EXPONENTS

SYNOPSIS

In general, for $x \in R$, we use the notation x^n to represent the product $x.x.x.....x(n-\text{ times})$. We call this natation exponential notation.

The natural number n in x^n is called the exponent (or index), the quantity x is called the base and x^n is called then n^{th} power of x. Throughout the chapter we restrict our discussion to the bases $x \in R$.

Examples : 1. $a^2b^3 = a.a.b.b.b$

2. $(2y)(2y)(2y) = (2.2.2.)(y.y.y)$

$= 2^3 y^3$

Let us agree to write a^1 as a.

Laws of Exponents

Multiplication Property :

For $m, n \in N$ and $x \in R$ we have $x^m.x^n = x^{m+n}$

By definition

$x^m.x^n = (x.x.x.... \text{ m} - \text{times})\ \ (x.x.x.... \text{ n} - \text{times})$

$= (x.x.x.....x)\ \ (m+n \text{ times})$

$= x^{m+n}$

The Power of a Power Property :

For any $m, n \in N, x \in R$ we have

$(x^m)^n = x^{mn}$

By the difinition

$(x^m)^n = x^m.x^m.x^m\ (n-times)$

$= x^{m+m+m+m....+m}\ (n-times)$

x^{mn}

The Power of a product Property :

For every $n \in N$ and $x, y \in R$ we have

$(xy)^n = x^n y^n$ (prove)

The Division Property :

For any $m, n \in N$ and $x \in R$, $x \neq 0$

We have

$$\frac{x^m}{x^n} = \begin{cases} x^{m-n} & if \quad m > n \\ 1 & if \quad m = n \\ \dfrac{1}{x^{n-m}} & if \quad m < n \end{cases}$$

By the difinition

$$\frac{x^m}{x^n} = \frac{x.x.x....x(m-times)}{x.x.x.....x(n-times)}$$

Case (i) : For $m > n$

$$\frac{x^m}{x^n} = \frac{(x.x.x.....x(n-times)(x.x.x.....x(m-n\ times)}{(x.x.x...x\ n-times)}$$

$$= \frac{x.x.x....m\ times}{x.x.x....n\ times}$$

$$= (x.x.x.....x)(m-n\ times)$$

$$= x^{m-n}$$

Case (ii) : When $m = n$

$$\frac{x^m}{x^n} = \frac{x^m}{x^m} = 1$$

Case (iii) : for $m < n$

$$\frac{x^m}{x^n} = \frac{(x.x.x....)(m-times)}{(x.x.x....x\ n-m\ times)(x.x.x....(n-m)times)}$$

$$= \frac{1}{x.x.x....x\ n-m\ times}$$

$$= \frac{1}{(x.x.x.....x)(n-m\ times)}$$

$$= \frac{1}{x^{n-m}}$$

Power of a Quotient Property:

For any $n \in N, \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, a, b \in R,\ b \neq 0$

Definition

For any $n \in N, x \in R, x \neq 0$ we definite $x^{-n} = \frac{1}{x^n}$

With this definition property 6.1.4 can be rewritten as follows:
In the first insance

for any $m, n \in N, m \neq n, x \in R$ and $x \neq 0$

$\frac{x^m}{x^n} = x^{m-n}$ ---(A)

If $m = n, \frac{x^m}{x^n} = \frac{x^m}{x^m} = 1$ -----(B)

gives x^0 which is equal to 1 by the case (ii) of property 6.1.4

$\therefore x^0 = 1 \ \forall \ x \neq 0 \in R$

WORK SHEET

Single Answer Type

1. The value of $5^{\frac{1}{4}} \times (125)^{0.25}$ is

   1) $\sqrt{5}$ 2) 5 3) $5\sqrt{5}$ 4) 25

2. The value of $\frac{1}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{1}{(32)^{\frac{-1}{5}}}$ is

   1) 102 2) 105 3) 107 4) 109

3. $\left(2.4 \times 10^3\right) \div \left(8 \times 10^{-2}\right) =$

   1) 3×10^{-5} 2) 3×10^4 3) 3×10^5 4) 30

4. $\left(\frac{1}{213}\right)^{\frac{-2}{3}} \div \left(\frac{1}{27}\right)^{\frac{-4}{3}} =$

