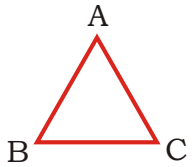
TRIANGLES

SYNOPSIS - 1

Introduction: A simple closed figure bounded by line segments is called a Polygon.

Triangle: A polygon with three sides is called a triangle. The symbol for triangle is 'Δ'

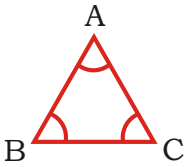
Perimeter of the traiangle: The sides are $\overline{AB}, \overline{BC}, \overline{CA}$. The sum of the measures of $\overline{AB}, \overline{BC}$ and $\overline{CA}$ is called the perimeter of the triangle.



Perimeter of the triangle ABC = BC + CA + AB.

Angles of the triangle: Observe the figure $\overline{BA}$ and $\overline{BC}$ are two line segment having the same end point B, which forms an angle. This angle is '$\angle B$'.

Similarly $\angle C$ and $\angle A$ are the other two angles.



$\therefore$ A triangle has three sides and three angles.

Totally these six are called six components (or) six parts of a triangle.

Note: The sum of the measures of the angles of a triangle is 180°.

INTERIOR AND EXTERIOR OF A TRIANGLE

a) Interior of a triangle: A point is said to be interior of a triangle, if it lies inside the triangle.

b) Lies on a triangle: A point lies on a triangle, if it lies on any one of its sides.

c) Exterior of a triangle: A point lies in the outside of a traingle, if it lies in the plane of the triangle, but neither on the triangle nor in the interior.

Note: A triangle divides a plane in which it lies into three parts.

CLASSIFICATION OF TRIANGLES

a) Classification of triangles according to the sides:

1) Equilateral triangle: A triangle whose sides are equal in length is called an 'equilateral triangle'. All the angles in the equilateral triangle are equal.

2) Isosceles triangle: A triangle in which two sides are equal in length is called 'Isosceles triangle'. In an isosceles triangle the unequal side is called the base of the triangle. The base angles of an isosceles triangle are congruent.

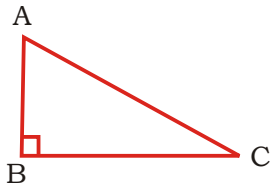
Note: Every equilateral triangle is isoceles.

3) Scalene triangle: If no two sides of a triangle are equal in length, it is called a Scalene triangle.

b) Classification of triangles according to the angles:

1) Acute angled triangle: If each angle of a triangle is an acute angle, then it is called an 'Acute angled triangle'. Measure all angles and observe all are less than 90°.

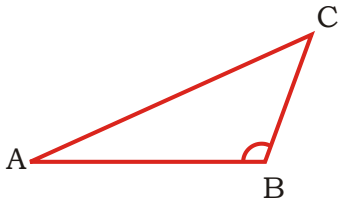
2) Right angled triangle: A triangle in which one of its angles is a right angle is called 'Right angled triangle'



In this triangle $\angle B = 90°$, therefore it is a right angled triangle.

In a right angled triangle, the opposite side of the right angle is called 'Hypotenuse'.

3) Obtuse angled triangle: A triangle containing an obtuse angle is called obtuse angled triangle.



In the figure $\angle B > 90°$, so it is an obtuse angled triangle.

CONCURRENT LINES IN A TRIANGLE:

Median: A line segment which joins a vertex of a triangle to the mid point of the opposite side is called median. The number of such line segments that can be drawn in the triangle are three. The median which joins the vertex A of a triangle to the mid point of side a is denoted by M_a it is given, in terms of the sides of a triangle, the formula $M_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Altitude: The perpendicular drawn from any vertex of the triangle to the opposite side or its extension is called altitude. The number of such line segments that can be drawn in the triangle are 3.

Note: In an obtuse angled triangle, two altitudes fall on the extensions of the sides outside the triangle, and the third altitude falls inside the triangle. In an acute angled triangle all three altitudes lie within the triangle. In a right angled triangle the legs serve as altitudes.

