CIRCLES
MIND MAPPING

Centre of Circle = O

Radius of Circle = OP = OQ = OM = ON = r

Diameter of Circle = PQ = d = 2r

Chord of Circle = AB

Sector of Circle = MON = Region

= between two radii and Corresponding are

Segment = ACB

Region between

Chord and Corresponding

Cyclic Quadrilateral :-
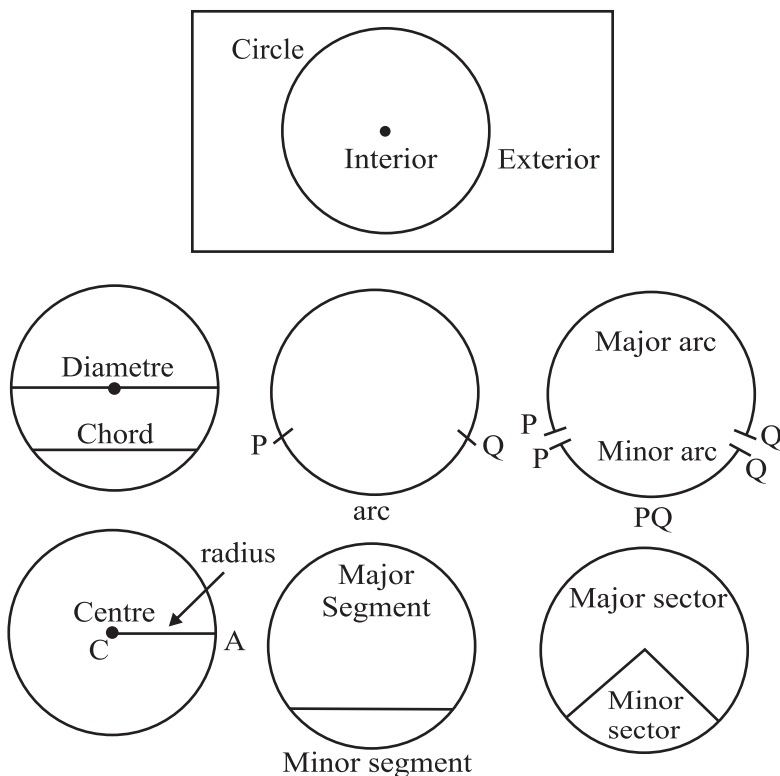
If the sum of pair of opposite angles

of quadrilateral is 180° = ☐PABQ

CIRCLES

• The collection of those points in a plane which are at a fixed distance from a given fixed point is called a circle. The fixed point is called centre of the circle and the fixed distance is called radius.

Circle and related Terms !

• There is one and only one circle passing through three non-collinear points.

• Equal chords of a circle subtends equal angles at centre.

• If angles subtended by chords at centre are equal then chords are equal.

• The perpendicular from centre to a chord of a circle, bisects the chord.

- The line joining the centre of a circle to the mid point of a chord is perpendicular to the chord.
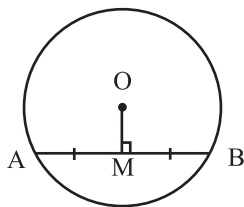
- Equal chords of a circle are equidistant from centre.

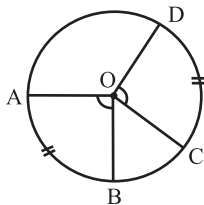- Chords equidistant from centre are equal in length.
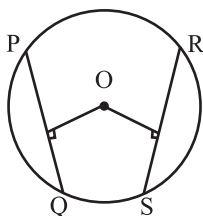
- If two chords of a circle are equal then corresponding arcs are equal.

