LINEAR EQUATIONS
SYNOPSIS - 1
FUNDAMENTAL CONCEPTS:

1) Expressions of the form 9 + 2, 11 - 7, $4 \times 20, \dfrac{33}{7}$ etc., are called ' **numerical expressions'**.

2) If two numerical expresssions are joined or connected by ' is equal to' (=) or ' is greater than' (>) or 'is less than' (<) etc., then they are called **'mathematical sentences'.**

Example: 9 + 3 **<** 15.

3) A mathematical sentence that can be verified either true or false but not both, is called a **'mathematical statement'.**

Example: 9 + 7 > 9.

4) A true mathematical statement containing the sign "=" is called as an **equality**.

Example: 9 × 2 = 18.

5) Sentences which cannot be verified either true or false are called **'open sentences'.**

Example: (i) 2x + 3 = 5,   (ii) 4x + 1 < 8

6) A particular number written in the place of a variable in an open sentence is called **'replacement'.**

EQUATION: If two numerical expressins are joined or connected by the symbol "is equal to(=)" , then the combination is called as an equation.

                                         or

An open sentence containing the sign '=' is called an equation.

L.H.S. and R.H.S. Notations : The sign of equality '=' in an equation divides it into two sides namely, the left hand side and the right hand side, written as L.H.S. and R.H.S. respectively.

Example: 6x + 2 = 9x – 5 here, L.H.S = 6x + 2   R.H.S = 9x -5

Solution of an Equation : The value of a variable which satisfies(L.H.S = R.H.S) the equation is called the solution or root of an equation.

Example: 6x + 2 = 9x - 4 . Here LHS = 6x + 2 and RHS = 9x -4

Above equation is true only when x = 2 i.e if x = 2, then

LHS = 6 × 2 + 2 = 14   RHS = 9×2 – 4 = 14 Hence, LHS = RHS

LINEAR EQUATION: An equation in which the highest power of a variable (index) is one called 'linear equation'.

Example: 2x + 3y + 5 = 0 and 9x – 7 = 2010

LINEAR EQUATION IN ONE VARIABLE: Linear equation which involves one variable is called 'linear equation in one variable' or 'simple equation'.

Example: (i) 4x -3 = x + 7   (ii) y + 7 = 11

Note: ax + b = 0 is the general form of linear equation in one variable, where $(a \neq 0)$ and a, b are real numbers.

L.H.S. AND R.H.S. NOTATIONS: The sign of equality '=' in an equation divides it into   two sides namely, the left hand side and the right hand side, written as L.H.S. and R.H.S. respectively.

Example: 7x + 8 = 9x - 10   here, L.H.S = 7x + 8 and R.H.S = 9x - 10

Solution of an Equation: The value of a variable which satisfies(L.H.S = R.H.S) the equation is called the solution or root of an equation.

Example: $2x + 6 = 10$ here, L.H.S. = $2x + 6$ R.H.S. = 10

Above equation is true only when x = 2 i.e., x = 2 $\Rightarrow$ L.H.S. = $2 \times 2 + 6 = 10$
R.H.S. = 10

$\therefore$ L.H.S = R.H.S.

$\therefore$ Root of $2x + 6 = 10$ is 2.

Note: Method of finding the solution for the given equation is called 'solving an equation'.

DOMAIN OF THE VARIABLE: The replacement set of the variable of an equation is called the 'Domain of the variable'.

Types of Finding the Solutions of Linear Equations :

1) Trail and error method   2) Guess method

3) Systematic method   4) Transposition mehod.

Trial and Error method: Finding the root or solution or true value of the given equation from its domain is called the Trial and error method.

Note: Under trial and error method, trying all values of the variable of the equation takes too much of time. It becomes impossible when the domain contains infinite   number of elements such as **N** or **W** or **Z**. So, guessing some times helps up to solve an equation easily.

Example: Domain of the equation $2x + 3 = 7$ is {1, 2, 3........}, where 2 is the root of   the given equation.

Systematic method for solving an equation:

In this case we compare the equation with a balance, by using the following rules.

1) We can add the same number on both sides of the equation.

2) We can subtract the same number from both sides of the equation.

3) We can multiply both sides of the equation with the same number.

4) We can divide both sides of the equation by the same non-zero number.

5) If $\dfrac{ax + b}{cx + d} = \dfrac{p}{q}$, then q(ax + b) = p(cx + d). This process is called Cross Multiplication.

Transposition: Any term of an equation may be taken to the other side with a change in its sign. This process is called transposition.