Perpendicular bisector: The line passing through the mid point of the side and perpendicular to the same side is called perpendicular bisector. The number of such lines that can be drawn in the triangle are 3.

Angular bisector: An angular bisector of triangle is the line segment which divides any angle into two equal halves.

Concurrent lines: Three or more lines passing through the same point are called concurrent lines. That common point is called point of concurrence.

1. **Centroid:** The point of concurrence of the medians of a triangle is called centroid. It is denoted by 'G'

   Note: 'G' divides AD in the ratio 2:1.

2. **Orthocentre:** The point of concurrence of the altitude of a triangle is called orthocentre. It is denoted by 'O' (or) 'H'

3. **Circumcentre:** The point of concurrence of perpendicular of the sides of a triangle is called circumcentre. It is denoted by 'S'.

   Note: Circumcentre is equidistance to its vertices.

   In a right angle triangle 's' is the mid point of the hypotenuse.

S.No.	Type of Triangle	Position of circum centre
1.	Acute	Interior of the triangle
2.	Obtuse	Exterior of the triangle
3.	Right	Mid point of hypotenuse

4. **Incentre:** The point of concurrent of internal angular bisectors of a triangle is called incentre. It is denoted by 'I'.

   Note: Incentre is equi distance to its sides.

5. **Excentre:** The point of concurrence of internal bisector of one angle and the external bisectors of other two angles is called excentre.

   Note: A triangle has three ex-centres.

WORK SHEET - 1

SINGLE ANSWER TYPE

1. A polygon with three sides is called a

   1) Line 2) Triangle 3) Curve 4) Square

2. The longest side in a right angled triangle is _____.

   1) Hypotenuse 2) Diagonal 3) Base 4) Adjacent side

3. The line segment which joins the vertex to midpoint of opposite side is

   1) Altitude 2) Median 3) Hypotenuse 4) Diagonal

4. In a right angle triangle, the other possibility of two angles are

   1) 60°, 30° 2) 40°, 45° 3) 0°, 90° 4) 40°, 10°

5. A triangle divides a plane into _____ parts.

   1) 2 2) 3 3) 4 4) 5

6. Each angle in an equilateral triangle is

   1) 40° 2) 50° 3) 60° 4) 70°

7. Altitude is the _____ drawn from any vertex of the triangle to the opposite side.

   1) Perpendicular 2) Parallel 3) Equal 4) Median

8. The line passing through the midpoint of the side and perpendicular to the same side is called _____

   1) Angular bisector 2) Perpendicular bisector

   3) Median 4) Altitude

9. The point of concurrence of angular bisectors of a triangle is

1) Circumcentre 2) Incentre 3) Excentre 4) Centroid

MULTI ANSWER TYPE

10. ΔABC is an isosceles triangle in which AD is the median

1) $\Delta ABC \cong \Delta ACD$ 2) BC = CD 3) $\angle ABC = \angle ACB$ 4) $\angle BAD = \angle CDA$

11. ABC is a right angled isosceles triangle and D is the midpoint of AC, $\overline{BD}$ is called

1) Altitude 2) Perpendicular bisector
3) Median 4) Angular bisector

REASONING ANSWER TYPE

12. *Statement-I*: In an equilateral triangle all sides are equal.
Statement-II: All angles are equal in an equilateral triangle.

1) Both Statements are true, Statement II is the correct explanation of Statement I.

2) Both Statements are true, Statement II is not correct explanation of Statement I.

3) Statement I is true, Statement II is false.

4) Statement I is false, Statement II is true.

13. *Statement-I*: In ΔABC, a = 8cm, b = 5cm, c = 5cm, then median is 3cm.
Statement-II: Formula to calculate the length median is $\frac{1}{2}\sqrt{2b^2+2c^2-a^2}$.

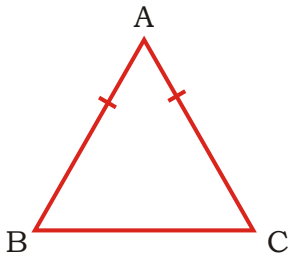
1) Both Statements are true, Statement II is the correct explanation of Statement I.

