NUMBER THEORY

SYNOPSIS - 1

Highest Common Factor :

i) The greatest number which is the common factor of two or more given numbers is called H.C.F. or G.C.D.

H.C.F. can be determined in different methods

H.C.F. of 144, 198 by prime factorisation method.

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

$198 = 2 \times 3 \times 3 \times 11 = 2 \times (3)^2 \times 11$

$\therefore$ G.C.D. of 144, 198 = $2 \times (3)^2 = 9 \times 2 = 18$

ii) H.C.F. of 144, 198 by division method

The G.C.D. of 144, 198 is 18, since the last divisor is 18.

i.e.,

```
144) 198 (1
     144
     ----
      54) 144 (2
          108
          ----
           36) 54 (1
               36
               ----
               18) 36 (2
                   36
                   ----
                    0
                   ----
```

Note : HCF of two distinct prime numbers is one.

HCF of two co-primes is one.

HCF of an even number and an odd number is one.

HCF of two consecutive even numbers is two.

L.C.M. :

The least common multiple of two or more numbers is the smallest number which is a multiple of each of the numbers.

i) **To find L.C.M. of 24, 36 and 40 by prime factorisation method**

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$

$\therefore$ L.C.M. of 24, 36, 40 is $2^3 \times 3^2 \times 5 = 360$.

ii) **To find L.C.M of 24, 36 and 40 by division method**

2	24, 36, 40
2	12, 18, 20
2	6, 9, 10
3	3, 9, 5
	1, 3, 5

$\therefore$ L.C.M. of 24, 36 and 40 is 2 × 2 × 2 × 3 × 3 × 5 = 8 × 9 × 5 = 360.

Note : If two numbers are relatively primes, then their LCM is equal to their product.
In the given two numbers if the first number is a multiple of second number, then their LCM is equal to the first number.
The least common multiple of two prime numbers is their prodduct.
The LCM of two numbers is neven less than either of the two numbers.

iii. **LCM and GCD of fractions**

The LCM and GCD of fractions can be determined by the following relations:

LCM of fractions $=\dfrac{\text{LCM of numerators}}{\text{GCD of denominators}}$

GCD of fractions $=\dfrac{\text{GCD of numerators}}{\text{LCM of denominators}}$

Example:

Find the HCF and LCM of $\frac{4}{5},\frac{2}{5}$ and $\frac{3}{4}$.

$$\text{LCM}\left(\frac{4}{5},\frac{2}{5},\frac{3}{4}\right)=\frac{LCM\left(4,2,3\right)}{HCF\left(5,5,4\right)}=\frac{4\times3}{1}=12$$

$$HCF=\left(\frac{4}{5},\frac{2}{5},\frac{3}{4}\right)=\frac{HCF\left(4,2,3\right)}{LCM\left(5,5,4\right)}=\frac{1}{5\times4}=\frac{1}{20}$$

Note: The given fractions should be reduced to their lowest terms, before finding the GCD or LCM.

Example:

If the LCM/GCD of $\frac{2}{4}$ and $\frac{6}{9}$ has to be found, first $\frac{2}{4}$ and $\frac{6}{9}$ have to be expressed as $\frac{1}{2}$ and $\frac{2}{3}$ and LCM/GCD should be found.

WORK SHEET - 1

Single Answer Type

1. The L.C.M of 24, 36 and 40 is
   1) 120 2) 240 3) 360 4) 480
2. The L.C.M of 22, 54, 108, 135 and 198 is
   1) 330 2) 1980 3) 5940 4) 11880
3. The L.C.M of $\frac{1}{3},\frac{5}{6},\frac{2}{9},\frac{4}{27}$ is
   1) $\frac{1}{54}$ 2) $\frac{10}{27}$ 3) $\frac{20}{3}$ 4) $\frac{3}{20}$
4. The L.C.M of $\frac{2}{3},\frac{3}{5},\frac{4}{7},\frac{9}{13}$ is
   1) 36 2) $\frac{1}{36}$ 3) $\frac{1}{1365}$ 4) $\frac{12}{455}$
5. The L.C.M of $2^3\times3^2\times5\times11$, $2^4\times3^4\times5^2\times7$ and $2^5\times3^3\times5^3\times7^2\times11$
   1) $2^3\times3^2\times5$ 2) $2^5\times3^4\times5^3$
   3) $2^3\times3^2\times5\times7\times11$ 4) $2^5\times3^4\times5^3\times7^2\times11$
6. The L.C.M of 3, 2.7 and 0.09 is
   1) 2.7 2) 0.27 3) 0.027 4) 27
7. The H.C.F of 2923 and 3239 is
   1) 37 2) 47 3) 73 4) 79
8. The H.C.F of 204, 1190 and 1145 is
   1) 17 2) 18 3) 19 4) 21
9. The H.C.F of $2^2\times3^3\times5^5$, $2^3\times3^2\times5^2\times7$ and $2^4\times3^4\times5\times7^2\times11$ is
   1) $2^2\times3^2\times5$ 2) $2^2\times3^2\times5\times7\times11$ 3) $2^4\times3^4\times5^5$ 4) $2^4\times3^4\times5^5\times7\times11$
10. The H.C.F of $2^4\times3^2\times5^3\times7$, $2^3\times3^3\times5^2\times7^2$ and $3\times5\times7\times11$ is
    1) 105 2) 1155 3) 2310 4) 27720
11. The H.C.F of $\frac{9}{10},\frac{12}{25},\frac{18}{35}$ and $\frac{21}{40}$ is
    1) $\frac{3}{5}$ 2) $\frac{252}{5}$ 3) $\frac{3}{2800}$ 4) $\frac{63}{700}$

12. The H.C.F of $\frac{2}{3}, \frac{8}{9}, \frac{64}{81}$ and $\frac{10}{27}$ is

1) $\frac{2}{3}$ 2) $\frac{2}{81}$ 3) $\frac{160}{3}$ 4) $\frac{160}{81}$

13. The H.C.F of 1.75, 5.6 and 7 is

1) 0.07 2) 0.7 3) 3.5 4) 0.35

14. The G.C.O of 1.08, 0.36 and 0.9 is

1) 0.03 2) 0.9 3) 0.18 4) 470.108

15. The ratio of two numbers is $3:4$ and their H.C.F is 4. Their L.C.M is

1) 12 2) 16 3) 24 4) 48

16. Three numbers are in the ratio $1:2:3$ and their H.C.F is 12. The numbers are

1) 4, 8, 12 2) 5, 10, 15 3) 10, 20, 30 4) 12, 24, 36

Multi Answer Type

17. If $A = 7^2 \times 9 \times 5^3$, $B = 7 \times 9^2 \times 5^2$ and $C = 7^3 \times 9^3 \times 5$ then

1) H.C.F of A, B, C is $7 \times 9 \times 5$ 2) G.C.D of A, B, C is $7^3 \times 9^3 \times 5^3$

3) L.C.M of A, B, C is $7^3 \times 9^3 \times 5^3$ 4) L.C.M of A, B, C is $7 \times 9 \times 5$

18. The L.C.M of two numbers is 48. The numbers are in the ratio $2:3$. Then

1) The numbers are 16, 48 2) The numbres are 16, 24

3) H.C.F of the is numbers is 8

4) Difference of the numbers is 8

19. The Sum of two numbers is 216 and their H.C.F is 27. Then the numbers are

1) 27, 189 2) 81, 135 3) 108, 108 4) 154, 762

20. Product of two Co-prime numbers is 117. Then

1) The numbers are 1, 117

2) The H.C.F of those two numbers is 1

3) The L.C.M of those two numbers is 117

4) The H.C.F of those two numbers is 117

Reasoning Answer Type

21. **Statement - 1:** L.C.M of 15, 24, 32 and 36 is 1440.

