 1$, then the sum of the infinite G. P., $a + ar + ar^2 + ... = \frac{a}{1-r}$

i.e., $S_\infty = \frac{a}{1-r}$

69. **Assertion:** If the third term of a G.P. is 4, then the product of its first five terms is 4^5.
Reason: Product of first five terms of a G.P. is given as $a\,(ar)\,(ar^2)\,(ar^3)\,(ar^4)$

70. **Assertion:** If a, b, c are in A.P., then b + c, c + a, a + b are in A.P.
Reason: If a, b, c are in A.P., then $10^a, 10^b, 10^c$ are in G.P.

71. **Assertion:** If $\frac{2}{3}, k, \frac{5}{8}$ are in A.P, then the value of k is $\frac{31}{48}$.
Reason: Three numbers a, b, c are in A.P. iff $2b = a + c$

72. **Assertion:** If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, then the value of m is 27.

Reason: 20^{th} term of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, ...$ is $\frac{5}{2^{20}}$

73. **Assertion:** The 20^{th} term of the series $2 \times 4 + 4 \times 6 + 6 \times 8 + ... +$ n terms is 1680.
Reason: If the sum of three numbers in A.P. is 24 and their product is 440. Then the numbers are 5, 8, 11 or 11, 8, 5.

74. **Assertion:** Sum of n terms of the A.P., whose k^{th} term is $5k + 1$, is $\frac{n(5n+7)}{2}$.
Reason: Sum of all natural numbers lying between 100 and 1000, which are multiples of 5, is 980.

75. **Assertion:** : The sum of n terms of two arithmetic progressions are in the ratio (7n + 1) : (4n + 17), then the ratio of their n^{th} terms is 7:4.
Reason: If $S_n = ax^2 + bx + c$, then $T_n = S_n - S_{n-1}$

76. Let sum of n terms of a series $S_n = 6n^2 + 3n + 1$.
Assertion: The series S_n is in A.P.
Reason: Sum of n terms of an A.P. is always of the form $an^2 + bn$.

77. **Assertion:** The arithmetic mean (A.M.) between two numbers is 34 and their geometric mean is 16.
The numbers are 4 and 64.

Reason: For two numbers a and b, A.M. = $A = \frac{a+b}{2}$

G.M = G = $\sqrt{ab}$.

78. **Assertion:** The ratio of sum of m terms to the sum of n terms of an A.P is $m^2 : n^2$. If T_k is the k^{th} term, then $T_5/T_2 = 3$.
Reason: For n^{th} term, $t_n = a + (n - 1)d$, where 'a' is first term and 'd' is common difference.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

79. Consider an infinite geometric series with first term *a* and common ratio *r*. If its sum is 4 and the second term is $\frac{3}{4}$, then :

(a) $a = \frac{4}{7}, r = \frac{3}{7}$ (b) $a = 2, r = \frac{3}{8}$

(c) $a = \frac{3}{2}, r = \frac{1}{2}$ (d) $a = 3, r = \frac{1}{4}$

80. If roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP, then its common difference is
(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

81. 4^{th} term from the end of the G.P. 3, 6, 12, 24., ..., 3072 is
(a) 348 (b) 843
(c) 438 (d) 384

82. If $a^x = b^y = c^z$, where a, b, c are in G.P. and a,b, c, x, y, $z \neq 0$; then $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in:
(a) A.P. (b) G.P. (c) H.P (d) None of these

83. The value of $3 - 1 + \frac{1}{3} - \frac{1}{9} + ...$ is equal to:
(a) $\frac{20}{9}$ (b) $\frac{9}{20}$ (c) $\frac{9}{4}$ (d) $\frac{4}{9}$

84. $5^{1+x} + 5^{1-x}, \frac{a}{2}, 5^{2x} + 5^{-2x}$ are in A.P., then the value of a is:
(a) $a < 12$ (b) $a \leq 12$
(c) $a \geq 12$ (d) None of these

85. The product of *n* positive numbers is unity, then their sum is :
(a) a positive integer (b) divisible by *n*
(c) equal to $n + \frac{1}{n}$ (d) never less than *n*

86. An infinite G.P has first term *x* and sum 5, then
(a) $x < -10$ (b) $-10 < x < 0$
(c) $0 < x < 10$ (d) $x > 10$

87. Sum of the first *n* terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$ is equal to :
(a) $2^n - n - 1$ (b) $1 - 2^{-n}$
(c) $n + 2^{-n} - 1$ (d) $2^n + 1$

88. In a G.P. of even number of terms, the sum of all terms is 5 times the sum of the odd terms. The common ratio of the G.P. is

(a) $\frac{-4}{5}$ (b) $\frac{1}{5}$
(c) 4 (d) None of these

89. The sum of 11 terms of an A.P. whose middle term is 30,

(a) 320 (b) 330 (c) 340 (d) 350

90. The first term of an infinite G.P. is 1 and each term is twice the sum of the succeeding terms. then the sum of the series is

(a) 2 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

91. There are four numbers of which the first three are in G.P. and the last three are in A.P., whose common difference is 6. If the first and the last numbers are equal then two other numbers are

(a) –2, 4 (b) –4, 2
(c) 2, 6 (d) None of these

92. If in a series $S_n = an^2 + bn + c$, where S_n denotes the sum of n terms, then

(a) The series is always arithmetic
(b) The series is arithmetic from the second term onwards
(c) The series may or may not be arithmetic
(d) The series cannot be arithmetic

93. If the sum of the first ten terms of an arithmetic progression is four times the sum of the first five terms, then the ratio of the first term to the common difference is :

(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

94. If the nth term of an arithmetic progression is $3n + 7$, then what is the sum of its first 50 terms?

(a) 3925 (b) 4100
(c) 4175 (d) 8200

95. Let x be one A.M and g_1 and g_2 be two G.Ms between y and z. What is $g_1^3 + g_2^3$ equal to ?

(a) xyz (b) xy^2z
(c) xyz^2 (d) 2xyz

96. What is the sum of the first 50 terms of the series $(1 \times 3) + (3 \times 5) + (5 \times 7) + ...$?

(a) 1,71,650 (b) 26,600
(c) 26,650 (d) 26,900

97. The A.M. of the series 1, 2, 4, 8, 16, ..., 2^n is :

(a) $\frac{2^n - 1}{n}$ (b) $\frac{2^{n+1} - 1}{n+1}$

(c) $\frac{2^n + 1}{n}$ (d) $\frac{2^n - 1}{n+1}$

98. The 10 th common term between the series $3 + 7 + 11 + ...$ and $1 + 6 + 11 + ...$ is

(a) 191 (b) 193 (c) 211 (d) None of these

99. A man saves ` 135/- in the first year, ` 150/- in the second year and in this way he increases his savings by ` 15/- every year. In what time will his total savings be ` 5550/-?

(a) 20 years (b) 25 years
(c) 30 years (d) 35 years

100. Let a, b, c, be in A.P. with a common difference d. Then $e^{1/c}$, $e^{b/ac}$, $e^{1/a}$ are in :

(a) G.P. with common ratio e^d
(b) G.P with common ratio $e^{1/d}$
(c) G.P. with common ratio $e^{d/(b^2-d^2)}$
(d) A.P.

101. If $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P. then $9^{ax+1}, 9^{bx+1}, 9^{cx+1}$, $x \neq 0$ are in :

(a) G.P. (b) G.P. only if x < 0
(c) G.P. only if x > 0 (d) None of these

102. The value of x + y + z is 15 if a, x, y, z, b are in A.P. while the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is $\frac{5}{3}$ if a, x, y, z, b are in H.P. Then the value of a and b are

(a) 2 and 8 (b) 1 and 9
(c) 3 and 7 (d) None of these

103. The A. M. between two positive numbers a and b is twice the G. M. between them. The ratio of the numbers is

(a) $(\sqrt{2} + 3) : (\sqrt{2} - 3)$
(b) $(2 + \sqrt{3}) : (2 - \sqrt{3})$
(c) $(\sqrt{3} + 1) : (\sqrt{3} - 1)$
(d) None of these

104. If S_n denotes the sum of n terms of a G.P. whose first term is a and the common ratio r, then value of $S_1 + S_3 + S_5 + ... + S_{2n-1}$ is

(a) $\frac{a}{1+r}\left[n + r.\frac{1-r^{2n}}{1-r^2}\right]$ (b) $\frac{2a}{1+r}\left[n + r.\frac{1-r^{2n}}{1+r^2}\right]$

(c) $\frac{a}{1+r}\left[n - r.\frac{1-r^{2n}}{1-r^2}\right]$ (d) $\frac{a}{1-r}\left[n - r.\frac{1-r^{2n}}{1-r^2}\right]$

105. If S_1, S_2 and S_3 denote the sum of first n_1, n_2 and n_3 terms respectively of an A.P., then value of

$\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2)$ is

(a) $\frac{1}{2}$ (b) 0 (c) $-\frac{1}{2}$ (d) $\frac{3}{2}$

106. Find the sum up to 16 terms of the series

$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + ...$

(a) 448 (b) 445 (c) 446 (d) None of these

107. The sum of the first n terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$

is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\frac{n^2(n+1)}{2}$

(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{3n(n+1)}{2}$

108. If sum of the infinite G.P. is $\frac{4}{3}$ and its first term is $\frac{3}{4}$ then its common ratio is :

(a) $\frac{7}{16}$ (b) $\frac{9}{16}$ (c) $\frac{1}{9}$ (d) $\frac{7}{9}$

109. If sixth term of a H. P. is $\frac{1}{61}$ and its tenth term is $\frac{1}{105}$, then the first term of that H.P. is

(a) $\frac{1}{28}$ (b) $\frac{1}{39}$ (c) $\frac{1}{6}$ (d) $\frac{1}{17}$

110. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are the p^{th}, q^{th}, r^{th} terms respectively of an A.P. then the value of $ab(p-q) + bc(q-r) + ca(r-p)$ is

(a) –1 (b) 2 (c) 0 (d) –2

111. If the sum of an infinitely decreasing GP is 3, and the sum of the squares of its terms is 9/2, then sum of the cubes of the terms is

(a) $\frac{107}{12}$ (b) $\frac{105}{17}$ (c) $\frac{108}{13}$ (d) $\frac{97}{12}$

112. If x,y,z are in G.P. and $a^x = b^y = c^z$, then

(a) $\log_b a = \log_a c$ (b) $\log_c b = \log_a c$

(c) $\log_b a = \log_c b$ (d) None of these

113. The sum to infinite term of the series

$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+ \ldots$ is

(a) 3 (b) 4 (c) 6 (d) 2

114. The fifth term of the H.P., $2, 2\frac{1}{2}, 3\frac{1}{3}, \ldots\ldots$ will be

(a) $5\frac{1}{5}$ (b) $3\frac{1}{5}$

(c) $\frac{1}{10}$ (d) 10

115. If the 7^{th} term of a H.P. is $\frac{1}{10}$ and the 12^{th} term is $\frac{1}{25}$, then the 20^{th} term is

(a) $\frac{1}{37}$ (b) $\frac{1}{41}$

(c) $\frac{1}{45}$ (d) $\frac{1}{49}$

116. The harmonic mean of $\frac{a}{1-ab}$ and $\frac{a}{1+ab}$ is

(a) $\frac{a}{\sqrt{1-a^2b^2}}$ (b) $\frac{a}{1-a^2b^2}$

(c) a (d) $\frac{1}{1-a^2b^2}$

117. If the arithmetic, geometric and harmonic means between two distinct positive real numbers be A, G and H respectively, then the relation between them is

(a) $A > G > H$ (b) $A > G < H$

(c) $H > G > A$ (d) $G > A > H$

118. If the arithmetic, geometric and harmonic means between two positive real numbers be A, G and H, then

(a) $A^2 = GH$ (b) $H^2 = AG$

(c) $G = AH$ (d) $G^2 = AH$

119. If b^2, a^2, c^2 are in A.P., then $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a}$ will be in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

120. If the arithmetic mean of two numbers be A and geometric mean be G, then the numbers will be

(a) $A \pm (A^2 - G^2)$

(b) $\sqrt{A} \pm \sqrt{A^2 - G^2}$

(c) $A \pm \sqrt{(A+G)(A-G)}$

(d) $\frac{A \pm \sqrt{(A+G)(A-G)}}{2}$

121. If $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$, then a, b, c are in

(a) A.P. (b) G.P.

(c) H.P. (d) In G.P. and H.P. both

122. If a, b, c are in A.P. and a, b, d in G.P., then a, a – b, d – c will be in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

123. If the ratio of H.M. and G.M. of two quantities is 12 : 13, then the ratio of the numbers is

(a) 1 : 2 (b) 2 : 3

(c) 3 : 4 (d) None of these

124. If the ratio of H.M. and G.M. between two numbers a and b is 4 : 5, then the ratio of the two numbers will be

(a) 1 : 2 (b) 2 : 1

(c) 4 : 1 (d) 1 : 4

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)**
2. **(b)** Here,
$t_3 = 4 \Rightarrow ar^2 = 4$
$\therefore$ Product of first five terms
$= a.\ ar.\ ar^2.\ ar^3.ar^4$
$= a^5 r^{10} = (ar^2)^5 = (4)^5$
3. **(b)** Let a, d be the first term and common difference respectively.
Therefore, $T_p = a + (p-1)d = q$ and ...(i)
$T_{p+q} = a + (p + q - 1)d = 0$ ...(ii)
Subtracting (i), from (ii) we get $qd = -q$
Substituting in (i), we get
$a = q - (p-1)(-1) = q + p - 1$
Now $T_q = a + (q-1)d = q + p - 1 + (q-1)(-1)$
$= p + q - 1 - q + 1 = p$
4. **(b)** Let x be the common difference of the A.P.
$a, b, c, d, e, f.$
$\therefore e = a + (5-1)x \quad [\because a_n = a + (n-1)d]$
$\Rightarrow e = a + 4x$ ...(i)
and $c = a + 2x$ ...(ii)
$\therefore$ Using equations (i) and (ii), we get
$e - c = a + 4x - a - 2x$
$\Rightarrow e - c = 2x = 2(d - c).$
5. **(b)** Let a be the first term and r be common ratio.
Fourth term of G.P. : $p = T_4 = ar^3$ ...(i)
Seventh term of G.P. : $q = T_7 = ar^6$ ...(ii)
Tenth term of G.P. : $r = T_{10} = ar^9$ ...(iii)
Equ. (i) × Equ. (iii) :
$pr = ar^3 \times ar^9 \Rightarrow pr = a^2r^{12} \Rightarrow pr = (ar^6)^2 \Rightarrow pr = q^2$
6. **(d)** $2a = 1 + P$ and $g^2 = P$
$\Rightarrow g^2 = 2a - 1 \Rightarrow 1 - 2a + g^2 = 0$
7. **(c)** $\frac{a}{b}$ or $\frac{b}{c}$
8. **(c)** First term + last term
9. **(b)** Let a be the first term and r be the common ratio so, general term of G.P is $T_n = ar^{n-1}$
As given,
$T_1 = x^{-4} = a$ and, $T_8 = ar^7 = x^{52} \therefore ar^7 = x^{52}$
$\Rightarrow x^{-4} r^7 = x^{52} \Rightarrow r^7 = x^{56}$
$\Rightarrow r^7 = (x^8)^7 \Rightarrow r = x^8$
$\therefore T_2 = ar^1 = x^{-4}.x^8$
$T_2 = x^4$
But $T_2 = t\,x \Rightarrow x^t = x^4 \Rightarrow t = 4$
10. **(a)** Let R be the common ratio of this GP and a be the first term. pth term is aR^{p-1}, qth term is aR^{q-1} and rth term is aR^{r-1}.
Since p, q and r are in G.P. then
$(aR^{q-1})^2 = aR^{p-1}.\ aR^{r-1}$
$\Rightarrow a^2R^{2q-2} = a^2R^{p+r-2}$
$\Rightarrow R^{2q-2} = R^{p+r-2}$
$\Rightarrow 2q - 2 = p + r - 2$
$\Rightarrow 2q = p + r \Rightarrow$ p, q, r are in A.P.
11. **(d)** None of the options a, b or c satisfy the condition.
12. **(a)** Let first term and common difference of an AP are a and d respectively.
Its p^{th} term $= a + (p-1)d = q$ ...(i)
and q^{th} term $= a + (q-1)d = p$ ...(ii)
Solving eqs. (i) and (ii), we find
$a = p + q - 1,\ d = -1$
13. **(a)** Given that a, b, c, are in GP.
Let r be common ratio of GP.
So, a = a, b = ar and $c = ar^2$
Also, given that a, 2b, 3c are in AP.
$\Rightarrow 2b = \frac{a+3c}{2}$
$\Rightarrow 4b = a + 3c$ ... (i)
From eq. (i)
$4ar = a + 3ar^2$
$\Rightarrow 3r^2 - 4r + 1 = 0$
$\Rightarrow 3r^2 - 3r - r + 1 = 0$
$\Rightarrow 3r(r-1) - 1(r-1) = 0$
$\Rightarrow (r-1)(3r-1) = 0$
$\Rightarrow r = 1$ or $r = \frac{1}{3}$
14. **(c)** As given 1, x, y, z, 16 are in geometric progression.
Let common ratio be r,
$x = 1.\ r = r$
$y = 1.\ r^2 = r^2$
$z = 1.\ r^3 = r^3$
and $16 = 1.r^4 \Rightarrow 16 = r^4$
$\Rightarrow r = 2$
$\therefore x = 1.r = 2,\ y = 1.r^2 = 4, z = 1.r^3 = 8$
$\therefore x + y + z = 2 + 4 + 8 = 14$
15. **(d)** Let a be the first term and r, the common ratio
First nine terms of a GP are a, ar, ar^2, ... ar^8.
$\therefore P = a.ar.\ ar^2 \ldots\ ar^8$
$= a^9.r^{1+2+\ldots+8}$
$= a^9.r^{\frac{8.9}{2}} = a^9r^{36}$
$= (ar^4)^9 = (T_5)^9$
= 9th power of the 5th term

16. (d) For a G. P, $a_{m+n} = p$ and $a_{m-n} = q$,
We know that $a_n = AR^{n-1}$ (in G.P.)
where A = first term and R = ratio

$\therefore\ a_{m+n} = p$

$\Rightarrow\ AR^{m+n-1} = p$ ...(i)

and $a_{m-n} = q$

$\Rightarrow\ AR^{m-n-1} = q$ ...(ii)

On multiply equations (i) and (ii), we have

$(AR^{m+n-1}).(AR^{m-n-1}) = pq$

$\Rightarrow\ A^2.R^{2(m-1)} = pq$

$\Rightarrow (AR^{m-1})^2 = pq$

$\Rightarrow AR^{m-1} = \sqrt{pq}$

$\Rightarrow\ a_m = \sqrt{pq}$

17. (c) The following consecutive terms

$\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ are in A.P because

$2\left(\frac{1}{1-x}\right) = \frac{1}{1+\sqrt{x}} + \frac{1}{1-\sqrt{x}} = \frac{2}{1-x}$

(i.e. 2b = a + c)

18. (b) Consider series $(\sqrt{2}+1), 1, (\sqrt{2}-1)$,........

$a = \sqrt{2}+1, r = \sqrt{2}-1$

Common ratios of this series are equal. Therefore series is in G.P.

19. (b) In G.P., let the three numbers be $\frac{a}{r}, a, ar$

If the middle number is double, then new numbers are in A.P.

i.e., $\frac{a}{r}, 2a, ar$, are in A.P.

$\therefore\quad 2a - \frac{a}{r} = ar - 2a$

$\Rightarrow\ a\left[2 - \frac{1}{r}\right] = a[r-2]$

$\Rightarrow\ 2 - \frac{1}{r} = r - 2$

$\Rightarrow\ r + \frac{1}{r} = 4$

$\Rightarrow\ r^2 - 4r + 1 = 0$

$\Rightarrow\ r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

$\because\quad r < 1$ not possible

$\therefore\quad r = 2 + \sqrt{3}$

20. (c) Given, $\frac{2n}{2}\{2.2 + (2n-1)3\} = \frac{n}{2}\{2.57 + (n-1)2\}$

or $2(6n+1) = 112 + 2n$ or $10n = 110$

$\therefore\ n = 11$

21. (d) Let the means be X_1, X_2, X_3, X_4 and the common difference be b; then $2, X_1, X_2, X_3, X_4, -18$ are in A.P.;

$\Rightarrow -18 = 2 + 5b \Rightarrow 5b = -20 \Rightarrow b = -4$

Hence, $X_1 = 2 + b = 2 + (-4) = -2;$

$X_2 = 2 + 2b = 2 - 8 = -6$

$X_3 = 2 + 3b = 2 - 12 = -10;$

$X_4 = 2 + 4b = 2 - 16 = -14$

The required means are –2, –6, –10, –14.

22. (c) We take third observation as w

So, $x = \frac{y+z+w}{3}$

$\Rightarrow\ 3x = y + z + w$

$\Rightarrow\ w = 3x - y - z$

23. (d) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...$ can be written as

$1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + ...$

[$\because$ This is a GP with first term = 1

and common ratio = $-\frac{1}{2}$]

So, sum of the series

$= \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$

24. (b) $1/(q+r), 1/(r+p), 1/(p+q)$ are in A.P.

$\Rightarrow\ \frac{1}{r+p} - \frac{1}{q+r} = \frac{1}{p+q} - \frac{1}{r+p}$

$\Rightarrow\ q^2 - p^2 = r^2 - q^2$

$\Rightarrow\ p^2, q^2, r^2$ are in A.P.

25. (b) As given $G = \sqrt{xy}$

$\therefore \frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} = \frac{1}{xy - x^2} + \frac{1}{xy - y^2}$

$= \frac{1}{x-y}\left\{-\frac{1}{x} + \frac{1}{y}\right\} = \frac{1}{xy} = \frac{1}{G^2}$

26. (c) First five terms of given geometric progression are a, ar, ar^2 , ar^3, ar^4

A.M. of these five terms

$= \frac{a + ar + ar^2 + ar^3 + ar^4}{5} = \frac{a(r^5-1)}{5(r-1)}$

27. (b) Since p, q, r are in A.P.

$\therefore\ q = \frac{p+r}{2}$ ...(i)

Since a is the G.M. between p, q

$\therefore\ a^2 = pq$ ...(ii)

Since b is the G.M. between q, r

$\therefore\ b^2 = qr$ ...(iii)

From (ii) and (iii)

$p = \frac{a^2}{q},\ r = \frac{b^2}{q}$

$\therefore$ (i) gives $2q = \frac{a^2}{q} + \frac{b^2}{q}$

$\Rightarrow\ 2q^2 = a^2 + b^2 \Rightarrow a^2, q^2, b^2$ are in A.P.

28. (d) $T_n = [n^{th}$ term of 1.3.5......] × [n^{th} term of 3.5.7....]

or $T_n = [1 + (n-1) \times 2] \times [3 + (n-1) \times 2]$

or $T_n = (2n-1)(2n+1) = (4n^2 - 1)$

$$S_n = \sum T_n = \sum (4n^2 - 1)$$

$$= 4.\sum n^2 - \sum 1$$

$$= \frac{4 \times n(n+1)(2n+1)}{6} - n = \frac{2}{3} n(n+1)(2n+1) - n$$

29. (d) $$\frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2} \Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

$$\Rightarrow \frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

For $\frac{a_6}{a_{21}}$, $p = 11, q = 41 \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$

30. (b) Let the series a, ar, ar^2, ... are in geometric progression.

given, $a = ar + ar^2$

$\Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1 \pm \sqrt{1 - 4 \times -1}}{2} \Rightarrow r = \frac{-1 \pm \sqrt{5}}{2}$$

$$\Rightarrow r = \frac{\sqrt{5} - 1}{2}$$ [∵ terms of G.P. are positive ∴ r should be positive]

31. (b) As per question,

$a + ar = 12$ …(i)

$ar^2 + ar^3 = 48$ …(ii)

$$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$$

(∵ terms are +ve and –ve alternately)

$\Rightarrow a = -12$

32. (a) Let C be the required harmonic mean such that

$\frac{a}{1-ab}, C, \frac{a}{1+ab}$ are in H.P.

$\Rightarrow \frac{1-ab}{a}, \frac{1}{C}, \frac{1+ab}{a}$ are in A.P.

$$\Rightarrow \frac{2}{C} = \frac{1-ab}{a} + \frac{1+ab}{a} \Rightarrow \frac{2}{C} = \frac{2}{a} \Rightarrow C = a.$$

33. (b) Arithmetic mean between a and b is given by $\frac{a+b}{2}$

$$\therefore \frac{a+b}{2} = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$$

$\Rightarrow 2a^{n+1} + 2b^{n+1} = a^{n+1} + a^n b + b^n a + b^{n+1}$

$\Rightarrow (a^{n+1} - a^n b) + (b^{n+1} - ab^n) = 0$

$\Rightarrow a^n(a-b) + b^n(b-a) = 0$

$\Rightarrow (a^n - b^n)(a-b) = 0$

$\Rightarrow a^n - b^n = 0 \quad (\because a - b \neq 0)$

$$\Rightarrow \left(\frac{a}{b}\right)^n = 1 \Rightarrow \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^0$$

$\Rightarrow n = 0$

34. (d) Let α and β be the root of equation

$x^2 - 10x + 11 = 0$

$\therefore \alpha + \beta = 10, \alpha\beta = 11$

$$\therefore HM = \frac{2\alpha\beta}{\alpha+\beta} = \frac{2.11}{10} = \frac{+22}{10} = \frac{11}{5}$$

35. (d) Let the means be $x_1, x_2, ...x_m$ so that $1, x_1, x_2, ...x_m$, 31 is an A.P. of (m + 2) terms.

Now, $31 = T_{m+2} = a + (m+1)d = 1 + (m+1)d$

$\therefore d = \frac{30}{m+1}$ Given : $\frac{x_7}{x_{m-1}} = \frac{5}{9}$

$$\therefore \frac{T_8}{T_m} = \frac{a + 7d}{a + (m-1)d} = \frac{5}{9}$$

$\Rightarrow 9a + 63d = 5a + (5m - 5)d$

$$\Rightarrow 4.1 = (5m - 68)\frac{30}{m+1}$$

$\Rightarrow 2m + 2 = 75m - 1020 \Rightarrow 73m = 1022$

$$\therefore m = \frac{1022}{73} = 14$$

36. (b) Since, S_n denote the sum of an A.P. series.

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$ where 'a' is the first term and 'd' is the common difference of an A.P.

Given, $S_{2n} = 3S_n$

Now, $S_{2n} = \frac{2n}{2}[2a + (2n-1)d]$

∴ From given equation, we have

$$\frac{2n}{2}[2a + (2n-1)d] = \frac{3n}{2}[2a + (n-1)d]$$

$\Rightarrow 2[2a + (2n-1)d] = 3[2a + (n-1)d]$

$\Rightarrow 4a + 2(2n-1)d = 6a + 3(n-1)d$

$\Rightarrow (4n-2)d = 2a + (3n-3)d$

$\Rightarrow 2a = (n+1)d$

Now, consider

$$\frac{S_{3n}}{S_n} = \frac{\frac{1}{2}(3n)[2a + (3n-1)d]}{\frac{1}{2}(n)[2a + (n-1)d]}$$

$$= \frac{\frac{3n}{2}[2a + (3n-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{3[2a + 3nd - d]}{[2a + nd - d]}$$

Put value of $2a = (n+1)d$, we get

$$\frac{S_{3n}}{S_n} = \frac{3[(n+1)d + 3nd - d]}{(n+1)d + nd - d}$$

$$= \frac{3[nd + d + 3nd - d]}{nd + d + nd - d} = \frac{3(4nd)}{2nd} = 6$$

STATEMENT TYPE QUESTIONS

37. **(d)** By definition, both the given statements are true.
38. **(b)**
39. **(c)** Both are statements are true.
40. **(b)** I. n^{th} term is $a_n = a + (n-1)d$
41. **(a)** II. Geometric mean of 'a' and 'b' = $\sqrt{ab}$
42. **(c)** All the given statements are true.
43. **(a)** Both the given statements are true
II. $a = 4, d = 5$
$a_n = 124 \Rightarrow a + (n-1)d = 124$
$\Rightarrow 4 + (n-1)5 = 124$
$\Rightarrow n = 25$
44. **(b)**
I. $a = 72, d = -2$
$a + (n-1)d = 40$
$\Rightarrow 72 + (n-1)(-2) = 40$
$\Rightarrow 2n = 34 \Rightarrow n = 17$
Hence, 17^{th} term is 40.
II. $a = 8 - 6i, d = -1 + 2i$
$a_n = (8 - 6i) + (n-1)(-1 + 2i)$
$= (9 - n) + i(2n - 8)$
a_n is purely real if $2n - 8 = 0 \Rightarrow n = 4$
Hence, 4^{th} term is purely real.
45. **(a)** I. $a = 3, d = 3$
$a + (n-1)d = 111 \Rightarrow 3 + (n-1)(3) = 111$
$\Rightarrow n = 37$
II. $a = 9, d = 3$
$a_n = a + (n-1)d = 9 + (n-1)3 = 3n + 6$
46. **(c)** I. $a.r^{n-1} = 5120 \Rightarrow 5(2^{n-1}) = 5120$
$\Rightarrow 2^{n-1} = 1024 \Rightarrow 2^{n-1} = 2^{10}$
$\Rightarrow n = 11$
II. Let α, β be the roots of the quadratic equation.
A.M. of $\alpha, \beta = \dfrac{\alpha+\beta}{2} = 8$;
G. M. of $\alpha, \beta = \sqrt{\alpha\beta} = 5 \Rightarrow \alpha\beta = 5^2$
$\alpha + \beta = 16, \ \alpha\beta = 25$
Equation whose roots are α, β, is
$x^2 - (\alpha+\beta)x + \alpha\beta = 0$
$x^2 - 16x + 25 = 0$

MATCHING TYPE QUESTIONS

47. **(a)** (A) $S_{20} = \dfrac{20}{2}[2 \times 1 + (20-1)3]$
$= 10 \times 59 = 590$
(B) $a + (n-1)d = 181$
$\Rightarrow 5 + (n-1)8 = 181 \Rightarrow n = 23$
$\therefore$ Required sum $= \dfrac{n}{2}[a + l] = \dfrac{23}{2}[5 + 181]$
$= 2139$
(C) 105, 112, 119, ..., 994
$a_n = 994 \Rightarrow a + (n-1)d = 994$
$\Rightarrow 105 + (n-1)7 = 994$
$\Rightarrow n = 128$
$\therefore$ Required sum $= \dfrac{128}{2}[2 \times 105 + (128-1)7]$
$= 70336$
(D) 252, 255, 258, ..., 999
$a_n = 999 \Rightarrow 252 + (n-1)3 = 999$
$\Rightarrow n = 250$
$S_n = \dfrac{250}{2}[252 + 999] = 156375$

48. **(d)** (A) $S_7 = a\left(\dfrac{r^7-1}{r-1}\right) = 3\left(\dfrac{2^7-1}{2-1}\right)$
$= 3(128-1) = 381$
(B) $S_{10} = 1\left[\dfrac{\left(\frac{1}{2}\right)^{10} - 1}{\left(\frac{1}{2}\right) - 1}\right] = 2\left(1 - \dfrac{1}{210}\right)$
$= \dfrac{1024-1}{512} = \dfrac{1023}{512}$
(C) $a = 2, r = 3, l = 4374$
Required sum $= \dfrac{lr - a}{r-1} = \dfrac{(4374 \times 3) - 2}{3-1}$
$= 6520$
(D) $S_n = 11 + 103 + 1005 + \ldots$ to n terms
$= (10+1) + (10^2+3) + (10^3+5) + \ldots + \{10^n + (2n-1)\}$
$= (10 + 10^2 + 10^3 + \ldots + 10^n) + \{1 + 3 + 5 + \ldots + (2n-1)\}$
$= \dfrac{10(10^n-1)}{(10-1)} + \dfrac{n}{2}(1 + 2n - 1)$
$= \dfrac{10}{9}(10^n - 1) + n^2$

49. **(c)** (A) $S = \dfrac{a}{1-r} = \dfrac{-\frac{5}{4}}{1-\left(-\frac{1}{4}\right)} = -1$
(B) $6^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\infty\right)} = 6^{\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)} = 6^1 = 6$
(C) $S_\infty = 6 \Rightarrow \dfrac{2}{1-r} = 6 \Rightarrow r = \dfrac{2}{3}$
(D) $a_n = 2[a_{n+1} + a_{n+2} + \ldots\infty] \ \forall \ n \in N$
$\Rightarrow a.r^{n-1} = 2[a.r^n + a.r^{n+1} + \ldots\infty]$
$= \dfrac{2.a.r^n}{1-r}$
$\Rightarrow 1 = \dfrac{2r}{1-r} \Rightarrow r = \dfrac{1}{3}$

50. (a) $a_n = n(n+2)$

For $n = 1$, $a_1 = 1(1+2) = 3$

For $n = 2$, $a_2 = 2(2+2) = 8$

For $n = 3$, $a_3 = 3(3+2) = 15$

For $n = 4$, $a_4 = 4(4+2) = 24$

For $n = 5$, $a_5 = 5(5+2) = 35$

Thus first five terms are 3, 8, 15, 24, 35.

51. (d) Here $a_n = \frac{2n-3}{6}$

Putting $n = 1, 2, 3, 4, 5$, we get

$$a_1 = \frac{2\times 1-3}{6} = \frac{2-3}{6} = \frac{-1}{6};$$

$$a_2 = \frac{2\times 2-3}{6} = \frac{4-3}{6} = \frac{1}{6};$$

$$a_3 = \frac{2\times 3-3}{6} = \frac{6-3}{6} = \frac{3}{6} = \frac{1}{2};$$

$$a_4 = \frac{2\times 4-3}{6} = \frac{8-3}{6} = \frac{5}{6};$$

and $a_5 = \frac{2\times 5-3}{6} = \frac{10-3}{6} = \frac{7}{6}$

$\therefore$ The first five terms are $-\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$ and $\frac{7}{6}$

INTEGER TYPE QUESTIONS

52. (a) Let a, be the first term and d, the common difference.

General term (n_{th} term) of the AP is

$T_n = a + (n-1)d$

As given, $T_j = a + (j-1)d = k$ ....(i)

$T_k = a + (k-1)d = j$ ....(ii)

Subtracting (ii) from (i), we get

$(j-k)d = k - j \Rightarrow d = -1$

On putting $d = -1$ in (i), we get

$a + (j-1)(-1) = k$

$\Rightarrow a = k + j - 1$

Now, $T_{k+j} = a + (k+j-1)d = k + j - 1 + [(k+j) - 1](-1)$

$= (k+j-1) - (k+j-1) = 0$

53. (c) $a_n = 2^n \Rightarrow a_3 = 2^3 = 8$

54. (b) For n = 1, $\frac{a_{n+1}}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$ $(\because a_1 = a_2 = 1)$

and $a_n = a_{n-1} + a_{n-2}$, $n > 2$ ...(A)

n = 3 in equation (A) $a_3 = a_2 + a_1 = 1 + 1 = 2$

for n = 2, $\frac{a_{n+1}}{a_n} = \frac{a_3}{a_2} = \frac{2}{1} = 2;$

55. (c) $a = 25$, $d = 22 - 25 = -3$. Let n be the no. of terms

Sum = 116 ; Sum $= \frac{n}{2}[2a + (n-1)d]$

$$116 = \frac{n}{2}[50 + (n-1)(-3)]$$

or $232 = n[50 - 3n + 3] = n[53 - 3n]$

$= -3n^2 + 53n$

$\Rightarrow 3n^2 - 53 + 232 = 0$

$\Rightarrow (n-8)(3n-29) = 0$

$\Rightarrow n = 8$ or $n = \frac{29}{3}$, $n \neq \frac{29}{3}$ $\therefore n = 8$

$\therefore$ Now, $T_8 = a + (8-1)d = 25 + 7\times(-3)$

$= 25 - 21$

$\therefore$ Last term = 4

56. (a) Let a be the first term and d be the common difference of A.P.

Sum of first p terms $= \frac{p}{2}[2a + (p-1)d]$ ... (i)

Sum of first q terms $= \frac{q}{2}[2a + (q-1)d]$ ... (ii)

Equating (i) & (ii)

$$\frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$$

Transposing the term of R.H.S to L.H.S

or $2a(p-q) + p(p-1) - q(q-1)d = 0$

$\Rightarrow 2a(p-q) + [(p^2 - q^2 - (p-q)d] = 0$

or $2a(p-q) + (p-q)[(p+q) - d] = 0$

$\Rightarrow (p-q)[2a + (p+q-1)d] = 0$

$\Rightarrow 2a + (p+q-1)d = 0$ ...(iii)

$(\because p \neq q)$

Sum of first $(p+q)$ term $= \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2} \times 0 = 0$

$\therefore 2a + (p+q-1)d = 0$ [from (iii)]

57. (a) A. M. between a and b $= \frac{a+b}{2}$

$$\therefore \frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$$

$2a^n + 2b^n = a^n + ab^{n-1} + a^{n-1}b + b^n$

$\Rightarrow a^n a^{n-1}b - ab^{n-1} + b^n = 0$

$\Rightarrow a^{n-1}(a-b) - b^{n-1}(a-b) = 0$

$\Rightarrow (a-b)(a^n - b^{n-1}) + b^n = 0$ $[\because a \neq 0]$

$\Rightarrow a^{n-1} - b^{n-1} = 0 \Rightarrow a^{n-1} = b^{n-1}$

$$\left(\frac{a}{b}\right)^{n-1} = 1 = \left(\frac{a}{b}\right)^0 \Rightarrow n - 1 = 0 \Rightarrow n = 1$$

58. **(b)** The angles of a polygon of n sides form an A.P. whose first term is 120° and common difference is 5°.

The sum of interior angles

$$= \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[2\times 120+(n-1)5]$$

$$= \frac{n}{2}[240+5n-5] = \frac{n}{2}(235+5n)$$

Also the sum of interior angles = 180 × n – 360

$$\therefore \frac{n}{2}(235+5n) = 180n - 360$$

Multiplying by $\frac{2}{5}$, $n(47+n) = 2(36n-72)$

$n(47+n) = 72n - 144$

$\Rightarrow n^2 + (47-72)n + 144 = 0$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n-16)(n-9) = 0$

$\Rightarrow n \neq 16 \therefore n = 9$

59. **(b)** $a = \frac{1}{3}$, $r = \frac{1/9}{1/3} = \frac{1}{9}\times\frac{3}{1} = \frac{1}{3}$

Let $T_n = \frac{1}{19683} \Rightarrow ar^{n-1} = \frac{1}{19683}$

$\Rightarrow \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$

$\Rightarrow \left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^9 \Rightarrow n = 9$

60. **(b)** Let n be the number of terms of the G.P. 3, 3^2, 3^3, ... makes the sum = 120

we have $a = 3$, $r = 3$

$S = \frac{a(r^n-1)}{r-1}$, $r > 1$; Sum $= \frac{3(3^n-1)}{3-1} = 120$

or $\frac{3}{2}(3^n-1) = 120$

Multiplying both sides by $\frac{3}{2}$

$\therefore 3^n - 1 = 80$

$\therefore 3^n = 80+1 = 81 = 3^4 \Rightarrow n = 4$

$\therefore$ Required number of terms of given G. P. is 4

61. **(b)** $f(1) = 3, f(x+y) = f(x)f(y)$

$f(2) = f(1+1) = f(1)f(1) = 3.3 = 9$

$f(3) = f(1+2) = f(1)f(2) = 3.9 = 27$

$f(4) = f(1+3) = f(1)f(3) = 3.27 = 81$

Thus we have

$$\sum_1^n f(x) = f(1)+f(2)+f(3)+........+f(n) = 120$$

$\Rightarrow 3+9+27+.....$ to n term = 120

or $\frac{3(3^n-1)}{3-1} = 120$ $[a=3, r=3]$

$\therefore \frac{3(3^n-1)}{2} = 120 \Rightarrow 3^n - 1 = 120\times\frac{2}{3} = 80$

$3^n = 80+1 = 81 = 3^4 \Rightarrow n = 4$

62. **(c)** Let the G.P. be a, ar, ar^2, ...

$S = a + ar + ar^2 + ...+$ to $2n$ term

$$= \frac{a(r^{2n}-1)}{r-1}$$

The series formed by taking term occupying odd places is $S_1 = a + ar^2 + ar^4 + ...$ to n terms

$$S_1 = \frac{a\left[(r^2)^n - 1\right]}{r^2-1} \Rightarrow S_1 = \frac{a(r^{2n}-1)}{r^2-1}$$

Now, $S = 5S_1$

or $\frac{a(r^{2n}-1)}{r-1} = 5\frac{a(r^{2n}-1)}{r^2-1}$

$\Rightarrow 1 = \frac{5}{r+1}$

$\Rightarrow r+1 = 5 \therefore r = 4$

63. **(d)** $a\left(\frac{r^n-1}{r-1}\right) = 5461 \Rightarrow \frac{4^n-1}{4-1} = 5461$

$\Rightarrow 4^n = 4^7$

$\Rightarrow n = 7$

64. **(d)** $T_m = a + (m-1)d = \frac{1}{n}$ ...(i)

$T_n = a + (n-1)d = \frac{1}{m}$ ...(ii)

(i) - (ii) $\Rightarrow (m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow d = \frac{1}{mn}$

From (i) $a = \frac{1}{mn} \Rightarrow a - d = 0$

ASSERTION - REASON TYPE QUESTIONS

65. **(a)** The numbers $\frac{-2}{7}, x, \frac{-7}{2}$ will be in G.P.

If $\frac{x}{-\frac{2}{7}} = \frac{-\frac{7}{2}}{x} \Rightarrow x^2 = -\frac{7}{2}\times-\frac{2}{7} = 1 \Rightarrow x = \pm 1$

66. **(a)** Here $a = x^3$, $r = \frac{x^5}{x^3} = x^2, x \neq \pm 1$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{x^3(1-x^{2n})}{(1-x^2)}$$

67. **(b)** **Assertion:** $a_n = 4n - 3$

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Reason: $a_n = (-1)^{n-1}.n^3$

$a_9 = (-1)^{9-1}.(9)^3 = (-1)^8(729) = 729$

68. **(b)** Both are true but Reason is not the correct explanation for the Assertion.

69. **(a)** **Assertion:** $a_3 = 4 \Rightarrow ar^2 = 4$

$\therefore$ Product of first five terms $= a\ (ar)\ (ar^2)\ (ar^3)\ (ar^4)$

$= a^5.r^{10} = (ar^2)^5 = 4^5$

70. **(b)** **Assertion:** b + c, c + a, a + b will be in A.P.

if (c + a) – (b + c) = (a + b) – (c + a)

i.e. if 2b = a + c

i.e. if a, b, c are in A.P.

Reason: $10^a, 10^b, 10^c$ are in G.P. if $\frac{10^b}{10^a} = \frac{10^c}{10^b}$

i.e. if $10^{b-a} = 10^{c-b}$

i.e. if $b - a = c - b \Rightarrow 2b = a + c$ which is true.

71. **(a)** **Assertion:** $2k = \frac{2}{3} + \frac{5}{8} = \frac{16+15}{24}$

$2k = \frac{31}{24}$

$k = \frac{31}{24 \times 2} = \frac{31}{48}$

72. **(b)** **Assertion:** Let the sum of n term is denoted by S_n

$\therefore\ S_n = 3n^2 + 5n$

Put $n = 1, 2.\ \ T_1 = S_1 = 3.1^2 + 5.1 = 3 + 5 = 8;$

$S_2 = T_1 + T_2 = 3.\ 2^2 + 5\ .\ 2 = 12 + 10 = 22$

$\therefore\ T_2 = S_2 - S_1 = 22 - 8 = 14$

$\therefore$ Common difference $d = T_2 - T_1 = 14 - 8 = 6$

$a = 8,\ d = 6$

m^{th} term $= a + (m-1)d = 164 \Rightarrow 8 + (m-1)\ .\ 6 = 164$

$6m + 2 = 164 \Rightarrow 6m = 164 - 2 = 162$

$\therefore\ m = \frac{162}{6} = 27$

Reason: $T_n = ar^{n-1}$

$T_{20} = \frac{5}{2}\left(\frac{1}{2}\right)^{20-1} = \frac{5}{2}.\frac{1}{2^{19}} = \frac{5}{2^{20}}$

73. **(b)** **Assertion:** First factor of the terms are 2, 4, 6, ...

$\therefore$ First factor of n^{th} term $= 2n$ …(i)

Second factor of the term are 4, 6, 8 ....

$\therefore$ Second factor of n^{th} term

$= 4 + (n-1)2 = 2(n+1)$ …(ii)

$\therefore\ n^{th}$ term of the given series

$= 2n \times 2(n+1) = 4n(n+1)$

$\therefore$ putting $n = 20$

20^{th} term of the given series $= 4 \times 20 \times 21$

$= 80 \times 21 = 1680$

Reason: Let three number in A.P be $a - d,\ a,\ a + d$

Their sum $= a - d + a + a + d = 24$

$\Rightarrow 3a = 24$

$\Rightarrow a = 8$

Their product $= (a - d)\ (a)\ (a + d) = 440$

$a(a^2 - d^2) = 440 \Rightarrow 8\ (64 - d^2) = 440$

$\Rightarrow\ 64 - d^2 = 55 \Rightarrow d^2 = 64 - 55 = 9$

$\Rightarrow\ d = \pm 3$

Hence, the numbers are 8 – 3, 8, 8 + 3 or 8 + 3, 8, 8 – 3

i.e., 5, 8, 11, or 11, 8, 5

74. **(c)** **Assertion:** $T_k = 5k + 1$ Putting $k = 1, 2$

$T_1 = 5 \times 1 + 1 = 5 + 1 = 6$;

$T_2 = 5 \times 2 + 1 = 10 + 1 = 11$

$\therefore\ d = T_2 - T_1 = 11 - 6 = 5$

$a = 6,\ d = 5$

Sum of n term $= \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 6 + (n-1)5]$

$= \frac{n}{2}[12 + 5n - 5] = \frac{n(5n+7)}{2}$

Reason: We have to find the sum

105 + 110 + 115 + ...+ 995

Let 995 = n^{th} term

$\therefore\ a + [n-1]d = 995$ or $105 + [n-1]5 = 995$

Dividing by 5,

$21 + (n-1) = 199$ or $n = 199 - 20 = 179$

$\therefore$ 105 + 110 + 115 + ...+ 995

$= \frac{n}{2}[2a + (n-1)d]$

$= \frac{179}{2}[2 \times 105 + (179 - 1)5]$

$= \frac{179}{2}[2 \times 105 + 5 \times 178] = 98450$

75. **(a)** $\because\ \frac{S_n}{S'_n} = \frac{(7n+1)}{(4n+17)} = \frac{n(7n+1)}{n(4n+17)}$

$\therefore\ S_n = (7n^2 + n)\lambda,\ S'_n = (4n^2 + 17n)\lambda$

Then, $\frac{T_n}{T'_n} = \frac{S_n - S_{n-1}}{S'_n - S'_{n-1}} = \frac{7(2n-1)+1}{4(2n-1)+17} = \frac{14n-6}{8n+13}$

$\Rightarrow T_n : T'_n = (14n - 6) : (8n + 13)$

76. **(d)** We have, $S_n = 6n^2 + 3n + 1$

$\therefore S_1 = 6 + 3 + 1 = 10$

$S_2 = 24 + 6 + 1 = 31$

$S_3 = 54 + 9 + 1 = 64$ and so on.

So, $T_1 = 10$

$T_2 = S_2 - S_1 = 31 - 10 = 21$

$T_3 = S_3 - S_2 = 64 - 31 = 33$

So, the sequence is 10, 21, 33,...

Now, 21 – 10 = 11 and 33 – 21 = 12 $\neq$ 11

$\therefore$ The given series is not in A.P.

So, Assertion is false and Reason is true.

77. (a) Let the numbers be a and b.

Then, $A.M.=\frac{a+b}{2}=34 \Rightarrow a+b=68$ ...(i)

Also, $G.M.=\sqrt{ab}=16 \Rightarrow ab=256$ ...(ii)

Now, $a-b=\pm\sqrt{(a+b)^2-4ab}$

$=\pm\sqrt{(68)^2-4\times 256}=\pm\sqrt{4624-1024}=\pm\sqrt{3600}$

$\Rightarrow a-b=\pm 60$

$\therefore$ a – b = 60 or a – b = – 60 ...(iii)

when a – b = 60, then solving (i) and (iii), we get a = 64 and b = 4.

Then, numbers are 64 and 4.

When a – b = –60, then solving (i) and (iii), we get a = 4, b = 64

$\therefore$ Numbers are 4 and 64.

78. (b) $\frac{S_m}{S_n}=\frac{m^2}{n^2}$ (given)

Also

$$\frac{T_m}{T_n}=\frac{S_m-S_{m-1}}{S_n-S_{n-1}}=\frac{m^2-(m-1)^2}{n^2-(n-1)^2}=\frac{2m-1}{2n-1}$$

Substituting m = 5 and n = 2, we get

$$\frac{T_5}{T_2}=\frac{2(5)-1}{2(2)-1}=\frac{9}{3}=3$$

CRITICALTHINKING TYPE QUESTIONS

79. (d) Since, sum = 4

and second term = $\frac{3}{4}$

$\Rightarrow \frac{a}{1-r}=4$, and $ar=\frac{3}{4}$

$\Rightarrow \frac{a}{1-\frac{3}{4a}}=4$

$\Rightarrow (a-1)(a-3)=0$

$\Rightarrow a=1$ or $a=3$

80. (c) Let roots be α,β,γ and $a=a-d, b=a, c=a+d$.

Then $\alpha+\beta+\gamma=3a=-(-12)\Rightarrow a=4$

$\alpha\beta\gamma=a(a^2-d^2)=-(-28)\Rightarrow d=\pm 3$

81. (d) Clearly, the given progression is a G.P. with common ratio $r=2$.

$\therefore$ 4th term from the end $=\ell\left(\frac{1}{r}\right)^{4-1}$

$=(3072)\left(\frac{1}{2}\right)^{4-1}=384$

82. (a) As given : $a^x=b^y=c^z$

Let, $a^x=b^y=c^z=k$ (say)

$\Rightarrow a=k^{1/x}, b=k^{1/y}, c=k^{1/z}$

As given : a, b, c are in G.P.

$\Rightarrow b^2=ac$

i.e., $k^{2/y}=k^{1/x}k^{1/z}=k^{\left(\frac{1}{x}+\frac{1}{z}\right)}$

$\Rightarrow \frac{2}{y}=\frac{1}{x}+\frac{1}{z}$

$\Rightarrow \frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P.

83. (c) The given sereis $3-1+\frac{1}{3}-\frac{1}{9}.......\infty$ is in G.P.

Its common ratio $r=-\frac{1}{3}$ and first term a = 3

$$S_\infty=\frac{a}{1-r}=\frac{3}{1+\frac{1}{3}}=\frac{3\times 3}{4}=\frac{9}{4}$$

84. (d) Given : $5^{1+x}+5^{1-x},\frac{a}{2},5^{2x}+5^{-2x}$ are in A.P.

We know that if a, b, c are in A.P. then 2b = a + c

$\therefore\ 2.\frac{a}{2}=5^{1+x}+5^{1-x}+5^{2x}+5^{-2x}$

$\Rightarrow a=5.5^x+5(5^x)^{-1}+(5^x)^2+(5^x)^{-2}$

Let $5^x=t$

$\therefore\ a=5t+\frac{5}{t}+t^2+\frac{1}{t^2}$

$\Rightarrow a=t^2+\frac{1}{t^2}+5\left(t+\frac{1}{t}\right)$

$\Rightarrow a=\left(t+\frac{1}{t}\right)^2-2+5\left(t+\frac{1}{t}\right)$

Put $t+\frac{1}{t}=A$

$\therefore\ a=A^2+5A-2$ [add & subtract $\left(\frac{b}{2a}\right)^2$]

$\Rightarrow a=\left[A^2+5A-\left(\frac{5}{2}\right)^2\right]+\left(\frac{5}{2}\right)^2-2$

$\Rightarrow a=\left(A-\frac{5}{2}\right)^2+\frac{17}{4}$

$\Rightarrow a\ge\frac{17}{4}$.

85. (d) Since, product of n positive number is unity.

$\Rightarrow x_1x_2x_3.......x_n=1$ ...(i)

Using A.M. $\ge$ GM

$\Rightarrow \frac{x_1+x_2+.....+x_n}{n}\ge(x_1x_2.....x_n)^{\frac{1}{n}}$

$\Rightarrow x_1+x_n+.......+x_n\ge n\,(1)^{\frac{1}{n}}$ [From eq^n (i)]

$\Rightarrow$ Sum of n positive number is never less than n.

86. (c) We know that, the sum of infinite terms of GP is

$$S_\infty = \begin{cases} \dfrac{a}{1-r}, & |r|<1 \\ \infty \ , & |r|\ge 1 \end{cases}$$

$\therefore\ S_\infty = \dfrac{x}{1-r} = 5 \quad (\because |r|<1)$

or, $1-r = \dfrac{x}{5}$

$\Rightarrow\ r = \dfrac{5-x}{5}$ exists only when $|r|<1$

i.e., $-1 < \dfrac{5-x}{5} < 1$

or, $-10 < -x < 0$

or, $0 < x < 10$

87. (c) Sum of the n terms of the series

$\dfrac{1}{2}+\dfrac{3}{4}+\dfrac{7}{8}+\dfrac{15}{16}+.......$ upto n terms, can be written as

$\left(1-\dfrac{1}{2}\right)+\left(1-\dfrac{1}{4}\right)+\left(1-\dfrac{1}{8}\right)+\left(1-\dfrac{1}{16}\right)....$ upto n terms

$= n - \left(\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+.....+n \text{ terms}\right)$

$= n - \dfrac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}$

$= n + 2^{-n} - 1$

88. (c) Let us consider a G.P. $a, ar, ar^2, ...$with $2n$ terms.

We have $\dfrac{a(r^{2n}-1)}{r-1} = \dfrac{5a\left[(r^2)^n-1)\right]}{(r^2-1)}$

(Since common ratio of odd terms will be r^2 and number of terms will be n)

$\Rightarrow\ \dfrac{a(r^{2n}-1)}{r-1} = 5\dfrac{a(r^{2n}-1)}{(r^2-1)}$

$\Rightarrow a(r+1) = 5a$, i.e., $r = 4$

89. (b) Middle term = 6^{th} term = 30

$\Rightarrow a + 5d = 30$

$S_{11} = \dfrac{11}{2}[2a+10d] = \dfrac{11}{2}\times 2[a+5d] = 11\times 30 = 330$

90. (c) Let the G.P. be $1, r, r^2, ... \infty$

Given $x_n = 2(x_{n+1}+x_{n+2}+.....\text{ to }\infty)$

$\therefore\ x_n = 2\dfrac{x_{n+1}}{1-r}$ [common ratio is r]

$\therefore\ \dfrac{x_{n+1}}{x_n} = \dfrac{1-r}{2} \Rightarrow r = \dfrac{1-r}{2} \quad \therefore r = \dfrac{1}{3}$

The sum of required series is

$1+\dfrac{1}{3}+\left(\dfrac{1}{3}\right)^2+.....\infty = \dfrac{1}{1-\frac{1}{3}} = \dfrac{3}{2}$

91. (b) Let the last three numbers in A.P. be a, $a+6$, $a+12$, then the first term is also $a+12$.

But $a+12, a, a+6$ are in G.P.

$\therefore a^2 = (a+12)(a+6) \Rightarrow a^2 = a^2+18a+72$

$\therefore\ a = -4.$

$\therefore$ The numbers are 8, –4, 2, 8.

92. (b) $S_n = an^2+bn+c$

$\therefore S_{n-1} = a(n-1)^2+b(n-1)+c$ for $n\ge 2$

$\therefore\ t_n = S_n - S_{n-1}$

$= a\{n^2-(n-1)^2\}+b\{n-(n-1)\}$

$= a(2n-1)+b$

$\therefore\ t_n = 2an+b-a,\ n\ge 2$

$\therefore\ t_{n-1} = 2a(n-1)+b-a$ for $n\ge 3$

$\therefore\ t_n - t_{n-1} = 2a(n-n+1) = 2a$ for $n\ge 3$

$\therefore\ t_3-t_2 = t_4-t_3 = ...2a$

Now $t_2-t_1 = (S_2-S_1)-S_1 = S_2-2S_1$

$= (a.2^2+b.2+c)-\{a.1^2+b.1+c\}$

$= 2a-c \ne 2a$

$\therefore$ Series is arithmetic from the second term onwards.

93. (a) Sum of n terms of A.P with first term = a and common difference, = d is given by

$S_n = \dfrac{n}{2}[2a+(n-1)d]$

$\therefore\quad S_{10} = 5[2a+9d]$

$S_5 = \dfrac{5}{2}[2a+4d]$

According to the given condition,

$S_{10} = S_5 \Rightarrow 5[2a+9d] = 4\times\dfrac{5}{2}[2a+4d]$

$\Rightarrow\ 2a+9d = 2[2a+4d]$

$\Rightarrow\ 2a+9d = 4a+8d \Rightarrow d = 2a$

$\Rightarrow\ \dfrac{a}{d} = \dfrac{1}{2} \Rightarrow a:d = 1:2$

94. (c) As given, n^{th} term is

$T_n = 3n+7$

Sum of n term, $S_n = \sum T_n$

$= \sum(3n+7) = 3\sum n + 7\sum 1$

$= \dfrac{3n(n+1)}{2}+7n = n\left[\dfrac{3n+3+14}{2}\right]$

$= n\left[\dfrac{3n+17}{2}\right]$

Sum of 50 terms $= S_{50} = 50\left[\dfrac{3\times 50+17}{2}\right]$

$= 50\left[\dfrac{167}{2}\right] = 25\times 167 = 4175$

95. (d) Since x is A.M

$\Rightarrow \quad x = \frac{y+z}{2},$

$\Rightarrow \quad 2x = y + z$ ...(i)

and y, g_1, g_2, z ....are in G.P.

$\Rightarrow \quad \frac{g_1}{y} = \frac{g_2}{g_1} = \frac{z}{g_2}$

$\Rightarrow \quad g_1^2 = g_2 y$

$\Rightarrow \quad g_1^3 = g_1 g_2 y$ ...(ii)

Also, $g_2^2 = g_1 z$

$g_2^3 = g_1 g_2 z$ ...(iii)

$\Rightarrow \quad g_1^2 g_2^2 = g_1 g_2 yz$

$\Rightarrow \quad yz = g_1 g_2$ ...(iv)

Adding equations (ii) and (iii)

$g_1^3 + g_2^3 = y g_1 g_2 + z g_1 g_2 = g_1 g_2 (y+z)$

$= yz.2x = 2xyz$

96. (a) The given series is

$(1 \times 3) + (3 \times 5) + (5 \times 7) + ...$

Its general term is given by

$T_n = (2n-1)(2n+1) = 4n^2 - 1$

Sum of series $= 4\Sigma n^2 - \Sigma 1$

$S_n = \frac{4n(n+1)(2n+1)}{6} - n$

$S_n = n\left[\frac{2(2n^2+3n+1)}{3} - 1\right]$

$S_n = n\left[\frac{4n^2+6n+2-3}{3}\right]$

$S_n = \left[\frac{n(4n^2+6n-1)}{3}\right]$

For sum of first 50 terms of the series, n = 50,

$S_{50} = \frac{50[4(50)^2 + 6(50) - 1]}{3}$

$= \frac{50 \times (10000 + 300 - 1)}{3}$

$= \frac{50 \times 10299}{3} = 171650$

97. (b) We know that $\text{A.M.} = \frac{S_n}{n+1}$

Given sequence 1, 2, 4, 8, 16,....., 2^n.

$\Rightarrow \quad S_n = 1 + 2 + 2^2 + 2^3 + 2^4 + ... + 2^n$

$= \frac{2^{n+1}-1}{2-1} = 2^{n+1} - 1 \quad \left[\because S_n = \frac{a(r^n-1)}{(r-1)}\right]$

$\therefore \quad \text{A.M.} = \frac{2^{n+1}-1}{n+1}.$

98. (a) The first common term is 11.

Now the next common term is obtained by adding L.C.M. of the common difference 4 and 5, i.e., 20.

Therefore, 10^{th} common term = T_{10} of the AP whose $a = 11$ and $d = 20$

$T_{10} = a + 9d = 11 + 9(20) = 191$

99. (a) Given statement makes an AP series where, a = ` 135, d = ` 15 and S_n = ` 5550

Let total savings be ` 5550 in n years

So, $S_n = \frac{n}{2}[2a + (n-1)d]$

$5550 = \frac{n}{2}[2 \times 135 + (n-1)15]$

$\Rightarrow \quad 11100 = n[270 + 15n - 15]$

$\Rightarrow \quad 15n^2 + 255n - 11100 = 0$

$\Rightarrow \quad n^2 + 17n - 740 = 0$

$\Rightarrow \quad n^2 + 37n - 20n - 740 = 0$

$\Rightarrow \quad (n+37)(n-20) = 0$

$n = 20 \quad (\because n \neq -37)$

100. (c) a, b, c are in A.P. $\Rightarrow \quad 2b = a + c$

Now,

$e^{1/c} \times e^{1/a} = e^{(a+c)/ac} = e^{2b/ac} = (e^{b/ac})^2$

$\therefore e^{1/c}, \ e^{b/ac}, e^{1/a}$ in G.P. with common ratio

$= \frac{e^{b/ac}}{e^{1/c}} = e^{(b-a)/ac} = e^{d/(b-d)(b+d)}$

$= e^{d/(b^2-d^2)}$

[$\because$ a, b, c are in A.P. with common difference d

$\therefore$ b – a = c – b = d]

101. (a) $\frac{2}{\sqrt{c}+\sqrt{a}} = \frac{1}{\sqrt{b}+\sqrt{c}} + \frac{1}{\sqrt{a}+\sqrt{b}}$

$= \frac{2\sqrt{b}+\sqrt{a}+\sqrt{c}}{(\sqrt{b}+\sqrt{c})(\sqrt{a}+\sqrt{b})}$

$\Rightarrow 2\sqrt{ab} + 2b + 2\sqrt{ac} + 2\sqrt{bc}$

$= 2\sqrt{bc} + 2\sqrt{ac} + c + 2\sqrt{ab} + a$

$\Rightarrow 2b = a + c$

$\therefore$ a, b, c, are in A.P.

$\Rightarrow$ ax, bx, cx, are in A.P.

$\Rightarrow$ ax + 1, bx + 1, cx + 1, are in A.P.

$\Rightarrow 9^{ax+1}, 9^{bx+1}, 9^{cx+1}$ are in G.P.

102. (b) As x, y, z, are A.M. of a and b

$\therefore x + y + z = 3\left(\frac{a+b}{2}\right)$

$\therefore 15 = \frac{3}{2}(a+b) \Rightarrow a + b = 10$ ...(i)

Again $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are A.M. of $\frac{1}{a}$ and $\frac{1}{b}$

$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{3}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\therefore \frac{5}{3}=\frac{3}{2}.\frac{a+b}{ab}$

$\Rightarrow \frac{10}{9}=\frac{10}{ab} \Rightarrow ab=9$ ...(ii)

Solving (i) and (ii), we get
$a=9,\ 1,\ b=1,9$

103. (b) Given $2\sqrt{ab}=\frac{a+b}{2}$

$\Rightarrow \sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}=4$

$\Rightarrow t^2-4t+5=0$, where $\sqrt{\frac{a}{b}}=t$

$\therefore \quad t=2\pm\sqrt{3} \Rightarrow \sqrt{\frac{a}{b}}=2\pm\sqrt{3}$

$\therefore \quad \frac{a}{b}=\frac{(2\pm\sqrt{3})^2}{4-3}=\frac{(2\pm\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}$

$\therefore$ a : b $= 2+\sqrt{3} : 2-\sqrt{3}$

or $2-\sqrt{3} : 2+\sqrt{3}$

104. (d) We have $S_n=\frac{a(1-r^n)}{1-r}$

$\therefore \quad S_{2n-1}=\frac{a}{1-r}[1-r^{2n-1}]$

Putting 1, 2, 3,........, n for n in it and summing up we have

$S_1+S_3+S_5+..........+S_{2n-1}$

$=\frac{a}{1-r}$ [(1 +1 + ...n term) – (r +r^3 + r^5 + ...n term)]

$=\frac{a}{1-r}\left[n-\frac{r\left\{1-(r^2)^n\right\}}{1-r^2}\right]=\frac{a}{1-r}\left[n-r.\frac{1-r^{2n}}{1-r^2}\right]$

105. (b) We have ,

$S_1=\frac{n_1}{2}[2a+(n_1-1)d] \Rightarrow \frac{2S_1}{n_1}=2a+(n_1-1)d$

$S_2=\frac{n_2}{2}[2a+(n_2-1)d] \Rightarrow \frac{2S_2}{n_2}=2a+(n_2-1)d$

$S_3=\frac{n_3}{2}[2a+(n_3-1)d] \Rightarrow \frac{2S_3}{n_3}=2a+(n_3-1)d$

$\therefore \frac{2S_1}{n_1}(n_2-n_3)+\frac{2S_2}{n_2}(n_3-n_1)+\frac{2S_3}{n_3}(n_1-n_2)$

$=[2a+(n_1-1)d](n_2-n_3)+[2a+(n_2-1)d](n_3-n_1)$
$+[2a+(n_3-1)d](n_1-n_2)=0$

106. (c) We have $t_n=\frac{1^3+2^3+3^3+.......+n^3}{1+3+5+.....\text{upto n terms}}$

$=\frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\frac{n}{2}\{2+2(n-1)\}}=\frac{\frac{n^2(n+1)^2}{4}}{n^2}$

$=\frac{(n+1)^2}{4}=\frac{n^2}{4}+\frac{n}{2}+\frac{1}{4}$

$\therefore \quad S_n=\Sigma t_n=\frac{1}{4}\Sigma n^2+\frac{1}{2}\Sigma n+\frac{1}{4}\Sigma 1$

$=\frac{1}{4}\frac{n(n+1)(2n+1)}{6}+\frac{1}{2}\frac{n(n+1)}{2}+\frac{1}{4}n$

$S_{16}=\frac{16.17.33}{24}+\frac{16.17}{4}+\frac{16}{4}=446$

107. (b) If n is odd, the required sum is

$1^2+2.2^2+3^2+2.4^2+......+2.(n-1)^2+n^2$

$=\frac{(n-1)(n-1+1)^2}{2}+n^2$

[$\because$ (n–1) is even
$\therefore$ using given formula for the sum of (n–1) terms.]