   1) $\frac{3}{4}$ 2) $\frac{2}{3}$ 3) $\frac{4}{9}$ 4) $\frac{1}{8}$

5. $(256)^{0.16} \times (256)^{0.09} =$

   1) 4 2) 16 3) 64 4) 256

6. $(0.04)^{-1.5} =$

1) 25 2) 125 3) 250 4) 625

7. The value of $\left(8^{-25} - 8^{-26}\right)$ is

1) 7×8^{-25} 2) 7×8^{-26} 3) 8×8^{-26} 4) 7×8^{-27}

8. The value of $\frac{(243)^{0.13} \times (243)^{0.07}}{(7)^{0.25} \times (49)^{0.075} \times (343)^{0.2}}$ is

1) $\frac{3}{7}$ 2) $\frac{7}{3}$ 3) $1\frac{3}{7}$ 4) $2\frac{2}{7}$

9. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$ then the value of x is

1) $\frac{1}{2}$ 2) 1 3) 2 4) $\frac{7}{2}$

10. If $2^{2n-1} = \frac{1}{8^{n-3}}$, then the value of 'n' is

1) 3 2) 2 3) -2 4) 0

11. If $5^a = 3125$, then the value of $5^{(a-3)}$ is

1) 25 2) 125 3) 625 4) 1625

12. If $5\sqrt{5} \times 5^3 \div 5^{\frac{-3}{2}} = 5^{a+2}$ then the value of 'a' is

1) 4 2) 5 3) 6 4) 8

13. $(18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^x$ then x is

1) 3.5 2) 4.5 3) 6 4) 7

14. $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^x$ then x is

1) 8.5 2) 13 3) 16 4) 17.5

15. The value of $\frac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$ is

1) 6^2 2) 6^4 3) 6 4) 6^3

16. The value of $\frac{2^{n+4} - 2 \times 2^n}{2 \times 2^{2n+3}} + 2^{-3} =$

1) 2^{n+1} 2) $\frac{9}{8} - 2^n$ 3) $-2^{n+1} + \frac{1}{8}$ 4) 1

17. The value of $\frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$ is

1) 3 2) 3^2 3) 3^n 4) 3^{n+2}

18. If $x = y^a$, $y = z^b$ and $z = x^c$ then the value abc is

1) 1 2) 2 3) 0 4) Data insufficient

19. The value of $\left(\frac{x^a}{x^b}\right)^{a^2+b^2+ab} \times \left(\frac{x^b}{x^c}\right)^{b^2+c^2+bc} \times \left(\frac{x^c}{x^a}\right)^{c^2+a^2+ca} =$

1) x^{abc} 2) $x^{(a+b+c)^3}$ 3) $x^{a^3+b^3+c^3}$ 4) 1

20. If $\left(\frac{1}{5}\right)^{3y} = 0.008$, then the value of $(0.25)^{-y}$ us

1) 0.25 2) 1 3) 4 4) 0.625

21. If $2^x = 4^y = 8^z$ and $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$, then the value of z is

1) $\frac{7}{16}$ 2) $\frac{7}{32}$ 3) $\frac{7}{48}$ 4) $\frac{7}{64}$

22. $\frac{1}{1+a^{n-m}} + \frac{1}{1+a^{m-n}} =$

1) 0 2) $\frac{1}{2}$ 3) 1 4) a^{m+n}

23. $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} =$

1) 0 2) 1 3) x^{a-b-c} 4) x^{a+b+c}

24. If $abc = 1$ then $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} =$

1) 0 2) 1 3) $\frac{1}{ab}$ 4) ab

25. If $3^{x-y} = 27$ and $3^{x+y} = 243$ then x is equal to

1) 0 2) 2 3) 4 4) 6

Multi Answer Type

26. Given that $10^{0.48} = x, 10.70 = y$ and $x^z = y^2$ then is greater than

1) 1.45 2) 1.88 3) 2.9 4) 3.7

27. If m and n are whole numbers such that $m^n = 121$ the value 7 $(m-1)^{n+1}$ is

1) 14400 2) 121 3) 1 4) 1000

28. If $x^{x^{\frac{3}{2}}} = \left(x^{\frac{3}{2}}\right)$ then the value of x is

1) $\frac{3}{2}$ 2) $\frac{9}{4}$ 3) $\left(\frac{2}{3}\right)^{-1}$ 4) $\left(\frac{4}{9}\right)^{-1}$

29. If $\frac{9^n \times 3^2 \times \left(3^{\frac{-n}{2}}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = 27$ then the value of $m-n$ is

1) a factor of 2012 2) a multiple of 2012
3) 0 4) 1

30. If $\left[\left(x+\frac{1}{y}\right)^a \left(x-\frac{1}{y}\right)^b\right] \div \left[\left(y+\frac{1}{x}\right)^a \left(y-\frac{1}{x}\right)^b\right]$ is equal to

1) $\left(\frac{x}{y}\right)^{a+b}$ 2) $\left(\frac{y}{x}\right)^{a+b}$ 3) $\left(xy^{-1}\right)^{a+b}$ 4) $\left(x^{-1}y\right)^{a+b}$

Comprehension Type

Writeup-1

If $a^m.a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$ then

34. $\left(\frac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\frac{x^c}{x^a}\right)^{c+a-b} \cdot \left(\frac{x^a}{x^b}\right)^{a+b-c} =$

1) x^{abc} 2) 1 3) $x^{ab+bc+ca}$ 4) x^{a+b+c}

35. $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a} =$

1) 0 2) x^{abc} 3) x^{a+b+c} 4) 1

36. $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} =$

1) 1 2) $x^{\frac{1}{abc}}$ 3) $\frac{1}{x^{ab+bc+ca}}$ 4) x^{abc}

Writeup-2

If $a^x = a^y$ then $x = y$

37. If $a^x = b^y = c^z$ and $b^2 = ab$ then y equals

1) $\frac{xz}{x+z}$ 2) $\frac{xz}{2(x-z)}$ 3) $\frac{xz}{2(z-x)}$ 4) $\frac{2xz}{x+z}$

38. If $2^x = 3^y = 6^{-z}$, then $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is equal to

1) 0 2) 1 3) $\frac{3}{2}$ 4) $\frac{-1}{2}$

39. If $x^{x\sqrt{x}} = \left(x\sqrt{x}\right)^x$ then value of x is

1) $\frac{9}{4}$ 2) $\frac{4}{9}$ 3) 8 4) 4

Matrix Matching Type

31.