Examples: i) $x - 5 = 7 \Rightarrow x - 5 + 5 = 7 + 5 \Rightarrow x = 12$

ii) $x + 2 = 4 \Rightarrow x + 2 - 2 = 4 - 2 \Rightarrow x = 2$

iii) $\dfrac{x}{7} = 12 \Rightarrow x = 12 \times 7 \Rightarrow x = 84$

iv) $2x = 40 \Rightarrow \dfrac{2x}{2} = \dfrac{40}{2} \Rightarrow x = 20$

Law of cancellation: If ac = bc provided c$\neq$ 0 then a = b i.e., both sides of an equality can be divided by the same non-zero number.

Note: A linear equation in one variable 'x' is solved by getting all terms containing 'x' on one side and every thing else on the other side.

WORK SHEET - 1

SINGLE ANSWER TYPE

1. If two numerical expressions are joined by '=' or '$\geq$' or '<' then they are known as
    1) mathematical sentences   2) Theorens
    3) Axioms   4) None of these

2. Sentences which cannot be verified for their truth or false hood are called _____ sentences
    1) true   2) false   3) open   4) closed

3. If $x + 9 = 12$, then the value of x
    1) 1   2) 2   3) 3   4) 4

4. If $123 > x + 11$, then one of the value sutiable to the variable x is
    1) 112   2) 113   3) 117   4) 111

5. If 4 times a number m is 96, then the value of $m \times 24$ is _____
    1) 576   2) 676   3) 558   4) 658

6. If $\dfrac{x}{4} + \dfrac{x+2}{3} + \dfrac{x+3}{2} = 0$ then the value of x
    1) - 1   2) - 3   3) - 2   4) - 4

7. If $x + 3 = 9, x + y = 14, y + z = 20$ then the value of $x + y + z$ is _____
    1) 6   2) 12   3) 20   4) 26

8. If $(x - 4)(x + 4) = (x + 4)(x - 7) + 33$, then the value of x is
    1) 7   2) 6   3) 5   4) 8

9. If $\dfrac{6}{3m+1} = \dfrac{9}{5m-3}$ then the value of $2m + 1$ is _____
    1) 15   2) 17   3) 19   4) 23

10. If $\dfrac{0.4^{2-3}}{1.5z + 10} = \dfrac{7}{5}$ then the value of z is
    1) 3   2) - 4   3) 1.3   4) - 10

11. If 24 is added to 6 times a number, then the result is 10 less than four times the number, then the number is _____
    1) –17   2) 27   3) –23   4) 19

12. Solution of $\dfrac{2}{3}x + 1 = 5$ is _____
    1) 2   2) 4   3) 6   4) 8

13. Solution of 0.5m = 0.65 is _____
    1) 13   2) 1.3   3) 130   4) 0.13

14. Finding the root or solution or true volue of the given equation from its domain is called _____.
   1) Trial and Error Method   2) Gauss Method
   3) Systematic Method   4) Transposition Method

15. If 11x + 5 = 137, then x = _____
   1) 11   2) 12   3) 13   4) 14

16. If $\dfrac{1}{m+1} = \dfrac{5}{8m-1}$, then the possible value of 'm' is _____
   1) 9   2) –5   3) –3   4) 2

17. If $\dfrac{2x}{3} - \dfrac{x-8}{6} = 2\dfrac{(2x+9)}{9}$, then the solution for x is _____
   1) 45   2) 52   3) 27   4) 39

18. If $\dfrac{x-3}{x+5} = \dfrac{x-2}{x+2}$, then the solution for x is _____
   1) 0   2) 1   3) 2   4) 4

19. IF $\dfrac{0.5m+4}{1.2m+6} = \dfrac{5}{3}$, then the possible value of 'm' is _____
   1) $\dfrac{11}{5}$ 2) –4   3) $\dfrac{9}{5}$ 4) –6

20. If (2 + x) : (7 + x) : (10 + x) : (25 + x), then 'x' is _____
   1) Even prime   2) Odd prime
   3) Leavst Natural number   4) Least whole number

21. If $5x - \dfrac{1}{3}(x+1) = 6\left(x + \dfrac{1}{30}\right)$, then the possible value of 'x' is _____
   1) $\dfrac{-2}{5}$ 2) $\dfrac{-4}{5}$ 3) $\dfrac{3}{5}$ 4) $\dfrac{-6}{5}$

