INTEGERS

SYNOPSIS - 1

Positive Integers:

The natural numbers 1, 2, 3, 4, 5,.... are also know as positive integers.

Thus, $Z^{+} = \{1, 2, 3, 4, 5, ...\} = N$, is the set of all positve integers.

Negative Integers:

Corresponding to every positve integer 1, 2, 3, 4, 5.... we define a negative integer, denoted by -1, -2, -3, -4, -5, ... such that

$1 + (-1) = 0, 2 + (-2) = 0, 3 + (-3) = 0$ and so on.

The set $Z^{-} = \{-1, -2, -3, -4, -5, ...\}$ is the set of all negative integers.

Remarks: The number 1 and -1 are called the opposites of each other. Similarly, the numbers 2 and -2 are called the opposites of each other and so on.

Integers: All natural numbers, negatives of natural numbers and 0, togetyher form the set Z or I of all integers.

Thus, Z = $\{...., -3, -2, -1, 0, 1, 2, 3, ...\} = Z^{-} \cup \{0\} \cup Z^{+}$.

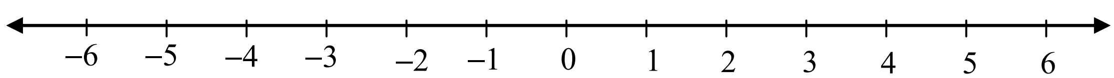
Remark: 0 is an integers which is neither positve nor negative.

Representation of integers on Number line.

Draw a line. Mark a point on this line.Label this point as 0.

Now, set off equal distances on the right as well as left of 0.

On the right hand side of 0, label the points of division as 1, 2, 3,....while on the left hand side of 0, label the points as -1,-2, -3,....as shown below.



The arrowheads on both sides of the number line indicate the continuation of integers indefinitely on each side.

Comparison of Integers: If we represent two integers on the number line, then the integer occuring on the right is greater than that occurring on the left.

As a consequence of it, we can make following observations:

i. 0 is less than every positive integer.
ii. 0 is greater than every negative integer.
iii. Every negative integer is less than every positive integer
iv. The greater is the integer, the lesser is its negative, i.e., $a > b \Rightarrow -a < -b$.

Absolute valueof an Integer: The absolute value of an integer ‘a’ is the numerical valueof ‘a’ regardless of its sign. We denote it by $|a|$.

Thus, $|-5| = 5, |5| = 5, |-9| = 9$ and $|9| = 9$, etc.

FUNDAMENTAL OPERATIONS ON INTEGERS

I. Addition of Integers: The addition of integers may be carried out by any of the following rules:

Rule 1: (Addition of two positive integers): The sum of two positive integers is a positive integer obtained by taking the sum of the numerical valuesof the addends.

Rule 2: (Addition of two negative integers): The sum of two negative integers is a negative integer obtained by taking the sum of the numerical values of the addends.

Rule 3: (Addition of a positive and negative integer): To add a positive and a negative integer, we find the difference between their numerical values of give the sign of the integer with the greater value to it.

Example: (i) $(+8)+(+7)=(+15)$

(ii) $(+6)+(+3)=(+9)$

(iii) $(-3)+(-4)=-(3+4)=(-7)$

(iv) $(-6)+(-7)=-(6+7)=(-13)$

(v) $(-3)+(+7)=+(7-3)=(+4)$

(vi) $(-23)+(+15)=-(23-15)=(-8)$

Properties of Addition of Integers:

1. Closure Property for Addition of Integers: The sum of two integers is always an integer. Thus, $a \in Z, b \in Z \Rightarrow a+b \in Z$.

   Example: (-7) and 29 are integers. Their sum is 22 which is also an integer.

2. Commutative Law for Addition of Integers: For any two integers a and b, we always have $a+b=b+a$.

   Example: $3+(-12)=(-9)$ and $(-12)+3=(-9)$

   $\therefore 3+(-12)=(-12)+3$

3. Associative Law for addition of Integers: For any three integers a, b and c, we have $(a+b)+c=a+(b+c)$.

   Example: For integers $(-3),(-6)$ and 5 we have

   $[(-3)+(-6)]+5=(-9)+5=(-4)$ and

   $(-3)+[(-6)+5]=(-3)+(-1)=(-4)$

Thus, $\left[(-3)+(-6)\right]+5=(-3)+\left[(-6)+5\right]$

4. Existence of Additive Identity: 0 is the additive identify for integers. So, for any integer 'a', we have.

$a+0=a$ and $0+a=a$

Example: (i) $3+0=0+3=3$

(ii) $(-7)+0=0+(-7)=(-7)$.

