AREAS OF PARALLELOGRAMS AND TRIANGLES

MIND-MAPPING

Same Base CD and Parallel Lines AQ || CD

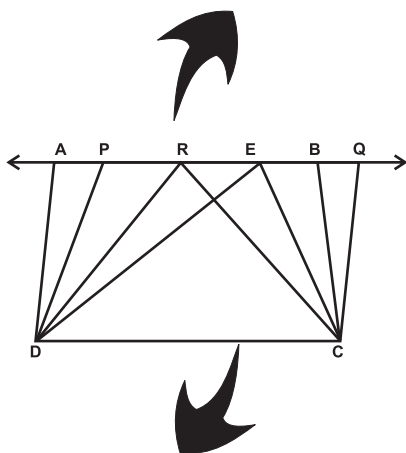Parallelograms :   ABCD and PQCD

ar (ABCD) = ar (PQCD)

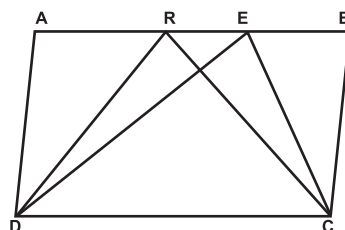Triangles on Same Base : $\triangle$RDC and $\triangle$EDC

ar ($\triangle$RDC) = ar ($\triangle$EDC)

$\therefore$ ar ($\triangle$RDC) = $\dfrac{1}{2}$ ar (ABCD) = $\dfrac{1}{2}$ ar (PQCD) = ar ($\triangle$EDC)

Same Base CD and
Parallel Lines AB || CD

Triangles on Same Base =
$\triangle$ RDC and $\triangle$ EDC

ar ($\triangle$ RDC) = ar ($\triangle$ EDC)

Also ar ($\triangle$ RDC) = $\dfrac{1}{2}$ ar (ABCD) =
ar ($\triangle$ EDC)
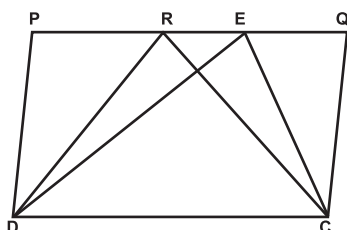
Same Base CD and Same Parallel
Lines PQ || DC
Triangles on same base
= $\triangle$ RDC and $\triangle$ EDC

ar ($\triangle$ RDC) = ar ($\triangle$ EDC)

Also ar ($\triangle$ RDC) = $\dfrac{1}{2}$ ar (PQCD)
= ar ($\triangle$ EDC)

KEY POINTS

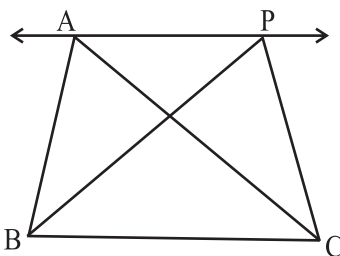1. Parallelograms on the same base and between same parallels are equal in area.

    Two parallelograms ABCD and EFCD on the same base DC and between same parallels AF and DC

    ar (ABCD) = ar (EFCD)
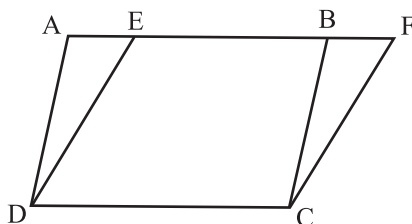
2. Two triangles on the same base and between the same parallels are equal in area.

    Two triangles ABC and PBC on the same base BC and between same Parallel lines BC and AP in the given figure then ar ($\triangle$ABC) = ar ($\triangle$PBC)

4. If a triangle and a parallelogram are on the same base and between the same parallels then the area of the triangle is half of the area of parallelogram.

    ar ($\triangle$LAB) = $\frac{1}{2}$ ar (ABCD)
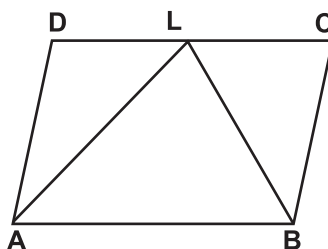
5. The median of a triangle divides it into two triangles of equal area.


Class IX - Mathematics

PART – A

1. Which of the following figures don't have equal areas if both the figures are on same base and between same parallels ?
   a) Two parallelograms
   b) One parallelograms and one rectangle
   c) Two Triangles
   d) One parallelogram and one triangle

2. Which statement is true ?
   a) Two congruent figures have always equal areas.
   b) Two figures having equal areas are always congruent.
   c) A triangle and a quadrilateral can be congruent.
   d) Two congruent figures have only some of its parts equal.