$=\left(\frac{n-1}{2}+1\right)n^2=\frac{n^2(n+1)}{2}$

108. (a) $S_\infty=\frac{a}{1-r}$ where 'a' be the first term and r be the common ratio of G.P.

$\therefore \quad \frac{4}{3}=\frac{3/4}{1-r}$

$\Rightarrow \quad 1-r=\frac{3/4}{4/3} \quad \Rightarrow \quad 1-\frac{9}{16}=r \quad \Rightarrow \quad r=\frac{7}{16}$

109. (c) Let six term of H.P. $=\frac{1}{61}$

$\Rightarrow$ six term of AP = 61
Similarly tenth term of A.P. = 105
Let first term of AP is a and common diff. = d
$\therefore \quad a+5d=61$
and $a+9d=105$
solving these equation, we get
$a=6,\ d=11$

Hence, first term of H.P. $=\frac{1}{6}$

110. (c) Let x be the first term and y be the c.d. of corresponding A.P., then

$\frac{1}{a}=x+(p-1)y$ ...... (i)

$\frac{1}{b}=x+(q-1)y$ ...... (ii)

$\frac{1}{c}=x+(r-1)y$ ...... (iii)

Multiplying (i), (ii) and (iii) respectively by abc (q – r), abc (r – p), abc (p – q) and then adding, we get, bc (q – r) + ca (r – p)+ ab (p – q) = 0

111. (c) Let the GP be a, ar, ar^2,........, where $0 < r < 1$.
Then, $a + ar + ar^2 + ... = 3$
and $a^2 + a^2r^2 + a^2r^4 + ... = 9/2$.

$$\Rightarrow \frac{a}{1-r} = 3 \text{ and } \frac{a^2}{1-r^2} = \frac{9}{2}$$

$$\Rightarrow \frac{9(1-r)^2}{1-r^2} = \frac{9}{2} \Rightarrow \frac{1-r}{1+r} = \frac{1}{2} \Rightarrow r = \frac{1}{3}$$

Putting $r = \frac{1}{3}$ in $\frac{a}{1-r} = 3$, we get a = 2

Now, the required sum of the cubes is

$$a^3 + a^3r^3 + a^3r^6 + ... = \frac{a^3}{1-r^3} = \frac{8}{1-(1/27)} = \frac{108}{13}$$

112. (c) x,y,z are in G.P. $\Rightarrow y^2 = xz$ ......(i)
We have, $a^x = b^y = c^z = \lambda$ (say)
$\Rightarrow x \log a = y \log b = z \log c = \log \lambda$

$$\Rightarrow x = \frac{\log\lambda}{\log a}, y = \frac{\log\lambda}{\log b}, z = \frac{\log\lambda}{\log c}$$

Putting x,y,z in (i), we get

$$\left(\frac{\log\lambda}{\log b}\right)^2 = \frac{\log\lambda}{\log a}\cdot\frac{\log\lambda}{\log c}$$

$(\log b)^2 = \log a . \log c$
or $\log_a b = \log_b c \Rightarrow \log_b a = \log_c b$

113. (a) We have

$$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...\infty \quad ....(i)$$

Multiplying both sides by $\frac{1}{3}$, we get

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...\infty \quad ....(ii)$$

Subtracting eqn. (ii) from eqn. (i), we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1-\frac{1}{3}} = \frac{4}{3} \times \frac{3}{2} \Rightarrow S = 3$$

114. (d) Series $2, 2\frac{1}{2}, 3\frac{1}{3}$, ... are in H.P.

$\Rightarrow \frac{1}{2}, \frac{2}{5}, \frac{3}{10}$, ... will be in A.P.

Now, first term $a = \frac{1}{2}$

and common difference $d = -\frac{1}{10}$

So, 5^{th} term of the A.P. $= \frac{1}{2} + (5-1)\left(-\frac{1}{10}\right) = \frac{1}{10}$.

Hence, 5^{th} term in H.P. is 10.

115. (d) Considering corresponding A.P.
$a + 6d = 10$ and $a + 11d = 25$
$\Rightarrow d = 3, a = -8$
$\Rightarrow T_{20} = a + 19d = -8 + 57 = 49$

Hence, 20^{th} term of the corresponding H.P. $= \frac{1}{49}$.

116. (c)

$$\text{H.M.} = \frac{2\left(\frac{a}{1-ab}\right)\left(\frac{a}{1+ab}\right)}{\frac{a}{1-ab} + \frac{a}{1+ab}}$$

$$= \frac{2\left(\frac{a^2}{1-a^2b^2}\right)}{\frac{a}{1-ab} + \frac{a}{1+ab}} = \frac{2a^2}{2a} = a.$$

117. (a) It is a fundamental concept.

118. (d) Let $A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$.

Then, $G^2 = ab$ ...(i)

and $AH = \left(\frac{a+b}{2}\right)\cdot\frac{2ab}{a+b} = ab$ ...(ii)

From (i) and (ii), we have $G^2 = AH$

119. (a) Given that b^2, a^2, c^2 are in A.P.
$\therefore a^2 - b^2 = c^2 - a^2$
$\Rightarrow (a - b)(a + b) = (c - a)(c + a)$

$$\Rightarrow \frac{1}{b+c} - \frac{1}{a+b} = \frac{1}{c+a} - \frac{1}{b+c}$$

$$\Rightarrow \frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a} \text{ are in A.P.}$$

120. (c) A.M. $= \frac{a+b}{2} = A$ and G.M. $= \sqrt{ab} = G$

On solving a and b are given by the values $A \pm \sqrt{(A+G)(A-G)}$.

Trick: Let the numbers be 1, 9. Then, A = 5 and G = 3. Now, put these values in options.

Here, (c) $\Rightarrow 5 \pm \sqrt{8 \times 2}$, i.e. 9 and 1.

121. (c) Since the reciprocals of a and c occur on R.H.S., let us first assume that a, b, c are in H.P.

So, that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b} = d$, say

$\Rightarrow \frac{a-b}{ab} = d = \frac{b-c}{bc} \Rightarrow a - b = abd$ and $b - c = bcd$

Now, L.H.S. $= -\frac{1}{a-b} + \frac{1}{b-c} = -\frac{1}{abd} + \frac{1}{bcd}$

$= \frac{1}{bd}\left(\frac{1}{c} - \frac{1}{a}\right) = \frac{1}{bd}(2d) \Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ = R.H.S.

$\therefore$ a, b, c are in H.P. is verified.

Aliter: $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$

$\frac{1}{b-a} - \frac{1}{c} = \frac{1}{a} - \frac{1}{b-c}$

$\Rightarrow \frac{c-b+a}{c(b-a)} = \frac{b-c-a}{a(b-c)} \Rightarrow -\frac{1}{c(b-a)} = \frac{1}{a(b-c)}$

$\Rightarrow ac - bc = ab - ac \Rightarrow b = \frac{2ac}{a+c}$

$\therefore$ a, b, c are in H.P.

122. (b) Given that a, b, c are in A.P.

$\Rightarrow b = \frac{a+c}{2}$ ... (i)

and $b^2 = ad$ ... (ii)

Hence, a, a – b, d – c are in G.P. because

$(a - b)^2 = a^2 - 2ab + b^2 = a(a - 2b) + ad$

$\Rightarrow a(-c) + ad = ad - ac$.

123. (d) Given that $\frac{\text{H.M.}}{\text{G.M.}} = \frac{12}{13}$

$\Rightarrow \frac{\frac{2ab}{a+b}}{\sqrt{ab}} = \frac{12}{13}$ or $\frac{a+b}{2\sqrt{ab}} = \frac{13}{12}$

$\Rightarrow \frac{(a+b)+2\sqrt{ab}}{(a+b)-2\sqrt{ab}} = \frac{13+12}{13-12} = \frac{25}{1}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{5^2}{1} \Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{5}{1}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})} = \frac{5+1}{5-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{6}{4} \Rightarrow \left(\frac{a}{b}\right)^{\frac{1}{2}} = \frac{6}{4} \Rightarrow a : b = 9 : 4$

124. (c) We have H.M. $= \frac{2ab}{a+b}$ and G.M. $= \sqrt{ab}$

So, $\frac{\text{H.M.}}{\text{G.M.}} = \frac{4}{5} \Rightarrow \frac{\frac{2ab}{(a+b)}}{\sqrt{ab}} = \frac{4}{5}$

$\Rightarrow \frac{2\sqrt{ab}}{(a+b)} = \frac{4}{5} \Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+4}{5-4} \Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{9}{1}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{3}{1}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})} = \frac{3+1}{3-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{4}{2} \Rightarrow \left(\frac{a}{b}\right) = 2^2 = 4$

$\Rightarrow a : b = 4 : 1$

CHAPTER 10

STRAIGHT LINES

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Slope of non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by :

(a) $m = \frac{y_2 - y_1}{x_2 - x_1}$ (b) $m = \frac{x_2 - x_1}{y_2 - y_1}$

(c) $m = \frac{x_2 + x_1}{y_2 + y_1}$ (d) $m = \frac{y_2 + y_1}{x_2 + x_1}$

2. If a line makes an angle α in anti-clockwise direction with the positive direction of x-axis, then the slope of the line is given by :

(a) $m = \sin\alpha$ (b) $m = \cos\alpha$

(c) $m = \tan\alpha$ (d) $m = \sec\alpha$

3. The point (x, y) lies on the line with slope m and through the fixed point (x_0, y_0) if and only if its coordinates satisfy the equation $y - y_0$ is equal to ....

(a) $m(x - x_0)$ (b) $m(y - x_0)$

(c) $m(y - x)$ (d) $m(x - y_0)$

4. If a line with slope m makes x-intercept d. Then equation of the line is :

(a) $y = m(d - x)$ (b) $y = m(x - d)$

(c) $y = m(x + d)$ (d) $y = mx + d$

5. The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by :

(a) $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$ (b) $d = \frac{|Ax_1 - By_1 + C|}{\sqrt{A^2 + B^2}}$

(c) $d = \frac{\sqrt{A^2 + B^2}}{|Ax_1 + By_1 + C|}$ (d) $d = \frac{\sqrt{A^2 + B^2}}{|Ax_1 - By_1 + C|}$

6. Distance between the parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, is given by:

(a) $d = \frac{\sqrt{A^2 + B^2}}{|C_1 - C_2|}$ (b) $d = \frac{\sqrt{A^2 - B^2}}{|C_1 - C_2|}$

(c) $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$ (d) $d = \frac{|C_1 + C_2|}{\sqrt{A^2 + B^2}}$

7. The inclination of the line $x - y + 3 = 0$ with the positive direction of x-axis is

(a) 45° (b) 135° (c) –45° (d) –135°

8. Slope of a line which cuts off intercepts of equal lengths on the axes is

(a) –1 (b) 0 (c) 2 (d) $\sqrt{3}$

9. Which of the following lines is farthest from the origin?

(a) $x - y + 1 = 0$ (b) $2x - y + 3 = 0$

(c) $x + 2y - 2 = 0$ (d) $x + y - 2 = 0$

10. Equation of the straight line making equal intercepts on the axes and passing through the point (2, 4) is :

(a) $4x - y - 4 = 0$ (b) $2x + y - 8 = 0$

(c) $x + y - 6 = 0$ (d) $x + 2y - 10 = 0$

11. A line passes through $P(1, 2)$ such that its intercept between the axes is bisected at P. The equation of the line is

(a) $x + 2y = 5$ (b) $x - y + 1 = 0$

(c) $x + y - 3 = 0$ (d) $2x + y - 4 = 0$

12. The tangent of angle between the lines whose intercepts on the axes are $a, -b$ and $b, -a$ respectively, is

(a) $\frac{a^2 - b^2}{ab}$ (b) $\frac{b^2 - a^2}{2}$

(c) $\frac{b^2 - a^2}{2ab}$ (d) None of these

13. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2) then the equation of the line will be

(a) $2x + 3y = 12$ (b) $3x + 2y = 12$

(c) $4x - 3y = 6$ (d) $5x - 2y = 10$

14. The intercept cut off by a line from y-axis twice than that from x-axis, and the line passes through the point (1, 2). The equation of the line is

(a) $2x + y = 4$ (b) $2x + y + 4 = 0$

(c) $2x - y = 4$ (d) $2x - y + 4 = 0$

15. Line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and $(x, 24)$. Find the value of x.

(a) 2 (b) 3 (c) 4 (d) 5

16. Let the perpendiculars from any point on the line $7x + 56y = 0$ upon $3x + 4y = 0$ and $5x - 12y = 0$ be p and p', then

(a) $2p = p'$ (b) $p = 2p'$

(c) $p = p'$ (d) None of these

17. The lines $x + 2y - 5 = 0$, $2x - 3y + 4 = 0$, $6x + 4y - 13 = 0$

(a) are concurrent.

(b) form a right angled triangle.

(c) form an isosceles triangle.

(d) form an equilateral triangle.

18. A triangle ABC is right angled at A has points A and B as (2, 3) and (0, –1) respectively. If $BC = 5$, then point C may be
(a) (–4, 2) (b) (4, –2) (c) (0, 4) (d) (0, –4)

19. The relation between a, b, a' and b' such that the two lines $ax + by = c$ and $a'x + b'y = c'$ are perpendicular is
(a) $aa' - bb' = 0$ (b) $aa' + bb' = 0$
(c) $ab + a'b' = 0$ (d) $ab - a'b' = 0$

20. The equation of a straight line which cuts off an intercept of 5 units on negative direction of y-axis and makes an angle of 120° with the positive direction of x-axis is
(a) $\sqrt{3}x + y + 5 = 0$ (b) $\sqrt{3}x + y - 5 = 0$
(c) $\sqrt{3}x - y - 5 = 0$ (d) $\sqrt{3}x - y + 5 = 0$

21. The equation of the straight line that passes through the point (3, 4) and perpendicular to the line $3x + 2y + 5 = 0$ is
(a) $2x + 3y + 6 = 0$ (b) $2x - 3y - 6 = 0$
(c) $2x - 3y + 6 = 0$ (d) $2x + 3y - 6 = 0$

22. Which one of the following is the nearest point on the line $3x - 4y = 25$ from the origin?
(a) (–1, –7) (b) (3, –4)
(c) (–5, –8) (d) (3, 4)

23. If the mid-point of the section of a straight line intercepted between the axes is (1, 1), then what is the equation of this line?
(a) $2x + y = 3$ (b) $2x - y = 1$
(c) $x - y = 0$ (d) $x + y = 2$

24. What is the angle between the two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$?
(a) 60° (b) 45° (c) 30° (d) 15°

25. If the points (x, y), (1, 2) and (–3, 4) are collinear, then
(a) $x + 2y - 5 = 0$ (b) $x + y - 1 = 0$
(c) $2x + y - 4 = 0$ (d) $2x - y + 10 = 0$

26. If p be the length of the perpendicular from the origin on the straight line $x + 2by = 2p$, then what is the value of b?
(a) $\frac{1}{p}$ (b) p (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

27. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is
(a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$
(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$
(d) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

28. The coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$ are
(a) (–6, 5) (b) (5, 6) (c) (–5, 6) (d) (6, 5)

29. The length of the perpendicular from the origin to a line is 7 and line makes an angle of 150° with the positive direction of y-axis then the equation of the line is
(a) $4x + 5y = 7$ (b) $-x + 3y = 2$
(c) $\sqrt{3}x - y = 10\sqrt{2}$ (d) $\sqrt{3}x + y = 14$

30. A straight line makes an angle of 135° with x-axis and cuts y-axis at a distance of – 5 from the origin. The equation of the line is
(a) $2x + y + 5 = 0$ (b) $x + 2y + 3 = 0$
(c) $x + y + 5 = 0$ (d) $x + y + 3 = 0$

31. The equation of a line through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$ is
(a) $2x + y - 5 = 0$ (b) $x - 3y + 6 = 0$
(c) $x + 2y - 7 = 0$ (d) $x + 3y + 8 = 0$

32. The lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other
(a) $a_1b_1 - b_1a_2 = 0$ (b) $a_1^2b_2 + b_1^2a_2 = 0$
(c) $a_1b_1 + a_2b_2 = 0$ (d) $a_1a_2 + b_1b_2 = 0$

33. If the coordinates of the points A and B be (3, 3) and (7, 6), then the length of the portion of the line AB intercepted between the axes is
(a) $\frac{5}{4}$ (b) $\frac{\sqrt{10}}{4}$ (c) $\frac{\sqrt{13}}{3}$ (d) None of these

34. The line $(3x - y + 5) + \lambda(2x - 3y - 4) = 0$ will be parallel to y-axis, if λ =
(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

35. The equation of a straight line passing through (–3, 2) and cutting an intercept equal in magnitude but opposite in sign from the axes is given by
(a) $x - y + 5 = 0$ (b) $x + y - 5 = 0$
(c) $x - y - 5 = 0$ (d) $x + y + 5 = 0$

36. The points A(1, 3) and C(5, 1) are the opposite vertices of rectangle. The equation of line passing through other two vertices and of gradient 2, is
(a) $2x + y - 8 = 0$ (b) $2x - y - 4 = 0$
(c) $2x - y + 4 = 0$ (d) $2x + y + 7 = 0$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

37. Consider the following statements about straight lines :
I. Slope of horizontal line is zero and slope of vertical line is undefined.
II. Two lines are parallel if and only if their slopes are equal.
III. Two lines are perpendicular if and only if product of their slope is – 1.
Which of the above statements are true ?
(a) Only I (b) Only II
(c) Only III (d) All the above

38. The distances of the point (1, 2, 3) from the coordinate axes are A, B and C respectively. Now consider the following equations:
I. $A^2 = B^2 + C^2$ II. $B^2 = 2C^2$
III. $2A^2C^2 = 13B^2$
Which of these hold(s) true?
(a) Only I (b) I and III (c) I and II (d) II and III

39. Consider the following statements.
I. Equation of the line passing through (0, 0) with slope m is $y = mx$
II. Equation of the x-axis is $x = 0$.

Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

40. Consider the following statements.
I. The distance between two points (x_1, y_1) and (x_2, y_2) is given by
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
II. The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2)
$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

41. Consider the following statements.
The three given points *A, B, C* are collinear i.e., lie on the same straight line, if
I. area of ΔABC is zero.
II. slope of AB = Slope of BC.
III. any one of the three points lie on the straight line joining the other two points.
Choose the correct option
(a) Only I is true (b) Only II is true
(c) Only III is true (d) All are true

42. Consider the following statements.
I. Slope of horizontal line is zero and slope of vertical line is undefined.
II. Two lines whose slopes are m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$
Choose the correct option.
(a) Both are true (b) Both are false
(c) Only I is true (d) Only II is true

43. Consider the following statements.
I. The length of perpendicular from a given point (x_1, y_1) to a line $ax + by + c = 0$ is
$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$
II. Three or more straight lines are said to be concurrent lines, if they meet at a point.
Choose the correct option
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

44. Consider the following statements.
I. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle then centroid is
$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
II. If the point $P(x, y)$ divides the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ (internally), then
$$x = \frac{mx_2 + nx_1}{m + n}, \; y = \frac{my_2 + ny_1}{m + n}$$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

45. Consider the following statements.
I. The equation of a straight line passing through the point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$
II. Equation of the y-axis is x = 0.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false.

46. Consider the following statements.
I. The equation of a straight line making intercepts *a* and *b* on *x* and *y*-axis respectively is given by
$\frac{x}{a} + \frac{y}{b} = 1$
II. If $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ be two parallel lines, then distance
between two parallel lines, $d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

47. Consider the following statements.
I. If (a, b), (c, d) and (a – c, b – d) are collinear, then bc – ad = 0
II. If the points A (1, 2), B (2, 4) and C (3, a) are collinear, then the length BC = 5 unit.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

48. Consider the following statements.
I. Centroid of a triangle is a point where angle bisectors meet.
II. If value of area after calculations is negative then we take its negative value.
Choose the correct option
(a) Only I is false (b) Only II is false
(c) Both are false (d) Both are true

49. Consider the following statements.
I. Two lines are parallel if and only if their slopes are equal.
II. Two lines are perpendicular if and only if product of their slopes is 1.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false.

50. Equation of a line is $3x - 4y + 10 = 0$
I. Slope of the given line is $\frac{3}{4}$.
II. x-intercept of the given line is $-\frac{10}{3}$.
III. y-intercept of the given line is $\frac{5}{2}$.
Choose the correct option.
(a) Only I and II are true
(b) Only II and III are true
(c) Only I and III are true
(d) All I, II and III are true

51. Consider the equation $\sqrt{3}x+y-8=0$

I. Normal form of the given equation is $\cos 30°x + \sin 30°\, y = 4$

II. Values of p and w are 4 and 30° respectively.

Choose the correct option.

(a) Only I is true (b) Only II is true

(c) Both are true (d) Both are false

52. Slope of the lines passing through the points

I. (3, – 2) and (– 1, 4) is $-\frac{3}{2}$

II. (3, – 2) and (7, – 2) is 0.

III. (3, – 2) and (3, 4) is 1.

Choose the correct option.

(a) Only I and III are true

(b) Only I and II are true

(c) Only II and III are true

(d) None of these

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

53. The value of x for which the points (x, –1), (2, 1) and (4, 5) are collinear, is

(a) 1 (b) 2 (c) 3 (d) 4

54. The distance of the point (–1, 1) from the line $12(x+6)=5(y-2)$ is

(a) 2 (b) 3 (c) 4 (d) 5

55. The perpendicular from the origin to the line $y = mx + c$ meets it at the point (–1, 2). Find the value of m + c.

(a) 2 (b) 3 (c) 4 (d) 5

56. The values of k for which the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is parallel to the x-axis, is

(a) 3 (b) 2 (c) 1 (d) 4

57. The line joining (–1, 1) and (5, 7) is divided by the line x + y = 4 in the ratio 1 : k. The value of 'k' is

(a) 2 (b) 4 (c) 3 (d) 1

58. If three points (h, 0), (a, b) and (0, k) lies on a line, then the value of $\frac{a}{h}+\frac{b}{k}$ is

(a) 0 (b) 1 (c) 2 (d) 3

59. Value of x so that 2 is the slope of the line through (2, 5) and (x, 3) is

(a) 0 (b) 1 (c) 2 (d) 3

60. What is the value of y so that the line through (3, y) and (2, 7) is parallel to the line through (– 1, 4) and (0, 6)?

(a) 6 (b) 7 (c) 5 (d) 9

61. Reduce the equation $\sqrt{3}x+y-8=0$ into normal form. The value of p is

(a) 2 (b) 3 (c) 4 (d) 5

62. The distance between the parallel lines 3x – 4y + 7 = 0 and 3x – 4y + 5 = 0 is $\frac{a}{b}$. Value of a + b is

(a) 2 (b) 5 (c) 7 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

63. **Assertion:** If θ is the inclination of a line l, then the slope or gradient of the line l is tan θ.

Reason: The slope of a line whose inclination is 90°, is not defined.

64. **Assertion:** The inclination of the line l may be acute or obtuse.

Reason: Slope of x-axis is zero and slope of y-axis is not defined.

65. **Assertion:** Slope of the line passing through the points (3, –2) and (3, 4) is 0.

Reason: If two lines having the same slope pass through a common point, then these lines will coincide.

66. **Assertion:** If A (– 2, –1), B (4, 0), C (3, 3) and D (–3, 2) are the vertices of a parallelogram, then mid-point of AC = Mid-point of BD

Reason: The points A, B and C are collinear ⇔ Area of ΔABC = 0.

67. **Assertion:** Pair of lines x + 2y – 3 = 0 and – 3x – 6y + 9 = 0 are coincident.

Reason: Two lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are coincident if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

68. **Assertion:** If the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, are parallel, then $\frac{a_1}{a_2}=\frac{b_1}{b_2}$.

Reason: If the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are perpendicular, then $a_1a_2-b_1b_2=0$.

69. **Assertion:** The equation of the line making intercepts a and b on x and y-axis respectively is

$$\frac{x}{a}+\frac{y}{b}=1$$

Reason: The slope of the line ax + by + c = 0 is $\frac{b}{a}$.

70. **Assertion:** The equation of a line parallel to the line ax + by + c = 0 is ax – by – λ = 0, where λ is a constant.

Reason: The equation of a line perpendicular to the line ax + by + c = 0 is bx – ay + λ = 0, where λ is a constant.

71. **Assertion:** The distance between the parallel lines 3x – 4y + 9 = 0 and 6x – 8y – 15 = 0 is $\frac{33}{10}$.

Reason: Distance between the parallel lines $Ax+By+C_1=0$ and $Ax+By+C_2=0$, is given by

$$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$$

72. **Assertion:** Equation of the horizontal line having distance 'a' from the x-axis is either $y = a$ or $y = -a$.
Reason: Equation of the vertical line having distance b from the y-axis is either $x = b$ or $x = -b$.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

73. In what ratio does the line $y - x + 2 = 0$ cut the line joining $(3, -1)$ and $(8, 9)$?
(a) 2 : 3 (b) 3 : 2 (c) 3 : –2 (d) 1 : 2

74. The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is:
(a) $\frac{3}{2}$ (b) $\frac{3}{10}$ (c) 6 (d) $\frac{9}{4}$

75. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y – intercept is:
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

76. If the area of the triangle with vertices (x, 0), (1, 1) and (0, 2) is 4 square unit, then the value of x is :
(a) – 2 (b) – 4 (c) – 6 (d) 8

77. The distance of the line $2x + y = 3$ from the point (–1, 3) in the direction whose slope is 1, is
(a) $\frac{2}{3}$ (b) $\frac{\sqrt{2}}{3}$
(c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{2\sqrt{5}}{3}$

78. The straight lines $x + 2y - 9 = 0$, $3x + 5y - 5 = 0$ and $ax + by = 1$ are concurrent if the straight line $35x - 22y + 1 = 0$ passes through :
(a) (a, b) (b) (b, a)
(c) (a, – b) (d) (– a, b)

79. The reflection of the point (4, – 13) in the line $5x + y + 6 = 0$ is
(a) (–1, –14) (b) (3, 4)
(c) (0, 0) (d) (1, 2)

80. If *a, b, c* are in A.P., then the straight lines $ax + by + c = 0$ will always pass through
(a) (1, – 2) (b) (1, 2)
(c) (– 1, 2) (d) (– 1, – 2)

81. What is the image of the point (2, 3) in the line $y = -x$?
(a) (–3, –2) (b) (– 3, 2)
(c) (–2, –3) (d) (3, 2)

82. If p be the length of the perpendicular from the origin on the straight line $ax + by = p$ and $b = \frac{\sqrt{3}}{2}$, then what is the angle between the perpendicular and the positive direction of x-axis?
(a) 30° (b) 45° (c) 60° (d) 90°

83. If (– 4, 5) is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is
(a) $x + 3y = 21$ (b) $2x - 3y = 7$
(c) $x + 7y = 31$ (d) $2x + 3y = 21$

84. A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). The co-ordinates of the point A is
(a) $\left(\frac{13}{5}, 0\right)$ (b) $\left(\frac{5}{13}, 0\right)$
(c) (–7, 0) (d) None of these

85. The vertices of a triangle ABC are (1, 1), (4, – 2) and (5, 5) respectively. Then equation of perpendicular dropped from C to the internal bisector of angle A is
(a) $y - 5 = 0$ (b) $x - 5 = 0$
(c) $2x + 3y - 7 = 0$ (d) None of these

86. The line L has intercepts a and b on the coordinate axes. When keeping the origin fixed, the coordinate axes are rotated through a fixed angle, then same line has intercepts p and q on the rotated axes, then
(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$
(c) $a^2 + \beta^2 = b^2 + q^2$ (d) $b^2 + q^2 = \frac{1}{b^2} + \frac{1}{q^2}$

87. The equation of two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point (1, – 10), then the equation of the third side is (are)
(a) $3x + y + 7 = 0$, $x - 3y - 31 = 0$
(b) $2x + y + 5 = 0$, $x - 2y + 3 = 0$
(c) $3x + y + 7 = 0$, $x + y = 0$
(d) $3x - y = 7$, $x + 3y = 15$

88. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for
(a) exactly one value of p
(b) exactly two values of p
(c) more than two values of p
(d) no value of p

89. The bisector of the acute angle formed between the lines $4x - 3y + 7 = 0$ and $3x - 4y + 14 = 0$ has the equation
(a) $x + y + 3 = 0$ (b) $x - y - 3 = 0$
(c) $x - y + 3 = 0$ (d) $3x + y - 7 = 0$

90. The equations of the lines which cuts off an intercept –1 from y-axis and equally inclined to the axes are
(a) $x - y + 1 = 0$, $x + y + 1 = 0$
(b) $x - y - 1 = 0$, $x + y - 1 = 0$
(c) $x - y - 1 = 0$, $x + y + 1 = 0$
(d) None of these

91. If the coordinates of the points A, B, C be (–1, 5), (0, 0) and (2, 2) respectively and D be the middle point of BC, then the equation of the perpendicular drawn from B to the line AD is
(a) $x + 2y = 0$ (b) $2x + y = 0$
(c) $x - 2y = 0$ (d) $2x - y = 0$

92. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(a) Above the x-axis at a distance of $\frac{3}{2}$ from it

(b) Above the x-axis at a distance of $\frac{2}{3}$ from it

(c) Below the x-axis at a distance of $\frac{3}{2}$ from it

(d) Below the x-axis at a distance of $\frac{2}{3}$ from it

93. Equation of angle bisector between the lines $3x + 4y - 7 = 0$ and $12x + 5y + 17 = 0$ are

(a) $\frac{3x + 4y - 7}{\sqrt{25}} = \pm\frac{12x + 5y + 17}{\sqrt{169}}$

(b) $\frac{3x + 4y + 7}{\sqrt{25}} = \frac{12x + 5y + 17}{\sqrt{169}}$

(c) $\frac{3x + 4y + 7}{\sqrt{25}} = \pm\frac{12x + 5y + 17}{\sqrt{169}}$

(d) None of these

94. The equation of the line which bisects the obtuse angle between the lines $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$, is

(a) $\left(4 - \sqrt{5}\right)x - \left(3 - 2\sqrt{5}\right)y + \left(2 - 4\sqrt{5}\right) = 0$

(b) $\left(4 + \sqrt{5}\right)x - \left(3 + 2\sqrt{5}\right)y + \left(2 + 4\sqrt{5}\right) = 0$

(c) $\left(4 + \sqrt{5}\right)x + \left(3 + 2\sqrt{5}\right)y + \left(2 + 4\sqrt{5}\right) = 0$

(d) None of these

95. Choose the correct statement which describe the position of the point $(-6, 2)$ relative to straight lines $2x + 3y - 4 = 0$ and $6x + 9y + 8 = 0$.

(a) Below both the lines (b) Above both the lines

(c) In between the lines (d) None of these

96. If A and B are two points on the line $3x + 4y + 15 = 0$ such that $OA = OB = 9$ units, then the area of the triangle OAB is

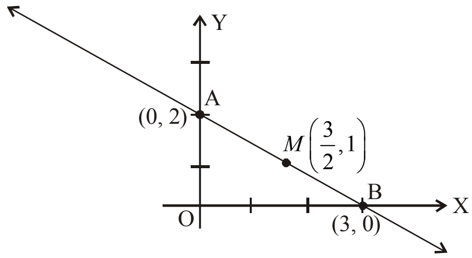
(a) 18 sq. units (b) $18\sqrt{2}$ sq. units

(c) $\frac{18}{\sqrt{2}}$ sq. units (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a) $m=\frac{y_2-y_1}{x_2-x_1}=\frac{y_1-y_2}{x_1-x_2},\ x_1\neq x_2$
2. (c) $m=\tan\alpha,\ \alpha\neq 90°$
3. (a) $y-y_0=m(x-x_0)$
4. (b) $y=m(x-d)$
5. (a) $d=\left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$
6. (c) $d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$
7. (a) The equation of the line $x-y+3=0$ can be rewritten as $y=x+3$
$\Rightarrow m=\tan\theta=1$ and hence $\theta=45°$.
8. (a) Equation of line in intercept form is $\frac{x}{a}+\frac{y}{a}=1$
($\because$ Intercept has equal length)
$\Rightarrow x+y=a$
$\Rightarrow y=-x+a$
$\Rightarrow$ slope $=-1$
9. (d) Let d_1, d_2, d_3, d_4 are distances of equations $x-y+1=0$, $2x-y+3=0$, $x+2y-2=0$ and $x+y-2=0$ respectively from the origin.
$$d_1=\left|\frac{-0+1}{\sqrt{1^2+(-1)^2}}\right|=\frac{1}{\sqrt{2}}$$
$$d_2=\left|\frac{2(0)-0+3}{\sqrt{2^2+(-1)^2}}\right|=\frac{3}{\sqrt{5}}$$
$$d_3=\left|\frac{1(0)+2(0)-2}{\sqrt{1^2+2^2}}\right|=\frac{2}{\sqrt{5}}$$
$$d_4=\left|\frac{0+0-2}{\sqrt{1^2+1^2}}\right|=\frac{2}{\sqrt{2}}=\sqrt{2}$$
Hence, line corresponding to d_4 (1.414) is farthest from origin.
10. (c) Let intercept on x-axis and y-axis be a and b respectively so that the equation of line is
$\frac{x}{a}+\frac{y}{b}=1$
But a = b [given]
so; x + y = a
Also it passes through (2, 4) (given)
Thus 2 + 4 = a
$\Rightarrow$ a = 6
Now, the reqd. equation of the straight line
x + y = 6
or, x + y – 6 = 0.
11. (d) We know that the equation of a line making intercepts a and b with x-axis and y-axis, respectively, is given by
$\frac{x}{a}+\frac{y}{b}=1.$
Here we have $1=\frac{a+0}{2}$ and $2=\frac{0+b}{2}$, which give $a=2$ and $b=4$. Therefore, the required equation of the line is given by
$\frac{x}{2}+\frac{y}{4}=1$ or $2x+y-4=0$
12. (c) Equations of lines are
$\frac{x}{a}+\frac{y}{-b}=1$ and $\frac{x}{b}+\frac{y}{-a}=1$
$\Rightarrow$ $bx-ay=ab$ and $ax-by=ab$
$\Rightarrow$ $m_1=\frac{b}{a}$ and $m_2=\frac{a}{b}$ (slopes)
$\therefore\ \tan\theta=\frac{\frac{b}{a}-\frac{a}{b}}{1+\frac{b}{a}\times\frac{a}{b}}=\frac{b^2-a^2}{2ab}$
13. (a) Equation of line AB is $\frac{y_2-y_1}{x_2-x_1}=\frac{y-y_1}{x-x_1}$



$\Rightarrow$ $-\frac{2}{3}=\frac{y-2}{x-3}$
$\Rightarrow$ $-2(x-3)=3(y-2)$
$\Rightarrow$ $2x+3y=12$
14. (a) Let the line make intercept 'a' on x-axis. Then, it makes intercept '$2a$' on y-axis. Therefore, the equation of the line is given by
$\frac{x}{a}+\frac{y}{2a}=1$
It passes through (1, 2), so, we have
$\frac{1}{a}+\frac{2}{2a}=1$ or $a=2$
Therefore, the required equation of the line is given by
$\frac{x}{2}+\frac{y}{4}=1$ or $2x+y=4$
15. (c) Slope of the line through the points (–2, 6) and (4, 8) is,
$m_1=\frac{8-6}{4-(-2)}=\frac{2}{6}=\frac{1}{3}$
Slope of the line through the points (8, 12) and (x, 24) is :
$m_2=\frac{24-12}{x-8}=\frac{12}{x-8}$

Since, two lines are perpendicular,
$m_1 m_2 = -1$, which gives

$$\frac{1}{3} \times \frac{12}{x-8} = -1$$

$\Rightarrow -x + 8 = 4 \Rightarrow 8 - 4 = x \Rightarrow x = 4$

16. (c) Any point on the line $7x + 56y = 0$ is

$$\left(x_1, -\frac{7x_1}{56}\right), \textit{i.e.}, \left(x_1, -\frac{x_1}{8}\right)$$

$\therefore$ The perpendicular distance p and p' are

$$p = \frac{3x_1 - \frac{4x_1}{8}}{5} = \frac{x_1}{2}$$

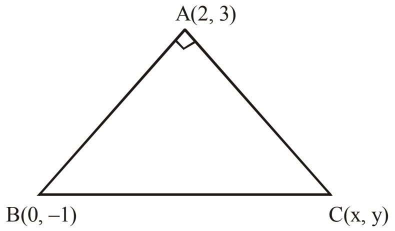
and $p' = \dfrac{5x_1 + \frac{12x_1}{8}}{13} = \dfrac{x_1}{2} \Rightarrow p = p'$

17. (b) Lines II and III are at right angles

$$\left[\because \left(\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1\right]$$

Lines I and II intersect at the point (1, 2) and (1, 2) does not belong to III. Hence, the lines are not concurrent, i.e., they form a right angled triangle.

18. (c) Slope of $AB = 2 \Rightarrow$ slope of $AC = -\dfrac{1}{2}$



$$\Rightarrow \frac{y-3}{x-2} = -\frac{1}{2} \Rightarrow x + 2y - 8 = 0 \quad \text{....(i)}$$

Also $x^2 + (y+1)^2 = 25$

$\Rightarrow (8-2y)^2 + (y+1)^2 = 25$ [from (i)]

$\Rightarrow y = 2$ *or* 4 and correspondingly $x = 4$ and $x = 0$.
Hence, C is (0, 4) or (4, 2).

19. (b) Slope of the line $ax + by = c$ is $\dfrac{-a}{b}$, and the slope of the line $a'x + b'y = c'$ is $\dfrac{-a'}{b'}$. The lines are perpendicular if $\left(\dfrac{-a}{b}\right)\left(\dfrac{-a'}{b'}\right) = -1$ or $aa' + bb' = 0$

20. (a) Here, m = tan 120º = tan (90 + 30º) = – cot 30º = $-\sqrt{3}$
and c = –5
So, the equation of the line is

$y = -\sqrt{3}\,x - 5$ [Using : $y = mx + c$]

$\Rightarrow \sqrt{3}\,x + y + 5 = 0$

21. (c) The equation of a line perpendicular to $3x + 2y + 5 = 0$ is
$2x - 3y + \lambda = 0$ ...(i)
This passes through the point (3, 4).
$\therefore 3 \times 2 - 3 \times 4 + \lambda = 0 \Rightarrow \lambda = 6$
Putting $\lambda = 6$ in (i), we get $2x - 3y + 6 = 0$, which is the required equation.

22. (b) Only two point A (–1, – 7) and B (3, 4) satisfy the given equation of the line 3x – 4y = 25
Distance of A (– 1, – 7) from the origin O.

$$= \sqrt{(0+1)^2 + (0+7)^2} = \sqrt{50} = 5\sqrt{2}$$

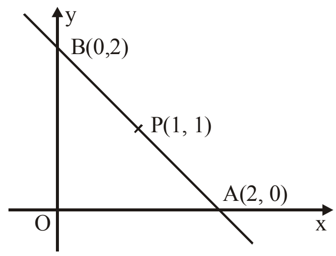
Distance of B (3, – 4) from the origin O.

$$= \sqrt{(0-3)^2 + (0+4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

The nearest point is (3, – 4)

23. (d) Let intercept on x-axis be a and that on y axis be b, the coordinate of these end points are (a, 0) and (b, 0).

Since, P(1, 1) is the mid point therefore $1 = \dfrac{a+0}{2}$ and $1 = \dfrac{0+b}{2} \Rightarrow a = 2, b = 2$.



So, equation of straight line is $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$\Rightarrow \dfrac{x}{2} + \dfrac{y}{2} = 1 \Rightarrow x + y = 2$

24. (a) The given lines are

$y = (2 - \sqrt{3})\, x + 5$

and $y = (2 + \sqrt{3})\, x - 7$

Therefore, slope of first line = $m_1 = 2 - \sqrt{3}$ and slope of second line = $m_2 = 2 + \sqrt{3}$

$$\therefore \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2 + \sqrt{3} - 2 + \sqrt{3}}{1 + (4-3)}\right|$$

$$= \left|\frac{2\sqrt{3}}{2}\right| = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3} = 60°$$

25. (a) If (x, y), (1, 2) and (–3, 4) are collinear then slope of line joining (x, 4) and (1, 2) is same as line joining points (1, 2) and (–3, 4) or line joining (x, 4) to (– 3, 4).

So, $\dfrac{2-y}{1-x} = \dfrac{4-2}{-3-1} = \dfrac{4-y}{-3-x}$

$\Rightarrow \dfrac{2-y}{1-x} = -\dfrac{1}{2} \Rightarrow \dfrac{y-2}{1-x} = \dfrac{1}{2}$

$\Rightarrow -4 + 2y = 1 - x \Rightarrow x + 2y - 5 = 0$

26 . (d) Length of perpendicular from the origin on the straight line x + 2by – 2p = 0 is

$$\left|\frac{0 + 2b \times 0 - 2p}{\sqrt{1^2 + (2b)^2}}\right| = p$$

or $p = \left|\dfrac{-2p}{\sqrt{1^2+4b^2}}\right|$ or $p^2 = \dfrac{4p^2}{1+4b^2}$

$\Rightarrow \dfrac{4}{1+4b^2} = 1$

$\Rightarrow 1+4b^2 = 4$ or $4b^2 = 3 \Rightarrow b^2 = \dfrac{3}{4}$

$\Rightarrow b = \pm\dfrac{\sqrt{3}}{2}$

$\Rightarrow b = \dfrac{\sqrt{3}}{2}$ matches with the given option.

27. (a) Let the required line be $\dfrac{x}{a}+\dfrac{y}{b}=1$ ....(i)

then $a+b=-1$ ....(ii)

(i) passes through (4, 3), $\Rightarrow \dfrac{4}{a}+\dfrac{3}{b}=1$

$\Rightarrow 4b+3a=ab$ ....(iii)

Eliminating b from (ii) and (iii), we get

$a^2-4=0 \Rightarrow a=\pm 2 \Rightarrow b=-3$ or 1

$\therefore$ Equations of straight lines are

$\dfrac{x}{2}+\dfrac{y}{-3}=1$ or $\dfrac{x}{-2}+\dfrac{y}{1}=1$

28. (b) Let (h, k) be the coordinates of the foot of the perpendicular from the point (2, 3) on the line $x+y-11=0$. Then, the slope of the perpendicular line is $\dfrac{k-3}{h-2}$. Again the slope of the given line $x+y-11=0$ is -1

Using the condition of perpendicularity of lines, we have

$\left(\dfrac{k-3}{h-2}\right)(-1)=-1$

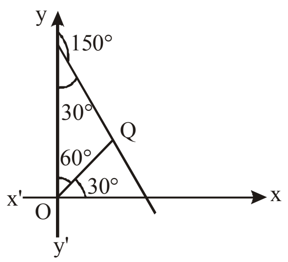
or $k-h=1$ ...(i)

Since (h, k) lies on the given line, we have,

$h+k-11=0$ or $h+k=11$ ...(ii)

Solving (i) and (ii), we get $h=5$ and $k=6$. Thus (5, 6) are the required coordinates of the foot of the perpendicular.

29. (d) Here p = 7 and $\alpha = 30°$



$\therefore$ Equation of the required line is

$x\cos 30° + y\sin 30° = 7$

or $x\dfrac{\sqrt{3}}{2}+y\times\dfrac{1}{2}=7$

or $\sqrt{3}x+y=14$

30. (c) The equation of a line making an angle θ with positive x-axis and cutting intercept c on y-axis is given by $y=\tan\theta\ x+c$

Here, $\theta=135° \Rightarrow \tan\theta=-1$ and $c=-5$

$\therefore y=-x-5 \Rightarrow x+y+5=0$

31. (a) Let the required line by method $P+\lambda Q=0$ be $(x-3y+1)+\lambda(2x+5y-9)=0$

$\therefore$ Perpendicular from (0, 0) $=\sqrt{5}$ gives

$\dfrac{1-9\lambda}{\sqrt{(1+2\lambda)^2+(5-3\lambda)^2}}=\sqrt{5}$

Squaring and simplifying, $(8\lambda-7)^2=0 \Rightarrow \lambda=7/8$

Hence the line required is

$(x-3y+1)+7/8\,(2x+5y-9)=0$

or $22x+11y-55=0 \Rightarrow 2x+y-5=0$

32. (d) The two lines having the slopes m_1 and m_2 are perpendicular iff $m_1.m_2=-1$

Now $a_1x+b_1y+c_1=0$

$\Rightarrow y=\dfrac{-a_1}{b_1}x-\dfrac{-c_1}{b_1} \Rightarrow$ slope $(m_1)=\dfrac{-a_1}{b_1}$

Similarly, $a_2x+b_2y+c_2=0$

Gives the slope, $m_2=\dfrac{-a_2}{b_2}$

Now, we know the lines $\perp$ when $m_1 . m_2=-1$

$\Rightarrow \dfrac{-a_1}{b_1}\cdot\dfrac{-a_2}{b_2}=-1$

$\Rightarrow a_1a_2=-b_1b_2 \Rightarrow a_1a_2+b_1b_2=0.$

33. (a) Equation of line AB is $y-3=\dfrac{6-3}{7-3}(x-3)$

$\Rightarrow 3x-4y+3=0 \Rightarrow \dfrac{x}{-1}+\dfrac{y}{\frac{3}{4}}=1$

Hence, required length is $\sqrt{(-1)^2+\left(\dfrac{3}{4}\right)^2}=\dfrac{5}{4}.$

34. (b) The given line can be written in this form

$(3+2\lambda)x+(-1-3\lambda)y+(5-4\lambda)=0$

It will be parallel to y-axis, if

$-1-3\lambda=0 \Rightarrow \lambda=-\dfrac{1}{3}.$

35. (a) Let the equation be $\dfrac{x}{a}+\dfrac{y}{-a}=1$

$\Rightarrow x-y=a$

But it passes through (–3, 2), hence a = –3 – 2 = –5.

Hence, the equation is x – y + 5 = 0.

36. (b) Mid point ≡ (3, 2). Equation is 2x – y – 4 = 0.

STATEMENT TYPE QUESTIONS

37. (d)

38. (d) Given: A = distance of point from x-axis

$A^2=2^2+3^2=4+9=13$

$B^2=3^2+1^2=9+1=10$

$C^2=1^2+2^2=1+4=5$

From above, we get
$B^2 = 10 = 2 \times 5 = 2C^2$
$\Rightarrow \; B^2 = 2C^2 \qquad [\because C^2 = 5]$
and $2A^2C^2 = 2.13.5 = 13.10 = 13B^2 \qquad [\because B^2 = 10]$
$\Rightarrow \; 2A^2C^2 = 13B^2$

39. (a) I. Equation of line is
$y - 0 = m(x - 0)$
$\Rightarrow y = mx$
II. Equation of the x-axis is $y = 0$.

40. (c) Both are true.

41. (d) All are true statements.

42. (a) Both the given statements are true.

43. (c)1

44. (c) Both the given statements are true.

45. (c) Both the given statements are true.

46. (c) Both the given statements are true.

47. (a) I. Let A, B and C having coordinates (a, b), (c, d) and {(a – c), (b – d)} respectively be the points
If these points are collinear then

$$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ a-c & b-d & 1 \end{vmatrix} = 0$$

On solving this expression we get
$1.\{a(d-b) - b(c-a)\} = 0$
$\Rightarrow ad - ab - bc + ab = 0$
$\Rightarrow bc - ad = 0$

II. Since the points are collinear.

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 3 & a & 1 \end{vmatrix} = 0$$

Expanding the above expression

$$\Rightarrow 1\begin{vmatrix} 4 & 1 \\ a & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 3 & a \end{vmatrix} = 0$$

$\Rightarrow (4 - a) - 2(2 - 3) + 1(2a - 12) = 0$
$\Rightarrow 4 - a + 2 + 2a - 12 = 0$
$\Rightarrow a - 6 = 0$
$\Rightarrow a = 6$
Thus, coordinates of C are (3, 6).

Thus, $BC = \sqrt{(3-2)^2 + (6-4)^2}$
$= \sqrt{1+4} = \sqrt{5}$ unit

48. (c) I. Centroid of a triangle is a point where medians meet.
II. If value of area after calculations is negative then we take its absolute value.

49. (a) II. Product of slopes = –1

50. (d) $y = \frac{3}{4}x + \frac{5}{2} \Rightarrow$ Slope $= \frac{3}{4}$
Also, $3x - 4y = -10$

$$\Rightarrow \frac{x}{-\frac{10}{3}} + \frac{y}{\frac{5}{2}} = 1$$

$\Rightarrow$ x-intercept $= \frac{-10}{3}$ and y-intercept $= \frac{5}{2}$

51. (c) $\sqrt{3}x + y - 8 = 0$
$\Rightarrow \frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$ (on dividing by 2)
$\Rightarrow \cos 30° \, x + \sin 30° y = 4$

52. (b) I. Slope $= \frac{4-(-2)}{-1-3} = \frac{-3}{2}$
II. Slope $= \frac{-2-(-2)}{7-3} = \frac{0}{4} = 0$
III. Slope $= \frac{4-(-2)}{3-3} = \frac{6}{0}$ which is not defined.

INTEGER TYPE QUESTIONS

53. (a) We have the points $A(x, -1)$, $B(2, 1)$, $C(4, 5)$.
A, B, C are collinear if the slope of *AB* = Slope of *BC*.

Slope of $AB = \frac{1+1}{2-x} = \frac{2}{2-x}$;

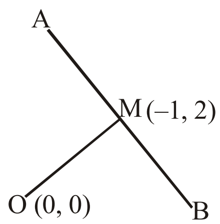
Slope of $BC = \frac{5-1}{4-2} = \frac{4}{2} = 2$

$\therefore \frac{2}{2-x} = 2$ or $2 - x = 1$ or $x = 1$

54. (d) The given line is $12(x + 6) = 5(y - 2)$
$\Rightarrow \; 12x + 72 = 5y - 10$
or $\; 12x - 5y + 72 + 10 = 0$
$\Rightarrow \; 12x - 5y + 82 = 0$
The perpendicular distance from (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{(ax_1 + by_1 + c)}{\sqrt{a^2 + b^2}}$.
The point (x_1, y_1) is (–1, 1), therefore, perpendicular distance from (–1, 1) to the line $12x - 5y + 82 = 0$ is

$$= \frac{|-12 - 5 + 82|}{\sqrt{12^2 + (-5)^2}} = \frac{65}{\sqrt{144 + 25}} = \frac{65}{\sqrt{169}} = \frac{65}{13} = 5$$

55. (b) Let the perpendicular *OM* is drawn from the origin to *AB*.



M is the foot of the perpendicular

Slope of $OM = \frac{2-0}{-1-0} = \frac{2}{-1}$;

Slope of $AB = m$

$OM \perp AB \; \therefore m \times (-2) = -1 \; \therefore m = \frac{1}{2}$

$M(-1, 2)$ lies on *AB* whose equation is

$y = mx + c$ or $y = \frac{1}{2}x + c$

$2 = \frac{1}{2} \times (-1) + c \Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2}$

$\therefore \quad m = \frac{1}{2}$ or $c = \frac{5}{2} \Rightarrow m + c = \frac{6}{2} = 3$

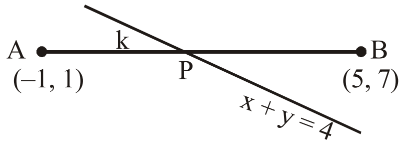
56. (a) Any line parallel to x-axis of the form $y = p$

i.e. coefficient of $x = 0$

$\therefore$ In equation $(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$

Coefficient of $x = k - 3 = 0 \therefore k = 3$

57. (a) The line joining the points A (–1, 1) and B(5, 7) is divided by $P(x, y)$ in the ratio $k : 1$



$\therefore$ Point P is $\left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right)$

This point lies on the line $x + y = 4$

$\therefore \frac{5k-1}{k+1} + \frac{7k+1}{k+1} = 4$

$\Rightarrow 5k - 1 + 7k + 1 = 4k + 4 \Rightarrow 8k = 4 \Rightarrow k = \frac{1}{2}$

$\therefore$ P divides AB in the ratio 1 : 2

58. (b) The given points are $A(h, 0)$, $B(a, b)$, $C(0, k)$, they lie on the same plane.

$\therefore$ Slope of AB = Slope of BC

$\therefore$ Slope of $AB = \frac{b-0}{a-h} = \frac{b}{a-h}$;

Slope of $BC = \frac{k-b}{0-a} = \frac{k-b}{-a}$

$\therefore \frac{b}{a-h} = \frac{k-b}{-a}$ or by cross multiplication

$-ab = (a-h)(k-b)$

or $-ab = ak - ab - hk + hb$

or $0 = ak - hk + hb$

or $ak + hb = hk$

Dividing by hk $\Rightarrow \frac{ak}{hk} + \frac{hb}{hk} = 1$ or $\frac{a}{h} + \frac{b}{k} = 1$

Hence proved.

59. (b) Slope of line through (2, 5) and (x, 3) is $\frac{3-5}{x-2}$

We have, $\frac{3-5}{x-2} = 2 \Rightarrow x = 1$

60. (d) Let A(3, y), B(2, 7), C(–1, 4) and D(0, 6) be the given points.

m_1 = slope of AB $= \frac{7-y}{2-3} = (y-7)$

m_2 = slope of CD $= \frac{6-4}{0-(-1)} = 2$

Since AB and CD are parallel

$\therefore \quad m_1 = m_2 \Rightarrow y = 9$.

61. (c) Given equation is

$\sqrt{3}x + y - 8 = 0$

Divide this by $\sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2$,

we get, $\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$

Which is in the normal form. Hence, p = 4.

62. (c) Given parallel lines are

$3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$

Required distance $= \frac{|7-5|}{\sqrt{(3)^2 + (-4)^2}} = \frac{2}{5}$

$\Rightarrow a = 2, b = 5$

ASSERTION - REASON TYPE QUESTIONS

63. (b) Assertion is correct and Reason is also correct

64. (b) Both the Assertion and Reason are true.

65. (c) Assertion is false and Reason is true.

Assertion: Slope $= \frac{4-(-2)}{3-3} = \frac{6}{0}$ which is not defined.

66. (b) Mid-point of AC $= \left(\frac{1}{2}, \frac{2}{2}\right) = \left(\frac{1}{2}, 1\right)$

Mid-point of BD $= \left(\frac{1}{2}, 1\right)$

67. (a) **Assertion:**

$a_1 = 1, b_1 = 2, c_1 = -3$

$a_2 = -3, b_2 = -6, c_2 = 9$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{-1}{3}$

So, the given lines are coincident.

68. (c) Assertion is correct but Reason is incorrect.

Correct Reason is given lines are perpendicular, if $a_1a_2 + b_1b_2 = 0$.

69. (c) Assertion is correct. Reason is incorrect.

Reason: The slope of the given line is $\frac{-a}{b}$.

70. (d) Assertion is incorrect Reason is correct.

Assertion: The equation of a line parallel to the line $ax + by + c = 0$ is $ax + by + \lambda = 0$ where λ is a constant.

71. (a) **Assertion:** $A = 3, B = -4$

$C_1 = 9, C_2 = -\frac{15}{2}$

$d = \frac{\left|-\frac{15}{2} - 9\right|}{\sqrt{9+16}} = \frac{\left|-\frac{33}{2}\right|}{5} = \frac{33}{10}$

72. (b) Both are correct.

CRITICALTHINKING TYPE QUESTIONS

73. (a) Let the point of intersection divide the line segment joining points, (3, – 1) and (8, 9) in k : 1 ratio then

The point is $\left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$

Since this point lies on the line $y - x + 2 = 0$

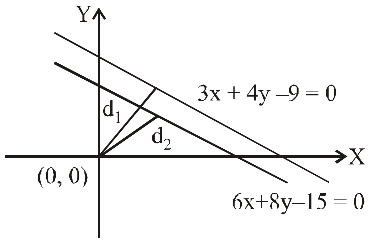
We have, $\frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0$

$\Rightarrow \frac{9k-1-8k-3}{k+1} + 2 = 0 \Rightarrow \frac{k-4}{k+1} + 2 = 0$

$\Rightarrow k-4+2k+2=0 \Rightarrow 3k-2=0$

$k=\frac{2}{3}:1$ i.e. $2:3$

74. (b) Let d_1 and d_2 be the distances of two lines $3x+4y-9=0$ and $6x+8y-15=0$ respectively from origin.



$\therefore \quad d_1=\frac{|3(0)+4(0)-9|}{\sqrt{3^2+4^2}} \Rightarrow d_1=\frac{9}{5}$

and $d_2=\frac{|6(0)+8(0)-15|}{\sqrt{36+64}}=\frac{15}{10}=\frac{3}{2}$

$\therefore$ distance between these lines is, $d=d_1-d_2$

$\Rightarrow \quad d=\frac{9}{5}-\frac{3}{2}=\frac{18-15}{10}=\frac{3}{10}$

75. (d) Given line is $3x+y=3$

Let the equation of line which is perpendicular to above line is

$x-3y+\lambda=0.$

This line is passing through point (2, 2)

$\therefore \quad 2-3\times 2+\lambda=0$

$\Rightarrow \quad 2-6+\lambda=0 \Rightarrow \lambda=4$

$\therefore$ Equation of line is $x-3y+4=0$

$\Rightarrow \quad 3y=x+4 \Rightarrow y=\frac{1}{3}x+\frac{4}{3}$

Compare the above equation with $y=mx+c$,

We get $c=\frac{4}{3}$

Thus, y –intercept is $\frac{4}{3}$.

76. (c) **Note:** If the vertices of a triangle are $A(a_1, b_1)$, $B(a_2, b_2)$ and $C(a_3, b_3)$, then the area of the triangle ABC

$$=\frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

Here in the given question:

we have A(x, 0), B(1, 1), C(0, 2).

and $\frac{1}{2}\begin{vmatrix} x & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix}=4$

$\Rightarrow \quad \frac{1}{2}[x(1-2)+1(2)]=4$

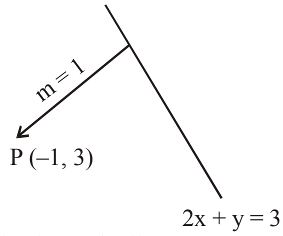
$\Rightarrow -x+2=8 \Rightarrow x=-6.$

77. (c) The equation of the line through (–1, 3) and having the slope 1 is

$$\frac{x+1}{\cos\theta}=\frac{y-3}{\sin\theta}=r.$$

Any point on this line at a distance r from $P(-1, 3)$ is

$(-1+r\cos\theta,\ 3+r\sin\theta)$



This point is on the line $2x+y=3$ if

$2(-1+r\cos\theta)+3+r\sin\theta=3$ ...(i)

But $\tan\theta=1; \Rightarrow \theta=45°$

(i) becomes,

$-2+2r.\frac{1}{\sqrt{2}}+3+r.\frac{1}{\sqrt{2}}=3$

$\Rightarrow \frac{3r}{\sqrt{2}}=2; \quad r=\frac{2\sqrt{2}}{3}$

Hence the required distance $=\frac{2\sqrt{2}}{3}$.

78. (a) Given equation of straingh lines are

$x+2y-9=0,\ 3x+5y-5=0$

and $ax+by-1=0$

They are concurrent, if

$-5+5b-2(-3+5a)-9(3b-5a)=0$

$\Rightarrow -5+5b+6-10a-27b+45a=0$

$\Rightarrow 35a-22b+1=0$

Thus, given straight lines are concurrent if the straight line $35x-22y+1=0$ passes through (a, b).

79. (a) Let (h, k) be the point of reflection of the given point (4, –13) about the line $5x+y+6=0$. The mid-point of the line segment joining points (h, k) and (4, –13) is given by

$$\left(\frac{h+4}{2}, \frac{k-13}{2}\right)$$

This point lies on the given line, so we have

$$5\left(\frac{h+4}{2}\right)+\frac{k-13}{2}+6=0$$

or $\quad 5h+k+19=0$ ...(i)

Again the slope of the line joining points (h, k) and (4, –13) is given by $\frac{k+13}{h-4}$. This line is perpendicular to the given line and hence

$$(-5)\left(\frac{k+3}{h-4}\right)=-1$$

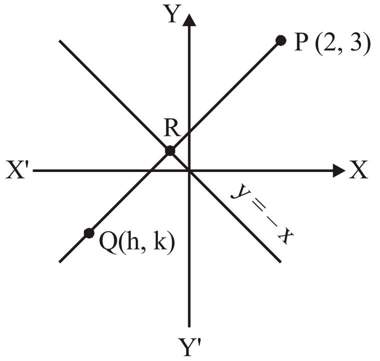
This gives $5k+65=h-4$

or $\quad h-5k-69=0$ ...(ii)

On solving (i) and (ii), we get $h=-1$ and $k=-14$. Thus the point (–1, – 14) is the reflection of the given point.

80. (a) (1, – 2)

81. (a) Let there be a point P(2,3) on cartesian plane. Image of this point in the line $y=-x$ will lie on a line which is perpendicular to this line and distance of this point from y = –x will be equal to distance of the image from this line.



Let Q be the image of p and let the co-ordinate of Q be (b, k)

Slope of line y = –x is –1

Line joining P, Q will be perpendicular to y = –x so, its slope = 1.

Let the equation of the line be y = x + c since this passes through point (2, 3)

$3 = 2 + c \Rightarrow c = 1$

and the equation y = x + 1

The point of intersection R lies in the middle of P & Q.

Point of intersection of line $y = -x$ and $y = x + 1$ is

$2y = 1, \Rightarrow y = \frac{1}{2}$ and $x = -\frac{1}{2}$

Hence, $\frac{h+2}{2} = -\frac{1}{2}$ and $\frac{k+3}{2} = \frac{1}{2}$

$\Rightarrow$ h = –3 and k = –2

So, the image of the point (2, 3) in the line $y = -x$ is (–3, –2).

82. (c) Equation of line is ax + by – p = 0, then length of perpendicular, from the origin is

$$p = \left|\frac{a\times 0 + b\times 0 - p}{\sqrt{a^2+b^2}}\right| \text{ or } p = \left|\frac{-p}{\sqrt{a^2+b^2}}\right|$$

$\Rightarrow a^2 + b^2 = 1$

$b = \frac{\sqrt{3}}{2}$ or $b^2 = \frac{3}{4} \Rightarrow a^2 + \frac{3}{4} = 1$

$a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$

[$a = -\frac{1}{2}$ not taken since angle is with + ve direction to x-axis.]

Equation is $\frac{1}{2}x + \frac{\sqrt{3}}{2}y = p$ or $x\cos 60° + y\sin 60° = p$

Angle = 60°

83. (c) One vertex of square is (– 4, 5) and equation of one diagonal is $7x - y + 8 = 0$

Diagonal of a square are perpendicular and bisect each other

Let the equation of the other diagonal be y = mx + c where m is the slope of the line and c is the y-intercept.

Since this line passes through (– 4, 5)

$\therefore \quad 5 = -4m + c \quad \ldots(i)$

Since this line is at right angle to the line $7x - y + 8 = 0$ or $y = 7x + 8$, having slope = 7,

$\therefore \quad 7 \times m = -1$ or $m = \frac{-1}{7}$

Putting this value of m in equation (i) we get

$5 = -4\times\left(\frac{-1}{7}\right) + c$

or $5 = \frac{4}{7} + c$ or $c = 5 - \frac{4}{7} = \frac{31}{7}$

Hence equation of the other diagonal is

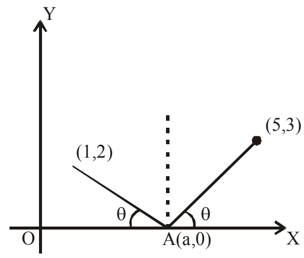
$y = -\frac{1}{7}x + \frac{31}{7}$

or $7y = -x + 31$

or $x + 7y - 31 = 0$

or $x + 7y = 31$.