5. Existence of Additive Inverse: For each integer 'a' there exists another integer $(-a)$ such that $a+(-a)=(-a)+a=0$. The integer $(-a)$ is called the opposite or negative or additive inverse of the integer 'a'.

Example: $8+(-8)=(-8)+8=0$

Therefore, the additive inverse of 8 is (-8)

Also, the additive inverse of (-8) is 8.

Remark: The additive inverse of 0 is 0.

II. Subtraction of Integers: To subtract one integer from another, we take the additive inverse of the integer to be subtracted and add it to the other integer. Thus, if a and b are two integers, then $a-b=a+(-b)$.

Example: Subtract (i) 9 from 2 (ii) -8 from 4 (iii) 12 from (-6) (iv) (-13) from (-26)

Solution:

(i) 2 - 9 = 2 + (additive inverse of 9) = $2+(-9)=(-7)$

(ii) $4-(-8)=4+$ (additive inverse of -8) $=4+8=12$

(iii) $(-6)-(12)=(-6)$ + (additive inverse of 12) $=(-6)+(-12)=(-18)$

(iv) $(-26)-(-13)=(-26)+$ (additive inverse of -13)

$=(-26)+13=(-13)$.

Properties of Subtraction of Integers:

1. Closure Property: The difference of two integers is always an integer, i.e., $a\in Z$, $b\in Z\Rightarrow(a-b)\in Z$

2. If 'a' is any integer, then $a-0=a$ and $0-a=-a$.

3. For any integer 'a', we have $-(-a)=a$, i.e., additive inverse of -*a* is *a.*

III. **Multiplication of Integers:** The multiplication of integers may be carried out by any of the following rules:

Rule 1: (Multiplication of two integers having like signs): The product of two integers having like signs is a positive integer obtained by multiplying their numerical values.

Rule 2: (Multiplication of two integers having unlike signs): The product of two integers having unlike signs is the negative of the integer obtained by multiplying the numerical values of the given integers.

Examples: (i) $(+5)\times(+3)=+(5\times3)=+15$

(ii) $(-6)\times(-4)=+(6\times4)=+24$

(iii) $(-4)\times(+3)=-(4\times3)=-12$

(iv) $(+5)\times(-4)=-(5\times4)=-20$

Properties of Multiplication of Integers:

1. Closure Property for Multiplication of Integers: The product of two integers is always an integer, i.e., $a\in Z, b\in Z \Rightarrow ab\in Z$.
2. Commutative Law for Multiplication of Integers: For any two integers a and b, we have $a\times b=b\times a$
3. Associative Law for Multiplication of Integers: For any three integers a, b and c, we have $(a\times b)\times c=a\times(b\times c)$.
4. Distributive Law of Multiplication over Addition: For any three integers a, b and c, we have $a\times(b+c)=(a\times b)+(a\times c)$.
5. Existence of Multiplicative Identify: The integer 1 is the multiplicative identify for integers. So, for any integer 'a' we have $a\times1=1\times a=a$.
6. Multiplicative property of 0: For any integer 'a' we have $a\times a=0\times a=0$.

IV. **Division of Integers:** The division of integers may be carried out by any of the following rules:

Rule 1: (Division of two integers having like signs): The quotient of two integers having like signs is a positive integer obtained by dividing the numerical value of the dividend with the numerical value of the divisor.

Rule 2: (Division of two integers having unlike signs): The quotient of two integers havingunlike signs is the negative of the integer obtained by dividing the numerical value of the dividend with the numerical value of the divisor.

Example: Divide (i) $+(20)$ by $(+5)$ (ii) (-30) by (-6)

(iii) $(+32)$ by (-8) (iv) (-45) by (9)

Solutions: (i) $(+20)+(+5)=+\left(\frac{20}{5}\right)=+4$ (Using Rule 1)

(ii) $(-30)+(-6)=+\left(\frac{30}{6}\right)=+5$ (Using Rule 1)

(iii) $(+32)+(-8)=-\left(\frac{32}{8}\right)=-4$ (Using Rule 2)

(iv) $(-45)+9=-\left(\frac{45}{9}\right)=-5$ (Using Rule 2)

Properties of Division of Integers:

1. If a and b are integers then $(a+b)$ is not necessarily an integer.

   Example: $4\in Z$ and $8\in Z$. But $\frac{4}{8}\notin Z$.
2. If 'a' is a non-zero integer, then $a\div a=1$
3. If 'a' is a non-zero integer, then $0\div a=0$, but $a\div 0$ is meaningless.
4. If 'a' is any integer, then $a\div 1=a$.
5. For unequal non-zero integers a and b, we have $a\div b\neq b\div a$.
6. For unequal non-zero integers a, b, c we have

   $(a\div b)\div c\neq a\div(b\div c)$ unless $c=1$.