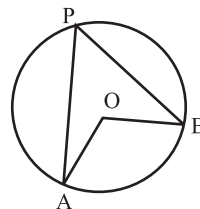- If arcs of a circle are equal then corresponding chords are also equal.
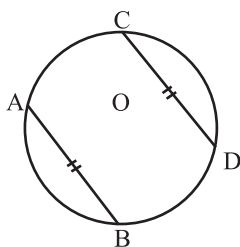
- Congruent arcs (or equal arcs) of a circle subtends equal angles at centre.
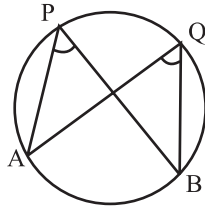
$$\Rightarrow \boxed{\angle AOB = \angle COD}$$

- The angle subtended by an arc at the centre of circle is twice the angle which is subtended at remaining part of the circle.

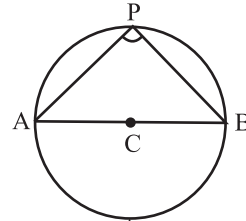$$\Rightarrow \boxed{\angle AOB = 2\angle APB}$$

- Any two angles in the same segment of the circle are equal.

$\Rightarrow$ $\boxed{\angle APB = \angle AQB}$
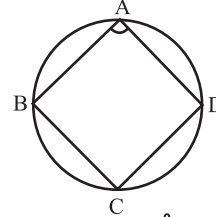
- Angle in semi circle is right angle.
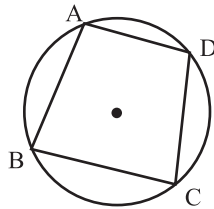
$\Rightarrow$ $\boxed{\angle APB = 90°}$

- In a cyclic quadrilateral the sum of opposite angles is 180^0.

$\Rightarrow$ $\boxed{\angle A + \angle C = 180°}$

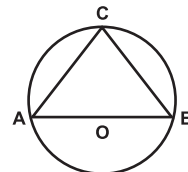$\Rightarrow$ $\boxed{\angle B + \angle D = 180°}$

- If sum of opposite angles of a quadrilateral is 180^0 then that quadrilateral is cyclic quadrilateral.

PART – A

1. In fig. AOB is a diameter of the circle and AC = BC the $\angle$CAB is equal to :
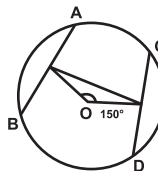   a) 30°
   b) 45°
   c) 60°
   d) 90°

2. In fig. AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If $\angle$POQ = 150° then $\angle$APQ is equal to
   a) 30°
   b) 75°
   c) 15°
   d) 60°

3. Angles in the same segment of a circle are
   a) Equal
   b) Complementary
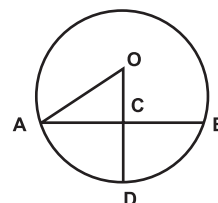   c) Supplementary
   d) Vertically Opposite Angles

4. In fig, if OA = 5cm, AB=8cm and OD is perpendicular to AB. Then CD is equal to :
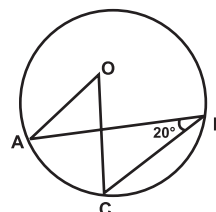   a) 2 cm
   b) 3 cm
   c) 4 cm
   d) 5 cm

5. The radius of a circle is 13cm and the length of one of its chords is 10cm. The distance of the chord from the centre is .
   a) 11.5 cm
   b) 12 cm
   c) √69 cm
   d) 23 cm

6. In fig. if ∠ABC = 20°, then ∠AOC is equal to
   a) 20°
   b) 40°
   c) 60°
   d) 10°
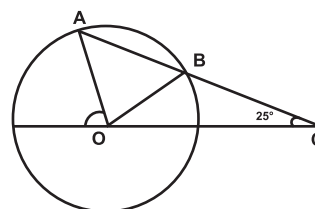

7. If AB = 12cm, BC=16cm and AB is perpendicular to BC, then the radius of the circle passing through the point A, B and C is :
   a) 6 cm
   b) 8 cm
   c) 10 cm
   d) 12 cm

