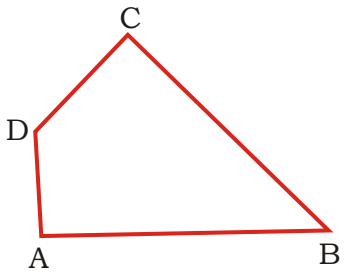
QUADRILATERALS

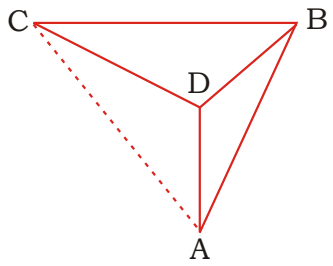
SYNOPSIS - 1

Quadrilateral: A quadrilateral is a closed figure formed by four line segments such that no two line segments cross each other except at their end points. Quadrilaterals are classified into two types.

i) Convex quadrilateral: A quadrilateral in which the measure of each interior angle is less than 180° is called a convex quadrilateral.



ii) Concave quadrilateral: A quadrilateral in which the measure of one of the interior angle is more than 180° is known as a concave quadrilateral.



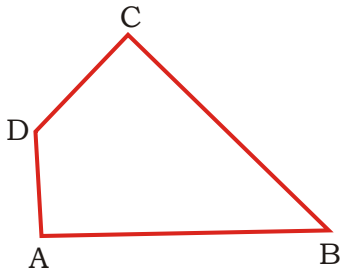
In a quadrilateral ABCD:

1. Four sides $\overline{AB}, \overline{BC}, \overline{DA}, \overline{CD}$.
2. Four angles $\angle A, \angle B, \angle C, \angle D$
3. Four vertices A, B, C, D.
4. Two diagonals $\overline{AC}, \overline{BD}$

(Diagonal: It is a line segment joining the opposite vertices of quadrilateral)

In a quadrillateral ABCD:

1. Adjacent sides: $\overline{AB}$ and $\overline{BC}$; $\overline{BC}$ and $\overline{CD}$; $\overline{CD}$ and $\overline{DA}$
2. Adjacent angles: $\angle A$ and $\angle B$; $\angle B$ and $\angle C$; $\angle C$and $\angle D$; $\angle D$ and $\angle A$
3. Opposite sides: $\overline{AB}$ and $\overline{CD}$; $\overline{AD}$ and $\overline{BC}$
4. Opposite angles: $\angle A$ and $\angle C$; $\angle B$and $\angle D$



Note:

1. The sum of the interior angles in a quadrilateral is 360°. (i.e., 4 right angles)

   Example: In a quadrilateral ABCD, $\angle A + \angle B + \angle C + \angle D = 360^0$

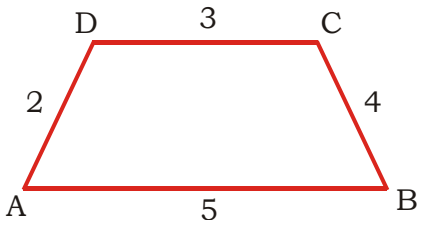
2. Each diagonal divides the quadrilateral into two triangles.

   Example: The diagonal AC divides the quadrilateral ABCD into ΔABC and ΔADC.

3. In general, convex quadrilateral is treated as quadrilateral
4. In a convex quadrilateral, both diagonals lie in the interior where as in the case of concave quadrilateral one diagonal lies in the interior and the other lies in the exterior.

Perimeter: Perimeter of a quadrilateral is the sum of the lengths of all sides of the quadrilateral.

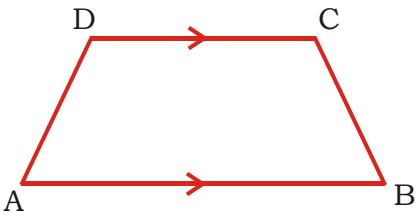
Example: The perimeter of the following quadrilateral = AB + BC + CD + DA
= 5 + 4 + 3 + 2 = 14 units.



