FACTORISATION

SYNOPSIS - 1

FACTORS

We already know that: $(3x+5)(3x-5)=(9x^2-25)$

We say that $(3x+5)$ and $(3x-5)$ are the factors of $(9x^2-25)$.

Thus, in a product of two or more expressions, each expression is called a factor of the product.

Factorisation: The factorisation of an algebraic expression means to express it as the product of monomials and the smallest degree polynomials.

Before taking up factorization, we would like to discuss about the factors and H.C.F. of monomials.

Factors of a Monimial

Example 1: Write all the possible factor sof $2xy^2$

Sol. The possible factors of 2 are 1, 2.

The possible factors of xy^2 are x, y, y^2, xy, xy^2

$\therefore$ All possible factors of $2xy^2$ are

$1, 2, x, 2x, y, 2y, y^2, 2y^2, xy, 2xy, xy^2$ and $2xy^2$.

H.C.F. of Monimials

The H.C.F. of two or more monomials is the product of the H.C.F. of the numerical coefficients and the common variables with smallest poewrs.

H.C.F. of monomials = (H.C.F. of numerical coefficients)
$\times$ (H.C.F. of literal coefficients)

Example 2: Find the H.C.F. of $4x^2y, 6xy^2, 8x^2y^2$.

Sol. We have

H.C.F. of $4x^2y, 6xy^2, 8x^2y^2$ = (H.C.F of 4, 6, 8) $\therefore$ (H.C.F. of x^2y, xy^2, x^2y^2)

= $2xy$

$\therefore$ H.C.F. of given monomials = $2xy$

Factorisation of Varoius types of polynomials

Case 1: When a monomial is the common factor of all the terms.

Step 1: Find the H.C.F. of all the terms of the given expression.

Step 2: Divide each term of the given expression by the H.C.F. so obtained. Enclose the quotients within a bracket and keep the common monomial outside the bracket.

Example : Factorise (i) $2x^2+4xy-6xy^2$ (ii) $2ax+3a^2y-a^3z$

Sol. (i) H.C.F. of $2x^2, 4xy$ and $6xy^2$

= (H.C.F. of 2, 4 and 6) $\times$ (H.C.F. of x^2, xy and xy^2) = 2x

$\therefore 2x^2 + 4xy - 6xy^2 = 2x\left(x + 2y - 3y^2\right)$

(ii) H.C.F. of $2ax, 3a^2y$ and a^3z

= (H.C.F. of 2, 3 and 1) $\times$ (H.C.F. of ax, a^2y and a^3z) = a

$\therefore\ 2ax + 3a^2y - a^3z = a\left(2x + 3ay - a^2z\right)$

Case 2: When the given algebraic expression has a common binomial.

In case of binomial is common, we can write the expression as the product of this binomial and the quotient obtained by dividing the given expression by this common binomial.

Example: Factorise $8(9x+5y)^2 - 12(9x+5y)$

Sol. $8(9x+5y)^2 - 12(9x+5y) = 4(9x+5y)\left[2(9x+5y) - 3\right]$

$= 4(9x+5y)(18x+10y-3)$

Case 3: When grouping gives rise to common factors

Step1: Arrange the terms of the given expression in groups in such a way that each group has a common factor.

Step2: Factorise each group.

Step3: Take out the factor which is common to both the groups.