8. In the given figure, AB is chord of a circle with centre O and AB is produced to C such that BC = OB. Also, CO is joined and produced to meet the circle in D. If ∠ACD = 25°, then ∠AOD ?
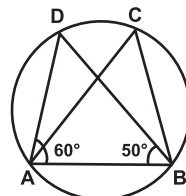   a) 50°
   b) 75°
   c) 90°
   d) 100°

9. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm the distance of AB from the center of the circle is :

a) 17 cm
b) 15 cm
c) 4 cm
d) 8 cm

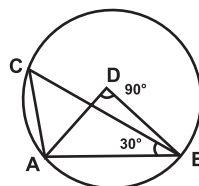10. In the given figure; ∠DAB = 60° and ∠ABD = 50° then ∠ACB = ?

a) 50°
b) 60°
c) 70°
d) 80°

11. In fig. ∠AOB = 90° and ∠ACB = 30°, then ∠CAO is equal to :

a) 30°
b) 45°
c) 90°
d) 60°

12. In the given figure O is the center of a circle and ∠BAC = 40°, then ∠OBC = ?
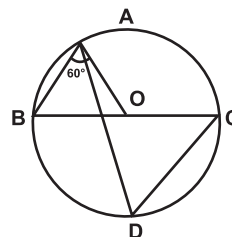
a) 40°
b) 50°
c) 80°
d) 20°
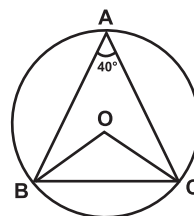
13. An equilateral triangle of side 9 cm is inscribed in a circle. The radius of the circle is :

a) 3 cm
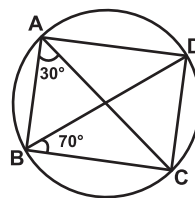b) $3\sqrt{2}$ cm
c) $3\sqrt{3}$ cm
d) 6 cm

14. In fig. BC is a diameter of the circle and ∠BAO = 60° Then ∠ADC is equal to :

a) 30°
b) 60°
c) 120°
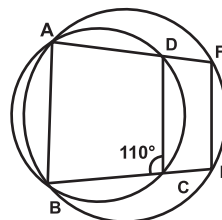d) 45°

15. In the given figure , the measure of ∠BCD is
    a) 80°   b) 30°
    c) 70°   d) 100°

16. In the given figure ABCD and ABEF are cyclic quadrilaterals. If ∠BCD = 110° then ∠BEF = ?
    a) 110°   b) 55°
    c) 90°   d) 70°

17. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and ∠ADC = 140°, then ∠BAC is equal to :
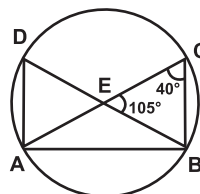   a) 80°   b) 30°
    c) 50°   d) 40°

18. The length of the chord which is at a distance of 12cm from the centre of a circle of radius 13 cm is :
    a) 5cm   b) 10 cm
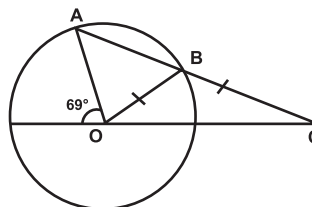   c) 12 cm   d) 13 cm

19. In the given figure, ∠ECB = 40° and ∠CEB = 105° Then, ∠EAD = ?
    a) 35°   b) 20°
    c) 50°   d) 40°

20. In the following figure, BC = radius OB. Then find the value of ∠OCB.
    a) 69°   b) 46°
   c) 92°   d) 23°

Fill in the blanks :-

21. A segment of a circle is the region between an arc and a _____ of the circle.

22. An arc of a circle is called a _____ if the ends of the arc on the ends of a diameter.

23. Two circles having the same centre and different radii are called _____.

Write T for True and F for False "

24. The degree measure of a semi circle is 180° (T / F)

25. A circle divides the plane into three parts. (T/F)

26. A circle can have only a finite number of equal chords . (T/F)

27. Write True or False and Justify your answer.

The angles subtended by a chord at any two points of a circle are equal.

