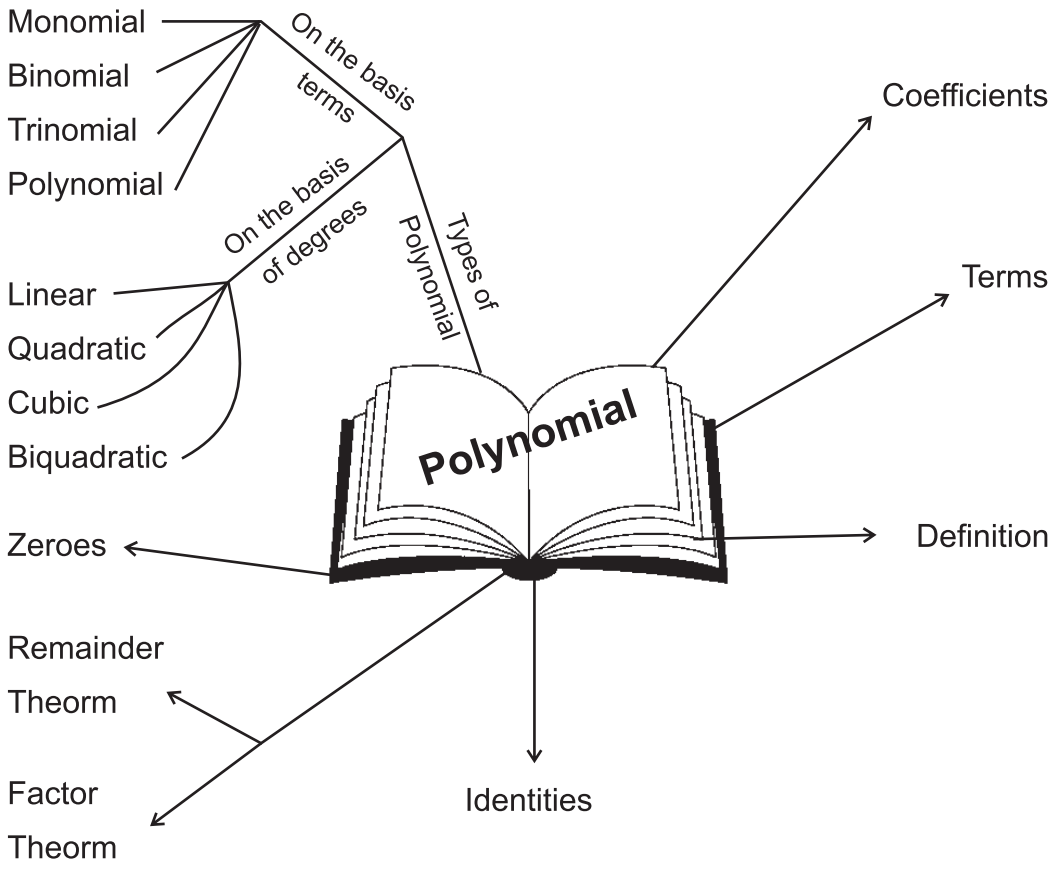
CHAPTER-2
POLYNOMIALS
MIND MAP



CHAPTER-2

POLYNOMIALS

KEY POINTS

1. A Polynomial p(x) in one variable x is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1x + a_0$, where

   (i) a_0, a_1, a_2 ..... a_n are constants and $a_n \neq 0$

   (ii) a_0, a_1, a_2..... a_n are respectively the coefficients of x^0, x^1, x^2, ... , x^n

   (iii) Each of a_nx^n, $a_{n-1}x^{n-1}$, $a_{n-2}x^{n-2}$, ... , a_2x^2, a_1x, a are called terms of the polynomial.

   (iv) n is called the degree of the polynomial where n is a non-negative integers.

2. **Degree of the Polynomial :** Highest power of x in the algebraic expression is called the degree of the polynomial.

3. **Different types of polynomials :**
   Generally, we divide the polynomials in the following categories :

   (i) Based on degrees

	Degree	Polynomial	General form	Examples
(a)	1	Linear	ax + b,	x + 1, 2x etc.
(b)	2	Quadratic	$ax^2 + bx + c$,	$4x^2 + 5x + \frac{2}{3}$ etc.
(c)	3	Cubic	$ax^3 + bx^2 + cx + d$,	$x^3 - 3x^2 + 5$ etc.
(d)	4	Biquadratic	$ax^4 + bx^3 + cx^2 + dx + e$,	$x^4 - 16$ etc.

   a, b, c, d, e are real constants and $a \neq 0$.