Example: Factorise $a^3 + a - 3a^2 - 3$

Sol. $a^3 + a - 3a^2 - 3 = \left(a^3 + a\right) - \left(3a^2 + 3\right)$

$= a\left(a^2+1\right) - 3\left(a^2+1\right)$

$= \left(a^2+1\right)(a-3)$

Case 4: When the given exression is expressible as the difference of two squares

Rule : Use the formula $\left(a^2 - b^2\right) = (a+b)(a-b)$

Example: Factorise $49x^2 - 16y^2$

Sol. $49x^2 - 16y^2 = (7x)^2 - (4y)^2$

$= (7x+4y)(7x-4y)$

WORK SHEET

SINGLE ANSWER TYPE

1. One of the factors of $4x^2y^2$ is =___
   1) 3x 2) 5xy 3) 2xy 4) 9
2. One of the factors of $-6pq^2r$ is _____
   1) 5pq 2)7qr 3) –2pq 4) $8r^2$
3. One of the factors of $24m^3n^5$ is =_____
   1) 1 2) 124 3) 42 4) m^4
4. The HCF of k×k×k×k×m×m×m×, k×k×l×l×m×m×m×m is _____
   1) k^2l^3 2) k^2l^2 3) k^2m^3 4)k^2m^3
5. $20x^4y^3, 16x^2y^4, 24x^3y^2$ are monomials, then its HCF is =_____
   1) $4x^2y^2$ 2) $4x^4y^2$ 3)$4x^2y^3$ 4)$4x^2y^4$
6. $28p^3q^2 - 56p^2q^3 + 16p^3q^2 - 32p^2q^3$ =_____
   1) $44p^2q^2(p + 2q)$ 2) $44p^2q^2(p - 2q)$ 3)44 $p^2q^2(-p - 2q)$ 4)$44p^2q^2(p + 2q)$
7. If $A = 4x^2(2x^2 - 4x + 1$ & $B = x^2(x^2 - 2x + 1)$, then A + 8B=_____
   1) $4x^2(4x^2 - 8x + 3)$ 2) $4x^2(x^2 - 2x + 1)$ 3) $4x^2(4x^2 - 2x + 1)$ 4)$4x^2(x^2 - 8x + 3)$
8. The HCF of $27x^2y^2z^2$, $-15x^2y^3z^4$, $3xyz^3$ is ___
   1) $3x^2y^2z^2$ 2)$15x^2y^2z^2$ 3) 3xyz 4)$3xyz^2$
9. $\left[\{2x(x-3)-3(x-3)\}\div(2x-3)\right]+\left[\left\{3(x-2)^2-5(x-2)\right\}\div(3x-11)\right]=$
   1) 2x–3 2)2x–5 3) 3x–2 4) 5x–2
10. If k is the HCF of $64x^2yz^3$, $162x^3y^2z$,$226xy^3z^2$ and *1* is the HCF of $39a^2bxy^3z$, $69b^2cxyz^3$, $129c^2ax^3yz$,then the HCF of k and *1* is=_____
    1) $2x^2yz$ 2) xyz 3) (abc)(xyz) 4) (ax)(by)(cy)
11. $\frac{16x^8y^5z^6}{2x^2y^4z^3}+\frac{20x^7y^8z^9}{5x^4y^2z^3}+\frac{8x^8y^6z^5}{x^2y^3z^2}$ is simplified, then the answer is
    1) $4x^3yz^3(2x^2y^2 + y^5z^2 + 2x^3)$ 2) $4x^3yz^3(2x^2y^2 + y^5z^2 + 2x^2)$
    3) $4x^3yz^3(2x^3y^2 + y^5z^3 + 2x^3)$ 4) $4x^3yz^3\ (2x^2y^2 + y^4z^2 + 2x^2)$

MULTI ANSWER TYPE

12. The factors of $16a^2b^4c^8$ are _____
    1) 2 2) abc 3) 4abc 4) $a^2b^2c^2$
13. One of the factors of $\frac{32x^8y^6z^5}{4x^2y^3z^2}$ is = ___
    1) $8x^2y^3$ 2) $4x^6z^2$ 3) y^4z^3 4) $(xyz)^3$
14. If $A = 144\times a^{1+2+3+.....+10} \times b^{1+2+....+5}$ and $B = 192a^{1+2++...+6}\times b^{1+2+...+18}$, then their HCF is
    1) $48a^{21}\times b^{15}$ 2) $2^3\times3\times a^{15}\times b^{21}$ 3) $24\times a^{15}\times b^{21}$ 4) $2^4\times3\times a^{21}\times b^{15}$

REASONING ANSWER TYPE

15. *Statement I :* One of the factors of $31pq^2r^3$ is pqr
Statement II : The number of factors of x^2y^4 are 12
1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

16. *Statement I :* If $A = 8x^3y^2+4x^2y^3+12xy^4$, $B=x^2y^3+8xy+12^3y^2$ & $C=16x^3y^2 - 16xy^4+20x^2y^3$ then $A+B - 4(C - B)= 8xy^2(3x^2+6xy+17y^2)$
Statement II : When a monomial is sthe common factor of all the terms then divide each term of the gives expression by the HCF and enclose the quotients with in a bracket keeping the common monomial outside the bracket.
1. Both Statements are true, Statement II is the correct explanation of Statement I.
2. Both Statements are true, Statement II is not correct explanation of Statement I.
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