3. $\triangle$ DEF is divided into two triangles $\triangle$DEM and $\triangle$ DFM of equal areas. Which of the following statement is true ?
   a) $\triangle$ DEM and $\triangle$ DFM have equal bases.
   b) ar $(\triangle$ DEM$) = \dfrac{1}{3}$ ar $(\triangle$ DEF$)$
   c) M is the mid - point of side EF.
   d) $\triangle$ DEM and $\triangle$ DFM and congruent.

4. The ratio of the areas of the triangle and a parallel between same parallels and on the same base is :
   a) 1 : 2   b) 4 : 1
   c) 2 : 1   d) 1 : 4

5. The area of a parallelogram *PQRS* is 36 cm². M is any point on the side RS. The area of $\triangle$PMQ is.
   a) 18 cm^2 b) 9 cm^2
   c) 36 cm^2 d) 12 cm^2

6. AY and BZ are the diagonals of a parallelogram ABYZ, intersecting at O. ar $(\triangle$ BYZ$) = ?$
   a) greater than ar $(\triangle$ABZ$)$
   b) is equal to ar $(\triangle$ BOA $+ \triangle$ BOY$)$
   c) more than ar $(\triangle$ BOA $+ \triangle$ BOY$)$
   d) less than ar $(\triangle$ BOA $+ \triangle$ BOY$)$

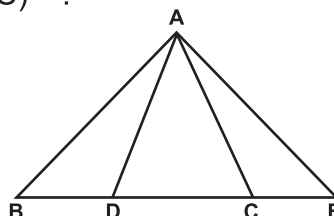7. AD is the median of $\triangle$ABC and E is any point on AD. Which of the
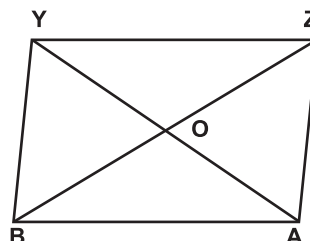
following statement is true ?

a) ar $(\triangle ABD) > $ ar $(\triangle ACD)$

b) ar $(\triangle ABD) < $ ar $(\triangle ACD)$

c) ar $(\triangle ABE) = $ ar $(\triangle CED)$

d) ar $(\triangle ABE) = $ ar $(\triangle ACE)$
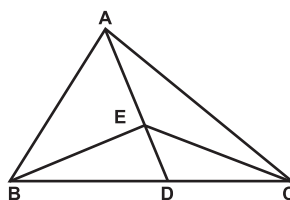
8. In the given figure ar $(\triangle ABC) = $ ar $(\triangle ADE)$. AC is the median of $\triangle ADE$. If ar $(ACE) = 14$ cm^2 then ar $(\triangle ABC) = $?

a) 14 cm^2 b) 7 cm^2

c) 21 cm^2 b) 28 cm^2

9. In the given figure ar $(DEFG) = 40$ cm^2, then ar $(HDG) + $ ar $(HEF) = $?

a) 10 cm^2 b) 20cm^2

c) 30 cm^2 d) 40cm^2
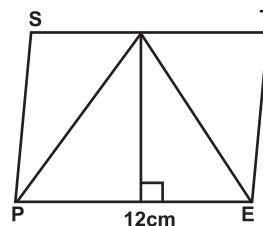
10. In the given figure PQRS is a parallelogram. Which of the following statements is true

a) QZ = RZ

b) ar $(\triangle PYS) = $ ar $(PQRS)$
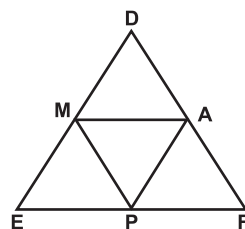
c) ar $(\triangle PQZ) = \frac{1}{2}$ ar $(PQRS)$

d) $\angle PQZ = \angle YRZ$
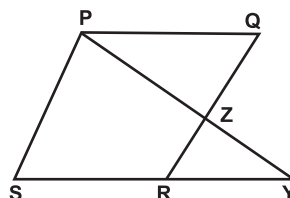
11. STEP is a parallelogram and ar $(STEP) = 84$ cm^2. The length of the altitude of $\triangle APE$ is

a) $\frac{7}{4}$ cm   b) $\frac{7}{2}$ cm

c) 7 cm   d) 14 cm

12. M, A and P are the mid-points of the sides DE, DF and EF of $\triangle$ DEF respectively. Which of the following statements is true ?