28. Through three collinear points a circle can be drawn.

29. If A, B, C and D are four points such that $\angle BAC = 45°$ and $\angle BDC = 45°$ then A, B, C, D are concyclic.

30. A circle of radius 3cm can be drawn through two points A, B such that AB = 6cm.

31. If the sum of a pair of opposite angles of a quadrilateral is 180^0, then quadrilateral is _____.

32. A round pizza is cut into 4 equal pieces. What does each piece represent?

33. AD is a diameter of a circle and AB is a chord if AD = 34cm, AB=30 cm then find the distance of AB from the centre of chord.

34. Given two concentric circles with centre O. A line cut the circle at A, B, C and D respectively. If AB = 10cm, then find the length of CD.

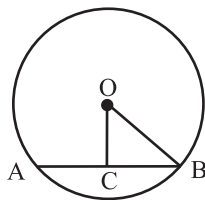35. Find y in given figure
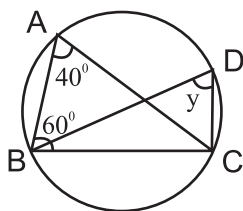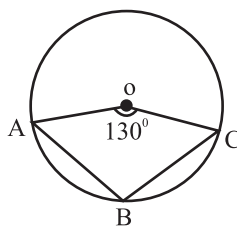
36. Find x
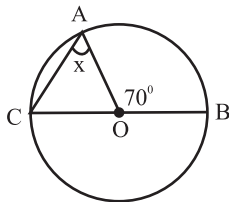
37. Find x
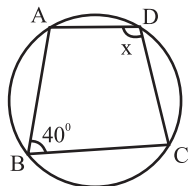
38. Diameter is the _____ Chord of a circle.

39. Circle having the same centre and different radii are called _____ circles.

40. In given figure OC is perpendicular segment drawn from centre O on chord AB. If OB = 5cm, and OC = 3cm then find length of AB.
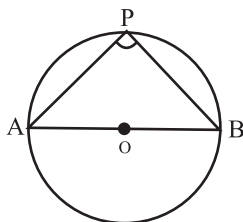

41. In given figure O is centre of circle.

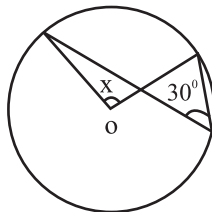   If ∠AOC = 130⁰ then find ∠ABC

42. In given figure AOB is diameter of circle & P is any point on the circle. Find ∠APB.
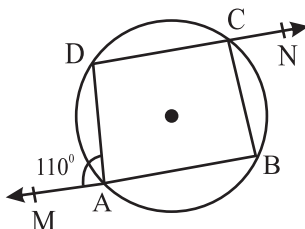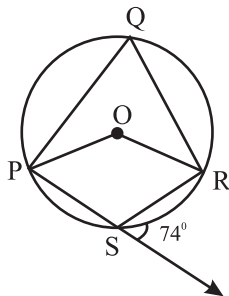
43. Find the value of x in given figure.