84. (a) Let the coordinates of A be (a, 0). Then the slope of the reflected ray is



$\frac{3-0}{5-a} = \tan\theta$ (say) $\quad \ldots(i)$

Then the slope of the incident ray

$= \frac{2-0}{1-a} = \tan(\pi - \theta)$. $\quad \ldots(ii)$

from (i) and (ii) $\tan\theta + \tan(\pi - \theta) = 0$

$\Rightarrow \frac{3}{5-a} + \frac{2}{1-a} = 0 \Rightarrow 3 - 3a + 10 - 2a = 0$

$\Rightarrow a = \frac{13}{5}$

Thus, the co-ordinates of A are $\left(\frac{13}{5}, 0\right)$.

85. (b) $AB = 3\sqrt{2}$, $AC = 4\sqrt{2}$, $BC = 5\sqrt{2}$

$\therefore \frac{AB}{AC} = \frac{3}{4}$. That is the internal bisector of angle A cuts the side BC in ratio 3 : 4 at D. The coordinates of D are

$\left(\frac{4\times 4 + 3\times 5}{4+3}, \frac{4\times -2 + 3\times 5}{4+3}\right) \equiv \left(\frac{31}{7}, 1\right)$

Slope of AD = 0

$\therefore$ Equation of perpendicular from C(5, 5) to AD is x = 5

86. (b) Since the line L has intercepts a and b on the coordinate axes, therefore its equation is

$\frac{x}{a} + \frac{y}{b} = 1 \quad \ldots(i)$

When the axes are rotated, its equation with respect to the new axes and same origin will become

$\frac{x}{p}+\frac{y}{q}=1$ ...(ii)

In both the cases, the length of the perpendicular from the origin to the line will be same.

$\therefore \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}$ or $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

87. (a) Third side passes through (1, – 10), so let its equation be $y+10=m(x-1)$

If it makes equal angle, say θ with given two sides, then

$\tan\theta=\frac{m-7}{1+7m}=\frac{m-(-1)}{1+m(-1)} \Rightarrow m=-3$ or 1/3

Hence possible equations of third side are

$y+10=-3(x-1)$ and $y+10=\frac{1}{3}(x-1)$

or $3x+y+7=0$ and $x-3y-31=0$

88. (a) If the lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line then these lines must be parallel to each other,

$\therefore m_1=m_2 \Rightarrow -\frac{p(p^2+1)}{-1}=-\frac{(p^2+1)^2}{p^2+1}$

$\Rightarrow (p^2+1)(p+1)=0 \Rightarrow p=-1$

$\therefore p$ can have exactly one value.

89. (c) On comparing given equations with $ax+by+c=0$

We get $a_1=4, a_2=3, b_1=-3, b_2=-4$

Now $a_1a_2+b_1b_2=(4\times3+3\times4)=24>0$ (Positive)

Since, $a_1a_2+b_1b_2$ is +ve

∴ Origin lies in obtuse angle

For acute angle, we find the bisector

Now, equation of bisectors of given lines are

$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$

The equation of the bisector is

$\left[\frac{4x-3y+7}{5}\right]=-\left[\frac{3x-4y+14}{5}\right] \Rightarrow x-y+3=0$

90. (c) Here, c = –1 and m = tan θ = tan 45° = 1

(Since the line is equally inclined to the axes, so θ = 45°)

Also, m = tan 135° = –1 ⇒ m = ± 1

∴ θ = 45° and 135°

Hence, equation of straight line is $y=\pm(1\cdot x)-1$

⇒ $x-y-1=0$ and $x+y+1=0$

91. (c) Here, D(1, 1), therefore, equation of line AD is given by $2x+y-3=0$. Thus, the line perpendicular to AD is $x-2y+k=0$ and it passes through B, so k = 0. Hence, required equation is $x-2y=0$.

92. (c) The lines passing through the intersection of the lines $ax+2by+3b=0$ and $bx-2ay-3a=0$ is

$ax+2by+3b+\lambda(bx-2ay-3a)=0$

$\Rightarrow (a+b\lambda)x+(2b-2a\lambda)y+3b-3\lambda a=0$ ... (i)

Line (i) is parallel to x-axis,

$\therefore a+b\lambda=0 \Rightarrow \lambda=\frac{-a}{b}=0$

Put the value of λ in (i),

$ax+2by+3b-\frac{a}{b}(bx-2ay-3a)=0$

$y\left(2b+\frac{2a^2}{b}\right)+3b+\frac{3a^2}{b}=0,$

$y\left(\frac{2b^2+2a^2}{b}\right)=-\left(\frac{3b^2+3a^2}{b}\right)$

$y=\frac{-3(a^2+b^2)}{2(b^2+a^2)}=\frac{-3}{2},\ y=-\frac{3}{2}$

So, it is $\frac{3}{2}$ unit below x-axis.

93. (a) By direct formulae,

$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$

$\frac{3x+4y-7}{\sqrt{3^2+4^2}}=\pm\frac{12x+5y+17}{\sqrt{(12)^2+(5)^2}}$

$\frac{3x+4y-7}{5}=\pm\frac{12x+5y+17}{13}.$

94. (a) The equations of the bisectors of the angles between the lines are $\frac{x-2y+4}{\sqrt{1+4}}=\pm\frac{4x-3y+2}{\sqrt{16+9}}$

Taking positive sign, then

$(4-\sqrt{5})x-(3-2\sqrt{5})y-(4\sqrt{5}-2)=0$ ... (i)

and negative sign gives

$(4+\sqrt{5})x-(2\sqrt{5}+3)y+(4\sqrt{5}+2)=0$ ... (ii)

Let θ be the angle between the line (i) and one of the given line, then

$\tan\theta=\left|\frac{\frac{1}{2}-\frac{4-\sqrt{5}}{3-2\sqrt{5}}}{1+\frac{1}{2}\cdot\frac{4-\sqrt{5}}{3-2\sqrt{5}}}\right|=\sqrt{5}+2>1$

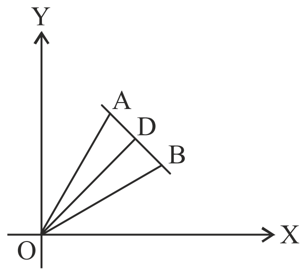
Hence, the line (i) bisects the obtuse angle between the given lines.

95. (a) $L \equiv 2x+3y-4=0,\ L_{(-6,2)}=-12+6-4<0$

$L'=6x+9y+8=0,\ L'_{(-6,2)}=-36+18+8<0$

Hence, the point is below both the lines.

96. (b) $OA=OB=9,\ OD=\frac{15}{\sqrt{25}}=3$



Therefore, $AB=2AD=2\sqrt{81-9}=2\sqrt{72}=12\sqrt{2}$

Hence, $\Delta=\frac{1}{2}(3\times12\sqrt{2})=18\sqrt{2}$ sq. units

CHAPTER

11

CONIC SECTION

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The equation of the circle which passes through the point (4, 5) and has its centre at (2, 2) is

(a) $(x-2)+(y-2)=13$ (b) $(x-2)^2+(y-2)^2=13$

(c) $(x)^2+(y)^2=13$ (d) $(x-4)^2+(y-5)^2=13$

2. Point (1, 2) relative to the circle $x^2+y^2+4x-2y-4=0$ is a/an

(a) exterior point

(b) interior point, but not centre

(c) boundary point

(d) centre

3. A conic section with eccentricity *e* is a parabola if:

(a) $e=0$ (b) $e<1$ (c) $e>1$ (d) $e=1$

4. For the ellipse $3x^2+4y^2=12$ length of the latus rectum is:

(a) 3 (b) 4 (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

5. The focal distance of a point on the parabola $y^2=12x$ is 4. What is the abscissa of the point ?

(a) 1 (b) – 1

(c) $2\sqrt{3}$ (d) – 2

6. What is the difference of the focal distances of any point on the hyperbola?

(a) Eccentricity

(b) Distance between foci

(c) Length of transverse axis

(d) Length of semi-transverse axis

7. The equation of an ellipse with foci on the *x*-axis is

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (b) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$

(c) $\frac{x}{a}+\frac{y}{b}=1$ (d) $\frac{a}{x}+\frac{b}{y}=1$

8. Length of the latus rectum of the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) $\frac{b^2}{a^2}$ (b) $\frac{2b}{a}$

(c) $\frac{2b^2}{a}$ (d) $\frac{2a^2}{b}$

9. The equation of a hyperbola with foci on the *x*-axis is

(a) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (b) $\frac{x}{a}-\frac{y}{b}=1$

(c) $\frac{a^2}{x^2}-\frac{b^2}{y^2}=1$ (d) $\frac{a}{x}-\frac{b}{y}=1$

10. Length of the latus rectum of the hyperbola :

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(a) $\frac{b^2}{a}$ (b) $\frac{2b^2}{a}$ (c) $\frac{a^2}{b}$ (d) $\frac{2a^2}{b}$

11. What is the length of the smallest focal chord of the parabola $y^2=4ax$?

(a) a (b) 2a (c) 4a (d) 8a

12. The equation of the hyperbola with vertices (3, 0), (–3, 0) and semi-latus rectum 4 is given by :

(a) $4x^2-3y^2+36=0$ (b) $4x^2-3y^2+12=0$

(c) $4x^2-3y^2-36=0$ (d) $4x^2+3y^2-25=0$

13. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

(a) $x^2-y^2=32$ (b) $\frac{x^2}{4}-\frac{y^2}{9}=1$

(c) $2x-3y^2=7$ (d) None of these

14. If the equation of a circle is

$(4a-3)x^2+ay^2+6x-2y+2=0$,

then its centre is

(a) (3, –1) (b) (3, 1) (c) (–3, 1) (d) None of these

15. The equation of the parabola with vertex at origin, which passes through the point (–3, 7) and axis along the *x*-axis is

(a) $y^2=49x$ (b) $3y^2=-49x$

(c) $3y^2=49x$ (d) $x^2=-49y$

16. The length of the semi-latus rectum of an ellipse is one third of its major axis, its eccentricity would be

(a) $\frac{2}{3}$ (b) $\sqrt{\frac{2}{3}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

17. The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is

(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$
(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$

18. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is

(a) $\frac{3}{5}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{\sqrt{5}}$

19. Eccentricity of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if it passes through point (9, 5) and (12, 4) is

(a) $\sqrt{3/4}$ (b) $\sqrt{4/5}$ (c) $\sqrt{5/6}$ (d) $\sqrt{6/7}$

20. The equation of the ellipse with focus at (± 5, 0) and $x = \frac{36}{5}$ as one directrix is

(a) $\frac{x^2}{36} + \frac{y^2}{25} = 1$ (b) $\frac{x^2}{36} + \frac{y^2}{11} = 1$
(c) $\frac{x^2}{25} + \frac{y^2}{11} = 1$ (d) None of these

21. The foci of the ellipse $25(x+1)^2 + 9(y+2)^2 = 225$ are at :

(a) (–1, 2) and (–1, –6) (b) (–2, 1) and (–2, 6)
(c) (–1, –2) and (–2, –1) (d) (–1, –2) and (–1, –6)

22. The eccentricity of the hyperbola $x^2 - 3y^2 = 2x + 8$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

23. The equation of the hyperbola with vertices at (0, ±6) and $e = \frac{5}{3}$ is

(a) $\frac{x^2}{36} - \frac{y^2}{64} = 1$ (b) $\frac{y^2}{36} - \frac{x^2}{64} = 1$
(c) $\frac{x^2}{64} - \frac{y^2}{36} = 1$ (d) $\frac{y^2}{64} - \frac{x^2}{36} = 1$

24. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x = 4$, then the equation of the ellipse is:

(a) $4x^2 + 3y^2 = 1$ (b) $3x^2 + 4y^2 = 12$
(c) $4x^2 + 3y^2 = 12$ (d) $3x^2 + 4y^2 = 1$

25. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

26. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is

(a) $5x^2 + 3y^2 - 48 = 0$ (b) $3x^2 + 5y^2 - 15 = 0$
(c) $5x^2 + 3y^2 - 32 = 0$ (d) $3x^2 + 5y^2 - 32 = 0$

27. The equation of the hyperbola whose foci are (– 2, 0) and (2, 0) and eccentricity is 2 is given by :

(a) $x^2 - 3y^2 = 3$ (b) $3x^2 - y^2 = 3$
(c) $-x^2 + 3y^2 = 3$ (d) $-3x^2 + y^2 = 3$

28. For what value of k, does the equation

$$9x^2 + y^2 = k(x^2 - y^2 - 2x)$$

represent equation of a circle ?

(a) 1 (b) 2 (c) –1 (d) 4

29. The eccentricity of the ellipse whose major axis is three times the minor axis is:

(a) $\frac{\sqrt{2}}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{2}{\sqrt{3}}$

30. The focal distance of a point on the parabola $y^2 = 8x$ is 4. Its ordinates are:

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

31. If the eccentricity and length of latus rectum of a hyperbola are $\frac{\sqrt{13}}{3}$ and $\frac{10}{3}$ units respectively, then what is the length of the transverse axis?

(a) $\frac{7}{2}$ unit (b) 12 unit (c) $\frac{15}{2}$ unit (d) $\frac{15}{4}$ unit

32. The equation of a circle with centre at (1, 0) and circumference 10π units is

(a) $x^2 + y^2 - 2x + 24 = 0$ (b) $x^2 + y^2 - x - 25 = 0$
(c) $x^2 + y^2 - 2x - 24 = 0$ (d) $x^2 + y^2 + 2x + 24 = 0$

33. If the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$, then

(a) transverse axis is along x-axis of length 6
(b) transverse axis is along y-axis of length 8
(c) conjugate axis is along y-axis of length 6
(d) None of the above

34. The length of transverse axis of the hyperbola $3x^2 - 4y^2 = 32$, is

(a) $\frac{8\sqrt{2}}{\sqrt{3}}$ (b) $\frac{16\sqrt{2}}{\sqrt{3}}$ (c) $\frac{3}{32}$ (d) $\frac{64}{3}$

35. The length of the transverse axis along x-axis with centre at origin of a hyperbola is 7 and it passes through the point (5, – 2). Then, the equation of the hyperbola is

(a) $\frac{4}{49}x^2 - \frac{196}{51}y^2 = 1$ (b) $\frac{49}{4}x^2 - \frac{51}{196}y^2 = 1$
(c) $\frac{4}{49}x^2 - \frac{51}{196}y^2 = 1$ (d) None of these

36. The equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13, is
(a) $25x^2 - 144y^2 = 900$ (b) $144x^2 - 25y^2 = 900$
(c) $144x^2 + 25y^2 = 900$ (b) $25x^2 + 144y^2 = 900$

37. Which one of the following points lies outside the ellipse $(x^2/a^2) + (y^2/b^2) = 1$?
(a) (a, 0) (b) (0, b)
(c) (– a, 0) (d) (a, b)

38. The equation of an ellipse with one vertex at the point (3, 1), the nearer focus at the point (1, 1) and $e = \frac{2}{3}$ is :
(a) $\frac{(x+3)^2}{36} + \frac{(y-1)^2}{20} = 1$ (b) $\frac{(x-3)^2}{20} + \frac{(y+1)^2}{36} = 1$
(c) $\frac{(x-3)^2}{36} + \frac{(y+1)^2}{20} = 1$ (d) $\frac{(x-3)^2}{36} + \frac{(y-1)^2}{20} = 1$

39. The vertex of the parabola $(x-4)^2 + 2y = 9$ is :
(a) (2, 8) (b) (7, 2) (c) $\left(4, \frac{9}{2}\right)$ (d) $\left(-4, -\frac{9}{2}\right)$

40. The equations of the lines joining the vertex of the parabola $y^2 = 6x$ to the points on it which have abscissa 24 are
(a) $y \pm 2x = 0$ (b) $2y \pm x = 0$
(c) $x \pm 2y = 0$ (d) $2x \pm y = 0$

41. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and e_2 is the eccentricity of the hyperbola passing through the foci of the ellipse and $e_1 e_2 = 1$, then equation of the hyperbola is :
(a) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (b) $\frac{x^2}{16} - \frac{y^2}{9} = -1$
(c) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (d) $\frac{x^2}{9} - \frac{y^2}{36} = 1$

42. A circle has radius 3 and its centre lies on the line $y = x - 1$. The equation of the circle, if it passes through (7, 3), is
(a) $x^2 + y^2 + 8x - 6y + 16 = 0$
(b) $x^2 + y^2 - 8x + 6y + 16 = 0$
(c) $x^2 + y^2 - 8x - 6y - 16 = 0$
(d) $x^2 + y^2 - 8x - 6y + 16 = 0$

43. The equation $y^2 + 3 = 2(2x + y)$ represents a parabola with the vertex at
(a) $\left(\frac{1}{2}, 1\right)$ and axis parallel to y-axis
(b) $\left(1, \frac{1}{2}\right)$ and axis parallel to x-axis
(c) $\left(\frac{1}{2}, 1\right)$ and focus at $\left(\frac{3}{2}, 1\right)$
(d) $\left(1, \frac{1}{2}\right)$ and focus at $\left(\frac{3}{2}, 1\right)$

44. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

45. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then
(a) $d^2 + (3b - 2c)^2 = 0$ (b) $d^2 + (3b + 2c)^2 = 0$
(c) $d^2 + (2b - 3c)^2 = 0$ (d) $d^2 + (2b + 3c)^2 = 0$

46. The eccentricity of the curve $2x^2 + y^2 - 8x - 2y + 1 = 0$ is:
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

47. The focus of the curve $y^2 + 4x - 6y + 13 = 0$ is
(a) (2, 3) (b) (–2, 3)
(c) (2, –3) (d) (–2, –3)

48. The eccentricities of the ellipse $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$, $\alpha > \beta$; and $\frac{x^2}{9} + \frac{y^2}{16} = 1$ are equal. Which one of the following is correct ?
(a) $4\alpha = 3\beta$ (b) $\alpha\beta = 12$
(c) $4\beta = 3\alpha$ (d) $9\alpha = 16\beta$

49. The vertex of the parabola $x^2 + 8x + 12y + 4 = 0$ is:
(a) (–4, 1) (b) (4, –1) (c) (–4, –1) (d) (4, 1)

50. The equation of the conic with focus at (1, – 1) directrix along $x - y + 1 = 0$ and with eccentricity $\sqrt{2}$ is
(a) $x^2 - y^2 = 1$ (b) $xy = 1$
(c) $2xy - 4x + 4y + 1 = 0$ (d) $2xy + 4x - 4y - 1 = 0$

51. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is
(a) (3, –4) (b) (–3, 4) (c) (–3, –4) (d) (3, 4)

52. A circle of radius 5 touches another circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5) then its equation is :
(a) $x^2 + y^2 + 18x + 16y + 120 = 0$
(b) $x^2 + y^2 - 18x - 16y + 120 = 0$
(c) $x^2 + y^2 - 18x + 16y + 120 = 0$
(d) None of these

53. The circle $x^2 + y^2 - 8x + 4y + 4 = 0$ touches :
(a) x-axis only (b) y-axis only
(c) both (a) and (b) (d) None of these

54. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct point, then
(a) $r > 2$ (b) $2 < r < 8$ (c) $r < 2$ (d) $r = 2$

55. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is
(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 3 (d) 2

56. The conic represented by
$x = 2(\cos t + \sin t)$, $y = 5(\cos t - \sin t)$ is
(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

57. Equation of the ellipse whose axes are along the coordinate axes, vertices are $(\pm 5, 0)$ and foci at $(\pm 4, 0)$ is
(a) $\frac{x^2}{16}+\frac{y^2}{9}=1$ (b) $\frac{x^2}{25}+\frac{y^2}{9}=1$
(c) $\frac{x^2}{4}+\frac{y^2}{25}=1$ (d) $\frac{x^2}{25}+\frac{y^2}{16}=1$

58. Equation of the hyperbola whose directrix is $2x + y = 1$, focus (1, 2) and eccentricity $\sqrt{3}$ is
(a) $7x^2 - 2y^2 + 12xy - 2x + 9y - 22 = 0$
(b) $5x^2 - 2y^2 + 10xy + 2x + 5y - 20 = 0$
(c) $4x^2 + 8y^2 + 8xy + 2x - 2y + 10 = 0$
(d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

59. I. Equation of conjugate hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
II. Length of latus rectum of the conjugate hyperbola is $\frac{2a^2}{b}$.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

60. I. The straight line passing through the focus and perpendicular to the directrix is called the axis of the conic section.
II. The points of intersection of the conic section and the axis are called vertices of the conic section.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

61. An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.
I. The two fixed points are called the foci of the ellipse.
II. The mid point of the line segment joining the foci is called the centre of the ellipse.
III. The end points of the major axis are called the vertices of the ellipse.
(a) Only I and II are correct.
(b) Only II and III are correct.
(c) Only I and III are correct.
(d) All are correct.

62. If the equation of the circle is $x^2 + y^2 - 8x + 10y - 12 = 0$, then
I. Centre of the circle is (4, –5).
II. Radius of the circle is $\sqrt{53}$.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

63. If equation of the ellipse is $\frac{x^2}{100}+\frac{y^2}{400}=1$, then
I. Vertices of the ellipse are $(0, \pm 20)$
II. Foci of the ellipse are $(0, \pm 10\sqrt{3})$
III. Length of major axis is 40.
IV. Eccentricity of the ellipse is $\frac{\sqrt{3}}{2}$.
(a) I and II are true. (b) III and IV are true.
(c) II, III, IV are true. (d) All are true.

64. If the equation of the hyperbola is $\frac{y^2}{9}-\frac{x^2}{27}=1$, then
I. the coordinates of the foci are $(0, \pm 6)$
II. the length of the latus rectum is 18 units.
III. the eccentricity is $\frac{4}{5}$.
(a) Only I is true. (b) Only II is true.
(c) Only I and II is true. (d) Only II and III is true.

65. Consider the following statements.
I. A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.
II. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.
(a) Only I is true (b) Only II is true.
(c) Both are true. (d) Both are false.

66. If the equation of the hyperbola is
$9y^2 - 4x^2 = 36$, then
I. the coordinates of foci are $(0, \pm\sqrt{13})$
II. the eccentricity is $\frac{2}{\sqrt{13}}$.
III. the length of the latus rectum is 8.
(a) Only I is true. (b) Only II is true.
(c) Only III is true. (d) None of them is true.

67. Consider the following statements.
I. The equation of a circle with centre (h, k) and the radius r is $(x - h)^2 + (y - k)^2 = r^2$.
II. The equation of the parabola with focus at (a, 0), a > 0 and directrix $x = -a$ is $y^2 = -4ax$
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Botha re false.

68. Consider the following statements.

I. Length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$

II. A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

69. Match the foci, centre, transverse axis, conjugate axis and vertices of hyperbola given in column-I with their corresponding meaning given in column-II

Column-I	Column-II
A. Foci	1. Mid-point of the line segment joining the foci.
B. Centre	2. Points at which the hyperbola intersects the transverse axis.
C. Transverse axis	3. Line through the foci.
D. Conjugate axis	4. Two fixed points.
E. Vertices	5. Line through the centre and perpendicular to the transverse axis

Codes

	A	B	C	D	E
(a)	4	3	1	5	2
(b)	1	4	3	5	2
(c)	4	1	5	3	2
(d)	4	1	3	5	2

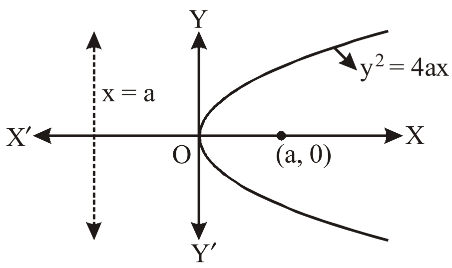
70.

Column - I	Column - II
(A) If e = 1, the conic is called	(1) Hyperbola
(B) If e < 1, the conic is called	(2) Parabola
(C) If e > 1, the conic is called	(3) Circle
(D) If e = 0, the conic is called	(4) Ellipse

Codes

	A	B	C	D
(a)	2	1	4	3
(b)	2	4	1	3
(c)	3	1	4	2
(d)	3	4	1	2

71. Match the columns for the parabola given in the graph.



Column - I	Column - II
(A) Eccentricity	(1) $x + a = 0$
(B) Focus	(2) 4a
(C) Equation of directrix	(3) $x - a = 0$
(D) Length of latus rectum	(4) (a, 0)
(E) Equation of latus rectum	(5) 1
(F) Equation of axis	(6) $y = 0$

Codes

	A	B	C	D	E	F
(a)	5	4	1	2	3	6
(b)	5	4	2	1	6	3
(c)	6	1	4	2	3	5
(d)	6	1	2	3	4	5

72.

Column - I (Centre and radius of circle)	Column - II (Equation of circle)
(A) Centre (–3, 2), radius = 4	(1) $x^2+y^2+4x-6y-3=0$
(B) Centre (–4, –5), radius = 7	(2) $x^2+y^2-4y=0$
(C) Centre (0, 2), radius = 2	(3) $x^2+y^2+8x+10y-8=0$
(D) Centre (–2, 3), radius = 4	(4) $(x+3)^2+(y-2)^2=16$

Codes

	A	B	C	D
(a)	4	2	3	1
(b)	1	2	3	4
(c)	1	3	2	4
(d)	4	3	2	1

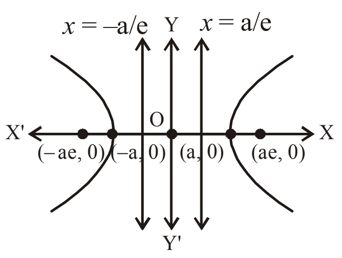
73. If the equation of ellipse is $9x^2 + 4y^2 = 36$, then

Column - I	Column - II
(A) The foci are	(1) $(0, \pm 3)$
(B) The vertices are	(2) $\frac{\sqrt{5}}{3}$
(C) The length of major axis is	(3) 6
(D) The eccentricity is	(4) $(0, \pm\sqrt{5})$

Codes

	A	B	C	D
(a)	4	1	3	2
(b)	2	1	3	4
(c)	4	3	1	2
(d)	2	3	1	4

74. Read the graph of the hyperbola. Match the column - I with column - II.



Column - I	Column - II
(A) Equation of the directrix is	(1) 2a
(B) Vertices are	(2) 2ae
(C) Foci are	(3) 2b
(D) Distance between foci is	(4) $(\pm a, 0)$
(E) Length of transverse axis is	(5) $x = \pm\frac{a}{e}$
(F) Length of conjugate axis is	(6) $(\pm ae, 0)$

Codes

	A	B	C	D	E	F
(a)	5	6	4	2	3	1
(b)	4	5	2	6	3	1
(c)	5	4	6	2	1	3
(d)	5	4	2	6	3	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

75. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is

(a) 9 (b) 1 (c) 5 (d) 7

76. Tangents are drawn from the point (–2, –1) to the parabola $y^2 = 4x$. If α is the angle between these tangent then the value of tan α is

(a) 3 (b) 4 (c) –5 (d) 5

77. The focal distance of a point on the parabola $y^2 - 12x$ is 4. The abscissa of this point is

(a) 0 (b) 1 (c) 2 (d) 4

78. Radius of the circle $(x + 5)^2 + (y – 3)^2 = 36$ is

(a) 2 (b) 3 (c) 6 (d) 5

79. The equation of the circle with centre (0, 2) and radius 2 is $x^2 + y^2 – my = 0$. The value of m is

(a) 1 (b) 2 (c) 4 (d) 3

80. The equation of parabola whose vertex (0, 0) and focus (3, 0) is $y^2 = 4ax$. The value of 'a' is

(a) 2 (b) 3 (c) 4 (d) 1

81. The equation of the hyperbola whose vertices are (± 2, 0) and foci are (± 3, 0) is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Sum of a^2 and b^2 is

(a) 5 (b) 4 (c) 9 (d) 1

82. For the parabola $y^2 = 8x$, the length of the latus-rectum is

(a) 4 (b) 2 (c) 8 (d) None of these

83. For the parabola $y^2 = –12x$, equation of directrix is x = a. The value of 'a' is

(a) 3 (b) 4 (c) 2 (d) 6

84. The foci of an ellipse are (±2, 0) and its eccentricity is $\frac{1}{2}$ then the equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{12}=1$. The value of 'a' is

(a) 3 (b) 4 (c) 6 (d) 2

85. The equation of the ellipse whose axes are along the co-ordinate axes, vertices are (±5, 0) and foci at (±4, 0), is $\frac{x^2}{25}+\frac{y^2}{b^2}=1$. The value of b^2 is

(a) 3 (b) 5 (c) 9 (d) 4

86. If y = 2x is a chord of the circle $x^2 + y^2 – 10x = 0$, then the equation of a circle with this chord as diameter, is $x^2 + y^2 – ax – by = 0$. Sum of a and b is

(a) 4 (b) 2 (c) 6 (d) 0

ASSERTION- REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

87. Assertion : Length of focal chord of a parabola $y^2 = 8x$ making an angle of 60° with x-axis is 32.

Reason : Length of focal chord of a parabola $y^2 = 4ax$ making an angle α with x-axis is 4a $\text{cosec}^2\,\alpha$.

88. Assertion : If $P\left(\frac{3\sqrt{3}}{2},1\right)$ is a point on the ellipse $4x^2 + 9y^2 = 36$. Circle drawn AP as diameter touches another circle $x^2 + y^2 = 9$, where $A \equiv (-\sqrt{5},0)$

Reason : Circle drawn with focal radius as diameter touches the auxiliary circle.

89. Assertion : Ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and $12x^2 - 4y^2 = 27$ intersect each other at right angle.

Reason : Whenever focal conics intersect, they intersect each other orthogonally.

90. Assertion : Centre of the circle $x^2 + y^2 – 6x + 4y – 12 = 0$ is (3, –2).

Reason : The coordinates of the centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are ($-\frac{1}{2}$ coefficient of x, $-\frac{1}{2}$ coefficient of y)

91. Assertion : Radius of the circle $2x^2 + 2y^2 + 3x + 4y + \frac{9}{8} = 0$ is 1.

Reason : Radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$\sqrt{\left(\frac{1}{2}\text{coeff. of x}\right)^2+\left(\frac{1}{2}\text{coeff. of y}\right)^2-\text{constant term}}$$

92. **Assertion :** Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola.

Reason : The equation of a hyperbola with foci on the y-axis is : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

93. **Assertion :** A hyperbola in which $a=b$ is called a rectangular hyperbola.

Reason : The eccentricity of a hyperbola is the ratio of the distances from the centre of the hyperbola to one of the foci and to one of the vertices of the hyperbola.

94. **Assertion :** Eccentricity of conjugate hyperbola is equal to $\sqrt{\frac{b^2+a^2}{b^2}}$

Reason : Equation of directrix of conjugate hyperbola is $y=\pm\frac{b}{e}$

95. **Assertion:** The area of the ellipse $2x^2+3y^2=6$ is more than the area of the circle $x^2+y^2-2x+4y+4=0$.

Reason: The length of semi-major axis of an ellipse is more than the radius of the circle.

96. Parabola is symmetric with respect to the axis of the parabola.

Assertion: If the equation has a term y^2, then the axis of symmetry is along the x-axis.

Reason: If the eqution has a term x^2, then the axis of symmetry is along the x-axis.

97. Let the centre of an ellipse is at (0, 0)

Assertion: If major axis is on the y-axis and ellipse passes through the points (3, 2) and (1, 6), then the equation of ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$.

Reason: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ is an equation of ellipse if major axis is along y-axis.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

98. The equation of the directrix of the parabola $y^2+4y+4x+2=0$ is :

(a) $x=-1$ (b) $x=1$ (c) $x=\frac{-3}{2}$ (d) $x=\frac{3}{2}$

99. The value of p such that the vertex of $y=x^2+2px+13$ is 4 units above the y-axis is

(a) 2 (b) ± 4 (c) 5 (d) ± 3

100. A parabola has the origin as its focus and the line $x=2$ as the directrix. Then the vertex of the parabola is at

(a) (0,2) (b) (1,0) (c) (0,1) (d) (2,0)

101. What is the radius of the circle passing through the points (0, 0), (a, 0) and (0, b) ?

(a) $\sqrt{a^2-b^2}$ (b) $\sqrt{a^2+b^2}$

(c) $\frac{1}{2}\sqrt{a^2+b^2}$ (d) $2\sqrt{a^2+b^2}$

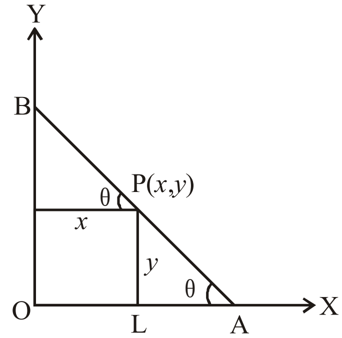
102. If (2, 0) is the vertex and the y-axis is the directrix of a parabola, then its focus is

(a) (0,0) (b) (–2,0) (c) (4,0) (d) (–4,0)

103. The latus rectum of parabola $y^2=5x+4y+1$ is:

(a) 10 (b) 5 (c) $\frac{5}{4}$ (d) $\frac{5}{2}$

104. A bar of given length moves with its extremities on two fixed straight lines at right angles. Any point of the bar describes



(a) parabola (b) ellipse

(c) hyperbola (d) circle

105. The equation of the circle, which touches the line $y=5$ and passes through (–1, 2) and (1, 2) is

(a) $9x^2+9y^2-60y+75=0$

(b) $9x^2+9y^2-60x-75=0$

(c) $9x^2+9y^2+60y-75=0$

(d) $9x^2+9y^2+60x+75=0$

106. Which points on the curve $x^2=2y$ are closest to the point (0, 5) ?

(a) $(\pm 2\sqrt{2},4)$ (b) $(\pm 2,2)$

(c) $(\pm 3,9/2)$ (d) $(\pm\sqrt{2},1)$

107. The latus rectum of the parabola $y^2=4ax$ whose focal chord is PSQ such that SP = 3 and SQ = 2 is given by :

(a) $\frac{24}{5}$ (b) $\frac{12}{5}$ (c) $\frac{6}{5}$ (d) $\frac{1}{5}$

108. The eccentric angles of the extremities of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are given by

(a) $\tan^{-1}\left(\pm\frac{ae}{b}\right)$ (b) $\tan^{-1}\left(\pm\frac{be}{a}\right)$

(c) $\tan^{-1}\left(\pm\frac{b}{ae}\right)$ (d) $\tan^{-1}\left(\pm\frac{a}{be}\right)$

109. The two conics $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ and $y^2=-\frac{b}{a}x$ intersect if and only if

(a) $0<a\le\frac{1}{2}$ (b) $0<b\le\frac{1}{2}$

(c) $b^2>a^2$ (d) $b^2<a^2$

110. A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$, then the equation of the pair of tangent are

(a) $x^2 + y^2 - 5xy = 0$ (b) $x^2 + y^2 + 2x + y = 0$

(c) $x^2 + y^2 - xy + 7 = 0$ (d) $2x^2 + 2y^2 + 5xy = 0$

111. Equation of the circle passing through the origin and through the points of intersection of the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ and the line $x + y - 1 = 0$ is

(a) $x^2 + y^2 - 20x + 15y = 0$

(b) $x^2 + y^2 + 33x + 33y = 0$

(c) $x^2 + y^2 - 22x - 16y = 0$

(d) $2x^2 + 2y^2 - 4x - 5y = 0$

112. Equation of the circle concentric with the circle $x^2 + y^2 - 3x + 4y - c = 0$ and passing through the point $(-1, -2)$, is

(a) $x^2 + y^2 - 3x - 4y = 0$

(b) $x^2 + y^2 - 3x + 4y = 0$

(c) $x^2 + y^2 + 3x + 4y = 0$

(d) $x^2 + y^2 - 7x + 7y = 0$

113. If the line $x + y = 1$ is a tangent to a circle with centre (2, 3), then its equation is

(a) $x^2 + y^2 + 2x + 2y + 5 = 0$

(b) $x^2 + y^2 - 4x - 6y + 5 = 0$

(c) $x^2 + y^2 - x - y + 3 = 0$

(d) $x^2 + y^2 + 5x + 2y = 0$

114. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then radius of the circle is

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{5}{2}$

115. A.M. of the slopes of two tangents which can be drawn from the point (3, 1) to the circle $x^2 + y^2 = 4$ is

(a) $\frac{2}{5}$ (b) $\frac{3}{4}$

(c) $\frac{3}{5}$ (d) $\frac{1}{7}$

116. Equation of the circle which passes through the intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ whose centre lies on $13x + 30y = 0$ is

(a) $x^2 + y^2 + 5x + y = 0$

(b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

(c) $2x^2 + 2y^2 + 3x - 4y = 0$

(d) $4x^2 + 4y^2 - 8x + 7y + 10 = 0$

117. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq.units. Then the equation of the circle is

(a) $x^2 + y^2 - 2x + 2y = 62$

(b) $x^2 + y^2 + 2x - 2y = 62$

(c) $x^2 + y^2 + 2x - 2y = 47$

(d) $x^2 + y^2 - 2x + 2y = 47$.

118. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameter of a circle of circumference 10π, then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y - 23 = 0$

(b) $x^2 + y^2 - 2x - 2y - 23 = 0$

(c) $x^2 + y^2 + 2x + 2y - 23 = 0$

(d) $x^2 + y^2 - 2x + 2y - 23 = 0$

119. Intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is

(a) $x^2 + y^2 + x - y = 0$ (b) $x^2 + y^2 - x + y = 0$

(c) $x^2 + y^2 + x + y = 0$ (d) $x^2 + y^2 - x - y = 0$

120. The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation:

(a) $x^2 - 6x - 10y + 14 = 0$ (b) $x^2 - 10x - 6y + 14 = 0$

(c) $y^2 - 6x - 10y + 14 = 0$ (d) $y^2 - 10x - 6y + 14 = 0$

121. Two circles $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ cut each other orthogonally, then :

(a) $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ (b) $2g_1g_2 - 2f_1f_2 = c_1 + c_2$

(c) $2g_1g_2 + 2f_1f_2 = c_1 - c_2$ (d) $2g_1g_2 - 2f_1f_2 = c_1 - c_2$

122. If the straight line $ax + by = 2$; $a, b \neq 0$ touches the circle $x^2 + y^2 - 2x = 3$ and is normal to the circle $x^2 + y^2 - 4y = 6$, then the values of a and b are

(a) $\frac{3}{2}, 2$ (b) $-\frac{4}{3}, 1$

(c) $\frac{1}{4}, 2$ (d) $\frac{2}{3}, -1$

123. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis . The locus of the other end of the diameter through A is

(a) $(y-q)^2 = 4px$ (b) $(x-q)^2 = 4py$

(c) $(y-p)^2 = 4qx$ (d) $(x-p)^2 = 4qy$

124. The value of λ does the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$ is/are

(a) $\pm 2\sqrt{2}$ (b) $2 \pm \sqrt{3}$ (c) ± 5 (d) $5 \pm \sqrt{2}$

125. The equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represents a hyperbola
(a) The length of the transverse axes is 4
(b) Length of latus rectum is 9
(c) Equation of directrix is $x = \frac{21}{5}$ and $x = -\frac{11}{5}$
(d) None of these

126. The sum of the minimum distance and the maximum distance from the point (4, – 3) to the circle $x^2 + y^2 + 4x - 10y - 7 = 0$ is
(a) 20 (b) 12 (c) 10 (d) 16

127. If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\theta = 4$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 16$, then the value of θ equals
(a) $\frac{\pi}{6}$ (b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

128. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are
(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$
(c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$

129. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is
(a) an ellipse (b) a circle
(c) a parabola (d) a hyperbola

130. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is
(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

131. If $x + y = k$ is normal to $y^2 = 12x$, then the value of k is
(a) 3 (b) 9
(c) –9 (d) –3

132. A rod AB of length 15 cm rests in between two coordinate axes in such a way that the end point A lies on x-axis and end point B lies on y-axis. A point P(x, y) is taken on the rod in such a way that AP = 6 cm. Then, the locus of P is a/an.
(a) circle (b) ellipse
(c) parabola (d) hyperbola

133. If a parabolic reflector is 20 cm in diameter and 5 cm deep, then the focus is
(a) (2, 0) (b) (3, 0)
(c) (4, 0) (d) (5, 0)

134. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Then, the length of supporting wire attached to the roadway 18 m from the middle is
(a) 10.02 m (b) 9.11 m
(c) 10.76 m (d) 12.06 m

135. The length of the line segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola where the line segment makes an angle θ to the x-axis is $\frac{4am}{n}$. Here, m and n respectively are
(a) sin θ, cos θ (b) cos θ, sin θ
(c) cos θ, $\sin^2\theta$ (d) $\sin^2\theta$, cos θ

136. An arch is in the form of semi-ellipse. It is 8 m wide and 2 m high at the centre. Then, the height of the arch at a point 1.5 m from one end is
(a) 1.56 m (b) 2.4375 m
(c) 2.056 m (d) 1.086 m

137. A man running a race course notes that sum of its distance from two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Then, the equation of the posts traced by the man is
(a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (b) $x^2 + y^2 = 25$
(c) $x^2 + y^2 = 9$ (d) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

138. The equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin is
(a) $x^2 + y^2 - 2x - 2y + 1 = 0$
(b) $x^2 + y^2 - 2x - 2y - 1 = 0$
(c) $x^2 + y^2 - 2x - 2y = 0$
(d) $x^2 + y^2 - 2x + 2y - 1 = 0$

139. Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on a circle for
(a) only one value of k (b) $0 < k < 1$
(c) $k < 0$ (d) all integral values of k

140. Find the equation of a circle which passes through the origin and makes intercepts 2 units and 4 units on x-axis and y-axis respectively.
(a) $x^2 + y^2 - 2x - 4y = 0$ (b) $x^2 + y^2 - 4y = 0$
(c) $x^2 + y^2 + 2x = 0$ (d) $x^2 + y^2 - 4x - 2y = 0$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** As the circle is passing through the point (4, 5) and its centre is (2, 2) so its radius is $\sqrt{(4-2)^2+(5-2)^2}=\sqrt{13}$.
Therefore, the required equation is
$(x-2)^2+(y-2)^2=13$

2. **(a)** We put the co-ordinates of the given point in the given equation of circle
$x^2+y^2+4x-2y-4=0$
At (1, 2)
$(1)^2+(2)^2+4(1)-2(2)-4=1+4+4-4-4=1>0$
$\Rightarrow$ Point (1, 2) lies out side the circle i.e, an exterior point.

3. **(d)** A conic section is a parabola if e = 1.

4. **(a)** We know that length of latus rectum of ellipse $=\dfrac{2b^2}{a}$
Given, $3x^2+4y^2=12\Rightarrow\dfrac{x^2}{4}+\dfrac{y^2}{3}=1$
$\Rightarrow a=2, b=\sqrt{3}$
$\therefore$ Length of latus rectum $=\dfrac{2\times3}{2}=3$

5. **(a)** Focal distance of a point (x_1, y_1) on the parabola is $y^2=4ax$ is equal to its distance from directrix $x+a=0$ is x_1+1.
For $y^2=12x$; $a=3$,
so $x_1+3=4$
$\Rightarrow$ $x_1=1$

6. **(c)** In case of hyperbola difference between two focal points from any point $P(x_1, y_1)$ of the hyperbola having eccentricity = e is equal to the length of transverse axis.
i.e., $S'P-SP=(ex_1+a)-(ex_1-a)$,
[where S' and S are two focal points = 2a]

7. **(a)** $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

8. **(c)** $\dfrac{2b^2}{a}$

9. **(a)** $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

10. **(b)** $\dfrac{2b^2}{a}$

11. **(c)** The length of smallest focal chord of the parabola is its latus rectum and for parabola $y^2=4ax$, it is 4a.

12. **(c)** We have $a=3$ and $\dfrac{b^2}{a}=4\Rightarrow b^2=12$
Hence, the equation of the hyperbola is $\dfrac{x^2}{9}-\dfrac{y^2}{12}=1$
$\Rightarrow 4x^2-3y^2=36\Rightarrow 4x^2-3y^2-36=0$

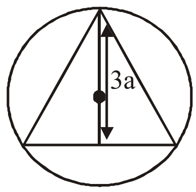
13. **(a)** $(-ae-ae)^2=(16)^2$
$\Rightarrow 4a^2e^2=256\Rightarrow a^2=32\ (\because e=\sqrt{2})$
Now, $e=\sqrt{\dfrac{a^2+b^2}{a^2}}\Rightarrow b^2=32$
$\therefore$ Required hyperbola is $x^2-y^2=32$

14. **(c)** Since the given equation represents a circle, therefore, $4a-3=a$ *i.e.,* $a=1$
($\because$ coefficients of x^2 and y^2 must be equal)
So, the circle becomes
$x^2+y^2+6x-2y+2=0$
$\therefore$ The coordinates of centre are (– 3, 1)

15. **(b)** Let a parabola with vertex at origin and axis along the x-axis be $y^2=4ax$. It passes through(–3, 7),
hence $(7)^2=4a(-3)\Rightarrow a=-\dfrac{49}{12}$.
$\therefore$ The required equation of the parabola is
$y^2=4\left(-\dfrac{49}{12}\right)x$ or $3y^2=-49x$.

16. **(c)** Let eq. of ellipse be $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$,
length of semi-latus rectum
$=\dfrac{b^2}{a}=\dfrac{a^2(1-e^2)}{a}=a(1-e^2)$
Given $a(1-e^2)=\dfrac{1}{3}(2a)$
$\Rightarrow 1-e^2=\dfrac{2}{3}\Rightarrow e^2=1-\dfrac{2}{3}=\dfrac{1}{3}\Rightarrow e=\dfrac{1}{\sqrt{3}}$

17. **(c)** Let equation of circle having centre (0, 0) be
$x^2+y^2=r^2$ ...(i)
Since, in an equilateral triangle, the centroid coincides with the centre of the circle.
$\therefore$ Radius of circle, $r=\dfrac{2}{3}(3a)=2a$



On putting r = 2a in (i), we get
$x^2+y^2=(2a)^2\Rightarrow x^2+y^2=4a^2$

18. **(a)** $2ae=6\Rightarrow ae=3$; $2b=8\Rightarrow b=4$
$b^2=a^2(1-e^2)$; $16=a^2-a^2e^2$
$\Rightarrow a^2=16+9=25\Rightarrow a=5$
$\therefore e=\dfrac{3}{a}=\dfrac{3}{5}$

19. **(d)** $e=\sqrt{1-\dfrac{1}{7}}=\sqrt{\dfrac{6}{7}}$

20. (b) We have $ae = 5$ [Since focus is $(\pm ae, 0)$]

and $\frac{a}{e} = \frac{36}{5}$ $\left[\text{since directrix is } x = \pm\frac{a}{e}\right]$

On solving we get $a = 6$ and $e = \frac{5}{6}$

$\Rightarrow b^2 = a^2(1-e^2) = 36\left(1-\frac{25}{36}\right) = 11$

Thus, the required equation of the ellipse is

$\frac{x^2}{36} + \frac{y^2}{11} = 1$.

21. (a) The given eq. is $25(x+1)^2 + 9(y+2)^2 = 225$

$\Rightarrow \frac{(x+1)^2}{9} + \frac{(y+2)^2}{25} = 1$

centre of the ellipse is $(-1, -2)$ and $a = 3$, $b = 5$, so that $a < b$.

$\Rightarrow 3 = 5\sqrt{1-e^2} \Rightarrow e^2 = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow e = \frac{4}{5}$

Hence, foci are $\left(-1, -2-5\times\frac{4}{5}\right)$ and $\left(-1, -2+5\times\frac{4}{5}\right)$,

i.e., foci are $(-1, -6)$ and $(-1, 2)$.

22. (c) The given equation reduces to

$\frac{(x-1)^2}{9} - \frac{y^2}{3} = 1$. Thus $a^2 = 9$, $b^2 = 3$

Using $b^2 = a^2(e^2 - 1)$,

we get $3 = 9(e^2 - 1) \Rightarrow e = \frac{2}{\sqrt{3}}$.

23. (b) Since the vertices are on the y-axis (with origin at the mid point), the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

As vertices are $(0, \pm 6)$, $a = 6$,

$b^2 = a^2(e^2 - 1) = 36\left(\frac{25}{9} - 1\right) = 64$, so the required equation of the hyperbola is

$\frac{y^2}{36} - \frac{x^2}{64} = 1$

24. (b) $e = \frac{1}{2}$. Directrix, $x = \frac{a}{e} = 4$

$\therefore\ a = 4 \times \frac{1}{2} = 2$

$\therefore\ b = 2\sqrt{1-\frac{1}{4}} = \sqrt{3}$

Equation of ellipse is

$\frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow 3x^2 + 4y^2 = 12$

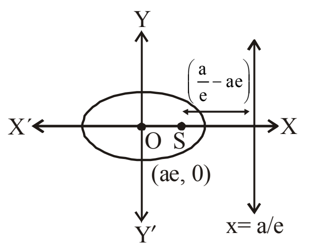
25. (a) Perpendicular distance of directrix from focus

$= \frac{a}{e} - ae = 4$

$\Rightarrow a\left(2 - \frac{1}{2}\right) = 4$

$\Rightarrow a = \frac{8}{3}$

$\therefore$ Semi-major axis $= 8/3$



26. (d) Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It passes through $(-3, 1)$

so $\frac{9}{a^2} + \frac{1}{b^2} = 1$ ...(i)

Also, $b^2 = a^2(1 - 2/5)$

$\Rightarrow 5b^2 = 3a^2$ ...(ii)

Solving (i) and (ii) we get $a^2 = \frac{32}{3}, b^2 = \frac{32}{5}$

So, the equation of the ellipse is $3x^2 + 5y^2 = 32$

27. (b) $ae = 2$ and $e = 2$

$\therefore\ a = 1$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = 1(4-1) \Rightarrow b^2 = 3$

Equation of hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow\ \frac{x^2}{1} - \frac{y^2}{3} = 1$

$\Rightarrow\ 3x^2 - y^2 = 3$

28. (d) The given equation $9x^2 + y^2 = k(x^2 - y^2 - 2x)$ can be written as $9x^2 + y^2 - kx^2 + ky^2 + 2kx = 0$

$\Rightarrow (9-k)x^2 + (1+k)y^2 + 2kx = 0$

This equation represents a circle, if coefficients of x^2 and y^2 are equal. so,

$9 - k = 1 + k$

$\Rightarrow 2k = 8 \Rightarrow k = 4$

29. (c) Let a be the major axis and b, the minor axis of the ellipse, then 3 minor axis = major axis.

$\Rightarrow\ 3b = a$

Eccentricity is given by

$b^2 = a^2(1 - e^2)$

$\Rightarrow\ b^2 = 9b^2(1 - e^2)$

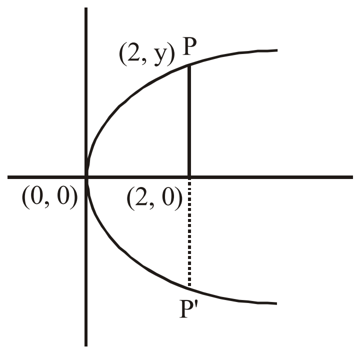
$\frac{1}{9} = (1 - e^2) \Rightarrow e^2 = \frac{8}{9} \Rightarrow e = \frac{2\sqrt{2}}{3}$

30. (d) Given parabola is $y^2 = 8x$ and focal distance = 4

Comparing this with standard parabola, $y^2 = 4ax$

$a = 2$, co-ordinate of focus is $(0, 2)$.

Focal distance of any point $(x, y) = x + 2$



$\Rightarrow\ x + 2 = 4 \Rightarrow x = 2$

$y^2 = 8x = 8 \times 2 = 16 \Rightarrow y = \pm 4$

31. (c) Length of latus rectum of a hyperbola is $\frac{2b^2}{a}$ where a is the half of the distance between two vertex of the hyperbola.

Latus rectum $= \frac{2b^2}{a} = \frac{10}{3}$

or, $b^2 = \frac{5a}{3}$ ...(i)

In case of hyperbola,

$b^2 = a^2(e^2 - 1)$ ...(ii)

Putting value of b^2 from equation (i) and $e = \frac{\sqrt{13}}{3}$ in equation (ii),

$\frac{5a}{3} = a^2\left(\frac{13}{9} - 1\right)$ or, $\frac{5a}{3} = \frac{4a^2}{9}$

$\Rightarrow$ $4a^2 - 15a = 0$ or, $a(4 - 15a) = 0$

$a \neq 0$, hence, $a = \frac{15}{4}$

Length of transverse axis $= 2a = 2 \times \frac{15}{4} = \frac{15}{2}$

32. (c) Centre (1, 0), circumference $= 10\pi$ (given)

$\therefore$ $2\pi r = 10\pi \Rightarrow r = 5$

So, equation of circle is $(x - 1)^2 + (y - 0)^2 = 25$

$\Rightarrow$ $x^2 + y^2 - 2x - 24 = 0$

33. (a) The foci are always on the transverse axis. It is the positive term whose denominator gives the transverse axis.

Thus, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ has transverse axis along x-axis of length 6.

34. (a) The given equation may be written as

$\frac{x^2}{32/3} - \frac{y^2}{8} = 1$ or $\frac{x^2}{\left(4\sqrt{2}/\sqrt{3}\right)^2} - \frac{y^2}{\left(2\sqrt{2}\right)^2} = 1$

Comparing the given equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get $a^2 = \left(\frac{4\sqrt{2}}{\sqrt{3}}\right)^2$ or $a = \frac{4\sqrt{2}}{\sqrt{3}}$. Therefore, length of transverse axis of a hyperbola $= 2a = 2 \times \frac{4\sqrt{2}}{\sqrt{3}} = \frac{8\sqrt{2}}{\sqrt{3}}$

35. (c) Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ represent the hyperbola. Then, according to the given condition, the length of transverse axis, i.e., $2a = 7 \Rightarrow a = \frac{7}{2}$.

Also, the point (5, – 2) lies on the hyperbola. So, we have $\frac{4}{49}(25) - \frac{4}{b^2} = 1$, which gives $b^2 = \frac{196}{51}$.

Hence, the equation of the hyperbola is

$\frac{4}{49}x^2 - \frac{51}{196}y^2 = 1$

36. (a) Conjugate axis is 5 and distance between foci = 13

$\Rightarrow 2b = 5$ and $2ae = 13$.

Now, also we know for hyperbola

$b^2 = a^2(e^2 - 1) \Rightarrow \frac{25}{4} = \frac{(13)^2}{4e^2}\left(e^2 - 1\right)$

$\Rightarrow$ $\frac{25}{4} = \frac{169}{4} - \frac{169}{4e^2}$ or $e^2 = \frac{169}{144}$ $\Rightarrow$ $e = \frac{13}{12}$

or $a = 6$, $b = \frac{5}{2}$ or hyperbola is $\frac{x^2}{36} - \frac{y^2}{\frac{25}{4}} = 1$

$\Rightarrow$ $25x^2 - 144y^2 = 900$

37. (d) The equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$

The point for which $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 > 0$ is outside ellipse.

Since, at (a, 0) : $1 + 0 - 1 = 0$

It lies on the ellipse.

At (0, b) : $0 + 1 - 1 = 0$

It lies on the ellipse.

At (– a, 0) : $1 + 0 - 1 = 0$

It lies on the ellipse.

At (a, b) : $1 + 1 - 1 > 0$

So, the point (a, b) lies outside the ellipse.

38. (d) Given, $e = \frac{2}{3}$

So, $a = \frac{3}{2}$ $(\because\ ae = 1)$

We know, $b^2 = a^2(1 - e^2)$

$\Rightarrow$ $b^2 = \frac{9}{4}\left[1 - \frac{4}{9}\right] = \frac{5}{4}$

So equation of the ellipse with vertex (3, 1) is

$\frac{(x-3)^2}{36} + \frac{(y-1)^2}{20} = 1$.

39. (c) The given parabola can be written as:

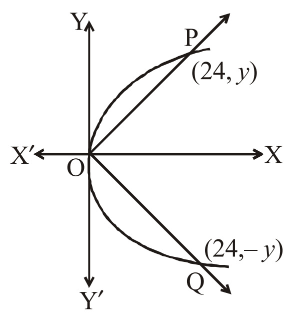
$(x - 4)^2 = -2(y - 9/2)$

which is of the form $x^2 = 4ay$

Thus, the vertex is (4, 9/2).

40. (b) Let P and Q be points on the parabola $y^2 = 6x$ and OP, OQ be the lines joining the vertex O to the points P and Q whose abscissa are 24 $\Rightarrow$ $y = \pm 12$.

Therefore, the coordinates of the points P and Q are (24, 12) and (24, – 12), respectively. Hence, the lines are $y = \pm\frac{12}{24}x \Rightarrow 2y = \pm x$.



41. (b) The eccentricity of $\frac{x^2}{16}+\frac{y^2}{25}=1$ is

$$e_1=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$

$\therefore\ e_2=\frac{5}{3}\ (\because e_1e_2=1)$

$\Rightarrow$ foci of ellipse = $(0, \pm 3)$

$\Rightarrow$ Equation of hyperbola is

$$\frac{x^2}{16}-\frac{y^2}{9}=-1$$

42. (d) Let the centre of the circle be (h, k).

Since the centre lies on the line $y = x - 1$

$\therefore\ k = h - 1$ …(i)

Since the circle passes through the point (7, 3), therefore, the distance of the centre from this point is the radius of the circle.

$\therefore\ 3=\sqrt{(h-7)^2+(k-3)^2}$

$\Rightarrow\ 3=\sqrt{(h-7)^2+(h-1-3)^2}$ [using (i)]

$\Rightarrow\ h = 7$ or $h = 4$

For $h = 7$, we get $k = 6$

and for $h = 4$, we get $k = 3$

Hence, the circles which satisfy the given conditions are

$(x-7)^2+(y-6)^2=9$

or $x^2+y^2-14x+12y+76=0$

and $(x-4)^2+(y-3)^2=9$

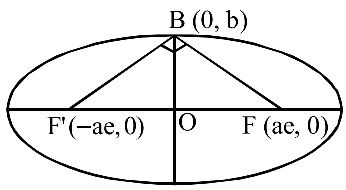
or $x^2+y^2-8x-6y+16=0$

43. (c) The given equation can be rewritten as $(y-1)^2 = 4\left(x-\frac{1}{2}\right)$ which is a parabola with its vertex $\left(\frac{1}{2}, 1\right)$ axis along the line $y = 1$, hence axis parallel to x-axis. Its focus is $\left(\frac{1}{2}+1, 1\right)$, i.e., $\left(\frac{3}{2}, 1\right)$.

44. (a) $\because \angle FBF' = 90° \Rightarrow FB^2 + F'B^2 = FF'^2$

$\therefore \left(\sqrt{a^2e^2+b^2}\right)^2+\left(\sqrt{a^2e^2+b^2}\right)^2=(2ae)^2$

$\Rightarrow 2(a^2e^2+b^2)=4a^2e^2\ \Rightarrow e^2=\frac{b^2}{a^2}$



Also $e^2=1-b^2/a^2=1-e^2$

$\Rightarrow 2e^2=1\ \Rightarrow\ e=\frac{1}{\sqrt{2}}$.

45. (d) Solving equations of parabolas

$y^2=4ax$ and $x^2=4ay$

we get (0, 0) and $(4a, 4a)$

Substituting in the given equation of line

$2bx+3cy+4d=0$,

we get $d = 0$ and $2b + 3c = 0$

$\Rightarrow d^2+(2b+3c)^2=0$

46. (b) The given curve is :

$2x^2-8x+y^2-2y+1=0$

$\Rightarrow\ 2(x^2-4x+4-4)+(y^2-2y+1)=0$

$\Rightarrow\ 2(x-2)^2-8+(y-1)^2=0$

$\Rightarrow\ 2(x-2)^2+(y-1)^2=8$

$\Rightarrow\ \frac{(x-2)^2}{4}+\frac{(y-1)^2}{8}=1$

This is equation of ellipse with centre (2, 1)

$\Rightarrow\ a^2=4, b^2=8$

Eccentricity $e=\sqrt{\frac{b^2-a^2}{b^2}}=\sqrt{\frac{8-4}{8}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$

47. (b) The given equation of curve is

$y^2+4x-6y+13=0$

which can be written as :

$y^2-6y+9+4x+4=0$

$\Rightarrow\ (y^2-6y+9)=-4(x+1)$

$\Rightarrow\ (y-3)^2=-4(x+1)$

Put $Y = y - 3$ and $X = x + 1$

On comparing $Y^2 = 4aX$

Length of focus from vertex, $a = -1$

At focus $X = a$ and $Y = 0$

$\Rightarrow\ x+1=-1\ \Rightarrow\ x=-2$

$y-3=0\ \Rightarrow\ y=3$

$\therefore$ Focus is $(-2, 3)$.

48. (a) Let eccentricity of both the parabolas be e.

Then in the given ellipse

$\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$

We have $a^2=\alpha^2, b^2=\beta^2$

$b^2=a^2(1-e^2)$

$\beta^2=\alpha^2(1-e^2)$ $(\because \alpha > \beta)$

$\Rightarrow \frac{\beta^2}{\alpha^2}=1-e^2$

$\Rightarrow e^2=1-\frac{\beta^2}{\alpha^2}$ …(i)

From equation $\frac{x^2}{9}+\frac{y^2}{16}=1$

$a^2=9, b^2=16$

Then $b^2=a^2(1-e^2)$, $b > a$

$\frac{16}{9}=1-e^2$

$e^2=1-\frac{16}{9}$ …(ii)

From equations (i) and (ii) we get

$1-\frac{16}{9}=1-\frac{\beta^2}{\alpha^2}$

$\Rightarrow \frac{16}{9}=\frac{\beta^2}{\alpha^2} \Rightarrow \frac{\beta}{\alpha}=\pm\frac{4}{3} \Rightarrow 4\alpha=3\beta$

or $4\alpha=-3\beta$

$4\alpha=3\beta$ is in the option.

49. (a) Given the equation of parabola

$x^2+8x+12y+4=0$

Make it perfect square

$\Rightarrow x^2+8x+16+12y+4-16=0$

$\Rightarrow (x+4)^2+12y-12=0$

$\Rightarrow (x+4)^2=-12(y-1)$

$\Rightarrow X^2=-12Y$

where $X=x+4$ and $Y=y-1$

vertex $X=0$ and $Y=0$

$\Rightarrow x+4=0$ and $y-1=0$

$\Rightarrow x=-4, y=1$ i.e., $(-4,1)$

50. (c) From the definition of conic; If P(x, y) is the point on a conic then ratio of its distance from focus to its distance from directrix is a fixed ratio e, called eccentricity.

Here focus is (1, – 1) and directrix is $x-y+1=0$.

Distance of this point from focus

$=\sqrt{(x-1)^2+(y+1)^2}$

Distance of this point from directrix. $=\left|\frac{x-y+1}{\sqrt{1^2+(-1)^2}}\right|$

So, from the definition of conic

$\sqrt{(x-1)^2+(y+1)^2}=e.\left|\frac{x-y+1}{\sqrt{2}}\right|$ ...(i)

Squaring both sides of equation (i), we get

$(x-1)^2+(y+1)^2=e^2.\frac{(x-y+1)^2}{2}$

$=(\sqrt{2})^2.\frac{(x-y+1)^2}{2}$ $\left[e=\sqrt{2}, \text{given}\right]$

$=(x-y+1)^2$

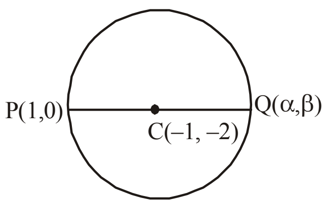
$\Rightarrow (x-1)^2+(y+1)^2=(x-y+1)^2$

$\Rightarrow x^2-2x+1+y^2+2y+1$

$=x^2-2xy+y^2+2x-2y+1$

$\Rightarrow 2xy-4x+4y+1=0.$

51. (c) The given circle is $x^2+y^2+2x+4y-3=0$



Centre (–1, –2)

Let $Q(\alpha, \beta)$ be the point diametrically opposite to the point $P(1, 0)$,

then $\frac{1+\alpha}{2}=-1$ and $\frac{0+\beta}{2}=-2 \Rightarrow \alpha=-3, \beta=-4$

So, Q is (–3, –4)

52. (b) We consider the options. Since, the required equation of circle has radius 5 and touches another circle at (5, 5)

$\therefore$ point (5, 5) satisfies the equation of required circle.

Point (5, 5) lies only on the circle

$x^2+y^2-18x-16y+120=0$

and also radius of this circle is 5.

53. (b) We have circle $x^2+y^2-8x+4y+4=0$

$x^2-8x+16+y^2+4y+4=-4+20$

$(x-4)^2+(y+2)^2=4^2$

Its centre is (4, – 2) and radius is 4.

Clearly this touches y-axis.

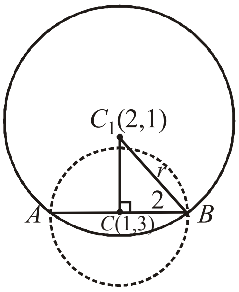
54. (b) $|r_1-r_2|<C_1C_2$ for intersection

$\Rightarrow r-3<5 \Rightarrow r<8$ ...(i)

and $r_1+r_2>C_1C_2$, $r+3>5 \Rightarrow r>2$ ...(ii)

From (i) and (ii), $2<r<8$.