1. Interior point of the quadrilateral: A point is said to be an interior point of the quadrilateral if it lies inside the quadrilateral.
2. Exterior point of the quadrilateral: A point is said to be an exterior point of the quadrilateral if it lies outside the quadrilateral.
3. Point on the quadrilateral: A point is said to be on the quadrilateral if it lies on any side of quadrilateral.

Trapezium: A Trapezium is a quadrilateral in which one pair of opposite sides are parallel.

Note: The parallel sides $(\overline{AB}, \overline{CD})$ are called the bases of the trapezium. And the other two sides are called its non-parallel sides (legs). $(\overline{BC}, \overline{AD})$

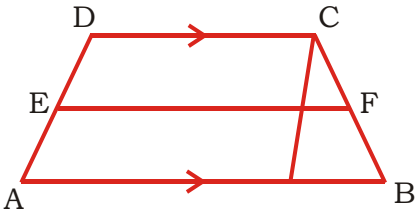


Median of Trapezium: A line segment formed by joining the midpoints of non parallel sides of a trapezium is called the median of a trapezium.

Note:

1. The length of the median of a trapezium is equal to half of the sum of the parallel sides.

   Example: If ABCD is a trapezium $(\overline{AB}, \overline{CD})$, then the median $\overline{EF} = \frac{1}{2}(\overline{AB} + \overline{CD})$



2. The median of a trapezium is parallel to the bases of the trapezium.

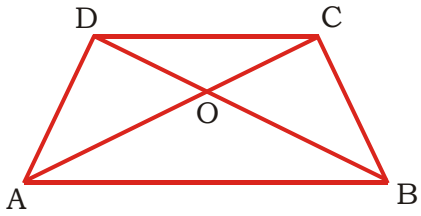
Altitude of a Trapezium: The perpendicular distance from vertex to the opposite base.

Properties:

i) Consider a trapezium ABCD in which $\overline{AB} \| \overline{CD}$. $\angle A + \angle D = 180°, \angle B + \angle C = 180°$.

ii) The diagonals of a trapezium divides each other proportionally.

Example: If the diagonals $(\overline{AC}$ and $\overline{BD})$ of a trapezium ABCD $(\overline{AB} \| \overline{CD})$ intersect at O then $\frac{DO}{OB} = \frac{CO}{OA}$.

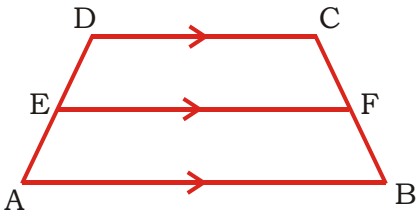


iii) If the diagonals are proportional in a quadrilateral then it forms a trapezium.

iv) Any line parallel to the parallel sides of the trapezium divides the non-

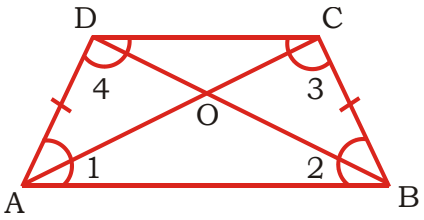
parallel sides proportionally.

Example: In a trapezium ABCD (AB || CD), if EF || AB, then $\frac{AE}{ED} = \frac{BF}{FC}$.



Note: Difference between trapezoid and trapezium
Trapezoid: A quadrilateral with no sides parallel
Trapezium: A quadrilateral with one pair of parallel sides

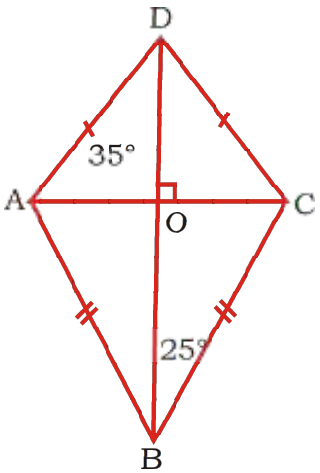
Isosceles Trapezium: A trapezium in which the non parallel sides are equal to each other is known as an Isosceles trapezium. In the Isosceles trapezium ABCD, $\overline{AB} \| \overline{CD}, AD = BC$



1. $\angle 1 + \angle 4 = 180^0$ and $\angle 2 + \angle 3 = 180^0$
2. Base angles are equal $(\angle 1 = \angle 2$ and $\angle 3 = \angle 4)$
3. The lengths of diagonals are equal (AC = BD)

