SET THEORY

SYNOPSIS - 1

1. **Introduction :** In everyday life, we have to deal with the collections of objects of one kind or the other.

For Example :

i) The collection of even natural numbers less than 12 i.e., the numbers 2, 4, 6, 8 and 10.

ii) The collection of vowels in the English alphabet i.e., the letters a, e, i, o, u.

iii) The collection of all natural numbers that divide 12 i.e., the numbers 1, 2, 3, 4, 6 and 12.

Definition : Any well defined collection of objects is called a set.

By *'well-defined collection'* we mean that given a set and an object, it must be possible to decide whether or not the object belongs to the set.

The objects are called the members or the elements of the set. Sets are usually denoted by capital letters and their members are denoted by small letters.

We write the elements of set with in the bracess { }.

If x is a member of the set S, we write $x \in S$ (read as x belongs to S) and if x is not a member of the set S, we write $x \notin S$ (read as x does not belong to S). If x and y both belongs to set S, we write $x, y \in S$.

Representation of Sets : There are two ways to represent a given set.

1) **Roster or Tabular Form or list form.** In this form, list all the members of the set, separate these by commas and enclose these within braces (curly brackets)

For example :

i) The set S of even natural numbers less than 12 in the tabular form is written as S = { 2, 4, 6, 8, 10 }. Note that $8 \in S$ while $7 \notin S$.

ii) The set S of prime natural numbers less than 20 in the tabular form is written as S = { 2, 3, 5, 7, 11, 13, 17, 19 }

iii) The set N of natural numbers in the tabular form is written as N = { 1, 2, 3, ... }, the dots indicating infinitely many missing positive integers.

2. **Set Builder or rule form :** In this form, write one or more (if necessary) variables (say x, y etc.) representing an arbitrary member of the set, this is followed by a statement or a property which must be satisfied by each member of the set.

For example :

i) The set S of even natural numbers less than 12 in the set builder form is written as S = { x/x is an even natural number less than 12}.

ii) The set of prime natural number less than 20 in the set builder form is written as {x/x is a prime natural number less than 20}.

The symbol '|' stands for the words 'such that' or 'where'. Sometimes we use the symbol ';' or ':' in place of the symbol '|'.

iii) The set N of natural numbers in the set builder form is written as N = { x :x is a natural number}.

Some Standard Sets : We enlist below some sets of numbers which are most commonly used in the study of sets :

i) The set of natural numbers (or positive integers). It is usually denoted by N.

i.e. N = { 1, 2, 3, 4, ... }

ii) The set of whole numbers. It is usually denoted by W. i.e. W = { 0, 1, 2, 3, ... }.

iii) The set of integers. It is usually denoted by Z . i.e. Z = { ...., -3, -2, -1, 0, 1, 2, 3,... }

iv) The set of rational numbers. It is usually denoted by Q i.e. Q = { x : x is a rational number } or $Q = \left\{ x : x = \frac{m}{n}, \text{where } m \text{ and } n \text{ are integers and } n \neq 0 \right\}$

v) The set of real numbers. It is usually denoted by R. i.e. R = { x : x is a real number } or R = { x : x is either a rational number or an irrational number }

Note :- $i = \sqrt{-1}$

Types of sets :

The Empty set : A set containing no element is called the empty set.

It is also called the *null set* or *void set* . There is only one such set. It is denoted by ϕ or by { }.

For example :

i) The collection of all integers whose square is less than 0 is the empty set.

($\because$ Square of an integer cannot be negative)

The order of the empty set is zero.

Singleton set : A set is said to be a singleton set if it contains only one element.

The set { 7 }, { – 15 } are singleton sets. { x : x + 4 = 0, $x \in Z$ } is a singleton set, because the set contains only one integer namely, –4.

The set { x : x + 4 = 0, $x \in N$ } is a null set, because there is no natural number which may satisfy the equation x + 4 = 0.

A set whose order is 1 is called a singleton set. Thus, a singleton set is a set which contains only one distinct element.

For example :

i) If A = { x : x is a positive divisor of 20 }, then n(A) = 6 as A = { 1, 2, 4, 5, 10, 20 }

ii) If B = { x : x is a positive even prime }, then n(B) = 1 as B = { 2 }. Note that B is a singleton set.

iii) If C = { x : x is an integer neither positive nor negative }, then n(C) = 1 as C = {0}. C is a singleton set.