Part – B

44. Prove that cyclic parallelogram is a rectangle.

45. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

46. In the following figure. Find the value of $\angle$BCN.
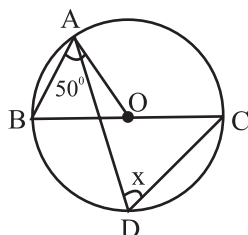

47. In the given figure. Find the value of reflex angle POR.
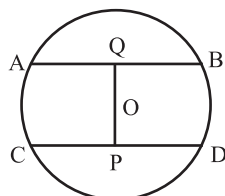

48. In given figure ABCD is a cyclic quadrilateral, chords AB and CD are produced to meet E, show that EA x EB = EC x ED.
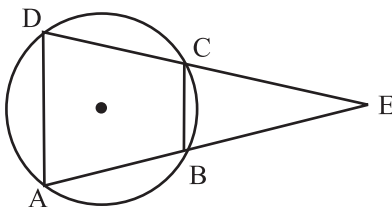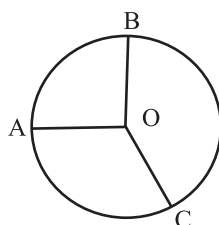
49. Find the value of x in figure if O is centre of circle and $\angle OAB = 50^0$.
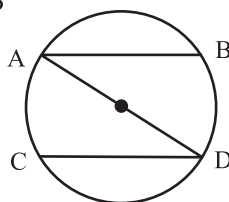

50. In the given figure, O is centre of the circle with radius 5 cm, $OP \perp CD$, $OQ \perp AB$, AB ‖ CD, AB = 6 cm and CD = 8 cm. Determine PQ.
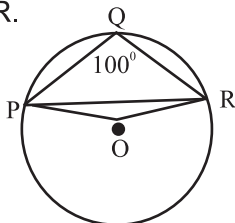

51. In the given figure, O is the centre of a circle, $\angle AOB = 90^0$, $\angle BOC = 120^0$, what is measure of $\angle ABC$?

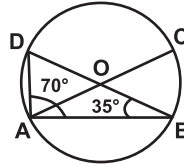52. In the given figure AB and CD are parallel chords if the length of arc AC = 14 cm. What is length of BD?

53. In given figure $\angle PQR = 100^0$ where P, Q & R are points on the circle with centre O. Find $\angle OPR$.

Class IX - Mathematics

54. In the given figure O is centre of circle. If ∠ABD = 35⁰ and ∠BAD = 70⁰, find ∠ACB.

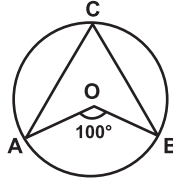55. Match the following Columns.

Column I   Column II

(a) Angle in a semicircle measures   (p) 40°

(b) In the given figure, O is the centre of   (q) 80°
    a circle. If ∠AOB = 100º, then ∠ACB = ?
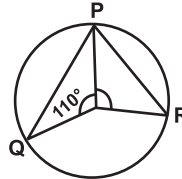
(c) In the given figure, O is the centre of   (r) 90°
    a circle. If ∠POR = 90° and ∠POQ = 110°
    then ∠QPR = ?
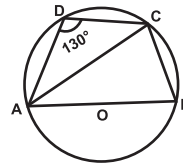
(d) In cyclic quadrilateral ABCD, it is given   (s) 50°
    that ∠ADC = 130° and AOB is a diameter of
    the circle through A, B, C and D, Then ∠BAC = ?
    The correct answer is
    a) _____   b) _____   c) _____   d) _____

56. MCQ based on synthesis
    Three statements are given below:

(i) If a diameter of a circle bisects each of the two chords of a circle,
    then the chords are parallel.

___

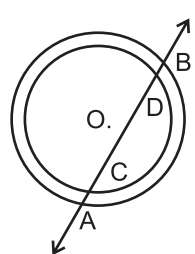(ii) Two circle of radii 10 cm and 17 cm intersect each other and the length of the common chord is 16 cm. Then, the distance between their centres is 23 cm.

(iii) L is the Line intersecting two concentric circles with centre O at point A, B, C and D as shown. Then AC = DB

Which is true ?

a) I and II

b) I and III

c) II and III

d) II only

PART – C
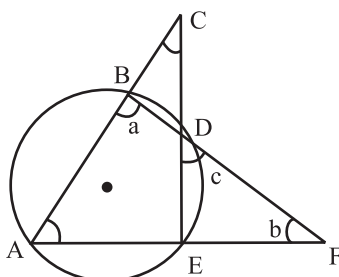
57. In the given figure, O is the centre of a circle prove that $\angle x + \angle y = \angle z$.
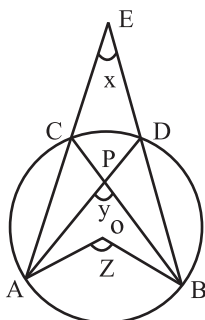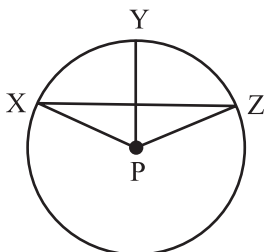