COMPREHENSION TYPE

If $A= 14x^2y$, $B = -3x^2y$, then

17. The factors of A are _____
1) 14 2) x^2y 3) 14xy 4) All of these

18. One of the factors of A+B is =____
1) x – y 2) 11(x – y) 3) 11xy 4) x(x – y)

19. One of factors of B is ____
1) xy^2 2) xy 3) 9 4) 6x

Comprehension Type:
If $A= (x^2+8x+12)\div(x+6)$, $B= (x^2+12x+32)\div(x+4)$, $C= (x^2+16x+48)\div(x+4)$, then

20. If A+B is simplified, then the answer is _____
1) 10 – 2x 2) 10x – 2 3) 2x + 10 4) 2x – 10

21. (A + B) + C=___
1) 3x + 22 2) 3x – 22 3) 22 – 3x 4) 22x + 3

22. If [(A + B) + C] × 8x is simplifed, then the answer is
1) 24x + 176 2)$24x^2$ + 176x 3) 24x – 176 4) $24x^2$ – 176

MATRIX MATCHING TYPE

23.

Column-I	Column-II
a) HCF of $7x^2$, $5y^2$ is ___	1) xy
b) HCF of $18x^2y$, $35xy^2$ is	2) 25bc
c) HCF of $25bc^2$ and $125b^2c$ is =_____	3) 4p
d) HCF of 8axp,4byp is=___	4) 1
	5) Product of x and y

INTEGER ANSWER TYPE

24. The number of integer factors of the monomial $3p^2q$ is =_____

WORK SHEET (KEY)				
1) 3	2) 3	3) 1	4) 3	5) 1
6) 2	7) 1	8) 4	9) 2	10) 2
11) 3	12) 1,2,3,4	13) 1,2,4	14) 1,4	15) 3
16) 4	17) 4	18) 3	19) 2	20) 3
21) 1	22) 2	23) 4,(1,5),2,3	24) 3	

1. Key: 3 ; Sol:- $4x^2y^2=(2xy)^2$ \ one of the factors of $4x^2y^2$is 2xy
2. Key: 3 ; Sol:- $-6pq^2r=(-2pq)(3qr)$ \ one the factors = –2pq (or)3qr
3. Key: 1 ; Sol:- $24m^3n^5 = (24mn)(m^2n^4)$ \ one the factors = 24mn
4. Key: 3 ; Sol:- $k\times k\times k\times k\times m\times m\times m= k^4\times m^3$; $k\times k\times l\times l\times m\times m\times m= k^2l^2m^4$
   \ The HCF of k^4m^3 of k^4m^3 and $k^2l^2m^4=k^2m^3$
5. Key: 1 ; Sol:- H.C.F = $4x^2y^2$
6. Key: 2 ; Sol:-$44p^3q^2 - 88p^2q^3 = 44p^2q^2 (p - 2q)$
7. Key: 1; Sol:- $8x^4 - 16x^3 + 4x^2 + 8x^4 - 16x^3 + 8x^2 = 16x^4 - 32x^3 + 12x^2$
   $= 4x^2 (4x^2 - 8x + 3)$
8. Key : 4 ; Sol: H.C.F. of $27x^2y^2z^2$, $-15x^2y^3z^4$, $3xyz^3 = 3xyz^2$.
9. Key : 2 ; Sol: $[\{2x(x - 3) - 3(x - 3\} \div (2x - 3)]$
   $= (2x - 3)(x - 3) \div (2x - 3) = x - 3.= [3(x - 2)^2 - 5(x - 2)] \div (3x - 11)$
   $= (x - 2)[3(x - 2)] \div (3x - 11)= x - 2$
   $\therefore$ $x - 3 + x - 2 = 2x - 5$.
10. Key : 2 ; Sol: k = H.C.F. of $64x^2yz^3$, $162x^3y^2z$, $226xy^3z^2$ = 2xyz.
    (Since H.C.F of 64, 162, 226 = 2)
    1 = H.C.F of $39a^2bxy^3z$, $69b^2cxyz^3$, $129c^2ax^3yz$ = 3xyz.
    Now H.C.F of k and *1* = H.C.F of (2xyz) & (3xyz) = xyz.
11. Key : 3 ; Sol: $\frac{16x^8y^5z^6}{2x^2y^4z^3}+\frac{20x^7y^8z^7}{5x^4y^2z^3}+\frac{8x^8y^6z^5}{x^2y^3z^2} = 8x^6yz^3 + 4x^3y^6z^6 + 8x^6y^3z^3$
    $= 4x^3yz^3 (2x^3 + y^5z^3 + 2x^3y^2)$
12. Key: (1,2,3,4) ; Sol:- $16a^2b^4c^8=2\times(a^2b^2c^2)\times(b^2c^6)=2\times4(abc)\times(2abc)(b^2c^6)$
    \ 2,abc, 4abc, $a^2b^2c^2$ are factors of $16a^2b^4c^8$
13. Key: (1,2,4) ; Sol:- $8x^6y^3z^3 = (8x^2y^3)(x^4y^3) = (4x^6z^2)(2y^3z) = (xyz)^3(8x^3)$
14. Key : (1, 4) ; Sol: H.C.F of A and B $= 48\cdot a^{1+2+3+4+5+6} \times b^{1+2+3+4+5}$
    $= 48 \times a^{21} \times b^{15} = 2^4 \times 3 \times a^{21} \times b^{15}$.