Finite and Infinite sets : A set is called finite if the process of counting of its different elements comes to an end; otherwise, it is called infinite. The empty set is taken as finite.

For example : i) The set S = { 2, 4, 6, 8 } is a finite set.

ii) The set of all students studying in a given school is a finite set.

iii) The set N of all natural numbers is an infinite set.

iv) The set of divisors of a given natural number is a finite set.

v) The set of all prime numbers is an infinite set.

Order of a finite set : The number of different elements in a finite set S is called order of S, it is denoted by O(S) or n(S).

Note : The order of an infinite set is not defined.

Equivalent sets : Two finite sets A and B are said to be equivalent written A ~ B (or $A \approx B$), iff they contain the same number of distinct elements i.e., iff n(A) = n(B).

For example :

i) The sets { 1 } and {2, 2, 2} are equivalent.

ii) The sets { 3, 4 } and { x : $x^2 = 4$ } are equivalent sets.

iii) The sets { a, b, c, d, e }, { 1, 2, 3, 4, 5 } and { a, e, i, o, u } are equivalent sets as each of these sets contains 5 distinct elements.

Equal sets :

Two sets A and B are said to be equal, written as A = B, iff every member of A is a member of B and every member B is a member of A.Remember that equal sets are always equivalent but equivalent sets may not be equal.

For example :

i) The sets { – 1, +1 } and { x : $x^2 = 1$ } are equal.

ii) The sets { 0, 0 } and { 3 } are not equal, but they are equivalent.

Cardinal number of set : The number of distrinct elements contained in a finite set is called its cardinal number and is denoted by n(A).

Examples : If A = {1, 2, 3, 4, 5} then n(A) = 5, if B = {1, 2, 3} then n(B) = 3.

WORKSHEET - 1

SINGLE ANSWER TYPE

1. The collection of rich persons in india is

   1) A null set 2) A finite set 3) A infinite set 4) Not a set

2. Which of the following is a set ?

   1) The collection of all talented persons 2) The collection of all beautiful flowers

   3) The collection of all peaple in hyderabad.

   4) The collection of beautiful girls in india.

3. Which of the following is a set ?

   1) The collection of all difficult problems in maths

   2) The collection of all difficult problems in physics

   3) The collection of all difficult problems in chemistry

   4) The collection of all topics in algebra.

4. Which of the following is a void set ?

   1) The prime numbers less than 100

   2) The collection of compositic numbers between 90 to 100

   3) Even primes in between 100 to 200

   4) Perfect numbers between 1 to 100

5. Which of the following is a null set ?

   1) $\{x : |x| < 1, x \in N\}$ 2) $\{x : |x| = 5, x \in N\}$

   3) $\{x : x^2 = 1, x \in z\}$ 4) $\{x : x^2 2x + 1 = 0, x \in R\}$

6. Which of the following is infinite set ?

   1) The natural numbers less than 1,00,000.

   2) The real numbers in between 2011 to 2013.

   3) The Collection of all 8th class students.

   4) None

7. The cardinal number of the set $A\{x : x + 1 < 12, x \in N\}$ is

   1) 9 2) 10 3) 11 4) 12

8. The cardinal number of the set containing the word “MATHEMATICS”

   1) 11 2) 10 3) 9 4) 8

9. The cardinal number of the set containing all primes less than 100.
1) 21 2) 22 3) 23 4) 24

10. The cardinal number of the set $A\{x : x^2 + 1 < 0, x \in R\}$
1) 0 2) 10 3) 1 4) 2

11. The set of all composite numbers is a
1) Finite set 2) Infinite set 3) Null set 4) Not set

12. If $Q = \left\{x : x = \frac{1}{y}, where\ \text{x, y} \in \text{N}\right\}$, then
1) $0 \in Q$ 2) $1 \in Q$ 3) $2 \in Q$ 4) $\frac{2}{3} \in Q$

13. Which of the following is a singleton set
1) The natural numbers less than 1
2) The whole numbers greater than 9
3) The even primes less than 100
4) The real numbers in between 1 and 3.