Column - I	Column - II
1) $(256)^{\frac{5}{4}} =$	p) $\frac{9}{4}$
2) $\sqrt{3}\left(\sqrt{8}\right)^{\frac{1}{3}} \div \left(\sqrt{\frac{2}{3}}\right) =$	q) 36
3) $\left(\frac{32}{243}\right)^{\frac{-4}{10}} =$	r) $\frac{4}{9}$
4) $\left(\frac{-1}{216}\right)^{\frac{-2}{3}} =$	s) 1024
	t) 3

32.

Column - I	Column - II
1) $\left(\sqrt{3}\right)^5 \times 9^2 = 3^n \times 3\sqrt{3}$ then n =	p) 7
2) If $2^{n+4} - 2^{n+2} = 3$ then n =	q) 5
3) If $2^{n-1} + 2^{n+1} = 320$ then n =	r) 27
4) If $3^x - 3^{x-1} = 18$ then x^x =	s) -2
	t) 9

33.

Column - I	Column - II
1) $\left(a^{m-n}\right)^{l}\times\left(a^{n-l}\right)^{m}\times\left(a^{l-m}\right)^{n}=$	p) $\frac{11}{5}$
2) If $5^{x+3}=(25)^{3x-4}$ then $x=$	q) $\frac{-2}{5}$
3) If $2^{x}\times 8^{\frac{1}{5}}=2^{\frac{1}{5}}$ then $x=$	r) $\frac{5}{3}$
4) If $2^{x}=\sqrt[3]{32}$ then $x=$	s) 1
	t) $(2012)^{0}$

Integer Answer Type

40. If $\left(\frac{9}{4}\right)^{x}\cdot\left(\frac{8}{27}\right)^{x-1}=\frac{2}{3}$ then the value of x is = _____

41. $\frac{(243)^{\frac{n}{5}}\times 3^{2n+1}}{9^{n}\times 3^{n-1}}=$ _____

42. If $2^{x-1}+2^{x+1}=1280$ then x = _____

43. If $a+b+c=0$ then the value of

$$\frac{1}{x^{b}+x^{-c}+1}+\frac{1}{x^{c}+x^{-a}+1}+\frac{1}{x^{a}+x^{-b}+1}=$$ _____

44. The value of $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^{3}\right]^{\frac{1}{4}}=$ _____

45. If $2^{2^{x}}=16^{2^{3x}}$ then the value of $x+10=$ _____

Verbal Resonaning

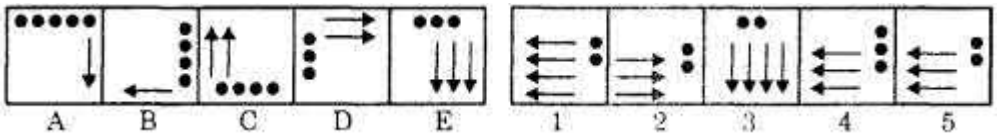
Directions : In each of the following questions, a number series is given with one term missing. Choose the correct alternative that will continue the same pattern and replace

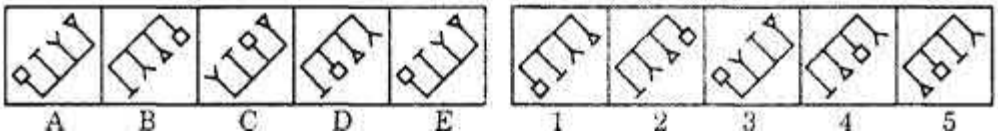
1. 196, 169, 144, 121, 101
   a) 101 b) 121 c) 169 d) 196
2. 25, 36, 49, 81, 121, 169, 225
   a) 36 b) 49 c) 169 d) 225
3. 5, 27, 61, 1221, 213, 340, 509
   a) 27 b) 61 c) 122 d) 509

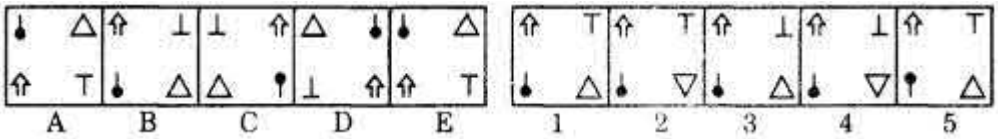
4. 16, 22, 30, 45, 52, 66
   a) 30 b) 45 c) 52 d) 66
5. 4, 10, 22, 46, 96, 190, 382
   a) 4 b)10 c) 96 d) 382
6. 105, 85, 60, 30, 0, –45, –90
   a) 105 b) 60 c) 0 d) –45
7. 380, 188, 92, 48, 20, 8, 2
   a) 8 b) 20 c) 48 d) 188
8. 56, 58, 62, 70, 84, 118, 182
   a) 58 b) 62 c) 84 d) 118
9. 1, 2, 4, 8, 16, 32, 64, 96
   a) 4 b) 32 c) 64 d) 96
10. 6, 15, 35, 77, 165, 221
    a) 35 b) 77 c) 165 d) 221
11. 46080, 3840, 384, 48, 24, 2, 1
    a) 384 b) 48 c) 24 d) 2
12. 10, 14, 28, 32, 64, 68, 132
    a) 28 b) 32 c) 64 d) 132
13. 8, 14, 26, 48, 98, 194, 386
    a) 14 b) 48 c) 98 d) 194
14. 1, 3, 10, 21, 64, 129, 356, 777
    a) 21 b) 129 c) 10 d) 356
15. 2, 6, 24, 96, 285, 568, 567
    a) 6 b) 24 c) 285 d) 567
16. 93, 309, 434, 498, 521, 533
    a) 309 b) 434 c) 498 d) 521
17. 1236, 2346, 3456, 4566, 5686
    a) 1236 b) 3456 c) 4566 d) 5686
18. 2, 3, 4, 4, 6, 8, 9, 12, 16
    a) 3 b) 6 c) 9 d) 12
19. 11, 5, 20, 12, 40, 26, 74, 54
    a) 5 b) 20 c) 40 d) 26
20. 5, 27, 61, 122, 213, 340, 509
    a) 27 b) 61 c) 455 d) 509