a) $\text{ar}(\triangle MPF) = 2\,\text{ar}(\triangle DEF)$

b) $\text{ar}(AMPF) = \frac{1}{2}\,\text{ar}(\triangle DEF)$

c) $\text{ar}(\triangle AMP) = \frac{1}{4}\,\text{ar}(\triangle DEF)$

d) $\text{ar}(AMPF) = \frac{1}{2}\,\text{ar}(\triangle DEF)$

13. STOP is a rectangle STAR is a parallelogram in the given figure. Which of the following statement is true ?

a) Perimeter (STAR) > Perimeter (STOP)

b) Perimeter (STAR) < Perimeter (STOP)

c) Perimeter (STAR) = Perimeter (STOP)

d) Perimeter (STAR) = $\frac{1}{2}$ Perimeter (STOP)
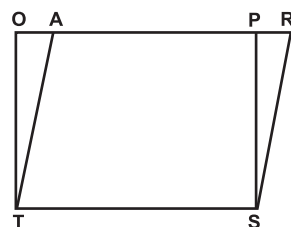
Fill in blanks :

14. The area of a parallelogram is the product of any of its sides and its corresponding _____

15. The area of parallelogram on the same base and between the same _____ are equal.

16. The diagonal of a parallelogram divides it into triangles having equal _____.

17. Area of trapezium = $\frac{1}{2}$ x height x _____

State True or False :

18. The median of a triangle divides it into two triangles of equal area.

19. The diagonals of a parallelogram are equal.

20. If both the diagonals of a quadrilateral divides it into four triangles of equal area, then the quadrilateral is a rhombus.
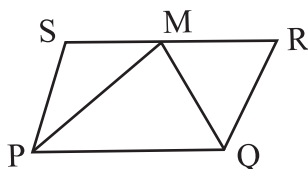
21.

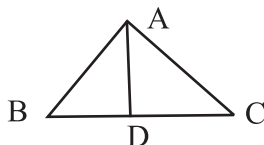If area of Parallelogram ABCD is 80 cm². Find the area of $\triangle APD$.
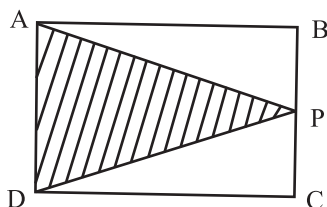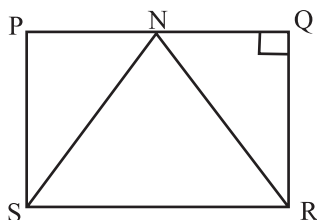
22. If area of Parallelogram PQRS is 88 cm² find K.
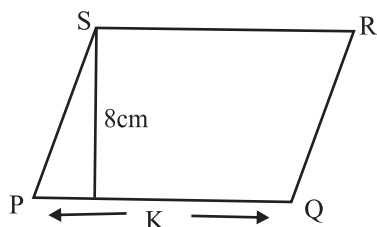
23. PQRS is a Parallelogram and PQM is a triangle. If area of PQM = 18cm². Find the area of PQRS.

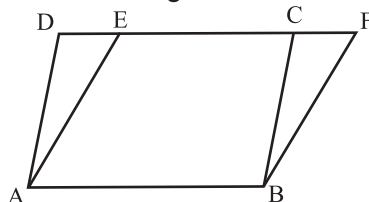24. In $\triangle ABC$, AD is median. If area of $\triangle ABD$ = 25cm² find the area of $\triangle ABC$.

25. In the given figure area of $\triangle SRN$ = 21cm² RQ = 6cm find PQ.

26. In the figure ABCD and ABFE are Parallelograms then find ar ($\triangle BCF$).

If ar (ABCE) = 18 cm²

ar (ABCD) = 25 cm²

27. If two parallelogram are on equal base and between the same parallels, then what is the ratio of their areas?

28. In $\triangle ABC$, D, E, F are respectively the mid points of the sides AB, BC and AC. Find ratio of the area of $\triangle DEF$ and area of $\triangle ABC$.

29. If the base of a parallelogram is 8 cm and its altitude is 5 cm then find its area.

30. If two triangles are on the same base and between the same parallels. Then find the ratio of area of the two triangles.