Kite: A quadrilateral having two pairs of equal adjacent sides but unequal opposite sides is called a kite. A kite ABCD with AB = BC & AD = CD

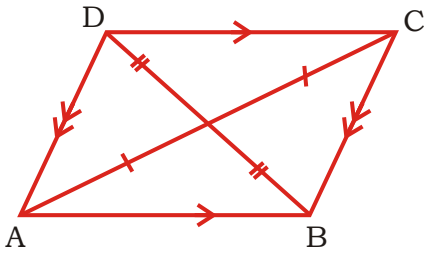
Properties:



1. The diagonals of kite are perpendicular to each other i.e., $\overline{BD} \perp \overline{AC}$.
2. OA = OC
3. $\angle A = \angle C$
4. Diagonal BD bisects $\angle B, \angle D$
5. Diagonal BD divides the kite into two congruent triangles.

Parallelogram:

A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram.



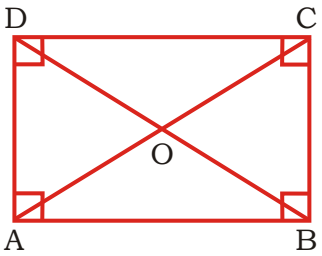
1. In a parallelogram ABCD two pairs of opposite sides are equal i.e., AB = CD; and BC = AD.
2. Opposite angles are equal i.e., $\angle A = \angle C; \angle B = \angle D$.
3. The diagonals of a parallelogram bisect each other (AO = OC, BO = OD).
4. In a paralllelogram each diagonal divides it into two congruent triangles.
5. In a parallelogram the sum of the adjacent angels is equal to 180°. i.e., They are supplementary

$(\angle A + \angle B = 180°; \angle A + \angle D = 180°; \angle C + \angle D = 180°; \angle B + \angle C = 180°)$

6. If a quadrilateral has two pairs of opposite sides are parallel and equal then it forms a parallelogram.
7. The sum of the squares of four sides of the parallelogram is equal to the sum of the squares of the diagonals.

   i.e., In a parallelogram ABCD: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Rectangle: If one of the angles of a parallelogram is a right angle, then all angles are right angles. Such a parallelogram is called a rectangle. (OR) A parallelogram in which one angle is a right angle is called a rectangle.



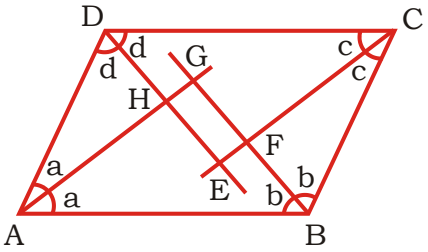
Properties:

A rectangle satisfies all the properties of parallelogram:

1) The length of the diagonals of a rectangle are equal.
2) Opposite sides are equal.
3) Opposite angles are equal.
4) Each diagonal divides it into congruent right angled triangles
e) The diagonals of a rectangle bisect each other.

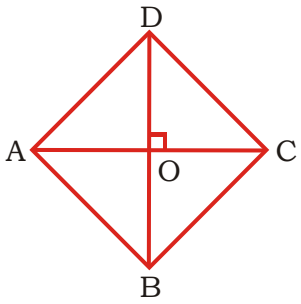
Note:

1. The bisectors of angles of a parallelogram forms a rectangle.



2. In a rectangle ABCD, O is an interior point of rectangle then $OA^2 + OC^2 = OB^2 + OD^2$
3. In a rectangle, the square of a diagonal is equal to the sum of the squares of the sides. i.e., In a rectangle ABCD, $AC^2 = AD^2 + DC^2$ or $AC^2 = AB^2 + BC^2$.

Rhombus: A parallelogram in which two adjacent sides are equal is called a rhombus.