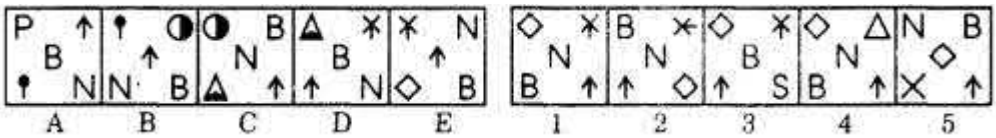
Non-Verbal Resonaning

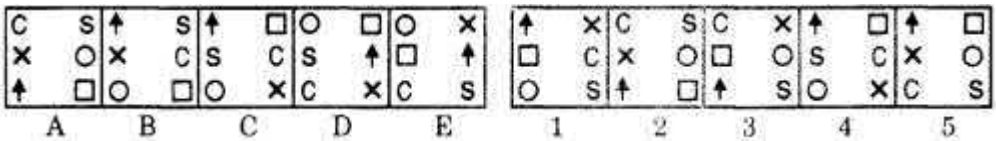
PROBLEM FIGURES ANSWER FIGURES

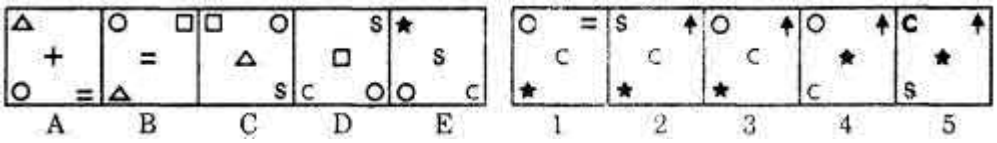
1. 

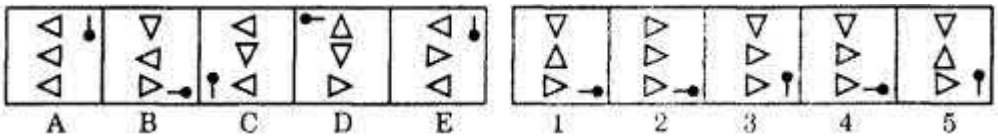
2. 

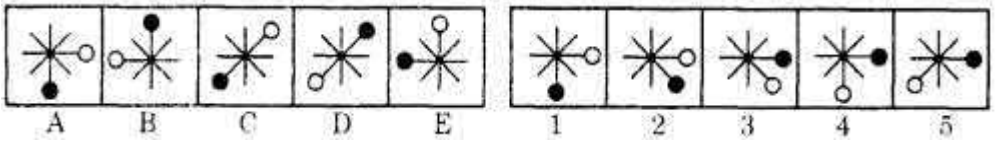
3. 

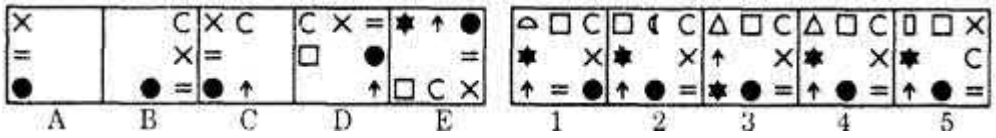
4. 

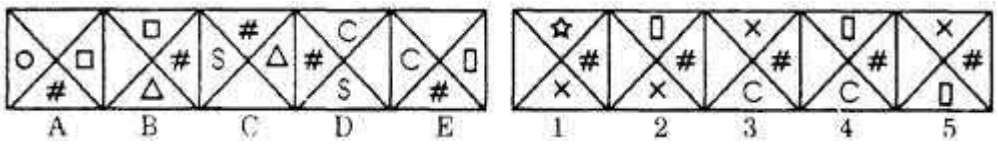
5. 

6. 

7. 

8. 

9. 

10. 

KEY & HINTS

WORK SHEET (KEY)				
1) 2	2) 1	3) 2	4) 3	5) 1
6) 2	7) 2	8) 1	9) 3	10) 2
11) 1	12) 1	13) 4	14) 2	15) 3
16) 4	17) 2	18) 1	19) 4	20) 3
21) 3	22) 3	23) 2	24) 2	25) 3
26) 1,2,3	27) 1,4	28) 2,4	29) 1,4	30) 1,3
31) 2	32) 4	33) 1	34) 4	35) 1
36) 1	37) A-S B-T C-P D-Q	38) A-Q B-S C-P D-R	39) A-ST B-P C-Q D-R	40) 4
41) 9	42) 9	43) 1	44) 5	45) 9

1. $5^{\frac{1}{4}} \times (125)^{0.25} = 5^{\frac{1}{4}} \times 5^{\frac{3}{4}} = 5$

Ans : (2)