MULTI ANSWER TYPE

22. If $P < 175 + 194$, then the value of the variable 'p' from the following is
   1) 369   2) 367   3) 366   4) 370

23. If $(18-8)^2 = P, (28-4)^2 = Q$ then P + Q is
   1) $24^2 + 10^2$ 2) 6 + 6   3) 26^2 4) 34^2

24. If $\dfrac{x+2}{x-2} = \dfrac{7}{3}$, then x = _____.
   1) 5   2) 10   3) $\sqrt{25}$ 4) $\sqrt{100}$

25. If $\dfrac{13k-1}{6} + \dfrac{2k+5}{3} = \dfrac{5}{9}$, then the possible value of 'K' is _____
   1) –1   2) 1   3) Least Natural number   4)Greatest negative integer

26. If $\frac{2}{3}(4x-1)-\left(2x-\frac{1+x}{3}\right)=\frac{x}{3}+\frac{4}{3}$, then the solution for 'x' is _____

   1) $1\frac{1}{2}$ 2) $2\frac{1}{2}$ 3) $\frac{3}{2}$ 4) $\frac{5}{2}$

REASONING ANSWER TYPE

27. *Statement I:* If $x-2=10$ then value of x is 12

   Statement II: If a number satisfies an equation, then the numbers is known as a root of the equation.
   1)   Both Statement-I and Statement-II are true.
   2)   Both Statement-I and Statement-II are false.
   3)   Statement I is true, Statement II is false.
   4)   Statement I is false, Statement II is true.

28. *Statement I:* If $2\frac{1}{2}-\frac{x-2}{4}=\frac{x}{2}$ then the value of x is 3

   Statement II: If $\frac{x+1}{3}-\frac{x-1}{4}=1$ then the value of x is $\frac{11}{12}$
   1)   Both Statement-I and Statement-II are true.
   2)   Both Statement-I and Statement-II are false.
   3)   Statement I is true, Statement II is false.
   4)   Statement I is false, Statement II is true.

29. Statement I : If 3x + 5 = 15x – 11, then x = 4/3
   Statement II : If the domain of the equation 7x + 3 = 17 is {1, 2, 3,---------},
   then '2' is the root of the opiven equation.
   1)   Both Statement-I and Statement-II are true.
   2)   Both Statement-I and Statement-II are false.
   3)   Statement I is true, Statement II is false.
   4)   Statement I is false, Statement II is true.

30. Statement I : If $\frac{5}{x+2}-\frac{3}{x-2}=\frac{3}{2}$, then the possible value of x is '0'

   Statement II : The value of the variable for which LHS = RHS is the root of the equation.
   1)   Both Statement-I and Statement-II are true.
   2)   Both Statement-I and Statement-II are false.
   3)   Statement I is true, Statement II is false.
   4)   Statement I is false, Statement II is true.

COMPREHENSION TYPE

Writeup-1:
   The root of the equation is satisfies the equation
31. If $3x+7=28$ then the root of the equation is
   1) 7   2) 8   3) 9   4) 6

32. If $3p - 5 = 19$ then the root is
    1) 2   2) 9   3) 8   4) 6

33. $2p - 5 = 5$ then the value of p is
    1) 2   2) 3   3) 4   4) 5

Writeup-2:

$$\frac{3x_1 - 1}{4\frac{1}{2}} - \frac{x_1 - 1}{2} = 0 \quad \text{and} \quad \frac{4}{5}\left(x_2 + \frac{5}{6}\right) - \frac{2}{3}\left(x_2 - \frac{1}{4}\right) = \frac{10}{9}$$

34. Then value of $3x_1 - 1$ is

    1) $-\frac{5}{3}$ 2) $\frac{3}{5}$ 3) - 6   4) 2

35. Value of $\dfrac{12x_2}{25}$ is

    1) 1   2) 2   3) 3   4) 4

36. Value of $(3x_1 - 1) + \dfrac{12x_2}{25}$ is

    1) - 5   2) 3   3) 4   4) None

Writeup-3:

Under trial and error method, trying all volues of the variable of the equation takes too much tirne. If becomes impossible when the domain contains infinte number of elements such as N or W or Z. so. guessing some times helps up to solve the equation easily.