WORK SHEET - 1

SINGLE ANSWER TYPE

1. The absolute value fo -8 is
   1) -8 2) 8 3) 0.8 4)8.1
2. 0 is greater than integer
   1) -ve integer 2) +ve integer 3) any integer 4) None
3. Compare +42 □ +23 is
   1)< 2) > 3) = 4) Ė
4. The value of 8+$|-4|$ is
   1) 4 2)-12 3) 12 4) -4
5. The value of $|-5|-|6|+|-8|$ is
   1) 19 2) -19 3) 7 4) None
6. The absolute value of 0 is
   1) 0 2) 1 3) not determined 4) None
7. The movement of units in the direction of left of zero is
   1)0 2) +ve 3) -ve 4) None
8. Two different natural numbers are such that their product is lessthan their sum, then one of the number must be
   1) 1 2) 2 3) 3 4) None
9. For an integer n, is n^3 if odd, then which of the following statements are true.
   I) n is odd II) n^2 is odd III) n^2 is even
   1) I only 2) II only 3) I and II only 4) I and III only
10. The smallest value of n, for which 2n +1 is not a prime number is
    1) 3 2) 4 3) 5 4) 1

11. The absolute value of x-10 if x is an integer greater than 10
1) x-10 2) x+10 3)10-x 4) -x-10

12 Write the integers -61,-36,3,-3,6,-6 in descending order
1) $-36>3>-3>-61>6>-6$ 2) $6>3>-3>-36>-6>-61$
3) $6>3>-3>-6>-36>-61$ 4) $-61>-36>-6>-3>3>6$

MULTI ANSWER TYPE

13. If R is an integer, then
1) R^2 is an integer 2) R^3 is an integer
3) 5R is an integer 4) $\frac{1}{R}$ is an integer

14. An integer may be
1) + ve 2) –ve 3) Zero 4) all of these

REASONING ANSWER TYPE

15. *Statement I* : -2 lies to the right of -5 on the number line.
Statement II: If a>b for two integers a lies to the right of b on the number line

1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

16. *Statement I* : The absolute value of -1991 is 1991
Statement II: The absolute value of an integer is the numerical value of the integer regardless of its sign

1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

COMPREHENSION TYPE

Absolute value of an integer x is defined by

$$|x| = \begin{cases} x : \text{if } x \text{ is Positive} \\ 0 : \text{if } x \text{ is Zero} \\ -x : \text{if } x \text{ is negative} \end{cases}$$

17. $|-516| =$
1) -516 2) 516 3) ± 516 4)615

18. $|100| + |-30| - |-676| =$
1) 746 2) 806 3) -546 4) -606

19. The absolute value of m is

1) m if m>0 2) m if m<0 3) -m if m>0 4)-m if -m<0

MATRIX MATCHING TYPE

20.

Column-I	Column-II
a) Set of integers	1) { 1,2,3,4,........................ }
b) Set of positive integers	2) { -1,-2,-3,-4.................... }
c) Set of negetive integers	3) { ....-2,-1,0,1,2.......... }
d) Set of non-negetive integers	4) { 0,1,2,3,4..................... }
	5) { ....-3,-2,-1,0 }

21.

Column-I	Column-II
a) Absolute value of -25 is	1) -357
b) $\lvert 357 \rvert$ is greate than	2) -10
c) Absolute value of zero is	3) 25
d) -110 is less than	4) 10
	5) 0

INTEGER ANSWER TYPE

22. In the set of positive integers, the least number is ___________

23. Absolute value of $\left\{ \dfrac{\lvert 25 \times (-4) \rvert - \lvert 125 \times (-16) \rvert}{\lvert (-50 \times 2) \rvert + \lvert (-45) \times 2 \rvert} \right\}$ is ________