Statement - 2: L.C.M of numbers is the least number which is divisible by each of the given number exactly.

1. Both Statements I and II are correct
2. Both statement I and II are incorrect
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

22. **Statement - 1:** Greatest number which divides 62, 132 and 237 to leave the same remainder in each Case is 35.

Statement - 2: Greatest number which divides a, b, c to leave the same remainder in each case is H.C.F of $(b-a),(c-b),(c-a)$

1. Both Statements I and II are correct
2. Both statement I and II are incorrect
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

Comprehension Type

Writeup-1

LCM of numbers is the least number which is divisible by each of the given numbers.

23. LCM of 48, 108 and 280 is

1) 15120 2) 16150 3) 3950 4) 46790

24. LCM of 1134 and 2106 is

1) 15672 2) 14742 3) 49353 4) 97426

25. LCM of 72, 108, 144, 162 is

1) 676 2) 1579 3) 1296 4) 1892

Writeup-2

Greatest number which divides a, b, c to leave the same remainder in each case is H.C.F of $(b-a),(c-b),(c-a)$.

26. Find the largest number which divides 32, 122 and 157 to leave same remainder in each case ?

1) 5 2) 15 3) 30 4) 20

27. Find the largest number that will divide 43, 91 and 183 so as to leave the same remainder in each case ?

1) 4 2) 7 3) 9 4) 13

28. Find the largest number that will divide 1305, 4665 and 6905 leaving the same remainder in each case then sum of the digits in that number is

1) 4 2) 5 3) 6 4) 8

Matrix Matching Type

29.

Column - I	Column - II
1) LCM of 168, 180 and 330	A) 1136
2) LCM of 16, 24, 36 and 54	B) 27720
3) LCM of 248 and 868	C) 5103
4) LCM of 567 and 729	D) 432
	E) 2601

30. Column - I

Column - I	Column - II
1) Common factor of 18 and 24	A) 3
2) GCD of 55 and 121 is	B) 6
3) Common factor of 38 and 57	C) 11
4) GCD of 3156 and 6	D) 19
	E) 1

Integer Answer Type

31. G.C.D of two Consecutive even numbers is ______

32. G.C.D of two Co-primes is ________

SYNOPSIS - 2

Relation between G.C.D and L.C.M. :

If 'a' and 'b' are any two natural numbers and L and G are respectively their L.C.M. and G.C.D. then a × b = L × G.

Eg 1 : If the G.C.D of two numbers is 16 and their product is 3072, then their

$$\text{L.C.M.} = \frac{\text{product of the given two numbers}}{\text{their G.C.D}} = \frac{3072}{16} = 192$$

Eg 2 : The L.C.M and G.C.D of two numbers respectively are 80 and 4. If one of the numbers is 16, then the other number is $b = \frac{L \times G}{a} = \frac{80}{16} = 5$

WORK SHEET - 2

Single Answer Type

1. The H.C.F of two number is 11 and their L.C.M is 693. If one of the numbers is 77 then the other number is
   1) 9 2) 99 3) 69 4) 63
2. The H.C.F of two numbers is 11 and their LCM is 7700. If one of the number is 275, then the other is
   1) 279 2) 283 3) 308 4) 318
3. The H.C.F and L.C.M of two numbers are 84 and 21 respectively. If the ratio of the two numbers is $1:4$, then the larger of the two numbers is
   1) 12 2) 48 3) 84 4) 108
4. The product of two numbers is 1320 and their H.C.F is 6. The L.C.M of the numbers is
   1) 220 2) 1314 3) 1326 4) 7920
5. If the G.C.D of two numbers is 1, theb their LCM is equal to their
   1) Sum 2) Quotient 3) Different 4) Product

6. The LCM of two numbers is 1320. Their GCD is 12. If one of the number is 132 then another number is
   1) 123 2) 12 3) 120 4) 1200
7. The greater number that exactly divides 105, 100, and 2436 is
   1) 3 2) 7 3) 11 4) 21
8. The greatest possible length which can be used to measure exactly the lengths 7m, 3m 85 cm, 12m 95cm
   1) 15 cm 2) 25 cm 3) 35 cm 4) 42 cm
9. The maximum number of students among then 1001 pens and 910 pencils can be distributed in Such a way that each student gets the same number of pens and same number of pencils is
   1) 91 2) 910 3) 1001 4) 1911
10. The greatest number which can divide 1356, 1868 and 2764 leaving the same remainder 12 in each case is
    1) 64 2) 124 3) 156 4) 260
11. The least number of five digits which is exactly divisible by 12, 15 and 18 is
    1) 10010 2) 10015 3) 10020 4) 10080
12. The smallest number which when diminished by 7 is divisible by 12, 16, 18, 21 and 28 is
    1) 1008 2) 1015 3) 10020 4) 10080
13. The least number which when increased by 5 is divisible by each one of 24, 32, 36 and 54 is
    1) 427 2) 859 3) 869 4) 4320
14. The least number, which when divided by 12, 15, 20 and 54 leaves in each case remainder of 8 is
    1) 504 2) 536 3) 544 4) 548
15. Six belts Commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?
    1) 4 2) 10 3) 15 4) 16

Multi Answer Type

16. The H.C.F and LCM of two numbers is 10 and $10^{12} \times 7^{2}$. If one of the number is 10^{12} then the
    1) Other number is $10^{10} \times 7^{2}$ 2) Ratio of numbers is $100:49$
    3) Other number is $10^{12} \times 7^{2}$ 4) Ratio of numbers is $1:49$
17. H.C.F and LCM of two numbers $16^{20}, 32^{56}$ are
    1) $32^{16}, 32^{56}$ 2) $16^{20}, 32^{56}$ 3) $4^{40}, 128^{40}$ 4) $2^{80}, 2^{280}$

Reasoning Answer Type

18. S.I : LCM of two numbers $2^{51}\times3^{18}$ and $2^{12}\times3^{42}\times5^{20}$ is $2^{51}\times3^{42}\times5^{20}$.

S.II : LCM of $a^p\times b^q\times c^r$ and $a^s\times b^t$ is $a^p\times b^q\times c^r$, if $p>s$ and $q>t$

1. Both Statements I and II are correct
2. Both statement I and II are incorrect
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

19. S.I : H.C.F of two numbers is 16 and their LCM is 160. If one of the numbers is 32, then the other number is 80.

S.I : Product of two numbers = Product of their HCF and LCM.

1. Both Statements I and II are correct
2. Both statement I and II are incorrect
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

Comprehension Type

Realation between HCF and LCM of two numbres is product of two number = Product of their HCF and LCM.