55. (c) The given circle is $x^2+y^2-2x-6y+6=0$ with centre $C(1, 3)$ and radius $=\sqrt{1+9-6}=2$. Let AB be one of its diameter which is the chord of other circle with centre at $C_1(2, 1)$.



Then in ΔC_1CB,

$C_1B^2=CC_1^2+CB^2$

$r^2=[(2-1)^2+(1-3)^2]+(2)^2$

$\Rightarrow r^2=1+4+4 \Rightarrow r^2=9 \Rightarrow r=3.$

56. (c) From given equations

$\frac{x}{2}=\cos t+\sin t$ ...(i)

$\frac{y}{5}=\cos t-\sin t$ ...(ii)

Eliminating t from (i) and (ii), we have

$\frac{x^2}{4}+\frac{y^2}{25}=2 \Rightarrow \frac{x^2}{8}+\frac{y^2}{50}=1$ which is an ellipse.

57. (b) Let the equation of the required ellipse be

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(i)

The coordinates of its vertices and foci are (± a, 0) and (± ae, 0) respectively.

$\therefore$ a = 5 and ae = 4 $\Rightarrow$ e = 4/5

Now, $b^2=a^2(1-e^2) \Rightarrow b^2=25\left(1-\frac{16}{25}\right)=9$

Substituting the values of a^2 and b^2 in (i), we get

$\frac{x^2}{25}+\frac{y^2}{9}=1$, which is the equation of the required ellipse.

58. (a) Let P (x, y) be any point on the hyperbola and PM is perpendicular from P on the directrix,
Then by definition, SP = e PM
$\Rightarrow (SP)^2 = e^2 (PM)^2$

$$\Rightarrow (x-1)^2 + (y-2)^2 = 3\left\{\frac{2x+y-1}{\sqrt{4+1}}\right\}^2$$

$\Rightarrow 5(x^2 + y^2 - 2x - 4y + 5)$
$= 3(4x^2 + y^2 + 1 + 4xy - 2y - 4x)$
$\Rightarrow 7x^2 - 2y^2 + 12xy - 2x + 9y - 22 = 0$
Which is the required hyperbola.

STATEMENT TYPE QUESTIONS

59. (c) Both are true statements.
60. (c) Both are true statements.
61. (d) By definition of ellipse, all statements are correct.
62. (c) The given equation is

$$x^2 + y^2 - 8x + 10y - 12 = 0$$

or $(x^2 - 8x) + (y^2 + 10) = 12$
or $(x^2 - 8x + 16) + (y^2 + 10y + 25) = 12 + 16 + 25$
or $(x-4)^2 + (y+5)^2 = 53$
Therefore, the given circle has centre at (4, –5) and radius $\sqrt{53}$.

63. (d) $\frac{x^2}{100} + \frac{y^2}{400} = 1$ is the equation of ellipse.
Major axis is along y-axis
$a^2 = 400, \therefore a = 20, b^2 = 100 \quad \therefore b = 10$

$c^2 = a^2 - b^2 = 400 - 100 = 300 \quad \therefore c = 10\sqrt{3}$

Vertices are $(0, \pm a)$ i.e., $(0, \pm 20)$

$\therefore$ Foci are $(0, \pm c)$ i.e., $(0, \pm 10\sqrt{3})$
Length of major axis $= 2a = 2 \times 20 = 40$
Length of minor axis $= 2b = 2 \times 10 = 20$

Eccentricity, $e = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$

Length of Latus rectum $= \frac{2b^2}{a} = \frac{2 \times 100}{20} = 10$

64. (c) Comparing the equation $\frac{y^2}{9} - \frac{x^2}{27} = 1$ with the standard equation.
we have, $a = 3, \; b = 3\sqrt{3}$

and $c = \sqrt{a^2 + b^2} = \sqrt{9 + 27} = \sqrt{36} = 6$
Therefore, the coordinates of the foci are (0, ± 6) and that of vertices are (0, ± 3). Also, the eccentricity

$e = \frac{c}{a} = \frac{5}{4}$ and the length of latus rectum $= \frac{2b^2}{a}$

$= \frac{2(3\sqrt{3})^2}{3} = \frac{54}{3} = 18$ units

65. (c) Both the statements are definitions.

66. (a) $9y^2 - 4x^2 = 36$ is the equation of hyperbola

i.e., $\frac{y^2}{4} - \frac{x^2}{9} = 1$
$\therefore a^2 = 4, b^2 = 9,$
$\therefore c^2 = a^2 + b^2 = 4 + 9 = 13, a = 2, b = 3, c = \sqrt{3}$
Axis is y-axis
Foci $(0, \pm\sqrt{13})$, vertices $= (0, \pm 2)$

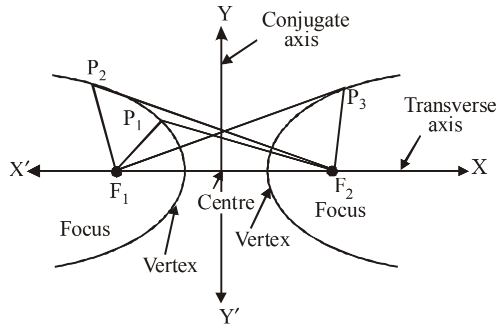
Eccentricity $= e = \frac{c}{a} = \frac{\sqrt{13}}{2}$,

Latus rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{2} = 9.$

67. (a) Only I is true.
68. (c) Both the statements are true.

MATCHING TYPE QUESTIONS

69. (d) The two fixed points are called the foci of the hyperbola. The mid-point of the line segment joining the foci is called the centre of the hyperbola. The line through the foci is called the transverse axis and the line through the centre and perpendicular to the transverse axis is called the conjugate axis. The points at which the hyperbola intersects the transverse axis are called the vertices of the hyperbola.



$P_1F_2 - P_1F_1 = P_2F_2 - P_2F_1 = P_3F_1 - P_3F_2$

70. (b) e = 1 $\Rightarrow$ Parabola
e < 1 $\Rightarrow$ Ellipse
e > 1 $\Rightarrow$ Hyperbola
e = 0 $\Rightarrow$ Circle

71. (a)

72. (d) (A) h = –3, k = 2, r = 4
Required circle is $(x+3)^2 + (y-2)^2 = 16$
(B) $(x^2 + 8x) + (y^2 + 10y) = 8$
$\Rightarrow (x^2 + 8x + 16) + (y^2 + 10y + 25) = 8 + 16 + 25$
$\Rightarrow (x+4)^2 + (y+5)^2 = 49$
(C) $(x-0)^2 + (y-2)^2 = 4$
$\Rightarrow (x^2 + y^2 + 4 - 4y = 4$
$\Rightarrow x^2 + y^2 - 4y = 0$

73. (a) $\frac{x^2}{4} + \frac{y^2}{9} = 1 \Rightarrow \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
$\Rightarrow a = 3, b = 2$

Now, $c = \sqrt{a^2 - b^2} = \sqrt{5}$, $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

Hence, foci $(0, \pm\sqrt{5})$, Vertices $(0, \pm 3)$,

Length of major axis = 6 units

Eccentricity $= \frac{\sqrt{5}}{3}$

74. (c) By definition of hyperbola, we have

$A \rightarrow 5; B \rightarrow 4; C \rightarrow 6; D \rightarrow 2; E \rightarrow 1; F \rightarrow 3$

INTEGER TYPE QUESTIONS

75. (d) $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$a = \sqrt{\frac{144}{25}}, b = \sqrt{\frac{81}{25}}, \quad e = \sqrt{1 + \frac{81}{144}} = \frac{15}{12} = \frac{5}{4}$

$\therefore$ Foci = $(\pm 3, 0)$

$\therefore$ foci of ellipse = foci of hyperbola

$\therefore$ for ellipse $ae = 3$ but $a = 4$,

$\therefore \quad e = \frac{3}{4}$

Then $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 16\left(1 - \frac{9}{16}\right) = 7$

76. (a) Any tangent to $y^2 = 4x$ is $y = mx + 1/m$

If it is drawn from $(-2, -1)$, then

$-1 = -2m + 1/m$

$\Rightarrow 2m^2 - m - 1 = 0$

If $m = m_1, m_2$ then $m_1 + m_2 = 1/2$,

$m_1 m_2 = -1/2$

$\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}$

$= \frac{\sqrt{1/4 + 2}}{1 - 1/2} = 3$

77. (b) $a = 3$

So, focal distance is $x + 3$.

$\therefore x + 3 = 4 \Rightarrow x = 1$

Hence, the abscissa = 1

78. (c) Comparing the equation of the circle

$(x+5)^2 + (y-3)^2 = 36$

with $(x-h)^2 + (y-k)^2 = 2$

$\therefore -h = 5$ or $h = -5, k = 3, r^2 = 36 \Rightarrow r = 6$

$\therefore$ Centre of the circle is $(-5, 3)$ and radius = 6

79. (c) Here $h = 0$, $k = 2$ and $r = 2$. Therefore, the required equation of the circle is

$(x-0)^2 + (y-2)^2 = (2)^2$

or $x^2 + y^2 - 4y + 4 = 4$

or $x^2 + y^2 - 4y = 0$

80. (b) Vertex (0, 0), Focus is (3, 0)

$a = 3$

$\therefore 4a = 12$

$\therefore$ Equation of parabola is $y^2 = 12x$

81. (c) Since the foci are on x-axis, the equation of the hyperbola is of the form

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Given : vertices are $(\pm 2, 0)$, $a = 2$

Also, since foci are $(\pm 3, 0)$, $c = 3$ and

$b^2 = c^2 - a^2 = 9 - 4 = 5$

Therefore, the equation of the hyperbola is

$\frac{x^2}{4} - \frac{y^2}{5} = 1$

82. (c) $y^2 = 8x \Rightarrow y^2 = 4.2.x \Rightarrow a = 2$

Length of the latus-rectum = 4a = 8

83. (a) $y^2 = -12x \Rightarrow 4a = 12 \Rightarrow a = 3$

So, equation of the directrix is $x = 3$.

84. (b) Coordinates of foci are $(\pm ae, 0)$

$\therefore ae = 2 \Rightarrow a.\frac{1}{2} = 2 \Rightarrow a = 4$

85. (c) Coordinates of vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$\therefore a = 5$ and $ae = 4 \Rightarrow e = \frac{4}{5}$

Now, $b^2 = a^2(1 - e^2) \Rightarrow b^2 = 25\left(1 - \frac{16}{25}\right) = 9$

86. (c) On solving $y = 2x$ and $x^2 + y^2 - 10x = 0$ simultaneously, we get $x = 0, 2$

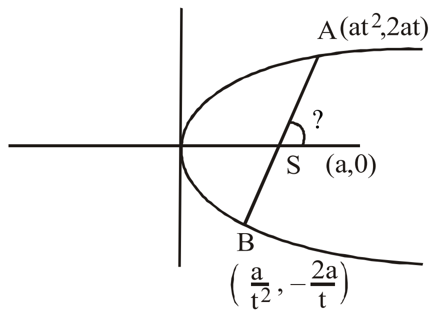
Putting $x = 0$ and $x = 2$ respectively in $y = 2x$, we get $y = 0$ and $y = 4$. Thus, the points of intersection of the given line and the circle are A (0, 0) and B (2, 4).

Required equation is $x^2 + y^2 - 2x - 4y = 0$

$a = 2, b = 4 \Rightarrow a + b = 6$

ASSERTION- REASON TYPE QUESTIONS

87. (d)



Let AB be a focal chord.

Slope of AB $= \frac{2t}{t^2 - 1} = \tan\alpha$

$\Rightarrow \tan\frac{\alpha}{2} = \frac{1}{t} \Rightarrow t = \cot\frac{\alpha}{2}$

Length of AB $= a\left(t + \frac{1}{t}\right)^2 = 4a\,\mathrm{cosec}^2\,\alpha$

$\Rightarrow$ Reason is correct but Assertion is false.

88. **(a)** The ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\therefore$ Auxiliary circle is $x^2+y^2=9$ and $(-\sqrt{5},0)$ and $(\sqrt{5},0)$ are focii.

$\therefore$ Assertion is true. Reason is true.

89. **(a)** $e=\frac{5}{3}$, $a=5$

$\therefore$ Focii are $(\pm 3, 0)$

For hyperbola $\frac{x^2}{\frac{27}{12}}-\frac{y^2}{\frac{27}{4}}=1$

$e=\sqrt{\frac{12+4}{4}}=2$, $a=\frac{3}{2}$

$\therefore$ focii are $(\pm 3, 0)$

$\therefore$ The two conics are confocal.

90. **(a)** Given circle is

$x^2+y^2-6x+4y-12=0$

$\text{Centre}=\left(-\frac{1}{2}\times(-6),-\frac{1}{2}\times 4\right)=(3,-2)$

91. **(a)** Given circle can be written as

$x^2+y^2+\frac{3}{2}x+2y+\frac{9}{16}=0$

$\text{Radius}=\sqrt{\left(\frac{3}{4}\right)^2+(1)^2-\frac{9}{16}}=1$

92. **(c)** Assertion is correct but Reason is incorrect.

93. **(d)** Assertion is incorrect. Reason is correct.

Assertion : A hyperbola in which a = b is called an equilateral hyperbola.

94. **(b)** Both Assertion and Reason are correct.

95. **(b)** Given ellipse is $\frac{x^2}{3}+\frac{y^2}{2}=1$, whose area is

$\pi\sqrt{3}\sqrt{2}=\pi\sqrt{6}$. Circle is $x^2+y^2-2x+4y+4=0$

or $(x-1)^2+(y+2)^2=1$.

Its area is π. Hence, Assertion is true.

Also, Reason is true (as length of semi-major axis $=\sqrt{3}>1$ (radius of circle) but it is not the correct explanation of Assertion.

96. **(c)** Parabola is symmetric with respect to the axis of the parabola. If the equation has a y^2 term, then the axis of symmetry is along the x-axis and if the equation has an x^2 term, then the axis of symmetry is along the y-axis.

97. **(a)** **Assertion**: Since, major axis is along y-axis. Hence, equation of ellipse will be of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \quad ...(i)$$

Given that (i) passes through the points (3, 2) and (1, 6) i.e, they will satisfy it

$$\therefore \frac{3^2}{b^2}+\frac{2^2}{a^2}=1 \Rightarrow \frac{9}{b^2}+\frac{4}{a^2}=1 \quad ...(ii)$$

and $\frac{1^2}{b^2}+\frac{6^2}{a^2}=1 \Rightarrow \frac{1}{b^2}+\frac{36}{a^2}=1$ ...(iii)

Multiplying (ii) by 9 and then subtracting (iii) from it,

we get $\frac{80}{b^2}=8 \Rightarrow b^2=\frac{80}{8}$ $\therefore b^2=10$

from eq (ii), we get $\frac{9}{10}+\frac{4}{a^2}=1 \Rightarrow \frac{4}{a^2}=1-\frac{9}{10} \Rightarrow a^2=40$

putting the value of $a^2=40$ and $b^2=10$ in (i), we get

$$\frac{x^2}{10}+\frac{y^2}{40}=1$$

CRITICALTHINKING TYPE QUESTIONS

98. **(d)** Given $y^2+4y+4x+2=0$

$\Rightarrow (y+2)^2+4x-2=0$

$\Rightarrow (y+2)^2=-4\left(x-\frac{1}{2}\right)$

Replace, $y+2=y$, $x-\frac{1}{2}=x$

we have, $y^2=-4x$

This is a parabola with directrix at $x=1$

$\Rightarrow x-\frac{1}{2}=1 \Rightarrow x=\frac{3}{2}$

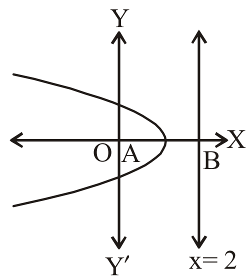
99. **(d)** Given, $y=x^2+2px+13$

$\Rightarrow y-(13-p^2)=(x+p)^2$

$\therefore$ vertex is at $(-p, 13-p^2)$

$\Rightarrow 13-p^2=4 \Rightarrow p^2=9 \Rightarrow p=\pm 3$

100. **(b)** Vertex of a parabola is the mid-point of focus and the point



where directrix meets the axis of the parabola.

Here focus is O (0, 0) and directxix meets the axis at B(2, 0)

$\therefore$ Vertex of the parabola is (1, 0)

101. **(c)** Let (h, k) be the centre of the circle.

Since, circle is passing through (0, 0), (a, 0) and (0, b), distance between centre and these points would be same and equal to radius.

Hence, $h^2+k^2=(h-a)^2+k^2=h^2+(k-b)^2$

$\Rightarrow h^2+k^2=h^2+k^2+h^2-2ah=h^2+k^2+b^2-2bk$

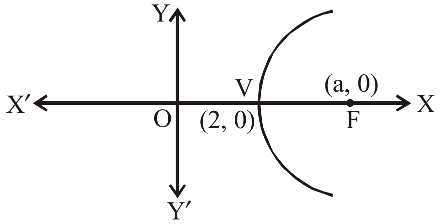
$\Rightarrow h^2+k^2=h^2+k^2+a^2-2ah$

$\Rightarrow h=\frac{a}{2}$

Similarly, $k=\frac{b}{2}$

$\therefore$ Radius of circle $=\sqrt{h^2+k^2}=\frac{1}{2}\sqrt{a^2+b^2}$

102. (c) Vertex is (2, 0). Since, y-axis is the directrix of a parabola.
$\therefore$ Equation of directrix is $x = 0$. So, axis of parabola is x-axis. Let the focus be (a, 0)



Distance of the vertex of a parabola from directrix = its distance from focus

So, $OV = VF \Rightarrow 2 = a - 2$

$\Rightarrow$ Focus is (4, 0)

103. (b) Given the equation of parabola:

$y^2 = 5x + 4y + 1$

$\Rightarrow y^2 - 4y = 5x + 1$

$\Rightarrow (y-2)^2 = 5x + 5 = 5(x+1)$

(By adding 4 on each side)

Put $y - 2 = Y$ and $x + 1 = X$

Then we get $Y^2 = 5X$

which is in the form of $y^2 = 4ax$

where 4a is the latus rectum

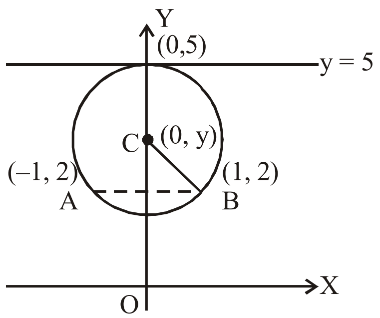
Thus length of latus rectum = 5.

104. (b) Let P (x, y) be any point on the bar such that PA = a and PB = b, clearly from the figure.

$x = OL = b\cos\theta$ and $y = PL = a\sin\theta$

This gives $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$. Which is an ellipse.

105. (a)



The centre of the circle is on the perpendicular bisector of the line joining (–1, 2) and (1, 2), which is the y-axis. The ordinate of the centre is given by

$$(5-y)^2 = 1 + (y-2)^2 \Rightarrow y = \frac{10}{3}$$

Hence, eq. of the circle is

$$x^2 + \left(y - \frac{10}{3}\right)^2 = \left(\frac{5}{3}\right)^2$$

$$\Rightarrow 9x^2 + 9y^2 - 60y + 75 = 0$$

106. (a) Since all of the points

$(\pm 2\sqrt{2}, 4), (\pm 2, 2), \left(\pm 3, \dfrac{9}{2}\right)$

and $(\pm\sqrt{2}, 1)$ lie on the curve $x^2 = 2y$

And the distance between $(\pm 2\sqrt{2}, 4)$ and $(0, 5)$ is shortest distance. Thus $(\pm 2\sqrt{2}, 4)$ on the curve are closest to the point (0, 5).

107. (a) We know the relationship between semi latus rectum and focal chord which is given as

$$\frac{2}{2a} = \frac{1}{SP} + \frac{1}{SQ} \Rightarrow \frac{2(SP)(SQ)}{SP + SQ} = 2a$$

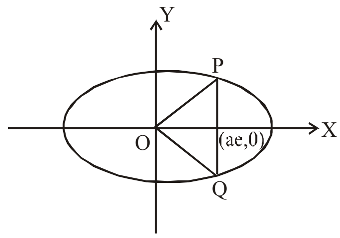
Given : SP = 3, SQ = 2

$$\therefore\ 2a = \frac{2(3)(2)}{3+2} \Rightarrow 2a = \frac{12}{5}$$

Now, latus rectum $= 2[2a] = \dfrac{24}{5}$.

108. (c) Let eq. of ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

If the latus rectum is PQ then for the points P and Q



$x = ae$, $\dfrac{y^2}{b^2} = 1 - e^2$

$\Rightarrow y^2 = b^2(1 - e^2) \Rightarrow y = \pm b\sqrt{1-e^2}$

Hence, P is $(ae, b\sqrt{1-e^2})$ and Q is $(ae, -b\sqrt{1-e^2})$.

If eccentric angle of the extremities be θ,

then, $a\cos\theta = ae$ and $b\sin\theta = \pm b\sqrt{1-e^2}$

$$\Rightarrow \tan\theta = \pm\frac{\sqrt{1-e^2}}{e} = \pm\left(\frac{b}{ae}\right) \Rightarrow \theta = \tan^{-1}\left(\pm\frac{b}{ae}\right)$$

109. (b) The x co-ordinates of the points of intersection are given by $\dfrac{x^2}{a^2} + \dfrac{1}{ab}x + 1 = 0$

and the roots are real if and only if

$$\frac{1}{a^2b^2} - \frac{4}{a^2} \ge 0 \Rightarrow \frac{1}{b^2} - 4 \ge 0$$

$$\Rightarrow 0 < b^2 \le \frac{1}{4} \Rightarrow 0 < b \le \frac{1}{2}$$

110. (d) Equation of pair of tangents is given by $SS_1 = T^2$,

or $S = x^2 + y^2 + 20(x + y) + 20$, $S_1 = 20$,

$T = 10(x + y) + 20 = 0$

$\therefore$ $SS_1 = T^2$

$\Rightarrow$ $20(x^2 + y^2 + 20(x + y) + 20) = 10^2(x + y + 2)^2$

$\Rightarrow$ $4x^2 + 4y^2 + 10xy = 0 \Rightarrow 2x^2 + 2y^2 + 5xy = 0$

111. (c) Let the required equation be

$(x^2+y^2-2x+4y-20)+\lambda(x+y-1)=0$

Since, it passes through (0, 0),so we have

$-20-\lambda=0 \Rightarrow \lambda=-20$

Hence the required equation is

$(x^2+y^2-2x+4y-20)-20(x+y-1)=0$

$\Rightarrow x^2+y^2-22x-16y=0$

112. (b) The equation of two concentric circles differ only in constant terms. So let the equation of the required circle be

$x^2+y^2-3x+4y+\lambda=0$

It passes through (–1, – 2), so we have

$1+4+3-8+\lambda=0 \Rightarrow \lambda=0,$

Hence required equation is $x^2+y^2-3x+4y=0$

113. (b) Radius of the circle = perpendicular distance of (2, 3) from $x+y=1$ is $\frac{4}{\sqrt{2}}=2\sqrt{2}$

∴ The required equation will be

$(x-2)^2+(y-3)^2=8 \Rightarrow x^2+y^2-4x-6y+5=0$

114. (a) The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x-4y+4=0$ and $3x-4y-\frac{7}{2}=0$ and so it is equal to

$$\frac{4}{\sqrt{9+16}}+\frac{7/2}{\sqrt{9+16}}=\frac{3}{2}$$

Hence radius is $\frac{3}{4}$.

115. (c) Any tangent to the given circle, with slope m is

$y=mx+2\sqrt{1+m^2}$

since it passes through the point (3, 1) ; so

$1=3m+2\sqrt{1+m^2}$

$\Rightarrow 4m^2+4=(3m-1)^2 \Rightarrow 5m^2-6m-3=0$

If $m=m_1, m_2$, then

AM of slopes $=\frac{1}{2}(m_1+m_2)=\frac{1}{2}(6/5)=3/5$

116. (b) The equation of required circle is $s_1+\lambda s_2=0$

$=x^2(1+\lambda)+y^2(1+\lambda)+x(2+13\lambda)-y\left(\frac{7}{2}+3\lambda\right)-\frac{25}{2}=0$

Centre $=\left(\frac{-(2+13\lambda)}{2},\frac{7/2+3\lambda}{2}\right)$

∵ Centre lies on $13x+30y=0$

$\Rightarrow -13\left(\frac{2+13\lambda}{2}\right)+30\left(\frac{7/2+3\lambda}{2}\right)=0 \Rightarrow \lambda=1$

117. (d) $\pi r^2=154 \Rightarrow r=7$

For centre:

On solving equation

$2x-3y=5 \,\&\, 3x-4y=7,$ we get $x=1, y=-1$

∴ centre = (1, –1)

Equation of circle, $(x-1)^2+(y+1)^2=7^2$

$x^2+y^2-2x+2y=47$

118. (d) Two diameters are along

$2x+3y+1=0$ and $3x-y-4=0$

solving we get centre (1, –1)

circumference $=2\pi r=10\pi$

$\therefore r=5.$

Required circle is, $(x-1)^2+(y+1)^2=5^2$

$\Rightarrow x^2+y^2-2x+2y-23=0$

119. (d) Solving $y=x$ and the circle

$x^2+y^2-2x=0$, we get

$x=0, y=0$ and $x=1, y=1$

∴ Extremities of diameter of the required circle are (0, 0) and (1, 1). Hence, the equation of circle is

$(x-0)(x-1)+(y-0)(y-1)=0$

$\Rightarrow x^2+y^2-x-y=0$

120. (d) The given circle is $x^2+y^2-6x+14=0$, centre (3, 3), radius = 2

Let (h, k) be the centre of touching circle. Then radius of touching circle = h [as it touches y-axis also]

∴ Distance between centres of two circles = sum of the radii of two circles

$\Rightarrow \sqrt{(h-3)^2+(k-3)^2}=2+h$

$\Rightarrow (h-3)^2+(k-3)^2=(2+h)^2$

$\Rightarrow h^2-6h+9+k^2-6k+9=4+4h+h^2$

$\Rightarrow k^2-10h-6k+14=0$

∴ locus of (h, k) is

$y^2-10x-6y+14=0$

121. (a) If two circles intersect at right angle i.e. the tangent at their point of intersection are at right angles, then the circles are called orthogonal circles.

The circles

$x^2+y^2+2gx+2fy+c=0$ and

$x^2+y^2+2g_1x+2f_1y+c_1=0$

are orthogonal, if

$2gg_1+2ff_1=c+c_1$

Thus, in the given question, the condition will be

$2g_1g_2+2f_1f_2=c_1+c_2$.

122. (b) Given $x^2+y^2-2x=3$

∴ Centre = (1, 0) and radius = 2

And $x^2+y^2-4y=6$

∴ Centre = (0, 2) and radius = $\sqrt{10}$.

Since line $ax+by=2$ touches the first circle.

$\therefore \frac{a(1)+b(0)-2}{\sqrt{a^2+b^2}}=2$ or $(a-2)=[2\sqrt{a^2-b^2}]$ ...(i)

Also the given line is normal to the second circle. Hence it will pass through the centre of the second circle.

∴ $a(0)+b(2)=2$ or $2b=2 \Rightarrow b=1$

Putting the value in equation (i) we get

$a-2=2\sqrt{a^2+1}$ or $(a-2)^2=4(a^2+1)$

or $a^2 + 4 - 4a = 4a^2 + 4$ or, $3a^2 + 4a = 0$

or $a(3a + 4) = 0$ or $a = 0, -4/3$

$\therefore$ values of a and b are –4/3, 1 respectively.

123. (d) Let the variable circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(i)

$\therefore p^2 + q^2 + 2gp + 2fq + c = 0$ ....(ii)

Circle (i) touches x-axis,

$\therefore g^2 - c = 0 \Rightarrow c = g^2$. From (ii)

$p^2 + q^2 + 2gp + 2fq + g^2 = 0$ ....(iii)

Let the other end of diameter through (p, q) be (h, k), then

$\frac{h+p}{2} = -g$ and $\frac{k+q}{2} = -f$

Put in (iii)

$$p^2 + q^2 + 2p\left(-\frac{h+p}{2}\right) + 2q\left(-\frac{k+q}{2}\right) + \left(\frac{h+p}{2}\right)^2 = 0$$

$\Rightarrow \quad h^2 + p^2 - 2hp - 4kq = 0$

$\therefore$ locus of (h, k) is $x^2 + p^2 - 2xp - 4yq = 0$

$\Rightarrow \quad (x - p)^2 = 4qy$

124. (c) $\because$ Equation of ellipse is $9x^2 + 16y^2 = 144$ or $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Comparing this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then we get $a^2 = 16$ and $b^2 = 9$ and comparing the line $y = x + \lambda$ with $y = mx + c$

$\therefore$ m = 1 and c = λ

If the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$, then $c^2 = a^2m^2 + b^2$

$\Rightarrow \lambda^2 = 16 \times 1^2 + 9 \Rightarrow \lambda^2 = 25$

$\therefore \lambda = \pm 5$

125. (c) We have, $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

$\Rightarrow 9(x^2 - 2x) - 16(y^2 - 2y) = 151$

$\Rightarrow 9(x^2 - 2x + 1) - 16(y^2 - 2y + 1) = 144$

$\Rightarrow 9(x - 1)^2 - 16(y - 1)^2 = 144$

$\Rightarrow \frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$

Shifting the origin at (1, 1) without rotating the axes

$\frac{X^2}{16} - \frac{Y^2}{9} = 1$, where $x = X + 1$ and $y = Y + 1$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

where $a^2 = 16$ and $b^2 = 9$

so the length of the transverse axes = 2a = 8

The length of the latus rectum = $\frac{2b^2}{a} = \frac{a}{2}$

The equation of the directrix, $x = \pm \frac{a}{e}$

$x - 1 = \pm \frac{16}{5} \Rightarrow x = \pm \frac{16}{5} + 1 \Rightarrow x = \frac{21}{5}$; $x = -\frac{11}{5}$

126. (a) Centre of the given circle $\equiv C(-2, 5)$

Radius of the circle CN = CT = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{2^2 + 5^2 + 7} = \sqrt{36} = 6$

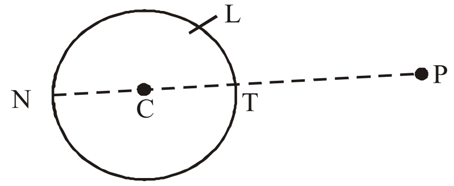
Distance between (4, – 3) and (– 2, 5) is

$PC = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

We join the external point, (4, – 3) to the centre of the circle (– 2, 5). Then PT is the minimum distance, from external point P to the circle and PN is the maximum distance. Minimum distance = PT = PC – CT = 10 – 6 = 4.

Maximum distance = PN = PC + CN = (10 + 6 = 16)

So, sum of minimum and maximum distance = 16 + 4 = 20.



127. (b) Given: $x^2 - y^2 \sec^2\theta = 4$ and $x^2 - \sec^2\theta + y^2 = 16$

$\Rightarrow \frac{x^2}{4} - \frac{y^2}{4\cos^2\theta} = 1$ and $\frac{x^2}{16\cos^2\theta} + \frac{y^2}{16} = 1$

According to problem

$\frac{4 + 4\cos^2\theta}{4} = 3\left(\frac{16 - 16\cos^2\theta}{16}\right)$

$\Rightarrow 1 + \cos^2\theta = 3(1 - \cos^2\theta) \Rightarrow 4\cos^2\theta = 2$

$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}$

128. (b) Any tangent to the parabola $y^2 = 8ax$ is

$y = mx + \frac{2a}{m}$ ...(i)

If (i) is a tangent to the circle, $x^2 + y^2 = 2a^2$ then,

$\sqrt{2}a = \pm\frac{2a}{m\sqrt{m^2 + 1}}$

$\Rightarrow m^2(1 + m^2) = 2 \Rightarrow (m^2 + 2)(m^2 - 1) = 0; \Rightarrow m = \pm 1$.

So from (i), $y = \pm(x + 2a)$.

129. (d) Tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

Given that $y = \alpha x + \beta$ is the tangent of hyperbola

$\Rightarrow m = \alpha$ and $a^2m^2 - b^2 = \beta^2$

$\therefore a^2\alpha^2 - b^2 = \beta^2$

Locus is $a^2x^2 - y^2 = b^2$ which is hyperbola.

130. (b) $\frac{dy}{dx} = 2x - 5 \therefore m_1 = (2x - 5)_{(2,0)} = -1$,

$m_2 = (2x - 5)_{(3,0)} = 1 \Rightarrow m_1m_2 = -1$

i.e. the tangents are perpendicular to each other.

131. (b) $y = mx + c$ is normal to the parabola

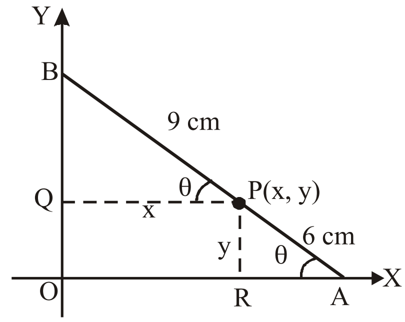
$y^2 = 4ax$ if $c = -2am - am^3$

Here $m = -1$, $c = k$ and $a = 3$

$\therefore \quad c = k = -2(3)(-1) - 3(-1)^3 = 9$

132. (b) Let AB be the rod making an angle θ with OX as shown in figure and P(x, y) the point on it such that AP = 6 cm. Since, AB = 15 cm, we have

PB = 9 cm



From P, draw PQ and PR perpendiculars on y-axis and x-axis, respectively.

From Δ PBQ, $\cos\theta = \frac{x}{9}$

From ΔPRA, $\sin\theta = \frac{y}{6}$

Since, $\cos^2\theta + \sin^2\theta = 1$

$$\left(\frac{x}{9}\right)^2 + \left(\frac{y}{6}\right)^2 = 1 \text{ or } \frac{x^2}{81} + \frac{y^2}{36} = 1$$

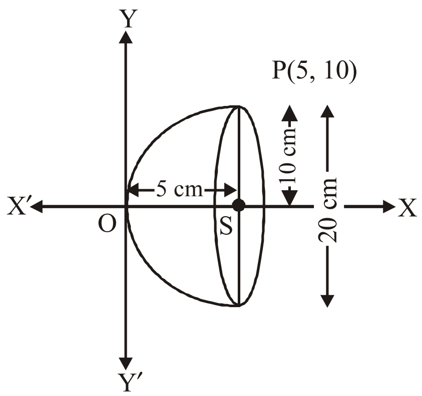
Thus, the locus of P is an ellipse.

133. (d) Taking vertex of the parabolic reflector at origin, x-axis along the axis of parabola. The equation of the parabola is $y^2 = 4ax$. Given depth is 5 cm, diameter is 20 cm.

$\because$ Point P(5, 10) lies on parabola.

$\therefore \ (10)^2 = 4a(5) \Rightarrow a = 5$

Clearly, focus is at the mid-point of given diameter.

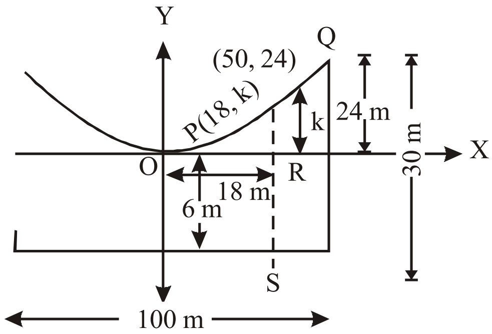


i.e., S = (5, 0)

134. (b) Since, wires are vertical. Let equation of the parabola is in the form

$x^2 = 4ay$ …(i)

Focus is at the middle of the cable and shortest and longest vertical supports are 6 m and 30 m and roadway is 100 m long.



$x^2 = 4ay$

Clearly, coordinate of Q(50, 24) will satisfy eq. (i)

$(50)^2 = 4a \times 24$

$\Rightarrow \ 2500 = 96a \Rightarrow a = \frac{2500}{96}$

Hence, from eq. (i), $x^2 = 4 \times \frac{2500}{96} y \Rightarrow x^2 = \frac{2500}{24} y$

Let PR = k m

$\therefore$ Point P(18, k) will satisfy the equation of parabola i.e.,

From eq. (i), $(18)^2 = \frac{2500}{24} k$

$\Rightarrow \ 324 = \frac{2500}{24} k$

$\Rightarrow \ k = \frac{324 \times 24}{2500} = \frac{324 \times 6}{625} = \frac{1944}{625}$

$\Rightarrow \ k = 3{:}11$

$\therefore$ Required length = 6 + k = 6 + 3.11 = 9.11 m (approx.)

135. (c) Let any point P(h, k) will satisfy

$y^2 = 4ax$ i.e, $k^2 = 4ah$ ...(i)

Let a line OP makes an angle θ from the x-axis.

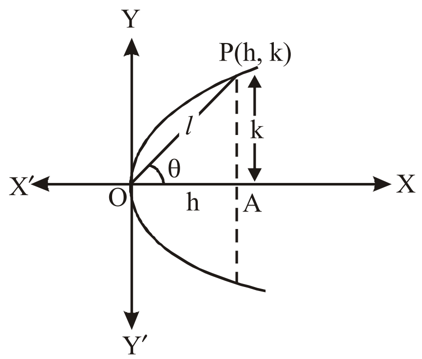
$\therefore$ In ΔOAP, $\sin\theta = \frac{PA}{OP}$

$\sin\theta = \frac{k}{l}$

$\Rightarrow \ k = l\sin\theta$

and $\cos\theta = \frac{OA}{OP}$

$\Rightarrow \ \cos\theta = \frac{h}{l} \Rightarrow h = \ell\cos\theta$



Hence, from eq.(i), we get

$l^2\sin^2\theta = 4a \times l\cos\theta$ (put $k = l\sin\theta$, $h = l\cos\theta$)

$\Rightarrow \ l = \frac{4a\cos\theta}{\sin^2\theta}$

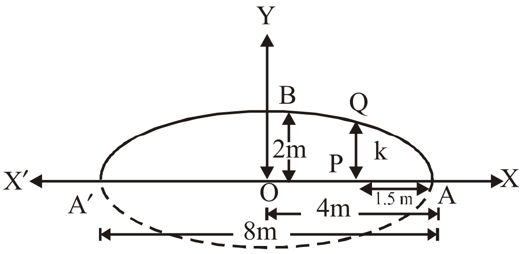
136. (a) Clearly, equation of ellipse takes the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \ldots(i)$$

Here, it is given 2a = 8 and b = 2 ⇒ a = 4, b = 2

Put the values of a and b in eq. (i), we get

$$\frac{x^2}{16}+\frac{y^2}{4}=1$$



Given, AP = 1.5 m

⇒ OP = OA – AP = 4 – 1.5

⇒ OP = 2.5 M

Let PQ = k

∴ Coordinate Q = (2.5, k) will satisfy the equation of ellipse.

i.e., $\frac{(2.5)^2}{16}+\frac{k^2}{4}=1 \Rightarrow \frac{6.25}{16}+\frac{k^2}{4}=1$

$\Rightarrow \frac{k^2}{4}=\frac{1}{1}-\frac{6.25}{16}=\frac{16-6.25}{16}$

$\Rightarrow \frac{k^2}{4}=\frac{9.75}{16} \Rightarrow k^2=\frac{9.75}{4}$

$\Rightarrow k^2 = 2.4375$

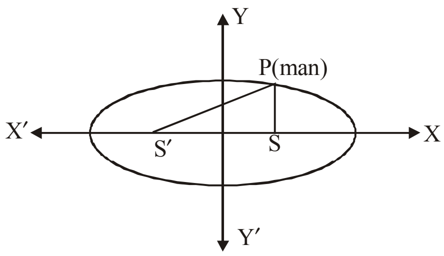
k = 1.56 m (approx.)

137. (a) Clearly, path traced by the man will be ellipse.

Given, SP + S′P = 10

i.e., 2a = 10

⇒ a = 5



Flag posts

Since, the coordinates of S and S′ are (c, 0) and (– c, 0), respectively. Therefore, distance between S and S′ is

2c = 8 ⇒ c = 4

∵ $c^2 = a^2 - b^2$

⇒ $16 = 25 - b^2 \Rightarrow b^2 = 25 - 16$

⇒ $b^2 = 9 \Rightarrow b = 3$

Hence, equation of path (ellipse) is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{25}+\frac{y^2}{9}=1 \; (\because a=5, b=3)$$

138. (a) Since the equation can be written as $(x-1)^2+(y-1)^2=1$ or $x^2+y^2-2x-2y+1=0$, which represents a circle touching both the axes with its centre (1, 1) and radius one unit.

139. (a) The equation of the circle through (1, 0), (0, 1) and (0, 0) is $x^2+y^2-x-y=0$

It passes through (2k, 3k)

So, $4k^2+9k^2-2k-3k=0$ or $13k^2-5k=0$

⇒ $k(13k-5)=0 \Rightarrow k=0$ or $k=\frac{5}{13}$

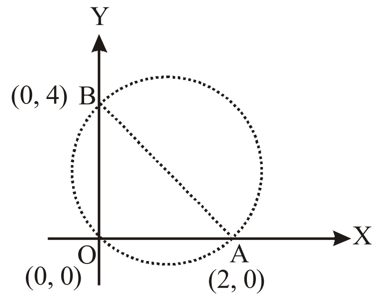
But k ≠ 0 [∵ all the four points are distinct)

∴ $k=\frac{5}{13}$.

140. (a) As the circle passes through origin and makes intercept 2 units and 4 units on x-axis and y-axis respectively, it passes through the points A(2, 0) and B(0, 4).

Since axes are perpendicular to each other, therefore, ∠AOB = 90° and hence AB becomes a diameter of the circle.

So, the equation of the required circle is



$(x-2)(x-0)+(y-0)(y-4)=0$

or $x^2+y^2-2x-4y=0$.

CHAPTER 12

INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. For every point $P(x, y, z)$ on the xy-plane,
(a) $x = 0$ (b) $y = 0$
(c) $z = 0$ (d) None of these

2. For every point $P(x, y, z)$ on the x-axis (except the origin),
(a) $x = 0, y = 0, z \neq 0$ (b) $x = 0, z = 0, y \neq 0$
(c) $y = 0, z = 0, x \neq 0$ (d) None of these

3. The distance of the point $P(a, b, c)$ from the x-axis is
(a) $\sqrt{b^2 + c^2}$ (b) $\sqrt{a^2 + c^2}$
(c) $\sqrt{a^2 + b^2}$ (d) None of these

4. Point (–3, 1, 2) lies in
(a) Octant I (b) Octant II
(c) Octant III (d) Octant IV

5. The three vertices of a parallelogram taken in order are (–1, 0), (3, 1) and (2, 2) respectively. The coordinate of the fourth vertex is
(a) (2,1) (b) (–2,1)
(c) (1,2) (d) (1,–2)

6. The point equidistant from the four points (0,0, 0), (3/2, 0, 0), (0,5/2, 0) and (0, 0, 7/2) is:
(a) $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{5}\right)$ (b) $\left(3, 2, \frac{3}{5}\right)$
(c) $\left(\frac{3}{4}, \frac{5}{4}, \frac{7}{4}\right)$ (d) $\left(\frac{1}{2}, 0, -1\right)$

7. The perpendicular distance of the point $P(6, 7, 8)$ from xy-plane is
(a) 8 (b) 7
(c) 6 (d) None of these

8. The ratio in which the join of points (1, –2, 3) and (4, 2, –1) is divided by XOY plane is
(a) 1 : 3 (b) 3 : 1
(c) –1 : 3 (d) None of these

9. The ratio in which the line joining the points (2,4, 5) and (3, 5, – 4) is internally divided by the xy-plane is
(a) 5 : 4 (b) 3 : 4
(c) 1 : 2 (d) 7 : 5

10. L is the foot of the perpendicular drawn from a point $P(6, 7, 8)$ on the xy-plane. The coordinates of point L is
(a) (6, 0, 0) (b) (6, 7, 0)
(c) (6, 0, 8) (d) None of these

11. If the sum of the squares of the distance of the point (x, y, z) from the points (a, 0, 0) and (–a , 0, 0) is $2c^2$, then which one of the following is correct?
(a) $x^2 + a^2 = 2c^2 – y^2 – z^2$ (b) $x^2 + a^2 = c^2 – y^2 – z^2$
(c) $x^2 – a^2 = c^2 – y^2 – z^2$ (d) $x^2 + a^2 = c^2 + y^2 + z^2$

12. The equation of set points P such that $PA^2 + PB^2 = 2K^2$, where A and B are the points (3, 4, 5) and (–1, 3, –7), respectively is
(a) $K^2 – 109$ (b) $2K^2 – 109$
(c) $3K^2 – 109$ (d) $4K^2 – 10$

13. The ratio in which the join of (2, 1, 5) and (3, 4, 3) is divided by the plane $(x + y – z) = \frac{1}{2}$ is:
(a) 3 : 5 (b) 5 : 7
(c) 1 : 3 (d) 4 : 5

14. The octant in which the points (– 3, 1, 2) and (– 3, 1, – 2) lies respectively is
(a) second, fourth (b) sixth, second
(c) fifth, sixth (d) second, sixth

15. Let L, M, N be the feet of the perpendiculars drawn from a point P(7, 9, 4) on the x, y and z-axes respectively. The coordinates of L, M and N respectively are
(a) (7, 0, 0), (0, 9, 0), (0, 0, 4) (b) (7, 0, 0), (0, 0, 9), (0, 4, 0)
(c) (0, 7, 0), (0, 0, 9), (4, 0, 0) (d) (0, 0, 7), (0, 9, 0), (4, 0, 0)

16. If a parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes, then the length of the diagonal is
(a) 7 units (b) 5 units
(c) 8 units (d) 3 units

17. The points A(4, – 2, 1), B(7, – 4, 7), C (2, – 5, 10) and D (– 1, – 3, 4) are the vertices of a
(a) tetrahedron (b) parallelogram
(c) rhombus (d) square

18. x-axis is the intersection of two planes are
(a) xy and xz (b) yz and zx
(c) xy and yz (d) None of these

19. The point (– 2, – 3, – 4) lies in the
(a) first octant (b) seventh octant
(c) second octant (d) eighth octant

20. A plane is parallel to yz-plane, so it is perpendicular to:
(a) x-axis (b) y-axis
(c) z-axis (d) None of these

21. The locus of a point for which x = 0 is
(a) xy-plane (b) yz-plane
(c) zx-plane (d) None of these

22. If L, M and N are the feet of perpendiculars drawn from the point P(3, 4, 5) on the XY, YZ and ZX-planes respectively, then
(a) distance of the point L from the point P is 5 units.
(b) distance of the point M from the point P is 3 units.
(c) distance of the point N from the point P is 4 units.
(d) All of the above.

23. If the point A (3, 2, 2) and B(5, 5, 4) are equidistant from P, which is on x-axis, then the coordinates of P are
(a) $\left(\frac{39}{4}, 2, 0\right)$ (b) $\left(\frac{49}{4}, 2, 0\right)$
(c) $\left(\frac{39}{4}, 0, 0\right)$ (d) $\left(\frac{49}{4}, 0, 0\right)$

24. The points (0, 7, 10), (– 1, 6, 6) and (– 4, 9, 6) form
(a) a right angled isosceles triangle
(b) a scalene triangle
(c) a right angled triangle
(d) an equilateral triangle

25. The point in YZ-plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1) is
(a) (0, 3, 1) (b) (0, 1, 3)
(c) (1, 3, 0) (d) (3, 1, 0)

26. Perpendicular distance of the point P(3, 5, 6) from y-axis is
(a) $\sqrt{41}$ (b) 6
(c) 7 (d) None of these

27. The coordinates of the point R, which divides the line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the ratio k : 1, are
(a) $\left(\frac{kx_2 - x_1}{1-k}, \frac{ky_2 - y_1}{1-k}, \frac{kz_2 - z_1}{1-k}\right)$
(b) $\left(\frac{kx_2 + x_1}{1+k}, \frac{ky_2 + y_1}{1+k}, \frac{kz_2 + z_1}{1+k}\right)$
(c) $\left(\frac{kx_2 + x_1}{1-k}, \frac{ky_2 + y_1}{1-k}, \frac{kz_2 + z_1}{1-k}\right)$
(d) None of these

28. The ratio, in which YZ-plane divides the line segment joining the points (4, 8, 10) and (6, 10, – 8), is
(a) 2 : 3 (externally) (b) 2 : 3 (internally)
(c) 1 : 2 (externally) (d) 1 : 2 (internally)

29. The ratio in which YZ-plane divides the line segment formed by joining the points (– 2, 4, 7) and (3, – 5, 8), is
(a) 2 : 3 (externally) (b) 2 : 3 (internally)
(c) 1 : 3 (externally) (d) 1 : 3 (internally)

30. If the origin is the centroid of a ΔABC having vertices A(a, 1, 3), B(– 2, b, – 5) and C(4, 7, c), then
(a) a = – 2 (b) b = 8
(c) c = – 2 (d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

31. P(a, b, c); Q (a + 2, b + 2, c – 2) and R (a + 6, b + 6, c – 6) are collinear.
Consider the following statements :
I. R divides PQ internally in the ratio 3 : 2
II. R divides PQ externally in the ratio 3 : 2
III. Q divides PR internally in the ratio 1 : 2
Which of the statements given above is/are correct ?
(a) Only I (b) Only II
(c) I and III (d) II and III

32. Consider the following statements
I. The x-axis and y-axis together determine a plane known as xy-plane.
II. Coordinates of points in xy-plane are of the form $(x_1, y_1, 0)$.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

33. Consider the following statement
I. Any point on *X*-axis is of the form $(x, 0, 0)$
II. Any point on *Y*-axis is of the form $(0, y, 0)$
III. Any point on *Z*-axis is of the form $(0, 0, z)$
Choose the correct option.
(a) Only I and II are true. (b) Only II and III are true.
(c) Only I and III are true. (d) All are true.

34. I. The distance of the point (x, y, z) from the origin is given by $\sqrt{x^2 + y^2 + z^2}$.
II. If a point *R* divides the line segment joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m : n$ externally, then

$$R = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

35. I. The (0, 7, – 10), (1, 6, – 6) and (4, 9, –6) are the vertices of an isosceles triangle.
II. Centroid of the triangle whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

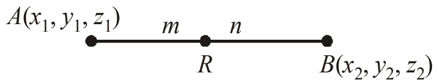
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

36. I. The coordinates of the mid-point of the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

II. If a point R divides the line segment joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m : n$ internally, then

$$R = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$



Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

37. Distance between the points (2, 3, 5) and (4, 3, 1) is $a\sqrt{5}$. The value of 'a' is
(a) 2 (b) 3 (c) 9 (d) 5

38. The perpendicular distance of the point $P(6, 7, 8)$ from xy-plane is
(a) 8 (b) 7
(c) 6 (d) None of these

39. The ratio in which the YZ-plane divide the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8) is 2 : m. The value of m is
(a) 2 (b) 3 (c) 4 (d) 1

40. Given that A(3, 2, – 4), B(5, 4 – 6) and C(9, 8, – 10) are collinear. Ratio in which B divides AC is 1 : m. The value of m is
(a) 2 (b) 3 (c) 4 (d) 5

41. If the origin is the centroid of the triangle with vertices A (2a, 2, 6), B (– 4, 3b, –10) and C (8, 14, 2c), then the sum of value of a and c is
(a) 0 (b) 1 (c) 2 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

42. **Assertion:** The coordinates of the point which divides the join of A (2, –1, 4) and B (4, 3, 2) in the ratio 2 : 3 externally is C (–2, –9, 8)
Reason : If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points, and let R be a point on PQ produced dividing it externally in the ratio $m_1 : m_2$. Then the coordinates of R are

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$$

43. **Assertion :** If three vertices of a parallelogram ABCD are A(3, –1, 2), B (1, 2, –4) and C (–1, 1, 2), then the fourth vertex is (1, –2, 8).
Reason : Diagonals of a parallelogram bisect each other and mid-point of AC and BD coincide.

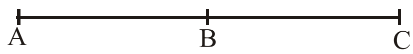
44. **Assertion :** The distance of a point P(x, y, z) from the origin O(0, 0, 0) is given by $OP = \sqrt{x^2 + y^2 + z^2}$.
Reason : A point is on the x-axis. Its y-coordinate and z-coordinate are 0 and 0 respectively.

45. **Assertion :** Coordinates (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.
Reason : Opposite sides of a parallelogram are equal and diagonals are not equal.

46. **Assertion :** If P (x, y, z) is any point in the space, then x, y and z are perpendicular distances from YZ, ZX and XY-planes, respectively.
Reason : If three planes are drawn parallel to YZ, ZX and XY-planes such that they intersect X, Y and Z-axes at (x, 0, 0), (0, y, 0) and (0, 0, z), then the planes meet in space at a point P(x, y, z).

47. **Assertion :** The distance between the points P(1, – 3, 4) and Q(– 4, 1, 2) is $\sqrt{5}$ units.
Reason : $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
where, P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2).

48. **Assertion :** Points (– 2, 3, 5), (1, 2, 3) and (7, 0, – 1) are collinear.
Reason : Three points A, B and C are said to be collinear, if AB + BC = AC (as shown below).



49. **Assertion :** Points (– 4, 6, 10), (2, 4, 6) and (14, 0, – 2) are collinear.
Reason : Point (14, 0, – 2) divides the line segment joining by other two given points in the ratio 3 : 2 internally.

50. **Assertion :** The XY-plane divides the line joining the points (– 1, 3, 4) and (2, – 5, 6) externally in the ratio 2 : 3.
Reason : For a point in XY-plane, its z-coordinate should be zero.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

51. What is the locus of a point which is equidistant from the points (1, 2, 3) and (3, 2, – 1) ?
(a) $x + z = 0$ (b) $x - 3z = 0$
(c) $x - z = 0$ (d) $x - 2z = 0$

52. What is the shortest distance of the point (1, 2, 3) from x-axis ?

(a) 1 (b) $\sqrt{6}$

(c) $\sqrt{13}$ (d) $\sqrt{14}$

53. The equation of locus of a point whose distance from the y-axis is equal to its distance from the point (2, 1, –1) is

(a) $x^2 + y^2 + z^2 = 6$ (b) $x^2 - 4x^2 + 2z^2 + 6 = 0$

(c) $y^2 - 2y^2 - 4x^2 + 2z + 6 = 0$ (d) $x^2 + y^2 - z^2 = 0$

54. ABC is a triangle and AD is the median. If the coordinates of A are (4, 7, – 8)and the coordinates of centroid of the triangle ABC are (1, 1, 1), what are the coordinates of D?

(a) $\left(-\frac{1}{2}, 2, 11\right)$ (b) $\left(-\frac{1}{2}, -2, \frac{11}{2}\right)$

(c) (–1, 2, 11) (d) (–5, –11, 19)

55. In three dimensional space the path of a point whose distance from the x-axis is 3 times its distance from the yz-plane is:

(a) $y^2 + z^2 = 9x^2$ (b) $x^2 + y^2 = 3z^2$

(c) $x^2 + z^2 = 3y^2$ (d) $y^2 - z^2 = 9x^2$

56. Let (3, 4, – 1) and (– 1, 2, 3) be the end points of a diameter of a sphere. Then, the radius of the sphere is equal to

(a) 2 units (b) 3 units

(c) 6 units (d) 7 units

57. Find the coordinates of the point which is three fifth of the way from (3, 4, 5) to (– 2, – 1, 0).

(a) (1, 0, 2) (b) (2, 0, 1)

(c) (0, 2, 1) (d) (0, 1, 2)

58. The coordinates of the points which trisect the line segment joining the points P(4, 2, – 6) and Q(10, – 16, 6) are

(a) (6, – 4, – 2) and (8, 10, – 2)

(b) (6, – 4, – 2) and (8, – 10, 2)

(c) (–6, 4, 2) and (– 8, 10, 2)

(d) None of these

59. If A(3, 2, 0), B(5, 3, 2) and C(– 9, 6, – 3) are three points forming a triangle and AD, the bisector of ∠BAC, meets BC in D, then the coordinates of the point D are

(a) $\left(\frac{17}{8}, \frac{57}{8}, \frac{17}{8}\right)$ (b) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

(c) $\left(\frac{8}{17}, \frac{8}{19}, \frac{17}{8}\right)$ (d) None of these

60. The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, – 1), then the vertices are

(a) (7, 2, 5), (3, 12, 17), (– 3, 4, – 7)

(b) (7, 2, 5), (3, 12, 17), (3, 4, 7)

(c) (7, 2, 5), (– 3, 11, 15), (3, 4, 8)

(d) None of the above

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(c)** On xy-plane, z-co-ordinate is zero.
2. **(c)** On x-axis, y and z-co-ordinates are zero.
3. **(a)** Let $(a, 0, 0)$ be a point on x-axis.

   Required distance $= \sqrt{(a-a^2)+(b-0)^2+(c-0)^2}$

   $= \sqrt{b^2+c^2}$
4. **(b)** $(-3, 1, 2)$ lies in second octant.
5. **(b)** Let $A(-1, 0)$, $B(3, 1)$, $C(2, 2)$ and $D(x, y)$ be the vertices of a parallelogram ABCD taken in order. Since, the diagonals of a parallelogram bisect each other.

   $\therefore$ Coordinates of the mid point of AC = Coordinates of the mid-point of BD

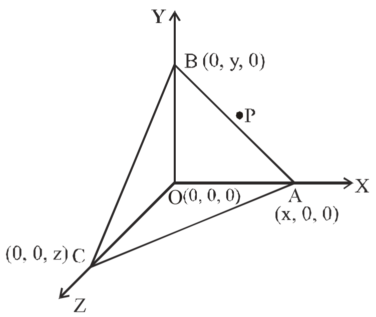
   $\Rightarrow \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{3+x}{2}, \frac{1+y}{2}\right)$

   $\Rightarrow \left(\frac{1}{2}, 1\right) = \left(\frac{3+x}{2}, \frac{y+1}{2}\right)$

   $\Rightarrow \frac{3+x}{2} = \frac{1}{2}$ and $\frac{y+1}{2} = 1$

   $\Rightarrow x = -2$ and $y = 1$.

   Hence the fourth vertex of the parallelogram is $(-2, 1)$
6. **(c)**



   We know the co-ordinate of P which is equidistant from four points A(x, 0, 0), B (0, y, 0), C (0, 0, z), O (0, 0, 0) is $\frac{1}{2}(x, y, z)$

   $\therefore$ Given: points are $(0, 0, 0)$, $\left(\frac{3}{2}, 0, 0\right)$, $\left(0, \frac{5}{2}, 0\right)$ and $\left(0, 0, \frac{7}{2}\right)$

   $\therefore$ Co-ordinate of point $P = \frac{1}{2}\left(\frac{3}{2}, \frac{5}{2}, \frac{7}{2}\right) = \left(\frac{3}{4}, \frac{5}{4}, \frac{7}{4}\right)$
7. **(a)** Let L be the foot of perpendicular drawn from the point $P(6, 7, 8)$ to the xy-plane and the distance of this foot L from P is z-coordinate of P, i.e., 8 units.
8. **(b)** Let $A(1, -2, 3)$ and $B(4, 2, -1)$. Let the plane XOY meet the line AB in the point C such that C divides AB in the ratio $k : 1$, then $C \equiv \left(\frac{4k+1}{k+1}, \frac{2k-2}{k+1}, \frac{-k+3}{k+1}\right)$.

   Since C lies on the plane XOY i.e. the plane $z = 0$,

   therefore, $\frac{-k+3}{k+1} = 0 \Rightarrow k = 3$.
9. **(a)** Let the line joining the points $(2, 4, 5)$ and $(3, 5, -4)$ is internally divided by the xy - plane in the ratio $k : 1$.

   $\therefore$ For xy plane, $z = 0$

   $\Rightarrow 0 = \frac{-k \times 4 + 5}{k+1} \Rightarrow 4k = 5 \Rightarrow k = \frac{5}{4}$.

   so, ratio is 5 : 4
10. **(b)** Since L is the foot of perpendicular from P on the xy-plane, z-coordinate is zero in the xy-plane. Hence, coordinates of L are $(6, 7, 0)$.
11. **(b)** Let the point be $P(x, y, z)$ and two points, $(a, 0, 0)$ and $(-a, 0, 0)$ be A and B

    As given in the problem,

    $PA^2 + PB^2 = 2c^2$

    so, $(x+a)^2+(y-0)^2+(z-0)^2+(x-a)^2+(y-0)^2+(z-0)^2=2c^2$

    or, $(x+a)^2 + y^2 + z^2 + (x-a)^2 + y^2 + z^2 = 2c^2$

    $\Rightarrow x^2 + 2a + a^2 + y^2 + z^2 + x^2 - 2a + a^2 + y^2 + z^2 = 2c^2$

    $\Rightarrow 2(x^2 + y^2 + z^2 + a^2) = 2c^2$

    $\Rightarrow x^2 + y^2 + z^2 + a^2 = c^2$

    $\Rightarrow x^2 + a^2 = c^2 - y^2 - z^2$
12. **(b)** Let the coordinates of point P be (x, y, z).

    Here, $PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$

    $PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$

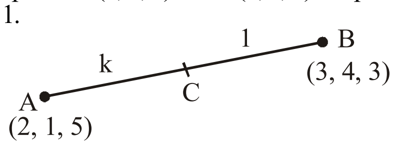
    By the given condition

    $PA^2 + PB^2 = 2K^2$

    We have

    $(x-3)^2+(y-4)^2+(z-5)^2+(x+1)^2+(y-3)^2+(z+7)^2=2K^2$

    i.e. $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = 2K^2 - 109$
13. **(b)** As given plane $x + y - z = \frac{1}{2}$ divides the line joining the points A(2, 1, 5) and B(3, 4, 3) at a point C in the ratio k : 1.



    Then coordinates of C

    $\left(\frac{3k+2}{k+1}, \frac{4k+1}{k+1}, \frac{3k+5}{k+1}\right)$

    Point C lies on the plane,

    $\Rightarrow$ Coordinates of C must satisfy the equation of plane.

So, $\left(\frac{3k+2}{k+1}\right)+\left(\frac{4k+1}{k+1}\right)-\left(\frac{3k+5}{k+1}\right)=\frac{1}{2}$

$\Rightarrow 3k+2+4k+1-3k-5=\frac{1}{2}(k+1)$

$\Rightarrow 4k-2=\frac{1}{2}(k+1)$

$\Rightarrow 8k-4=k+1 \Rightarrow 7k=5$

$\Rightarrow k=\frac{5}{7}$

Ratio is 5 : 7.

14. (d) The point (– 3, 1, 2) lies in second octant and the point (– 3, 1, – 2) lies in sixth octant.

15. (a) Since L is the foot of perpendicular from P on the x-axis, its y and z-coordinates are zero. So, the coordinates of L is (7, 0, 0). Similarly, the coordinates of M and N are (0, 9, 0) and (0, 0, 4), respectively.

16. (a) Length of edges of the parallelopiped are 5 – 2, 9 – 3, 7 – 5 i.e., 3, 6, 2.

$\therefore$ Length of diagonal is $\sqrt{3^2+6^2+2^2}$ = 7 units.

17. (b) Here, the mid-point of AC is

$\left(\frac{4+2}{2},\frac{-2-5}{2},\frac{1+10}{2}\right)=\left(3,-\frac{7}{2},\frac{11}{2}\right)$

and that of BD is

$\left(\frac{7-1}{2},\frac{-4-3}{2},\frac{7+4}{2}\right)=\left(3,-\frac{7}{2},\frac{11}{2}\right)$

So, the diagonals AC and BD bisect each other.

$\Rightarrow$ ABCD is a parallelogram.

As $|AB|=\sqrt{3^2+2^2+6^2}=7$ and

$|AD|=\sqrt{5^2+1^2+3^2}=\sqrt{35}\neq|AB|$,

Therefore, ABCD is not a rhombus and naturally, it cannot be a square.

18. (a)

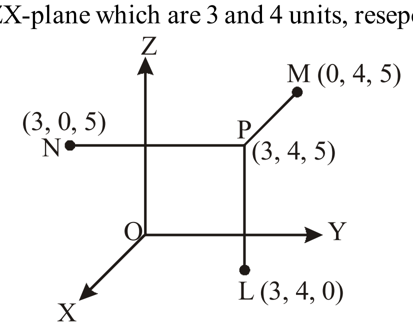
19. (b) The point (– 2, – 3, – 4) lies on negative of x, y and z-axis.

$\therefore$ It lies in seventh octant.

20. (a) A plane is parallel to yz-plane which is always perpendicular to x-axis.

21. (b) For yz-plane x = 0, locus of point for which x = 0 is yz-plane.

22. (d) L is the foot of perpendicular drawn from the point P(3, 4, 5) to the XY-plane. Therefore, the coordinates of the point L is (3, 4, 0). The distance between the point (3, 4, 5) and (3, 4, 0) is 5. Similarly, we can find the lengths of the foot of perpendiculars on YZ and ZX-plane which are 3 and 4 units, resepctively.