2) Both Statements are true, Statement II is not correct explanation of Statement I.

3) Statement I is true, Statement II is false.

4) Statement I is false, Statement II is true.

COMPREHENSION TYPE



A triangle with two equal sides is called an isosceles triangle.
triangle ABC is an isosceles triangle.

14. In the above triangle $\overline{AB}$ ..... $\overline{AC}$

1) = 2) $\neq$ 3) > 4) <

15. In the above triangle. If AB = AC then

1) $\angle A = \angle B$ 2) $\angle B = \angle C$ 3) $\angle A = \angle C$ 4) $\angle A = \angle B = \angle C$

16. In an isosceles triangle how many angles are equal
1) 1 2) 2 3) 4 4) Zero

MATRIX MATCHING TYPE

17.

Column - I	Column - II
a) In an equilateral triangle orthocentre, centroid, circumcentre, incentre are	1) Exterior of triangles
b) In an isosceles triangle orthocentre, centroid, circumcentre, in centre are	2) Coincides
c) Circumcentre in an acute angled triangles lies	3) Interior of triangles
d) Circum centre in an obtuse angled triangles lies	4) Collinear
	5) Non-Collinear

18.

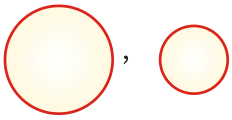
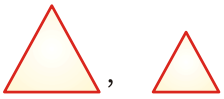
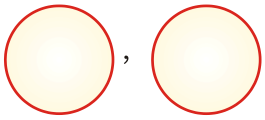
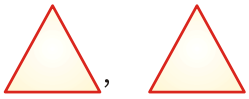
Column - I	Column - II
a) Triangle	1) 6 sides
b) Square	2) 4 Sides
c) Pentagon	3) 5 Sides
d) Hexagon	4) 3 sides
	5) 7 sides

INTEGER ANSWER TYPE

19. The number of components in a triangle is _____.

SYNOPSIS - 2

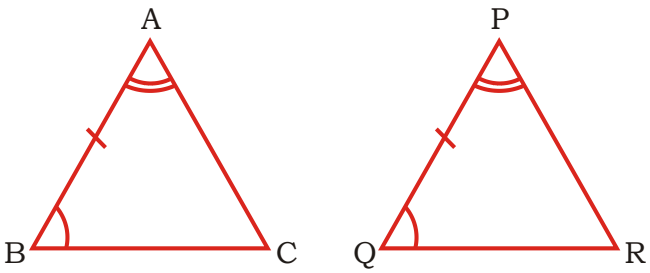
Similarity and Congruency of triangles:

Similarity	
1. Shape is same Examples: a) _____, _____ b)  c) 	1. Shape & size both are same Examples: a) _____, _____ b)  c) 
2. Represented with '~' 3. Ratio of corresponding sides are equal	2. Represented with '≅' 3. Measures of corresponding sides & areas are also equal

i. **S.A.S Axiom:** Two triangles are congruent if and only if two sides and the included angle of one triangle are equal to the corresponding sides and the included angle of the other.

ii. **A.S.A Axiom:** Two triangles are congruent if and only if any two angles and the included side of one triangle are equal to the corresponding angles and included side of the other triangle.

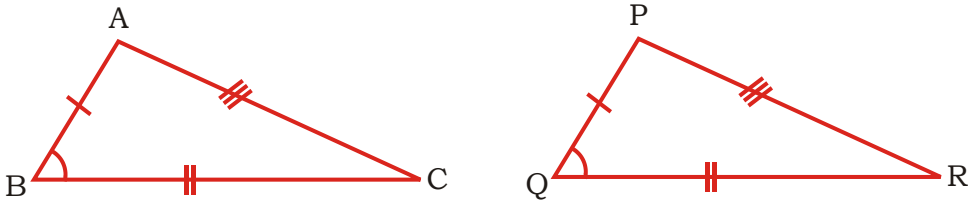
Example :



where $\Delta ABC \cong \Delta PQR$

iii. **S.S.S Axiom:** Two triangles are congruent if and only if three sides of one triangle are equal to the corresponding sides of the other triangle.