20. The LCM of 248 and 868 is 1736, Then HCF is

1) 248 2) 124 3) 868 4) 1736

21. The product of two numbers is 15870 and their HCF is 23. Then LCM is

1) 238 2) 158 3) 700 4) 690

22. The HCF of two numbers is 31 and their LCM is 1488. If one of the numbers is 186. Then the other number is

1) 248 2) 134 3) 736 4) 688

Matrix Matching Type

23. LCM and GCD of two numbers p and q are l and m respectively.

Column - I	Column - II
1) $p\times q$	A) m/p
2) p/m	B) $l\times m$
3) q/l	C) m/q
4) l/p	D) l/q
	E) q/m

Integer Answer Type

24. The product of LCM and GCD of the numbers 2 and 3 is ________

25. The product of numbers is 24 and LCM of the numbers is 12. Then their GCD is ________.

SYNOPSIS - 3

Factorial of the given positive integer N.
Find the number of zeros at the end of N!

1. **Factorial of the given positive integer N:**
Factorial of N is the product of the first N natural numbers.
that is N (factorial) $=1\times2\times3\times4\times5\times6\times.......\times N$.
N (factorial) is denoted by N!
All the factorial valuies of N will end in zeros when N is atleast 5.
It becomes interesting to find the number of zeros at the end of the factorial of the given number N.

2. **How to find the number of zeros at the end of N!?**
Let us understand the definition of 'Greatest Integer Function'
The greatest integer function (or floor function) will round any number down to the nearest integer. The notation for the greatest integer function is shown here. That is '[N]'.
Example: $[0.7]=0$ and $[-1.8]=-2$
Let N! is given to us.
To find the number of zxeros at the end of it.
The formula to find is $\left[\frac{N}{5}\right]+\left[\frac{N}{5^2}\right]+\left[\frac{N}{5^3}\right]+......+\left[\frac{N}{5^n}\right]$ where $5^n < N$.

WORK SHEET - 3

Single Answer Type

1. The value of $2!\times3!$ is
1) 6 2) 12 3) 18 4) 24

2. The value of $\frac{10!}{8!2!}$ is
1) 45 2) 90 3) 10 4) 0

3. The value of $\frac{(n+1)!}{1!}-\frac{n!}{(n-1)!}$ is
1) n 2) $n+1$ 3) $n-1$ 4) 1

4. If $(n+1)!=25(n!)$ then $n=$
1) 25 2) 24 3) 23 4) 22

5. If $n+n!=n^2$ then the value of $n=$
1) 0 2) 1 3) 2 4) 3

6. If $n+n!=n^3$ then the value of n =
1) 2 2) 3 3) 4 4) 5
7. L.C.M. of 3!, 6! is
1) 480 2) 680 3) 720 4) 860
8. L.C.M. of 5 and 5! is
1) 5 2) 5! 3) $5\times5!$ 4) 4!
9. The H.C.F. of 12! and 21! is
1) 12! 2) 21! 3) $12!\times21!$ 4) Does not exist
10. The no. of zero's at the end of 100! is
1) 22 2) 23 3) 24 4) 25
11. The highest power of 5 in 100! is
1) 22 2) 23 3) 24 4) 25
12. The no. of zeros at the end of 25! is
1) 5 2) 6 3) 7 3) 8
13. The highest power of 15 in 100! is
1) 48 2) 24 3) 28 4) 21
14. The highest power of 6 in 50! is
1) 21 2) 22 3) 23 4) 24
15. The no. of zeros at the end of 63! is
1) 11 2) 12 3) 13 4) 14

Multi Answer Type

16. If $a=17!$ and $b=71!$ then
1) L.C.M. of a, b is $17!$ 2) L.C.M. of a,b is $71!$
3) H.C.F. of a, b is $17!$ 4) H.C.F. of a, b is $71!$
17. Which of the following is correct?
1) $n!=n(n-1)!$ 2) $(n+1)!=(n+1)n(n-1)(n-2)!$
3) $\frac{(n+2)!}{(n+1)!}=n+2$ 4) $\frac{n!}{(n+1)!}=n+1$
18. The value of $(1.2.3....9)(11.12.13.....19)....(21.22.....29)$ =
1) $\frac{29!}{2\times10^2}$ 2) $\frac{30!}{6\times10^3}$ 3) $\frac{29!}{6\times10^3}$ 4) $\frac{30!}{2\times10^2}$
19. $1!+2!+3!+4!+5!+......+2012!$ is a/an
1) Odd number 2) Multiple of 3 3) Even number 4) Multiple of 2
20. Which of the following is correct?
1) 50! end with 12 zeros 2) 2012! end with 50! zeros
3) 108! end with 25 zeros 4) 104! end with 25 zeros

21. Which of the following is not wrong?
1) Highest power of 4 in 100! is 32
2) Highest power of 2 in 50! is 46
3) Highest power of 6 in 120! is 58
4) Highest power of 12 in 100! is 32

Reasoning Answer Type

22. Statement - I : If $n! - n = n$, then the value of n is 3
Statement - II : $0! = 1$
1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

23. Statement - I : L.C.M. of 4! and 8! is 4!
Statement - II : L.C.M. of $m!, n!$ is $m!$ if $m > n$
1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

24. Statement - I : H.C.F. of 3! and 6! is 3!
Statement - II : H.C.F. of m! and n! is m! if $m < n$
1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

25. Statement - I : The no.of zeroes does 6250! end with is 1562
Statement - II : No. of zeros at the end of N! is $\left[\frac{N}{5}\right] + \left[\frac{N}{5^2}\right] + \left[\frac{N}{5^3}\right] + \ldots\ldots + \left[\frac{N}{5^n}\right]$.
Here $5^n \le N$
1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

26. Statement - I : Highest power of 3 in 10! is 4
Statement - II : Highest power of a prime P in N! is
$\left[\frac{N}{P}\right]+\left[\frac{N}{P^2}\right]+\left[\frac{N}{P^3}\right]+.....+\left[\frac{N}{P^n}\right]$. Here $P^n \le N$
1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

Comprehension Type

Writeup-1

If $m^n - m = (m-n)!$ where $m > n > 1$ and $m = n^2$

27. Then the value of $m =$
1) 3 2) 6 3) 9 4) 12

28. The value of $m^2 + n^2$ is
1) 272 2) 20 3) 90 4) 80

29. The value of $m^3 + n^3$ is
1) 756 2) 842 3) 953 4) 645

Writeup-2

The value of $\frac{(n+1)!}{n!} = n+1$ and $n! = n(n-1)!$

30. The value of $1! + \frac{2!}{1!} + \frac{3!}{2!} + \frac{4!}{3!} + ... + \frac{n!}{(n-1)!}$ is
1) $\frac{n(n+1)}{2}$ 2) $\frac{(n-1)n}{2}$ 3) $\frac{n(n+1)(2n+1)}{6}$ 4) $\frac{n^2(n+1)^2}{4}$

31. The value of $\frac{(n+3)!}{(n+2)!} + \frac{(n+5)!}{(n+4)!} = 28$ then $n =$
1) 20 2) 15 3) 12 4) 10

32. The value of $10! \div 8! \times 2!$ is
1) 90 2) 180 3) 45 4) 12

Matrix Matching Type

33. Match the following

Column - I	Column - II
1) L.C.M. of 2011!, 2012! is	a) 2011
2) H.C.F. of 2011!, 2012! is	b) 2012
3) L.C.M. of m!, n! if $m>n$ is	c) $m!$
4) H.C.F. of $m!,n!$ if $m<n$ is	d) $n!$
	e) $\frac{4024}{2}$

34.