   Note : A polynomial of degree five or more than five does not have any particular name. Such a polynomial is usually called a polynomial of degree five or six or ... etc.

   (ii) Based on Number of Terms:

	No. of Terms	Polynominal	Examples
(a)	1	Monomial	5, 3x, $\frac{1}{3}y$ etc.
(b)	2	Binomial	$\sqrt{3} + 6x$, $x - 5y$, $x^2 + 2$ etc.
(c)	3	Trinomial	$\sqrt{2}x^2 + 4x + 2$, $5y^4 + 2y + 6$ etc.

Note : A polynomial having four or more than four terms does not have particular name. These are simply called polynomials.

(iii) Zero degree polynomial or non-zero constant polynomial.

Any non-zero number (constant) is regarded as polynomial of degree zero or zero degree polynomial. i.e., $p(x) = a$ where $a \neq 0$ is a zero degree polynomial, since we can write $p(x) = a$,

as $p(x) = ax^0$

e.g., $5 = 5x^0$, $\frac{\sqrt{7}}{2} = \frac{\sqrt{7}}{2} x^0$

(iv) Zero Polynomial : A polynomial whose all coefficients are zero is called as zero polynomial i.e., $p(x) = 0$. The degree of zero polynomial is not defined or we can not determine the degree of zero polynomial.

4. For a polynomial $p(x)$ if $p(a) = 0$ where a is a real number we say that 'a' is a zero of the polynomial.

5. If $p(x)$ is any polynomial of degree greater than or equal to 1 and $p(x)$ is divided by a linear polynomial $x – a$, then the remainder is $p(a)$. This is called remainder theorem.

6. If $p(x)$ is a polynomial of degree ≥ 1 and 'a' is any real number then

(i) $(x – a)$ is a factor of $p(x)$, if $p(a) = 0$ and

(ii) $p(a) = 0$ if $(x – a)$ is a factor of $p(x)$.

This is called factor theorem.

7. A polynomial of degree 'n' can have at most n zeroes.

- Some algebraic identities :–

(i) $(x+y)^2 = x^2 + 2xy + y^2$

(ii) $(x–y)^2 = x^2 – 2xy + y^2$

(iii) $x^2–y^2 = (x+y)(x–y)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x+y)^3 = x^3 + y^3 + 3xy\,(x+y) = x^3 + y^3 + 3x^2y + 3xy^2$

(vii) $(x-y)^3 = x^3 - y^3 - 3xy\,(x-y) = x^3 - y^3 - 3x^2y + 3xy^2$

(viii) $x^3 + y^3 = (x+y)\,(x^2 - xy + y^2)$

ix) $x^3 - y^3 = (x-y)\,(x^2 + xy + y^2)$

x) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)\,(x^2 + y^2 + z^2 - xy - yz - zx)$

$$= \frac{1}{2}\,(x+y+z)\left\{(x-y)^2 + (y-z)^2 + (z-x)^2\right\}$$

xi) If $x+y+z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

POLYNOMIALS

1. Find $ax^p + bx^q + c$ to be polynomial p & q are :

   a) Rational Numbers b) Natural Numbers

   c) Real Numbers d) Whole Numbers

2. Which of the following is/are polynomial (s) :

   a) $\sqrt{7x} + 5$ b) $\sqrt{7}\,x + 5$

   c) $\dfrac{\sqrt{7}x + 5}{\sqrt{7}x - 5}$ d) $\dfrac{5x^{5/2} + 3x^{3/2}}{x}$

3. Choose the correct option for a polynomial :

   i) $3x+2$ ii) $7x+1 = 0$

   iii) $5x^4 + 3x^2 + 1 = 0$ iv) $x^3 + 3x^2 + 1$

   a) i) & ii) b) i) & iii)

   c) ii) & iv) c) i) & iv)