31. In given figure. If area of parallelogram ABCD is 30 cm^2 then find ar $(\triangle ADE)$ + ar $(\triangle BCE)$
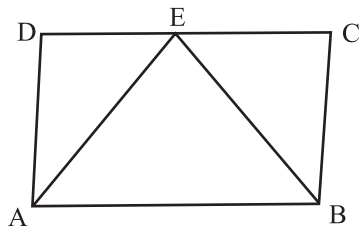

Part – B

32. Show that the median of a triangle divides it into two triangles of equal areas.

33. P and Q are any two points lying on the side DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

34. If the ratio of altitude and area of the parallelogram is 2:11 then find the length of the base of parallelogram.

35. In figure if PQRS is a parallelogram in which PQ=12cm, ST=9cm, QM=6cm, ST $\perp$ PQ, QM $\perp$ SP then find length of SP.
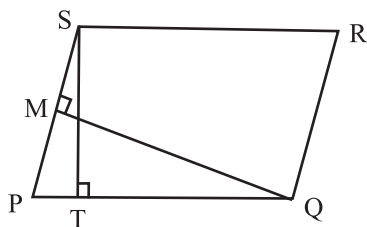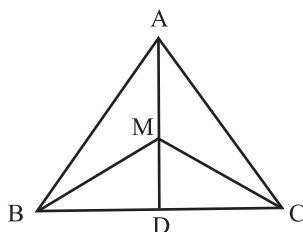

36. In given fig. ABCD is a square whose diagonals are interesting at O. If OD = 2 cm then find the length of AB.
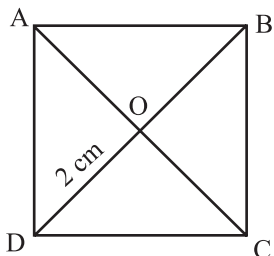
37. Show that the diagonals of a parallelogram divides it into four triangles of equal area.

38. M is any point on the median AD of △ABC. Show that ar (AMB) = ar (AMC).

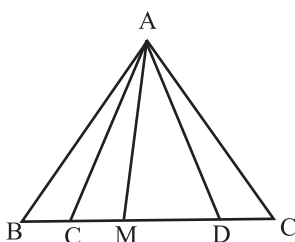39. If D, E and F are respectively the mid points of sides BC, CA, and AB of △ABC show that.

    i)   BDEF is a parallelogram.

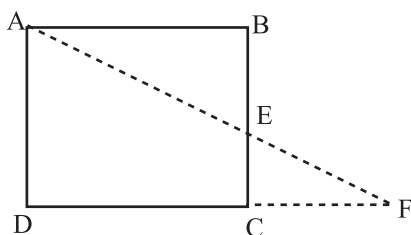   ii) ar (DEF) = 1/4 ar (△ABC)

40. In the given figure BC = CD = DE
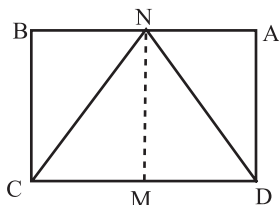
    M is the mid point of CD then find the area of △AMC.

41. ABCD is a parallelogram. Through point A, a line AEF is drawn to meet BC at E. DC produced to meet at F. Show that ar (△BEF) = ar (△DCE).
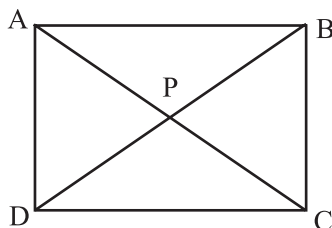
42. In the given figure, the area of parallelogram ABCD is 40 cm². If MN is a median of △CDN then find the area of △NDM.

Part-C

43. In the figure, P is the point in the interior of parallelogram ABCD then show that

   (i)   ar (APB) + ar (PCD) = 1/2 ar (ABCD)

    (ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

44. ABCD is a trapezium in which the AB $\parallel$ DC. If diagonal AC and BD intersect at O. Prove that ar (AOD) = ar (BOC).

45. ABCD is a parallelogram whose diagonals AC and BD intersect at O. A line through O intersects AB at P and DC at Q. Prove that ar ($\triangle$POA) = ar ($\triangle$QOC).

46. Diagonal PR and QS of quadrilateral PQRS intersects at T such that

    PT = TR and PS = QR, show that

    ar ($\triangle$PTS) = ar ($\triangle$RTQ).