Column - I	Column - II
1) If $\frac{(n-1)!}{(n-2)!}=11$ then $n=$	a) 1
2) If $\frac{(n-2)!}{(n-3)!}=\frac{(n-2)!}{(n-1)!}\times 2$ then $n=$	b) 2
3) $(n-1)!\div n!\times n=$	c) 12
4) $n!+n=2n$ then n =	d) 3
	e) Multiple of 3

Integer Answer Type

35. If $(n+1)!=90(n-1)!$ then $n=$
36. L.C.M. of 2! and 3! is
37. H.C.F. of 2! and 2012! is
38. If $\frac{(n+3)!}{(n+2)!}+\frac{(n+5)!}{(n+4)!}+\frac{(n+8)!}{(n+7)!}=25$ then $n=$

SYNOPSIS - 4

UNITS DIGIT

Look at the following:

1 × 5 = 5

3 × 5 = 15

5 × 5 = 25

7 × 5 = 35

9 × 5 = 45

11 × 5 = 55

....

....

i.e., if the number whose last digit is 5, is multiplied by any odd number, the unit digit of the product will always be 5.

For example 13 ×15 = 195, 19 × 35 = 665 etc.

Now, 2 × 5 = 10

4 × 5 = 20

6 × 5 = 30

8 × 5 = 40

10 × 5 = 50

12 × 5 = 60

....

....

i.e., if the number whose last digit is 5, is multiplied by any even number (including zero), the unit digit of the product is always zero.

For example 82 × 15 = 1230, 156 × 45 = 7020

62 × 13 × 65 = 52390 etc.

Now we will discuss that the unit digit of the resultant value depends upon the unit digits of all the participating numbers

i.e., 12 + 17 + 13 + 47 = 89

Thus it is clear that the unit digit of the resultant value 89 depends upon the unit digits 2, 7, 3, 7

Similarly, 6 × 7 × 9 = 378

3 × 7 × 8 = 168

So we can find out the unit digit of the resultant value only by solving the unit digits of the given expression.

5. Cyclicity :

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$
$3^7 = 2187$
$3^8 = 6561$
$3^9 = 19683$ etc.

Similarly, $4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1024$
$4^6 = 4096$

Thus we can say that the unit digit follows a periodic pattern that is after a particular period it repeats in a cyclic form.

The unit digit of 2^1, 2^5, 2^9, 2^{13}....... is the same which is 2.

Similarly 2^2, 2^6, 2^{10}, 2^{14},...... etc. has the same unit digit which is 4,

Again the last digit of 3^1 is 3.
and the last digit of 3^2 is 9.
and the last digit of 3^3 is 7
and the last digit of 3^4 is 1.
and the last digit of 3^5 is 3.
and the last digit of 3^6 is 9.
and the last digit of 3^7 is 7.
and the last digit of 3^8 is 1.

Thus the last digit must follow a pattern. It can be seen that the last digit of 2 repeats after every four steps and the last digit of 4 repeats after every 2 steps.

Note:-

1. The last digit (or unit digits) of 0, 1, 5 and 6 is always the same irrespective of their powers raised on them.
2. The last digit 4 and 9 follows the pattern of odd-even i.e., their period is 2.
3. The last digit of 2, 3, 7, 8 repeats after every 4 steps i.e., their cyclic period

6. Sum of 'n' natural numbers is $\frac{n(n+1)}{2}$

7. Sum of squares of 'n' natural numbers is $\frac{n(n+1)(2n+1)}{6}$

8. Sum of cubes of 'n' natural numbers is $\frac{n^2(n+1)^2}{4}$

Congruences : If a and b are two integers and m is a positive integer, then a is said to be congruent to b modulo m if m divides a – b denoted by m | (a – b).

In notation form we express it as $a \equiv b$ mod m or $a - b \equiv 0$ mod m

Note :

1) $a \equiv b$ mod m, then m | (a – b) or (a – b) is a multiple of m.

2) If m | (a – b) [m does not divide (a – b), then a is said to be incongruent to b mod m and this fact is expressed as a is not congruent to b mod m.

3) If m | a, then $a \equiv 0$ mod m

For example :

1) $13 \equiv 1$ mod 4 ($\because$ 4 | (13 – 1) = 12)

2) $4 \equiv -1$ mod 5 ($\because$ 5 | 4 – (–1)) = 5)

3) $12 \equiv 0$ mod 4 ($\because$ 4 | 12)

4) 17 is not congruent to 3 mod 5 ($\because$ 5 | (17 – 3)).

Properties :

1. If $a \equiv b$ mod m, then
   i) a + c $\equiv$ b + c mod m
   ii) $ac \equiv bc$ mod m, where c is any integer

2. If $a \equiv b$ mod m and $c \equiv d$ mod m, then
   i) $a + c \equiv b + d$ mod m
   ii) $a - c \equiv b - d$ mod m
   iii) $ac \equiv bd$ mod m

3. If $a \equiv b$ mod m, then $a^k \equiv b^k$ mod m for every positive integer k.

WORK SHEET - 4

Single Answer Type

1. Unit digit in the sum of $1!+2!+3!+....+2012!$ is
   1) 1 2) 2 3) 3 4) 0
2. Unit digit in the product of $213\times512\times717\times81$ is
   1) 2 2) 3 3) 6 4) 4
3. Unit digit $2^{2012}+3^{2012}$ is
   1) 6 2) 7 3) 8 4) 9
4. Unit digit of the product $312\times215\times727\times829$ is
   1) 6 2) 4 3) 3 4) 0
5. The remainder when 3^{2012} is divisible by 4
   1) 0 2) 1 3) 2 4) 3
6. The remainder when 2^{2012} is divisible by 7
   1) 2 2) 3 3) 4 4) 6
7. The last digit in $(2012)^{2012}$ is
   1) 2 2) 4 3) 6 4) 8
8. The last digit of 43^{17} is
   1) 2 2) 6 3) 9 4) 3
9. The last digit of $222^{888}+888^{222}$ is
   1) 0 2) 2 3) 8 4) 4
10. The last digit of the expression $1^1+2^2+3^2+.....+100^2$ is
    1) 0 2) 1 3) 2 4) 4
11. $(1!+2!+3!+4!)\equiv P(\bmod 10)$. Then P is
    1) 1 2) 2 3) 3 4) 4
12. If $A=2^2\times3^2, B=2^3\times3^3$ if $AB\equiv P(\bmod 10)$ then the value of P is
    1) 4 2) 6 3) 12 4) 18
13. If $12\equiv P(\bmod 5)$ then the least positive value of P is
    1) 0 2) 7 3) 2 4) 12
14. If $a\equiv b(\bmod m)$ then the least positive value of P is
    1) 0 2) 1 3) 2 4) 3
15. If $3x+4\equiv-2\ (\bmod 3)$ then the least positive value of P is
    1) 0 2) 1 3) 2 4) 3