4. The terms of the polynomial $x^3 - 4x^2 - 3x + 2$ are :

   a) 1, –4, –3, 2 b) $x^3 - 4x^2 - 3x + 2$

   c) $+x^3, -4x^2, -3x, 2$ d) $x^3, 4x^2, 3x, 2$

5. Coefficient of x^2 in $(x^2 - 1)(x - 2)$ is :

   a) 2 b) –2

   c) –1 d) +1

6. The degree of the polynomial $\sqrt{5}$ is :

   a) 1/2 b) 0

   c) 1 d) –1

7. If deg (f(x)) = 5 & deg (g(x)) = 4 then deg [f(x) – g(x)] is.

   a) 5 b) 4

   c) 1 d) 9

8. Degree of cubic polynomial with two terms is –

   a) 0 b) 1

   c) 2 d) 3

9. Degree of zero polynomial is :

a) 0 b) 1

c) 2 d) Not defined

10. Degree of non zero constant polynomial is –

a) 0 b) 1

c) –1 d) Not defined

11. The zero (s) of the polynomial x^2-8 is (are) :–

a) 8 b) $2\sqrt{2}$

c) $2\sqrt{2}, -2\sqrt{2}$ d) $\sqrt{8}$

12. The zero (s) of the polynomial z^2-2z is (are) –

a) 0 b) 2z

c) 2 d) 0, 2

13. If (px+q) is a factor of the polynomial h(x) then which one is true :

a) $h\left(\frac{-p}{q}\right)=0$ b) $h\left(\frac{p}{q}\right)=0$

c) $h\left(\frac{q}{p}\right)=0$ d) $h\left(\frac{-q}{p}\right)=0$

14. Let h(x) be a polynomial such that $h\left(-\frac{1}{3}\right)=0$, then one of the factors of h (x) is –

a) 3x–1 b) 3x+1

c) x–3 d) x+3

15. If y+2m is a factor of $y^5-4m^2y^3+2y+2m+3$ then value of m is –

a) $\frac{2}{3}$ b) $\frac{3}{2}$

c) 1 d) $\frac{-3}{2}$

16. If $\left(\frac{1}{16}p^2-q\right)=\left(\frac{1}{4}p-11\right)\left(\frac{1}{4}p+11\right)$ then q is –

a) 11 b) 1

c) 121 d) $\frac{11}{4}$

17. If $3x = a + b + c$ then $(x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c)$ is

a) $a+b+c$ b) 0

c) 1 d) $3(x-a)(x-b)(x-c)$

18. If $p + q + r = 9$ then $(3-p)^3 + (3-q)^3 + (3-r)^3$ is :–

a) $3(3-p)(3-q)(3-r)$ b) 0

c) 1 d) $-3(3-p)(3-q)(3-r)$

19. If $(x-1)(x-2)(x+c) = x^3 + ax^2 + bc + 5\times2\times1$ then c will be

a) 1 b) 2

c) 5 d) –5

20. If $(x+2)(x-5) = x^2 + (a + b)x + a \times b$ then value of (a+b) is

a) 3 b) –3

c) 7 d) –10

Fill in the blanks:–

21. $49^3 - 30^3 + (.....)^3 = 3 \times 49 \times 30 \times 19$

22. The polynomial containing two non zero terms is called .....

23. The polynomial containing exactly two non zero, zeroes has ..... degree .....

24. If $l(x) = 4x+1$ then $l(-6) - l(-5)$ is .....

25. If $p(x) = x^3 - 2x^2 + x + 1$ then $p(0) \times p(-1) =$ .....

26. If $q(x) = x^2 - 3x + 2$ then $p(1) + p(-1) - p(0)$ is .....

27. If side of a square is (x+2y–z) units then area of the square is .....

28. If $x^2 + mx - 30 = (x-5)(x+6)$ then m is .....

29. A quadratic polynomial can be written as the product of ..... linear polynomials.

30. If the factors of $5x^2 - 18x + 9$ are (ax+b), (x+b) then the values of a & b are ..... & ..... respectively.

31. In the polynomial $x^3 - 5x$, the expressions x^4 & –5x are called ..... of the polynomial

32. When a polynomial q(x) is divided by (x–2) & the remainder q(2) = 0 then (x–2) is a ..... of the polynomial.

33. Write True of False :

i) Every polynomial is also an equation.

ii) Every polynomial is binomial.

iii) A binomial may have degree 5.

iv) If 2 is a zero of a polynomial q(x) then 2 is also a zero of $2 \times q(x)$.

v) If (x-a) is a factor of polynomial p(x) then a is a zero of $a \times p(x)$.

vi) x=3 is a zero of the polynomial $x^3 - 3x + x - 3$.

vii) 2, 1 and –1 all are zeroes of $x^2 - x - 2$.

viii) (x+1) is a factor of $x^n + 1$ only if n is odd positive integer.

ix) When $(p^2 - p - 29)$ is divided by (p–6) the remainder is 1.

x) The remainder theorm is true only when the divisor of the polynomial is linear polynomial.

34.