WORK SHEET - 2

SINGLE ANSWER TYPE

1. The sum of -373, 20, -3, -5 and 5 is

1)–356 2) 356 3) 406 4) –406

2. For all non-zero integers a, b, c $a \times (b - c) = (a \times b) - (a \times c)$ is the property over]

1) Multiplication 2) Addition 3) Division 4) substraction

3. The value of $(-2) \times (5) \times (-30)$ is

1) 300 2) –300 3) 30 4) –7

4. The quotient of $35 \div (-7)$ is

1) 5 2) –5 3) 7 4) –7

5. The value of $-37 - (-15) - 2$ is

1) 24 2)–24 3) 54 4) –54

6. The value of $(22) \times (-3) \times (-2) \times (-1)$ is

1)28 2) –28 3)132 4)–132

7. The value of $3+(-3)+3+(-3)+\ldots\ldots\ldots(305 \text{ times })$

1) 3 2) –3 3) 0 4) None

8. The sum of two integers is -300. If one of them is 60 then the other numbers is

1)360 2)–360 3)240 4)–240

9. The product of two numbers is -180. If one of the numbers is 12. Find the other numbers ?

1) 15 2) –15 3) –192 4) 168

10. $7\left[3\left\{2-6(6 \text{ of } -2)\right\}\right]=$

1) –172 2) 180 3)172 4) 0

11 The sum of two numbers is 40 and their difference is 4. The ratio of the numbers is

1) 11:9 2) 11:18 3)21:19 4) 22:9

12 A number is doubled and the sum of that two numbers is equal to 54. Find the numbers?

1) 9, 18 2) 18, 36 3) 16, 32 4) 16, 28

MULTI ANSWER TYPE

13. If 'a' and 'b' are two integers then

1) a+b is an integer 2) a-b is not an integer

3) $a \times b$ is an integer 4) $a \div b$ is an integer

14. For any three integers a, b, c

1) $(a+b)+c=a+(b+c)$ 2) $(a \times b)\times c=a(b \times c)$

3) $(a-b)-c=a-(b-c)$ 4) $(a \div b)\div c \neq a \div (b \div c)$

REASONING ANSWER TYPE

15. *Statement I* : For any integer x, we have $x+0=0+x=0$.
Statement II: '0' is called identity element under addition for integers.
1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

16. *Statement I* : The product of negative and positive integer is a negativeinteger.
Statement II: Division by zero is not defined.
1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

COMPREHENSION TYPE

'0' is identity element under addition of integers.
'1' is identity element under multiplication of integers.
For any integer a, additive inverse is -a and multiplicative inverse does not exist in integers.

17. $513627 + 0 = 0 + 513627 =$
1)0 2)513627 3) 1027254 4) -513627

18. $987654321 \times 1 = 1 \times 987654321 =$
1)1 2)987654321 3)0 4)123456789

19. Additive inversse of –9032673908 is 1)
9032673908 2)-9032673908 3)0 4)1

MATRIX MATCHING TYPE

20.

Column-I	Column-II
a) $x + y =$	1) $y - x$
b) $x \times (y \times z) =$	2) $x + (y + z)$
c) $x - y =$	3) $y + x$
d) $(x + y) + z =$	4) $(y - x)$
	5) $(x \times y) \times z$

21.

Column-I	Column-II
a) $-51 \times (16 + 13)$	1) $-51 + (16 + 13)$
b) $(-51 + 16) \times 13$	2) $(-51 + 16) - 13$
c) $(-51 + 16) + 13$	3) $(-51) \times 13 + 16 \times 13$
d) $-51 + (16 - 13)$	4) $(-51) \times 16 + (-51) \times 13$
	5) $-51 \times (16 \times 13)$

INTEGER ANSWER TYPE

22. $1 - 1 + 1 - 1 + 1 - 1 + \ldots\ldots\ldots\ldots(101 \text{ times}) =$ ___________

23. Product of three integers is 139750. And product of two of them is –2150 then the other number is _______

WORK SHEET - 3

SINGLE ANSWER TYPE

1. $(-1)^{19} + (2)^{2} + (3)^{3} =$
1)31 2)32 3) 30 4) 35

2. $(-3)^{5} + (-2)^{3} =$
1) -250 2) -251 3)–252 4) –253

3. If $(-2) \times (-2) \times (-2) \ldots\ldots$ 101 times is simplified, then the index form is
1) $(-2)^{101}$ 2) 2^{101} 3) $(-2)^{-101}$ 4) $(101)^{-2}$

4. Find the sum of cube of 10 and square of 21

1) 1451 2) 1461 3) 1441 4) 1431

5. The value of -8 raised to power of 3

1) 512 2) 256 3) –256 4) –512

6. If $48 = 2 \times 2 \times 2 \times 2 \times 3$ then $48 \times 3 \times 3 \times 3$ in power form

1) $2^4 \times 3^4$ 2) $2^4 \times 3$ 3) $2^4 \times 3^3$ 4) $2^3 \times 3^4$

7. The cube of (–16) is

1) 3096 2) –4096 3) –2096 4) –1096

8. If $7 \times 7 \times 7 \ldots\ldots\ldots$ x times $= 7^{70}$ and $8 \times 8 \times 8 \ldots\ldots\ldots$ y times $= 8^{71}$ then x, y are