15. Key: 3 ; Sol:- The factors of x^2y^4 are x,y, $xy,x^2,x^2y,y^2,xy^2,x^2y^2,y^3,xy^3,x^2y^3,y^4,xy^4,x^2y^4$

\ Number of factors are 14 \ st-2 is false

Celarly pqr is factors of $31pq^2r^3$ \ st-1 is true .

16. Key : 4 ; Sol: Clearly St-2: is true

St-1: A + B $= (8x^3y^2 + 4x^2y^3 + 12xy^4) + (x^2y^3 + 8xy^4 + 12x^3y^2)$

$= 20x^3y^2 + 5x^2y^3 + 20xy^4$

C – B $= (16x^3y^2 – 16xy^4 + 20x^2y^3) – (x^2y^3 + 8xy4 + 12x^3y^2)$

$= 4x^3y^2 – 24xy^4 + 19x^2y^3$

$\therefore$ (A + B) – 4(C – B)

$= [20x^3y^2 – 5x^2y^3 + 20xy^4] – 4[4x^3y^2 – 24xy^4 + 19x^2y^3]$

$= 4x^3y^2 + 71x^2y^3 + 116xy^4$

$= xy^2(4x^2 + 71xy + 116y^2)$ -------> St-1 is false.

17. Key: 4 ; Sol:- Given A = $14x^2y$, \ The factors of A are $14,x^2y,14xy$.....etc

18. Key: 3 ; Sol:- A+B = $(14x^2y)+(–3x^2y)=11x^2y$

19. Key: 2 ; Sol:- B = $–3x^2y = (xy)(–3x)$

20. Key : 3 ; Sol: A = $(x^2 + 8x + 12) \div (x + 6) = (x + 6)(x + 2) \div (x + 6) = x + 2$

B = $(x^2 + 12x + 32) \div (x + 4) = (x + 8)(x + 4) \div (x + 4) = x + 8$

$\therefore$ A + B = x + 2 + x + 8= 2x + 10.

21. Key : 1 ; Sol: C = $(x^2 + 16x + 48) \div (x + 4) = (x + 4)(x + 12) \div (x + 4) = x + 12$.

$\therefore$ (A + B) + C = 2x + 10 + x + 12 = 3x + 22.

22. Key : 2 ; Sol: [(A + B) + C] × 8x = (3x + 22) × 8x = $24x^2 + 176x$

23. Key: a–4,b–(1,5), c–2,d–3

a) HCF of $7x^2$ & $5y^2$ is 1

b) The highest common factors of $18x^2y,35xy^2$ is xy

c) HCF of $25bc^2$ and $125b^2c$ is 25bc

d) HCF of 8axp, 4byp is 4p

24. Key:3 ; Sol:- The integer factors of $3p^2q$ are 1,3