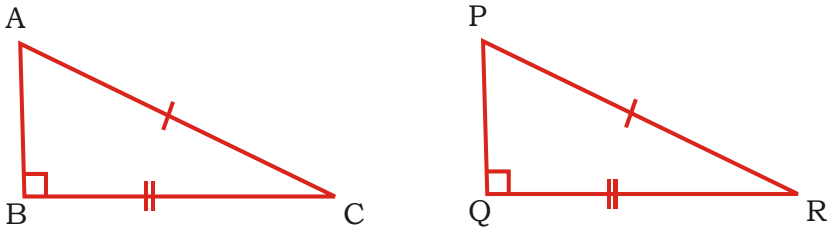
Example:



where $\Delta ABC \cong \Delta PQR$

iv. **R.H.S Axiom:** Two right angled triangles are congruent if and only if the hypotenuse and a side of one triangle are equal to the corresponding hypotenuse and side of another right angled triangle.

Example:

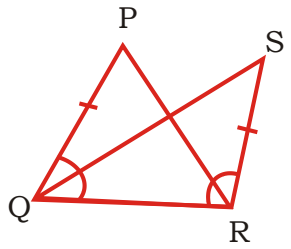


where $\Delta ABC \cong \Delta PQR$

WORK SHEET - 2

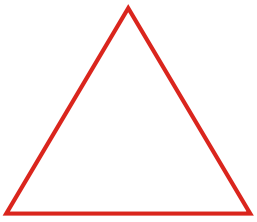
SINGLE ANSWER TYPE

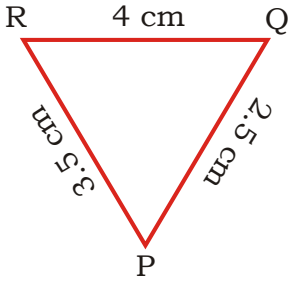
1. In ΔPQR and ΔSQR, $\overline{PQ} = \overline{SR}$ and $\angle PQR = \angle QRS$, then



1) $\Delta PQR \sim \Delta SRQ$ 2) $\Delta PQR \cong \Delta SRQ$ 3) $\Delta PQR \neq \Delta SRQ$ 4) None of these

2. State the axiom under which $\Delta ABC \cong \Delta PQR$

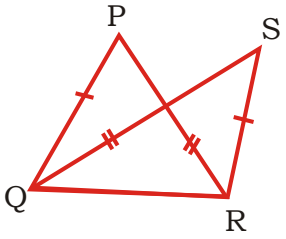




1) S.A.S 2) A.S.A 3) S.S.S 4) R.H.S

MULTI ANSWER TYPE

3. In the given figure PQ = SR and PR = SQ, then



1) $\Delta PQR \cong \Delta SRQ$ 2) $\angle PQR = \angle SRQ$ 3) PQ = SQ 4) $\angle PQR = \angle QSR$

REASONING ANSWER TYPE

4. *Statement-I*: In an isosceles triangle the angles opposite to the equal sides are equal.

Statement-II: R.H.S axiom.

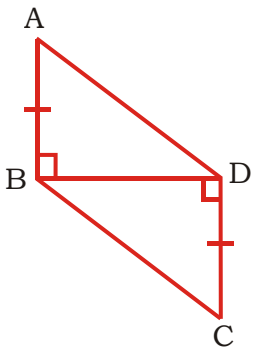
1) Both the statements are true, statement II is correct explanation of statement I

2) Both the statements are true, statement II is not correct explanation of statement I

3) Statement I is true, statement II is false

4) Statement I is false, statement II is true.