1) x = 71; y = 70 2) x = 71; y = 71 3) x = 70; y = 71 4) x = 70; y = 70

9. Write $\frac{-27}{512}$ in power notation

1) $\left(\frac{-3}{8}\right)^2$ 2) $\left(\frac{-3}{8}\right)^3$ 3) $\left(\frac{-3}{8}\right)^4$ 4) $\left(\frac{-3}{8}\right)^5$

10. Simplify $\left(3^5 \times 5^2 \times 2^4\right) \div 2^3 \times 3^2$

1) $3^3 \times 5^2 \times 2^2$ 2) $3^3 \times 5^2 \times 2$ 3) $5^2 \times 3 \times 2$ 4) $5^2 \times 3^2 \times 2$

11. Simplify $(-2)^4 \times (-4)^2 \times 4^2$

1) 4096 2) 1596 3) 1024 4) 1156

MULTI ANSWER TYPE

12. $(-2) \times (-2) \times (-2) \times (-2) \times (-2) =$

1) $(-2)^5$ 2) $5^{(-2)}$ 3) -2^5 4) $5 \times (-2)$

13. $215 \times 128 = 27520$ then $(-27520) \div (-215)$

1) 128 2) $|-128|$ 3) 215 4) 27520

REASONING ANSWER TYPE

14. *Statement I* : In $(25)^{1000}$, 1000 is called index, and 25 is called base.

Statement II: In a^n, a is called base and n is called index.

1. Both Statements are true, Statement II is the correct explanation of Statement I.

2. Both Statements are true, Statement II is not correct explanation of Statement I.

3. Statement I is true, Statement II is false.

4. Statement I is false, Statement II is true.

15. *Statement I* : The value of -5 raised to the power of 5 is 3125

Statement II: a is raised to the power of m means a^m

1. Both Statements are true, Statement II is the correct explanation of Statement I.

2.Both Statements are true, Statement II is not correct explanation of Statement I.

3.Statement I is true, Statement II is false.

4.Statement I is false, Statement II is true.

COMPREHENSION TYPE

If x is an integer, the procuct x.x.x........... x (n times)= x^n power of a number is means multiplication of a number by itself several times.

16. $11 \times 11 \times 11$..............(p times) = 11^{25} then p is

1) 11   2) 25   3) 11×25 4) 25 + 11

17. $(-9) \times (-9) \times (-9)$..............(99 times) = $(-9)^q$ then q is

1) –9   2) 9   3) 99   4)–99

MATRIX MATCHING TYPE

18.

Column-I	Column-II
a) $15^{10} - 15^6$	1) 15^{16}
b) $15^{10} \div 15^6$	2) 15^4
c) $15^{10} \times 15^6$	3) $-(15^6 - 15^{10})$
d) $15^{10} + 15^6$	4) $15^6 + 15^{10}$
	5) 15^0

19.

Column-I	Column-II
a) Second power of -10	1) 10^{100}
b) 100^{th} power of 10	2) 100
c) Product -4 and square of 5	3) –100
d) 10^{th} power of 100	4) 100^{10}
	5) 10^{10}

INTEGER ANSWER TYPE

20. Sumof cube of 3 and square of 9 is ____

21. Quotient of $-(5^{91} \times 7^{76} \times 3^{13})$ and $-(7^{75} \times 5^{90} \times 3^{12})$ is _____

KEY

WORK SHEET – 1 (KEY)

1) 2	2) 1	3) 2	4) 3	5) 3
6) 1	7) 3	8) 1	9) 3	10) 2
11) 1	12) 3	13) 1,2,3	14) 1,2,3,4	15) 1
16) 1	17) 2	18) 3	19) 1	20) A-3 B-1 C-2 D-1,4
21) A-3 B-1,2,3,4,5 C-5 D-2,3,4,5	22) 1	23) 10		

WORK SHEET – 2 (KEY)

1) 1	2) 4	3) 1	4) 2	5) 2
6) 4	7) 1	8) 2	9) 2	10) 4
11) 1	12) 2	13) 1,3	14) 1,2,4	15) 1
16) 2	17) 2	18) 2	19) 1	20) A-3 B-5 C-4 D-2
21) A-4 B-3 C-1 D-2	22) 1	23) -65		

WORK SHEET – 3 (KEY)

1) 3	2) 2	3) 1	4) 3	5) 4
6) 1	7) 2	8) 3	9) 2	10) 2
11) 1	12) 1,3	13) 1,2	14) 1	15) 4
16) 1	17) 4	18) 2	19) 3	20) A-3 B-2 C-1 D-4
21) A-2 B-1 C-3 D-4	22) 108	23) 105		