Multi Answer Type

16. If $P = 222^{888} + 888^{222}$ then

1) Last digit of P is 0 2) Last digit of 222^{888} is 6

3) Last digit of 888^{222} is 3 4) Last digit of P is 9

17. If $2x+1=15, 3y-2=16$ if $x \equiv P(\bmod 5)$, $y = q(\bmod 5)$ then

1) $P=2$ 2) $P=1$ 3) $Q=1$ 4) $Q=2$

18. If $a \equiv b(\bmod m)$ and 'K' is any constant then

1) $a+K \equiv b+K(\bmod m)$ 2) $aK \equiv bK(\bmod m)$

3) $a-K \equiv b-K(\bmod m)$ 4) $a+K \equiv b(\bmod m)$

19. If $a \equiv b(\bmod m)$ and $c \equiv d(\bmod m)$ the

1) $a+c \equiv (b+d)(\bmod m)$ 2) $a-c \equiv b-d(\bmod m)$

3) $ac \equiv bd(\bmod m)$ 4) $a+c \equiv (b-d)(\bmod m)$

20. Which of the following statements is true?

1) $45 \equiv -5(\bmod 10)$ 2) $157 \equiv 7(\bmod 15)$ 3) $17 \equiv 3(\bmod 5)$ 4) $(72 \equiv 8)(\bmod 9)$

21. Which of the following is correct?

1) The unit digit of $(2012)^{2012} + (2013)^{2012}$ is 7

2) The remainder when $93 \times 35 \times 2012 \div 7$ is zero

3) The remainder when $82 \times 47 \times 38 \div 3$ is one

4) The unit digit of $(5213)^{100}$ is one

Reasoning Answer Type

22. Statement - 1: $15 \equiv 3(\bmod 4)$ the true

Statement - 2: If $a \equiv b(\bmod m)$ then $\frac{m}{a-b}$

1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

23. Statement - 1: The unit digit of 10^{100} is 0

Statement - 2: The unit digit of 6^{216} is 6

1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

24. Statement - 1: The unit digit of 9^{2012} is 1

Statement - 2: The cyclicity of 9 in finding unit digit is 4

1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

25. Statement - 1: Sum of the first 100 natural numbers is 5050

Statement - 2: Sum of squares of 'n' natural numbers is $\frac{n(n+1)(2n+1)}{6}$

1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

Comprehension Type

Writup-1

If a and b are two integers and m is positive integer, then a is said to be congruent to b module n. If m divides a- b denoted by $\frac{m}{a-b}$. In rotation we can express it as $a \equiv b(\mod m)$ or $a-b \equiv 0 \ (\mod m)$

26. The remainder when 5^{500} is divided by 13 is

1) 1 2) 3 3) 7 4) 9

27. If $100 \equiv x(\mod 7)$, the least positive value of x is

1) 1 2) 2 3) 3 4) 4

28. If $121 \equiv (10-x)(\mod 5)$ then the least positive value of x is

1) 1 2) 2 3) 3 4) 4

Writup-2

The last digit (or unit digits) of 0, 1, 5, 6 is always the same irrespective of their powers raised on them. The last digit 4 and 9 follows the pattern of odd-even. i.e. their period is 2 the last digit of 2, 3, 7, 8 represents after every 4 steps i.e. their cyclic period is 4

29. The unit digit of 777^{777} is

1) 0 2) 1 3) 3 4) 7

30. $\left(53^{53}-33^{33}\right)$ is divisible by

1) 3 2) 4 3) 9 4) 10

31. The unit digit in the (diamal) representation of 11^{2012} is

1) 1 2) 2 3) 4 4) 6

Matrix Matching Type

32. Match the following

Column - I	Column - II
1) Unit's digits in the product of $274\times378\times577\times313$ is	a) 3
2) Unit's digits in the product of $(3127)^{173}$ is	b) 2
3) If n is odd, then $n\left(n^2-1\right)$ is	c) 7
4) The digit in units place of the product $81\times82\times......\times89$	d) 8
	e) 0

33.

Column - I	Column -II
1) $3x\equiv5\left(\text{mod}\,6\right)$ then $x=$	a) 3
2) The remainder when $2^{3!}\div5$ is	b) 11
3) 483.2718 is divisible by	c) 2
4) $7x\equiv5\left(\text{mod}\,8\right)$ then x is	d) 3
	e) No solution

Integer Answer Type

34. The remainder when $(2)^{2012}\div7$ is

35. The remainder when $3^{100}\div7$ is

36. Last digit of the sum of $1!+2!+..........+10!=$

WORK SHEET – 1 (KEY)				
1) 3	2) 3	3) 3	4) 1	5) 4
6) 4	7) 4	8) 1	9) 1	10) 1
11) 3	12) 2	13) 4	14) 3	15) 4
16) 4	17) 1,3	18) 2,3,4	19) 1,2	20) 1,2,3
21) 1	22) 1	23) 1	24) 2	25) 3
26) 1	27) 1	28) 1	29) 1-b 2-d 3-a 4-c	30) 1-abe 2-c 3-de 4-b
31) 2	32) 1			

1.

2	24 — 36 — 40
2	12 — 18 — 20
2	6 — 9 — 10
3	3 — 9 — 5
	1 — 3 — 5

L.C.M $= 2\times2\times2\times3\times3\times5 = 360$

3. Required L.C.M $= \dfrac{\text{L.C.M. of 1, 5, 2, 4}}{\text{H.C.F. of 3, 6, 9, 27}} = \dfrac{20}{3}$

4. Required L.C.M $= \dfrac{\text{L.C.M. of 2, 3, 4, 9}}{\text{H.C.F. of 3, 5, 7, 13}} = \dfrac{36}{1} = 36$

5. L.C.M = Product of highest powers of prime factors $= 2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$

6. Given numbers are 3.00, 2.70 and 0.09 L.C.M of 300, 270 and 9 is 2700
   $\therefore$ L.C.M of given numbers $= 27.00 = 27$
7. 2923
8. $204 = 2^2 \times 3 \times 17; 1190 = 2 \times 5 \times 7 \times 17; 1445 = 5 \times 17^2$
   $\therefore$ H.C.F = 17.
10. H.C.F = Product of lowest powers of
    Common factors $= 3 \times 5 \times 7 = 105$
11. $= \dfrac{\text{H.C.F of 9, 12, 18, 21}}{\text{L.C.M of 10, 25, 35, 40}} = \dfrac{3}{2800}$
12. Required H.C.F $= \dfrac{\text{H.C.F of 2, 8, 64, 10}}{\text{L.C.M of 3, 9, 81, 27}} = \dfrac{2}{81}$
13. Given numbers with two decimal places are 1.75, 5.60 and 7.00 with out decimal places these numbers are 175, 560 and 700 whose H.C.F is 35
    $\therefore$ H.C.F of given numbers $= 0.35$
14. Given numbers are 1.08, 0.36 and 0.90 H.C.F of 108, 36 and 90 is 18
    $\therefore$ H.C.F of given numbers $= 0.18$