47. In the figure, ABC and ABD are two triangles on the same base AB. If line segment CD bisects AB at O show that ar ($\triangle$ABC) = ar ($\triangle$ABD).
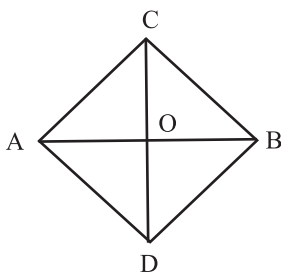
48. In given figure AD is median of $\triangle$ABC. Prove that ar ($\triangle$ABD) = ar ($\triangle$ACD).
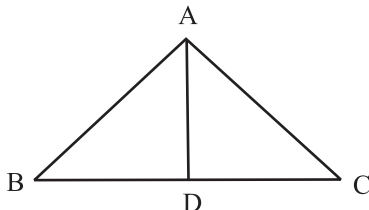
Part – D

49. Prove that parallelogram on the same base and between same parallels are equal in area.

50. Prove that the two triangles on the same base and between the same parallels are equal in area.

51. If a triangle and parallelogram are on the same base and between the same parallels then prove that the area of triangle is equal to the half the area of parallelogram. Using this find ar (△CMD).
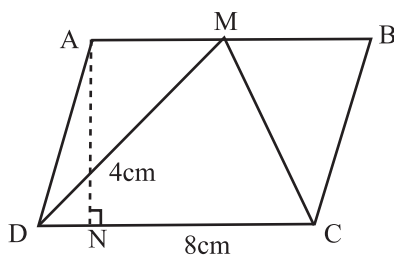
52. XY is a line parallel to side BC of a triangle ABC. If BE ‖ AC and CF ‖ AB meet XY at E and F respectively show that ar (ABE) = ar (ACF).

53. If E, F, G and H are respectively the mid points of the sides of a parallelograms ABCD. Show that ar (EFGH) = 1/2 ar (ABCD).

54. There is a plot in a village in the shape of a quadrilateral ABCD. Head of the village wants to get floor cemented so as to use it for panchayat meetings.

    Later he decided to construct playground of shape △ABP for children. If AC ‖ DP then

    (a) Prove than ar (ABCD) = ar (ABP)

    (b) area   (ABCD) = 2x _____
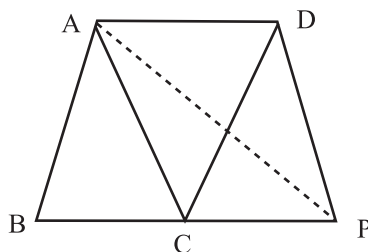
55. A farmer has a square plot of land where he wants to grow five different crops at a time. On half of the area in the middle he want to grow different crops.

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a) Explain by diagram how he can divide the area to fulfill his purpose.

b) For same base and between the same parallels write the relation between area of triangle and parallelogram formed.

56. In the adjoining figure, the point D divides the side BC of $\triangle ABC$ in the ratio m:n. Prove that ar $(\triangle ABD)$: ar $(\triangle ADC) = m : n$.
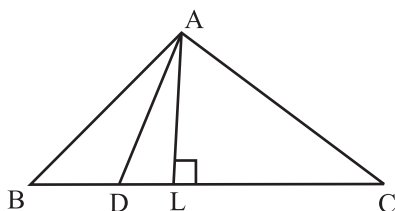

57. ABCD is a parallelogram. E is a point on BA such that BE= 2EA and F is a point on DC such that DF = 2 FC. Prove that AECF is a parallelogram whose area is one third of the area of parallelogram ABCD.
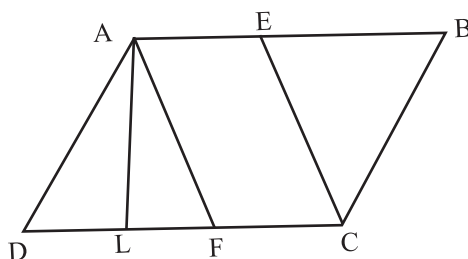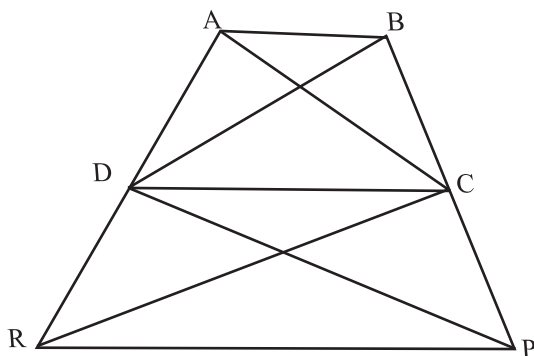