14. If $A = \{S, H, I, V, A\}$, $B = \{V, A, I, S, H\}$ then A, B are
1) Equivalent 2) Equal 3) Not sets 4) Unequal

15. If $A = \{x : \ x^2 - 4 < 0, \ x \in z\}$, $B = \{N, A, G, A\}$ Then A, B are
1) Equal 2) Equivalent 3) Unequal 4) None

MULTI ANSWER TYPE

16. Which of the following sets are singleton sets ?
1) $\{x : 6x + 9 = 0, x \in Q\}$ 2) $\{x : x^2 + 1 = 0, x \in R\}$
3) $\{x : x < 2, x \in N\}$ 4) $\{x : 2x^2 - 8 = 0, x \in N\}$

17. Which of the following is not a singleton set ?
1) $\{x : |x| = 9, x \in z\}$ 2) $\{x : x^2 + 2x + 1 = 0, x \in N\}$
3) $\{x : x^2 = 101, x \in N\}$ 4) $\{x : x + 12 = 10, x \in N\}$

18. $A = \{1, 2, 3, 4\}$, $B = \{x : \ x - 5 < 0, \ x \in N\}$ then A, B are
1) equal 2) Equivalent 3) Finite 4) Unequal

19. If $x = \{a, b, c, d\}$ then which of the following are not Correct
1) $a \in x$ 2) $\{a, b\} \in x$ 3) $b \in x$ 4) $\{c\} \in x$

20. Which of the following set having cardinal number is infinite.
 1) Set of rational is infinite. 2) Set of Comptex numbers
 3) Set of real numbers between 2012 and 2013.
 4) Set of all odd numbers.

COMPREHENSION TYPE

Writeup-1

21. If $A=\{5,9,13,17,21\}$ then set builder form of A is
 1) $A=\{n(n+1)\ /\ n<5,\ n\in N\}$ 2) $A=\{4n+1\ /\ n\le 5,\ n\in N\}$
 3) $A=\{4n+1\ /\ n<5,\ n\in N\}$ 4) $\{A=2n+1\ /\ n\ge 2,\ n\in N\}$
22. The no. of prime numbers in A is
 1) 5 2) 4 3) 3 4) 2
23. The no. of composite numbers in A is
 1) 5 2) 4 3) 3 4) 2

Writeup-2

If Variable are representing an arbitrary member of the set, this is followed by a statement is called set builder or rule form of a set.

24. If $A=\{1,3,5,7,9\}$ then rule form of A is
 1) $\{x:\ x=2n+1,\ n\in N,\ n\le 5\}$ 2) $\{x:\ x=2n+1,\ n\in W,\ n<5\}$
 3) $\{x:\ x=2n-1,\ n\in N,\ n\le 5\}$ 4) $\{x:\ x=2n-1,\ n\in N,\ n<5\}$
25. If $A=\{2,4,9,16,25\}$ then set builder form of A is
 1) $\{x:\ x=n^2,\ n\in N,\ n<5\}$ 2) $\{x:\ x=(n+1)^2,\ n\in W,\ n\le 5\}$
 3) $\{x:\ x=n^2,\ n\in z,\ n\le 5\}$ 4) $\{x:\ x=(n+1)^2,\ n\in W,\ n<5\}$
26. If $A=\{2,9,28,65\}$ then set builder form of A is
 1) $\{x:\ x=n^3+1,\ n\in N,\ n\le 4\}$ 2) $\{x:\ x=n^3+1,\ n\in N,\ n<4\}$
 3) $\{x:\ x=(n+1)^3+1,\ n\in W,\ n\le 4\}$ 4) $\{x:\ x=n^3-1,\ n\in N,\ n\le 4\}$

REASONING ANSWER TYPE

27. S.I : The set of all Consonants in the English alphabet is a finite set
 S.II : A set Containing finite number of elements is called a finite set.
 1. Both Statements I and II are correct
 2. Both statement I and II are incorrect
 3. Statement I is true, Statement II is false.
 4. Statement I is false, Statement II is true.