2. $\frac{1}{\left(6^3\right)^{\frac{-2}{3}}} + \frac{1}{\left(4^4\right)^{\frac{-3}{4}}} + \frac{1}{\left(2^5\right)^{\frac{-1}{5}}}$

$= 36 + 64 + 2$

$= 102$

Ans : (1)

3. $\frac{2.4 \times 10^3}{8 \times 10^{-2}} = 3 \times 10^4$

Ans : (2)

4. $(216)^{\frac{2}{3}} \div (27)^{\frac{4}{3}} = \left(6^3\right)^{\frac{2}{3}} \div \left(3^3\right)^{\frac{4}{3}} = \frac{36}{81} = \frac{4}{9}$

Ans : (3)

5. $(256)^{0.16+0.09} = (256)^{0.25} = \left(4^4\right)^{\frac{1}{4}} = 4$

Ans : (1)

6. $\left(\frac{4}{100}\right)^{\frac{-3}{2}} = \left(\frac{1}{25}\right)^{\frac{-3}{2}} = \left(5^2\right)^{\frac{3}{2}} = 125$

Ans : (2)

7. $8^{-26}(8-1) = 7 \times 8^{-26}$

Ans : (2)

8. $\frac{(243)^{0.13+0.07}}{7^{0.25}.7^{2\times0.075}.7^{3\times0.2}} = \frac{\left(3^5\right)^{0.2}}{7^{(0.25+0.15+0.6)}}$

$= \frac{3}{7}$

Ans : (1)

9. $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{a}{b}\right)^{3-x} \quad \Rightarrow \quad x-1 = 3-x$

$2x = 4$

$x = 2$

Ans : (3)

10. $2^{2n-1} = \left(2^{-3}\right)^{n-3} \quad \Rightarrow \quad 2n-1 = -3n+9$

$5n = 10$

$n = 2$

Ans : (2)

11. $5^9 = 3125 = 5^5 \Rightarrow a = 5$

$5^{a-3} = 5^{5-3} = 5^2$

Ans : (1)

12. $5^{\frac{3}{2}} \times 5^3 \times 5^{\frac{3}{2}} = 5^{a+2}$

$5^6 = 5^{a+2}$

$6 = a+2$

$a = 4$

Ans : (1)

13. $\left(\frac{18}{27}\right)^{3.5} \times 6^{3.5} = 2^x$

$\left(\frac{18 \times 6}{27}\right)^{3.5} = 2^x$

$2^{2 \times 3.5} = 2^x$

$\therefore x = 7$

Ans : (4)

14. $\frac{\left(5^2\right)^{7.5} \times 5^{2.5}}{\left(5^3\right)^{1.5}} = 5^x$

$\Rightarrow 5^{15+2.5-4.5} = 5^x$

$5^{13} = 5^x$

$\therefore x = 13$

Ans : (2)

15. $\frac{6^{\frac{2}{3}} \times 6^{\frac{7}{3}}}{6^{\frac{6}{3}}} = 6^{3-2} = 6$

Ans : (3)

16. $\frac{2^n\left(2^4 - 2\right)}{2^n.2.2^3} + 2^{-3} \Rightarrow \frac{14}{2 \times 8} + \frac{1}{8} = \frac{7}{8} + \frac{1}{8} = 1$

Ans : (4)

17. $\frac{\left(3^5\right)^{\frac{n}{5}}.3^{2n+1}}{3^{2n}.3^{n-1}} = \frac{3^{3n+1}}{3^{3n-1}} = 3^2 = 9$

Ans : (2)

18. $x = y^a$

$= \left(z^b\right)^a$

$= z^{ab}$

$= \left(x^c\right)^{ab}$

$= x^{abc}$

$\therefore abc = 1$

Ans : (1)

19. $x^{(a-b)(a^2+b^2+ab)} \cdot x^{(b-c)(b^2+c^2+bc)} \cdot x^{(c-a)(c^2+a^2+ca)}$

$\Rightarrow x^{a^3-b^3} . x^{b^3-c^3} . x^{c^3-a^3}$

$\Rightarrow x^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= x^0$

$= 1$

Ans (1)

20. $\left(\frac{1}{5}\right)^{3y} = \left(\frac{8}{1000}\right) = \frac{1}{125}$

$\left(\frac{1}{5}\right)^{3y} = \left(\frac{1}{5}\right)^3$

$\therefore 3y = 3$

$y = 1$

$\therefore (0.25) = \left(\frac{1}{4}\right)^{-1} = 4$

Ans : (3)

21. $2^x = 2^{2y} = 2^{3z} \quad \Rightarrow \quad x = 2y = 3z$

$\therefore \frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$

$\Rightarrow \frac{1}{6z} + \frac{1}{6z} + \frac{1}{6z} = \frac{24}{7} \Rightarrow \frac{1}{2z} = \frac{24}{7}$

$\Rightarrow z = \frac{7}{48}$

Ans : (3)

22. $\frac{1}{1+\frac{a^n}{a^m}} + \frac{1}{1+\frac{a^m}{a^n}} \Rightarrow \frac{a^m}{a^m+a^n} + \frac{a^n}{a^n+a^m} \Rightarrow \frac{a^m+a^n}{a^m+a^n} = 1$

Ans : (3)