37. 18a + 216 = 0, then a $\in$ _____
    1) Z   2) N   3) W   4) All of these

38. Solution for 2m + 8 = 2 is _____
    1) -5 2) -1 3) -3 4) -2

39. In x + 3 = 5, the root x = 2 is a _____.

MATRIX MATCHING TYPE

40. Column-I   Column-II

    a) $x_1 - 9 = 11$ 1) 42

    b) $x_2 \times 3 = 63$ 2) 19

    c) $x_3 + 3 = 22$ 3) 20

    d) $x_4 \div 6 = 7$ 4) 21

41. Column-I   Column-II

a) $\dfrac{1}{2}(x+1)+\dfrac{1}{3}(x+4)=\dfrac{1}{6}(x-1)$ 1) 9

b) $\dfrac{1}{3}(x+5)-\dfrac{1}{2}(x+2)=\dfrac{1}{3}(x+5)$ 2) - 2

c) $\dfrac{2}{3}(x-4)=5-\dfrac{1}{6}(x+1)$ 3) - 7/3

d) $\dfrac{2}{5}(x-6)+(x+1)=\dfrac{7}{2}(x+1)$ 4) - 3

42. Column-I   Column-II
a) If $3(x-3)=5(2x+1)$, then x = ____   1) 1
b) If $2(x-5)+3(x-2)=8+7(x-4)$___   2) –2
   then x = _____.

c) If $\dfrac{3y-2}{7}-\dfrac{5y-8}{4}=\dfrac{1}{4}$, then y = ____   3) –1

d) If $\dfrac{5x-3}{2}-\dfrac{3x-2}{3}=\dfrac{2}{3}$, then x = ____   4) 2

                                                   5) –3
43. Column-I   Column-II

a) 12 is a root of the equation_____   1) $\dfrac{3x+5}{2x+1}=\dfrac{1}{3}$

b) 8 is a root of the equation _____   2) $\dfrac{x}{4}+\dfrac{x}{6}=x-7$

c) '–2' is a root of the equation _____   3) $\dfrac{2x}{3}+1=\dfrac{7}{3}$

d) '2' is a root of the equation _____   4) $\dfrac{y+6}{4}+\dfrac{y-3}{5}=\dfrac{5y-4}{8}$

                                                5) $5x+7=2x+1$

INTEGER ANSWER TYPE

44. If $\dfrac{m}{5}=6$ and $p-4=8$ then the value of $m+p=$_____

45. If $\dfrac{2x+1}{3x-1}=\dfrac{3}{2}$, then the possible value of 'x' is _____

SYNOPSIS - 2

Simple equations are useful in solving simple practical problems, these problems are generally given in words.

STEPS TO BE FOLLOWED IN SOLVING A VERBAL PROBLEMS:

i) Read the problem carefully and note down the asked quantity.

ii) Select a letter x or y or z etc., to represent the asked quantity.

iii) Write an equation in the letter selected, showing the relationship in the problem.

iv) Solve the equation obtained in step (iii).

v) Check the answer to make sure that it satisfies the given conditions in the problem.

To solve practical problems given in words, we must first change the word statements into symbolic statements.

Example–1: 5 added to a certain number gives 11. Find the number.

Solution: Let the required number be 'x'

If 5 is added to 'x' we get 5 + x.

According to the problem this is equal to 11

∴ Equation is 5 + x = 11

By guess method or trial and error method, we find that x = 6 is a solution.

Veritication: If 5 is added to 6 we get 11, satistying the condition given in the problem

Example–2: One number is 3 times the another number. If 15 is added to both the numbers, then one of the new numbers becomes twice that of the other new number. Find the numbers.

Solution: Let one number be 'x'.

Then the other number = 3x.

By the given condition, one number +15 = 2 (other number + 15)

i.e., $3x + 15 = 2(x + 15) \Rightarrow 3x + 15 = 2x + 30 \Rightarrow 3x - 2x = 30 - 15 \Rightarrow x = 15$.

Hence, one number is 15 and the other number = $3 \times 15 = 45$.

WORK SHEET - 2

SINGLE ANSWER TYPE

1. The solution for a + b = 5 is _____.

    1) (1, 4)   2) (5, 3)   3) (6, 7)   4) (4, 2)

2. If 2k + 3 = 3k + 7, then k = _____.

    1) –1   2) –2   3) –3   4) –4

3. If $\dfrac{8-3x}{5x-3} = \dfrac{2}{3}$, then x = _____.

    1) $\dfrac{86}{19}$ 2) $\dfrac{28}{19}$ 3) $\dfrac{58}{19}$ 4) $\dfrac{75}{19}$

4. If 0.5x _ (0.8 – 0.2x) = 0.2 – 0.3x, then x = _____ .

    1) 0   2) 1   3) 2   4) 3

5. Two complementary angles differ by 10°, the larger angle is = _____.
   1) 60°   2) 50°   3) 64°   4) 54°

6. A dealer earned a profit of 5% by selling a radio for Rs. 714. Then the cost price of the radio is Rs _____.
   1) 680   2) 540   3) 720   4) 960