COMPREHENSION TYPE

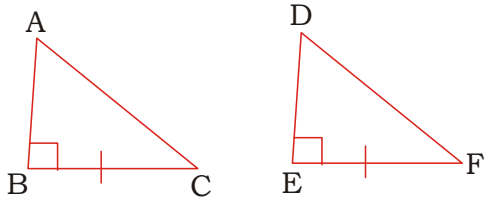
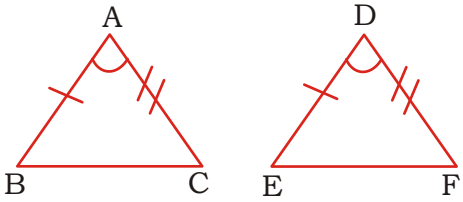
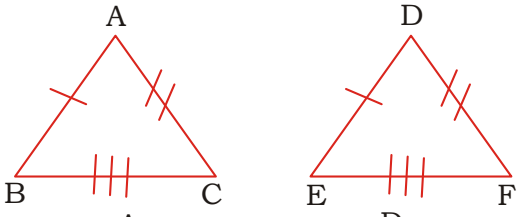
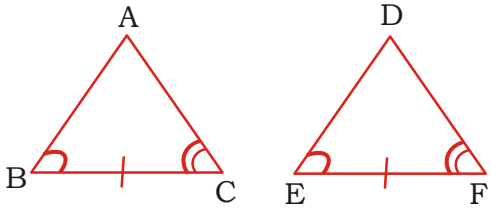


In the adjoining figure $AB \perp BD, CD \perp BD$ and AB = CD

5. $\angle ABD =$
   1) $\angle DBC$ 2) $\angle BDC$ 3) $\angle BAD$ 4) $\angle BCD$
6. $\Delta ABD \cong \Delta BCD$, then
   1) $\angle A \neq \angle C$ 2) $\angle B = \angle A$ 3) BD = CD 4) AD = BC
7. $\Delta ABD \cong \Delta CDB, \angle BDA =$
   1) $\angle BCD$ 2) $\angle DBC$ 3) $\angle BDC$ 4) $\angle DAB$

MATRIX MATCHING TYPE

8.

Column - I	Column - II
a) 	1) S.A.S
b) 	2) S.S.S
c) 	3) R.H.S
d) 	4) A.S.A
	5) A. A. A

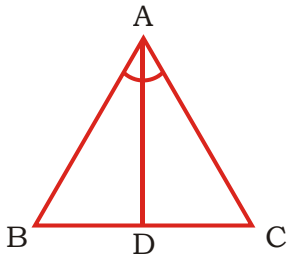
INTEGER ANSWER TYPE

9. $\Delta LMN \cong \Delta XZY$, $\angle M = 70°$, then $\angle Z =$ _____

SYNOPSIS - 3

PROPERTIES:

1. In a right angled triangle, hypotenuse^2 = side^2 + side^2 (Pythagoras theorem).
2. Centroid divides the median in the ratio 2:1 from the vertex.
3. In an equilateral triangle, G, O, S and I are coincide.
4. Angular bisector divides the opposite side in the ratio of other two sides



$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

5. S, G, O and I are collinear in an isosceles triangle.
6. S, G and O are collinear in any triangle and this line is called Euler's line.
7. G divides OS in the ratio 2:1
8. If G is centroid and A, B, C are vertices, then

$$GA^2 + GB^2 + GC^2 = \frac{1}{3}\left(AB^2 + BC^2 + CA^2\right)$$

9. Exterior angle is equal to the sum of opposite interior angles
10. If 'O' is a point inside the triangle, then $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

WORK SHEET - 3

SINGLE ANSWER TYPE

1. Pythagoras theorem states that

   1) Side = side+hypotenuse
   2) Side^2 = hypotensuse + side^2
   3) hypotensuse^2 = side^2 + side^2
   4) hypotensuse^2 = side^2 – side^2

2. In an isosceles triangle

   1) S,G,O and I are collinear
   2) Only S,O,I are collinear
   3) Only G,O,I are collinear
   4) Only G and I are collinear