23. $\frac{x^a}{x^a\left(1+x^{b-a}+x^{c-a}\right)} + \frac{x^b}{x^b\left(1+x^{a-b}+x^{c-b}\right)} + \frac{x^c}{x^c\left(1+x^{b-c}+x^{a-c}\right)}$

$$\Rightarrow \frac{x^a}{x^a+x^b+x^c}+\frac{x^b}{x^b+x^a+x^c}+\frac{x^c}{x^c+x^b+x^a}$$

$$\Rightarrow \frac{x^a+x^b+x^c}{x^a+x^b+x^c}=1$$

Ans : (2)

24. $\dfrac{1}{1+a+b^{-1}}+\dfrac{b^{-1}}{b^{-1}\left(1+b+c^{-1}\right)}+\dfrac{a}{a\left(1+c+a^{-1}\right)}$

$$\Rightarrow \frac{1}{1+a+b^{-1}}+\frac{b^{-1}}{b^{-1}1+bc^{-1}}+\frac{a}{a+ac+1}$$

$$\Rightarrow \frac{1}{1+a+b^{-1}}+\frac{b^{-1}}{b^{-1}+1+a}+\frac{a}{a+b^{-1}+1}$$

$$\Rightarrow \frac{1+b^{-1}+a}{1+a+b^{-1}}=1$$

25. $3^{x-y}=3^3$ $\qquad$ $3^{x+y}=3^5$

$x-y=3$ ---- (1) $\qquad$ $x+y=5$ ----- (2)

By adding (1) & (2)

$$2x=8$$

$$x=4$$

Ans : (3)

26. $x=10^{0.48},\quad y=10^{0.70}$

$x^z=y^2$

$\left(10^{0.48}\right)^z=\left(10^{0.70}\right)^2$

$\therefore 0.48\times z=2\times 0.70$

$z=\dfrac{2\times 0.70}{0.48}=\dfrac{35}{12}=3.91..$

Ans : (1), (2), (3)

27. $m^n=121$

$\therefore m^n=11^2 \quad or \quad m^n=\left(121\right)^1$

$\therefore m=11,n=2 \qquad m=121,n=1$

$\therefore \left(m-1\right)^{n+1}=10^3 \qquad \left(121-1\right)^{1+1}=\left(120\right)^2$

$=1000 \qquad =14400$

Ans : (1), (4).

28. $x^{x^{\frac{3}{2}}} = x^{\frac{3x}{2}}$

$\therefore x^{\frac{3}{2}} = \frac{3x}{2}$

$\frac{x^{\frac{3}{2}}}{x} = \frac{3}{2}$

$x^{\frac{3}{2}-1} = \frac{3}{2}$

$x^{\frac{1}{2}} = \frac{3}{2}$ $\quad\quad$ $\therefore x - \frac{9}{4}$

Ans : (2), (4)

29. $\frac{3^{2n}.3^2.3^n - \left(3^3\right)^n}{3^{3m}.2^3} = 3^{-3}$

$\therefore \frac{3^{3n}\left(9-1\right)}{3^{3m}.2^3} = 3^{-3}$

$\therefore m - n = 1$

Ans : (1), (4)

30. $\frac{\frac{\left(xy+1\right)^a}{y^a}.\frac{\left(xy-1\right)^b}{y^b}}{\frac{\left(xy+1\right)^a}{y^a}.\frac{\left(xy-1\right)^b}{y^b}} = \left(\frac{x}{y}\right)^{a+b}$

Ans : (1), (3)

31. 1) $\left(4^4\right)^{\frac{5}{4}} = 4^5 = 1024$

2) $\sqrt{3}.\sqrt{2} \times \frac{\sqrt{3}}{\sqrt{2}} = 3$

3) $\left[\left(\frac{2}{3}\right)^5\right]^{\frac{-4}{10}} = \left(\frac{2}{3}\right)^{-2} = \frac{9}{4}$

4) $\left[\left(\frac{-1}{6}\right)^3\right]^{\frac{-2}{3}} = \left(\frac{-1}{6}\right)^{-2} = 36$

$\therefore (1) \to s, \quad (2) \to t, \quad (3) \to p, \quad (4) \to q$

32. 1) $\left(3^{\frac{1}{2}}\right)^5 .3^4 = 3^n .3^{\frac{3}{2}}$

$3^{\frac{5}{2}+4} = 3^{n+\frac{3}{2}}$

$\therefore \frac{5}{2} + 4 = n + \frac{3}{2}$

$n = 2$

2) $2^n\left(2^4 - 2^2\right) = 3$

$2^n(12) = 3$

$2^n = 2^{-2}$

$\therefore n = -2$

3) $2^n\left(\frac{1}{2} + 2\right) = 320$

$2^n \times \frac{5}{2} = 320$

$2^n = 64 \times 2 = 2^7$

$\therefore n = 7$

4) $3^x\left(1 - \frac{1}{3}\right) = 18$

$3^x\left(\frac{2}{3}\right) = 18$

$3^x = 3^3$

$\therefore x = 3$

$\therefore x^x = 27$

$\therefore (1) \to q, \quad (2) \to s, \quad (3) \to p, \quad (4) \to r$

33. 1) $a^{ml-nl+nm-lm+\ln-\ln}$

$a^0 = 1$

2) $5^{x+3} = 5^{6x-8}$

$\therefore x+3 = 6x-8$

$5x = 11$

$x = \frac{11}{5}$

3) $2^x = 2^{\frac{1}{5}-\frac{3}{5}}$

$2^x = 2^{\frac{-2}{5}}$

$\therefore x = 2^{\frac{-2}{5}}$

4) $2^x = 2^{\frac{5}{3}}$

$\therefore x = \frac{5}{3}$

$\therefore (1) \to s,t, \quad (2) \to p, \quad (3) \to q, \quad (4) \to r$

34. $x^{(b-c)(b+c-a)} . x^{(c-a)(c+a-b)} . x^{(a-b)(a+b-c)}$

$= x^0 = 1$

Ans : (2)