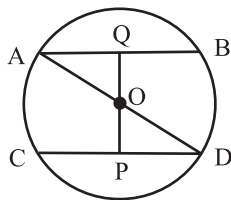58. If two non parallel sides of a trapezium are equal prove that it is cyclic quadrilateral.

59. In the given figure determine a, b & c if $\angle BCD = 43^0$, $\angle BAF = 62^0$.

60. In the figure P is the centre prove that $\angle XPZ = 2 (\angle XZP + \angle YXZ)$

61. In the given figure AD is diameter of the circle whose centre is O and AB || CD prove that AB = CD.

62. In an equilateral triangle, prove that the centroid and the circum centre coincide.

63. In the given figure A, B, C and D, E, F are two sets of collinear points. Prove that AD || CF.
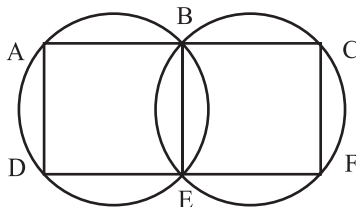
64. In given figure, O is centre of circle and $\angle DAB = 50^0$, calculate the value of x and y.
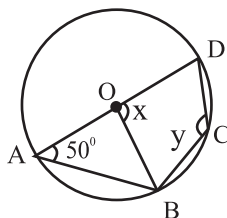
65. If two equal chords of a circle intersect within the circle prove that the segment of one chord is equal to corresponding segment of other chord.

66. Prove that if a pair of opposite angles of a quadrilateral is supplementary then the quadrilateral is cyclic.

Part – D

67. Bisector of angle A, B and C of a ∆ABC intersect its circum circle at D, E and F respectively, prove that the angles of a triangle DEF are

$$90^0 - \frac{1}{2} A, \quad 90^0 - \frac{1}{2} B, \quad 90^0 - \frac{1}{2} C$$

68. Find the sum of the angles in the four segments exterior to a cyclic quadrilateral.

69. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.
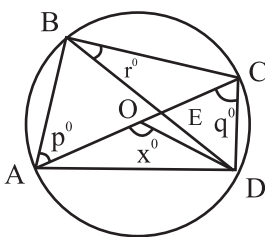
$$\angle ABC = \frac{1}{2} [\angle DOE - \angle AOC]$$

70. In the given figure O is centre of the circle of radius 5 cm, $OP \perp CD$, $AB \parallel CD$

    AB = 6 cm and CD = 8 cm

    Determine PQ
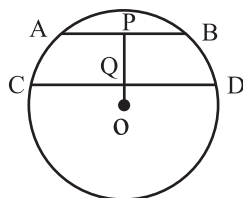
71. In the adjoining figure AC is diameter of a circle with centre O and chord $BD \perp AC$, intersecting each other at E. Find out the values of p, q, r in terms of x, if $\angle AOD = x^0$, $\angle BAC = p^0$, $\angle ACD = q$.

72. During a practical activity in maths lab students were using circular geo board. The angle subtended by an arc at the centre is $(2a+50^0)$. Pallavi calculated $\angle BAC$ as $(a+25^0)$.

   a)   Is her finding correct ? Justify it.

    b)   Find $\angle BAC$ if $a = 30^0$

    c)   What will be the value of $\angle BOC$ for $a = 15^0$

    d)   If $a = 30°$ then find the measure of Reflex $\angle BOC$.
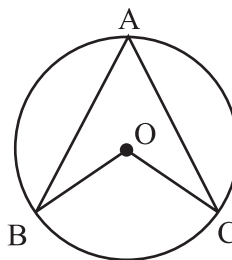
73. Show that if two chords of a circle bisect each other, they must be diameters of the circle.

74. Prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

75. Prove that there is one and only one circle can pass through three non-collinear points.

76. In the given figure OPQR is a square. A circle drawn with centre O cuts the square in X and Y. Prove that QX = QY.
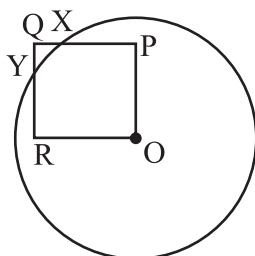
77. Prove that the opposite angles of a cyclic quadrilateral are supplementary.

78. In the given figure, AB is a diameter of a circle (o, r) and chord CD = radius oc. If AC and BD when produced meet at P. Prove that $\angle APB$ is constant.
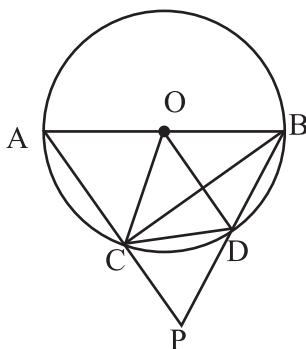

79. Prove that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

80. In the given figure, AB is a chord of a circle with centre O and AB is produced to C such that BC = OB. Also, CO is joined and produced to meet the circle in D. If $\angle ACD = y^0$ and $\angle AOD = x^0$. Prove that x = 3y.
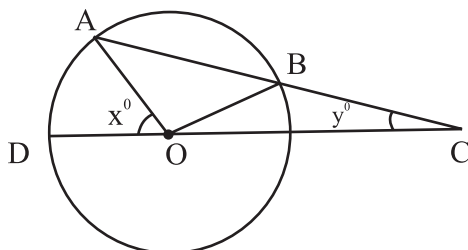
81. Two circles whose centres are O and O' intersect at P. Through P, a line l parallel to OO', intersecting the circle at C and D is drawn. Prove that CD = 2OO'.
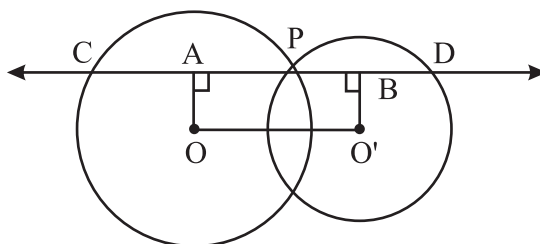
82. AB and CD are two parallel chords of a circle which are on opposite sides of the centre O such that AB = 10cm, CD = 24cm and the distance between AB and CD is 17 cm. Find the radius of the circle.
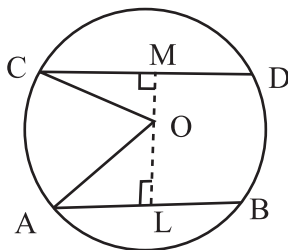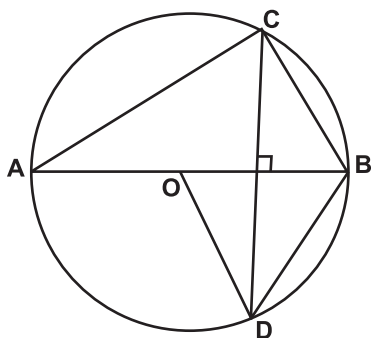
PART-D

83. AB and AC are two chords of a circle of radius r such that AB = 2 AC. If p and q are the distance of AB and AC from the centre, Prove that $4q^2 = P^2 + 3r^2$

84. In figure, O is the centre of the circle, $\angle$ BCO = 30° Find x and y.
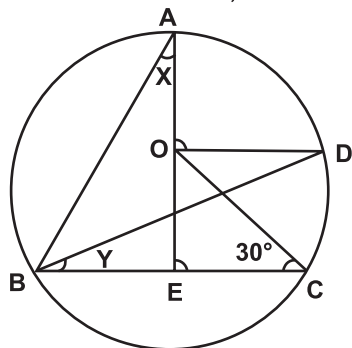
85. In figure, O is the centre of the circle, BD = OD and CD $\perp$ AB, Find $\angle$CAB.