28. S.I : The set builder form of $A=\left\{-1,\frac{1}{2},\frac{-1}{3},\frac{1}{4},\frac{-1}{5},..\right\}$ is $A=\left\{\frac{(-1)^n}{n}\Big/ n\in N\right\}$

S.II : $(-1)^n=-1$ if n is odd, $(-1)^n=1$, if n is even

1. Both Statements I and II are correct
2. Both statement I and II are incorrect
3. Statement I is true, Statement II is false.
4. Statement I is false, Statement II is true.

MATRIX MATCHING TYPE

29.

Column - I	Column - II
1) The Collection of good books in Library	p) Set
2) $\{1,3,5,7,9\}$	q) Not a sest
3) $\{2n/n\in N, n\le 5\}$	r) Roster form
4) The collection of all even numbers	s) Set builder form
	t) $\{2,4,6,8,10\}$

30.

Column - I	Column - II
1) $A=\left\{0,\frac{1}{3},\frac{1}{2},\frac{3}{5}\right\}$	p) Singleton set
2) $A=\{2,5,10,17\}$	q) $n(A)=4$
3) $A=\{x:\ x\in z,\ x^2-4=0\}$	r) $\left\{x:\ x=\frac{n-1}{n+1},\ n\in N,\ n\le 4\right\}$
4) $A=\{x:\ x\in W,\ x^2+1<2\}$	s) $A=\{x:\ x=n^2+1,\ n\in N,\ n\le 4\}$
	t) $n(A)=2$

INTEGER ANSWER TYPE

31. A = set of vowels in first 10 alphabets then $n(A)=$_______

32. If A = set of letters in the word MISSISSIPI then $n(A)=$ _______

33. If $A=\{2^x<50, x\in N\}$ then $n(A)=$ _______

34. If A = Set of even primes less than 2012 then $n(A)=$_______

SYNOPSIS - 2

Subsets:

Let A, B be two sets such that every member of A is a member of B, then A is called a subset of B, it is written as $A \subseteq B$.

Thus, $A \subseteq B$ iff (read as 'if and only if') $x \in A \Rightarrow x \in B$.

If $\exists$ (read as 'there exists') atleast one element in A which is not a member of B, then A is not a subset of B and we write it as $A \not\subset B$.

For example:

i) Let A = {–1, 2, 5} and B = {3, – 1, 2, 7, 5}, then $A \subseteq B$. Note that $B \not\subset A$

ii) The set of all even natural numbers is a subset of the set of natural numbers.

Some properties of subsets :

i) The null set is subset of every set. Let A be any set.

$\phi \subseteq A$, as there is no element in ϕ which is not in A.

ii) Every set is subset of itself. Let A be any set.

$\therefore\ x \in A \Rightarrow x \in A \qquad \therefore\ A \subseteq A$.

iii) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. Let $x \in A$.

$\therefore\ x \in B \qquad (\because A \subseteq B)$

$\therefore\ x \in C \qquad (\because B \subseteq C)$

$\therefore\ A \subseteq C$.

iv) A = B iff $A \subseteq B$ and $B \subseteq A$. Let A = B.

$\therefore\ x \in A \Rightarrow x \in B \qquad (\because A = B)$

$\therefore\ A \subseteq B$. Similarly, $x \in B \Rightarrow x \in A \qquad (\because A = B)$

$\therefore\ B \subseteq A$. Conversely, let $A \subseteq B$ and $B \subseteq A$.

$\therefore\ x \in A \Rightarrow x \in B \quad (\because A \subseteq B)$ and $x \in B \Rightarrow x \in A \quad (\because B \subseteq A)$ $\therefore$ A = B

Note:-

1. Two sets A and B are equal iff $A \subseteq B$ and $B \subseteq A$.
2. Since every element of a set A belongs to A, it follows that every set is a subset of itself.

Proper subset: Let A be a subset of B. We say that A is a proper subset of B if $A \neq B$ i.e., if there exists atleast one element in B which does not belong to A. A subset, which is not proper, is called an improper subset.

Observe that every set is an improper subset of itself. If a set A is non-empty, then the null set is a proper subset of A.

Ex 1 : If A = { 1, 2, 3 }, then proper subsets of A are ϕ, { 1 }, { 2 } , { 3 }, { 1, 2 }, { 1, 3 }, { 2, 3 }

Ex 2 : A = { 1, 2, 3, 4, 5 }, B = { 2, 3, 4 }

Every element of B i.e., 2, 3 and 4 is also an element of A.