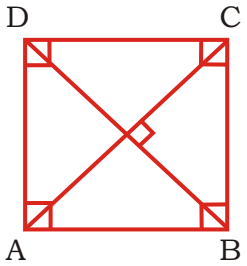


Properties:

1) Each diagonal of a rhombus divides it into two congruent isosceles triangles.
2) Opposite angles are equal and the sum of any two adjacent angles is 180°.
3) The diagonals bisect each other perpendicularly.
4) The diagonals AC bisects $\angle A$ and $\angle C$; the diagonal BD bisects $\angle B$ and $\angle D$.

Square:

A rectangle in which adjacent sides are equal is called a square. (OR)
A rhombus in which one of its angles is a right anlge is called a square.



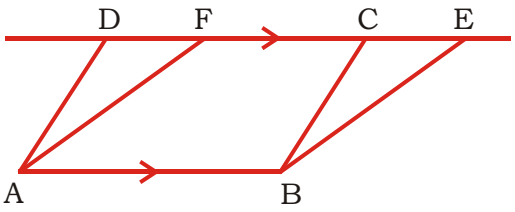
Properties:

1) All sides are equal.
2) Each angle is equal to 90°
3) The diagonals are equal and are mutually perpendicular bisectors.
4) Each diagonal divides the square into two congruent right angled isosceles triangles.

e) The quadrilateral formed by joining successively the midpoints of sides of a square is a square.

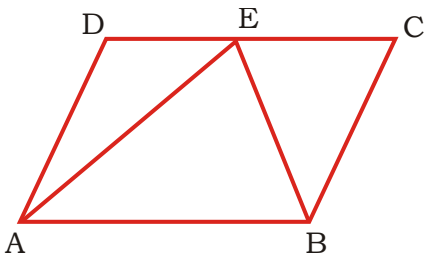
Note:

1. If the diagonals of a parallelogram are equal and right bisectors of each other then it is a square.
2. In a rhombus ABCD the diagonal $\overline{AC}$ bisects $\angle A$ and $\angle C$; the diagonal $\overline{BD}$ bisects $\angle B$ and $\angle D$.
3. The diagonals of a rhombus divide it into four congruent right angled triangles.
4. Parallelograms on equal base and between the same parallel lines are equal in area.



Area of $||^{gm}$ ABCD = Area of $||^{gm}$ ABEF

5. If a triangle and a parallelogram are on the same base and between the same parallel lines then the area of the triangle is equal to half that of the parallelogram.



Area of $\Delta ABE = \frac{1}{2}\left(\text{Area of} ||^{gm} ABCD\right)$.

6. The quadrilateral formed by joining the midpoints of sides of rectangle successively is a rhombus.
7. The quadrilateral formed by joining successively the midpoints of sides of rhombus is a rectangle.

WORK SHEET

SINGLE ANSWER TYPE

1. A simple closed figure formed by the four line segments is called a
   1) polygon 2) Triangle 3) Quadrilateral 4) None of these
2. In a quadrilateral ABCD, AB + BC + CD + DA is called
   1) Area 2) perimeter 3) volume 4) None of these
3. In a quadrilateral the line segment joining any two opposite vertices is called a
   1) Ray 2) Radius 3) diagonal 4) None of these
4. The sura of the angles in a quadrilateral is
   1) 180^0 2) 360^0 3) right angles 4) both B & C
5. The angles of a quadrilateral are x^0, $x^0 - 10^0$, $x^0 + 30^0$ and $2x^0$ then the least angle in a quadrilateral is
   1) 15 B)45 C)38^0 4) 68^0
6. In a quadrilateral the angles are in the ratio of 3:4:5:6 then the least angle is
   1) 50^0 2) 45^0 3) 60^0 4) None
7. In a quadrilateral one pair of opposite sides are parallel then it is called a
   1) parallelogram 2) Rhombus
   3) Rectangle 4) Trapezium
8. A quadrilateral in which both pair of opposite sides are parallel then it is called a
   1) parallelogram 2) rectangle
   3) Rhombus 4) None
9. In a parallelogram any two adjacent angles are
   1) equal 2) complimentary 3) supplementary 4) None
10. In a parallelogram diagonal divides the parallelogram in to two triangles.
    1) congruent 2) similar 3) equal 4) None
11. The length of the diagonals in a rectangle are
    1) equal 2) bisect each other
    3) Both A & B 4) not equal
12. A parallelogram in which two adjacent sides are equal is called a
    1) Rhombus 2) Rectangle 3) Square 4) Trapezium
13. Opposite angles in a rhombus are equal and the sum of any two adjacent Angles are
    1) 1800 2) supplementary 3) Complimentary 4) Both A & B
14. A rectangle in which adjacent sides are equal is called a
    1) Rhombus 2) Parallelogram 3) rectangle 4) square