15. Let the number be $3\times$ and $4\times$ then , their
    H.C.F $= \times .50, \times = 4$
    so the numbers are 12 and 16
    L.C.M of 12 and 16 $= 48$
16. Let the required numbers be $\times$, $2\times$ and $3\times$ then their H.C.F $= \times$. So, $\times = 12$
    $\therefore$ The numbers are 12, 24 and 36
17. $A = 7^2 \times 9 \times 5^3, \quad B = 7 \times 9^2 \times 5^2, \quad C = 7^3 \times 9^3 \times 5$
    $\therefore$ H.C.F $= 7 \times 9 \times 5$ (Least powers)
    L.C.M $= 7^3 \times 9^3 \times 5^3$ (Greatest powers)
18. Numbers are $2x, 3x$
    $\therefore$ L.C.M is $6x = 48$
    $x = 8$
    $\therefore$ The numbers are 16, 24
    H.C.F of 16, 24 is 8
    Difference of 16, 24 is also 8
19. Let the required number be 27a and 276.
    Then, $27a + 27b = 216 \Rightarrow a + b = 8$.

Now, Co-primes with Sum 8 is (1, 7) and (3, 5)

$\therefore$ Required numbers are $(27\times1, 27\times7)$ and $(27\times3, 27\times5)$

$\therefore 27,189$ or $81,135$

20. $\therefore$ The numbers are 1, 117
G.C.D of Co-primes is 1.
L.C.M is product of Co-primes.
21. (1)
22. (1)

G.C.D of $(132-62),(237-132),(237-62)$

G.C.D of $70,105,175=35$

26. G.C.D of $(122-32),(157-122),(157-32)$

G.C.D of $90,35,125=5$

27. G.C.D of $(91-43),(183-91),(103-43)$

G.C.D of $48,92,140$ is 4.

28. G.C.D of $(4665-1305),(6905-4665),6905-(1305)$
G.C.D is 4.

29. $(1)\rightarrow B,\quad (2)\rightarrow D,\quad (3)\rightarrow A,\quad (4)\rightarrow C$

30. $(1)\rightarrow A,B,E,\quad (2)\rightarrow C,\quad (3)\rightarrow D,E,\quad (4)\rightarrow B$

W0RK SHEET – 2 (KEY)

1) 2	2) 3	3) 3	4) 1	5) 4
6) 3	7) 2	8) 3	9) 1	10) 1
11) 4	12) 2	13) 2	14) 4	15) 4
16) 1,2	17) 1,2,3,4	18) 1	19) 1	20) 2
21) 4	22) 1	23) 1-b 2-d 3-a 4-e	24) 6	25) 2

1. Other number $=\frac{11\times 693}{77}=99$

2. Other number $=\frac{11\times 7700}{275}=308$

3. Let the number be x and 4x.

$\therefore \quad x\times 4x \quad = \quad 84\times 21$

$x^2=\frac{84\times 21}{4}=21\times 21$

$\therefore$ Larger number $4x=84$

4. L.C.M $=\frac{\text{Product of numbers}}{\text{H.C.F}}=\frac{1320}{6}=220$

5. (4)

6. Other number $=\frac{1320\times 12}{132}=120$

7. H.C.F of 2436 and 1001 is 7.
Also, H.C.F of 105 and 7 is 7.
$\therefore$ H.C.F of 105, 1001, 2436 is 7.

8. Required length $=$ H.C.F of 700 cm, 385 cm, 1295 cm

$=35$ cm

9. Required number of students $=$ H.C.F of 1001 and 910

$=91$

10. Required number $=$ H.C.F of $(1356-12),(1868-12)$ and $(2764-12)$

$=$ H.C.F of 1344, 1856 and 2752

$=64$

11. Least number of 5 digits is 10000.
L.C.M of 12, 15 and 18 is 180.
On dividing 10000 by 180, the remainder is 100.

$\therefore$ Required number $=10000+(180-100)=10080$

12. Required number $=(\text{L.C.M of } 12, 16, 18, 21, 28)+7$

13. Required number $=(\text{L.C.M of } 24, 32, 36, 56)-5$

$=859$

14. Required number $=(\text{L.C.M of } 12, 15, 20, 54)+8$

$540+8$

$= 548$

15. L.C.M of 2, 4, 6, 8, 10, 12 is 120
    So, the bells will toll together after every 120 seconds.
    i.e. 2 minutes.
    In 30 minutes, they will toll together, $\frac{30}{2}+1=16$ times.

16. Othe number $=\frac{10^{10}\times10^{12}\times7^{2}}{10^{12}}=10^{10}\times7^{2}$

    $\therefore$ Ratio of number is $10^{12}:10^{10}\times7^{2}$

    $100:49$

17. Given number are $16^{20}, 32^{56}$

    $\therefore$ H.C.F is $16^{20}=32^{16}=4^{40}=2^{80}$

    L.C.M is $32^{56}=128^{40}=2^{280}$

18. (1)
19. (1)
20. $\therefore$ H.C.F $=\frac{\text{Product of numbers}}{\text{L.C.M}}$

    $=\frac{248\times868}{1736}=124$

21. L.C.M $=\frac{15870}{23}=690$

22. Other number $=\frac{1488\times31}{186}=248$

23. $(1)\to B,\quad (2)\to D,\quad (3)\to A,\quad (4)\to E$

24. $\text{L.C.M}\times\text{G.C.D}=\text{Product of number}$

    $=2\times3$

    $=6$

25. G.C.D $=\frac{\text{Product of numbers}}{\text{L.C.M}}$

    $=\frac{24}{12}$

    $=2$

W0RK SHEET – 3 (KEY)				
1) 2	2) 1	3) 4	4) 2	5) 4
6) 4	7) 3	8) 2	9) 1	10) 3
11) 3	12) 2	13) 2	14) 2	15) 4
16) 2,3	17) 1,2,3	18) 1,2	19) 1,2	20) 1,2,3
21) 1,2,3,4	22) 1	23) 4	24) 1	25) 1
26) 1	27) 3	28) 3	29) 1	30) 1
31) 4	32) 2	33) 1-bc 2-a 3-c d-c	34) 1-c,e 2-d,e 3-a 4-a,b	35) 9
36) 6	37) 2	38) 3		

1. $2!\times 3! = 2\times 6 = 12$

2. $\dfrac{10\times 9\times 8!}{8!\times 2} = 45$

3. $(n+1)-n=1$

4. $(n+1)n! = 25.n!$

$n+1=25$

$n=24$

5. $n+n(n-1)! = n^2$

$1+(n-1)! = n$

$(n-1)! = (n-1)$

$(n-2)! = 1$

6. By verification $n=5$

7. L.C.M. of 3!, 6! = Greatest of $(3!,6!) = 6! = 720$

9. H.C.F $(12!, 21!) =$ Least of $(12!, 21!) = 12!$

10. No. of zeros = $\left[\frac{100}{5}\right]+\left[\frac{100}{5^2}\right]+\left[\frac{100}{5^3}\right] = 20+4+0 = 24$

11. Highest power of 5 in 100! = $\left[\frac{100}{5}\right]+\left[\frac{100}{5^2}\right]+\left[\frac{100}{5^3}\right] = 24$

13. $15 = 3\times 5$

$\therefore$ Heighest power of 3 in 100!