58. In the adjoining figure, two parallelogram ABCD and AEFB are drawn on opposite sides of AB. Prove that

ar($\square$ABCD) + ar($\square$ AEFB) = ar ($\square$ EFCD)
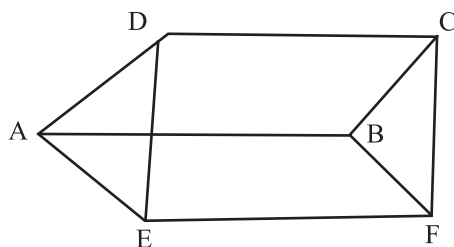
59. In the given figure BC || XY, BX || CA and AB || YC. Prove that ar (△ABX) = ar (△ACY)
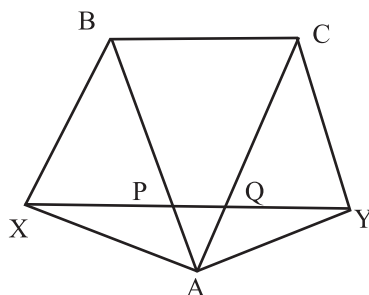
60. In the given figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilateral ABCD and DCPR are trapeziums.

AREAS OF PARALLELOGRAMS TRIANGLES
ANSWERS

1. d) One parallelogram and one triangle

2. a) Two congruent figures have always equal areas.

3. c) M is the mid - point of side EF.

4. a) 1 : 2

5. a) 18 cm^2

6. b) is equal to ar ($\triangle$ BOA + $\triangle$ BOY)

7. d) ar ($\triangle$ ABE) = ar ($\triangle$ ACE)

8. b) 28 cm^2

9. b) 20cm^2

10. d) $\angle$PQZ = $\angle$YRZ

11. c) 7 cm

12. b) ar (AMPF) = $\frac{1}{2}$ ar ($\triangle$ DEF)

13. A) Per (STAR) > Per (STOP)

14. altitude

15. parallels

16. areas

17. distance between the parallels

18. True

19. False

20. False

21. 40 cm^2

22. 11 cm

23. 36 cm^2

24. 50 cm^2

25. 7 cm

26. 7 cm^2

27. $1 : 1$

28. $1 : 4$

29. 40 cm^2

30. $1 : 1$

31. 15 cm^2

34. $\dfrac{11}{2}$ units

35. 18 cm

36. $\sqrt{8}$ cm

39. $\dfrac{1}{6} \triangle ABC$

42. 10 cm^2

51. 16 cm^2

54. ar (ADPC) = 2 x ar ($\triangle$ACD)

55. Area of triangle =
     $\dfrac{1}{2}$ x area of parallelogram
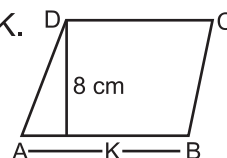
AREAS OF PARALLELOGRAMS & TRIANGLES
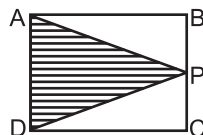
Time : 50 Min.   M.M. 20
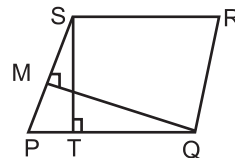
1. If area of parallelogram ABCD is 96 cm^2, find K.

2. If area of parallelogram ABCD is 60 cm^2. Find area of $\triangle$APD.

3. Show that the median of a triangle divides it into two triangles of equal area.

4. In figure if PQRS is a parallelogram in which PQ = 12 cm, ST = 9 cm QM = 6 cm, ST $\perp$ PQ, QM $\perp$ SP, then find length of SP.
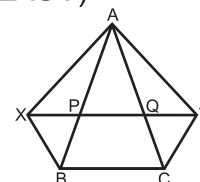
5. The base BC of $\triangle$ABC is divided at D. Such that BD = $\frac{1}{2}$ DC. Prove that ar ($\triangle$ABD) = $\frac{1}{3}$ ar ($\triangle$ABC)

6. ABCD is a parallelogram and O is a point in the interior, Prove that

    ar ($\triangle$AOB) + ar ($\triangle$COD) = ar ($\triangle$AOD) + ar ($\triangle$BOC)
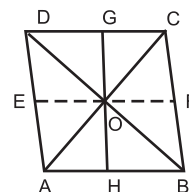
7. In the adjoining figure, PQ is a line parallel to the side BC to $\triangle$ABC. If BX $\parallel$ CA and Cy $\parallel$ BA meet the line PQ produced in X and Y respectively. Show that ar ($\triangle$ABX) = ar ($\triangle$ACY)

8. Prove that parallelogram on the same base and between same parallels are equal in area.