CIRCLES

ANSWERS

1. b) 45°
2. b) 75°
3. a) Equal
4. a) 2 cm
5. b) 12 cm
6. b) 40°
7. c) 10 cm
8. b) 75°
9. d) 8 cm
10. c) 70°
11. d) 60°
12. b) 50°
13. c) 3 √3 cm
14. b) 60°
15. a) 80°
16. a) 110°
17. c) 50°
18. b) 10 cm
19. a) 35°
20. d) 23°
21. Chord
22. Semicircle
23. Concentric
24. True (T)
25. True (T)
26. False (F)
27. False, If two points lie in the same segment (major or minor) only, then the angles will be equal otherwise they are not equal.
28. False, Because a circle through two points cannot pass through a point which is collinear to these two points.
29. True, Angles in the same segment.
30. True, Because AB will be the diameter
31. Cyclic Quadrilateral
32. Sector
33. 8 cm

34. 10 cm
35. $y = 40°$
36. $x = 35°$
37. $x = 140°$
38. longest
39. concentric
40. 8 cm
41. 115°
42. 90°
43. 60°
45. 30°, 150°
46. 70°
47. 212°
49. 50°
50. 7 cm
51. 75°
52. 14 cm
53. 10°
54. 75°
55. $(A) - (R), (B) - (S)$
    $(C) - (Q), (D) - (P)$
56. (B) I and III (Distance between centres $= 21$ cm $\neq 23$ cm
59. $a = 105°, b = 13°, c = 62°$
64. $x = 100°, y = 130°$
68. 540°
70. 1 cm
71. $p = 90° - \dfrac{1}{2}x$

    $q = \dfrac{1}{2}x$

    $r = 90° - \dfrac{1}{2}x$

72. i) yes
    ii) 55°
    iii) 80°
    iv) 250°
82. 13 cm
84. $x = 30°, y = 15°$
85. 30°

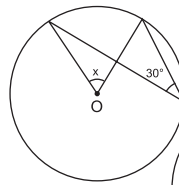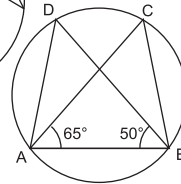Time : 50 Min.   **Circles**   M.M. 20

1.   Find the value of x in the given figure   (1)
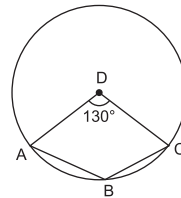
2.   In the given figure : $\angle DAB = 60°$ and $\angle ABD = 50°$. then $\angle ACB = ?$ (1)

3.   In given figure O is the centre of circle. If $\angle AOC = 130°$ then find $\angle ABC$.   (2)

4.   Prove that equal chords of a circle subtend equal angles at the centre.   (2)
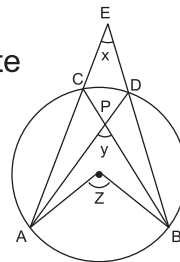
5.   Prove that the sum of either pair of the opposite angle s of a cyclic quadrilateral is 180°.   (3)
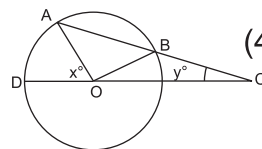
6.   In the given figure, O is the centre of a circle prove that

     $\angle x + \angle y = \angle z$ (3)

7.   In the given figure, AB is a chord of a circle with centre O and AB is produced to C. Such that BC = OB Also, CO is joined and produced to meet the circle in D.   (4)

     If $\angle ACD = y°$ and $\angle AOD = x°$. Prove that x = 3y.

8.   Prove that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.   (4)