15. A quadrilateral becomes a parallelogram
    1) Both pairs of opposite sides are equal
    2) one pair of opposite sides are equal and parallel
    3) It's diagonals bisect each other.
    4) All of these
16. A quadrilateral becomes a rectangle if
    1) All its angles are right angles.
    2) Its diagonals are equal and bisect each other
    3) Both pairs of opposite sides are equal and parallel
    4) All of these.
17. A quadrilateral becomes a rhombus if
    1) All its sides are equal and opposite sides are parallel.
    2) Its diagonals bisect each other at right angles
    3) Both A (an4) B
    4) None
18. A quadrilateral becomes a square if
    1) All its sides are equal and one angle is 90^0.
    2) All its angles are equal and the diagonals intersect at right angles.
    3) All sides are equal and the diagonals are equal
    4) All of these.
19. A quadrilateral in which diagonals are equal and bisect each other perpendicularly is a
    1) square 2) Rhombus which is not a square
    3) Rectangle which is not a square 4) None of these
20. In a parallelogram ABCD, $\angle A = 50^0$, then $\angle B, \angle C$ and $\angle D$ are respectively
    1) $50^0, 130^0, 130^0$ 2) $130^0, 50^0, 130^0$
    3) $130^0, 130^0, 50^0$ 4) $130^0, 50^0, 50^0$
21. To construct a rhombus how many independent measurements are required
    1) one 2) two 3) three 4) four
22. To construct a trapezium how many independent measurements are required
    1) one 2) two 3) three 4) four
23. The basic objects of the nature which we see, and they do not require any proof then we call them as
    1) denned objects 2) undefined objects
    3) redefined objects 4) none
24. The terms which are defined with the help of the basic terms are said to be
    1) defined terms 2) undefined terms
    3) redefined terms 4) all
25. The axioms must be_____
    1) consistent 2) inconsistent3) both A, B 4) none
26. Any theorem is a statement.
    1) conditional 2) unconditional 3) both A, B 4) none

MULTI ANSWER TYPE

27. The sum of the interior angles in a quadrilateral is
    1) 180° 2) 360°
    3) 4 (right angle) 4) Two right angles

28. In a parallelogram ABCD if $\angle A = 115^0$ then the value of $\angle B, \angle C, \angle D$ is
    1) $\angle B = 65^0, \angle C = 115^0, \angle D = 65^0$ 2) $\angle B = 115^0, \angle C = 65^0, \angle D = 65^0$
    3) $\angle C = 115^0, \angle B = \angle D = 65^0$ 4) $\angle C = \angle A$ and $\angle B = \angle D$

REASONING ANSWER TYPE

29. *Statement-I*: In the isosceles trapezium ABCD. The length of diagonals are equal (AC = BD).

    Statement-II: $\overline{AB} \,||\, \overline{CD}$ and AD = BC.

    1) Both Statements are true, Statement II is the correct explanation of Statement I.
    2) Both Statements are true, Statement II is not correct explanation of Statement I.
    3) Statement I is true, Statement II is false.
    4) Statement I is false, Statement II is true.

30. *Statement-I*: A parallelogram in which two adjacent sides are equal is called a rhombus.

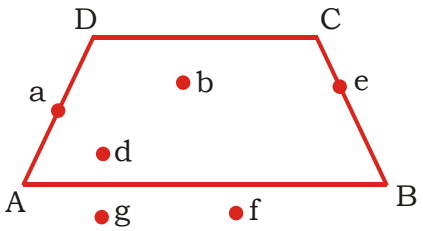
    Statement-II: A closed figure bounded by four lines segments is called a quadrilateral.

    1) Both Statements are true, Statement II is the correct explanation of Statement I.
    2) Both Statements are true, Statement II is not correct explanation of Statement I.
    3) Statement I is true, Statement II is false.
    4) Statement I is false, Statement II is true.