$\therefore B \subset A$

Further we note that there are two more elements that are in A and not in B. They are 1 and 5. Then $A \neq B$. In such circumstances we say that B is a proper subset of A.

Ex 2 : $N \subset W \subset Z \subset Q \subset R$

Remark 1 : If $A \subseteq B$ then every element of A is in B and there is a chance that A may be equal to B i.e., every element of B is A, but if $A \subset B$, then every element of A is in B and there is no chance that A may be equal to B i.e., there will exist at least one element in B which is not in A.

$\therefore \quad A \subset B \Rightarrow A \subseteq B, A \neq B$ i.e., $A \subseteq B, B \not\subset A$.

Remark 2 : If $A \subseteq B$, we may have $B \subseteq A$, but if $A \subset B$, we cannot have $B \subset A$.

Power Set:

The set formed by all the subsets of a given set A is called the power set of A, it is usually denoted by P(A).

For example:

i) Let A = {0}, then P(A) = {ϕ, {0}}. Note that n(P(A)) = 2 = 2^1.

ii) Let A = {a, b}, then P(A) = {ϕ, {a}, {b}, {a, b}}. Note that n(P(A)) = 4 = 2^2

iii) Let A = {1, 2, 3}, then P(A) = {ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, A}.

Note that O(P (A)) = 8 = 2^3. In all these examples, we have observed that n(P(A)) = $2^{n(A)}$.

Rule to write down the power set of a finite set A:

First of all write ϕ.

Next, write down singleton subsets each containing only one element of A.

In the next step write all the subsets which contain two elements from the set A.

Continue this way and in the end write A itself as A is also a subset of A.

Enclose all these subsets in braces to get the power set of A.

Comparable Sets : Two sets A and B are said to be comparable iff either $A \subset B$ or $B \subset A$.

For example :

i) The sets A = {1, 2} and B = {1, 2, 4, 5} are comparable as $A \subset B$.

ii) The sets A = {0, 1, 3} and B = {1, 3} are comparable as $B \subset A$.

iii) The sets A = {–1, 1} and B = {x : x^2 = 1} are comparable as $A \subset B$ and also $B \subset A$.

Clearly, equal sets are always comparable. However, comparable sets may not be equal

Universal Set:

In any application of the theory of sets, all sets under investigation are regarded as subsets of fixed set. We call this set the universal set, it is usually denoted by X or U or ξ.

OPERATIONS OF SETS

UNION OF SETS

The union of two sets A and B is the set of all those elements, which are either in A or in B (including those which are in both)

In symbolic form, union of two sets A and B is denoted as, $A \cup B$. It is read as "A union B".

Clearly, $x \in A \cup B \Rightarrow x \in A \text{ or } x \in B$.

And, $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$.

It is evident from definition that $A \subseteq A \cup B; B \subseteq A \cup B$

SOLVED EXAMPLES

(i) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$

(ii) $A = \{1, 3, 5\}$, $B = \{1, 2, 3\}$

Solution :- (i) We have, $A \cup B = \{a, e, i, o, u\} \cup \{a, b, c\}$

$\Rightarrow A \cup B = \{a, b, c, e, i, o, u\}$

Here, the common element a has been taken only once, while writing $A \cup B$.

(ii) We have $A \cup B = \{1, 3, 5\} \cup \{1, 2, 3\} \Rightarrow A \cup B = \{1, 2, 3, 5\}$

Here, the common elements 1 and 3 have been taken only once, while writing $A \cup B$.

UNION OF THREE OR MORE SETS

The union of $n(n \geq 3)$ for sets A_1, A_2........., A_n is defined as the set of all those elements which are in A_i $(1 \leq i \leq n)$ for atleast one value of i. The union of A_1, A_2, A_3,........A_n is denoted $\bigcup_{i=1}^{n} A_i$ In symbols, we write

$\bigcup_{i=1}^{n} A_i = \{ x : x \in A_i$ for at least one value of i, $1 \leq i \leq n\}$

SOLVED EXAMPLE

Example :- If A = {1, 2, 3, 4} B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}, find