23. (d) The point on the x-axis is of the form P(x, 0, 0). Since, the points A and B are equidistant from P. Therefore, $PA^2 = PB^2$,

i.e., $(x-3)^2+(0-2)^2+(0-2)^2$
$=(x-5)^2+(0-5)^2+(0-4)^2$

$\Rightarrow \quad 4x = 25+25+16-17$ i.e., $x=\frac{49}{4}$

Thus, the point P on the x-axis is $\left(\frac{49}{4},0,0\right)$ which is equidistant from A and B

24. (a) Let P(0, 7, 10), Q(– 1, 6, 6) and R(– 4, 9, 6) be the vertices of a triangle

Here, $PQ=\sqrt{1+1+16}=3\sqrt{2}$

$QR=\sqrt{9+9+0}=3\sqrt{2}$

$PR=\sqrt{16+4+16}=6$

Now, $PQ^2+QR^2=\left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2$
$=18+18=36=(PR)^2$

Therefore, ΔPQR is a right angled triangle at Q. Also, OQ = QR, Hence, ΔPQR is a right angled isosceles triangle.

25. (b) Since x-coordinate of every point in YZ-plane is zero. Let P(0, y, z) be a point on the YZ-plane such that PA = PB = PC.

Now, PA = PB

$\Rightarrow \quad (0-2)^2+(y-0)^2+(z-3)^2$
$=(0-0)^2+(y-3)^2+(z-2)^2$,

i.e., $z-3y=0$

and PB = PC

$\Rightarrow \quad y^2+9-6y+z^2+4-4z=y^2+z^2+1-2z$,

i.e., $3y+z=6$

On simplifying the two equations, we get y = 1 and z = 3. Here, the coordinate of the point P are (0, 1, 3).

26. (d) Let M is the foot of perpendicular from P on the y-axis, therefore its x and z-coordinates are zero. The coordinates of M is (0, 5, 0). Therefore, the perpendicular distance of the point P from y-axis $=\sqrt{3^2+6^2}=\sqrt{45}$.

27. (b) The coordinates of the point R which divides PQ in the ratio k : 1 where coordinates of P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2) are obtained by taking $k=\frac{m}{n}$ in the coordinates of the point R which divides PQ internally in the ratio m : n, which are as given below.

$$\left(\frac{kx_2+x_1}{1+k},\frac{ky_2+y_1}{1+k},\frac{kz_2+z_1}{1+k}\right)$$

28. (a) Let YZ-plane divides the line segment joining A (4, 8, 10) and B(6, 10, –8) at P(x, y, z) in the ratio k : 1. Then, the coordinates of P are

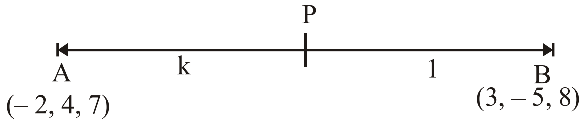
$$\left(\frac{4+6k}{k+1},\frac{8+10k}{k+1},\frac{10-8k}{k+1}\right)$$

Since, P lies on the YZ-plane, its x-coordinates is zero.

i.e., $\frac{4+6k}{k+1}=0$ or $k=-\frac{2}{3}$

Therefore, YZ-plane divides AB externally in the ratio 2 : 3

29. (b) The given points are A(– 2, 4, 7) and B(3, – 5, 8).
Let the point P(0, y, z) in YZ-plane divides AB in the ratio k : 1. Then,



x-coordinate of point $P = \dfrac{mx_2 + nx_1}{m+n}$

$\dfrac{k \times 3 + 1 \times (-2)}{k+1} = 0$ (∵ x-coordinate of P is zero)

$\Rightarrow \quad 3k - 2 = 0$

$\Rightarrow \quad k = \dfrac{2}{3}$

$\Rightarrow \quad k : 1 = 2 : 3$

∴ YZ-plane divides the segment internally in the ratio 2 : 3

30. (a) For centroid of ΔABC,

$$x = \frac{a-2+4}{3} = \frac{a+2}{3}$$

$$y = \frac{1+b+7}{3} = \frac{b+8}{3}$$

and $$z = \frac{3-5+c}{3} = \frac{c-2}{3}$$

But given centroid is (0, 0, 0).

$$\therefore \quad \frac{a+2}{3} = 0 \Rightarrow a = -2$$

$$\frac{b+8}{3} = 0 \Rightarrow b = -8$$

$$\frac{c-2}{3} = 0 \Rightarrow c = 2$$

STATEMENT TYPE QUESTIONS

31. (d) Given that P (a, b, c), Q (a + 2, b + 2, c – 2) and R (a + 6, b + 6, c – 6) are collinear, one point must divide, the other two points externally or internally.
Let R divide P and Q in ratio k : 1 so,
taking on x-coordinates

$$\frac{k(a+2)+a}{k+1} = a+6$$

$\Rightarrow \quad k(a+2) + a = (k+1)(a+6)$

$\Rightarrow \quad ka + 2k + a = ka + 6k + a + 6 \Rightarrow -4k = 6$

or $\quad k = -\dfrac{3}{2}$

Negative sign shows that this is external division in ratio 3 : 2. So, R divides P and Q externally in 3 : 2 ratio.
Putting this value for y - and z - coordinates satisfied :
for y - coordinate

$$\frac{3(b+2)-2b}{3-2} = 3b + 6 - 2b = b + 6$$

and for z-coordinate :

$$\frac{3(c-2)-2c}{3-2} = \frac{3c-6-2c}{1} = c - b$$

Statement II is correct.
Also, let Q divide P and R in ratio p : 1 taking an x-coordinate:

$$\frac{p(a+6)+a}{p+1} = a+2$$

$$\frac{p.a + 6p + a}{p+1} = a+2$$

$\Rightarrow \quad pa + 6p + a = pa + a + 2p + 2$

$\Rightarrow \quad 4p = 2 \Rightarrow p = \dfrac{1}{2}.$

Positive sign shows that the division is internal and in the ratio 1 : 2
Verifying for y - and z- coordinates, satisfies this results.
For y coordinate,

$$\frac{(b+6)\times 1 + 2b}{3} = \frac{3b+6}{3} = b+2$$

and for z-coordinate,

$$\frac{c-6+2c}{3} = \frac{3c-6}{3} = c-2$$

values are satisfied.
So, statement III is correct.

32. (c) 33. (d)

34. (c) Both the statements are true.

35. (c) Both the statements are true.

36. (c) Both the given statements are true.

INTEGER TYPE QUESTIONS

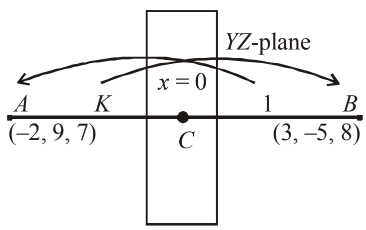
37. (a) The given points are (2, 3, 5) and (4, 3, 1).
∴ Required distance

$$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} = \sqrt{4+0+16}$$

$$= \sqrt{20} = 2\sqrt{5}$$

38. (a) Let *L* be the foot of perpendicular drawn from the point *P*(6, 7, 8) to the *xy*-plane and the distance of this foot *L* from *P* is *z*-coordinate of *P*, i.e., 8 units.

39. (b) Let the points be *A*(–2, 4, 7) and *B*(3, –5, 8) on *YZ*-plane, *x*-coordinate = 0.



Let the ratio be *K* : 1.
The coordinates of *C* are

$$\left(\frac{3K-2}{K+1}, \frac{-5K+4}{K+1}, \frac{8K+7}{K+1}\right)$$

Clearly $\dfrac{3K-2}{K+1} = 0 \Rightarrow 3K = 2 \Rightarrow K = \dfrac{2}{3}$

Hence required ratio is 2 : 3.

40. (a) Suppose B divides AC in the ratio λ : 1.

$$\therefore \quad B = \left(\frac{9\lambda+3}{\lambda+1}, \frac{8\lambda+2}{\lambda+1}, \frac{-10\lambda-4}{\lambda+1}\right) = (5, 4, -6)$$

$\Rightarrow \dfrac{9\lambda+3}{\lambda+1}=5, \dfrac{8\lambda+2}{\lambda+1}=4, \dfrac{-10\lambda-4}{\lambda+1}=-6$

$\Rightarrow \lambda=\dfrac{1}{2}$

So, required ratio is 1 : 2.

41. (a) Centriod of Δ ABC are $\left(\dfrac{2a+4}{3}, \dfrac{3b+16}{3}, \dfrac{2c-4}{3}\right)$

Given centroid = (0, 0, 0)

$\therefore \quad 2a+4=0, 16+3b=0, 2c-4=0$

$\Rightarrow \quad a=-2, b=\dfrac{-16}{3}, c=2$

Hence, a + c = 0

ASSERTION - REASON TYPE QUESTIONS

42. (a) Assertion :

$x=\dfrac{2\times4-3\times2}{2-3}, y=\dfrac{2\times3-3(-1)}{2-3},$

$z=\dfrac{2\times2-3\times4}{2-3}$

$\Rightarrow x=-2, y=-9, z=8$

43. (a) Since diagonals of a parallelogram bisect each other therefore, mid-point of AC and BD coincide.

$\therefore \quad (1,0,2)=\left(\dfrac{x+1}{2}, \dfrac{y+2}{2}, \dfrac{z-4}{2}\right)$

$\Rightarrow \quad \dfrac{x+1}{2}=1, \dfrac{y+2}{2}=0, \dfrac{z-4}{2}=2$

$\Rightarrow \quad x=1, y=-2, z=8$

44. (b) Both Assertion and Reason is correct.

45. (a) Assertion: The given points are $A(-1, 2, 1)$, $B(1, -2, 5)$, $C(4, -7, 8)$ and $D(2, -3, 4)$, then by distance formula

$AB=\sqrt{(1+1)^2+(-2-2)^2+(5-1)^2}$

$=\sqrt{4+16+16}=\sqrt{36}=6$

$BC=\sqrt{(4-1)^2+(-7+2)^2+(8-5)^2}$

$=\sqrt{9+25+9}=\sqrt{43}$

$CD=\sqrt{(2-4)^2+(-3+7)^2+(4-8)^2}$

$=\sqrt{4+16+16}=\sqrt{36}=6$

$DA=\sqrt{(-1-2)^2+(2+3)^2+(1-4)^2}$

$=\sqrt{9+25+9}=\sqrt{43}$

$AC=\sqrt{(4+1)^2+(-7-2)^2+(8-1)^2}$

$=\sqrt{25+81+49}=\sqrt{155}$

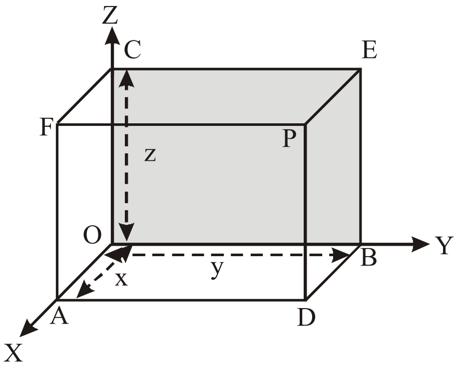
$BD=\sqrt{(2-1)^2+(-3+2)^2+(4-5)^2}$

$=\sqrt{1+1+1}=\sqrt{3}$

Now, since $AB = CD$ and $BC = DA$ i.e., opposite sides are equal and $AC \neq BD$ i.e. the diagonals are not equal. So, points are the vertices of parallelogram.

46. (b) Assertion : Through, the point P in the space, we draw three planes, parallel to the coordinates planes, meeting the X-axis, Y-axis and Z-axis in the points A, B and C, respectively. We observe that OA = x, OB = y and OC = z. Thus, if P(x, y, z) is any point in the space, then x, y and z are perpendicular distances from YZ, ZX and XY-planes, respectively.

Reason : Given x, y and z, we locate the three points A, B and C on the three coordinate axes. Through the points A, B and C, we draw planes parallel to the YZ-plane, ZX-plane and XY-plane, respectively. The point of intersection of these three planes, namely ADPF, BDPE and CEPF is obviously the point P, corresponding to the ordered triplet (x, y, z).



47. (d) The distance PQ between the points P(1, – 3, 4) and Q(– 4, 1, 2) is

$PQ=\sqrt{(-4-1)^2+(1+3)^2+(2-4)^2}$

$=\sqrt{25+16+4}=\sqrt{45}=3\sqrt{5}$ units

48. (a) The given points are

A(– 2, 3, 5), B(1, 2, 3), C(7, 0, – 1)

Distance between A and B

$AB=\sqrt{(-2-1)^2+(3-2)^2+(5-3)^2}$

$=\sqrt{(-3)^2+(1)^2+(2)^2}=\sqrt{9+1+4}=\sqrt{14}$

Distance between B and C

$BC=\sqrt{(1-7)^2+(2-0)^2+(3+1)^2}$

$=\sqrt{(-6)^2+(2)^2+(4)^2}$

$=\sqrt{36+4+16}=\sqrt{56}=2\sqrt{14}$

Distance between A and C

$AC=\sqrt{(-2-7)^2+(3-0)^2+(5+1)^2}$

$=\sqrt{(-9)^2+(3)^2+(6)^2}$

$=\sqrt{81+9+36}=\sqrt{126}=3\sqrt{14}$

Clearly, AB + BC = AC

Hence, the given points are collinear.

49. (c) Let A(– 4, 6, 10), B(2, 4, 6) and C(14, 0, – 2) be the given points. Let the point P divides AB in the ratio k : 1. Then, coordinates of the point P are

$$\dfrac{2k-4}{k+1}, \dfrac{4k+6}{k+1}, \dfrac{6k+10}{k+1}$$

Let us examine whether for some value of k, the point P coincides with point C.

On putting $\frac{2k-4}{k+1}=14$, we get $k=-\frac{3}{2}$

When $k=-\frac{3}{2}$, then $\frac{4k+6}{k+1}=\frac{4\left(-\frac{3}{2}\right)+6}{-\frac{3}{2}+1}=0$

and $\frac{6k+10}{k+1}=\frac{6\left(-\frac{3}{2}\right)+10}{-\frac{3}{2}+1}=-2$

Therefore, C (14, 0, – 2) is a point which divides AB externally in the ratio 3 : 2 and is same as P. Hence A, B, C are collinear.

50. (a) Suppose xy-plane divides the line joining the given points in the ratio $\lambda : 1$. The coordinates of the points of division are $\left[\frac{2\lambda-1}{\lambda+1},\frac{-5\lambda+3}{\lambda+1},\frac{6\lambda+4}{\lambda+1}\right]$. Since, the points lies on the XY-plane.

$\therefore \quad \frac{6\lambda+4}{\lambda+1}=0 \Rightarrow \lambda=\frac{-2}{3}$

CRITICALTHINKING TYPE QUESTIONS

51. (d) Let (h, k, ℓ) be the point which is equidistant from the points (1, 2, 3) and (3, 2, –1)

$\Rightarrow \sqrt{(h-1)^2+(k-2)^2+(\ell-3)^2}$

$=\sqrt{(h-3)^2+(k-2)^2+(\ell+1)^2}$

$\Rightarrow (h-1)^2+(\ell-3)^2=(h-3)^2+(\ell+1)^2$

$\Rightarrow h^2+1-2h+\ell^2-6\ell+9=h^2-6h+9+\ell^2+2\ell+1$

$\Rightarrow -2h-6\ell=-6h+2\ell$

$\Rightarrow 6h-2h-6\ell-2\ell=0 \Rightarrow 4h-8\ell=0$

$\Rightarrow h-2\ell=0$

Putting h = x and ℓ = z

We get locus of points (h, k, ℓ)

as, x – 2z = 0

52. (c) Any point on x-axis has y = z = 0

Distance of the point (1, 2, 3) from x-axis is the distance between point (1, 2, 3) and point (1, 0, 0)

$=\sqrt{(1-1)^2+(2-0)^2+(3-0)^2}=\sqrt{2^2+3^2}$

$=\sqrt{4+9}=\sqrt{13}$

53. (c) The variable point is $P(x, y, z)$.

Its distance from the y-axis $=\sqrt{x^2+z^2}$

Its distance from (2, 1, –1)

$=\sqrt{(x-2)^2+(y-1)^2+(z+1)^2}$

Given

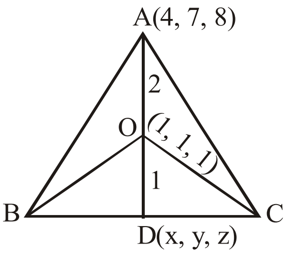
$\sqrt{x^2+z^2}=\sqrt{(x-2)^2+(y-1)^2+(z+1)^2}$

$\Rightarrow y^2-2y-4x+2z+6=0$

54. (b) Let coordinates of D be (x, y, z)

Co-ordinates of centroid is (1, 1, 1), and of A, is (4, 7, 8)

Centroid divides median in 2 : 1 ratio



So, $\frac{AO}{OD}=2:1$

For x :

$1=\frac{2\times x+1\times 4}{1+2}$

$\Rightarrow x=-1/2$

For y :

$1=\frac{2y+1\times 7}{1+2} \Rightarrow y=-2$

For z :

$1=\frac{2\times z+1\times -8}{3} \Rightarrow z=+11/2$

$\therefore$ Coordinates of D are (–1/2, –2, 11/2)

55. (a) Let $P(x_1, y_1, z_1)$ be the point.

Then distance of P from x-axis $=\sqrt{y_1^2+z_1^2}$

Given plane is x = 0 (yz-plane)

Distance of $P(x_1, y_1, z_1)$ from yz-plane is $\frac{x_1}{\sqrt{1}}$

From the given condition,distance of P from x-axis = 3 × distance of P from yz-plane

$\sqrt{y_1^2+z_1^2}=3x_1$

Squaring, $y_1^2+z_1^2=9x_1^2$

Thus, path of $P(x_1, y_1, z_1)$ is got by putting x, y, z in place of x_1, y_1, z_1 as $y^2+z^2=9x^2$

56. (b) Let P(3, 4, – 1) and Q(– 1, 2, 3) be the end points of the diameter of a sphere.

$\therefore$ Length of diameter = PQ

$=\sqrt{(-1-3)^2+(2-4)^2+(3+1)^2}$

$=\sqrt{16+4+16}=\sqrt{36}=6$ units

$\therefore$ Radius $=\frac{6}{2}$ = 3 units

57. (d) Let A = (3, 4, 5), B = (– 2, – 1, 0) and P(x, y, z) be the required point. As P is three-fifth of the way from A to B, we have

$AP=\frac{3}{5}AB \Rightarrow AP=\frac{3}{5}(AP+PB)$

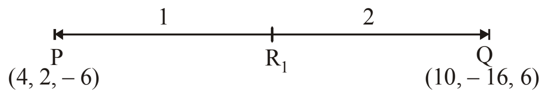
$\Rightarrow 5AP=3AP+3PB \Rightarrow \frac{AP}{PB}=\frac{3}{2}$

$\Rightarrow$ P divides [AB] in the ratio 3 : 2

$\therefore P\equiv\left(\frac{3\times(-2)+2\times 3}{3+2},\frac{3\times(-1)+2\times 4}{3+2},\frac{3\times 0+2\times 5}{3+2}\right)$

$\Rightarrow$ P = (0, 1, 2)

58. (b) Let the points R_1 and R_2 trisects the line PQ i.e., R_1 divides the line in the ratio 1 : 2.

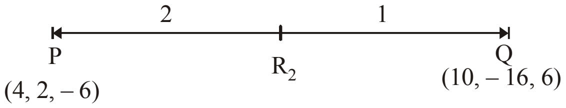


$$\Rightarrow R_1 = \left(\frac{1\times10+2\times4}{1+2}, \frac{1\times(-16)+2\times2}{1+2}, \frac{1\times6+2\times(-6)}{1+2}\right)$$

$$= \left(\frac{10+8}{3}, \frac{-16+4}{3}, \frac{6-12}{3}\right) = \left(\frac{18}{3}, \frac{-12}{3}, \frac{-6}{3}\right)$$

$= (6, -4, -2)$

Again, let the point R_2 divides PQ internally in the ratio 2 : 1. Then.



$$\Rightarrow R_2 = \left(\frac{2\times10+1\times4}{2+1}, \frac{2\times(-16)+1\times2}{2+1}, \frac{2\times6+1\times(-6)}{2+1}\right)$$

$$= \left(\frac{20+4}{3}, \frac{-32+2}{3}, \frac{12-6}{3}\right) = \left(\frac{24}{3}, \frac{-30}{3}, \frac{6}{3}\right)$$

$= (8, -10, 2)$

59. **(b)** $AB = \sqrt{(5-3)^2+(3-2)^2+(2-0)^2} = \sqrt{4+1+4} = 3$

$AC = \sqrt{(-9-3)^2+(6-2)^2+(-3-0)^2}$

$= \sqrt{144+16+9} = 13$

Since, AD is the bisector of $\angle BAC$, we have

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{3}{13}$$

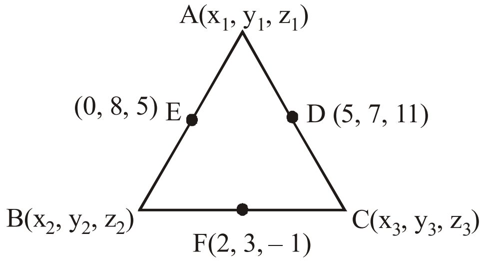
i.e., D divides BC in the ratio 3 : 13.

Hence, the coordinates of D are

$$\left(\frac{3(-9)+13(5)}{3+13}, \frac{3(6)+13(3)}{3+13}, \frac{3(-3)+13(2)}{3+13}\right)$$

$$= \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$$

60. (a) Let the vertices of a triangle be $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$.



Since D, E and F are the mid-points of AC, BC and AB

$$\therefore \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = (0, 8, 5)$$

$\Rightarrow x_1 + x_2 = 0, y_1 + y_2 = 16, z_1 + z_2 = 10$ …(i)

$$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right) = (2, 3, -1)$$

$\Rightarrow x_2 + x_3 = 4, y_2 + y_3 = 6, z_2 + z_3 = -2$ …(ii)

and $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right) = (5, 7, 11)$

$\Rightarrow x_1 + x_3 = 10, y_1 + y_3 = 14, z_1 + z_3 = 22$ …(iii)

On adding eqs. (i), (ii) and (iii), we get

$2(x_1 + x_2 + x_3) = 14, 2(y_1 + y_2 + y_3) = 36.$

$2(z_1 + z_2 + z_3) = 30,$

$\Rightarrow x_1 + x_2 + x_3 = 7, y_1 + y_2 + y_3 = 18,$

$z_1 + z_2 + z_3 = 15$ …(iv)

On solving eqs. (i), (ii), (iii) and (iv), we get

$x_3 = 7, x_1 = 3, x_2 = -3$

$y_3 = 2, y_1 = 12, y_2 = 4$

and $z_3 = 5, z_1 = 17, z_2 = -7$

Hence, vertices of a triangle are (7, 2, 5), (3, 12, 17) and (– 3, 4, – 7).

CHAPTER 13

LIMITS AND DERIVATIVE

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The limit of $f(x) = x^2$ as x tends to zero equals
(a) zero (b) one (c) two (d) three

2. Consider the function $f(x) = \begin{cases} 1, & x \le 0 \\ 2, & x > 0 \end{cases}$
Then, left hand limit and right hand limit of f (x) at x = 0, are respectively
(a) 1, 2 (b) 2, 1 (c) 1,1 (d) 2, 2

3. The value of $\lim_{x \to -1} \left[\frac{x^2 - 1}{x^2 + 3x + 2} \right]$ is
(a) 2 (b) –2 (c) 0 (d) –1

4. The value of $\lim_{x \to 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{1+x}$ is
(a) 2 (b) –2 (c) 1 (d) –1

5. Evaluate $\lim_{x \to 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$
(a) 1 (b) 2 (c) –1 (d) –2

6. Value of $\lim_{x \to 5} \frac{1 - \sqrt{x-4}}{x-5}$ is
(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) does not exist

7. $\lim_{x \to 0} \frac{\sqrt{1+x+x^2} - 1}{x} =$
(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 0 (d) ∞

8. If $f(x) = \begin{cases} x^2 + 1, & x \ge 1 \\ 3x - 1, & x < 1 \end{cases}$, then the value of $\lim_{x \to 1} f(x)$ is
(a) 2 (b) –2 (c) 1 (d) –1

9. The value of $\lim_{x \to 0} \frac{\sqrt{(1+x^2)} - \sqrt{1-x^2}}{x^2}$ is
(a) 1 (b) –1 (c) 0 (d) does not exist

10. If $f(t) = \frac{1-t}{1+t}$, then the value of f' (1/t) is
(a) $\frac{-2t^2}{(t+1)^2}$ (b) $\frac{2t}{(t+1)^2}$ (c) $\frac{2t^2}{(t-1)^2}$ (d) $\frac{-2t^2}{(t-1)^2}$

11. Let f and g be two functions such that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then, which of the following is incomplete?
(a) $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
(b) $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
(c) $\lim_{x \to a} [f(x) \,.\, g(x)] = \lim_{x \to a} f(x) \,.\, \lim_{x \to a} g(x)$
(d) $\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

12. The derivative of the function f (x) = x is
(a) 0 (b) 1 (c) ∞ (d) None of these

13. The derivative of sin x at x = 0 is
(a) 0 (b) 2 (c) 1 (d) 3

14. The derivative of the function f (x) = 3x at x = 2 is
(a) 0 (b) 1 (c) 2 (d) 3

15. The derivative of f (x) = 3 at x = 0 and at x = 3 are
(a) negative (b) zero
(c) different (d) not defined

16. Derivative of f at x = a is denoted by
(a) $\left.\frac{d}{dx} f(x)\right|_a$ (b) $\left.\frac{df}{dx}\right|_a$
(c) $\left(\frac{df}{dx}\right)_{x=a}$ (d) All of these

17. If a is a non-zero constant, then the derivative of x + a is
(a) 1 (b) 0
(c) a (d) None of these

18. The derivative of $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$ is
(a) $\frac{2}{(1+x)^2}$ (b) $\frac{-2}{(1-x)^2}$
(c) $\frac{-1}{(1-x)^2}$ (d) $\frac{3}{(1-x)^2}$

19. The derivative of $4\sqrt{x} - 2$ is
(a) $\frac{1}{\sqrt{x}}$ (b) $2\sqrt{x}$ (c) $\frac{2}{\sqrt{x}}$ (d) $\sqrt{x}$

20. If a and b are fixed non-zero constants, then the derivative of $(ax+b)^n$ is
(a) $n(ax+b)^{n-1}$ (b) $na(ax+b)^{n-1}$
(c) $nb(ax+b)^{n-1}$ (d) $nab(ax+b)^{n-1}$

21. The derivative of $\sin^n x$ is
(a) $n \sin^{n-1} x$ (b) $n \cos^{n-1} x$
(c) $n \sin^{n-1} x \cos x$ (d) $n \cos^{n-1} x \sin x$

22. The derivative of $(x^2+1)\cos x$ is
(a) $-x^2 \sin x - \sin x - 2x \cos x$
(b) $-x^2 \sin x - \sin x + 2 \cos x$
(c) $-x^2 \sin x - x \sin x + 2 \cos x$
(d) $-x^2 \sin x - \sin x + 2 x \cos x$

23. The derivative of $f(x) = \tan(ax+b)$ is
(a) $\sec^2(ax+b)$ (b) $b \sec^2(ax+b)$
(c) $a \sec^2(ax+b)$ (d) $ab \sec^2(ax+b)$

24. If $f(x) = x \sin x$, then $f'\left(\frac{\pi}{2}\right)$ is equal to
(a) 0 (b) 1 (c) -1 (d) $\frac{1}{2}$

25. The derivative of function $6x^{100} - x^{55} + x$ is
(a) $600x^{100} - 55x^{55} + x$ (b) $600x^{99} - 55x^{54} + 1$
(c) $99x^{99} - 54x^{54} + 1$ (d) $99x^{99} - 54x^{54}$

26. $\lim_{x \to 0} \frac{x}{\tan x}$ is
(a) 0 (b) 1 (c) 4 (d) not defined

27. Derivative of $\log_x x$ is
(a) 0 (b) 1 (c) $\frac{1}{x}$ (d) x

28. Derivative of $e^{3 \log x}$ is
(a) e^x (b) $3x^2$ (c) $3x$ (d) $\log x$

29. Derivative of $x^2 + \sin x + \frac{1}{x^2}$ is
(a) $2x + \cos x$ (b) $2x + \cos x + (-2)x^{-3}$
(c) $2x - 2x^{-3}$ (d) None of these

30. Derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ is
(a) $\frac{1}{x^2}$ (b) $1 - \frac{1}{x^2}$ (c) 1 (d) $1 + \frac{1}{x^2}$

31. If $f(x) = \alpha x^n$, then $\alpha =$
(a) $f'(1)$ (b) $\frac{f'(1)}{n}$ (c) $n \cdot f'(1)$ (d) $\frac{n}{f'(1)}$

32. Derivative of $x \sin x$
(a) $x \cos x$ (b) $x \sin x$
(c) $x \cos x + \sin x$ (d) $\sin x$

33. Value of $\lim_{x \to 0} \frac{a^{\sin x} - 1}{\sin x}$ is
(a) $\log a$ (b) $\sin x$ (c) $\log(\sin x)$ (d) $\cos x$

34. $\lim_{x \to 0} \frac{2\sin^2 3x}{x^2}$ is equal to :
(a) 12 (b) 18 (c) 0 (d) 6

35. $\lim_{\theta \to 0} \frac{\sin m^2\theta}{\theta}$ is equal to :
(a) 0 (b) 1 (c) m (d) m^2

36. Derivative of the function $f(x) = 7x^{-3}$ is
(a) $21x^{-4}$ (b) $-21x^{-4}$ (c) $21x^4$ (d) $-21x^4$

37. If $f(x) = 2\sin x - 3x^4 + 8$, then $f'(x)$ is
(a) $2\sin x - 12x^3$ (b) $2\cos x - 12x^3$
(c) $2\cos x + 12x^3$ (d) $2\sin x + 12x^3$

38. Derivative of the function $f(x) = (x-1)(x-2)$ is
(a) $2x + 3$ (b) $3x - 2$
(c) $3x + 2$ (d) $2x - 3$

39. If $\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right]$ exists, then which one of the following correct ?
(a) Both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist
(b) $\lim_{x \to a} f(x)$ need not exist but $\lim_{x \to a} g(x)$ must exist
(c) Both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ need not exist
(d) $\lim_{x \to a} f(x)$ must exist but $\lim_{x \to a} g(x)$ need not exist

40. The value of $\lim_{x \to 0} \frac{1 + \frac{x}{3} - 1 + \frac{x}{3}}{x}$ is
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

41. The value of $\lim_{x \to 0} \frac{\cos x}{\pi - x}$ is
(a) π (b) $-\pi$ (c) $\frac{1}{\pi}$ (d) $-\frac{1}{\pi}$

42. Let $3f(x) - 2f(1/x) = x$, then $f'(2)$ is equal to
(a) $\frac{2}{7}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{7}{2}$

43. What is the derivative of
$f(x) = \frac{7x}{(2x-1)(x+3)}$?
(a) $-\frac{3}{(x+3)^2} - \frac{2}{(2x-1)^2}$ (b) $-\frac{3}{(x+3)^2} - \frac{1}{(2x-1)^2}$
(c) $\frac{3}{(x+3)^2} + \frac{1}{(2x-1)^2}$ (d) $\frac{3}{(x+3)^2} + \frac{2}{(2x-1)^2}$

44. As $x \to a$, $f(x) \to l$, then l is called .... of the function f(x),
(a) limit (b) value
(c) absolute value (d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

45. Consider the function $g(x) = |x|, x \neq 0$.
Then
I. g (0) is not defined.
II. $\lim_{x \to 0} g(x)$ is not defined.

Which of the following is/are true?
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

46. Consider the function $h(x) = \frac{x^2-4}{x-2}, x \neq 2$
Then,
I. h (2) is not defined.
II. $\lim_{x\to 2} h(x) = 4$.
Which of the following is/are true?
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

47. Which of the following is/are true?
I. $\lim_{x\to 1}\left[\frac{x^{15}-1}{x^{10}-1}\right] = \frac{3}{2}$
II. $\lim_{x\to 0}\left[\frac{\sqrt{1+x}-1}{x}\right] = \frac{1}{2}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

48. Which of the following is/are true?
I. $\lim_{x\to 0}\frac{\sin x}{x} = 1$
II. $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

49. Which of the following is/are true?
I. $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}$ (where $a+b+c \neq 0$) is 1.
II. $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is $\frac{1}{4}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

50. $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ is equal to
I. $a^2\sin a + 2a\cos a$ II. $a^2\cos a + 2a\sin a$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

51. Which of the following is/are true?
I. The derivative of $f(x) = \sin 2x$ is $2(\cos^2 x - \sin^2 x)$.
II. The derivative of $g(x) = \cot x$ is $-\text{cosec}^2 x$.
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

52. Which of the following is/are true?
I. The derivative of $x^2 - 2$ at $x = 10$ is 18.
II. The derivative of 99x at x = 100 is 99.
III. The derivative of x at x = 1 is 1.
(a) I, II and III are true (b) I and II are true
(c) II and III are true (d) I and III are true

53. Which of the following is/are true?
I. The derivative of $y = 2x - \frac{3}{4}$ is 2.
II. The derivative of $y = (5x^3 + 3x - 1)(x - 1)$ is $20x^3 + 15x^2 + 6x - 4$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

54. Which of the following is/are true?
I. The derivative of $f(x) = x^3$ is x^2
II. The derivative of $f(x) = \frac{1}{x^3}$ is $\frac{-1}{x^2}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

55. Which of the following is/are true?
I. The derivative of $-x$ is -1.
II. The derivative of $(-x)^{-1}$ is $\frac{1}{x^2}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

56. Which of the following is/are true?
I. The derivative of sin (x + a) is cos (x + a), where *a* is a fixed non-zero constant.
II. The derivative of cosec x cot x is $\text{cosec}^3 x - \cot^2 x \,\text{cosec}\, x$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Borh I and II are false

57. Which of the following is/are true?
I. The derivative of $f(x) = 1 + x + x^2 + ... + x^{50}$ at x = 1 is 1250.
II. The derivative of $f(x) = \frac{x+1}{x}$ is $\frac{1}{x^2}$.
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

58. Consider the following limits which holds for function *f* and *g*:
I. $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
II. $\lim_{x\to a}[f(x).g(x)] = \lim_{x\to a} f(x).\lim_{x\to a} g(x)$
III. $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
Which of the above are true ?
(a) Only I (b) Only II
(c) Only III (d) All of the above

59. Consider the following derivatives which holds for function *u* and *v*.
I. $(u \pm v)' = u' \pm v'$ II. $(uv)' = uv' + vu'$
III. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
Which of the above holds are true ?
(a) Only I (b) Only II
(c) Only III (d) All of these

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

60.

Column-I	Column-II
A. $\lim_{x\to a^-} f(x)$	1. left hand limit of *f* at *a*
B. $\lim_{x\to a^+} f(x)$	2. limit of *f* at *a*
C. $\lim_{x\to a} f(x)$	4. right hand limit of *f* at *a*

Codes

	A	B	C
(a)	3	1	2
(b)	1	3	2
(c)	1	2	3
(d)	2	3	1

61.

Column-I (Limts)	Column-II (Values)
A. $\lim_{x\to 3} x+3$	1. π
B. $\lim_{x\to\pi}\left(x-\frac{22}{7}\right)$	2. 6
C. $\lim_{r\to 1}\pi r^2$	3. $\frac{19}{2}$
D. $\lim_{x\to 4}\left(\frac{4x+3}{x-2}\right)$	4. $\frac{-1}{2}$
E. $\lim_{x\to -1}\left(\frac{x^{10}+x^5+1}{x-1}\right)$	5. $\pi-\frac{22}{7}$

Codes

	A	B	C	D	E
(a)	5	2	1	4	3
(b)	2	5	1	3	4
(c)	5	2	1	3	4
(d)	2	5	3	1	4

62.

Column-I (Limits)	Column-II (Values)
A. $\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$	1. 4
B. $\lim_{x\to 0}\frac{\cos x}{\pi-x}$	2. $\frac{1}{\pi}$
C. $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$	3. $\frac{a+1}{b}$
D. $\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$	4. 0
E. $\lim_{x\to 0} x\sec x$	5. 1
F. $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}$ $(a, b, a+b\neq 0)$	

Codes

	A	B	C	D	E	F
(a)	2	2	1	3	5	4
(b)	2	2	3	1	4	5
(c)	2	2	1	4	3	5
(d)	2	2	1	3	4	5

63.

Column-I (Functions)	Column-II (Derivatives)
A. cosec x	1. 5 cos x + 6 sin x
B. 3 cot x + 5 cosec x	2. – 3 cosec^2 x – 5 cosec x cot x
C. 5 sin x – 6 cos x + 7	3. 2 $\sec^2$ x – 7 sec x tan x
D. 2 tan x – 7 sec x	4. – cot x cosec x

Codes

	A	B	C	D
(a)	4	1	2	3
(b)	4	2	3	1
(c)	2	4	1	3
(d)	4	2	1	3

64.

Column-I (Functions)	Column-II (Derivatives)
A. f (x) = 10x	1. 2x
B. f (x) = x^2	2. $-\frac{1}{x^2}$
C. f(x) = a for fixed real no. a	3. 0
D. f (x) = $\frac{1}{x}$	4. 10

Codes

	A	B	C	D
(a)	4	1	3	2
(b)	1	4	3	2
(c)	4	1	2	3
(d)	4	3	1	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

65. If value of $\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}$ is equal to $\frac{1}{a\sqrt{2}}$ then 'a' equals

(a) 1 (b) 2 (c) 3 (d) 4

66. If value of $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ is equal to $\frac{2\sqrt{3}}{m}$, where m is equal to

(a) 2 (b) 8 (c) 9 (d) 3

67. $\lim_{x\to\pi/2}$ (sec x – tan x) is equal to

(a) 0 (b) 2 (c) 1 (d) 3

68. Suppose $f(x)=\begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax, & x>1\end{cases}$

and if $\lim_{x\to 1} f(x) = f(1)$ then the value of a + b is

(a) 0 (b) 2 (c) 4 (d) 8

69. If $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$ is equal to p cos q, then sum of p and q is

(a) 2 (b) 1 (c) 3 (d) 4

70. If $f(x) = |x| - 5$, then the value of $\lim_{x\to 5} f(x)$ is

(a) 9 (b) 1 (c) 0 (d) None of these

71. If value of $\lim_{x\to 0}\frac{\sin x}{x(1+\cos x)}$ is equal to $\frac{a}{2}$ then the value of 'a' is

(a) 0 (b) 1 (c) 2 (d) 3

72. Value of $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}$ is

(a) 1 (b) 2 (c) 4 (d) None of these

73. If $f(x)=\begin{cases} 2x+3, & x\le 0 \\ 3(x+1), & x>0\end{cases}$ then the value of $\lim_{x\to 0} f(x)$ is

(a) 0 (b) 6 (c) 2 (d) 3

74. Let $f(x)=\begin{cases} x+2, & x\le -1 \\ cx^2, & x>-1 \end{cases}$

If $\lim_{x\to -1} f(x)$ exists, then c is equal to

(a) 1 (b) 0 (c) 2 (d) 3

75. If value of $\lim_{x\to\frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$ is $a\sqrt{2}$, then the value of 'a' is

(a) 2 (b) 3 (c) 4 (d) 5

76. If $\lim_{x\to 2} \frac{x^n-2^n}{x-2}=80$ and $n\in N$, then the value of 'n' is

(a) 2 (b) 3 (c) 4 (d) 5

77. $\lim_{x\to 0} \frac{\sin 2x}{x}$ is equal to

(a) 2 (b) 0 (c) 1 (d) 3

78. If $f(x)=x^n$ and $f'(1)=10$, then the value of 'n' is

(a) 1 (b) 5 (c) 9 (d) 10

79. If $\lim_{x\to 5} \frac{x^k-5^k}{x-5}=500$, then k is equal to :

(a) 3 (b) 4 (c) 5 (d) 6

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

80. **Assertion:** $\lim_{x\to 0} \frac{\sin ax}{bx}=\frac{a}{b}$

Reason: $\lim_{x\to 0} \frac{\sin ax}{\sin bx}=\frac{b}{a}$ $(a, b\neq 0)$

81. **Assertion:** $\lim_{x\to 0} (\operatorname{cosec} x-\cot x)=0$

Reason: $\lim_{x\to\frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}}=1$

82. **Assertion:** If a and b are non-zero constants, then the derivative of $f(x)=ax+b$ is a.

Reason: If a, b and c are non-zero constants, then the derivative of $f(x)=ax^2+bx+c$ is $ax+b$.

83. Let $a_1, a_2, a_3, ..., a_n$ be fixed real numbers and define a function $f(x)=(x-a_1)(x-a_2)...(x-a_n)$, then

Assertion: $\lim_{x\to a_1} f(x)=0$.

Reason: $\lim_{x\to a} f(x)=(a-a_1)(a-a_2)...(a-a_n)$, for some $a\neq a_1, a_2, ..., a_n$.

84. **Assertion:** Suppose f is real valued function, the derivative of 'f' at x is given by $f'(x)=\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Reason: If $y=f(x)$ is the function, then derivative of 'f' at any x is denoted by $f'(x)$.

85. **Assertion.** For the function

$f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+...+\frac{x^2}{2}+x+1$, $f'(1)=100f'(0)$.

Reason: $\frac{d}{dx}(x^n)=n.x^{n-1}$.

86. **Assertion:** $\lim_{x\to 0} (1+3x)^{1/x}=e^3$.

Reason: Since $\lim_{x\to 0} (1+x)^{1/x}=e$.

87. **Assertion:** $\lim_{x\to 0} \log_e\left(\frac{\sin x}{x}\right)=0$

Reason: $\lim_{x\to 0} f(g(x))=f(\lim_{x\to 0} g(x))$.

88. **Assertion:** $\lim_{x\to 0} \frac{\tan x^0}{x^0}=1$ where x^0 means x degree.

Reason: If $\lim_{x\to 0} f(x)=l$, $\lim_{x\to 0} g(x)=m$, then $\lim_{x\to 0}\{f(x)g(x)\}=lm$

89. **Assertion:** Derivative of $f(x)=x|x|$ is $2|x|$.

Reason: For function u and v, $(uv)'=uv'+vu'$.

90. **Assertion:** Let $\lim_{x\to a} f(x)=l$ and $\lim_{x\to a} g(x)=m$. If l and m both exist, then $\lim_{x\to a} (fg)(x)=\lim_{x\to a} f(x).\lim_{x\to a} g(x)=lm$

Reason: Let f be a real valued function defined by $f(x)=x^2+1$, then $f'(2)=4$.

91. **Assertion:** Derivative of $f(x)=2$ is zero.

Reason: Differentiation of a constant function is zero.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

92. Evaluate $\lim_{x\to 0} \frac{\sin^2 2x}{x^2}$.

(a) 4 (b) –4 (c) sin x (d) cos x

93. The value of $\lim_{x\to 0} \frac{x^3 \cot x}{1-\cos x}$ is

(a) 1 (b) –2 (c) 2 (d) 0

94. The value of $\lim_{x\to 0} \frac{x(e^x-1)}{1-\cos x}$ is

(a) 0 (b) 2 (c) –2 (d) does not exist

95. $\lim_{x\to 0} \frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(a) 1 (b) –1 (c) zero (d) does not exist

96. $\lim_{x\to 0} \frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ is

(a) 2 (b) –2 (c) 1/2 (d) –1/2

97. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ equals

(a) $-\pi$ (b) π (c) $\pi/2$ (d) 1

98. The value of $\lim_{\theta\to -\frac{\pi}{4}} \frac{\cos\theta+\sin\theta}{\theta+\frac{\pi}{4}}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{-\pi}{4}$ (c) $-\sqrt{2}$ (d) $\sqrt{2}$

99. If $f(x)=\dfrac{x+|x|}{x}$, then the value of $\lim_{x\to 0} f(x)$ is
(a) 0 (b) 2
(c) does not exist (d) None of these

100. f(x) is a function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$ and h (x) is a function such that $h(x) = [f(x)]^2 + [g(x)]^2$ and h (5) = 11, then the value of h (10) is
(a) 5 (b) –5 (c) –11 (d) 11

101. Let α and β be the distinct roots of $ax^2 + bx = c = 0$, then

$\lim_{x\to\alpha} \dfrac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(a) $\dfrac{a^2}{2}(\alpha-\beta)^2$ (b) 0

(c) $\dfrac{-a^2}{2}(\alpha-\beta)^2$ (d) $\dfrac{1}{2}(\alpha-\beta)^2$

102. If $\lim_{x\to 0} \dfrac{((a-n)nx-\tan x)\sin nx}{x^2} = 0$, where n is non-zero real number, then a is equal to

(a) 0 (b) $\dfrac{n+1}{n}$ (c) n (d) $n+\dfrac{1}{n}$

103. If sin y = x sin (a + y), then value of dy/dx is

(a) $\dfrac{\sin(a+y)}{\sin a}$ (b) $\dfrac{\sin^2(a+y)}{\sin a}$

(c) $\sin^2(a+y)$ (d) $\dfrac{\cos^2(a+y)}{\sin a}$

104. If $y = x\tan\dfrac{x}{2}$, then value of $(1+\cos x)\dfrac{dy}{dx} - \sin x$ is
(a) –x (b) x^2 (c) x (d) None

105. Differential coefficient of $\dfrac{x\sin x}{1+\cos x}$ is

(a) $\dfrac{-x-\sin x}{1+\cos x}$ (b) $\dfrac{x-\sin x}{1+\cos x}$ (c) $\dfrac{x+\sin x}{1-\cos x}$ (d) $\dfrac{x+\sin x}{1+\cos x}$

106. If $x^3 + y^3 = 3xy$, then value of $\dfrac{dy}{dx}$ is

(a) $\dfrac{x-y^2}{x^2-y}$ (b) $\dfrac{x^2-y}{x-y^2}$ (c) $\dfrac{x^2-y^2}{x-y}$ (d) $\dfrac{x-y}{x^2-y^2}$

107. If $y = ax^{n+1} + bx^{-n}$, then value of $x^2\dfrac{d^2y}{dx^2}$ is
(a) n (n + 1) y (b) $n^2(n+1)y$
(c) 2n (n – 1) y (d) None of these

108. If $y = f\left(\dfrac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, then value of $\dfrac{dy}{dx}$ is

(a) $\sin\left(\dfrac{2x+1}{x^2+1}\right)^2\left[\dfrac{2+2x-2x^2}{x^2+1}\right]$

(b) $\sin\left(\dfrac{2x-1}{x^2-1}\right)^2\left[\dfrac{2+2x-2x^2}{x^2+1}\right]$

(c) $\sin\left(\dfrac{2x-1}{x^2+1}\right)^2\left[\dfrac{2+2x-2x^2}{x^2+1}\right]$

(d) $\sin\left(\dfrac{2x+1}{x^2-1}\right)^2\left[\dfrac{2+2x-2x^2}{x^2+1}\right]$

109. $\lim_{x\to 2}\left[\dfrac{1}{x-2}-\dfrac{2(2x-3)}{x^3-3x^2+2x}\right]$ is equal to

(a) $\dfrac{1}{2}$ (b) $\dfrac{-1}{2}$ (c) 1 (d) – 2

110. If $\lim_{x\to 3}\dfrac{x^n-3^n}{x-3} = 108$, the positive integer n is equal to
(a) 3 (b) 5 (c) 2 (d) 4

111. $\lim_{x\to 0}\dfrac{\cos ax-\cos bx}{\cos cx-1}$ is equal to $\dfrac{m}{n}$, where m and n are respectively
(a) $a^2 + b^2, c^2$ (b) $c^2, a^2 + b^2$
(c) $a^2 - b^2, c^2$ (d) $c^2, a^2 - b^2$

112. $\lim_{x\to 1}[x-1]$, where [.] is greatest integer function, is equal to
(a) 1 (b) 2 (c) 0 (d) does not exist

113. $\lim_{x\to 0}\dfrac{\tan x-\sin x}{\sin^3 x}$ is equal to

(a) $\dfrac{1}{2}$ (b) 0 (c) 1 (d) Not defined

114. If a, b are fixed non-zero constant, then the derivative of $\dfrac{a}{x^4}-\dfrac{b}{x^2}+\cos x$ is ma + nb – p, where

(a) $m = 4x^3, n = \dfrac{-2}{x^3}, p = \sin x$

(b) $m = \dfrac{-4}{x^5}, n = \dfrac{2}{x^3}, p = \sin x$

(c) $m = \dfrac{-4}{x^5}, n = \dfrac{-2}{x^3}, p = -\sin x$

(d) $m = 4x^3, n = \dfrac{2}{x^3}, p = -\sin x$

115. If a is a fixed non-zero constant, then the derivative of $\dfrac{\sin(x+a)}{\cos x}$ is

(a) $\dfrac{\cos a}{\cos^2 x}$ (b) $\dfrac{-\cos a}{\cos^2 x}$ (c) $\dfrac{\sin a}{\cos^2 x}$ (d) $\dfrac{-\sin a}{\cos^2 x}$

116. $\lim_{x\to 4}\dfrac{|x-4|}{x-4}$ is equal to
(a) 1 (b) 0 (c) – 1 (d) does not exist

117. Let $f(x) = \begin{cases} \dfrac{k\cos x}{\pi-2x}, & \text{when } x \neq \dfrac{\pi}{2} \\ 3, & \text{when } x = \dfrac{\pi}{2} \end{cases}$

If $\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\dfrac{\pi}{2}\right)$, then k is equal to
(a) 2 (b) 4 (c) 6 (d) 8

118. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 0 (d) None of these

119. If $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ for all $x, y \in R$ $(xy \neq 1)$ and $\lim_{x\to 0}\frac{f(x)}{x} = 2$. Then, $f'\left(\frac{1}{\sqrt{3}}\right)$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{6}$ (d) $\frac{3}{2}$

120. If f be a function given by $f(x) = 2x^2 + 3x - 5$. Then, $f'(0) = mf'(-1)$, where m is equal to

(a) -1 (b) -2 (c) -3 (d) -4

121. For the function

$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... \frac{x^2}{2} + x + 1,$

$f'(1) = mf'(0)$, where m is equal to

(a) 50 (b) 0 (c) 100 (d) 200

122. Evaluate: $\lim_{x\to\pi/6}\frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$

(a) 3 (b) -3
(c) 1 (d) -1

123. The function $u = e^x \sin x$, $v = e^x \cos x$ satisfy the equation

(a) $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ (b) $\frac{d^2u}{dx^2} = 2v$

(c) $\frac{d^2v}{dx^2} = -2u$ (d) All of these

124. If $f(x) = \begin{cases} |x|+1, & x<0 \\ 0, & x=0 \\ |x|-1, & x>0 \end{cases}$ then $\lim_{x\to a} f(x)$ exists for all

(a) $a \neq 1$ (b) $a \neq 0$ (c) $a \neq -1$ (d) $a \neq 2$

125. Evaluate : $\lim_{x\to a}\frac{(x+2)^{5/3} - (a+2)^{5/3}}{x-a}$.

(a) $\frac{-5}{3}(a+2)^{2/3}$ (b) $\frac{5}{3}(a-2)^{2/3}$

(c) $\frac{5}{3}(a+2)^{-2/3}$ (d) $\frac{5}{3}(a+2)^{2/3}$

126. $\lim_{x\to 0}\sqrt{\frac{x - \sin x}{x + \sin^2 x}}$ is equal to

(a) 1 (b) 0 (c) ∞ (d) None of these

127. What is the value of $\lim_{x\to 0}\frac{x\sin 5x}{\sin^2 4x}$?

(a) 0 (b) $\frac{5}{4}$ (c) $\frac{5}{16}$ (d) $\frac{25}{4}$

128. If $\lim_{x\to 0}\frac{a^x - x^a}{x^a - a^a} = -1$, then a is equal to:

(a) -1 (b) 0 (c) 1 (d) 2

129. The value of $\lim_{x\to 2}\frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$ is

(a) $\frac{1}{8\sqrt{3}}$ (b) $\frac{1}{4\sqrt{3}}$ (c) 0 (d) None of these

130. The value of $\lim_{x\to 2a}\frac{\sqrt{x-2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$ is

(a) $\frac{1}{\sqrt{a}}$ (b) $\frac{1}{2\sqrt{a}}$ (c) $\frac{\sqrt{a}}{2}$ (d) $2\sqrt{a}$

131. $\lim_{x\to\frac{\pi}{2}}\frac{\left[1 - \tan\left(\frac{x}{2}\right)\right][1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right)\right][\pi - 2x]^3}$ is

(a) ∞ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

132. $\lim_{x\to 2}\left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x-2}\right)$ is equal to

(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$

(c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

133. Let $f : R \to [0, \infty)$ be such that $\lim_{x\to 5} f(x)$ exists and $\lim_{x\to 5}\frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$. Then $\lim_{x\to 5} f(x)$ equals :

(a) 0 (b) 1 (c) 2 (d) 3

134. The value of $\lim_{x\to 0}\frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$ is at $\tan x = 3$, is

(a) 0 (b) 1 (c) 2 (d) 3

135. $\lim_{x\to 0}\frac{x\sqrt[3]{z^2 - (z-x)^2}}{\left(\sqrt[3]{8xz - 4x^2} + \sqrt[3]{8xz}\right)^4}$ is equal to

(a) $\frac{z}{2^{11/3}}$ (b) $\frac{1}{2^{23/3}.z}$ (c) $2^{21/3}z$ (d) None of these

136. $\lim_{h\to 0}\left(\frac{1}{h\sqrt[3]{8+h}} - \frac{1}{2h}\right)$ equals to

(a) $-\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{48}$ (d) $-\frac{1}{48}$

137. The value of $\lim_{x\to 0}\frac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(a) 1/5 (b) 1/6 (c) 1/4 (d) 1/2

138. The value of $\lim_{x\to 0}\frac{1 - \cos x + 2\sin x - \sin^3 x - x^2 + 3x^4}{\tan^3 x - 6\sin^2 x + x - 5x^3}$ is

(a) 1 (b) 2 (c) -1 (d) -2

139. A function f is said to be a rational function, if $f(x) = \frac{g(x)}{h(x)}$, where g (x) and h (x) are polynomials such that $h(x) \neq 0$, then

(a) $h(a) \neq 0 \Rightarrow \lim_{x\to a} f(x) = \frac{g(a)}{h(a)}$

(b) $h(a) = 0$ and $g(a) \neq 0 \Rightarrow \lim_{x\to a} f(x)$ does not exist

(c) Both (a) and (b) are true

(d) Both (a) and (b) are false.

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(a)** Given function $f(x) = x^2$. Observe that as x takes values very close to 0, the value of f (x) also approaches towards 0.

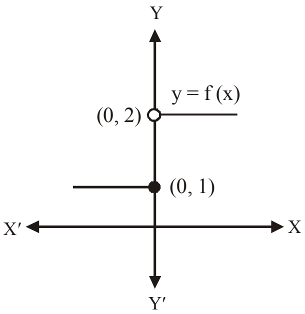
We say $\lim_{x\to 0} f(x) = 0$

(i.e, the limit of f (x) as x tends to zero equals zero).

2. **(a)** Given function $f(x) = \begin{cases} 1, & x \le 0 \\ 2, & x > 0 \end{cases}$

Graph of this function is shown below. It is clear that the value of f at 0 dictated by values of f (x) with $x \le 0$ equals 1, i.e. the left hand limit of f (x) at x = 0 is

$\lim_{x\to 0^-} f(x) = 1$



Similarly, the value of f at x = 0 dictated by values of f (x) with x > 0 equals 2, i.e., the right hand limit of f (x) at x = 0 is

$$\lim_{x\to 0^+} f(x) = 2$$

In this case the right and left hand limits are different, and hence we say that the limit of f (x) as x tends to zero does not exist (even though the function is defined at 0).

3. **(b)** Limit $= \lim_{x\to -1} \frac{(x-1)(x+1)}{(x+2)(x+1)} = \frac{-1-1}{-1+2} = -2$

4. **(a)** From direct substitution $\frac{\sqrt{1+0}+\sqrt{1-0}}{1+0} = \frac{2}{1} = 2$

5. **(a)** Limit $= \lim_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)} = \lim_{x\to 0} \frac{\sqrt{1+x}+\sqrt{1-x}}{2} = 1$

6. **(c)** $\lim_{x\to 5} \frac{1-\sqrt{x-4}}{x-5} = \lim_{x\to 5} \frac{1-\sqrt{x-4}}{x-5} \cdot \frac{1+\sqrt{x-4}}{1+\sqrt{x-4}}$

$$= \lim_{x\to 5} \frac{1-x+4}{(x-5)(1+\sqrt{x-4})} = \lim_{x\to 5} \frac{-(x-5)}{(x-5)(1+\sqrt{x-4})}$$

$$= \lim_{x\to 5} \frac{-1}{(1+\sqrt{x-4})} = \frac{-1}{(1+\sqrt{5-4})} = \frac{-1}{2}$$

7. **(a)** By rationalisation of numerator, given expression

$$= \lim_{x\to 0} \frac{\sqrt{1+x+x^2}-1}{x} \cdot \frac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+1}$$

$$= \lim_{x\to 0} \frac{1+x+x^2-1}{x\left(\sqrt{1+x+x^2}+1\right)} = \lim_{x\to 0} \frac{x(1+x)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$= \lim_{x\to 0} \frac{1+x}{\sqrt{1+x+x^2}+1} = \frac{1}{2}$$

8. **(a)** Left hand limit $= \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (3x-1) = 3.1-1 = 2$

and Right hand limit $= \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^2+1)$

$$= 1^2 + 1 = 2$$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 2$

So $\lim_{x\to 1} f(x) = 2$

9. **(a)** $\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x^2} \cdot \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$

$$= \lim_{x\to 0} \frac{1+x^2-1+x^2}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}$$

$$= \lim_{x\to 0} \frac{2x^2}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)} = \frac{2}{\sqrt{1}+\sqrt{1}} = \frac{2}{2} = 1$$

10. **(a)** $f'(t) = \frac{d}{dt}\left[\frac{1-t}{1+t}\right] = \frac{(1+t)(-1)-(1-t)\times(1)}{(1+t)^2}$

$$= \frac{-1-t-1+t}{(1+t)^2} = \frac{-2}{(1+t)^2}$$

$$f'[1/t] = \frac{-2}{\left(1+\frac{1}{t}\right)^2} = \frac{-2t^2}{(t+1)^2}$$

11. **(d)** Let f and g be two functions such that both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. Then,

(i) Limit of sum of two functions is sum of the limits of the functions i.e.,

$\lim_{x\to a} [f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.

(ii) Limit of difference of two functions is difference of the limits of the functions, i.e.,

$\lim_{x\to a} [f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$.

(iii) Limit of product of two functions is product of the limits of the functions, i.e.,

$$\lim_{x\to a}[f(x).g(x)] = \lim_{x\to a} f(x).\lim_{x\to a} g(x).$$

(iv) Limit of quotient of two functions is quotient of the limits of the functions (whenever the denominator is non-zero), i.e.,

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \text{ if } \lim_{x\to a} g(x) \neq 0$$

12. (b) It is easy to see that the derivative of the function f (x) = x is the constant function 1. This is because

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{x+h-x}{h} = \lim_{h\to 0} 1 = 1$$

13. (c) Let f (x) = sin x. Then,

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(0+h)-\sin(0)}{h} = \lim_{h\to 0}\frac{\sin h}{h} = 1$$

14. (d) We have,

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{3(2+h)-3(2)}{h}$$

$$= \lim_{h\to 0}\frac{6+3h-6}{h} = \lim_{h\to 0}\frac{3h}{h} = \lim_{h\to 0} 3 = 3.$$

The derivative of the function f (x) = 3x at x = 2 is 3.

15. (b) Since, the derivative measures the change in the function, intuitively it is clear that the derivative of the constant function must be zero at every point.

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{3-3}{h} = \lim_{h\to 0}\frac{0}{h} = 0$$

Similarly, $f'(3) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0}\frac{3-3}{h} = 0$

16. (d) The derivative of f at x = a is denoted by

$$\left.\frac{d}{dx}f(x)\right|_a \text{ or } \left.\frac{df}{dx}\right|_a \text{ or even } \left(\frac{df}{dx}\right)_{x=a}$$

17. (a) Let y = x + a

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 1 + 0 = 1$$

18. (b) Let $y = \frac{1+\frac{1}{x}}{1-\frac{1}{x}} \Rightarrow y = \frac{x+1}{x-1}$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(x-1)\frac{d}{dx}(x+1)-(x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{(x-1)(1+0)-(x+1)(1-0)}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{(x-1)^2} = \frac{-2}{(1-x)^2}$$

19. (c) Let $y = 4\sqrt{x}-2 \Rightarrow y = 4x^{1/2}-2$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 4.\frac{1}{2}x^{\frac{1}{2}-1} - 0 = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{x}}$$

20. (b) Let $y = (ax+b)^n$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = n(ax+b)^{n-1}\frac{d}{dx}(ax+b) = n(ax+b)^{n-1}a$$

$$\Rightarrow \frac{dy}{dx} = na(ax+b)^{n-1}$$

21. (b) Let $y = \sin^n x \Rightarrow y = (\sin x)^n$

Differentiating y w.r.t.x, we get

$$\frac{dy}{dx} = n(\sin x)^{n-1}\frac{d}{dx}(\sin x) \Rightarrow \frac{dy}{dx} = n(\sin x)^{n-1}\cos x$$

22. (d) Let $y = (x^2+1)\cos x$,

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = (x^2+1)\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}(x^2+1)$$

(by product rule)

$$= (x^2+1)(-\sin x) + \cos x(2x) = -x^2\sin x - \sin x + 2x\cos x$$

23. (c) We have, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0}\frac{\tan[a(x+h)+b]-\tan(ax+b)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sin(ax+ah+b)}{\cos(ax+ah+b)}-\frac{\sin(ax+b)}{\cos(ax+b)}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sin(ax+ah+b)\cos(ax+b)-\sin(ax+b)\cos(ax+ah+b)}{}}{h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0}\frac{a\sin(ah)}{a.h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0}\frac{a}{\cos(ax+b)\cos(ax+ah+b)}\lim_{h\to 0}\frac{\sin ah}{ah}$$

[as h → 0, ah → 0]

$$= \frac{a}{\cos^2(ax+b)} = a\sec^2(ax+b)$$

24. **(b)** $\because f(x)=x\sin x$

$\Rightarrow \quad f'(x)=\frac{d}{dx}(x\sin x)$

$=\sin x\frac{d}{dx}x+x\frac{d}{dx}\sin x=\sin x+x\cos x$

$\Rightarrow f'\left(\frac{\pi}{2}\right)=\sin\frac{\pi}{2}+\frac{\pi}{2}\cos\frac{\pi}{2}=1$

25. **(b)** If given function is $6x^{100}-x^{55}+x$. Then, the derivative of function is $6.100.x^{99}-55.x^{54}+1$ or $600x^{99}-55x^{54}+1$

26. **(b)** $\lim\limits_{x\to 0}\frac{x}{\tan x}=1$

27. **(a)** We have,

$\frac{d}{dx}(\log_x x)=\frac{d}{dx}(1)=0 \quad [\because \log_x x=1]$

28. **(b)** We have,

$\frac{d}{dx}(e^{3\log x})=\frac{d}{dx}(e^{\log x^3})=\frac{d}{dx}(x^3)=3x^2 \quad [\because e^{\log k}=k]$

29. **(b)** We have,

$\frac{d}{dx}\left\{x^2+\sin x+\frac{1}{x^2}\right\}=\frac{d}{dx}(x^2+\sin x+x^{-2})$

$=\frac{d}{dx}(x^2)+\frac{d}{dx}(\sin x)+\frac{d}{dx}(x^{-2})$

$=2x+\cos x+(-2)x^{-3}$

30. **(b)** We have,

$\frac{d}{dx}\left\{\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2\right\}=\frac{d}{dx}\left\{x+\frac{1}{x}+2\right\}$

$=\frac{d}{dx}(x)+\frac{d}{dx}(x^{-1})+\frac{d}{dx}(2)=1+(-1)x^{-2}+0=1-\frac{1}{x^2}$

31. **(b)** We have, $f(x)=\alpha x^n$

Differentiating both sides w.r.t. x, we obtain

$\frac{d}{dx}\{f(x)\}=\frac{d}{dx}(\alpha x^n)$

$\Rightarrow \quad f'(x)=\alpha\frac{d}{dx}(x^n)\Rightarrow f'(x)=\alpha n.x^{n-1}$

Putting x = 1 on both sides, we get

$f'(1)=\alpha.n\Rightarrow\alpha=\frac{f'(1)}{n}$

32. **(c)** We have,

$\frac{d}{dx}(x\sin x)=x.\frac{d}{dx}(\sin x)+\sin x.\frac{d}{dx}(x)$

$=x\cos x+\sin x\cdot 1=x\cos x+\sin x.$

33. **(a)** We have,

$\lim\limits_{x\to 0}\frac{a^{\sin x}-1}{\sin x}=\lim\limits_{y\to 0}\frac{a^y-1}{y}=\log a$, where $y=\sin x$

$[\because x\to 0\Rightarrow y=\sin x\to 0]$

34. **(b)** Consider $\lim\limits_{x\to 0}\frac{2\sin^2 3x}{x^2}$

$=2.\lim\limits_{x\to 0}\left[\frac{\sin 3x}{x}\right]^2=2.\lim\limits_{x\to 0}\left[3\frac{\sin 3x}{3x}\right]^2$

$=2.9.\lim\limits_{x\to 0}\left(\frac{\sin 3x}{3x}\right)^2=18\times 1=18$

35. **(d)** Consider $\lim\limits_{\theta\to 0}\frac{\sin m^2\theta}{\theta}=\lim\limits_{\theta\to 0}\left(\frac{\sin m^2\theta}{m^2\theta}\right).m^2=1\times m^2=m^2$

36. **(b)** $f(x)=7(-3)x^{-3-1}=-21x^{-4}$.