$=\left[\frac{100}{3}\right]+\left[\frac{100}{3^2}\right]+\left[\frac{100}{3^3}\right]+\left[\frac{100}{3^4}\right]$

$=33+11+3+1=48$

Heighest power of 5 in 100! = 24

$\therefore \min(24,48)=24$

18. $\frac{(1.2.3...9)10.(11,12....19)20.(21.12....29)}{10.20} = \frac{29!}{2\times 10^2}$

19. $(1!+2!+3!+4!)+5!+........+2012!$

$(1+2+6+24)+(5!+.......+2012!)$

$33+(\text{also multiple of } 3)$ which is odd

27. Because of $m>n>1, m=n^2$

By verification; If $n=2$ then $m=4$

$\therefore 4^2-4\neq(4-2)!$

If $n=3$, then $m=9$

$9^3-9=(9-3)!$

$729-9=6!$

$720=6!$

$\therefore m=9, n=3$

28. $m^2+n^2=9^2+3^2=90$

29. $m^3+n^3=9^3+3^3+729+27=756$

30. $1+2+3+.......+n=\frac{n(n+1)}{2}$

31. $n+3+n+5=28$

$2n = 28.8$

$n = 10$

32. $\frac{10!}{8!} \times 2 = \frac{10 \times 9 \times 8!}{8!} \times 2 = 180$

33. L.C.M. of 2011!, 2012! is 2012!
H.C.F of 2011!, 2012! is 2011!
L.C.M. of m!, n! is m! if m = n
H.C.F of m!, n! is m! if m < n

34. a) $n - 1 = 11 \Rightarrow n = 12$

b) $(n-1)! = 2(n-3)!$

$(n-1)(n-2) = 2$

$\therefore n = 3$

c) $\frac{(n-1)!}{n!} \times n = \frac{(n-1)!}{n(n-1)!} \times n = 1$

d) $n! = n$

$(n-1)! = 1$

$n - 1 = 0 \qquad n - 1 = 1$

$n = 1 \qquad n = 2$

35. $(n+1)n(n-1) = 90(n-1)!$

$\therefore n(n+1) = 90$

$\therefore n = 9$

38. $n + 3 + n + 5 + n + 8 = 25$

$3n + 16 = 25$

$3n = 9$

$n = 3$

WORK SHEET – 4 (KEY)				
1) 3	2) 1	3) 2	4) 4	5) 2
6) 3	7) 3	8) 4	9) 1	10) 1
11) 3	12) 2	13) 3	14) 2	15) 2
16) 2,3,4	17) 1,3	18) 1,2,3	19) 1,2,3	20) 1,2
21) 1,2,3,4	22) 1	23) 1	24) 3	25) 1
26) 1	27) 2	28) 4	29) 2	30) 4
31) 1	32) 1-b 2-c 3-a,d 4-c	33) 1-e 2-a 3-b 4-d	34) 4	35) 4
36) 3				

1. $1!+2!+3!+4!+5!+....+2012!$

   $1+2+6+24+120+......$

   $33+120+720+.....=$ This sum ends with 3.

2. Unit digit of the product $213\times512\times717\times81$ is same as unit digit of the product $3\times2\times7\times1$ which is 2.

3. Cyclicity of 2, 3 is 4.
   2012 is exactly divisible by 4.

   $\therefore$ Unit digit of $2^{2012}=6$

   Unit digit of $3^{2012}=1$

   $\therefore$ Sum is 7.

4. $2\times5\times7\times9$ in this product end with 0.

5. $3^{2012}\div4$

   $$\frac{\left(3^2\right)^{1006}}{4}=\left(1\right)^{1006}=1$$

6. $2^{2012} \div 7 = \frac{\left(2^3\right)^{670}}{7}.2^2 = \frac{1\times 4}{7} = 4$

8. Last digit of 43^{17} is same as 3^{17} cyclicity of 3 is 4.

$\therefore$ Last digit is 3.

9. Last digit of 222^{888} is same as 2^{888}.

$\therefore$ Unit digit is 6.

Last digit of 888^{222} is same as 8^{212}

$\therefore$ Unit digit is 4

$\therefore$ Sum end with 0.

10. The unit digit of the whole expression will be the equal to the unit digit of the sum of the unit digits of the expression.

Now adding the unit digits of $1^2 + 2^2 + ... + 10^2$

We get $1+4+9+6+5+6+9+4+1+0 = 45$

Hence, the unit digit of $1^2 + 2^2 + ...10^2$ is 5.

Now, since there are 10 similar columns of numbers which will yield the same unit digit 5. Here the sum of the unit digits of all the 10 columns is 50

$(5+5+.....+5)$

$\therefore$ Unit digit of the given expression is zero.

11. $33 \equiv P(\text{mod}10)$

$33-P$ is divisible by 10

$\therefore P = 3$

12. $AB = 2^2 \times 3^2 + 2^3 \times 3^3 = 2^5 \times 3^5 = 6^5 = 7776$

$7776 \equiv 6(\text{mod}10)$

$\therefore P = 6$

13. $12 \equiv P(\text{mod}\,5)$

$\therefore P = 2$

15. $3x+6$ is divisible by 3

$\therefore x = 1, 2, 3......$

$\therefore$ Least value = 1

17. $2x+1=15$ $3y-2=16$

$x=7$ $y=6$

$7 \equiv 2(\text{mod}\,5)$ $6 \equiv 1(\text{mod}\,5)$

$\therefore P = 2 Q = 1$

21. 1) Unit digit is $(2012)^{2012}+(2013)^{2012}$ is same as $2^{2012}+3^{2012}$

$\therefore$ Unit digit is 6 + 1 = 7

2) 35 is exactly divisible by 7

$\therefore$ Remainder is zero

3) $\frac{2\times7\times8}{3}=\frac{2\times1\times2}{3}=\frac{4}{3}=1$

4) Unit digit of $(5213)^{100}$ is same as 3^{100}.

Cyclicity of 3 is 4.

$\therefore$ Unit digit is 1.

26. We know that $5^4=625=13\times48+1$

$\therefore 5^4\equiv1(\text{mod}13)$

$\left(5^4\right)^{125}=1^{125}(\text{mod}13)$

$5^{500}\equiv1(\text{mod}13)$

$\therefore$ remainder is 1.

27. $100-x$ is divisible by 7.

$\therefore x=2$

28. $(121)-(10-x)$ is divisible by 5

$(111+x)$ is divisible by 5.

$\therefore x=4$

29. Unit digit of 777^{777} is same as 7^{777}

Cyclicity of 7 is 4.

$\therefore$ Unit digit is 1.

30. Unit digit of 53^{53} is 3

Unit digit of 33^{33} is 3

$\therefore$ Difference of these two ends with 0

$\therefore$ Given expression is divisible by 10.

31. Unit 2 digit of 11^{2012} is same as 1^{2012}

$\therefore$ Unit digit is 1.