3. G divides OS in the ratio

   1) 1 : 2
   2) 2 : 1
   3) 1:3
   4) 3:1

MULTI ANSWER TYPE

4. In a right angled triangle

   1) hypotensuse^2 = side^2 + side^2
   2) side^2 = hypotensuse^2 – side^2
   3) side^2 = hypotensuse^2 + side^2
   4) hypotensuse^2 = side^2 – side^2

5. Which of the following are the sides of right angled triangle

   1) 3cm, 4cm, 5cm
   2) 8cm, 15cm, 17cm
   3) 7cm, 24cm, 25cm
   4) 5cm, 8cm, 11cm

6. If G is centroid and A,B,C are vertices then

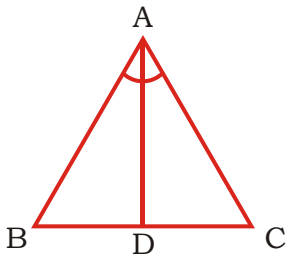
1) $3GA^2 = AB^2 + BC^2 + AC^2$

2) $GA^2 + GB^2 + GC^2 = \frac{1}{3}\left(AB^2 + BC^2 + CA^2\right)$

3) $AB^2 + BC^2 + AC^2 = 3\left(GA^2 + GB^2 + GC^2\right)$ 4) $3AB^2 = GA^2 + GB^2 - GC^2$

REASONING ANSWER TYPE

7. *Statement-I*: In the given figure $\frac{BD}{DC} = \frac{AB}{AC}$



Statement-II: Angular bisector divides the opposite side in the ratio of other two sides.

1) Both the statements are true, statement II is correct explanation of statement I

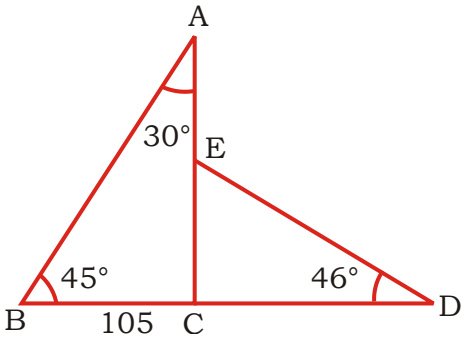
2) Both the statements are true, statement II is not correct explanation of statement I

3) Statement I is true, statement II is false

4) Statement I is false, statement II is true.

COMPREHENSION TYPE

Observe the given figure and calculate



8. $\angle ACD =$

1) 75° 2) 115° 3) 105° 4) 100°

9. $\angle CED =$

1) 95° 2) 59° 3) 121° 4) 115°

10. $\angle AED =$

1) 111° 2) 105° 3) 121° 4) 115°

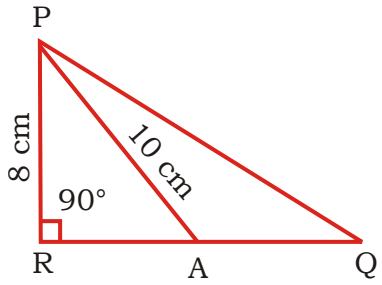
MATRIX MATCHING TYPE

11.

Column - I	Column - II
a) Pythagoras Theorem	1) Isosceles Triangle
b) Medians intersect point	2) Right angled triangle
c) G,O,S and I coinside	3) Centroid
d) S,G,O and I are collinear	4) Equilateral triangle
	5) Scalene triangle

INTEGER ANSWER TYPE

12. In the given figure, ΔPQR, $\angle R = 90°$. If A is the midpoint of side QR, PR = 8cm and PA = 10cm, then $(PQ)^2$ is _____

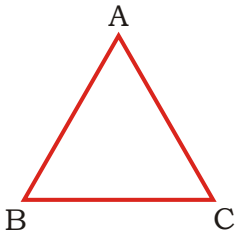


SYNOPSIS - 4

PROPERTIES OF TRIANGLES:

I. Inequalities of a triangle:

1. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
2. The difference of the lengths of any two sides of a triangle is smaller than the length of the third side. In the adjacent triangle ABC

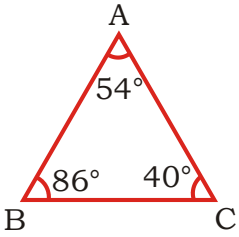


$AB + AC > BC, AC + BC > AB, BC + AB > AC$

$AB - AC < BC, AC - BC < AB, BC - AB < AC$

II. Relation between sides and angles of a triangle :

1. In a triangle, the angle opposite to longer side is the greatest angle.
2. In a triangle, the angle opposite to shorter side is the smallest angle.