37. **(b)** $f'(x)=2\cos x-12x^3$

38. **(d)** Applying product rule,

$f'(x)=(x-1)\frac{d}{dx}(x-2)+(x-2)\frac{d}{dx}(x-1)$

$=x-1+x-2=2x-3$

39. **(a)** For $\lim\limits_{x\to a}\left[\frac{f(x)}{g(x)}\right]$ to exist, then both $\lim\limits_{x\to a}f(x)$ and $\lim\limits_{x\to a}g(x)$ must exist.

40. **(a)** $\lim\limits_{x\to 0}\frac{1+\frac{x}{3}-1+\frac{x}{3}}{x}=\lim\limits_{x\to 0}\frac{2x}{3x}=\frac{2}{3}$

41. **(c)** $\lim\limits_{x\to 0}\frac{\cos x}{\pi-x}=\frac{1}{\pi-0}=\frac{1}{\pi}$

42. **(b)** $3f(x)-2f\left(\frac{1}{x}\right)=x \quad \ldots(i)$

Put $x=\frac{1}{x}$, then $3f\left(\frac{1}{x}\right)-2f(x)=\frac{1}{x} \quad \ldots(ii)$

Solving (i) and (ii), we get

$5f(x)=3x+\frac{2}{x}\Rightarrow f'(x)=\frac{3}{5}-\frac{2}{5x^2}$

$\therefore \; f'(2)=\frac{3}{5}-\frac{2}{20}=\frac{1}{2}$

43. (a) Given function is $f(x)=\dfrac{7x}{(2x-1)(x+3)}$

Breaking into partial fraction

We get, $f(x)=\dfrac{1}{2x-1}+\dfrac{3}{x+3}$

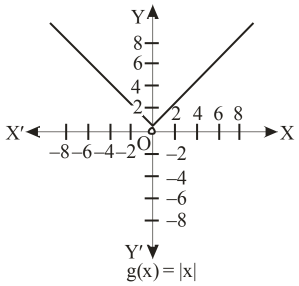
Differentiating w.r.t. x, we get

$$f'(x)=-\frac{2}{(2x-1)^2}-\frac{3}{(x+3)^2}$$

44. (a)

STATEMENT TYPE QUESTIONS

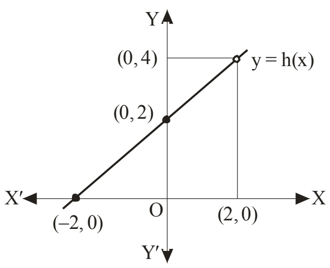
45. (b) Given function $g(x)=|x|, x\neq 0$. Observe that g (0) is not defined. Now, on computing the value of g (x) for values of x very near to 0, we see that the value of g(x) moves towards 0. So, $\lim_{x\to 0} g(x)=0$. This is intuitively clear from the graph of $y=|x|$ for $x\neq 0$.



46. (a) Given, the following function.

$h(x)=\dfrac{x^2-4}{x-2}, x\neq 2$

Now, on computing the value of h (x) for values of x very near to 2 (but not at x = 2), we get all these values are near to 4.



This is somewhat strengthened by considering the graph of the function y = h (x).

47. (a) I. Given,

$$\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}=\lim_{x\to 1}\left[\frac{x^{15}-1}{x-1}\div\frac{x^{10}-1}{x-1}\right]$$

$$=\lim_{x\to 1}\left[\frac{x^{15}-1}{x-1}\right]\div\lim_{x\to 1}\left[\frac{x^{10}-1}{x-1}\right]$$

$$=15(1)^{14}\div 10(1)^{9}=15\div 10=\frac{3}{2}$$

II. Put y = 1 + x, so that y → 1 as x → 0.

$$\text{Then, }\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{y\to 1}\frac{\sqrt{y}-1}{y-1}$$

$$=\lim_{y\to 1}\frac{y^{\frac{1}{2}}-1^{\frac{1}{2}}}{y-1}=\frac{1}{2}(1)^{\frac{1}{2}-1}=\frac{1}{2}$$

48. (a) I. $\lim_{x\to 0}\dfrac{\sin x}{x}=1$ (Standard Result)

II. Let us recall the trigonometric identity

$1-\cos x=2\sin^2\left(\dfrac{x}{2}\right)$.

$$\text{Then, }\lim_{x\to 0}\frac{1-\cos x}{x}=\lim_{x\to 0}\frac{2\sin^2\left(\frac{x}{2}\right)}{x}$$

$$=\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}.\sin\left(\frac{x}{2}\right)=\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}.\lim_{x\to 0}\sin\left(\frac{x}{2}\right)$$

$=1.0=0$

Observe that, we have implicity used the fact that $x\to 0$ is equivalent to $\dfrac{x}{2}\to 0$. This may be justified by putting $y=\dfrac{x}{2}$.

49. (b) I. Given, $\lim_{x\to 1}\dfrac{ax^2+bx+c}{cx^2+bx+a}=\dfrac{a\times(1)^2+b\times 1+c}{c\times(1)^2+b\times 1+a}$

$$=\frac{a+b+c}{c+b+a}=1$$

II. $\lim_{x\to -2}\dfrac{\frac{1}{x}+\frac{1}{2}}{x+2}=\lim_{x\to -2}\dfrac{(2+x)}{2x(x+2)}$

$$=\lim_{x\to -2}\frac{1}{2x}=\frac{1}{2(-2)}=-\frac{1}{4}$$

50. (c) We have $\lim_{h\to 0}\dfrac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$

$$=\lim_{h\to 0}\frac{(a^2+h^2+2ah)[\sin a\cos h+\cos a\sin h]-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\left[\frac{a^2\sin a(\cos h-1)}{h}+\frac{a^2\cos a\sin h}{h}+(h+2a)(\sin a\cos h+\cos a\sin h)\right]$$

$$=\lim_{h\to 0}\left[\frac{a^2\sin a\left(-2\sin^2\frac{h}{2}\right)}{\frac{h^2}{2}}.\frac{h}{2}\right]+\lim_{h\to 0}\frac{a^2\cos a\sin h}{h}+\lim_{h\to 0}(h+2a)\sin(a+h)$$

$= a^2\sin a\times 0+a^2\cos a(1)+2a\sin a=a^2\cos a+2a\sin a$.

51. (a) I. Recall the trigonometric rule $\sin 2x=2\sin x\cos x$. Thus,

$$\frac{df(x)}{dx}=\frac{d}{dx}(2\sin x\cos x)=2\frac{d}{dx}(\sin x\cos x)$$

$= 2[(\sin x)'\cos x+\sin x(\cos x)']$

$= 2[(\cos x)\cos x+\sin x(-\sin x)]$

$= 2(\cos^2 x-\sin^2 x)$

II. $g(x) = \cot x = \frac{\cos x}{\sin x}$

$\Rightarrow \frac{d}{dx}(g(x)) = \frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$

$= \frac{(\cos x)'(\sin x) - (\cos x)(\sin x)'}{(\sin x)^2}$

$= \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$

$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = -\operatorname{cosec}^2 x$

52. (c) I. Let $f(x) = x^2 - 2$, we have

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{[(x+h)^2 - 2] - (x^2 - 2)}{h}$

$= \lim_{h \to 0} \frac{x^2 + h^2 + 2xh - 2 - x^2 + 2}{h}$

$= \lim_{h \to 0} \frac{h(h+2x)}{h} = 0 + 2x = 2x$

At $x = 10$, $f'(10) = 2 \times 10 = 20$

II. Let $f(x) = 99x$

We have $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{99(x+h) - 99x}{h}$

$= \lim_{h \to 0} \frac{99x + 99h - 99x}{h} = \lim_{h \to 0} \frac{99h}{h} = 99$

At $x = 100$, $f'(100) = 99$

III. Let $f(x) = x$

We have, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{x + h - x}{h} \Rightarrow f'(x) = \lim_{h \to 0} \frac{h}{h} = 1$

At $x = 1$, $f'(1) = 1$

53. (b) I. We have, $y = 2x - \frac{3}{4}$

Differentiating y w.r.t. x, we get

$\frac{dy}{dx} = 2 \times 1 - 0 = 2$

II. We have, $y = (5x^3 + 3x - 1)(x - 1)$

Differentiating y w.r.t x, we get

$\frac{dy}{dx} = (5x^3 + 3x - 1)\frac{d}{dx}(x-1) + (x-1)\frac{d}{dx}(5x^3 + 3x - 1)$

$= (5x^3 + 3x - 1)(1 - 0) + (x - 1)(5 \times 3x^2 + 3 \times 1 - 0)$

$= (5x^3 + 3x - 1) + (x - 1)(15x^2 + 3)$

$= 5x^3 + 3x - 1 + 15x^3 + 3x - 15x^2 - 3$

$= 20x^3 - 15x^2 + 6x - 4$

54. (d) I. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

$= \lim_{h \to 0} \frac{x^3 + h^3 + 3xh(x+h) - x^3}{h}$

$= \lim_{h \to 0} (h^2 + 3x(x+h)) = 3x^2$

II. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h} \quad \left[\because f(x) = \frac{1}{x^3}\right]$

$= \lim_{h \to 0} \frac{x^3 - (x+h)^3}{(x+h)^3 x^3 h}$

$= \lim_{h \to 0} \frac{x^3 - [x^3 + h^3 + 3xh(x+h)]}{(x+h)^3 x^3 h}$

$= \lim_{h \to 0} \frac{-h^3 - 3xh(x+h)}{(x+h)^3 x^3 h}$

$= \lim_{h \to 0} \frac{-h[h^2 + 3x(x+h)]}{(x+h)^3 x^3 h} = \frac{-3}{x^4}$

55. (a) I. Let $f(x) = -x$

We have, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ (by first principle)

$= \lim_{h \to 0} \frac{-(x+h) - (-x)}{h} = \lim_{h \to 0} \frac{-x - h + x}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{-h}{h} = -1$

II. Let $f(x) = (-x)^{-1}$

$\Rightarrow f(x) = -\frac{1}{x}$

We have, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ (by first principle)

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{-\frac{1}{x+h} + \frac{1}{x}}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{1}{x} - \frac{1}{x+h}}{h} = \lim_{h \to 0} \frac{x + h - x}{x(x+h)h}$

$= \lim_{h \to 0} \frac{h}{x(x+h)h} = \frac{1}{x(x+0)} = \frac{1}{x^2}$

56. (b) I. Let $y = \sin(x + a)$

$y = \sin x \cos a + \cos x \sin a$

$[\because \sin(A + B) = \sin A \cos B + \cos A \sin B]$

Differentiating y w.r.t. x, we get

$\frac{dy}{dx} = \cos a \frac{d}{dx}(\sin x) + \sin a \frac{d}{dx}(\cos x)$

$= \cos a \cos x - \sin a \sin x = \cos(x + a)$

II. Let $y = \operatorname{cosec} x \cot x$

Differentiating y w.r.t. x, we get

$\frac{dy}{dx} = \operatorname{cosec} x \frac{d}{dx}(\cot x) + \cot x \frac{d}{dx}(\operatorname{cosec} x)$

$= -\operatorname{cosec} x \operatorname{cosec}^2 x + \cot x(-\operatorname{cosec} x \cot x)$

$= -\operatorname{cosec}^3 x - \cot^2 x \operatorname{cosec} x$

57. **(d)** I. The derivative of the function is $1 + 2x + 3x^2 + ... + 50x^{49}$. At x = 1 the value of this function equals to

$1 + 2(1) + 3(1)^2 + ... + 50(1)^{49} = 1 + 2 + 3 + ... + 50$

$= \frac{(50)(51)}{2} = 1275$

II. Clearly, this function is defined everywhere except at x = 0. We use the quotient rule with u = x + 1 and v = x. Hence, u′ = 1 and v′ = 1. Therefore,

$$\frac{df(x)}{dx} = \frac{d}{dx}\left(\frac{x+1}{x}\right) = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

$$= \frac{1(x) - (x+1)1}{x^2} = -\frac{1}{x^2}$$

58. **(d)** **59.** **(d)**

MATCHING TYPE QUESTIONS

60. **(b)** We say $\lim_{x\to a^-} f(x)$ is the expected value of *f* at *x* = *a* given the values of *f* near *x* to the left of *a*. This value is called the left hand limit of *f* at *a*.

Now, $\lim_{x\to a^+} f(x)$ is the expected value of *f* at *x* = *a* given the values of *f* near *x* to the right of *a*. This value is called the right hand limit of f (x) at *a* and if the right and left hand limits coincide, we call that common value as the limit of f(x) at *x* = *a* and denote it by $\lim_{x\to a} f(x)$.

61. **(b)** A. $\lim_{x\to 3} x + 3 = 3 + 3 = 6$

B. $\lim_{x\to\pi}\left(x - \frac{22}{7}\right) = \pi - \frac{22}{7}$

C. $\lim_{r\to 1} \pi r^2 = \pi \times (1)^2 = \pi$

D. $\lim_{x\to 4}\frac{4x+3}{x-2} = \frac{4\times 4+3}{4-2} = \frac{19}{2}$

E. $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1} = \frac{(-1)^{10}+(-1)^5+1}{-1-1} = \frac{1-1+1}{-2} = \frac{-1}{2}$

62. **(d)** A. Given, $\lim_{x\to\pi}\frac{\sin(\pi - x)}{\pi(\pi - x)}$

Let $\pi - x = h$, As $x \to \pi$, then $h \to 0$

$$\therefore \lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)} = \lim_{h\to 0}\frac{\sin h}{\pi h} = \lim_{h\to 0}\frac{1}{\pi}\times\frac{\sin h}{h}$$

$$= \frac{1}{\pi}\times 1 = \frac{1}{\pi} \quad \left(\because \lim_{h\to 0}\frac{\sin h}{h} = 1\right)$$

B. Given $\lim_{x\to 0}\frac{\cos x}{\pi - x}$

Put the limit directly, we get $\frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$

C. Given, $\lim_{x\to 0}\frac{\cos 2x - 1}{\cos x - 1} = \lim_{x\to 0}\frac{1-\cos 2x}{1-\cos x} = \lim_{x\to 0}\frac{2\sin^2 x}{2\sin^2\frac{x}{2}}$

$$\left(\because 1-\cos 2x = 2\sin^2 x \text{ and } 1-\cos x = 2\sin^2\frac{x}{2}\right)$$

Multiplying and dividing by x^2 and then multiplying by $\frac{4}{4}$ in the numerator,

$$= \lim_{x\to 0}\frac{\sin^2 x}{x^2}\times\frac{4\times\frac{x^2}{4}}{\sin^2\frac{x}{2}} = \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\times\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)^2\times 4$$

$= 1 \times 1 \times 4 = 4$

D. Given, $\lim_{x\to 0}\frac{ax + x\cos x}{b\sin x}$

Dividing each term by x, we get

$$= \lim_{x\to 0}\frac{\frac{ax}{x}+\frac{x\cos x}{x}}{\frac{b\sin x}{x}} = \lim_{x\to 0}\frac{a+\cos x}{b\left(\frac{\sin x}{x}\right)}$$

$$= \frac{a+\cos 0}{b\times 1} = \frac{a+1}{b} \quad \left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$

E. $\lim_{x\to 0} x \sec x = 0 \times \sec 0 = 0 \times 1 = 0$

F. $\lim_{x\to 0}\frac{\sin ax + bx}{ax + \sin bx}$

Dividing each term by x,

$$= \lim_{x\to 0}\frac{\frac{\sin ax}{x}+\frac{bx}{x}}{\frac{ax}{x}+\frac{\sin bx}{x}} = \lim_{x\to 0}\frac{\frac{a\sin ax}{ax}+b}{a+\frac{b\sin bx}{bx}}$$

$$= \frac{a\times 1+b}{a+b\times 1} = \frac{a+b}{a+b} = 1 \quad \left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$

63. **(d)** A. Let $y = \text{cosec } x = \frac{1}{\sin x}$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\sin x\frac{d}{dx}(1) - (1)\frac{d}{dx}(\sin x)}{\sin^2 x}$$

$$= \frac{\sin x\times 0 - 1\times\cos x}{\sin^2 x}$$

$$= \frac{0-\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x}\times\frac{1}{\sin x}$$

$$\Rightarrow \frac{dy}{dx} = -\cot x \text{ cosec } x$$

B. Let y = 3 cot x + 5 cosec x

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = -3\text{ cosec}^2 x - 5\text{ cosec } x\cot x$$

C. Let y = 5 sin x – 6 cos x + 7

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 5\cos x - 6(-\sin x) + 0 = 5\cos x + 6\sin x$$

D. Let y = 2 tan x – 7 sec x

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 2\sec^2 x - 7\sec x\tan x$$

64. (a) A. Since, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{10(x+h)-10(x)}{h}=\lim_{h\to 0}\frac{10h}{h}$

$=\lim_{h\to 0}(10)=10$

B. We have, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$

$=\lim_{h\to 0}(h+2x)=2x$

C. We have, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{a-a}{h}=\lim_{h\to 0}\frac{0}{h}=0$ (as $h\neq 0$)

D. We have, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h}$

$=\lim_{h\to 0}\frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right]=\lim_{h\to 0}\frac{1}{h}\left[\frac{-h}{x(x+h)}\right]$

$=\lim_{h\to 0}\frac{-1}{x(x+h)}=\frac{-1}{x^2}$

INTEGER TYPE QUESTIONS

65. (b) $\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}\times\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}=\lim_{x\to 0}\frac{(2+x)-2}{x\left[\sqrt{2+x}+\sqrt{2}\right]}$

$=\lim_{x\to 0}\frac{1}{\sqrt{2+x}+\sqrt{2}}=\frac{1}{2\sqrt{2}}$

66. (c) $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}$

$=\lim_{x\to a}\frac{(a+2x)-3x}{\left(\sqrt{3a+x}-2\sqrt{x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$

Again rationalizing, we get

$=\lim_{x\to a}\frac{(a-x)\left[\sqrt{3a+x}+2\sqrt{x}\right]}{\left(\sqrt{a+2x}+\sqrt{3x}\right)(3a-3x)}=\frac{4\sqrt{a}}{6\sqrt{3a}}$

$=\frac{2\sqrt{3}}{9}$

67. (a) Put $y=\frac{\pi}{2}-x$

$\therefore \lim_{x\to\pi/2}(\sec x-\tan x)=\lim_{y\to 0}\left[\sec\left(\frac{\pi}{2}-y\right)-\tan\left(\frac{\pi}{2}-y\right)\right]$

$=\lim_{y\to 0}[\text{cosec } y-\cot y]=\lim_{y\to 0}\left[\frac{1-\cos y}{\sin y}\right]$

$=\lim_{y\to 0}\frac{2\sin^2\frac{y}{2}}{2\sin\frac{y}{2}\cos\frac{y}{2}}=\lim_{\frac{y}{2}\to 0}\tan\frac{y}{2}=0$

68. (c) $\lim_{x\to 1}f(x)=f(1)$

i.e. RHL = LHL = f (1)

$\Rightarrow \lim_{x\to 1^+}f(x)=\lim_{x\to 1^{-1}}f(x)=4$

$\lim_{h\to 0}f(1+h)=\lim_{h\to 0}f(1-h)=4$

$\Rightarrow \lim_{h\to 0}b-a(1+h)=\lim_{h\to 0}a+b(1-h)=4$

$\Rightarrow b-a(1+0)=a+b(1-0)=4$

$\Rightarrow b-a=4$ and $b+a=4$

On solving, we get $a=0,\ b=4$

69. (d) $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$

$=\lim_{x\to 0}\frac{2\cos\frac{(2+x+2-x)}{2}\sin\frac{(2+x-2+x)}{2}}{x}$

$=\lim_{x\to 0}\frac{(2\cos 2)\sin x}{x}=2\cos 2$

$\Rightarrow p=2$ and $q=2$.

70. (c) At $x=5$, RHL $=\lim_{x\to 5^+}f(x)$

$=\lim_{h\to 0}f(5+h)=\lim_{h\to 0}|5+h|-5=0$

L.H.L $=\lim_{x\to 5}f(x)=\lim_{h\to 0}f(5-h)$

$=\lim_{h\to 0}|5-h|-5=0$

Hence, RHL = LHL $=\lim_{x\to 5}f(x)=0$

71. (b) $\lim_{x\to 0}\frac{\sin x}{x(1+\cos x)}=\lim_{x\to 0}\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{x\left[2\cos^2\frac{x}{2}\right]}$

$=\lim_{x\to 0}\frac{\tan x/2}{2.\frac{x}{2}}=\frac{1}{2}.\lim_{\frac{x}{2}\to 0}\frac{\tan\frac{x}{2}}{\frac{x}{2}}=\frac{1}{2}$

72. (b) $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}=\lim_{x\to 0}\frac{\sin 4x}{4x}\times\frac{4x}{\sin 2x}\times\frac{2x}{2x}$

$=\lim_{x\to 0}\frac{\sin 4x}{4x}\times\frac{2x}{\sin 2x}\times\frac{4x}{2x}=\frac{4}{2}=2$

$(\because x\to 0\Rightarrow 4x\to 0 \text{ and } 2x\to 0)$

73. (d) At $x=0$,

RHL $=\lim_{x\to 0^+}f(x)=\lim_{h\to 0}f(0+h)=\lim_{h\to 0}3(0+h+1)=3$

LHL $=\lim_{x\to 0^-}f(x)=\lim_{h\to 0}f(0-h)=\lim_{h\to 0}2(0-h)+3=3$

Hence, RHL = LHL $=\lim_{x\to 0}f(x)=3$

74. (a) At $x=-1$, limit exists.

$\therefore$ RHL = LHL

$\Rightarrow \lim_{x\to -1^+}f(x)=\lim_{x\to -1^-}f(x)$

$\Rightarrow \lim_{h\to 0}f(-1+h)=\lim_{h\to 0}f(-1-h)$

$\Rightarrow \lim_{h\to 0}c(-1+h)^2=\lim_{h\to 0}(-1-h+2)$

$\Rightarrow c(-1+0)^2=1-0\Rightarrow c=1$

75. **(d)** We have

$$\lim_{x\to\frac{\pi}{4}}\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{2^{\frac{5}{2}}-[(\cos x+\sin x)^2]^{\frac{5}{2}}}{2-(1+\sin 2x)}=\lim_{x\to\frac{\pi}{4}}\frac{(1+\sin 2x)^{\frac{5}{2}}-2^{\frac{5}{2}}}{(1+\sin 2x)-2}$$

$$=\lim_{t\to 2}\frac{t^{\frac{5}{2}}-2^{\frac{5}{2}}}{t-2},\text{ where } t=1+\sin 2x=\frac{5}{2}\times(2)^{\frac{5}{2}-1}=5\sqrt{2}$$

76. **(d)** $\lim_{x\to 2}\frac{x^n-2^n}{x-2}=80$

$\Rightarrow n.2^{n-1}=80 \Rightarrow n.2^{n-1}=5.2^{5-1} \Rightarrow n=5$

77. **(a)** $\lim_{x\to 0}\frac{\sin 2x}{x}=\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2=2.\lim_{x\to 0}\frac{\sin 2x}{2x}=2\times 1=2.$

78. **(d)** Let $f(x)=x^n$

$f'(x)=n.x^{n-1}$

$f'(1)=n.1^{n-1}=n$

$10=n$

79. **(b)** Let $\lim_{x\to 5}\frac{x^k-5^k}{x-5}=500$

By using $\lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}$, we have

$k.5^{k-1}=500$

Now, put $k=4$, we get

$4.5^{4-1}=500 \Rightarrow 4.5^3=500$

which is true.

$\therefore\ k=4$

ASSERTION- REASON TYPE QUESTIONS

80. **(c)** Assertion is correct but Reason is incorrect.

81. **(c)** Assertion is correct

$$\lim_{x\to 0}\left[\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right]=\lim_{x\to 0}\frac{1-\cos x}{\sin x}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}=\lim_{x\to 0}\tan\frac{x}{2}=0$$

82. **(c)** **Assertion** is correct but Reason is incorrect.

Reason: $f(x)=ax^2+bx+c$

$f'(x)=2ax+b$

83. **(b)** Both Assertion and Reason are correct but reason is not the correct explanation.

84. **(b)** Both Assertion and Reason are correct.

85. **(a)** We know that $\frac{d}{dx}(x^n)=nx^{n-1}$

$$\therefore \text{For } f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+.....+\frac{x^2}{2}+x+1$$

$$f'(x)=\frac{100x^{99}}{100}+99\frac{x^{98}}{99}+.....+\frac{2x}{2}+1$$

$=x^{99}+x^{98}+..........+x+1$

Now, $f'(1)=1+1+$..........to 100 term $=100$

$f'(0)=1$

$\therefore f'(1)=100\times 1=100f'(0)$

Hence, $f'(1)=100f'(0)$

86. **(a)** $\lim_{x\to 0}(1+3x)^{1/x}=\lim_{x\to 0}\left[\left(1+3x^{1/3x}\right)\right]^3=e^3$

because $\lim_{x\to 0}(1+x)^{1/x}=e$

87. **(c)** Obviously Assertion is true, but Reason is not always true.

Consider, $f(x)=[x]$ and $g(x)=\sin x$.

88. **(b)** $\because\ \lim_{x\to 0}\frac{\tan x^0}{x^0}=\lim_{x\to 0}\frac{\tan\left(\frac{\pi x}{180}\right)}{\left(\frac{\pi x}{180}\right)}=1$

and $\lim_{x\to 0}\{f(x)g(x)\}=\left(\lim_{x\to 0}f(x)\right)\left(\lim_{x\to 0}g(x)\right)=lm$

89. **(a)** **Assertion:** Let $u=x$, $v=|x|$

90. **(b)** Both Assertion and Reason are correct.

Reason: $f'(2)=\lim_{h\to 0}\frac{\{(2+h)^2+1\}-\{2^2+1\}}{h}$

$$=\lim_{h\to 0}\frac{h^2+4h}{h}=\lim_{h\to 0}h+4=4 \Rightarrow f'(2)=4$$

91. **(a)**

CRITICALTHINKING TYPE QUESTIONS

92. **(a)** $\lim_{x\to 0}\frac{\sin^2 2x}{x^2}$

$$=\lim_{x\to 0}\frac{(2\sin x\cos x)^2}{x^2}=4\lim_{x\to 0}\frac{\sin^2 x}{x^2}.\cos^2 x=4$$

93. **(c)** $\lim_{x\to 0}\frac{x^3\cot x}{1-\cos x}=\lim_{x\to 0}\left(\frac{x^3\cot x}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}\right)$

$$=\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^3\times\lim_{x\to 0}\cos x\times\lim_{x\to 0}(1+\cos x)=2$$

94. **(b)** $\lim_{x\to 0}\frac{x(e^x-1)}{1-\cos x}=\lim_{x\to 0}\frac{2x^2(e^x-1)}{4\sin^2\frac{x}{2}}$

$$=2\lim_{x\to 0}\left[\frac{(x/2)^2}{\sin^2(x/2)}\right]\left(\frac{e^x-1}{x}\right)=2$$

95. **(d)** $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}=\lim_{x\to 0}\frac{\sqrt{1-(1-2\sin^2 x)}}{\sqrt{2}x};$

$$=\lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x}=\lim_{x\to 0}\frac{|\sin x|}{x}$$

The limit of above does not exist as LHS $=-1\neq$ RHL $=1$

96. **(c)** Given expression can be written as

$$\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{4\sin^4 x}$$

$$= \lim_{x\to 0} \frac{x}{4\sin^4 x}\left[\frac{2\tan x}{1-\tan^2 x} - 2\tan x\right]$$

$$= \lim_{x\to 0} \frac{2x\tan x}{4\sin^4 x}\left[\frac{1-1+\tan^2 x}{1-\tan^2 x}\right]$$

$$= \lim_{x\to 0} \frac{2x\tan^3 x}{4\sin x^4(1-\tan^2 x)}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{x}{\sin x}.\frac{1}{\cos^3 x}.\frac{1}{1-\tan^2 x} = \frac{1}{2}.1.\frac{1}{1^3}.\frac{1}{1-0} = \frac{1}{2}$$

97. (b) $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin(\pi - \pi\sin^2 x)}{x^2}$

$[\because \sin(\pi-\theta) = \sin\theta]$

$$= \lim_{x\to 0} \frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x} \times \frac{(\pi\sin^2 x)}{x^2} = \pi$$

98. (d) Put $\theta + \frac{\pi}{4} = h$ or $\theta = -\frac{\pi}{4} + h$

$$\text{Limit} = \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{4}-h\right) - \sin\left(\frac{\pi}{4}-h\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{4}-h\right) - \cos\left(\frac{\pi}{4}+h\right)}{h}$$

$$= \lim_{h\to 0} \frac{2\sin\frac{\pi}{4}.\sin h}{h} = \sqrt{2}$$

99. (c) $\text{LHL} = \lim_{h\to 0} \frac{-h+|h|}{-h} = \lim_{h\to 0} (0) = 0$

$\text{RHL} = \lim_{h\to 0} \frac{h+|h|}{h} = 2$

LHL $\neq$ RHL $\Rightarrow$ limit does not exist

100. (d)
$$\begin{aligned} h'(x) &= 2f(x)f'(x) + 2g(x)g'(x) \\ &= 2f(x)g(x) + 2g(x)f''(x) \\ &= 2f(x)g(x) - 2f(x)g(x) \\ &= 0 \quad [\because f''(x) = -f(x)] \end{aligned}$$

$\Rightarrow \quad h(x) = c \Rightarrow h(10) = h(5) = 11$

101. (a) Given limit $= \lim_{x\to\alpha} \frac{1-\cos a(x-\alpha)(x-\beta)}{(x-\alpha)^2}$

$$= \lim_{x\to\alpha} \frac{2\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha} \frac{2}{(x-\alpha)^2} \times \frac{\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}} \times \frac{a^2(x-\alpha)^2(x-\beta)^2}{4}$$

$$= \frac{a^2(\alpha-\beta)^2}{2}.$$

102. (d) We are given that

$$\lim_{x\to 0} \frac{[(a-n)nx - \tan x]\sin nx}{x^2} = 0$$

where n is non zero real number

$$\Rightarrow \lim_{x\to 0} n.\frac{\sin nx}{nx}\left[\left\{(a-n)n - \frac{\tan x}{x}\right\}\right] = 0$$

$$\Rightarrow 1.n[(a-n)n-1] = 0 \Rightarrow a = \frac{1}{n} + n$$

103. (b) $\sin y = x\sin(a+y)$

$$\therefore x = \frac{\sin y}{\sin(a+y)}$$

Differentiating the function with respect to y

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

104. (c) Let $y = x\tan\frac{x}{2} \Rightarrow \frac{dy}{dx} = 1.\tan\frac{x}{2} + x.\sec^2\frac{x}{2}.\frac{1}{2}$

$$= \tan\frac{x}{2} + \frac{x}{2}\sec^2\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} + \frac{x}{2\cos^2\frac{x}{2}}$$

$$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2} + x}{2\cos^2\frac{x}{2}} = \frac{\sin x + x}{1+\cos x}$$

$$\Rightarrow (1+\cos x)\frac{dy}{dx} - \sin x = x$$

105. (d) $\frac{d}{dx}\left(\frac{x\sin x}{1+\cos x}\right)$

$$= \frac{(1+\cos x)(\sin x + x\cos x) - (x\sin x)(0-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\sin x(1+\cos x) + x\cos x + x(\cos^2 x + \sin^2 x)}{(1+\cos x)^2}$$

$$= \frac{(x+\sin x)(1+\cos x)}{(1+\cos x)^2} = \frac{x+\sin x}{1+\cos x}$$

106. (b) Differentiating w.r.t. x,

$$3x^2 + 3y^2\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$$

$$\Rightarrow 3(x^2-y) = 3\frac{dy}{dx}(x-y^2) \Rightarrow \frac{dy}{dx} = \frac{x^2-y}{x-y^2}$$

107. (a) $y = ax^{n+1} + bx^{-n}$

$$\frac{dy}{dx} = (n+1)ax^n - n\,bx^{-n-1}$$

$$\frac{d^2y}{dx^2} = (n+1)n\,ax^{n-1} + n(n+1)bx^{-n-2}$$

$$\therefore x^2\frac{d^2y}{dx^2} = (n+1)na.x^{n+1} + n(n+1)bx^{-n}$$

$$= n(n+1)[ax^{n+1} + bx^{-n}] = n(n+1)y$$

108. (c) We have, $y = f\left(\frac{2x-1}{x^2+1}\right)$

$$\Rightarrow \frac{dy}{dx} = f'\left(\frac{2x-1}{x^2+1}\right).\left[\frac{(x^2+1)2-(2x-1).2x}{(x^2+1)^2}\right]$$

$$= \sin\left(\frac{2x-1}{x^2+1}\right)^2.\left[\frac{2+2x-2x^2}{(x^2+1)}\right]$$

$$\left[\because f'(x) = \sin x^2, \therefore f'\left(\frac{2x-1}{x^2+1}\right) = \sin\left(\frac{2x-1}{x^2+1}\right)^2\right]$$

109. (b) We have,

$$\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x^3-3x^2+2x}\right]$$

$$= \lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x(x-1)(x-2)}\right]$$

$$= \lim_{x\to 2}\left[\frac{x(x-1)-2(2x-3)}{x(x-1)(x-2)}\right]$$

$$= \lim_{x\to 2}\left[\frac{x^2-5x+6}{x(x-1)(x-2)}\right]$$

$$= \lim_{x\to 2}\left[\frac{(x-2)(x-3)}{x(x-1)(x-2)}\right] \quad (x-2\neq 0)$$

$$= \lim_{x\to 2}\left[\frac{x-3}{x(x-1)}\right] = \frac{-1}{2}$$

110. (d) We have, $\lim_{x\to 3}\frac{x^n-3^n}{x-3} = n(3)^{n-1}$

Therefore, $n(3)^{n-1} = 108 = 4(27) = 4(3)^{4-1}$

On comparing, we get $n = 4$

111. (c) We have, $\lim_{x\to 0}\frac{\cos ax-\cos bx}{\cos cx-1}$

$$= \lim_{x\to 0}\frac{2\sin\left[\frac{(a+b)}{2}x\right]\sin\left(\frac{(a-b)x}{2}\right)}{2\sin^2\left(\frac{cx}{2}\right)}$$

$$= \lim_{x\to 0}\frac{\sin\frac{(a+b)x}{2}.\sin\frac{(a-b)x}{2}}{x^2}.\frac{x^2}{\sin^2\frac{cx}{2}}$$

$$= \lim_{x\to 0}\frac{\sin\frac{(a+b)x}{2}}{\frac{(a+b)x}{2}.\left(\frac{2}{a+b}\right)}.\frac{\sin\frac{(a-b)x}{2}}{\frac{(a-b)x}{2}.\frac{2}{a-b}}.\frac{\left(\frac{cx}{2}\right)^2\times\frac{4}{c^2}}{\sin^2\frac{cx}{2}}$$

$= \left(\frac{a+b}{2}\times\frac{a-b}{2}\times\frac{4}{c^2}\right) = \frac{a^2-b^2}{c^2}$. Hence m and n are $a^2 - b^2$ and c^2 respectively.

112. (d) Since, RHL $= \lim_{x\to 1^+}[x-1] = 0$

and LHL $= \lim_{x\to 1^-}[x-1] = -1$

Hence, at $x = 1$ limit does not exist.

113. (a) We have, $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x} = \lim_{x\to 0}\frac{\sin x\left(\frac{1}{\cos x}-1\right)}{\sin^3 x}$

$$= \lim_{x\to 0}\frac{1-\cos x}{\cos x\sin^2 x} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\left(4\sin^2\frac{x}{2}.\cos^2\frac{x}{2}\right)} = \frac{1}{2}$$

114. (b) Let $y = \frac{a}{x^4}-\frac{b}{x^2}+\cos x$

$\Rightarrow \quad y = ax^{-4} - bx^{-2} + \cos x$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = a\frac{d}{dx}(x^{-4})-b\frac{d}{dx}(x^{-2})+\frac{d}{dx}(\cos x)$$

$$= a(-4)x^{-4-1} - b(-2)x^{-2-1}(-\sin x)$$

$$= -\frac{4a}{x^5}+\frac{2b}{x^3}-\sin x \qquad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

115. (a) Let $y = \frac{\sin(x+a)}{\cos x} = \frac{\sin x\cos a+\cos x\sin a}{\cos x}$

[$\because \sin(A+B) = \sin A\cos B + \cos A\sin B$]

$$= \frac{\sin x\cos a}{\cos x}+\frac{\cos x\sin a}{\cos x} = \cos a\tan x + \sin a$$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \cos a\frac{d}{dx}(\tan x)+\frac{d}{dx}(\sin a)$$

$$= \cos a\sec^2 x + 0 = \frac{\cos a}{\cos^2 x}$$

116. (d) Let $f(x) = \frac{|x-4|}{x-4}$

At $x=4$, RHL $= \lim_{x\to 4^+}f(x) = \lim_{h\to 0}f(4+h) = \lim_{h\to 0}\frac{|4+h-4|}{(4+h-4)}$

$$= \lim_{h\to 0}\left(\frac{4+h-4}{4+h-4}\right) = 1$$

At $x = 4$, LHL $= \lim_{x\to 4^-}f(x) = \lim_{h\to 0}f(4-h)$

$$= \lim_{h\to 0}\frac{|4-h-4|}{(4-h-4)} = \lim_{h\to 0}\frac{-(4-h-4)}{(4-h-4)} = -1$$

$\because$ RHL $\neq$ LHL

$\therefore$ Hence, at $x = 4$, limit does not exist.

117. (c) Given, $f(x) = \begin{cases}\frac{k\cos x}{\pi-2x}, & x\neq\frac{\pi}{2}\\ 3, & x = \frac{\pi}{2}\end{cases}$

Since, $\lim_{x\to\frac{\pi}{2}}f(x) = f\left(\frac{\pi}{2}\right) \Rightarrow \lim_{x\to\frac{\pi}{2}}f(x) = 3$

$$\Rightarrow \lim_{h\to 0}f\left(\frac{\pi}{2}+h\right) = 3 \Rightarrow \lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)} = 3$$

$\Rightarrow \quad \lim_{h\to 0}\frac{-k\sin h}{\pi-\pi-2h}=3 \Rightarrow \lim_{h\to 0}\frac{-k\sin h}{-2h}=3$

$\Rightarrow \quad \frac{k}{2}\times\lim_{h\to 0}\frac{\sin h}{h}=3 \Rightarrow \frac{k}{2}\times 1=3$

$\Rightarrow \quad k=6 \quad \left(\because \lim_{h\to 0}\frac{\sin h}{h}=1\right)$

118. (d) We have, $f(x)=|\cos x-\sin x|$

$$\Rightarrow \quad f(x)=\begin{cases}\cos x-\sin x, \text{ for } 0<x\le\frac{\pi}{4}\\ \sin x-\cos x, \text{ for } \frac{\pi}{4}<x<\frac{\pi}{2}\end{cases}$$

Clearly, $Lf'\left(\frac{\pi}{4}\right)=\left\{\frac{d}{dx}(\cos x-\sin x)\right\}_{\text{at } x=\frac{\pi}{4}}$

$=(-\sin x-\cos x)_{x=\frac{\pi}{4}}=-\sqrt{2}$

and $Rf'\left(\frac{\pi}{4}\right)=\left\{\frac{d}{dx}(\sin x-\cos x)\right\}_{\text{at } x=\frac{\pi}{4}}$

$=(\cos x+\sin x)_{x=\frac{\pi}{4}}=\sqrt{2}$

$\because \quad Lf'\left(\frac{\pi}{4}\right)\neq Rf'\left(\frac{\pi}{4}\right)$

$\therefore \quad f'\left(\frac{\pi}{4}\right)$ doesn't exist.

119. (d) $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$ ...(i)

Putting $x=y=0$, we get $f(0)=0$

Putting $y=-x$, we get $f(x)+f(-x)=f(0)=0$

$\Rightarrow \quad f(-x)=-f(x)$ ...(ii)

Also, $\lim_{x\to 0}\frac{f(x)}{x}=2$ ...(iii)

Now, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

[using eq. (ii) $-f(x)=f(-x)$]

$$f'(x)=\lim_{h\to 0}\frac{f\left(\frac{x+h-x}{1-(x+h)(-x)}\right)}{h}$$

[using eq. (i)]

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\left[\frac{f\left(\frac{h}{1+x(x+h)}\right)}{h}\right]$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{f\left(\frac{h}{1+xh+x^2}\right)}{\left(\frac{h}{1+xh+x^2}\right)}\times\left[\frac{1}{1+xh+x^2}\right]$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{f\left(\frac{h}{1+xh+x^2}\right)}{\left(\frac{h}{1+xh+x^2}\right)}\times\lim_{h\to 0}\frac{1}{1+xh+x^2}$$

$\left[\text{using} \lim_{x\to 0}\frac{f(x)}{x}=2\right]$

$\Rightarrow \quad f'(x)=2\times\frac{1}{1+x^2}=\frac{2}{1+x^2}$

$\Rightarrow \quad f'\left(\frac{1}{\sqrt{3}}\right)=\frac{2}{1+\frac{1}{3}}=\frac{2}{4/3}=\frac{6}{4}=\frac{3}{2}$

120. (c) We first find the derivatives of $f(x)$ at $x=-1$ and at $x=0$.

We have,

$f'(-1)=\lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h}$

$=\lim_{h\to 0}\frac{[2(-1+h)^2+3(-1+h)-5]-[2(-1)^2+3(-1)-5]}{h}$

$=\lim_{h\to 0}\frac{2h^2-h}{h}=\lim_{h\to 0}(2h-1)=2(0)-1=-1$

and $f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{[2(0+h)^2+3(0+h)-5]-[2(0)^2+3(0)-5]}{h}$

$=\lim_{h\to 0}\frac{2h^2+3h}{h}=\lim_{h\to 0}(2h+3)=2(0)+3=3$

Clearly, $f'(0)=-3f'(-1)$

121. (c) Given, $f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+...+\frac{x^2}{2}+x+1$

$\Rightarrow \quad f'(x)=\frac{100x^{99}}{100}+\frac{99x^{98}}{99}+...+\frac{2x}{2}+1+0$

$[\because f(x)=x^n \Rightarrow f'(x)=nx^{n-1}]$

$\Rightarrow \quad f'(x)=x^{99}+x^{98}+...+x+1$ ...(i)

Putting $x=1$, we get

$f'(1)=\underbrace{(1)^{99}+1^{98}+...+1+1}_{100 \text{ times}}=\underbrace{1+1+1...+1+1}_{100 \text{ times}}$

$\Rightarrow \quad f'(1)=100$ ...(ii)

Again, putting $x=0$, we get

$f'(0)=0+0+...+0+1$

$\Rightarrow \quad f'(0)=1$ ...(iii)

From eqs. (ii) and (iii), we get

$f'(1)=100f'(0)$

Hence, $m=100$

122. (b) We have,

$\lim_{x\to\pi/6}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}=\lim_{x\to\pi/6}\frac{(2\sin x-1)(\sin x+1)}{(2\sin x-1)(\sin x-1)}$

$=\lim_{x\to\pi/6}\frac{\sin x+1}{\sin x-1}=\frac{\frac{1}{2}+1}{\frac{1}{2}-1}=-3$

123. (d) We have, $u=e^x\sin x$

$\Rightarrow \quad \frac{du}{dx}=e^x\sin x+e^x\cos x=u+v$

$v=e^x\cos x$

$\Rightarrow \quad \frac{dv}{dx}=e^x\cos x-e^x\sin x=v-u$

$\therefore$ Consider $v\frac{du}{dx}-u\frac{dv}{dx}=v(u+v)-u(v-u)=u^2+v^2$

$\frac{d^2u}{dx^2}=\frac{du}{dx}+\frac{dv}{dx}=u+v+v-u=2v$

and $\frac{d^2v}{dx^2}=\frac{dv}{dx}-\frac{du}{dx}=(v-u)-(v+u)=-2u$

124. (b) Given, $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}=\begin{cases}-x+1, & x<0\\ 0, & x=0\\ x-1, & x>0\end{cases}$

Let us first check the existence of limit of f (x) at x = 0.
At x = 0,

$$\text{RHL}=\lim_{x\to0^+}f(x)=\lim_{h\to0}f(0+h)=\lim_{h\to0}(0+h)-1$$
$$=\lim_{h\to0}h-1=0-1=-1$$
$$\text{LHL}=\lim_{x\to0^-}f(x)=\lim_{h\to0}f(0-h)$$
$$=\lim_{h\to0}-(0-h)+1$$
$$=\lim_{h\to0}h+1=0+1=1$$

$\Rightarrow$ RHL $\neq$ LHL
$\Rightarrow$ At x = 0, limit does not exist.
Note that for any a < 0 or a > 0, $\lim_{x\to a}f(x)$ exists,
as for a < 0, $\lim_{x\to a}f(x)=\lim_{x\to a}-x+1=-a+1$ exists and for a > 0, $\lim_{x\to a}f(x)=\lim_{x\to a}x-1=a-1$ exists. Hence, $\lim_{x\to a}f(x)$ exists for all $a\neq 0$.

125. (d) $\lim_{x\to a}\dfrac{(x+2)^{5/3}-(a+2)^{5/3}}{x-a}=\lim_{x\to a}\dfrac{(x+2)^{5/3}-(a+2)^{5/3}}{(x+2)-(a+2)}$

$$=\lim_{y\to b}\frac{y^{5/3}-b^{5/3}}{y-b},$$

where $x+2=y$, $a+2=b$. and when $x\to a, y\to b$

$$=\frac{5}{3}b^{5/3-1}=\frac{5}{3}b^{2/3}=\frac{5}{3}(a+2)^{2/3}.$$

126. (b) $\lim_{x\to0}\sqrt{\dfrac{x-\sin x}{x+\sin^2 x}}=\lim_{x\to0}\sqrt{\dfrac{1-\dfrac{\sin x}{x}}{1+\dfrac{\sin^2 x}{x}}}$

$$=\lim_{x\to0}\sqrt{\frac{1-\dfrac{\sin x}{x}}{1+\left(\dfrac{\sin x}{x}\right)\sin x}}=\sqrt{\frac{1-1}{1+1\times0}}=0$$

127. (c) $\lim_{x\to0}\dfrac{x\sin 5x}{\sin^2 4x}$

[multiply denominator and numerator with x]
We get,

$$\lim_{x\to0}\frac{x^2\sin 5x}{x\sin^2 4x}=\lim_{x\to0}\frac{\sin 5x}{x}\cdot\frac{x^2}{\sin^2 4x}$$

Rearranging to bring a standard form, we get,

$$\lim_{x\to0}\frac{5\sin 5x}{5x}\cdot\frac{(4x)^2}{16\sin^2 4x}$$
$$=\frac{5}{16}\left(\lim_{x\to0}\frac{\sin 5x}{5x}\right)\cdot\frac{1}{\lim_{x\to0}\left(\dfrac{\sin 4x}{4x}\right)^2}=\frac{5}{16}$$

128. (c) As given $\lim_{x\to0}\dfrac{a^x-x^a}{x^a-a^a}=-1$

Applying limit, we have

$$\frac{1-0}{0-a^a}=-1\quad(\because\ a^0=1)$$
$$\Rightarrow\quad\frac{1}{-a^a}=-1\Rightarrow a^a=1$$

Taking log on both the sides
$a\log a=0 \Rightarrow a=0$ or $\log a=0$
$a\neq0 \Rightarrow \log a=0 \Rightarrow a=1$

129. (a) The required limit

$$=\lim_{x\to2}\frac{[1+\sqrt{2+x}-3]}{(x-2)\left[\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right]}\ \text{(on rationalizing)}$$
$$=\lim_{x\to2}\frac{\left(\sqrt{x+2}-2\right)\left(\sqrt{x+2}+2\right)}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{x+2}+2\right)}$$
$$=\lim_{x\to2}\frac{(x+2)-4}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{x+2}+2\right)}$$
$$=\lim_{x\to2}\frac{1}{\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{x+2}+2\right)}$$
$$=\frac{1}{2\sqrt{3}}\cdot\frac{1}{4}=\frac{1}{8\sqrt{3}}$$

130. (b) $\lim_{x\to2a}\dfrac{\sqrt{x-2a}}{\sqrt{x^2-4a^2}}+\dfrac{\sqrt{x}-\sqrt{2a}}{\sqrt{x^2-4a^2}}$

$$=\lim_{x\to2a}\frac{1}{\sqrt{x+2a}}+\frac{\sqrt{x}-\sqrt{2a}}{\sqrt{(x-2a)(x+2a)}}$$
$$=\frac{1}{2\sqrt{a}}+\lim_{x\to2a}\frac{\sqrt{x}-\sqrt{2a}}{\sqrt{\left(\sqrt{x}-\sqrt{2a}\right)\left(\sqrt{x}+\sqrt{2a}\right)(x+2a)}}$$
$$=\frac{1}{2\sqrt{a}}+\lim_{x\to2a}\frac{\sqrt{\sqrt{x}-\sqrt{2a}}}{\sqrt{\left(\sqrt{x}+\sqrt{2a}\right)(x+2a)}}=\frac{1}{2\sqrt{a}}+0$$

131. (d) $\lim_{x\to\frac{\pi}{2}}\dfrac{\tan\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)\cdot(1-\sin x)}{(\pi-2x)^3}$

Let $x=\dfrac{\pi}{2}+y;\ y\to0$

$$=\lim_{y\to0}\frac{\tan\left(-\dfrac{y}{2}\right)\cdot(1-\cos y)}{(-2y)^3}=\lim_{y\to0}\frac{-\tan\dfrac{y}{2}\left(2\sin^2\dfrac{y}{2}\right)}{(-8)\cdot\dfrac{y^3}{8}\cdot8}$$
$$=\lim_{y\to0}\frac{1}{32}\frac{\tan\dfrac{y}{2}}{\left(\dfrac{y}{2}\right)}\cdot\left[\frac{\sin y/2}{y/2}\right]^2=\frac{1}{32}$$

132. (d) $\lim_{x\to 2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}=\lim_{x\to 2}\frac{\sqrt{2}\,|\sin(x-2)|}{x-2}$

$$\underset{(\text{at } x=2)}{\text{L.H.L}} = -\lim_{x\to 2}\frac{\sqrt{2}\sin(x-2)}{(x-2)}=-1$$

$$\underset{(\text{at } x=2)}{\text{R.H.L}} = \lim_{x\to 2}\frac{\sqrt{2}\sin(x-2)}{(x-2)}=1$$

Thus $\underset{(\text{at } x=2)}{\text{L.H.L}} \neq \underset{(\text{at } x=2)}{\text{R.H.L}}$

Hence, $\lim_{x\to 2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}$ does not exist.

133. (d) $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$

$\Rightarrow \lim_{x\to 5}[(f(x))^2-9]=0 \Rightarrow \lim_{x\to 5}f(x)=3$

134. (c) Consider $\lim_{x\to 0}\frac{\tan^2 x-2\tan x-3}{\tan^2 x-4\tan x+3}$

$$=\lim_{x\to 0}\frac{\tan^2 x-3\tan x+\tan x-3}{\tan^2 x-3\tan x-\tan x+3}$$

$$=\lim_{x\to 0}\frac{(\tan x+1)(\tan x-3)}{(\tan x-1)(\tan x-3)}=\lim_{x\to 0}\frac{\tan x+1}{\tan x-1}$$

Now, at $\tan x=3$, we have

$$\lim_{\tan x\to 3}\frac{\tan x+1}{\tan x-1}=\frac{3+1}{3-1}=\frac{4}{2}=2$$

135. (b) $\lim_{x\to 0}\frac{x\sqrt[3]{z^2-(z-x)^2}}{\left(\sqrt[3]{8xz-4x^2}+\sqrt[3]{8xz}\right)^4}$

$$=\lim_{x\to 0}\frac{x\sqrt[3]{2xz-x^2}}{(\sqrt[3]{x}\sqrt[3]{8z-4x}+\sqrt[3]{8z}\sqrt[3]{x})^4}$$

$$=\lim_{x\to 0}\frac{x^{4/3}\sqrt[3]{2z-x}}{x^{4/3}[\sqrt[3]{8z-4x}+\sqrt[3]{8z}]^4}$$

$$=\frac{\sqrt[3]{2z}}{\left[2\sqrt[3]{8z}\right]^4}=\frac{1}{2^{23/3}.z}$$

136. (d) $\lim_{h\to 0}\frac{2-\sqrt[3]{8+h}}{2h.\sqrt[3]{8+h}}$

$$\lim_{h\to 0}\frac{8-(8+h)}{2h.\sqrt[3]{8+h}\{8^{2/3}+8^{1/3}.(8+h)^{1/3}+(8+h)^{2/3}\}}=-\frac{1}{48}$$

137. (b) $\frac{2\cos\left(\frac{\sin x+x}{2}\right).\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$

$$=\lim_{x\to 0}2\left[\frac{\sin\left(\frac{\sin x+x}{2}\right)}{\left(\frac{\sin x+x}{2}\right)}\right]\left[\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\right]$$

$$\times\left[\frac{1}{2\left(\frac{1}{\frac{\sin x}{x}+1}\right).3\frac{x^3}{(x-\sin x)}}\right]=\frac{1}{6}$$

138. (b)

139. (c) A function f is said to be a rational function, if

$f(x)=\frac{g(x)}{h(x)}$, where g (x) and h (x) are polynomials such that h (x) $\neq$ 0. Then,

$$\lim_{x\to a}f(x)=\lim_{x\to a}\frac{g(x)}{h(x)}=\frac{\lim_{x\to a}g(x)}{\lim_{x\to a}h(x)}=\frac{g(a)}{h(a)}$$

If h(a) = 0, there are two scenarios - (i) when g (a) $\neq$ 0 and (ii) when g (a) = 0. In case I, the limit does not exist. In case II, we can write g (x) = $(x-a)^k g_1(x)$, where k is the maximum of powers of (x – a) in g (x). Similarly, h (x) = $(x-a)^l h_1(x)$ as h(a) = 0. Now, if k > l, then

$$\lim_{x\to a}f(x)=\frac{\lim_{x\to a}g(x)}{\lim_{x\to a}h(x)}=\frac{\lim_{x\to a}(x-a)^k g_1(x)}{\lim_{x\to a}(x-a)^l h_1(x)}$$

$$=\frac{\lim_{x\to a}(x-a)^{k-l}g_1(x)}{\lim_{x\to a}h_1(x)}=\frac{0.g_1(a)}{h_1(a)}=0$$

If k < l, the limit is not defined.

CHAPTER

14

MATHEMATICAL REASONING

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Which of the following is a statement?
(a) Open the door. (b) Do your home work.
(c) Switch on the fan. (d) Two plus two is four.

2. Which of the following is a statement?
(a) May you live long!
(b) May God bless you!
(c) The sun is a star.
(d) Hurrah! we have won the match.

3. Which of the following is not a statement?
(a) Please do me a favour. (b) 2 is an even integer.
(c) $2+1=3$. (d) The number 17 is prime.

4. Which of the following is not a statement?
(a) 2 is an even integer.
(b) $2+1=3$.
(c) The number 17 is prime.
(d) $x+3=10, x \in R$.

5. Which of the following is the converse of the statement?
"If Billu secure good marks, then he will get a bicycle."
(a) If Billu will not get bicycle, then he will not secure good marks.
(b) If Billu will get a bicycle, then he will secure good marks.
(c) If Billu will get a bicycle, then he will not secure good marks.
(d) If Billu will not get a bicycle, then he will secure good marks.

6. The connective in the statement :
"$2+7>9$ or $2+7<9$" is
(a) and (b) or (c) $>$ (d) $<$

7. The connective in the statement :
"Earth revolves round the Sun and Moon is a satellite of earth" is
(a) or (b) Earth (c) Sun (d) and

8. The negation of the statement
"A circle is an ellipse" is
(a) An ellipse is a circle.
(b) An ellipse is not a circle.
(c) A circle is not an ellipse.
(d) A circle is an ellipse.

9. The contrapositive of the statement
"If 7 is greater than 5, then 8 is greater than 6" is
(a) If 8 is greater than 6, then 7 is greater than 5.
(b) If 8 is not greater than 6, then 7 is greater than 5.
(c) If 8 is not greater than 6, then 7 is not greater than 5.
(d) If 8 is greater than 6, then 7 is not greater than 5.

10. The negation of the statement :
"Rajesh or Rajni lived in Bangalore" is
(a) Rajesh did not live in Bangalore or Rajni lives in Bangalore.
(b) Rajesh lives in Bangalore and Rajni did not live in Bangalore.
(c) Rajesh did not live in Bangalore and Rajni did not live in Bangalore.
(d) Rajesh did not live in Bangalore or Rajni did not live in Bangalore.

11. The statement
"If x^2 is not even, then x is not even" is converse of the statement
(a) If x^2 is odd, then x is even.
(b) If x is not even, then x^2 is not even.
(c) If x is even, then x^2 is even.
(d) If x is odd, then x^2 is even.

12. Which of the following is the conditional $p \to q$?
(a) q is sufficient for p. (b) p is necessary for q.
(c) p only if q. (d) if q, then p.

13. Which of the following statement is a conjunction?
(a) Ram and Shyam are friends.
(b) Both Ram and Shyam are tall.
(c) Both Ram and Shyam are enemies.
(d) None of the above.

14. The false statement in the following is
(a) $p \wedge (\sim p)$ is contradiction
(b) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a contradiction
(c) $\sim(\sim p) \Leftrightarrow p$ is a tautology
(d) $p \vee (\sim p) \Leftrightarrow$ is a tautology

15. $\sim(p \vee (\sim p))$ is equal to
(a) $\sim p \vee q$ (b) $(\sim p) \wedge q$
(c) $\sim p \vee \sim p$ (d) $\sim p \wedge \sim p$

16. If $(p \wedge \sim r) \Rightarrow (q \vee r)$ is false and q and r are both false, then p is
(a) True (b) False
(c) May be true or false (d) Data insufficient

17. $\sim((\sim p)\wedge q)$ is equal to
(a) $p\vee(\sim q)$ (b) $p\vee q$
(c) $p\wedge(\sim q)$ (d) $\sim p\wedge\sim q$

18. Which of the following is true?
(a) $p\Rightarrow q\equiv\sim p\Rightarrow\sim q$
(b) $\sim(p\Rightarrow\sim q)\equiv\sim p\wedge q$
(c) $\sim(\sim p\Rightarrow\sim q)\equiv\sim p\wedge q$
(d) $\sim(\sim p\Leftrightarrow q)\equiv[\sim(p\Rightarrow q)\wedge\sim(q\Rightarrow p)]$

19. Which of the following is not a statement?
(a) Please do me a favour (b) 2 is an even integer
(c) $2+1=3$ (d) The number 17 is prime

20. Which of the following is not a statement?
(a) Roses are red
(b) New Delhi is in India
(c) Every square is a rectangle
(d) Alas! I have failed

21. The inverse of the statement $(p\wedge\sim q)\rightarrow r$ is
(a) $\sim(p\vee\sim q)\rightarrow\sim r$ (b) $(\sim p\wedge q)\rightarrow\sim r$
(c) $(\sim p\vee q)\rightarrow\sim r$ (d) None of these

22. Negation of the statement $(p\wedge r)\rightarrow(r\vee q)$ is
(a) $\sim(p\wedge r)\rightarrow\sim(r\vee q)$ (b) $(\sim p\vee\sim r)\vee(r\vee q)$
(c) $(p\wedge r)\wedge(r\wedge q)$ (d) $(p\wedge r)\wedge(\sim r\wedge\sim q)$

23. The sentence "There are 35 days in a month" is
(a) a statement (b) not a statement
(c) may be statement or not (d) None of these

24. Which of the following is a statement?
(a) Everyone in this room is bold
(b) She is an engineering student
(c) $\sin^2\theta$ is greater than 1/2
(d) Three plus three equals six

25. The sentence "New Delhi is in India", is
(a) a statement (b) not a statement
(c) may be statement or not (d) None of the above

26. The negation of the statement "$\sqrt{2}$ is not a complex number" is
(a) $\sqrt{2}$ is a rational number
(b) $\sqrt{2}$ is an irrational number
(c) $\sqrt{2}$ is a complex number
(d) None of the above

27. Which of the following is/are connectives?
(a) Today (b) Yesterday
(c) Tomorrow (d) "And", "or"

28. The contrapositive of the statement "If p, then q", is
(a) If q, then p (b) If p, then ~ q
(c) If ~ q, then ~ p (d) If ~ p, then ~ q

29. The contrapositive of the statement, ' If I do not secure good marks then I cannot go for engineering', is
(a) If I secure good marks, then I go for engineering.
(b) If I go for engineering then I secure good marks.
(c) If I cannot go for engineering then I donot secure good marks.
(d) None.

30. Which of the following is not a statement?
(a) Every set is a finite set
(b) 8 is less than 6
(c) Where are you going?
(d) The sum of interior angles of a triangle is 180 degrees

31. If $p\Rightarrow(\sim p\vee q)$ is false, the truth values of p and q are respectively
(a) F, T (b) F, F (c) T, T (d) T, F

32. Which of the following statement is a conjunction?
(a) Ram and Shyam are friends.
(b) Both Ram and Shyam are tall.
(c) Both Ram and Shyam are enemies.
(d) None of the above.

33. $p\Rightarrow q$ can also be written as
(a) $p\Rightarrow\sim q$ (b) $\sim p\vee q$
(c) $\sim q\Rightarrow\sim p$ (d) None of these

34. Which of the following is an open statement?
(a) Good morning to all (b) Please do me a favour
(c) Give me a glass of water (d) x is a natural number

35. If p, q, r are statement with truth vales F, T, F respectively, then the truth value of $p\rightarrow(q\rightarrow r)$ is
(a) false (b) true
(c) true if p is true (d) none

36. If $p\Rightarrow(q\vee r)$ is false, then the truth values of p, q, r are respectively
(a) T, F, F (b) F, F, F (c) F, T, T (d) T, T, F

37. A compound statement p or q is false only when
(a) p is false
(b) q is false
(c) both p and q are false
(d) depends on p and q

38. A compound statement p and q is true only when
(a) p is true (b) q is true
(c) both p and q are true (d) none of p and q is true

39. A compound statement $p\rightarrow q$ is false only when
(a) p is true and q is false
(b) p is false but q is true
(c) at least one of p or q is false
(d) both p and q are false

40. If p : Pappu passes the exam,
q : Papa will give him a bicycle.
Then, the statement 'Pappu passing the exam, implies that his papa will give him a bicycle' can be symbolically written as
(a) $p\rightarrow q$ (b) $p\leftrightarrow q$ (c) $p\wedge q$ (d) $p\vee q$

41. If Ram secures 100 marks in maths, then he will get a mobile. The converse is
(a) If Ram gets a mobile, then he will not secure 100 marks
(b) If Ram does not get a mobile, then he will secure 100 marks
(c) If Ram will get a mobile, then he secures 100 marks in maths
(d) None of these

42. In mathematical language, the reasoning is of ———— types.
(a) one (b) two (c) three (d) four

43. "Paris is in England" is a ________
(a) statement (b) sentence
(c) both 'a' and 'b' (d) neither 'a' nor 'b'

44. "The sun is a star" is a __________
(a) statement (b) sentence
(c) both 'a' and 'b' (d) neither 'a' nor 'b'

45. The negation of a statement is said to be a __________
(a) statement (b) sentence
(c) negation (d) ambiguous

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

46. Consider the following statements
Statement-I: The negation of the statement "The number 2 is greater than 7" is "The number 2 is not greater than 7".
Statement-II: The negation of the statement "Every natural number is an integer" is "every natural number is not an integer".
Choose the correct option.
(a) Only Statement I is true (b) Only Statement II is true
(c) Both Statement are true (d) Both Statement are false

47. Consider the following statements
Statement- I: The words "And" and "or" are connectives.
Statement-II: "There exists" and "For all" are called quantifiers.
(a) Only Statement I is true (b) Only Statement II is true
(c) Both Statement are true (d) Both Statement are false

48. Consider the following statements.
I. $x + y = y + x$ is true for every real numbers x and y.
II. There exists real numbers x and y for which $x + y = y + x$.
Choose the correct option.
(a) I and II are the negation of each other.
(b) I and II are not the negation of each other.
(c) I and II are the contrapositive of each other.
(d) I is the converse of II.

49. Read the following statements.
Statement: If x is a prime number, then x is odd.
I. **Contrapositive form :** If a number x is not odd, then x is not a prime number.
II. **Converse form :** If a number x is odd, then it is a prime number.
Choose the correct option.
(a) Both I and II are true. (b) Only I is true.
(c) Only II is true. (d) Neither I nor II true.