35. $\left(x^{a-b}\right)^{a+b} . \left(x^{b-c}\right)^{b+c} . \left(x^{c-a}\right)^{c+a}$

$\Rightarrow x^{a^2-b^2} . x^{b^2-c^2} . x^{c^{2-a^2}} \Rightarrow x^0$

$= 1$

Ans : (4)

36. $\left(x^{a-b}\right)^{\frac{1}{ab}} . \left(x^{b-c}\right)^{bc} . \left(x^{c-a}\right)^{ca}$

$= x^{\frac{1}{ab}(a-b)+\frac{1}{bc}(b-c)+\frac{1}{ca}(c-a)}$

$= x^{\frac{1}{b}-\frac{1}{a}+\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}} = x^0 = 1$

Ans : (1)

37. Let $a^x = b^y = c^z = k$

$\therefore a = k^{\frac{1}{x}}, \quad b = k^{\frac{1}{y}}, \quad c = k^{\frac{1}{z}}$

$\therefore b^2 = ac$

$$\Rightarrow \left(k^{\frac{1}{y}}\right)^2 = \left(k^{\frac{1}{x}}\right).\left(k^{\frac{1}{z}}\right)$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\therefore y = \frac{2xz}{x+z}$$

Ans : (4)

38. Let $2^x = 3^y = 6^{-z} = k$

$$\therefore k^{\frac{1}{x}} = 2, \quad k^{\frac{1}{y}} = 3, \quad k^{\frac{-1}{z}} = 6$$

$$k^{\frac{-1}{z}} = 6 = 2\times 3 = k^{\frac{1}{x}}.k^{\frac{1}{y}} \Rightarrow \frac{-1}{z} = \frac{1}{x} + \frac{1}{y} \qquad \therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

Ans : (1)

39. $x^{x^{\frac{3}{3}}} = x^{\frac{3}{2}x}$

$$\therefore x^{\frac{3}{2}} = \frac{3x}{2}$$

$$\therefore x = \frac{9}{4}$$

Ans : (1)

40. $\left(\frac{3}{2}\right)^{2x}.\left(\frac{2}{3}\right)^{3x-3} = \frac{2}{3}$

$$\left(\frac{2}{3}\right)^{3x-3-2x} = \left(\frac{2}{3}\right)^1$$

$$\therefore x - 3 = 1$$

$$x = 4$$

41. $\frac{\left(3^5\right)^{\frac{n}{5}} \times 3^{2n+1}}{3^{2n}.3^{n-1}} = \frac{3^{3n+1}}{3^{3n-1}} = 3^2 = 9$

42. $2^x\left(\frac{1}{2} + 2\right) = 1280$

$$2^x = \frac{1280\times 2}{2} = 256\times 2 = 2^9$$

$\therefore x = 9$

43. Given $a+b+c=0$

$$\frac{1}{x^b + x^{-c} + 1} + \frac{1}{x^c + x^{b+c} + 1} + \frac{1}{x^{-b-c} + x^{-b} + 1}$$

$$\Rightarrow \frac{1}{x^b + x^{-c} + 1} + \frac{x^{-c}}{x^{-c}\left(x^c + x^{b+c} + 1\right)} + \frac{x^b}{x^b\left(x^{-b-c} + x^{-b} + 1\right)}$$

$$\Rightarrow \frac{1 + x^{-c} + x^b}{1 + x^{-c} + x^b} = 1$$

Ans : (1)

44. $\left[5(2+3)^3\right]^{\frac{1}{4}}$

$= \left(5^4\right)^{\frac{1}{4}}$

$= 5$

45. $2^{2^x} = 2^{4.2^{3x}}$

$\therefore 2^x = 2^2.2^{3x}$

$2^x = 2^{3x+2}$

$3x + 2 = x$

$\therefore 2x = -2 \Rightarrow x = -1$

$\therefore x + 10 = -1 + 10$

$= 9$

VERBAL REASONING (KEY)				
1) A	2) A	3) A	4) B	5) C
6) C	7) C	8) C	9) D	10) C
11) C	12) D	13) B	14) D	15) B
16) D	17) D	18) C	19) C	20) A

1. (a) : The sequence is $\left(14^2\right), \left(13^2\right), \left(12^2\right)\left(11^2\right)\left(10^2\right)$.

So, 101 is wrong and must be replaced by $\left(10^2\right)$ *i.e.*100.

2. (a) :The correct sequence is $5^2, 7^2, 9^2, 11^2, 13^2, 15^2$. So 36 is wrong.

3. (a) :The terms of the series are

$(2^3-3), (3^3-3), (4^3-3), (5^3-3), (6^3-3), (7^3-3).$

So, 27 is wrong and must be replaced (3^3-3) *i.e.*24.

4. (b) :The correct pattern is $+6, +9, +10, +12, +14.$

So, 45 is wrong and must be replaced by $(30+10)$ *i.e.*40.

5. (c) :The correct pattern is $+6, +12, +24, +48, +96, +192.$

So, 96 is wrong and must be replaced by $(46+48)$ *i.e.*94.