In the above triangle ABC, $\angle C < \angle A < \angle B$ then
AC is the longest side and AB is smallest side.

WORK SHEET - 4

MULTI ANSWER TYPE

1. Which of the following are measurements of the sides of a triangle

1) 2cm, 5cm, 7cm 2) 6cm, 7cm, 7cm 3) 3cm, 4cm, 5cm 4) 5cm,7cm,9cm

REASONING ANSWER TYPE

2. *Statement-I*: Two angles in a triangle are 80°, 70°, then the third angle is right angle.

Statement-II: In a triangle the sum of three angles is equal to two right angles.

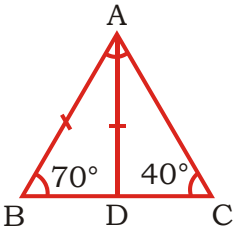
1) Both Statements are true, Statement II is the correct explanation of Statement I.

2) Both Statements are true, Statement II is not correct explanation of Statement I.

3) Statement I is true, Statement II is false.

4) Statement I is false, Statement II is true.

COMPREHENSION TYPE



In the given figure D is a point on BC of ΔABC such that

AB = AD, $\angle B = 70^0$ and $\angle C = 40^0$

3. In the ΔABC, which of the following is false

1) AB > CD 2) AB = AC 3) AC > AB 4) AC = BC

4. In the ΔABC, $\angle ADC =$

1) $\angle DAB - \angle ABC$ 2) 110° 3) 70° 4) 80°

5. In the ΔABC, $\angle A =$

1) 70° 2) 80° 3) 60° 4) 35°

MATRIX MATCHING TYPE

6.

Column - I	Column - II
a) In ΔPQR, PQ + QR	1) $\angle P < \angle R$
b) In ΔPQR, PQ – QR	2) Is greater than PR
c) In ΔPQR, $\angle P > \angle Q$	3) $\overline{QR} > \overline{PR}$
d) In ΔPQR, $\overline{PQ} > \overline{QR}$	4) Is less than PR
	5) Is equal to PR

INTEGER ANSWER TYPE

7. In a triangle ABC, AB = 8cm, BC = 9cm, and CA = x cm, then the number of integral values of x is _____.

KEY & HINTS

WORK SHEET – 1 (KEY)				
1) 2	2) 1	3) 2	4) 1	5) 2
6) 3	7) 1	8) 2	9) 2	10) 3
11) 1,2,3,4	12) 1	13) 1	14) 1	15) 2
16) 2	17) 2,4,3,1	18) 4,2,3,1	19) 6	

13. Length of the median = $\frac{1}{2}\sqrt{100-64} = 3$

WORK SHEET – 2 (KEY)				
1) 2	2) 3	3) 1,2	4) 1	5) 2
6) 4	7) 2	8) 3,1,2,4	9) 70	

WORK SHEET – 3 (KEY)				
1) 3	2) 1	3) 2	4) 1,2	5) 1,2,3
6) 2,3	7) 1	8) 1	9) 2	10) 3
11) 2,3,4,1	12) 206			

SOLUTIONS

5. $5^2 = 3^2 + 4^2$
   $17 = 8^2 + 15^2$ so A,B,C are right angled triangles
   $25^2 = 7^2 + 24^2$

8. $\angle ACD = \angle A + \angle B = 30 + 45 = 75°$

9. $\angle CED = 180° - 75° - 46° = 59°$

10. $\angle AED = \angle ACD + \angle D = 75° + 46° = 121°$