50. Consider the following statements.
I. A sentence is called a statement, if it is either true or false.
II. The sentence, "Today is a windy day", is not a statement.
Choose the correct option.
(a) Both I and II are true. (b) Only I is true.
(c) Only II is true. (d) Both I and II are false.

51. Consider the following statements
I. "Every rectangle is a square" is a statement.
II. "Close the door" is not a statement.
Choose the correct option.
(a) Only I is false. (b) Only II is false.
(c) Both are true. (d) Both are false.

52. Consider the following statements.
I. If a number is divisible by 10, then it is divisible by 5.
II. If a number is divisible by 5, then it is divisible by 10.
Choose the correct option.
(a) I is converse of II.
(b) II is converse of I.
(c) I is not converse of II.
(d) Both 'a' and 'b' are true.

53. Consider the following sentences.
I. She is a Mathematics graduate.
II. There are 40 days in a month.
Choose the correct option.
(a) Only I is a statement.
(b) Only II is a statement.
(c) Both are the statements.
(d) Neither I nor II is statement.

54. Consider the following sentences.
I. "Two plus three is five" is not a statement.
II. "Every square is a rectangle." is a statement.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

55. Consider the following
I. New Delhi is in Nepal.
II. Every relation is a function.
III. Do your homework.
Choose the correct option.
(a) I and II are statements.
(b) I and III are statements.
(c) II and III are statements.
(d) I, II and III are statements.

56. Consider the following sentences.
I. "Two plus two equals four" is a true statement.
II. "The sum of two positive numbers is positive" is a true statement.
III. "All prime numbers are odd numbers" is a true statement.
Choose the correct option.
(a) Only I is false.
(b) Only II is false.
(c) All I, II and III are false
(d) Only III is false.

57. The component statements of the statement "The sky is blue or the grass is green" are
I. p : The sky is blue.
q : The sky is not blue.
II. p : The sky is blue.
q : The grass is green.
Choose the correct option.
(a) I and II are component statements.
(b) Only I is component statement.
(c) Only II is component statement.
(d) Neither I nor II is component statement.

58. Consider the following statements.
I. If a statement is always true, then the statement is called "tautology".
II. If a statement is always false, then the statement is called "contradiction".

Choose the correct option.
(a) Both the statements are false.
(b) Only I is false.
(c) Only II is false.
(d) Both the statements are true.

59. If $p \to q$ is a conditional statement, then its
I. Converse : $q \to p$
II. Contrapositive : $\sim q \to \sim p$
III. Inverse : $\sim p \to \sim q$
Choose the correct option.
(a) Only I and II are true.
(b) Only II and III are true.
(c) Only I and III are true.
(d) All I, II and III are true.

60. Consider the following statement.
"If a triangle is equiangular, then it is an obtuse angled triangle."
This is equivalent to
I. a triangle is equiangular implies that it is an obtuse angled triangle.
II. for a triangle to be obtuse angled triangle it is sufficient that it is equiangular.
Choose the correct option.
(a) Both are correct. (b) Both are incorrect.
(c) Only I is correct. (d) Only II is correct.

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

61. **Assertion:** The compound statement with 'And' is true if all its component statements are true.
Reason: The compound statement with 'And' is false if any of its component statements is false.

62. **Assertion:** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Reason: $\sim(p \leftrightarrow \sim q)$ is a tautology

63. **Assertion:** "Mathematics is difficult", is a statement.
Reason: A sentence is a statement, if it is either true or false but not both.

64. **Assertion:** The sentence "8 is less than 6" is a statement.
Reason: A sentence is called a statement, if it is either true or false but not both.

65. **Assertion:** $\sim(p \vee q) \equiv \sim p \wedge \sim q$
Reason: $\sim(p \wedge q) \equiv \sim p \vee \sim q$

66. **Assertion:** $\sim(p \to q) \equiv p \wedge \sim q$
Reason: $\sim(p \leftrightarrow q) \equiv (p \vee \sim q) \wedge (q \wedge \sim p)$

67. **Assertion:** The contrapositive of $(p \vee q) \to r$ is $\sim r \to \sim p \wedge \sim q$.
Reason: If $(p \wedge \sim q) \to (\sim p \vee r)$ is a false statement, then respective truth values of p, q and r are F, T, T.

68. **Assertion:** If $p \to (\sim p \vee q)$ is false, the truth values of p and q are respectively F, T.
Reason: The negation of $p \to (\sim p \vee q)$ is $p \wedge \sim q$.

69. **Assertion :** The negation of $(p \vee \sim q) \wedge q$ is $(\sim p \wedge q) \vee \sim q$.
Reason : $\sim(p \to q) \equiv p \wedge \sim q$

70. **Assertion :** The denial of a statement is called negation of the statement.
Reason : A compound statement is a statement which can not be broken down into two or more statements.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

71. Which of the following is the conditional $p \to q$?
(a) q is sufficient for p (b) p is necessary for q
(c) p only if q (d) if q, then p

72. The converse of the statement "If $x > y$, then $x + a > y + a$" is
(a) If $x < y$, then $x + a < y + a$ (b) If $x + a > y + a$, then $x > y$
(c) If $x < y$, then $x + a > y + a$ (d) If $x > y$, then $x + a < y + a$

73. The statement "If x^2 is not even, then x is not even" is converse of the statement
(a) If x^2 is odd, then x is even
(b) If x is not even, then x^2 is not even
(c) If x is even, then x^2 is even
(d) If x is odd, then x^2 is even

74. The statement p: For any real numbers x, y if $x = y$, then $2x + a = 2y + a$ when $a \in Z$.
(a) is true
(b) is false
(c) its contrapositive is not true
(d) None of these

75. Which of the following is a statement?
(a) x is a real number (b) Switch off the fan
(c) 6 is a natural number (d) Let me go

76. Which of the following statement is false?
(a) A quadratic equation has always a real root
(b) The number of ways of seating 2 persons in two chairs out of n persons is P(n, 2)
(c) The cube roots of unity are in GP
(d) None of the above

77. The negation of the statement "A circle is an ellipse" is
(a) an ellipse is a circle (b) an ellipse is not a circle
(c) a circle is not an ellipse (d) a circle is an ellipse

78. Which of the following is not a negation of the statement "A natural number is greater than zero".
(a) A natural number is not greater than zero.
(b) It is false that a natural number is greater than zero.
(c) It is false that a natural number is not greater than zero.
(d) None of the above

79. For the statement "17 is a real number or a positive integer", the "or" is
(a) inclusive (b) exclusive
(c) Only (a) (d) None of these

80. The contrapositive of statement "If Chandigarh is capital of Punjab, then Chandigarh is in India" is
(a) "If Chandigarh is not in India, then Chandigarh is not the capital of Punjab".
(b) "If Chandigarh is in India, then Chandigarh is capital of Punjab".
(c) "If Chandigarh is not capital of Punjab, then Chandigarh is not the capital of India".
(d) "If Chandigarh is capital of Punjab, then Chandigarh is not in India".

81. Let p : I am brave,
q : I will climb the Mount Everest.
The symbolic form of a statement,
'I am neither brave nor I will climb the mount Everest' is
(a) $p \wedge q$ (b) $\sim(p \wedge q)$ (c) $\sim p \wedge \sim q$ (d) $\sim p \wedge q$

82. Let p : A quadrilateral is a parallelogram
q : The opposite side are parallel
Then the compound proposition
'A quadrilateral is a parallelogram if and only if the opposite sides are parallel' is represented by
(a) $p \vee q$ (b) $p \rightarrow q$ (c) $p \wedge q$ (d) $p \leftrightarrow q$

83. Let p: Kiran passed the examination,
q: Kiran is sad
The symbolic form of a statement "It is not true that Kiran passed therefore he is said" is
(a) $(\sim p \rightarrow q)$ (b) $(p \rightarrow q)$
(c) $\sim(p \rightarrow \sim q)$ (d) $\sim(p \leftrightarrow q)$

84. Which of the following is true?
(a) $p \Rightarrow q \equiv \sim p \Rightarrow \sim q$
(b) $\sim(p \Rightarrow \sim q) \equiv \sim p \wedge q$
(c) $\sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$
(d) $\sim(\sim p \Leftrightarrow q) \equiv [\sim(p \Rightarrow q) \wedge \sim(q \Rightarrow p)]$

85. If p and q are true statement and r, s are false statements, then the truth value of $\sim[(p \wedge \sim r) \vee (\sim q \vee s)]$ is
(a) true (b) false
(c) false if p is true (d) none

86. Consider the following statements
***P*:** Suman is brilliant
***Q*:** Suman is rich
***R*:** Suman is honest
The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as
(a) $\sim(Q \leftrightarrow (P \wedge \sim R))$ (b) $\sim Q \leftrightarrow \sim P \wedge R$
(c) $\sim(P \wedge \sim R) \leftrightarrow Q$ (d) $\sim P \wedge (Q \leftrightarrow \sim R)$

87. Let S be a non-empty subset of R. Consider the following statement :
P : There is a rational number $x \in S$ such that $x > 0$.
Which of the following statement is the negation of the statement P ?
(a) There is no rational number $x \in S$ such than $x \leq 0$.
(b) Every rational number $x \in S$ satisfies $x \leq 0$.
(c) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational.
(d) There is a rational number $x \in S$ such that $x \leq 0$.

88. The false statement in the following is
(a) $p \wedge (\sim p)$ is contradiction
(b) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a contradiction
(c) $\sim(\sim p) \Leftrightarrow p$ is a tautology
(d) $p \vee (\sim p) \Leftrightarrow$ is a tautology

89. The propositions $(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$ is
(a) Tautology and contradiction
(b) Neither tautology nor contradiction
(c) Contradiction
(d) Tautology

90. Truth value of the statement 'It is false that 3 + 3 = 33 or 1 + 2 = 12' is
(a) T (b) F
(c) both T and F (d) 54

91. If $\sim q \vee p$ is F, then which of the following is correct?
(a) $p \leftrightarrow q$ is T (b) $p \rightarrow q$ is T
(c) $q \rightarrow p$ is T (d) $p \rightarrow q$ is F

92. If p, q are true and r is false statement, then which of the following is true statement?
(a) $(p \wedge q) \vee r$ is F
(b) $(p \wedge q) \rightarrow r$ is T
(c) $(p \vee q) \wedge (p \vee r)$ is T
(d) $(p \rightarrow q) \leftrightarrow (p \rightarrow r)$ is T

93. Which of the following is true?
(a) $p \wedge \sim p \equiv T$ (b) $p \vee \sim p \equiv F$
(c) $p \rightarrow q \equiv q \rightarrow p$ (d) $p \rightarrow q \equiv (\sim q) \rightarrow (\sim p)$

94. Consider the following statements
p : x, y ∈ Z such that x and y are odd.
q : xy is odd. Then,
(a) $p \Rightarrow q$ is true (b) $\sim q \Rightarrow p$ is true
(c) Both (a) and (b) (d) None of these

95. Consider the following statements
p : A tumbler is half empty.
q : A tumbler is half full.
Then, the combination form of "p if and only if q" is
(a) a tumbler is half empty and half full
(b) a tumbler is half empty if and only if it is half full
(c) Both (a) and (b)
(d) None of the above

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (d) 'Two plus two is four', is a statement.
2. (c) "The sun is a star" is a statement.
3. (a) "Please do me a favour" is not a statement.
4. (d) $x + 3 = 10, x \in R$ is not a statement.
5. (b) Since the statement $q \to p$ is the converse of the statement $p \to q$.
6. (b) Connective word is 'or'.
7. (d) Connective word is 'and'.
8. (c) Negation : A circle is not an ellipse.
9. (c) If 8 is not greater than 6, then 7 is not greater than 5.
10. (c) Rajesh did not live in Bangalore and Rajni did not live in Bangalore.
11. (b) If x is not even, then x^2 is not even.
12. (c) p only if q.
13. (d)
14. (b) $p \Rightarrow q$ is logically equivalent to $\sim q \Rightarrow \sim p$
 $\therefore$ $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tautology but not a contradiction.
15. (b) $\sim (p \vee (\sim q)) \equiv \sim p \wedge \sim (\sim q) \equiv (\sim p) \wedge q$
16. (a) Given result means $p \wedge \sim r$ is true, $q \vee r$ is false.
17. (a) $\sim ((\sim p) \wedge q) \equiv \sim (\sim p) \vee \sim q \equiv p \vee (\sim q)$
18. (c) $\sim (p \Rightarrow q) \equiv p \wedge \sim q$
 $\therefore$ $\sim (\sim p \Rightarrow \sim q) \equiv \sim p \wedge \sim (\sim q) \equiv \sim p \wedge q$
 Thus $\sim (\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$
19. (a) "Please do me a favour" is not a statement.
20. (d) "Alas! I have failed" is not a statement.
21. (c) The inverse of the proposition $(p \wedge \sim q) \to r$ is
 $\sim (p \wedge \sim q) \to \sim r$
 $\equiv \sim p \vee \sim (\sim q) \to \sim r$
 $\equiv \sim p \vee q \to \sim r$
22. (d) We know that $\sim (p \to q) \equiv p \wedge \sim q$
 $\therefore \sim ((p \wedge r) \to (r \vee q)) \equiv (p \wedge r) \wedge [\sim (r \vee q)]$
 $\equiv (p \wedge r) \wedge (\sim r \wedge \sim q)$
23. (a) We know that a month has 30 or 31 days. It is false to say that a month has 35 days. Hence, it is a statement.
24. (d) (a) Everyone in this room is bold. This is not a statement because from the context, it is not clear which room is referred here and the term 'bold' is not clearly defined.
 (b) She is an engineering student. This is also not a statement because it is not clear, who she is.
 (c) $\sin^2\theta$ is greater than 1/2. This is not a statement because we cannot say whether the sentence is true or not.
 (d) We know that, 3 + 3 = 6. It is true. Hence, the sentence is a statement.
25. (a) "New Delhi is in India" is true. So, it is a statement.
26. (c) In negative statement, if word not is not given in the statement, then insert word 'not' in the statement. If word 'not' is given in the statement, then remove word 'not' from the statement.
 $\therefore$ The negation of the given statement is "$\sqrt{2}$ is a complex number".
27. (d) Some of the connecting words which are found in compound statement like "And", "or", etc, are often used in mathematical statements. These are called connectives.
28. (c) Contrapositive statement is
 "If $\sim q$, then $\sim p$."
29. (b)
30. (c) "Where are you going?" is not a statement.
31. (d) $p \Rightarrow (\sim p \vee q)$ is false means p is true and $\sim p \vee q$ is false.
 $\Rightarrow$ p is true and both ~p and q are false
 $\Rightarrow$ p is true and q is false.
32. (d)
33. (b) $p \Rightarrow q \equiv \sim p \vee q$
34. (d) 35. (b)
36. (a) $p \Rightarrow q$ is false only when p is true and q is false.
 $\therefore$ $p \Rightarrow q$ is false when p is true and $q \vee r$ is false, and $q \vee r$ is false when both q and r are false.
37. (c) It is a property.
38. (c) It is a property.
39. (a)
40. (a) Let p : Pappu pass the exam
 q : Papa will give him a bicycle.
 $\therefore$ Symbolic form is $p \to q$.
41. (c) Let p : Ram secures 100 marks in maths
 q : Ram will get a mobile
 Converse of $p \to q$ is $q \to p$
 i.e. If Ram will get a mobile, then he secures 100 marks in maths.
42. (b) In mathematical language, the reasoning is of two types.
43. (a) "Paris is in England" is a statement.
44. (c)
45. (a)

STATEMENT TYPE QUESTIONS

46. (c) I. The given statement is "The number 2 is greater than 7". Its negation is "The number 2 is not greater than 7".
 II. The given statement is "Every natural number is an integer". Its negation is "Every natural number is not an integer".
47. (c) The words "And" and "or" are called connectives and "There exists" and "For all" are called quantifiers.
48. (b) Statement I and II are not the negation of each other.

49. **(a)** Both contrapositive and converse statements are true.

50. **(a)** Today is a windy day. It is not clear that about which day it is said. Thus, it cannot be concluded whether it is true or false. Hence, it is not a statement.

51. **(c)** Both the statements I and II are true. "Every rectangle is a square" is false. So, it is a statement. "Close the door" is an order. So, it is not a statement.

52. **(d)** I and II are converse of each other.

53. **(b)** Only II is a statement.

54. **(a)** "Two plus three is five" is not a statement.

55. **(a)** Only I and II are statements.

56. **(d)** Statement I and II are correct but III is not correct.

57. **(c)** Only II is component statement.

58. **(d)** By definition, I and II both are true.

59. **(d)** All the statements are true.

60. **(a)** Given statement is equivalent to I and II both.

ASSERTION - REASON TYPE QUESTIONS

61. **(b)** Consider the following compound statements

p : A point occupies a position and its location can be determined.

The statement can be broken into two component statements as

q : A point occupies a position.

r : Its location can be determined.

Here, we observe that both statements are true.

Let us look at another statement.

p : 42 is divisible by 5, 6 and 7.

This statement has following component statements

q : 42 is divisible by 5.

r : 42 is divisible by 6.

s : 42 is divisible by 7.

Here, we know that the first is false, while the other two are true.

∴ p is false in this case.

Thus we can conclude that

1. The compound statement with 'and' is true, if all its component statements are true.
2. The compound statement with 'and' is false, if any of its component statement is false (this includes the case that some of its component statements are false or all of its component statements are false.)

62. **(c)** The truth table for the logical statements, involved in statement 1, is as follows :

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

We observe the columns for $\sim (p \leftrightarrow \sim q)$ and $p \leftrightarrow q$ are identical, therefore

$\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

But $\sim (p \leftrightarrow \sim q)$ is not a tautology as all entries in its column are not T.

∴ Statement-1 is true but statement-2 is false.

63. **(d)** Reason is correct but Assertion is not correct.

64. **(a)** Both are correct. Reason is correct explanation. We know that 8 is greater than 6.

65. **(b)** Assertion and Reason, both are correct but Reason is not the correct explanation for the Reason.

66. **(c)** Assertion is correct but Reason is not correct.

Reason: $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

67. **(c)** Assertion is correct. Reason is incorrect.

Reason : Truth values are T, F, F.

68. **(d)** Assertion is incorrect. Reason is correct.

69. **(b)** Both are correct but Reason is not the correct explanation for the Assertion.

70. **(c)** Assertion is correct but Reason is incorrect.

"A compound statement is a statement which is made up of two or more simple statements."

CRITICALTHINKING TYPE QUESTIONS

71. **(c)** $p \to q$ is the same as "p only if q".

72. **(b)** Converse statement is

"If $x + a > y + a$, then $x > y$".

73. **(b)** Converse statement is

"If x is not even, then x^2 is not even".

74. **(a)** We prove the statement p is true by contrapositive method and by direct method.

Direct method: For any real number x and y,

$x = y$

$\Rightarrow \quad 2x = 2y$

$\Rightarrow \quad 2x + a = 2y + a$ for some $a \in Z$

Contrapositive method: The contrapositive statement of p is "For any real numbers x, y if $2x + a \neq 2y + a$, where $a \in Z$, then $x \neq y$."

Given, $2x + a \neq 2y + a$

$\Rightarrow \quad 2x \neq 2y$

$\Rightarrow \quad x \neq y$

Hence, the given statement is true.

75. **(c)** (a) "x is a real number" is an open statement. So, this is not a statement.

(b) "Switch off the fan" is not a statement, it is an order.

(c) "6 is a natural number" is a true sentence. So, it is a statement

(d) "Let me go!" (optative sentence). So, it is not a statement.

76. **(a)** (a) It is false. (b) It is true. (c) It is true.

77. **(c)** The negation of statement "A circle is an ellipse" is "A circle is not an ellipse".

78. **(c)** The negation of given statement can be

(i) A natural number is not greater than zero.

(ii) It is false that a natural number is greater than zero.

∴ "It is false that a natural number is not greater than zero" is not a negation of the given statement.

79. **(a)** Inclusive "or". 17 is a real number or a positive integer or both.

80. (a) The contrapositive statement is "If Chandigarh is not in India, then Chandigarh is not the capital of Punjab".

81. (c) **82. (d)** **83. (b)**

84. (c) $\sim(p \Rightarrow q) \equiv p \wedge \sim q$

$\therefore \sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge \sim(\sim q) \equiv \sim p \wedge q$

Thus $\sim(\sim p \Rightarrow \sim q) \equiv \sim p \wedge q$

85. (b)

86. (a) Suman is brilliant and dishonest if and only if Suman is rich is expressed as

$Q \leftrightarrow (P \wedge \sim R)$

Negation of it will be $\sim(Q \leftrightarrow (P \wedge \sim R))$

87. (b) P : there is a rational number $x \in S$ such that $x > 0$

$\sim$P : Every rational number $x \in S$ satisfies $x \le 0$

88. (b) $p \Rightarrow q$ is logically equivalent to $\sim q \Rightarrow \sim p$

$\therefore (p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tautology but not a contradiction.

89. (c)

p	$\sim p$	$p \Rightarrow \sim p$	$\sim p \Rightarrow p$	$(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$
T	F	F	T	F
F	T	T	F	F

90. (a) p : 3 + 3 = 33, q : 1 + 2 = 12

Truth values of both p and q is F.

$\therefore$ $\sim(F \vee F) \equiv \sim F \equiv T$

91. (b)

p	q	$\sim q$	$\sim q \vee p$	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$
T	T	F	T	T	T	T
T	F	T	T	F	F	T
F	T	F	F	F	T	F
F	F	T	T	T	T	T

Alternate Method:

$\sim q \vee p$: F

$\therefore$ $\sim q$ is F, p is F

i.e. q is T, p is F

$\therefore$ $p \rightarrow q \equiv F \rightarrow T \equiv T$

92. (c) $(p \vee q) \wedge (p \vee r)$

$\equiv (T \vee T) \wedge (T \vee F)$

$\equiv T \wedge T$

$\equiv T$

93. (d) $(\sim q) \rightarrow (\sim p)$ is contrapositive of $p \rightarrow q$ and both convey the same meaning.

94. (a) Let p : x, y $\in$ Z such that x and y are odd.

q : xy is odd.

To check the validity of the given statement, assume that if p is true, then q is true.

p is true means that x and y are odd integers. Then,

x = 2m + 1, for some integer m.

y = 2n + 1, for some integer n.

Thus, xy = (2m + 1) (2n + 1)

= 2(2mn + m + n) + 1

This shows that xy is odd. Therefore, the given statement is true.

Also, if we assume that q is not true. This implies that we need to consider the negation of the statement q. This gives the statement.

$\sim q$: product xy is even.

This is possible only, if either x or y is even. This shows that p is not true. Thus, we have shown that $\sim q \Rightarrow \sim p$

Note: The above problem illustrates that to prove $p \Rightarrow q$. It is enough to show $\sim q \Rightarrow \sim p$ which is the contrapositive of the statement $p \Rightarrow q$.

95. (b) The given statements are

p : A tumbler is half empty.

q : A tumbler is half full.

We know that, if the first statement happens, then the second happens and also if the second happens, then the first happens. We can express this fact as

If a tumbler is half empty, then it is half full.

If a tumbler is half full, then it is half empty.

We combine these two statements and get the following. A tumbler is half empty, if and only if it is half full.

CHAPTER 15

STATISTICS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The measure of dispersion is:
(a) mean deviation (b) standard deviation
(c) quartile deviation (d) all (a) (b) and (c)

2. The observation which occur most frequently is known as :
(a) mode (b) median
(c) weighted mean (d) mean

3. The reciprocal of the mean of the reciprocals of n observation is the :
(a) geometric mean (b) median
(c) harmonic mean (d) average

4. The median of 18, 35, 10, 42, 21 is
(a) 20 (b) 19 (c) 21 (d) 22

5. While dividing each entry in a data by a non-zero number *a*, the arithmetic mean of the new data:
(a) is multiplied by *a* (b) does not change
(c) is divided by *a* (d) is diminished by *a*

6. The mode of the following series 3, 4, 2, 1, 7, 6, 7, 6, 8, 6, 5 is
(a) 5 (b) 6
(c) 7 (d) 8

7. The coefficient of variation is computed by:
(a) $\frac{\text{mean}}{\text{standard deviation}}$ (b) $\frac{\text{standard deviation}}{\text{mean}}$
(c) $\frac{\text{mean}}{\text{standard deviation}} \times 100$ (d) $\frac{\text{standard deviation}}{\text{mean}} \times 100$

8. If you want to measure the intelligence of a group of students,which one of the following measures will be more suitable?
(a) Arithmetic mean (b) Mode
(c) Median (d) Geometric mean

9. In computing a measure of the central tendency for any set of 51 numbers, which one of the following measures is well-defined but uses only very few of the numbers of the set?
(a) Arithmetic mean (b) Geometric mean
(c) Median (d) Mode

10. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is
(a) 6 (b) 7 (c) 8 (d) 10

11. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b ?
(a) $a = 0, b = 7$ (b) $a = 5, b = 2$
(c) $a = 1, b = 6$ (d) $a = 3, b = 4$

12. Find the mean deviation about the mean for the data
4, 7, 8, 9, 10, 12, 13, 17
(a) 3 (b) 24 (c) 10 (d) 8

13. Find the mean deviation about the mean for the data.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

(a) 6 (b) 7.3 (c) 8 (d) 6.32

14. Find the mean and variance for the following data
6, 7, 10, 12, 13, 4, 8, 12
(a) mean = 9, variance = 9.25
(b) mean = 3, variance = 7.5
(c) mean = 7, variance = 12
(d) mean = 9, variance = 12.5

15. The method used in Statistics to find a representative value for the given data is called
(a) measure of skewness
(b) measure of central tendency
(c) measure of dispersion
(d) None of the above

16. The value which represents the measure of central tendency, is/are
(a) mean (b) median (c) mode (d) All of these

17. The number which indicates variability of data or observations, is called
(a) measure of central tendency
(b) mean
(c) median
(d) measure of dispersion

18. Which of the following is/are used for the measures of dispersion?
(a) Range (b) Quartile deviation
(c) Standard deviation (d) All of these

19. We can grouped data into .... ways.
(a) three (b) four (c) two (d) None of these

20. When tested, the lives (in hours) of 5 bulbs were noted as follows
1357, 1090, 1666, 1494, 1623
The mean deviations (in hours) from their mean is
(a) 178 (b) 179 (c) 220 (d) 356

21. Number which is mean of the squares of deviations from mean, is called ....
(a) standard deviation (b) variance
(c) median (d) None of these

22. The variance of n observations $x_1, x_2, ..., x_n$ is given by
(a) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$ (b) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$
(c) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + \bar{x})$ (d) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + \bar{x})^2$

23. The measure of variability which is independent of units, is called
(a) mean deviation (b) variance
(c) standard deviation (d) coefficient of variation

24. If $x_1, x_2, ..., x_n$ are n values of a variable X and $y_1, y_2, ..., y_n$ are n values of variable Y such that $y_i = ax_i + b$; i = 1, 2, ..., n, then write Var(Y) in terms of Var(X).
(a) var (Y) = var (X) (b) var (Y) = a var (X)
(c) var (Y) = a^2 var (X) (d) var (X) = a^2 var (Y)

25. If X and Y are two variates connected by the relation $Y = \frac{aX+b}{c}$ and Var (X) = σ^2, then write the expression for the standard deviation of Y.
(a) $\left|\frac{a}{c}\right|$ (b) $\left|\frac{a}{c}\right|\sigma$ (c) $|a \cdot c|$ (d) $|a \cdot c|\sigma$

26. Variance of the numbers 3, 7, 10,18, 22 is equal to
(a) 12 (b) 6.4 (c) $\sqrt{49.2}$ (d) 49.2

27. The mean deviation from the mean of the following data :

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	8	15	16	6

is
(a) 10 (b) 10.22 (c) 9.86 (d) 9.44

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

28. Consider the following data which represents the runs scored by two batsmen in their last ten matches as
Batsman A : 30, 91, 0, 64, 42, 80, 30, 5, 117, 71
Batsman B : 53, 46, 48, 50, 53, 53, 58, 60, 57, 52
Which of the following is/are true about the data?
I. Mean of batsman A runs is 53.
II. Median of batsman A runs is 42.
III. Mean of batsman B runs is 53.
IV. Median of batsman B runs is 53.
(a) Only I is true (b) I and III are true
(c) I, III and IV are true (d) All are true

29. Which of the following is/are true about the range of the data?
I. It helps to find the variability in the observations on the basis of maximum and minimum value of observations.
II. Range of series = Minimum value – Maximum value.
III. It tells us about the dispersion of the data from a measure of central tendency.
(a) Only I is true (b) II and III are true
(c) I and II are true (d) All are true

30. **Statement I :** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3.5.
Statement II : The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

31. Consider the following data

Size	20	21	22	23	24
Frequency	6	4	5	1	4

I. Mean of the data is 22.65.
II. Mean deviation of the data is 1.25.
III. Mean of the data is 21.65.
IV. Mean deviation of the data is 2.25.
(a) I and II are true (b) II and III are true
(c) I and IV are true (d) III and IV are true

32. Consider the following data

Marks obtained	10	11	12	14	15
Number of students	2	3	8	3	4

I. Median of the data is 11.
II. Median of the data is 12.
III. Mean deviation about the median is 2.25.
IV. Mean deviation about the median is 1.25.
(a) I and III are true (b) I and IV are true
(c) II and III are true (d) II and IV are true

33. Consider the following data
6, 8, 10, 12, 14, 16, 18, 20, 22, 24
I. The variance of the data is 33.
II. The standard deviation of the data is 4.74.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

34. **Statement-I :** The mean and variance for first n natural numbers are $\frac{n+1}{2}$ and $\frac{n^2+1}{12}$, respectively.
Statement-II : The mean and variance for first 10 positive multiples of 3 are 16.5 and 74.25. respectively.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

35. Consider the following frequency distribution

Class	0–10	10–20	20–30	30–40	40–50
Frequency	5	8	15	16	6

I. Mean of the data is 27.
II. Mean of the data is 32.
III. Variance of the data is 132
IV. Variance of the data is 164
(a) II and IV are true (b) I and IV are true
(c) II and III are true (d) I and III are true

36. **Statement-I:** The series having greater CV is said to be less variable than the other.
Statement-II: The series having lesser CV is said to be more consistent than the other.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

37. If $\bar{x}_1$ and σ_1 are the mean and standard deviation of the first distribution and $\bar{x}_2$ and σ_2 are the mean and standard deviation of the second distribution.
I. CV (1st distribution) = $\frac{\sigma_1}{\bar{x}_1}\times 100$
II. CV (2nd distribution) = $\frac{\bar{x}_2}{\sigma_2}\times 100$
III. For $\bar{x}_1 = \bar{x}_2$, the series with lesser value of standard deviation is said to be more variable than the other.
IV. For $\bar{x}_1 = \bar{x}_2$, the series with greater value of standard deviation is said to be more consistent than the other.
(a) Only I is true (b) III and IV are true
(c) I, III and IV are true (d) All are true

38. If $\bar{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, ..., x_n$, then which of the following are true for the observations $ax_1, ax_2, ax_3, ..., ax_n$?
I. Mean of the observations is $\frac{\bar{x}}{a}$.
II. Variance of the observations is $\frac{\sigma^2}{a^2}$.
III. Mean of the observations is $a\bar{x}$.
IV. Variance of the observations is $a^2\sigma^2$.
(a) I and II are true (b) I and IV are true
(c) II and III are true (d) III and IV are true

39. Following are the marks obtained, out of 100 by two students Raju and Sita in 10 tests.

Raju	25	50	45	30	70	42	36	48	35	60
Sita	10	70	50	20	95	55	42	60	48	80

I. Raju is more intelligent.
II. Sita is more intelligent.
III. Raju is more consistent.
IV. Sita is more consistent.
(a) I and IV are true (b) II and III are true
(c) I and III are true (d) II and IV are true

40. If for a distribution $\sum(x-5)=3$, $\sum(x-5)^2=43$ and the total number of items is 18.
Statement-I: Mean of the distribution is 4.1666.
Statement-II: Standard deviation of the distribution is 1.54.
(a) Only Statement I is true
(b) Only Statement II is true
(c) Both statements are true
(d) Both statements are false

41. Consider the following statements :
I. Measures of dispersion Range, Quartile deviation, mean deviation, variance, standard deviation are measures of dispersion
Range = Maximum value – minimum values
II. Mean deviation for ungrouped data
M.D. $(\bar{x}) = \frac{\mathbf{S}|x_i-\bar{x}|}{n}$
M.D. $(M) = \frac{\mathbf{S}|x_i-M|}{n}$
III. Mean deviation for grouped data
M.D. $(\bar{x}) = \frac{\Sigma f_i|x_i-\bar{x}|}{N}$
M.D. $(M) = \frac{\mathbf{S} f_i|x_i-M|}{N}$
where $N = \mathbf{S} f_i$
Which of the above statements are true?
(a) Only (I) (b) Only (II)
(c) Only (III) (d) All of the abvoe

42. Consider the following statements :
I. Mode can be computed from histogram
II. Median is not independent of change of scale
III. Variance is independent of change of origin and scale.
Which of these is / are correct ?
(a) (I), (II) and (III) (b) Only (II)
(c) Only (I) and (II) (d) Only (I)

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

43.

Column - I	**Column - II**
(A) Mean deviation about the median for the data 3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21, is	(1) $\frac{\sigma}{\bar{x}}\times 100$
(B) Mean deviation about the median for the data 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17, is	(2) $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2}$
(C) The standard deviation of n observations $x_1, x_2, \ldots, x_n$ is given by	(3) 2.33
(D) The coefficient of variation (CV) is defined as	(4) 5.27

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	4	3	2	1
(c)	3	4	2	1
(d)	4	3	1	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

44. The average of 5 quantities is 6, the average of three of them is 4, then the average of remaining two numbers is :
(a) 9 (b) 6
(c) 10 (d) 5

45. The mean deviation from the median of the following data is

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	5	3	6	2

(a) 14 (b) 10
(c) 5 (d) 7

46. Consider the following frequency distribution

x	A	2A	3A	4A	5A	6A
f	2	1	1	1	1	1

where, A is a positive integer and has variance 160. Then the value of A is.
(a) 5 (b) 6 (c) 7 (d) 8

47. Coefficient of variation of two distribution are 50% and 60% and their standard deviation are 10 and 15, respectively. Then, difference of their arithmetic means is
(a) 3 (b) 4 (c) 5 (d) 6

48. The mean of 5 observation is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6, then difference of the other two observations is
(a) 5 (b) 4 (c) 6 (d) 9

49. Consider the following data.
36, 72, 46, 42, 60, 45, 53, 46, 51, 49
Then the mean deviation about the median for the data is
(a) 6 (b) 8 (c) 7 (d) None of these

50. Given N = 10, $\Sigma x = 60$ and $\Sigma x^2 = 1000$. The standard deviation is
(a) 6 (b) 7 (c) 8 (d) 9

51. The standard deviation of 5 scores 1, 2, 3, 4, 5 is $\sqrt{a}$. The value of 'a' is
(a) 2 (b) 3 (c) 5 (d) 1

52. The variance of the data 2, 4, 6, 8, 10 is
(a) 8 (b) 7 (c) 6 (d) None of these

53. The range of set of observations 2, 3, 5, 9, 8, 7, 6,5, 7, 4, 3 is
(a) 6 (b) 7 (c) 4 (d) 5

54. The mean deviation from the mean for the set of observations –1, 0, 4 is
(a) 3 (b) 2 (c) 1 (d) None of these

55. The S. D of 15 items is 6 and if each item is decreased or increased by 1, then standard deviation will be
(a) 5 (b) 6 (c) 7 (d) None of these

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

56. **Assertion :** Sum of absolute values of

$$\text{Mean of deviations} = \frac{\text{Deviations}}{\text{Number of observations}}$$

Reason : Sum of the deviations from mean $(\bar{x})$ is 1.

57. **Assertion :** $\text{Mean of deviations} = \dfrac{\text{Product of deviations}}{\text{No. of observations}}$

Reason : To find the dispersion of values of x from mean $\bar{x}$, we take absolute measure of dispersion.

58. Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.
Assertion : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.
Reason : Arithmetic mean of $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.

59. **Assertion :** The range is the difference between two extreme observations of the distribution.
Reason : The variance of a variate X is the arithmetic mean of the squares of all deviations of X from the arithmetic mean of the observations.

60. **Assertion :** The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is 2.57
Reason : For individual observation,

$$\text{Mean deviation } (\bar{X}) = \frac{\sum |x_i - \bar{x}|}{n}$$

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

61. The mean of six numbers is 30. If one number is excluded, the mean of the remaining numbers is 29. The excluded number is
(a) 29 (b) 30 (c) 35 (d) 45

62. Mean of 20 observations is 15.5 Later it was found that the observation 24 was misread as 42. The corrected mean is:
(a) 14.2 (b) 14.8 (c) 14.0 (d) 14.6

63. The mean of a set of 20 observation is 19.3. The mean is reduced by 0.5 when a new observation is added to the set. The new observation is
(a) 19.8 (b) 8.8 (c) 9.5 (d) 30.8

64. The observations 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95 are arranged in ascending order. What is the value of x if the median of the data is 63?
(a) 61 (b) 62 (c) 62.5 (d) 63

65. The mean of 13 observations is 14. If the mean of the first 7 observations is 12 and that of the last 7 observations is 16, what is the value of the 7th observation ?
(a) 12 (b) 13 (c) 14 (d) 15

66. The mean and variance for first n natural numbers are respectively

(a) $\text{mean} = \dfrac{n+1}{2}$, $\text{variance} = \dfrac{n^2-1}{12}$

(b) $\text{mean} = \dfrac{n-1}{2}$, $\text{variance} = \dfrac{n^2+1}{12}$

(c) $\text{mean} = \dfrac{n^2-1}{12}$, $\text{variance} = \dfrac{n+1}{2}$

(d) $\text{mean} = \dfrac{n^2+1}{2}$, $\text{variance} = \dfrac{n-1}{2}$

67. Find the mean and standard deviation for the following data :

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

(a) mean = 6.59, S.D = 19 (b) mean = 8, S.D = 19
(c) mean = 19, S.D = 6.59 (d) mean = 19, S.D = 6

68. The median of a set of 9 distinct observation is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set is
(a) increased by 2
(b) decreased by 2
(c) two times the original median
(d) remains the same as that of original set

69. The mean deviation from the mean of the set of observations – 1, 0 and 4 is
(a) 3 (b) 1 (c) – 2 (d) 2

70. Variance of the data 2, 4, 5, 6, 8, 17 is 23.33. Then, variance of 4, 8, 10, 12, 16, 34 will be
(a) 23.33 (b) 25.33 (c) 93.32 (d) 98.32

71. The mean of 100 observations is 50 and their standard deviation is 5. The sum of squares of all observation is
(a) 50000 (b) 250000 (c) 252500 (d) 255000

72. Consider the following data
1, 2, 3, 4, 5, 6, 7, 8, 9, 10
If 1 is added to each number, then variance of the numbers so obtained is
(a) 6.5 (b) 2.87 (c) 3.87 (d) 8.25

73. Consider the first 10 positive integers. If we multiply each number by (– 1) and then add 1 to each number, the variance of the numbers so obtained is
(a) 8.25 (b) 6.5 (c) 3.87 (d) 2.87

74. Coefficient of variation of two distributions are 50 and 60 and their arithmetic means are 30 and 25, respectively. Then, difference of their standard deviations is
(a) 0 (b) 1 (c) 1.5 (d) 2.5

75. The sum of the squares of deviations for 10 observations taken from their mean 50 is 250. Then, the coefficient of variation is
(a) 10% (b) 40%
(c) 50% (d) None of these

76. If $n = 10$, $\bar{x} = 12$ and $\sum x_i^2 = 1530$, then the coefficient of variation is
(a) 35% (b) 42% (c) 30% (d) 25%

77. The variance of 20 observations is 5. If each observation is multiplied by 2, then the new variance of the resulting observation is
(a) $2^3 \times 5$ (b) $2^2 \times 5$
(c) 2×5 (d) $2^4 \times 5$

78. Let a, b, c, d and e be the observations with mean m and standard deviation s. The standard deviation of the observations a + k, b + k, c + k, d + k and e + k is
(a) s (b) ks (c) s + k (d) s/k

79. Let x_1, x_2, x_3, x_4 and x_5 be the observations with mean m and standard deviations. Then, standard deviation of the observations kx_1, kx_2, kx_3, kx_4 and kx_5 is
(a) k + 5 (b) π / k (c) ks (d) s

80. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b ?
(a) $a = 0, b = 7$ (b) $a = 5, b = 2$
(c) $a = 1, b = 6$ (d) $a = 3, b = 4$

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is
(a) $\frac{11}{2}$ (b) 6 (c) $\frac{13}{2}$ (d) $\frac{5}{2}$

82. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?
(a) mean (b) median
(c) mode (d) variance

83. If mean of the n observations $x_1, x_2, x_3, \ldots x_n$ be $\bar{x}$, then the mean of n observations $2x_1 + 3, 2x_2 + 3, 2x_3 + 3, \ldots, 2x_n + 3$ is
(a) $3\bar{x} + 2$ (b) $2\bar{x} + 3$ (c) $\bar{x} + 3$ (d) $2\bar{x}$

84. If the mean of n observations $1^2, 2^2, 3^2, \ldots, n^2$ is $\frac{46n}{11}$, then n is equal to
(a) 11 (b) 12 (c) 23 (d) 22

85. If the mean of four observations is 20 and when a constant c is added to each observation, the mean becomes 22. The value of c is :
(a) – 2 (b) 2 (c) 4 (d) 6

86. The arithmetic mean of a set of observations is $\bar{x}$. If each observation is divided by α then it is increased by 10, then the mean of the new series is:
(a) $\frac{\bar{x}}{\alpha}$ (b) $\frac{\bar{x} + 10}{\alpha}$
(c) $\frac{\bar{x} + 10\alpha}{\alpha}$ (d) $\alpha\bar{x} + 10$

87. The mean of n items is $\bar{X}$. If the first item is increased by 1, second by 2 and so on, the new mean is :
(a) $\bar{X} + \frac{x}{2}$ (b) $\bar{X} + x$
(c) $\bar{X} + \frac{n+1}{2}$ (d) none of these

88. The coefficient of variation from the given data

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	2	10	8	4	6

is :
(a) 50 (b) 51.9 (c) 48 (d) 51.8

89. Coefficient of variation of two distribution are 60 and 70, and their standard deviations are 21 and 16, respectively. What are their arithmetic means?
(a) 35, 22.85 (b) 22.85, 35.28
(c) 36, 22.85 (d) 35.28, 23.85

90. Standard deviation for first 10 natural numbers is
(a) 5.5 (b) 3.87
(c) 2.97 (d) 2.87

91. In a batch of 15 students, if the marks of 10 students who passed are 70, 50, 95, 40, 60, 70, 80, 90, 75, 80 then the median marks of all the 15 students is:
(a) 40 (b) 50 (c) 60 (d) 70

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(d)** The measure of dispersion is mean deviation, standard deviation and quartile deviation.

2. **(a)** We know that the observation which occur most frequently is known as mode.

3. **(c)** Let $x_1, x_2, ... x_n$ be n observation.
Now, reciprocals of n observations are
$\frac{1}{x_1}, \frac{1}{x_2}, ..., \frac{1}{x_n}$
Now, mean of the reciprocals of n observation.
$$= \frac{\frac{1}{x_1}+\frac{1}{x_2}+.......+\frac{1}{x_n}}{n}$$
Now, reciprocal of mean of the reciprocals of n observations
$$= \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+.......+\frac{1}{x_n}} = \text{Harmonic mean}$$

4. **(c)** First we write all the observation as 10, 18, 21, 35, 42.
Since, number of observation = 5 (odd)
$$\therefore \text{ Median} = \left(\frac{n+1}{2}\right)^{th} \text{observation}$$
$$= \left(\frac{6}{2}\right) = \text{3rd observation} = 21$$

5. **(c)** While dividing each entry in a data by a nonzero number a, the arithmetic mean of the new data is divided by a.

6. **(b)** Since 6 occurs most of the times in the given series.
$\therefore$ Mode of the given series = 6

7. **(d)** Coefficient of variation $= \frac{\text{standard deviation}}{\text{Mean}} \times 100$

8. **(b)** To measure the intelligence of a group of students mode will be more suitable.

9. **(d)** Mode is the required measure.

10. **(c)** Mode of the data is 8 as it is repeated maximum number of times.

11. **(d)** Mean of a, b, 8, 5, 10 is 6
$$\Rightarrow \frac{a+b+8+5+10}{5} = 10 \Rightarrow a+b=7 \quad \ldots\text{(i)}$$
Variance of a, b, 8, 5, 10 is 6.80
$$\Rightarrow \frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5} = 6.80$$
$$\Rightarrow a^2-12a+36+(1-a)^2+21=34 \quad \text{[using eq. (i)]}$$
$$\Rightarrow 2a^2-14a+24=0 \Rightarrow a^2-7a+12=0$$
$$\Rightarrow a=3 \text{ or } 4 \Rightarrow b=4 \text{ or } 3$$
$\therefore$ The possible values of a and b are $a = 3$ and $b = 4$ or, $a = 4$ and $b = 3$

12. **(a)** Arithmetic mean $\bar{x}$ of 4, 7, 8, 9, 10, 12, 13, 17 is
$$\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$$
$$\sum|x_i - \bar{x}| = 6+3+2+1+0+2+3+7=24$$
$\therefore$ Mean deviation about mean
$$= \text{M.D. } (\bar{x}) = \frac{\sum|x_i-\bar{x}|}{n} = \frac{24}{8} = 3$$

13. **(d)**

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
Total	**25**	**350**		**158**

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{350}{25} = 14$$
Mean deviation from the mean
$$= \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{158}{25} = 6.32$$

14. **(a)** Mean $\bar{x} = \frac{\sum x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	6 – 9	$(-3)^2$
7	7 – 9	$(-2)^2$
10	10 – 9	1^2
12	12 – 9	3^2
13	13 – 9	4^2
4	4 – 9	$(-5)^2$
8	8 – 9	$(-1)^2$
12	12 – 9	$(-3)^2$

$$\Sigma(x_i - \bar{x})^2 = 9+4+1+9+16+25+1+9=74$$
$$\text{Variance} = \frac{\Sigma(x_i-\bar{x})^2}{\Sigma f_i} = \frac{74}{8} = 9.25$$

15. **(b)** We know that, Statistics deals with data collected for specific purposes. We can make decisions about the data by analysing and interpreting it. We have studied methods of representing data graphically and in tabular form. This representation reveals certain salient features or characteristics of the data. We have also studied the methods of finding a representative value for the given data. This value is called the measure of central tendency.

16. **(d)** Mean (arithmetic mean), median and mode are three measures of central tendency. A measure of central tendency gives us a rough idea, where data points are centred.

17. **(d)** Variability is another factor which is required to be studied under Statistics. Like 'measure of central tendency' we want to have a single number to describe variability. This single number is called a 'measure of dispersion'.

18. **(d)** The dispersion or scatter in a data is measured on the basis of the observations and the types of the measure of central tendency, used there. There are following measure of dispersion.
(i) Range; (ii) Quartile deviation; (iii) Mean deviation; (iv) Standard deviation

19. **(c)** We know that, data can be grouped into two ways
(i) Discrete frequency distribution.
(ii) Continuous frequency distribution.

20. **(a)** $\text{Mean}(\bar{x}) = \frac{1357+1090+1666+1494+1623}{5} = \frac{7230}{5}$
$= 1446$

$$\text{Mean deviation} = \sum_{i=1}^{5}|x_i - \bar{x}|$$

$$= \frac{\left[|1357-1446|+|1090-1446|+|1666-1446| + |1494-1446|+|1623-1446|\right]}{5}$$

$$= \frac{89+356+220+48+177}{5} = \frac{890}{5} = 178$$

21. **(b)** This number, i.e., means of the squares of the deviations from mean is called the variance and is denoted by σ^2 (read as sigma square).

22. **(b)** The variance of n observations $x_1, x_2, \ldots x_n$ is given by

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

23. **(d)** We have studied about some types of measures of dispersion. The mean deviation and the standard deviation have the same units in which the data are given. Whenever we want to compare the variability of two series with same mean, which are measured in different units, we do not merely calculate the measures of dispersion but we require such measures which are independent of the units. The measure of variability which is independent of units, is called coefficient of variation (denoted as CV).

24. **(c)** Var (Y) = a^2 Var (X)

25. **(b)** $\left|\frac{a}{c}\right|\sigma$

26. **(d)** The mean of the given items

$$\bar{x} = \frac{3+7+10+18+22}{5} = 12$$

$$\text{Hence, variance} = \frac{1}{n}\sum(x_i - \bar{x})^2$$

$$= \frac{1}{5}\left\{(3-12)^2+(7-12)^2+(10-12)^2+(18-12)^2+(22-12)^2\right\}$$

$$= \frac{1}{5}\{81+25+4+36+100\} = \frac{246}{5} = 49.2$$

27. **(d)** Construct the following table taking assumed mean $a = 25$.

Class	x_i	f_i	$u_i = \frac{x_i - a}{10}$	$f_i u_i$	$\lvert x_i - 27\rvert$	$f_i \lvert x_i - 27\rvert$
0–10	5	5	–2	–10	22	110
10–20	15	8	–1	–8	12	96
20–30	25	15	0	0	2	30
30–40	35	16	1	16	8	128
40–50	45	6	2	12	18	108
Total	50			10		472

$$\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times c = 25 + \frac{10}{50} \times 10 = 27,$$

and mean deviation (about mean)

$$= \frac{\sum f_i |x_i - 27|}{\sum f_i} = \frac{472}{50} = 9.44.$$

STATEMENT TYPE QUESTIONS

28. **(c)** The runs scored by two batsmen in their last ten matches are as follows
Batsman A: 30, 91, 0, 64, 42, 80, 30, 5, 117, 71
Batsman B: 53, 46, 48, 50, 53, 53, 58, 60, 57, 52
Clearly, the mean and median of the data are

	Batsman A	Batsman B
Mean	53	53
Median	53	53

We calculate the mean of a data (denoted by $\bar{x}$), i.e.,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Also, the median is obtained by first arranging the data in ascending or descending order and applying the rules
Mean for batsman A

$$= \frac{30+91+0+64+42+80+30+5+117+71}{10} = \frac{530}{10} = 53$$

Mean for batsman B

$$= \frac{53+46+48+50+53+53+58+60+57+52}{10} = \frac{530}{10} = 53$$

To apply the formula to obtain median first arrange the data in ascending order

For batsman A	0	5	30	30	42	64	71	80	91	117
For batsman B	46	48	50	52	53	53	53	57	58	60

Here, we have n = 10 which is even number. So median is the mean of 5th and 6th observations.

$$\text{Median for batsman A} = \frac{42+64}{2} = \frac{106}{2} = 53$$

$$\text{Median for batsman B} = \frac{53+53}{2} = \frac{106}{2} = 53$$

29. **(a)** Consider the data [given in above question] of runs scored by two batsmen A and B, we had some idea of variability in the scores on the basis of minimum and maximum runs in each series.

To obtain a single number for this, we find the difference of maximum and minimum values of each series. This difference is called the range of the data.
In case of batsman A, Range = 117 – 0 = 117
and for batsman B, Range = 60 – 46 = 14
Clearly, range of A > range of B. Therefore, the scores are scattered or dispersed in case of A, while for B these are close to each other.
Thus, range of a series
= Maximum value – Minimum value
The range of data gives us a rough idea of variability or scatter but does not tell about the dispersion of the data from a measure of central tendency.

30. **(d)** I. Mean of the given series

$$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$$

$$= \frac{4+7+8+9+10+12+13+17}{8} = 10$$

x_i	$\lvert x_i - \bar{x}\rvert$
4	$\lvert 4-10\rvert = 6$
7	$\lvert 7-10\rvert = 3$
8	$\lvert 8-10\rvert = 2$
9	$\lvert 9-10\rvert = 1$
10	$\lvert 10-10\rvert = 0$
12	$\lvert 12-10\rvert = 2$
13	$\lvert 13-10\rvert = 3$
17	$\lvert 17-10\rvert = 7$
$\sum x_i = 80$	$\sum \lvert x_i - \bar{x}\rvert = 24$

$\therefore$ Mean deviation about mean $= \frac{\sum \lvert x_i - \bar{x}\rvert}{n} = \frac{24}{8} = 3$

II. Mean of the given series

$$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$$

$$= \frac{38+70+48+40+42+55+63+46+54+44}{10} = 50$$

x_i	$\lvert x_i - \bar{x}\rvert$
38	$\lvert 38-50\rvert = 12$
70	$\lvert 70-50\rvert = 20$
48	$\lvert 48-50\rvert = 02$
40	$\lvert 40-50\rvert = 10$
42	$\lvert 42-50\rvert = 08$
55	$\lvert 55-50\rvert = 05$
63	$\lvert 63-50\rvert = 13$
46	$\lvert 46-50\rvert = 04$
54	$\lvert 54-50\rvert = 04$
44	$\lvert 44-50\rvert = 06$
$\sum x_i = 500$	$\sum \lvert x_i - \bar{x}\rvert = 84$

$\therefore$ Mean deviation about mean $= \frac{\sum \lvert x_i - \bar{x}\rvert}{n} = \frac{84}{10} = 8.4$

31. **(b)** Let us write the given data in tabular form and calculate the required values to find mean deviation about the mean as

x_i	f_i	$f_i x_i$	$\lvert d_i\rvert = \lvert x_i - \bar{x}\rvert = \lvert x_i - 21.65\rvert$	$f_i \lvert d_i\rvert$
20	6	120	1.65	9.90
21	4	84	0.65	2.60
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
Total	20	433		25.00

Mean $(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = 21.65$

Hence, mean of the data is 21.65

Mean deviation $= \frac{\sum f_i \lvert x_i - \bar{x}\rvert}{\sum f_i} = \frac{25}{20} = 1.25$

Hence, mean deviation of the data is 1.25

32. **(d)** Total number of students (n) = 2 + 3 + 8 + 3 + 4 = 20

$$\text{Median of numbers} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th}\text{term} + \left(\frac{n}{2}+1\right)^{th}\text{term}\right]$$

$$= \frac{1}{2}\left[\left(\frac{20}{2}\right)^{th}\text{term} + \left(\frac{20}{2}+1\right)^{th}\text{term}\right]$$

$$= \frac{1}{2}\left[10^{th}\text{term} + 11^{th}\text{term}\right]$$

Marks obtained	f_i	cf	$\lvert d_i\rvert = \lvert x_i - 12\rvert$	$f_i \lvert d_i\rvert$
10	2	2	2	4
11	3	5	1	3
12	8	13	0	0
14	3	16	2	6
15	4	20	3	12
Total	**20**		$\sum f_i \lvert d_i\rvert = 25$	**25**

Median $= \frac{12+12}{2} = 12$

Mean deviation $= \frac{\sum f_i \lvert d_i\rvert}{\sum f_i} = \frac{25}{20} = 1.25$

33. **(a)** From the given data we can form the following table. The mean is calculated by step-deviation method taking 14 as assumed mean. The number of observations is n = 10.

x_i	$d_i = \frac{x_i - 14}{2}$	Deviations from mean $(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	–4	–9	81
8	–3	–7	49
10	–2	–5	25
12	–1	–3	9
14	0	–1	1
16	1	1	1
18	2	3	9
20	3	5	25
22	4	7	49
24	5	9	81
	5		330

$\therefore$ Mean $(\bar{x})$ = Assumed mean + $\dfrac{\sum_{i=1}^{n} d_i}{n} \times h$

$= 14 + \dfrac{5}{10} \times 2 = 15$

Variance (σ^2) = $\dfrac{1}{n}\sum_{i=1}^{10}(x_i - \bar{x})^2 = \dfrac{1}{10} \times 330 = 33$

$\therefore$ Standard deviation (σ) = $\sqrt{33} = 5.74$

34. **(b)** I. First n natural numbers are 1, 2, 3, 4, ..., n.

x_i	x_i^2
1	1^2
2	2^2
3	3^2
4	4^2
.	.
.	.
.	.
n	n^2
Total = $\dfrac{n(n+1)}{2}$	$\dfrac{n(n+1)(2n+1)}{6}$

$\therefore$ Mean = $\dfrac{\sum x_i}{n}$

$\therefore \bar{x} = \dfrac{n(n+1)}{2n} = \dfrac{n+1}{2}$

Variance = $\dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2$

$= \dfrac{n(n+1)(2n+1)}{6n} - \left[\dfrac{n(n+1)}{2n}\right]^2$

$= \dfrac{(n+1)(2n+1)}{6} - \dfrac{(n+1)^2}{4}$

$= \dfrac{(n+1)}{2}\left[\dfrac{2n+1}{3} - \dfrac{n+1}{2}\right]$

$= \dfrac{(n+1)}{2}\left[\dfrac{4n+2-3n-3}{6}\right]$

$= \left(\dfrac{n+1}{2}\right)\left[\dfrac{n-1}{6}\right] = \dfrac{n^2-1}{12}$

II. First 10 positive multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

x_i	x_i^2
3	9
6	36
9	81
12	144
15	225
18	324
21	441
24	576
27	729
30	900
Total = 165	**3465**

Mean $(\bar{x}) = \dfrac{\sum x}{n} = \dfrac{165}{10} = 16.5$

Variance = $\dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2 = \dfrac{3465}{10} - \left(\dfrac{165}{10}\right)^2$

$= 346.5 - (16.5)^2 = 346.5 - 272.25 = 74.25$

35. **(d)**

Class	Frequency (f_i)	Mid value (x_i)	Deviation from mean $d_i = \dfrac{x_i - 25}{10}$	d_i^2	$f_i d_i$	$f_i d_i^2$
0 – 10	5	5	–2	4	–10	20
10 – 20	8	15	–1	1	–8	8
20 – 30	15	25	0	0	0	0
30 – 40	16	35	1	1	16	16
40 – 50	6	45	2	4	12	24
Total	**50**				**10**	**68**

Mean $(\bar{x}) = A + \dfrac{\sum f_i d_i}{\sum f_i} \times h = 25 + \dfrac{10}{50} \times 10$

$= 25 + \dfrac{100}{50} = 25 + 2 = 27$

Variance = $\left[\dfrac{\sum f_i d_i^2}{\sum f_i} - \left(\dfrac{\sum f_i d_i}{\sum f_i}\right)^2\right] \times h^2$

$= \left[\dfrac{68}{50} - \left(\dfrac{10}{50}\right)^2\right] \times (10)^2 = \dfrac{[68 \times 50 - 100]}{50 \times 50} \times 100$

$= \dfrac{(3400 - 100)}{50} \times 2 = \dfrac{3300 \times 2}{50} = \dfrac{6600}{50} = 132$

36. **(b)** For comparing the variability or dispersion of two series, we calculate the coefficient of variance for each series. The series having greater CV is said to be more variable than the other. The series having lesser CV is said to be more consistent than the other.

37. **(a)** If $\bar{x}_1$ and σ_1 are the mean and standard deviation of the first distribution, and $\bar{x}_2$ and σ_2 are the mean and standard deviation of the second distribution.

Then, CV (1st distribution) = $\dfrac{\sigma_1}{\bar{x}_1} \times 100$

and CV (2nd distribution) = $\dfrac{\sigma_2}{\bar{x}_2} \times 100$

Given, $\bar{x}_1 = \bar{x}_2 = \bar{x}$ (say)

Therefore, CV (1st distribution) = $\dfrac{\sigma_1}{\bar{x}} \times 100$ ...(i)

and CV (2nd distribution) = $\dfrac{\sigma_2}{\bar{x}} \times 100$ ...(ii)

It is clear from Eqs. (i) and (ii) that the two CVs can be compared on the basis of values of σ_1 and σ_2 only. Thus, we say that for two series with equal means, the series with greater standard deviation (or variance) is called more variable or dispersed than the other. Also, the series with lesser value of standard deviation (or variance) is said to be more consistent than the other.

38. **(d)** Mean of $ax_1, ax_2, ..., ax_n$

$= \dfrac{ax_1 + ax_2 + \ldots + ax_n}{n} = a\left(\dfrac{x_1 + x_2 + x_3 + \ldots + x_n}{n}\right)$

$= a\bar{x} \quad \left(\because \quad \bar{x} = \frac{x_1 + x_2 + ... + x_n}{n}\right)$

Variance of $ax_1, ax_2, \ldots, ax_n$

$= \frac{\sum(ax_i - a\bar{x})^2}{n}$

$= \frac{(ax_1 - a\bar{x})^2 + (ax_2 - a\bar{x})^2 + ... + (ax_n - a\bar{x})^2}{n}$

$= \frac{a^2\left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_n - \bar{x})^2\right]}{n}$

$= a^2 \frac{\sum(x_i - \bar{x})^2}{n} = a^2\sigma^2 \left[\because \sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}\right]$

39. **(b)** For Raju

x_i	$d_i = x_i - 45$	d_i^2
25	–20	400
50	5	25
45	0	0
30	–15	225
70	25	625
42	–3	9
36	–9	81
48	3	9
35	–10	100
60	15	225
	–9	**1699**

Standard deviation $(\sigma) = \sqrt{\frac{169.9}{10} - \left(\frac{-9}{10}\right)^2} = \sqrt{169.9 - 0.81}$

$= \sqrt{169.09} = 13$

Mean $(\bar{x}) = 45 - \left(\frac{9}{10}\right) = 45 - 0.9 = 44.1$

For Sita

x_i	$d_i = x_i - 55$	d_i^2
10	–45	2025
70	15	225
50	–5	25
20	–35	1225
95	40	1600
55	0	0
42	–13	169
60	5	25
48	–7	49
80	25	625
	–20	**5968**

Mean $(\bar{x}) = 55 - \frac{20}{10} = 53$

Standard deviation $(\sigma) = \sqrt{\frac{5968}{10} - (-10)^2} = \sqrt{496.8} = 22.28$

Coefficient of variation of both Raju and Sita are

For Raju

$= \frac{\sigma}{\bar{x}} \times 100 = \frac{13}{44.1} \times 100 = \frac{1300}{44.1} = 29.47$

For Sita

$= \frac{\sigma}{\bar{x}} \times 100 = \frac{22.28}{53} \times 100 = \frac{2228}{53} = 42.04$

Since, CV of Sita > CV of Raju

Also, mean of Sita > mean of Raju

Hence, Raju is more consistent, but Sita is more intelligent.

40. **(b)** Given, $\sum(x - 5) = 3$

$\therefore \quad \sum x - \sum 5 = 3$

$\Rightarrow \quad \sum x - 5 \times 18 = 3 \quad (\because \; n = 18)$

$\Rightarrow \quad \sum x = 3 + 90 \;\Rightarrow\; \sum x = 93$

Now, $\sum(x - 5)^2 = 43$

$\Rightarrow \quad \sum\left(x^2 + 25 - 10x\right) = 43$

$\Rightarrow \quad \sum x^2 + \sum 25 - 10\sum x = 43$

$\Rightarrow \quad \sum x^2 + 25 \times 18 - 10 \times 93 = 43$

$\Rightarrow \quad \sum x^2 = 43 + 930 - 450$

$\Rightarrow \quad \sum x^2 = 973 - 450 \;\Rightarrow\; \sum x^2 = 523$

Now, mean $= \frac{\sum x}{n} = \frac{93}{18} = 5.16$

and $SD(\sigma) = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{523}{18} - \left(\frac{93}{18}\right)^2}$

$= \sqrt{\frac{523 \times 18 - 93 \times 93}{18 \times 18}} = \frac{1}{18}\sqrt{9414 - 8649}$

$= \frac{1}{18}\sqrt{765} = \frac{27.66}{18} = 1.54$

41. **(d)**

42. **(c)** Only first (I) and second (II) statements are correct.

MATCHING TYPE QUESTIONS

43. **(b)** (A) Given data is

3, 3, 4, 5, 7, 9, 10, 12, 18, 19, 21.

Median (M) = 6^{th} obs = 9

$|x_i - M|$ are 6, 6, 5, 4, 2, 0, 1, 3, 9, 10, 12

$\therefore \quad \sum_{i=1}^{11} |x_i - M| = 58$

M.D (M) $= \frac{1}{11} \times 58 = 5.27$

(B) Data in ascending order is

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

Median $= \frac{6^{th} \text{ obs} + 7^{th} \text{ obs}}{2} = \frac{13 + 14}{2} = \frac{27}{2}$

$= 13.5$

Now, $\sum|x_i - M| = 28$

$\therefore \quad$ M.D (M) $= \frac{28}{12} = 2.33$

INTEGER TYPE QUESTIONS

44. **(a)** Let a_1, a_2, a_3, a_4 and a_5 be five quantities

Then $a_1 + a_2 + a_3 + a_4 + a_5 = 30$ (given)

Also given that $a_1 + a_2 + a_3 = 12$
Now $a_4 + a_5 = 18$
Thus the average of a_4 and a_5 will be

$\frac{a_4 + a_5}{2} = \frac{18}{2} = 9$.