COMPREHENSION TYPE

Writeup:1

A point is said be an interior point of the quadrilateral if it lies inside the quadrilateral. If it lies outside, it is exterior. If it lies on any side it is said to be on the quadrilateral from the given figure

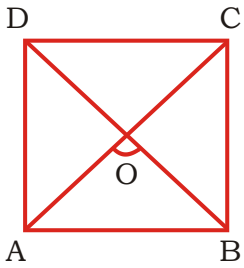


31. Interior points of the quadrilateral ABCD
    1) a, b 2) c, d 3) b, d 4) e, f

32. The points a and e are
    1) Interior 2) Exterior
    3) On the quadrilateral 4) do not exist

33. Exterior points of ABCD are
1) g, f 2) a, d 3) d, e 4) b, e

Writeup:2



The lengths of the diagonals of a rhombus are AC = 24 cm and BD = 18 cm respectively

34. Area of the rhombus is
1) 216 cm^2 2) 316 cm^2 3) 416 cm^2 4) 250 cm^2

35. In the above figure ABCD, AO + OB =
1) 24 2) 21 3) 18 4) 9

36. In the above figure ABCD, $\angle C = 60^0$, then $\angle D$ =
1) 40^0 2) 60^0 3) 120^0 4) 140^0

MATRIX MATCHING TYPE

37.

Column - I	Column - II
1) In ABCD, $\angle A + \angle C = 180°; \angle B + \angle D =$	1) Concave quadrilateral
2) Sum of interior angles in a quadrilateral	2) 180°
3) Interior angle < 180°	3) 360°
4) Interior angle > 180°	4) Convex quadrilateral
	5) 90°

38.

Column - I	Column -II
1) A parallelogram having all sides are equal is called	1) Kite
2) A quadrilateral in which two opposite sides are parallel is called	2) Trapezium
3) A quadrilateral in which two pair of adjecent sides are equal is called	3) Rhombus
4) A parallelogram each of whose angles measures 90^0 is called	4) Rectangle
	5) Parallelogram

INTEGER ANSWER TYPE

39. In the trapezium ABCD, AB is parallel to CD and EF is a median. BC = 6 cm., EF = 4cm, then AD is _____

40. Each side of a rhombus is 10cm long and one of its diagonals measures 16cm then the length of the other diagonal is _____

WORK SHEET (KEY)				
1) 3	2) 2	3) 3	4) 2	5) 4
6) 3	7) 4	8) 1	9) 3	10) 1
11) 1	12) 1	13) 1	14) 4	15) 4
16) 4	17) 3	18) 4	19) 1	20) 2
21) 2	22) 4	23) 2	24) 1	25) 1
26) 1	27) 2,3	28) 1,3,4	29) 1	30) 2
31) 3	32) 3	33) 1	34) 1	35) 2
36) 3	37) 2,3,4,1	38) 3,2,1,4	39) 2	40) 12

28. A trapezium in which the non parallel sides are equal to each other is known as an isosceles trapezium.

In the isosceles trapezium ABCD. $\overline{AB} \,||\, \overline{CD}$, AD = BC

then the length of diagonals are equal (AC = BD)

27. Hint: In a parallelogram ABCD, we know that

$\angle A = \angle C$ and $\angle B = \angle D$

$\therefore \angle A = \angle C = 115°$ and

$\angle B = \angle D = 65°$ or

in ABCD $\angle A + \angle B = 180°$

$115° + \angle B = 180°$

$\angle B = 180° - 115° = 65°$

$\therefore \quad \angle C = \angle A = 115°, \ \angle D = \angle B = 65°$

34. Area = $1/2\ d_1 d_2 = 1/2.24.18 = 216\ cm^2$.

35. OA + OB = $\frac{AC}{2} + \frac{BD}{2} = 21$.

36. In rhombus adjacent angles are suplementary.

40. $\Delta AOB.\ x^2 + y^2 = 10^2 \Rightarrow x^2 = 36 \Rightarrow x = 6$

$\therefore x + x = 6 + 6 = 12$