7. If $\dfrac{3}{x-2} - \dfrac{2}{x-3} = \dfrac{4}{x-3} - \dfrac{3}{x-1}$, then the root of equation is _____

   1) $\dfrac{5}{3}$ 2) $\dfrac{3}{2}$ 3) $\dfrac{7}{5}$ 4) $\dfrac{8}{3}$

MULTI ANSWER TYPE

8. In x + 3 = 5, x = 2 is called
   1) Root of the equation   2) Zero of the inequation
   3) Solution of equation   4) All of these

9. The value of 'x' which satisfis the equation 2.4 (3 – x) – 0.6 (2x – 3) = 0 is_____.

   1) $\dfrac{5}{2}$ 2) $\dfrac{3}{2}$ 3) 2.5   4) 1.5

10. The ages of A and B are in the ratio 5 : 3. After 6 years their ages will be in the ratio 7 : 5, then the resent ages of A and B respectively are _____.
    1) 10 years   2) 20 years   3) 15 years   4) 9 years

11. Three numbers are in the ratio 4 : 5 : 6 and their sum is 135. Then the numbers are _____
    1) 36   2) 45   3) 54   4) 63

REASONING ANSWER TYPE

12. Statement I : The volue of 'x' which satisfies the equation 0.3x + 0.4 = 0.28x + 1.16 is 38.
    Statement II : The value of the variable which makes the equation a true statement is called the solution or root of the equation.
    1)   Both Statement-I and Statement-II are true.
    2)   Both Statement-I and Statement-II are false.
    3)   Statement I is true, Statement II is false.
    4)   Statement I is false, Statement II is true.

13. Statement I : Sum of two consecutive multiples of 3 is 69 one of the numbers is 33
    Statement II : Mathematical representation of " the sum of two consecutive multiples of 3 is 69 " is x + (x + 3) = 69 where 'x' is multiples of '3'.
    1)   Both Statement-I and Statement-II are true.
    2)   Both Statement-I and Statement-II are false.
    3)   Statement I is true, Statement II is false.
    4)   Statement I is false, Statement II is true.

COMPREHENSION TYPE

If father is 7 times as old as his son. Two years ago, the father was 13 times as old as his son, then

14. Age of son, 2 years age is _____
    1) 2 years　　　　2) 4 years　　　　3) 6 years　　　　4) 8 years

15. Age of father, 3 years age is _____
    1) 20 years　　　　2) 23 years　　　　3) 25 years　　　　4) 28 years

16. The present age of father is _____
    1) 23 years　　　　2) 25 years　　　　3) 28 years　　　　4) 31 years

MATRIX MATCHING TYPE

17. Column-I　　　　　　　　　　　　Column-II

    a) $\dfrac{3x-1}{5} - \dfrac{x}{7} = \dfrac{3}{35}$　　　　　1) 1

    b) $\dfrac{2-x}{3} - \dfrac{2x}{6} + \dfrac{1}{3}$　　　　　2) 3

    c) $\dfrac{2}{7}(x-9) + \dfrac{x}{3} = 3$　　　　　3) 5/8

    d) $(6.5)\,x + \dfrac{x}{3} = (0.25)\,x + 7$ _____　　4) 3/4

    　　　　　　　　　　　　　　　　　　5) 12

INTEGER ANSWER TYPE

18. The root of the equation $t - (2t + 5) - 5(1 - 2t) = 2(3 + 4t) - 3(t - 4)$ is _____.

WORK SHEET – 1 (KEY)				
1) 1	2) 3	3) 3	4) 4	5) 1
6) 3	7) 4	8) 4	9) 3	10) 4
11) 1	12) 3	13) 2	14) 1	15) 2
16) 4	17) 2	18) 2	19) 2	20) 1
21) 1	22) 2,3	23) 1,3	24) 1,3	25) 1,4
26) 2,4	27) 1	28) 4	29) 1	30) 4
31) 1	32) 3	33) 4	34) 3	35) 1
36) 1	37) 1	38) 3	39) 4	40) 3,4,2,1
41) 4,2,1,3	42) 2,4,4,1	43) 2,4,(1,5),3	44) 42	45) 1

WORK SHEET – 2 (KEY)				
1) 1	2) 4	3) 1	4) 2	5) 2
6) 1	7) 1	8) 1,3	9) 1,3	10) 3,4
11) 1,2,3	12) 5	13) 1	14) 1	15) 3
16) 3	17) 3,1,2,5	18) 7		