45. (d)

Class interval	f_i	x_i	cf	$\|d_i\|=\|x_i-14\|$	$f_i\|d_i\|$
0 – 6	4	3	4	11	44
6 – 12	5	9	9	5	25
12 – 18	3	15	12	1	3
18 – 24	6	21	18	7	42
24 – 30	2	27	20	13	26
	$\Sigma f_i=20$				$\Sigma f_i\|d_i\|=140$

Median number $= \frac{n}{2} = \frac{20}{2} = 10$ ($\because$ n is even number)

$\therefore$ Median class = 12 – 18

Median $= l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 12 + \frac{6}{3}(10-9) = 12 + 2 = 14$

Mean deviation $= \frac{\sum f_i |d_i|}{\sum f_i} = \frac{140}{20} = 7$

Hence, mean deviation about the median is 7.

46. (c)

x_i	f_i	$f_i x_i$	$f_i x_i^2$
A	2	2A	$2A^2$
2A	1	2A	$4A^2$
3A	1	3A	$9A^2$
4A	1	4A	$16A^2$
5A	1	5A	$25A^2$
6A	1	6A	$36A^2$
Total	**7**	**22A**	$\mathbf{92A^2}$

$\because \sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$

$\Rightarrow 160 = \frac{92A^2}{7} - \left(\frac{22A}{7}\right)^2$

$\Rightarrow 160 = \frac{92A^2}{7} - \frac{484A^2}{49} \Rightarrow 160 = \frac{92 \times 7A^2 - 484A^2}{49}$

$\Rightarrow 160 \times 49 = 644A^2 - 484A^2 \Rightarrow 160A^2 = 7840$

$\Rightarrow A^2 = \frac{7840}{160} \Rightarrow A^2 = 49 \Rightarrow A = \pm 7$

A = 7 as A is a positive integer.

47. (c) We have,
CV of 1st distribution $(CV_1) = 50$
CV of 2nd distribution $(CV_2) = 60$
$\sigma_1 = 10$ and $\sigma_2 = 15$

We know that, $CV = \frac{\sigma}{\bar{x}} \times 100$

$\therefore CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100 \Rightarrow 50 = \frac{10}{\bar{x}_1} \times 100$

$\Rightarrow \bar{x}_1 = \frac{10 \times 100}{50} = 20$

Also, $CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$\Rightarrow 60 = \frac{15 \times 100}{\bar{x}_2} \Rightarrow \bar{x}_2 = \frac{15 \times 100}{60} \Rightarrow \bar{x}_2 = 25$

Thus, $\bar{x}_2 - \bar{x}_1 = 25 - 20 = 5$

48. (a) Let the other two observations be x and y.
Therefore, the series is 1, 2, 6, x, y.

Now, mean $(\bar{x}) = 4.4 = \frac{1+2+6+x+y}{5}$

or $22 = 9 + x + y$

Therefore, $x + y = 13$ …(i)

Also, variance $(\sigma^2) = 8.24 = \frac{1}{n}\sum_{i=1}^{5}(x_i - \bar{x})^2$

i.e., $8.24 = \frac{1}{5}[(3.4)^2 + (2.4)^2 + (1.6)^2 + x^2 + y^2 - 2 \times 4.4(x+y) + 2 \times (4.4)^2]$

or $41.20 = 11.56 + 5.76 + 2.56 + x^2 + y^2 - 8.8 \times 13 + 38.72$

Therefore, $x^2 + y^2 = 97$ …(ii)

But from eq. (i), we have

$x^2 + y^2 + 2xy = 169$ …(iii)

From eqs. (ii) and (iii), we have

$2xy = 72$ …(iv)

On subtracting eq, (iv) from eq. (ii), we get

$x^2 + y^2 - 2xy = 97 - 72$

i.e. $(x - y)^2 = 25$ or $x - y = \pm 5$ …(v)

So, from eqs. (i) and (v), we get
$x = 9, y = 4$ when $x - y = 5$
or $x = 4, y = 9$ when $x - y = -5$
Thus, the remaining observations are 4 and 9.
Required difference = 5

49. (c) The given data is 36, 72, 46, 42, 60, 45, 53, 46, 51, 49
Arranging the data in ascending order,
36, 42, 45, 46, 46, 49, 51, 53, 60, 72
Number of observation = 10 (even)
Median (M)

$= \frac{\left(\frac{N}{2}\right)^{th} \text{observation} + \left(\frac{N}{2}+1\right)^{th} \text{observation}}{2}$

$= \frac{\left(\frac{10}{2}\right)^{th} \text{observation} + \left(\frac{10}{2}+1\right)^{th} \text{observation}}{2}$

$= \frac{5^{th}\text{observation} + 6^{th}\text{observation}}{2} = \frac{46+49}{2} = 47.5$

x_i	$\|x_i - M\|$
36	$\|36 - 47.5\| = 11.5$
42	$\|42 - 47.5\| = 5.5$
45	$\|45 - 47.5\| = 2.5$
46	$\|46 - 47.5\| = 1.5$
46	$\|46 - 47.5\| = 1.5$
49	$\|49 - 47.5\| = 1.5$
51	$\|51 - 47.5\| = 3.5$
53	$\|53 - 47.5\| = 5.5$
60	$\|60 - 47.5\| = 12.5$
72	$\|72 - 47.5\| = 24.5$
	$\Sigma\|x_i - M\| = 70$

$\therefore$ Mean deviation about median

$= \frac{\sum |x_i - M|}{n} = \frac{70}{10} = 7$

50. **(c)** $\sigma^2 = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2 = \frac{1000}{10} - \left(\frac{60}{10}\right)^2 = 100 - 36 = 64$

$\sigma = \sqrt{64} = 8$

51. **(a)** Mean $(\bar{x}) = \frac{1+2+3+4+5}{5} = 3$

$S.D = \sigma = \sqrt{\frac{1}{5}(1+4+9+16+25) - 9} = \sqrt{11-9} = \sqrt{2}$

52. **(a)** $\bar{x} = \frac{2+4+6+8+10}{5} = 6$

Hence, variance $= \frac{1}{n}\sum (x_i - \bar{x})^2$

$= \frac{1}{5}\{(16)+4+0+4+16\} = \frac{40}{5} = 8$

53. **(b)** Range = Maximum observation – Minimum observation $= 9 - 2 = 7$

54. **(b)** Mean $= \frac{-1+0+4}{3} = 1$

M. D (about mean) $= \frac{|-1-1| + |0-1| + |4-1|}{3} = 2$

55. **(b)** If each item of a data is increased or decreased by the same constant, the standard deviation of the data remains unchanged.

ASSERTION - REASON TYPE QUESTIONS

56. **(c)** Assertion is correct. It is a formula.
Reason is incorrect.
Sum of the deviations from mean $(\bar{x})$ is zero.

57. **(d)** Assertion is incorrect but Reason is correct.

58. **(c)** If each observation is multiplied by k, mean gets multiplied by k and variance gets multiplied by k^2.
Hence the new mean should be $2\bar{x}$ and new variance should be $k^2\sigma^2$.
So Assertion is true and Reason is false.

59. **(b)** Both Assertion and Reason are correct but Reason is not the correct explanation for Assertion.

60. **(a)** Mean $(\bar{X}) = \frac{2+9+9+3+6+9+4}{7} = \frac{42}{7} = 6$

$MD(\bar{X}) = \frac{\sum |x_i - \bar{x}|}{n} = \frac{4+3+3+3+0+3+2}{7} = \frac{18}{7} = 2.57$

CRITICALTHINKING TYPE QUESTIONS

61. **(c)** Sum of 6 numbers $= 30 \times 6 = 180$
Sum of remaining 5 numbers $= 29 \times 5 = 145$
$\therefore$ Excluded number $= 180 - 145 = 35$.

62. **(d)** Sum of 20 observations $= 20 \times 15.5 = 310$
Corrected sum $= 310 - 42 + 24 = 292$
So, corrected Mean $= \frac{292}{20} = 14.6$

63. **(b)**

64. **(b)** Given observations are 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95.
Number of observations = 10
As per definition

$$\text{median} = \frac{\text{value of } \frac{10}{2} \text{th term} + \text{value of} \left(\frac{10}{2}+1\right) \text{th term}}{2}$$

$$= \frac{\text{value of 5th term} + \text{value of 6th term}}{2}$$

$$= \frac{x + x + 2}{2} = \frac{2(x+1)}{2} = x + 1$$

But Median = 63, is given.
So, $63 = x + 1 \Rightarrow x = 62$

65. **(c)** Total sum of 13 observations $= 14 \times 13 = 182$
Sum of 14 observation $= 7 \times 12 + 7 \times 16$
$= 84 \times 112 = 196$
So, the 7th observation $= 196 - 182 = 14$

66. **(a)** The first n natural numbers are 1, 2, 3, ...n

Their mean, $\bar{x} = \frac{1+2+3+4+...+n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$

[$\because$ The sum of Ist n natural numbers is $\frac{n(n+1)}{2}$]

Now, Variance $= \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

$= \frac{1}{n}\left[\sum (x_i^2 - 2\bar{x}x_i + \bar{x}^2)\right] = \frac{\sum x_i^2}{n} - 2\bar{x}\frac{\sum x_i}{n} + \frac{\bar{x}^2.n}{n}$

$= \frac{\sum x_i^2}{n} - 2\bar{x}^2 + \bar{x}^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

[Since frequency of each variate is one]

$\because \sum x_i^2 = \frac{n(n+1)(2n+1)}{6}$

$\therefore$ Variance $= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{(n+1)}{2}\right)^2$

$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$

$= (n+1)\left(\frac{2n+1}{6} - \frac{n+1}{4}\right) = \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$

67. **(c)** Calculation for Mean and Standard Deviation

x_i	f_i	$f_i x_i$	$f_i x_i^2$
6	2	12	72
10	4	40	400
14	7	98	1372
18	12	216	3888
24	8	192	4608
28	4	112	3136
30	3	90	2700
130	$\Sigma f_i = 40$	$\Sigma f_i x_i = 760$	$\Sigma f_i x_i^2 = 16176$

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{760}{40} = 19$

S.D. $= \sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2} = \sqrt{\frac{16176}{40} - \left(\frac{760}{40}\right)^2}$

$= \sqrt{404.4 - 361} = \sqrt{43.4} = 6.59$.

Hence, Mean = 19, S.D. = 6.59.

68. **(d)** After arranging the terms in ascending order median is the $\left(\frac{n+1}{2}\right)^{th}$ term i.e., 5th term.
Here, we increase largest four observations of the set which will come after 5th term.
Hence, median remains the same as that of original set.

69. (d) Mean$\left(\bar{x}\right)=\frac{-1+0+4}{3}=1$

$\therefore$ MD $=\frac{\sum\left|x_i-\bar{x}\right|}{n}=\frac{|-1-1|+|0-1|+|4-1|}{3}=2$

70. (c) When each observation is multiplied by 2, then variance is also multiplied by 2.
We are given, 2, 4, 5, 6, 8, 17.
When each observation multiplied by 2, we get 4, 8, 10, 12, 16, 34.
$\therefore$ Variance of new series $=2^2\times$ Variance of given data $=4\times 23.33=93.32$

71. (c) We have $n=100$, $\bar{x}=50$, $\sigma=5, \sigma^2=25$
We know that

$$\sigma^2=\frac{\sum x_i^2}{n}-\left(\frac{1}{n}\sum x_i\right)^2$$

$$\Rightarrow\quad 25=\frac{\sum x_i^2}{100}-(50)^2\Rightarrow 2500=\sum x_i^2-250000$$

$$\Rightarrow\quad \sum x_i^2=252500$$

72. (d) We have the following numbers
1, 2, 3, 4, 5, 6, 7, 8, 9, 10
If 1 is added to each number, we get
2, 3, 4, 5, 6, 7, 8, 9, 10, 11
Sum of these numbers, $\sum x_i=2+3+...+11=65$
Sum of squares of these numbers.

$\sum x_i^2=2^2+3^2+...+11^2=505$

Variance $\left(\sigma^2\right)=\frac{\sum x_i^2}{n}-\left(\frac{\sum x_i}{n}\right)^2$

$=\frac{505}{10}-(6.5)^2=50.5-42.25=8.25$

73. (a) First ten positive integers are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.
Sum of these numbers $\left(\sum x_i\right)=1+2+...+10=55$
Sum of squares of these numbers $\left(\sum x_i^2\right)$
$=1^2+2^2+\ldots+10^2=385$

Standard deviation $(\sigma)=\sqrt{\frac{\sum x_i^2}{n}-\left(\frac{1}{n}\sum x_i\right)^2}$

$=\sqrt{\frac{385}{10}-(5.5)^2}=\sqrt{38.5-30.25}=\sqrt{8.25}$

$\therefore$ Variance $(\sigma^2)=8.25$

74. (a) We know that,
Coefficient of variation $=\frac{\sigma}{\bar{x}}\times 100$

$\therefore$ CV of 1st distribution $=\frac{\sigma_1}{30}\times 100$

$\Rightarrow\quad 50=\frac{\sigma_1}{30}\times 100$ [CV of 1st distribution = 50 (given)]

$\Rightarrow\quad s_1=15$

Also, CV of 2nd distribution $=\frac{\sigma_2}{25}\times 100$

$\Rightarrow\quad 60=\frac{\sigma_2}{25}\times 100\Rightarrow\sigma_2=\frac{60\times 25}{100}\Rightarrow\quad\sigma_2=15$

Thus, $\sigma_1-\sigma_2=15-15=0$

75. (a) We have

$\sum\left(x_i-\bar{x}\right)^2=250$

$n=10$ and $\bar{x}=50$

$$\therefore\quad\sigma^2=\left[\frac{\sum_{i=1}^{10}\left(x_i-\bar{x}\right)^2}{n}\right]\Rightarrow\sigma^2=\left[\frac{250}{10}\right]$$

$\Rightarrow\quad\sigma=\sqrt{25}=5$

Coefficient of variation $=\frac{\sigma}{\bar{x}}\times 100=\frac{5}{50}\times 100=10\%$

76. (d) We have, $n=10$, $\bar{x}=12$ and $\sum x_i^2=1530$

$$\therefore\quad\sigma^2=\frac{1}{10}\left(\sum_{i=1}^{10}x_i^2\right)-\left(\frac{1}{10}\sum_{i=1}^{10}x_i\right)^2$$

$\Rightarrow\quad\sigma^2=\frac{1530}{10}-(12)^2\Rightarrow\sigma^2=153-144$

$\Rightarrow\quad\sigma^2=9\Rightarrow\sigma=3$

Coefficient of variation $=\frac{\sigma}{\bar{x}}\times 100=\frac{3}{12}\times 100=25\%$

77. (b) Let the observations be $x_1, x_2, ..., x_{20}$ and $\bar{x}$ be their mean. Given that, variance = 5 and n = 20. We know that,

Variance $\left(\sigma^2\right)=\frac{1}{n}\sum_{i=1}^{20}(x_i-\bar{x})^2$

i.e. $5=\frac{1}{20}\sum_{i=1}^{20}(x_i-\bar{x})^2$ or $\sum_{i=1}^{20}(x_i-\bar{x})^2=100$ ...(i)

If each observation is multiplied by 2 and the new resulting observations are y_i, then

$y_i=2x_i$ i.e., $x_i=\frac{1}{2}y_i$

Therefore, $\bar{y}=\frac{1}{n}\sum_{i=1}^{20}y_i=\frac{1}{20}\sum_{i=1}^{20}2x_i=2.\frac{1}{20}\sum_{i=1}^{20}x_i$

i.e., $\bar{y}=2\bar{x}$ or $\bar{x}=\frac{1}{2}\bar{y}$

On substituting the values of x_i and $\bar{x}$ in eq. (i), we get

$\sum_{i=1}^{20}\left(\frac{1}{2}y_i-\frac{1}{2}\bar{y}\right)^2=100$ i.e., $\sum_{i=1}^{20}(y_i-\bar{y})^2=400$

Thus, the variance of new observations

$=\frac{1}{20}\times 400=20=2^2\times 5$

78. (a) We know that, if any constant is added in each observation, then standard deviation remains same.
$\therefore$ The standard deviation of the observations $a+k, b+k, c+k, d+k, e+k$ is s.

79. (c) Standard deviation is dependent on change of scale. Therefore, the standard deviation of $kx_1, kx_2, kx_3, kx_4, kx_5$ is ks.

80. (d) Mean of a, b, 8, 5, 10 is 6

$\Rightarrow\quad\frac{a+b+8+5+10}{5}=6\Rightarrow a+b=7$ ...(i)

Variance of a, b, 8, 5, 10 is 6.80

$\Rightarrow \frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5} = 6.80$

$\Rightarrow a^2 - 12a + 36 + (1-a)^2 + 21 = 34$ [using eq. (i)]

$\Rightarrow 2a^2 - 14a + 24 = 0 \Rightarrow a^2 - 7a + 12 = 0$

$\Rightarrow a = 3$ or $4 \Rightarrow b = 4$ or 3

$\therefore$ The possible values of a and b are $a = 3$ and $b = 4$ or, $a = 4$ and $b = 3$

81. (a) $\sigma_x^2 = 4, \sigma_y^2 = 5, x = 2, y = 4$

$\frac{1}{5}\sum x_i^2 - (2)^2 = 4;\ \frac{1}{5}\sum y_i^2 - (4)^2 = 5$

$\sum x_i^2 = 40; \sum y_i^2 = 105 \Rightarrow \sum\left(x_i^2 + y_i^2\right) = 145$

$\Rightarrow \sum(x_i + y_i) = 5(2) + 5(4) = 30$

Variance of combined data

$= \frac{1}{10}\sum\left(x_i^2+y_i^2\right) - \left(\frac{1}{10}\sum(x_i+y_i)\right)^2 = \frac{145}{10} - 9 = \frac{11}{2}$

82. (d) If initially all marks were x_i then $\sigma_i^2 = \frac{\sum_i (x_i - \bar{x})^2}{N}$

Now each is increased by 10

$\sigma_i^2 = \frac{\sum_i [(x_i+10)-(\bar{x}+10)]^2}{N} = \frac{\sum_i (x_i - \bar{x})^2}{N} = \sigma_i^2$

Hence, variance will not change even after the grace marks were given.

83. (b) Required mean $= \frac{1}{n}\sum_{i=1}^{n}(2x_i + 3)$

$= \frac{2}{n}\left(\sum_{i=1}^{n} x_i\right) + \frac{3n}{n} = 2\left\{\frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\right\} + 3 = 2\bar{x} + 3$

84. (a) Mean of n observations is

$\frac{1^2+2^2+3^2+\ldots+n^2}{n} = \frac{n(n+1)(2n+1)}{6n}$

From the description of the problem:

$\frac{(n+1)(2n+1)}{6} = \frac{46n}{11}$

$\Rightarrow 11 \times (2n^2 + 3n + 1) = 6 \times 46n$

$\Rightarrow 22n^2 + 33n + 11 = 276n \Rightarrow 22n^2 + 243n + 11 = 0$

$\Rightarrow 22n^2 - 242n - n + 11 = 0$

$\Rightarrow 22n(n-11) - 1(n-11) = 0$

$\Rightarrow (n-11)(22n-1) = 0$

Now, $22n - 1 = 0 \Rightarrow n = \frac{1}{22}$

which is discarded as n cannot be a fraction.

$\therefore n - 11 = 0 \Rightarrow n = 11$

85. (b) Mean of four observations = 20

$\therefore$ total observations $= 20 \times 4 = 80$

When add c in each observation total observation will be $80 + 4c$, then new mean = 22

$\therefore$ According to the question,

$80 + 4c = 22 \times 4 \Rightarrow 80 + 4c = 88 \Rightarrow 4c = 8 \Rightarrow c = 2$

86. (c) Let x be a set of observations given as

$x = a_1, a_2, \ldots\ldots, a_n$

Then $\bar{x} = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$

If now each observation is divided by α, then

$\frac{\frac{a_1}{\alpha} + \frac{a_2}{\alpha} + \ldots\ldots + \frac{a_n}{\alpha} + 10n}{n} = \frac{\bar{x}}{\alpha} + 10 = \frac{\bar{x} + 10\alpha}{\alpha}$

87. (c) Let the items be $a_1, a_2, \ldots\ldots, a_n$.

then $\bar{X} = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$

Now, according to the given condition:

$\bar{X} = \frac{(a_1+1)+(a_2+2)+\ldots\ldots+(a_n+n)}{n}$

$= \bar{X} + \frac{1+2+3+\ldots\ldots+n}{n} = \bar{X} + \frac{n(n+1)}{2n}$

(using sum of n natural nos.)

$= \bar{X} + \frac{n+1}{2}$.

88. (c)

Classmid-value	(x)	f	f x	d = x–M	d^2	fd^2
0 – 10	5	2	10	–20.7	428.49	856.98
10 – 20	15	10	150	– 10.7	114.49	1144.9
20 – 30	25	8	200	– 0.7	0.49	3.92
30 – 40	35	4	140	9.3	86.49	345.96
40 – 50	45	6	270	19.3	372.49	2234.94
		Σf = 30	Σfx = 770			Σfd^2 = 4586.7

Now, M (A.M) $= \frac{\Sigma fx}{\Sigma f} = \frac{770}{30} = 25.7$

Now, standard deviation (S.D)

$= \sqrt{\frac{\Sigma fd^2}{\Sigma f}} = \sqrt{\frac{4586.70}{30}} = 12.36$

$\therefore$ Coeff of SD $= \frac{SD}{M} = \frac{12.36}{25.7} = 0.480$

$\therefore$ Coeff of variation = Coeff of S.D × 100

$= 0.480 \times 100 = 48$.

89. (a) C.V. (1st distribution) = 60, $\sigma_1 = 21$

C.V. (2nd distribution) = 70, $\sigma_2 = 16$

Let $\bar{x}_1$ and $\bar{x}_2$ be the means of 1st and 2nd distribution, respectively, Then

C.V. (1st distribution) $= \frac{\sigma_1}{\bar{x}_1} \times 100$

$\therefore 60 = \frac{21}{\bar{x}_1} \times 100$ or $\bar{x}_1 = \frac{21}{60} \times 100 = 35$

and C.V. (2nd distribution) $= \frac{\sigma_2}{\bar{x}_2} \times 100$

i.e., $70 = \frac{16}{\bar{x}_2} \times 100$ or, $\bar{x}_2 = \frac{16}{70} \times 100 = 22.85$

90. (d) Variance $= \frac{(10)^2 - 1}{12} = \frac{99}{12}$

$\therefore$ S.D $= \sqrt{\frac{99}{12}} = \sqrt{8.25} = 2.87$

91. (c) As given : marks of 10 students out of 15 in the ascending order are 40, 50, 60, 70, 70, 75, 80, 80, 90, 95

Total number of terms = 15 and 5 students who failed are below 40 marks, median $= \left(\frac{n+1}{2}\right)$th term

$= \left(\frac{15+1}{2}\right)^{th}$ term $= 8^{th}$ term $= 60$

CHAPTER 16

PROBABILITY-I

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If $\frac{1+4p}{4}, \frac{1-p}{2}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then value of p is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

2. Which of the following cannot be the probability of an event?

(a) 2/3 (b) – 1/5 (c) 15% (d) 0.7

3. Probability of an event can be

(a) – 0.7 (b) $\frac{11}{9}$ (c) 1.001 (d) 0.6

4. In an experiment, the sum of probabilities of different events is

(a) 1 (b) 0.5 (c) – 2 (d) 0

5. In rolling a dice, the probability of getting number 8 is

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

6. In a simultaneous throw of 2 coins, the probability of having 2 heads is:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) $\frac{1}{6}$

7. The probability of getting sum more than 7 when a pair of dice are thrown is:

(a) $\frac{7}{36}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) None of these

8. The probability of raining on day 1 is 0.2 and on day 2 is 0.3. The probability of raining on both the days is

(a) 0.2 (b) 0.1 (c) 0.06 (d) 0.25

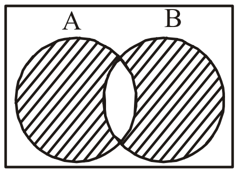
9. If A and B are two events, such that

$$P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}, P(A^c) = \frac{2}{3}$$

where A^c stands for the complementary event of A, then P(B) is given by:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

10. In the following Venn diagram circles A and B represent two events:



The probability of the union of shaded region will be

(a) $P(A) + P(B) - 2P(A \cap B)$
(b) $P(A) + P(B) - P(A \cap B)$
(c) $P(A) + P(B)$
(d) $2P(A) + 2P(B) - P(A \cap B)$

11. A single letter is selected at random from the word "PROBABILITY". The probability that the selected letter is a vowel is

(a) $\frac{2}{11}$ (b) $\frac{3}{11}$ (c) $\frac{4}{11}$ (d) 0

12. A bag contains 10 balls, out of which 4 balls are white and the others are non-white. The probability of getting a non-white ball is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

13. The dice are thrown together. The probability of getting the sum of digits as a multiple of 4 is:

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{5}{9}$

14. If the probabilities for *A* to fail in an examination is 0.2 and that for *B* is 0.3, then the probability that either *A* or *B* fails as

(a) $> .5$ (b) 0.5 (c) $\leq .5$ (d) 0

15. If $\frac{2}{11}$ is the probability of an event, then the probability of the event 'not A', is

(a) $\frac{9}{11}$ (b) $\frac{11}{2}$ (c) $\frac{11}{9}$ (d) $\frac{2}{11}$

16. An experiment is called random experiment, if it

(a) has more than one possible outcome
(b) is not possible to predict the outcome in advance
(c) Both (a) and (b)
(d) None of the above

17. An event can be classified into various types on the basis of the
(a) experiment (b) sample space
(c) elements (d) None of the above

18. An event which has only ...... sample point of a sample space, is called simple event.
(a) two (b) three (c) one (d) zero

19. If an event has more than one sample point, then it is called a/an
(a) simple event (b) elementary event
(c) compound event (d) None of these

20. When the sets A and B are two events associated with a sample space. Then, event '$A \cup B$' denotes
(a) A and B (b) Only A (c) A or B (d) Only B

21. If A and B are two events, then the set $A \cap B$ denotes the event
(a) A or B (b) A and B (c) Only A (d) Only B

22. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number", Then, E and F are
(a) mutually exclusive
(b) exhaustive
(c) mutually exclusive and exhaustive
(d) None of the above

23. Let S = {1, 2, 3, 4, 5, 6} and E = {1, 3, 5}, then $\overline{E}$ is
(a) {2,4} (b) {3,6} (c) {1,2,4} (d) {2,4,6}

24. If A and B are two events, then which of the following is true?
(a) $P(A \cup B) = P(A) + P(B)$
(b) $P(A \cup B) = P(A) + P(B) - \sum P(\omega_i), \forall \omega_i \in A \cap B$
(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(d) Both (b) and (c)

25. A coin is tossed once, then the sample space is
(a) {H} (b) {T} (c) {H, T} (d) None of these

26. A set containing the numbers from 1 to 25. Then, the set of event getting a prime number, when each of the given number is equally likely to be selected, is
(a) {2, 3, 7, 11, 13, 17}
(b) {1, 2, 3, 7, 11, 19}
(c) {2, 5, 7, 9, 11, 13, 17, 19, 23}
(d) {2, 3, 5, 7, 11, 13, 17, 19, 23}

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

27. Let S be a sample space containing outcomes $\omega_1, \omega_2, \omega_3, ...,$ ω_n i.e., $S = \{\omega_1, \omega_2, ..., \omega_n\}$.
Then, which of the following is true?
I. $0 \le P(\omega_i) \le 1$ for each $\omega_i \in S$
II. $P(\omega_1) + P(\omega_2) + ... P(\omega_n) = 1$
III. For any event A, $P(A) = \sum P(\omega_i)$, $\omega_i \in A$
(a) Only I (b) Only II (c) Only III (d) All of these

28. Which of the following is true?
I. If the empty set ϕ and the sample space describe events, then ϕ is an impossible event.
II. In the above statement, the whole sample space S is called the sure event.
(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Both I and II are false

29. Consider the experiment of rolling a die. Let A be the event 'getting a prime number' and B be the event 'getting an odd number'.
Then, which of the following is true?
I. A or B = $A \cup B$ = {1, 2, 3}
II. A and B = $A \cap B$ = {3, 5}
III. A but not B = A – B = {2}
IV. Not A = A′ = {1, 5, 6}
(a) Only I is true (b) Only II is true
(c) II and III is true (d) Only IV is true

30. A letter is chosen at random from the word 'ASSASSINATION'.
I. The probability that letter is a vowel is $\frac{6}{13}$.
II. The probability that letter is a consonant is $\frac{7}{13}$.
(a) Only I is correct.
(b) Both I and II are correct.
(c) Only II is correct.
(d) Both are incorrect.

31. A die is thrown.
I. The probability of a prime number will appear is $\frac{1}{2}$.
II. The probability of a number more than 6 will appear is 1.
(a) Only I is correct.
(b) Only II is correct.
(c) Both I and II are correct.
(d) Both I and II are incorrect.

32. A card is selected from a pack of 52 cards.
I. The probability that card is an ace of spades, is $\frac{2}{52}$.
II. The probability that the card is black card, is $\frac{26}{52}$.
(a) Only I is false. (b) Only II is false.
(c) Both I and II are false. (d) Both I and II are true.

33. A die is rolled, let E be the event "die shows 4" and F be the event "die shows even number". Then
I. E and F are mutually exclusive.
II. E and F are not mutually exclusive.
(a) Only I is true. (b) Only II is true.
(c) Neither I nor II is true. (d) Both I and II are true.

34. Consider the following statements.

I. If an event has only one sample point of the sample space is called a simple event.

II. A sample space is the set of all possible outcomes of an experiment.

(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Both I and II are false.

35. Consider the following statements.

I. If an event has more than one sample point it is called a compound event.

II. A set of events is said to be mutually exclusive if the happening of one excludes the happening of the other i.e. $A \cap B = \phi$.

III. An event having no sample point is called null or impossible event.

(a) I and II are true (b) II and III are true.
(c) I, II and III are true. (d) None of them are true.

36. Consider the following statements.

I. P(A or B) = P $(A \cup B)$ = P(A) + P(B), where A and B are two mutually exclusive events.

II. P(not 'A') = 1 – P(A) = $P(\bar{A})$, where $P(\bar{A})$ denotes the probability of not happening the event A.

III. $P(A \cap B)$ = Probability of simultaneous occurrence of A and B.

(a) I, II are true but III is false.
(b) I, III are true but II is false.
(c) II, III are true but I is false.
(d) All three statements are true.

37. Two dice are thrown. The events *A*, *B* and *C* are as follows:

A : getting an even number on the first die.
B : getting an odd number on the first die.
C : getting the sum of the numbers on the dice ≤ 5.

Then,

I. A′ : getting an odd number on the first die

II. A and B = $A \cap B = \phi$

III. B and C = $B \cap C = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(a) Only I and II is false.
(b) Only II and III is false.
(c) All I, II and III are false.
(d) All I, II and III are true.

38. If *A* and *B* are events such that *P*(*A*) = 0.42, *P*(*B*) = 0.48 and *P*(*A* and *B*) = 0.16. then,

I. P(not A) = 0.58

II. P(not B) = 0.52

III. P(A or B) = 0.47

(a) Only I and II are correct.
(b) Only II and III are correct.
(c) Only I and III are true.
(d) All three statements are correct.

39. If *E* and *F* are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, then,

I. P(E or F) = $\frac{5}{8}$

II. P(not E and not F) = $\frac{3}{8}$

(a) Only I is true. (b) Only II is true.
(c) Both I and II are true. (d) Neither I nor II is true.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

40. A die is thrown. Then, match the events of column-I with their respective sample points in column-II.

Column - I	Column - II
A. a number less than 7.	1. $\{3, 4, 5, 6\}$
B. a number greater than 7.	2. $\{6\}$
C. a multiple of 3.	3. $\{1,2,3\}$
D. a number less than 4.	4. $\{3,6\}$
E. an even number greater than 4.	5. $\{\ \}$
F. a number not less than 3.	6. $\{1,2,3,4,5,6\}$

Codes

	A	B	C	D	E	F
(a)	6	5	4	3	2	1
(b)	1	2	3	4	5	6
(c)	5	6	4	3	2	1
(d)	3	4	5	6	2	1

41. A die is thrown. If A, B, C, D, E and F are events described in above question. Then, match the events of column-I with their respective sample points in column-II.

Column - I	Column - II
A. $A \cup B$	1. $\{1,2\}$
B. $A \cap B$	2. ϕ
C. $B \cup C$	3. $\{1,2,3\}$
D. $E \cap F$	4. $\{1,2,4,5\}$
E. $D \cap E$	5. $\{6\}$
F. $A - C$	6. $\{3,6\}$
G. $D - E$	7. $\{1,2,3,4,5,6\}$
H. $E \cap F'$	
I. F'	

Codes

	A	B	C	D	E	F	G	H	I
(a)	1	2	7	3	4	5	6	4	2
(b)	7	2	6	5	2	4	3	2	1
(c)	1	2	4	7	5	3	4	2	1
(d)	6	5	4	1	2	3	6	5	3

42.

Column-I	Column-II
A. If E_1 and E_2 are the two mutually exclusive events, then	1. $E_1 \cap E_2 = E_1$
B. If E_1 and E_2 are the mutually exclusive and exhaustive events, then	2. $(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$
C. If E_1 and E_2 have common outcomes, then	3. $E_1 \cap E_2 = \phi$, $E_1 \cup E_2 = S$
D. If E_1 and E_2 are two events such that $E_1 \subset E_2$, then	4. $E_1 \cap E_2 = \phi$

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	4	3	2	1
(c)	2	3	4	1
(d)	1	4	2	3

43. Match the proposed probability under column I with the appropriate written description under column II.

Column-I (Probability)	Column-II (Written description)
A. 0.95	1. An incorrect assignment
B. 0.02	2. No chance of happening
C. – 0.3	3. As much chance of happening as not
D. 0.5	4. Very likely to happen
E. 0	5. Very little chance of happening

Codes

	A	B	C	D	E
(a)	4	5	1	3	2
(b)	1	2	3	4	5
(c)	3	2	4	5	1
(d)	5	2	3	4	1

44. A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$.

Then, match the terms of column-I with terms of column-II.

Column-I	Column-II
A. $P(A \cup B)$	1. 0.34
B. $P(A' \cap B')$	2. 0.19
C. $P(A \cap B')$	3. 0.12
D. $P(B \cap A')$	4. 0.88

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	1	2	3	4
(c)	2	3	4	1
(d)	3	2	1	4

45.

Column -I	Column - II
A. Three coins are tossed once. The probability of getting all heads,is	1. $\frac{5}{12}$
B. Two coins are tossed simultaneously. The probability of getting exactly one head, is	2. $\frac{2}{3}$
C. A die is thrown. The probability of getting a number less than or equal to 4, is	3. $\frac{1}{8}$
D. Two dice are thrown simultaneously. The probability of getting the sum as a prime number is	4. $\frac{1}{2}$

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	4	3	2	1
(c)	4	3	1	2
(d)	3	4	2	1

46. Let A, B and C are three arbitrary events, then match the columns and choose the correct option from the codes given below.

Column -I (events)	Column - II (Symbolic form)
A. Only A occurs	1. $\bar{A} \cap \bar{B} \cap \bar{C}$
B. Both A and B, but no C occur	2. $A \cap B \cap C$
C. All three events occur	3. $A \cap \bar{B} \cap \bar{C}$
D. At least one occur	4. $A \cup B \cup C$
E. None occurs	5. $A \cap B \cap \bar{C}$

Codes

	A	B	C	D	E
(a)	3	2	5	1	4
(b)	3	5	2	4	1
(c)	3	5	4	2	1
(d)	1	5	4	2	3

47.

Column -I	Column - II
A. A possible result of a random experiment is called	1. Complementary event
B. The set of outcomes is called the	2. An event
C. Any subset E of a sample space S is called	3. Sample space

D. For every event A, there corresponds another event A′ called the _____ to A	4. Outcome

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	4	2	3	1
(c)	1	3	2	4
(d)	1	2	3	4

48. Let A and B be two events related to a random experiment.

Column -I	**Column - II**
A. $P(A \cup B)$	1. Probability of non-occurrence of A.
B. $P(A \cap B)$	2. $\frac{\text{No. of fav. outcome}}{\text{Total outcome}}$
C. $P(\bar{A})$	3. Probability that at least one of the events occur.
D. P(A)	4. Probability of simultaneous occurrence of A and B.

Codes

	A	B	C	D
(a)	3	4	1	2
(b)	1	2	3	4
(c)	3	4	2	1
(d)	3	1	4	2

49.

Column -I (Experiment)	**Column - II (Sample space)**
A. A coin is tossed three times.	1. {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}
B. A coin is tossed two times.	2. {HH, HT, T1, T2, T3, T4, T5, T6}
C. A coin is tossed and a die is thrown.	3. {HHH, HHT, HTH, THH, THT, TTH, HTT, TTT}
D. Toss a coin and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once.	4. {HH, TT, HT, TH}

Codes

	A	B	C	D
(a)	3	4	2	1
(b)	4	3	1	2
(c)	3	4	1	2
(d)	4	3	2	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

50. Two dice are thrown simultaneously. The probability of obtaining a total score of seven is $\frac{1}{m}$. The value of 'm' is

(a) 3 (b) 2 (c) 6 (d) 9

51. A coin is tossed 3 times, the probability of getting exactly two heads is $\frac{m}{8}$. The value of 'm' is

(a) 1 (b) 2 (c) 3 (d) 4

52. A die is thrown. Let *A* be the event that the number obtained is greater than 3. Let *B* be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(a) $\frac{3}{5}$ (b) 0 (c) 1 (d) $\frac{2}{5}$

53. In a simultaneous toss of two coins, the probability of getting exactly 2 tails is $\frac{m}{n}$. The value of m + n is

(a) 1 (b) 4 (c) 5 (d) 2

54. A die is thrown. The probability of getting a number less than or equal to 6 is

(a) 6 (b) 1 (c) 2 (d) 5

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

55. **Assertion :** Probability of getting a head in a toss of an unbiased coin is $\frac{1}{2}$.

Reason : In a simultaneous toss of two coins, the probability of getting 'no tails' is $\frac{1}{4}$.

56. **Assertion :** In tossing a coin, the exhaustive number of cases is 2.

Reason : If a pair of dice is thrown, then the exhaustive number of cases is $6 \times 6 = 36$.

57. **Assertion :** A letter is chosen at random from the word NAGATATION. Then, the total number of outcomes is 10.

Reason : A letter is chosen at random from the word 'ASSASSINATION' Then, the total number of outcomes is 13.

58. Consider a single throw of die and two events.

A = the number is even = {2, 4, 6}

B = the number is a multiple of 3 = {3, 6}

Assertion : $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$

Reason : $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - \frac{2}{3} = \frac{1}{3}$

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

59. In a school there are 40% science students and the remaining 60% are arts students. It is known that 5% of the science students are girls and 10% of the arts students are girls. One student selected at random is a girl. What is the probability that she is an arts student?

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{5}$ (d) $\frac{3}{5}$

60. A bag contains 10 balls, out of which 4 balls are white and the others are non-white. The probability of getting a non-white ball is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

61. In a leap year the probability of having 53 Sundays or 53 Mondays is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

62. A fair die is thrown once. The probability of getting a composite number less than 5 is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) 0

63. The probability that a two digit number selected at random will be a multiple of '3' and not a multiple of '5' is

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$ (c) $\frac{1}{15}$ (d) $\frac{4}{90}$

64. Three identical dice are rolled. The probability that the same number will appear on each of them is:

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{18}$ (d) $\frac{3}{28}$

65. The probability that a card drawn from a pack of 52 cards will be a diamond or king is:

(a) $\frac{1}{52}$ (b) $\frac{2}{13}$ (c) $\frac{4}{13}$ (d) $\frac{1}{13}$

66. Events A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$

The set of possible values of x are in the interval is

(a) $[0, 1]$ (b) $\left[\frac{1}{3}, \frac{1}{2}\right]$

(c) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (d) $\left[\frac{1}{3}, \frac{13}{3}\right]$

67. A coin is tossed repeatedly until a tail comes up for the first time. Then, the sample space for this experiment is

(a) {T, HT, HTT}
(b) {TT, TTT, HTT, THH}
(c) {T, HT, HHT, HHHT, HHHHT, ...}
(d) None of the above

68. The probability that a randomly chosen two-digit positive integer is a multiple of 3, is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

69. If M and N are any two events, the probability that atleast one of them occurs is ...

(a) $P(M)+P(N)-2P(M\cap N)$
(b) $P(M)+P(N)-P(M\cap N)$
(c) $P(M)+P(N)+P(M\cap N)$
(d) $P(M)+P(N)+2P(M\cap N)$

70. If $P(A\cup B)=P(A\cap B)$ for any two events A and B, then

(a) $P(A)=P(B)$ (b) $P(A)>P(B)$
(c) $P(A)<P(B)$ (d) None of these

71. If A and B are mutually exclusive events, then

(a) $P(A)\le P(\overline{B})$ (b) $P(A)\ge P(\overline{B})$
(c) $P(A)<P(B)$ (d) None of these

72. If A, B and C are three mutually exclusive and exhaustive events of an experiment such that 3P(A) = 2P(B) = P(C), then P(A) is equal to ...

(a) $\frac{1}{11}$ (b) $\frac{2}{11}$ (c) $\frac{5}{11}$ (d) $\frac{6}{11}$

73. A coin is tossed twice. Then, the probability that atleast one tail occurs is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$

74. While shuffling a pack of 52 playing cards, 2 are accidentally dropped. The probability that the missing cards to be of different colours is

(a) $\frac{29}{52}$ (b) $\frac{1}{2}$ (c) $\frac{26}{51}$ (d) $\frac{27}{51}$

75. In a leap year, the probability of having 53 Sundays or 53 Mondays is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

76. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is

(a) 0.39 (b) 0.25
(c) 0.11 (d) None of these

77. If, $P(B)=\frac{3}{4}, P(A\cap B\cap\overline{C})=\frac{1}{3}$

and $P(\overline{A}\cap B\cap\overline{C})=\frac{1}{3}$, then $P(B\cap C)$ is

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{15}$ (d) $\frac{1}{9}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(a)** $\frac{1+4p}{4}, \frac{1-p}{2}, \frac{1-2p}{2}$ are probabilities of the three mutually exclusive events, then

$0 \le \frac{1+4p}{4} \le 1,\ 0 \le \frac{1-p}{2} \le 1,\ 0 \le \frac{1-2p}{2} \le 1$

and $0 \le \frac{1+4p}{4} + \frac{1-p}{2} + \frac{1-2p}{2} \le 1$

$\therefore -\frac{1}{4} \le p \le \frac{3}{4}, -1 \le p \le 1, -\frac{1}{2} \le p \le \frac{1}{2}, \frac{1}{2} \le p \le \frac{5}{2}$

$\therefore \frac{1}{2} \le p \le \frac{1}{2}$

[The intersection of above four intervals]

$\therefore p = \frac{1}{2}$

2. **(b)**

3. **(d)** Probability of an event always lies between 0 and 1. (both inclusive)

4. **(a)**

5. **(a)** Number 8 does not represent on dice.

6. **(a)** Let S be the sample space.

Since, simultaneously we throw 2 coins

S = {HH, HT, TH, TT}

$\therefore n(S) = 2^2$

Now, Let E be the event getting 2 heads i.e. HH

$\therefore n(E) = 1$

Thus, required prob $= \frac{n(E)}{n(S)} = \frac{1}{4}$

7. **(b)** Here $n(S) = 6^2 = 36$

Let E be the event "getting sum more than 7" i.e. sum of pair of dice = 8, 9, 10, 11, 12

$$\text{i.e. } E = \begin{Bmatrix} (2,6) & (3,5) & (4,4) & (5,3) & (6,2) \\ (3,6) & (4,5) & (5,4) & (6,3) & \\ (4,6) & (5,5) & (6,4) & & \\ (5,6) & (6,5) & (6,6) & & \end{Bmatrix}$$

$\therefore\ n(E) = 15$

$\therefore$ Required prob $= \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

8. **(d)**

9. **(b)** From the given problem :

$P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}$

$P(A^c) = \frac{2}{3} = 1 - P(A)$

$\Rightarrow P(A) = 1 - \frac{2}{3} = \frac{1}{3}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(B) = P(A \cup B) + P(A \cap B) - P(A)$

$= \frac{3}{4} + \frac{1}{4} - \frac{1}{3} = 1 - \frac{1}{3} = \frac{2}{3}$

10. **(b)** From the given Venn diagram the shadow region is $n(A) + n(B) - n(A \cap B)$.
Probability of the union of the shaded region is $P(A) + P(B) - P(A \cap B)$

11. **(c)** Required probability $= \frac{1+2+1}{11} = \frac{4}{11}$

12. **(b)** Total no. of balls = 10
No. of white balls = 4
No. of non-white balls = 10 – 4 = 6

So, Required prob $= \frac{6}{10} = \frac{3}{5}$

13. **(c)** Total exhaustive cases $= 6^2 = 36$
Following 9 pairs are favourable as the sum of their digits are multiple of 4
i.e., 4 or 8 or 12
(1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)

$\therefore$ Required probability $= \frac{9}{36} = \frac{1}{4}$

14. **(c)**

15. **(a)** Let $P(A) = \frac{2}{11}$;

$P(\text{not } A) = 1 - P(A) = 1 - \frac{2}{11} = \frac{9}{11}$

16. **(c)** In our day-to-day life, we perform many activities which have a fixed result no matter any number of times they are repeated. Such as, given any triangle, without knowing the three angles, we can definitely say that the sum of measure of angles is 180°.
When a coin is tossed it may turn up a head or a tail, but we are not sure which one of these results will actually be obtained. Such experiments are called random experiments.

17. **(c)** Events can be classified into various types on the basis of the elements they have.

18. **(c)** If an event E has only one sample point of a sample space, then it is called a simple (or elementary) event.
In a sample space containing n distinct elements, there are exactly n simple events.
For example, in the experiment of tossing two coins, a sample space is
S = {HH, HT, TH, TT}
There are four simple event corresponding to this sample space. There are E_1 = {HH}, E_2 = {HT}, E_3 = {TH} and E_4 = {TT}

19. (c) If an event has more than one sample point, then it is called a compound event.
For example, in the experiment of "tossing a coin thrice" the events
E: 'exactly one head appeared'
F: 'atleast one head appeared'
G: 'atmost one head appeared' etc.
are all compound events. The subsets of associated with these events are
E = {HTT, THT, TTH}
F = {HTT, THT, TTH, HHT, HTH, THH, HHH]
G = {TTT, THT, HTT, TTH}
Each of the above subsets contain more than one sample point, hence they are all compound events.

20. (c) Recall that union of two sets A and B denoted by $A \cup B$ contains all those elements which are either in A or in B or in both. When the sets A and B are two events associated with a sample space, then $A' \cup B'$ is the event 'either A or B' or both'. This event $A' \cup B'$ is also called 'A or B'.
Therefore, event 'A or B' = $A \cup B$
$= \{\omega : \omega \in A \text{ or } \omega \in B\}$

21. (b) We know that, intersection of two sets $A \cap B$ is the set of those elements which are common to both A and B, i.e., which belong to both 'A and B'.
Thus, $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$
For example, in the experiment of 'throwing a die twice' Let A be the event 'score on the first throw is 6' and B is the event 'sum of two scores is atleast 11'. Then,
A = {(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.
and B = {(5, 6), (6, 5), (6, 6)}
So, $A \cap B = \{(6,5),(6,6)\}$

22. (d) Let E = The die shows 4 = {4}
F = The die shows even number
= {2, 4, 6}
$\therefore\ E \cap F = \{4\} \neq \phi$
Hence, E and F are not mutually exclusive.

23. (d) Given that S = {1, 2, 3, 4, 5, 6} and E = {1, 3, 5}
Then, $\overline{E} = S - E = \{2,4,6\}$

24. (d) To find the probability of event 'A or B', i.e., $P(A \cup B)$. If S is sample space for tossing of three coins, then
S = {HHT, HHH, HTH, HTT, THH, THT, TTH, TTT}
Let A = {HHT, HTH, THH} and B = {HTH, THH, HHH} be two events associated with 'tossing of a coin thrice'.
Clearly, $A \cup B$ = {HHT, HTH, THH, HHH}
Now, $P(A \cup B)$ = P(HHT) + P(HTH) + P(THH) + P(HHH)
If all the outcomes are equally likely, then
$$P(A \cup B) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$
Also, P(A) = P(HHT) + P(HTH) + P(THH) = $\frac{3}{8}$
and P(B) = P(HTH) + P(THH) + P(HHH) = $\frac{3}{8}$
Therefore, $P(A) + P(B) = \frac{3}{8} + \frac{3}{8} = \frac{6}{8}$
It is clear that $P(A \cup B) \neq P(A) + P(B)$
The points HTH and THH are common to both A and B. In the computation of P(A) + P(B) the probabilities of points HTH and THH, i.e., the elements of $A \cap B$ are included twice. Thus, to get the probability of $P(A \cup B)$ we have to subtract the probabilities of the sample points in $A \cap B$ from P(A) + P(B).
i.e., $P(A \cup B) = P(A) + P(B) - \sum P(\omega_i), \forall \omega_i \in A \cap B$
$= P(A) + P(B) - P(A \cap B)$
Thus, we observe that,
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

25. (c) A coin is tossed once, then the sample space is S = {H, T}

26. (d) Let the set be S
Then, S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}.
Now, let the event E = Getting a prime number when each of the given number is equally likely to be selected
E = {2, 3, 5, 7, 11, 13, 17, 19, 23}

STATEMENT TYPE QUESTIONS

27. (d) Let S be the sample space containing outcomes $\omega_1, \omega_2, \ldots, \omega_n$ i.e., $S = \{\omega_1, \omega_2, \ldots, \omega_n\}$
It follows from the axiomatic definition of probability that
I. $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$
II. $P(\omega_1) + P(\omega_2) + \ldots + P(\omega_n) = 1$
III. For any event A, $P(A) = \sum P(\omega_i), \omega_i \in A$.

28. (c) The empty set ϕ and the sample space S describe events. Infact ϕ is called and impossible event and S, i.e., the whole sample space is called the sure event.

29. (c) Here, S = {1, 2, 3, 4, 5, 6}, A = {2, 3, 5} and B = {1, 3, 5}
Obviously
I. 'A or B' = $A \cup B$ = {1, 2, 3, 5}
II. 'A and B' = $A \cap B$ = {3, 5}
III. 'A but not B' = A – B = {2}
IV. 'not A' = A′ = {1, 4, 6}

30. (b) The word '*ASSASSINATION*' has 13 letters in which there are 6 vowels viz. *AAAIIO* and 7 consonants *SSSSNNT*.
$\therefore$ $n(S)$ = 13, No. of vowels = 6
$\therefore$ Probability of chosing a vowel = $\frac{6}{13}$
No. of consonants = 7
$\therefore$ Probability of chosing a consonant = $\frac{7}{13}$

31. (a) In this case, the possible outcomes are 1, 2, 3, 4, 5 and 6.
Total number of possible outcomes = 6.
I. Number of outcomes favourable to the event "a prime number" = 3 (*i.e.*, 2, 3, 5)

$P(\text{prime number}) = \frac{3}{6} = \frac{1}{2}$

II. Number of outcomes favourable to the event "a number more than 6" = 0

$P(\text{a number more than 6}) = \frac{0}{6} = 0$

32. **(a)** I. There is 1 ace of spade.

$\therefore n(A) = 1,\ n(S) = 52$

Probability that the card drawn is an ace of spade $= \frac{n(A)}{n(S)} = \frac{1}{52}$

II. There are 26 black cards.

$n(A) = 26,\ n(S) = 52$

Probability of getting a black card

$= \frac{26}{52} = \frac{1}{2}$

33. **(b)** When we throw a die, it can result in any one of the six number 1, 2, 3, 4, 5, 6

and $S = \{1, 2, 3, 4, 5, 6\}$

E (die shows 4) = {4}

F (die shows even number) = {2, 4, 6}

$\therefore\ E \cap F = \{4\} \Rightarrow E \cap F \neq \phi$

$\Rightarrow$ E and F are not mutually exclusive.

34. **(c)** By Definition, both the given statements are correct.

35. **(c)** By Definition, all the three statements are correct.

36. **(d)** By definition, All the three statements are true.

37. **(d)** B : getting an odd number on the first die.

= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

C : {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}

38. **(a)** I. $P(\text{not } A) = 1 - 0.42 = 0.58$

II. $P(\text{not } B) = 1 - P(B) = 1 - 0.48 = 0.52$

III. $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16 = 0.74$

39. **(c)** I. $P(E \text{ or } F) = P(E \cup F)$

$= P(E) + P(F) - P(E \cap F)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$

II. not E and not F = $E' \cap F' = (E \cup F)'$

$\therefore P$ (not E and not F)

$= P(E \cup F)' = 1 - P(E \cup F) = 1 - \frac{5}{8} = \frac{3}{8}$

MATCHING TYPE QUESTIONS

40. **(a)** A. a number less than 7 = {1, 2, 3, 4, 5, 6}

B. a number greater than 7 = {} = f($\because$ the maximum number on a die is 6, so there cannot be a number on die greater than 7).

C. a multiple of 3 = {3, 6}.

D. a number less than 4 = {1, 2, 3}

E. an even number greater than 4 = {6}

F. a number not less than 3 = {3, 4, 5, 6}

41. **(b)** A. Now, $A \cup B$ = The elements which are in A or B

$= \{1, 2, 3, 4, 5, 6\} \cup \phi = \{1, 2, 3, 4, 5, 6\}$

B. $A \cap B$ = The elements which are common in both A and B.

$= \{1, 2, 3, 4, 5, 6\} \cap \phi = \phi$

C. $B \cup C$ = The elements which are in both B and C.

$= \{\ \} \cup \{3, 6\} = \{3, 6\}$

D. $E \cap F$ = The elements which are common in both E and F.

$= \{6\} \cap \{3, 4, 5, 6\} = \{6\}$

E. $D \cap E$ = The elements which are common in both D and E.

$= \{1, 2, 3\} \cap \{6\} = \phi$

F. $A - C$ = The elements which are in A but not in C

$= \{1, 2, 3, 4, 5, 6\} - \{3, 6\} = \{1, 2, 4, 5\}$

G. $D - E$ = The elements which are in D but not in E

$= \{1, 2, 3\} - \{6\} = \{1, 2, 3\}$

H. $E \cap F' = E \cap (U - F) = E \cap [\{1, 2, 3, 4, 5, 6\} - (3, 4, 5, 6\}]$

$[\because U = \{1, 2, 3, 4, 5, 6\}] = \{6\} \cap \{1, 2\}] = \phi$

I. and $F' = (U - F) = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5, 6\} = \{1, 2\}$

42. **(b)** A. If E_1 and E_2 are two mutually exclusive events, then $E_1 \cap E_2 = \phi$

B. If E_1 and E_2 are the mutually exclusive and exhaustive events, then $E_1 \cap E_2 = \phi$ and $E_1 \cup E_2 = S$

where, S is the sample space for the events E_1 and E_2.

C. If E_1 and E_2 have common outcomes, then $(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$

D. If E_1 and E_2 are two events such that $E_1 \subset E_2$ and $E_1 \cap E_2 = E_1$

43. **(a)** A. Probability = 0.95

That means it is very likely to happen.

B. Probability = 0.02

That mean it is very little chance of happening.

C. Proabability = – 0.3

We know that, $0 \le P(E) \le 1$

So, it is an incorrect assignment.

D. Probability = 0.5

That means as much chance of happening as not

E. Probability = 0

That means no chance of happening.

44. **(a)** Using the relation,

A. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.54 + 0.69 - 0.35$

$= 1.23 - 0.35 = 0.88$

B. $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - 0.88 = 0.12$

C. $P(A \cap B') = P(\text{A only})$

$= P(A) - P(A \cap B)$

$= 0.54 - 0.35 = 0.19$

D. $P(B \cap A') = P(\text{B only})$

$= P(B) - P(B \cap A) = 0.69 - 0.35 = 0.34$

45. (d) A. S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
n(S) = 8
E = {HHH}, n(E) = 1

$\therefore$ Required Prob $= \frac{1}{8}$

B. S = {HH, HT, TH, TT} $\Rightarrow$ n(S) = 4
E = {HT, TH} $\Rightarrow$ n(E) = 2

$\therefore$ Required prob $= \frac{2}{4} = \frac{1}{2}$

C. S = {1, 2, 3, 4, 5, 6} $\Rightarrow$ n(S) = 6
E = {1, 2, 3, 4} $\Rightarrow$ n(E) = 4

$\therefore$ Required prob $= \frac{4}{6} = \frac{2}{3}$

D. $S = \left\{\begin{matrix} (1,1),(1,2),\ldots,(1,6) \\ (2,1),(2,2),\ldots,(2,6) \\ (3,1),(3,2),\ldots,(3,6) \\ (4,1),(4,2),\ldots,(4,6) \\ (5,1),(5,2),\ldots,(5,6) \\ (6,1),(6,2),\ldots,(6,6) \end{matrix}\right\}$

E = event "getting sum as 2, 3, 5, 7, 11
E = {(1,1), (1,2), (2,1), (1,4), (4,1), (2,3), (3,2), (1,6), (2,5), (3,4), (6,1), (5,2), (4,3), (5, 6), (6,5)}
n(S) = 36, n(E) = 15

$\therefore$ Required Prob $= \frac{15}{36} = \frac{5}{12}$

46. (b) By Algebra of Events
47. (a) By the definitions.
48. (a) **49. (c)**

INTEGER TYPE QUESTIONS

50. (c) When two dice are thrown then there are 6 × 6 exhaustive cases $\therefore$ n = 36. Let A denote the event "total score of 7" when 2 dice are thrown then
A = {(1, 6), (2, 5), (3, 4), (4,3), (5, 2), (6, 1)}.
Thus there are 6 favourable cases.
$\therefore$ m = 6

By definition $P(A) = \frac{m}{n}$

$\therefore P(A) = \frac{6}{36} = \frac{1}{6}.$

51. (c) The sample space (S) of toss of 3 coins will be given as:

H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$n(S) = 2^3 = 8$
Let E be the event of getting exactly 2 heads
$\therefore$ n(E) = 3
Thus the probability of getting exactly 2 heads

$= \frac{n(E)}{n(S)} = \frac{3}{8}.$

52. (c) $A \equiv$ number is greater than 3

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

$B \equiv$ number is less than 5 $\Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$

$A \cap B \equiv$ number is greater than 3 but less than 5.

$\Rightarrow P(A \cap B) = \frac{1}{6}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = \frac{3+4-1}{6} = 1$

53. (c) Exactly 2 tails can be obtained in one way i.e. TT. So, favourable number of elementary events = 1

Hence, required probability $= \frac{1}{4}$

$\Rightarrow$ m = 1, n = 4 and m + n = 5.

54. (b) Since every face of a die is marked with a number less than or equal to 6. So, favourable number of elementary events = 6

$\therefore$ Prob $= \frac{6}{6} = 1$

ASSERTION - REASON TYPE QUESTIONS

55. (b) Assertion : S = {H, T}
number of favourable event = 1

$\therefore$ Probability $= \frac{1}{2}$ (i.e., H)

Reason : S = {HH, HT, TH, TT}
E = {HH}

Probability $= \frac{n(E)}{n(S)} = \frac{1}{4}$

56. (b) Both Assertion and Reason is correct.
57. (b) Both Assertion and Reason are correct.
58. (b) Both Assertion and Reason are correct but Reason is not the correct explanation.

CRITICALTHINKING TYPE QUESTIONS

59. (b) Let there be 100 students.
So, there are 40 students of science and 60 students of arts.
5% of 40 = 2 science students (girls)
10% of 60 = 6 science students (girls)
Total girls students = 8
If a girl is chosed then

P (arts) $= \frac{6}{8} = \frac{3}{4}$

60. (b) Total no. of balls = 10
No. of white balls = 4
No. of non-white balls = 10 – 4 = 6

So, Required prob $= \frac{6}{10} = \frac{3}{5}$

61. (b) Since a leap year has 366 days and hence 52 weeks and 2 days. The 2 days can be SM, MT, TW, WTh, ThF, FSt, St.S.

Therefore, P(53 Sundays or 53 Mondays) $= \frac{3}{7}$

62. (b) [**Hint:** The outcomes are 1, 2, 3, 4, 5, 6. Out of these, 4 is the only composite number which is less than 5].

63. (b) 24 out of the 90 are two digit numbers which are divisible by '3' and not by '5'.
The required probability is therefore,

$\frac{24}{90} = \frac{4}{15}$.

64. (b) Total out comes

$$= \left\{\begin{matrix} (1,1,1),(1,1,2)........(1,1,6) \\ .... \\ .... \\ (6,6,1)................(6,6,6) \end{matrix}\right\}$$

i.e. n (S) = $6^3 = 6 \times 6 \times 6$
E = {(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)}
$\Rightarrow$ n (E) = 6

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{6}{6\times6\times6} = \frac{1}{36}$

65. (c) Total no. of cards = 52
13 cards are diamonds and 4 cards are king.
There is only one card which is a king of diamond.

$\therefore$ P (card is diamond) $= \frac{13}{52}$

P (card is king) $= \frac{4}{52}$

P (card is king of diamond) $= \frac{1}{52}$

$\therefore$ P (card is diamond or king)

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

66. (b) $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$, $P(C) = \frac{1-2x}{2}$

$\because$ For any event $E, 0 \le P(E) \le 1$

$\Rightarrow 0 \le \frac{3x+1}{3} \le 1,\ \ 0 \le \frac{1-x}{4} \le 1$ and $0 \le \frac{1-2x}{2} \le 1$

$\Rightarrow -1 \le 3x \le 2, -3 \le x \le 1$ and $-1 \le 2x \le 1$

$\Rightarrow -\frac{1}{3} \le x \le \frac{2}{3}$ and $-3 \le x \le 1$,

and $-\frac{1}{2} \le x \le \frac{1}{2}$

Also for mutually exclusive events A, B, C,

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$\Rightarrow P(A \cup B \cup C) = \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2}$

$\therefore\ 0 \le \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$0 \le 13 - 3x \le 12 \Rightarrow 1 \le 3x \le 13$

$\Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$

Considering all inequations, we get

$\max\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \le x \le \min\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\}$

$\Rightarrow \frac{1}{3} \le x \le \frac{1}{2} \quad \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2}\right]$

67. (c) The sample space is
S = {T, HT, HHT, HHHT, HHHHT, ...}

68. (b) 2-digit positive integers are 10, 11, 12, ... 99. Thus, there are 90 such numbers. Since, out of these, 30 numbers are multiple of 3, therefore, the probability that a randomly chosen positive 2-digit ingeter is a multiple of 3, is $\frac{30}{90} = \frac{1}{3}$.

69. (b) Given that, M and N are two events, then the probability that atleast one of them occurs is

$P(M \cup N) = P(M) + P(N) - P(M \cap N)$

70. (a) Given that, $P(A \cup B) = P(A \cap B)$

$\Rightarrow A = B \Rightarrow P(A) = P(B)$

71. (a) Given that A and B are two mutually exclusively events Then,

$P(A\cup B)=P(A)+P(B) \quad [\because (A\cap B)=\phi]$

since, $P(A\cup B)\le 1$

$\therefore P(A)+P(B)\le 1$

$\Rightarrow P(A)+1-P(\overline{B})\le 1$

$\Rightarrow P(A)\le P(\overline{B})$

72. (b) Let $3P(A) = 2P(B) = P(C) = p$ which gives

$P(A)=\frac{p}{3}, P(B)=\frac{p}{2}$ and $P(C)=p$

Now, since A, B, C are mutually exclusive and exhaustive events, we have

$P(A)+P(B)+P(C)=1$

$\Rightarrow \frac{p}{3}+\frac{p}{2}+p=1 \Rightarrow p=\frac{6}{11}$

Hence, $P(A)=\frac{p}{3}=\frac{2}{11}$

73. (d) The sample space is S = {HH, HT, TH, TT}

Let E be the event of getting atleast one tail

$\therefore$ E = {HT, TH, TT}

$\therefore$ Required probability p

$=\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}=\frac{n(E)}{n(S)}=\frac{3}{4}$

74. (c) There are 26 red cards and 26 black cards i.e., total number of cards = 52

P(both cards of different colours)

$=P(B)P(R)+P(R)P(B)$

$=\frac{26}{52}\times\frac{26}{51}+\frac{26}{52}\times\frac{26}{51}=2\left(\frac{26}{52}\times\frac{26}{51}\right)=\frac{26}{51}$

75. (b) Since, a leap year has 366 days and hence 52 weeks and 2 days. The 2 days can be SM, MT, TW, WTh, ThF, FSt, StS.

Therefore, P(53 Sundays or 53 Mondays) = $\frac{3}{7}$

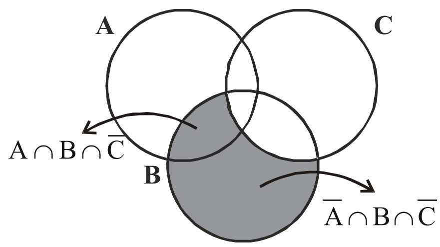
76. (a) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.25+0.50-0.14=0.61$

$\therefore P(A'\cap B')=P((A\cup B)')=1-P(A\cup B)$

$=1-0.61=0.39$

77. (a) From venn diagram, we can see that



$P(B\cap C)=P(B)-P(A\cap B\cap\overline{C})-P(\overline{A}\cap B\cap\overline{C})$

$=\frac{3}{4}-\frac{1}{3}-\frac{1}{3}=\frac{1}{12}.$