	Column I		Column II
i)	Degree of the polynomial $0.x^4 + 4x^3 - 2x + 3$	a)	$(100-3)^2$
ii)	Factors of $(x+y)^3 - (x^3+y^3)$	b)	0
iii)	97^2 can be solved as	c)	3
iv)	Zero (s) of $(x-2)^2 - (x+2)^2$	d)	3, x, y, (x+y)

35.

	Column I		Column II
i)	103×103	a)	0
ii)	If $\frac{x}{y} + \frac{y}{x} = 2$ then value of $(x-y)^2$ is	b)	1
iii)	Number of zeros of $px + q$	c)	–1
iv)	the value of K when $(-x^{140} - 2x^{151} + K)$ is divided by $(x+1)$	d)	$(100+3)^3$

36. Check whether $q(x)$ is a multiple of $r(x)$ or not.

Where $q(x) = 2x^3 - 11x^2 - 4x + 5$, $r(x) = 2x+1$

37. Show that $(x-5)$ is a factor of $x^3-3x^2-4x-30$.

38. Evaluate by using suitable identity : $(997)^3$

39. Find the zeroes of the polynomial $p(x) = x\,(x-2)\,(x+3)$

40. Find the quotient when $3x^2-7x-6$ is divided by (x–3)

41. Factories $8x^3 + \sqrt{27}\,y^3$.

42. If $p(x) = x + 9$, then find $p(x) + p(-x)$

43. Find the product without multiplying directly 106×94.

44. Expand using suitable identity $(2x-3y+z)^2$

45. Find the value of $(351)^2 - (350)^2$.

PART (C)

46. Factories : $64a^2 + 96ab + 36b^2$

47. Factories : $x^3 + 6x^2 + 11x + 6$

48. If $x^2 + y^2 = 49$ and $x-y = 3$, then find the value of x^3-y^3.

49. Simplify : $(5a-2b)\,(25a^2+10ab+4b^2)-(2a+5b)(4a^2-10ab+25b^2)$

50. Find the sum of remainders when x^3-3x^2+4x-4 is divided by (x–1) and (x+2).

51. Find the product $\left(p - \frac{1}{p}\right)\left(p + \frac{1}{p}\right)\left(p^2 + \frac{1}{p^2}\right)\left(p^4 + \frac{1}{p^4}\right)$

52. Factories : $7\sqrt{2}\,k^2 - 10k - 4\sqrt{2}$.

53. Simplify : $(3x-4y)^3 - (3x+4y)^3$

54. Use appropriate identity, expand $(2a)^3 + b^3 + (3c)^3 - 18abc$.

55. Simplify : $(x+y+z)^2-(x-y-z)^2$.

56. Factories : $125x^3 + 8y^3 + z^3 - 30xyz$.

57. $x+2$ is a factor of polynomial $ax^3 + bx^2 + x - 2$ and the remainder 4 is obtained on dividing this polynomial by $(x-2)$. Find the value of a and b.

58. If the polynomial $ax^3 + 4x^2 + 3x-4$ & $x^3-4x + a$ leave the same remainder when divided by (x–3). Find a

59. If $\left(\frac{9}{10}\right)^3 - \left(\frac{2}{5}\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{x}{50}$, find x

60. If (x–3) and $\left(x - \frac{1}{3}\right)$ are factors of the polynomial $px^2 + 3x + r$, show that p = r.

61. i) Using identity, find the value of $(-7)^3 + (5)^3 + (2)^3$.

ii) Find dimensions of cube whose volume is given by the expression $4x^2 + 14x + 6$.

62. Give possible expression for the length and breadth of each of the following rectangles if.

i) Area = $(x^2 + 5\sqrt{5}x + 30)$ sq. unit.

ii) Area = $(24x^2 - 26x - 8)$ sq. unit.

63. A literacy campaign was organised by Class IX girl students under NSS. Students made $(x–5)$ rows and $(3x–4)$ columns for the rally. Write the total number of students in the form of a polynomial.

64. Under tree plantation programme students of Class IX planted total $(3x^2 – 4x – 4)$ trees in school.

If total number of students is the class are (x – 2) then find out number of trees planted by each students. (Assuming each student planted equal number of trees).

65. If a + b + c = 0, find the value of

$$\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab}$$

66. Simplify :

$$\frac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$$

67. Factories:

$(2a–b–c)^3 + (2b–c–a)^3 + (2c–a–b)^3$

68. If the polynomial $4x^3–16x^2+ax+7$ is exactly divisible by $x–1$, then find the value of a. Hence factorise the polynomial.