32. $1-b, 2-c, 3-a, d, 4-e$

33. $1-e, 2-a, 3-b, 4-d$

VERBAL REASONING (KEY)				
1) B	2) C	3) C	4) A	5) D
6) D	7) D	8) A	9) D	10) C
11) A	12) C	13) B	14) C	15) B
16) D	17) B	18) B	19) B	20) B
21) C	22) C			

1. (b) : $R \xrightarrow{+3} U \xrightarrow{+3} X \xrightarrow{+3} A \xrightarrow{+3} D \xrightarrow{+3} G$
2. (c) : $B \xrightarrow{+2} D \xrightarrow{+2} F \xrightarrow{+3} I \xrightarrow{+3} L \xrightarrow{+4} P \xrightarrow{+4} (T)$
3. (c) : $H \xrightarrow{+1} I \xrightarrow{+2} K \xrightarrow{+3} N \xrightarrow{+4} (R)$
4. (a) : $A \xrightarrow{+6} G \xrightarrow{+5} L \xrightarrow{+4} P \xrightarrow{+3} S \xrightarrow{+2} (U)$
5. (d) : $A \xrightarrow{+3} D \xrightarrow{+4} H \xrightarrow{+5} M \xrightarrow{+6} (S) \xrightarrow{+7} Z$
6. (d) : $Z \xrightarrow{-5} U \xrightarrow{-4} Q \xrightarrow{-3} (N) \xrightarrow{-2} L$
7. (d) : $(Z \to Y \to X) \xrightarrow{-3} (U \to T \to S) \xrightarrow{-3} (P \to O \to N) \xrightarrow{-3} (K \to (J) \to (I))$
8. (a) : $Z \xrightarrow{-2} X \xrightarrow{-5} S \xrightarrow{-10} I \xrightarrow{-17} R \xrightarrow{-26} R \xrightarrow{-37} (G) \xrightarrow{-50} (I)$
9. (d) : The given sequence is a combination of two series :

   I. 1st, 3rd, 5th, 7th, 9th, 11th, terms $i.e. A, B, C, D, E, ?$

   II. 2nd, 4th, 6th, 8th, 10th terms $i.e. B, D, F, H\,?$

   Clearly, I consists of consecutive letters while II consists of alternate letters. So, the missing letter in I is F, while that in II is J.

   So, the missing terms $i.e. 10th$ and $11th$ terms are J and F respectively.
10. (c) : The given sequence is a combination of two series :

    I. C, F, I, L, O, ? and

    II. Z, X, V, T, ?

    The pattern in I is : $C \xrightarrow{+3} F \xrightarrow{+3} I \xrightarrow{+3} L \xrightarrow{+3} O \xrightarrow{+3} (R)$

    The pattern in II is : $Z \xrightarrow{-2} X \xrightarrow{-2} V \xrightarrow{-2} T \xrightarrow{-2} (R)$
11. (a) :The given sequence is a combination of two series :

    I. Z, W, T, Q, ? and II. S, O, K, G, ?

The pattern in I is : $Z \xrightarrow{-3} W \xrightarrow{-3} T \xrightarrow{-3} Q \xrightarrow{-3} (N)$

The pattern in II is : $S \xrightarrow{-4} O \xrightarrow{-4} K \xrightarrow{-4} G \xrightarrow{-4} (C)$

12. (c) : The number of letters in the terms of the given series increases by one at each step.
The first letter of each term is two steps ahead of the last letter of the preceding term. However, each term consists of consecutive letters in order.

13. (b) : 1st letter : $A \xrightarrow{+1} B \xrightarrow{+1} C \xrightarrow{+1} (D)$

2nd letter : $Z \xrightarrow{-6} T \xrightarrow{-6} N \xrightarrow{-6} (H) \xrightarrow{-6} B$

14. (c) : 1st letter : $A \xrightarrow{+2} C \xrightarrow{+3} F \xrightarrow{+4} (J)$

2nd letter : $Z \xrightarrow{-2} X \xrightarrow{-3} U \xrightarrow{-4} (Q)$

15. (b) : 1st letter : $a \xrightarrow{+6} g \xrightarrow{+6} (m) \xrightarrow{+6} s \xrightarrow{+6} y$

2nd letter : $j \xrightarrow{+6} p \xrightarrow{+6} (v) \xrightarrow{+6} b \xrightarrow{+6} h$

3rd letter : $s \xrightarrow{+6} y \xrightarrow{+6} e \xrightarrow{+6} k \xrightarrow{+6} q$

16. (d) : 1st letter : $B \xrightarrow{+2} D \xrightarrow{+2} F \xrightarrow{+2} (H)$

2nd letter : $M \xrightarrow{+1} N \xrightarrow{+1} O \xrightarrow{+1} (P)$

3rd letter : $X \xrightarrow{-1} W \xrightarrow{-2} U \xrightarrow{-1} (T)$

17. (b) : 1st letter : $A \xrightarrow{+3} D \xrightarrow{+4} H \xrightarrow{+5} M \xrightarrow{+6} S \xrightarrow{+7} (Z)$

2nd letter : $B \xrightarrow{+5} G \xrightarrow{+6} M \xrightarrow{+7} T \xrightarrow{+8} B \xrightarrow{+9} (K)$

3rd letter : $D \xrightarrow{+7} K \xrightarrow{+8} S \xrightarrow{+9} B \xrightarrow{+10} L \xrightarrow{+11} (W)$

18. (b) :1st letter : $W \xrightarrow{-3} T \xrightarrow{-3} Q \xrightarrow{-3} (N)$

2nd letter : $F \xrightarrow{+1} G \xrightarrow{+1} H \xrightarrow{+1} (I)$

3rd letter : $B \xrightarrow{+2} D \xrightarrow{+3} G \xrightarrow{+4} (K)$

19. (b) :1st letter : $U \xrightarrow{-2} (S) \xrightarrow{-4} O \xrightarrow{-2} M \xrightarrow{-4} I$

2nd letter : $P \xrightarrow{-8} (H) \xrightarrow{-4} D \xrightarrow{-2} B \xrightarrow{-1} A$

3rd letter : $I \xrightarrow{+1} (J) \xrightarrow{+6} P \xrightarrow{+1} Q \xrightarrow{+6} W$

20. (b) :1st letter : $A \xrightarrow{+1} B \xrightarrow{+2} D \xrightarrow{+3} G \xrightarrow{+4} (K)$

2nd letter : $Y \xrightarrow{-3} V \xrightarrow{-4} R \xrightarrow{-5} M \xrightarrow{-6} (G)$

3rd letter : $D \xrightarrow{+2} F \xrightarrow{+2} H \xrightarrow{+2} J \xrightarrow{+2} (L)$

21. (c) : Each term consists of consecutive letters in order. The number of letters in the terms goes on increasing by one at each step. Also, there is a gap of one letter between the last letter of the first term and the first letter of the second term; a gap of two letters between the last letter of the second term and the first letter of the third terml; and so on. So, there should be a gap of three letters between the last letter of the third term and the first letter of the desired term.

22. (c) : One letter from the beginning and one from the end of a term are removed, one by one, in alternate steps.