6. (c) :The correct pattern is $-20, -25, -30, ...$

So, 0 is wrong and must be replaced by $(30-35)$ *i.e.*-5.

7. (c) :The correct pattern is $-192, -96, -48, -24, -12, -6.$

So, 48 is wrong and must be replaced by $(92-48)$ *i.e.*44.

8. (c) :The correct pattern is $+2, +4, +8, +16, +32, +64,$ *i.e.*$+2, +2^2, +2^3, +2^4, +2^5, +2^6$.

So, 84 is wrong and must be replaced by $(70+16)$ *i.e.*86.

9. (d) :Each term of the series is obtained by multiplying the preceding term by 2.

So, 96, is wrong and must be replaced by (64×2) *i.e.*128.

10. (c) :The terms of the series are products of two consecutive prime numbers *i.e.*$(2\times 3), (3\times 5), (5\times 7), (7\times 11), ...$

So, 165 is wrong and must be replaced by $(1536\div 4)$ *i.e.*384 .

11. (c) :The correct pattern is $\times 2+1, \times 3+1, \times 2+1, \times 3+1, ...$

So, 24 is wrong and must be replaced by $(48\div 6)$ *i.e.*8.

12. (d) :The correct pattern is $+4, \times 2, +4, \times 2, ...$

So, 132 is wrong and must be replaced by (68×2) *i.e.*136.

13. (b) :The correct pattern is $\times 2-2.$

So, 48 is wrong and must be replaced by $(26\times 2-2)$ *i.e.*50.

14. (d) :The correct pattern is $\times 2+1, \times 3+1, \times 2+1, \times 3+1, ...$
So, 356 is wrong and must be replaced by

15. (b) :The correct pattern is $\times 6-6, \times 5-5, \times 4-4, ...$

So, 24 is wrong and must be replaced by $(6\times5-5)$ *i.e.*25.

16. (d) :The correct pattern is $+6^3,+5^3,+4^3,+3^3,.....$

So, 521 is wrong and must be replaced by $(498+3^3)$ *i.e.*525.

17. (d) :The correct pattern in the series is $+1110.$

So, 5686 is wrong and must be replaced by $(4566+1110)$ *i.e.*5676.

18. (c) : The given sequence is a combination of three series :

I. 1st, 4th, 7th terms *i.e.*2,4,9,.....

II. 2nd, 5th, 8th, terms *i.e.*3,6,12,....

III. 3rd, 6th, 9th terms *i.e.*4,8,16,....

In each one of I, II and III, each term is twice the preceding term.

So, 9 is wrong and must be replaced by (4×2) *i.e.*8.

19. (c) : The given sequence is a combination of two series :
I. 11, 20, 40, 74 and II. 5, 12, 26, 54

The correct pattern in I is $+9,+18,+36,....$

So, 40 is wrong and must be replaced by $(20+18)$ *i.e.*38.

20. (a) : We have :

I : 5 27 61 122 213 340 509

II : 22 34 61 91 127 169

III : 12 27 30 36 42

IV : 15 3 6 6

Clearly, the given series becomes a triangular-pattern series if each term in IV is 6.

Then, in III, 27 must be replaced by $(30\text{-}60)$ *i.e.*$24\,\&\,12$ by $(24-6)$ *i.e.*18.

Again, in II, 34 must be replaced by $(61-24)$ *i.e.*37 and 22 by

$(37-18)$ *i.e.*19.

Thus in the given series, 27 is a cinbubatuib if two series :

NON-VERBAL REASONING (KEY)				
1) 5	2) 2	3) 3	4) 4	5) 3
6) 3	7) 4	8) 4	9) 4	10) 2

1. (5) : In each step, all the existing elements move to the adjacent side (of the square boundary) in a CW direction. The number of black circles decreases by one in first, third, fifth, ... steps and the number of arrows increases by one in second, fourth, sixth, ... steps.
2. (2) : Similar figure reappears in every second step. Each time the first figure reappears, the elements interchange positions in the order : . And, each time the second figure reappears, the elements interchange positions in the order :
3. (3) : The elements interchange positions in the sequences and aternately. In each step, the elements that reaches the encircled position gets vertically inverted.
4. (4) : The elements move in the sequences and alternately. Also, the element at the encircled position is replaced by a new one in each step.
5. (3) : The elements move in the sequences and alternately.
6 (3) : In the first step, the elements move in the sequence and in each subsequent step, the elements move in the sequence obtained by rotating the previous sequence through 90^0 CW. Also, in each step, the element that reaches the encircled position, gets replaced by a new element.
7. (4) : The lowermost triangle gets laterally inverted in each step. The middle triangle rotates 90^0 ACW in every second step. The upper triangle rotates through 90^0 ACW, 90^0 CW, 90^0 CW, 90^0 ACW, 90^0 CW, 90^0 CW, ... sequentially. The pin rotates 90^0 ACW and moves to the adjacent corner in a CW direction in each step.
8. (4) : The black circle moves four steps ACW and three steps ACW alternately while the white circle moves four steps CW and three steps CW alternately.
9. (4) : In each step, all the existing symbols reverse their order and a new symbol is added after the existing symbols at the ACW-end. Also, all the symbols shift five, four, three, two and one spaces CW sequentially.
10 (2) : All the elements move one space ACW in each steps. Also, in one step, the CW-end element is replaced by a new element and in the next step, the ACW-end element is replaced by a new element.