69. If $p, q, \& r$ are all non zero and $p+q+r = 0$, prove that

$$\frac{p^2}{qr} + \frac{q^2}{rp} + \frac{r^2}{pq} = 3$$

70. Factories : $9x^3 – 27x^2 – 100x + 300$

71. If $(x+4)$ is a factor of the polynomial $x^3 - x^2 - 14x + 24$, find the other factors.

72. If $\frac{x}{y} + \frac{y}{x} = -1$ where $x \neq 0, y \neq 0$ then find the value of $x^3 - y^3$.

73. Simplify : $\frac{155 \times 155 + 155 \times 55 + 55 \times 55}{155 \times 155 \times 155 - 55 \times 55 \times 55}$

CHAPTER-2
POLYNOMIALS
ANSWERS

1. d) Whole Number
2. b) $\sqrt{7}\,x + 5$
3. d) (i) & (iv)
4. c) $x^3, -4x^2, -3x, 2$
5. b) –2
6. b) 0
7. a) 5
8. d) 3
9. d) Not defined
10. a) 0
11. c) $\pm 2\sqrt{2}$
12. d) 0, 2
13. $h\left(\frac{-q}{p}\right) = 0$
14. b) 3x + 1
15. b) 3/2
16. c) 121
17. b) 0
18. a) 3(3–p) (3–q) (3–r)
19. c) 5
20. b) –3
21. 19
22. Binomial
23. 2

23. 2

24. – 4

25. – 5

26. 4

27. $(x + 2y – z)^2 = x^2 + 4y^2 + z^2 + 4xy – 4yz – 2xz$

28. 1

29. Two

30. 5 and 3

31. Terms

32. Factor

33. i) False
    ii) False
    iii) True
    iv) True
    v) True
    vi) False
    vii) False
    viii) True
    ix) True
    x) True

34. i) c
    ii) d
    iii) a
    iv) b

35. i) d
    ii) a
    iii) b
    iv) c

36. 110

37. Hint Put $x = 5$

38. 991026973

39. 0, 2, –3

40. $(3x+2)$

41. $(2x + \sqrt{3}\, y\,)\ (4x^2 – 2\sqrt{3}\, xy + 3y^2)$

42. 18

43. Hint : (100+6) (100–6) = 9964

44. $4x^2 + 9y^2 + z^2 – 12xy – 6yz + 4xz$

45. 701

46. $(8a + 6b)^2$

47. (x+1) (x+2) (x+3)

48. 207

49. $117a^3 – 133b^3$

50. – 34

51. $p^8 – \frac{1}{p^8}$

52. $(K – \sqrt{2})\ (7\sqrt{2}\, K + 4)$

53. $– 8y(16y^2 + 27x^2)$ or $–128y^3 – 216\, x^2y$

54. –

55. 4xy + 4zx

56. $(5x + 2y + z)\ (25y^2 + 4y^2 + z^2 – 10xy – 2yz – 5zx)$

57. a = 0, b = 2

58. a = –1 Hint p(3) = q (3)

59. x = 27 (Use a+b+c = 0, $a^3 + b^3 + c^3 = 3abc$)

60. –

61. i) – 210

    ii) 2 ; (x + 3) ; (2x + 1)

62. i) $(x + 2\sqrt{5})(x + 3\sqrt{5})$

ii) $(4x + 1), (6x - 8)$

63. $3x^2 - 19x + 20$

64. $(3x + 2)$

65. 3

66. $(a+b)(b+c)(c+b)$

67. $3(2a-b-c)(2b-c-a)(2c-a-b)$

68. $a = 5, (x-1)(2x+1)(2x-7)$

70. $(3x+10)(x-3)(3x-10)$

71. $(x-3)(x-2)$

72. 0

73. 0.01

Practice Test
POLYNOMIALS

Time : 50 Min. M.M. 20

1. Is $(x^2)^{\frac{1}{2}} + 2\sqrt{5}a$ a polynomial? (1)

2. Show that x = 1 is a zero of the polynomial $3x^3 - 4x^2 + 8x - 7$. (1)

3. Find the zeroes of the polynomial $x^2 - 4x + 3$ (2)

4. If x + y + z = 6, xy + yz + zx = 11. Find the value of $x^2 + y^2 + z^2$ (2)

5. If 3x – 4 is a factor of the polynomial $p(x) = 2x^3 - 11x^2 + kx - 20$, find the value k (3)

6. Factorise : $a^2 + b^2 + 2(ab + bc + ca)$ (3)

7. If a + b + c = 0 then find the value of (4)
$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$$

8. Factorise $x^3 - 23x^2 + 142x - 120$ by using factor theorem. (4)