(i) $A \cup B$ (ii) $A \cup B \cup C$ (iii) $B \cup C \cup D$

Solution :- (i) We have, $A \cup B = \{1,2,3,4\} \cup \{3,4,5,6\}$

$\Rightarrow A \cup B = \{1,2,3,4,5,6\}$

(ii) We have, $A \cup B \cup C = \{1,2,3,4\} \cup \{3,4,5,6\} \cup \{5,6,7,8\}$

$\Rightarrow A \cup B \cup C = \{1,2,3,4,5,6,7,8\}$

(iii) We have, $B \cup C \cup D = \{3,4,5,6\} \cup \{5,6,7,8\} \cup \{7,8,9,10\}$

$\Rightarrow B \cup C \cup D = \{3,4,5,6,7,8,9,10\}$

INTERSECTION OF SETS

The intersection of two sets A and B is the set of all those elements which belong to both A and B.

Symbolically, we write $A \cap B$ = {x : $x \in A$ and $x \in B$} and read as "A intersection B".

Let $x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$ and $x \notin A \cap B \Leftrightarrow x \notin A$ or $x \notin B$

It is evident from the definition that $A \cap B \subseteq A$, $A \cap B \subseteq B$

SOLVED EXAMPLES

Example 1 : (i) If A = {3,5,7,9,11}, B = {7,9,11,13}, find $A \cap B$.

(ii) If A = {a, b, c}, B = ϕ, find $A \cap B$.

Solution. (i) We have, $A \cap B$ = {3,5,7,9,11}$\cap${7,9,11,13}

Since, 7,9,11 are the only elements which are common to both the sets A and B.

$\Rightarrow A \cap B = \{7,9,11\}$

(ii) We have, $A \cap B = \{a,b,c\} \cap \phi = \phi$. Since, there is no common element.

Example 3. Let A = {x : x is a natural number} and B = { x : x is an even natural number}.

Find $A \cap B$

Solutions : We have, A = { x : x is a natural number}. A = {1, 2, 3, 4,......}

B = {x : x is an even natural number}. B = {2, 4, 6,.......}

We observe that 2, 4, 6,..... are the elements which are common to both the sets A and B. Hence, $A \cap B$ = {2, 4, 6.......} = B.

INTERSECTION OF MORE SETS

The intersection of $n(n \geq 3)$ sets $A_1, A_2, A_3 \ldots\ldots\ldots A_n$ is defined as the set of all those elements which are in $A_i (1 \leq i \leq n)$ for each i.

The intersection of $A_1, A_2, \ldots\ldots, A_n$ is denoted by $\bigcap_{i=1}^{n} A_i$ =

In symbols, we write, $\bigcap_{i=1}^{n} A_i = \{ x : x \in A_i \text{ for all } i, 1 \le i \le n \}$

DISJOINT SETS : Two sets A and B are said to be disjoint, if $A \cap B = \phi$. If $A \cap B \neq \phi$, then A and B are said to be intersecting sets or overlapping sets. e.g. Let A = {1, 2, 3}, B = {a, b, c} $\Rightarrow A \cap B = \phi$, hence A and B are disjoint.

Solved example :

1. Which of the following pairs of sets are disjoint ?

   i) A = {1, 2, 3, 4}, B = { x : x is a natural number and $4 \le x \le 6$ }.

   ii) A = { a, e, i, o, u}, B= { c, d, e, f}

   iii) A = {x : x is an even integer}, B = {x : x is an odd integer}

Solution :-

i) We have, A = {1, 2, 3, 4}; B = {x : x is a natural number and $4 \le x \le 6$ }

$\Rightarrow$ B = { 4, 5, 6 } $\Rightarrow A \cap B$ = { 1, 2, 3, 4 } $\cap$ { 4, 5, 6 } = { 4 }

Here, $A \cap B \neq \phi$, hence A and B are not disjoint sets, but are intersecting sets.

ii) We have, $A \cap B$ = { a, e, i, o, u } $\cap$ { c, d, e, f } = { e }

Here, $A \cap B \neq \phi$, hence A and B are not disjoint sets but are intersecting sets.

iii) We have, A = { x : x is an even integer} = { ..... – 2, 0, 2, ...}

and B = { x : x is odd integer } = { ...–3, –1, 1, 3, ...}

Hence, $A \cap B = \phi$, hence A and B are disjoint sets

WORKSHEET - 2

SINGLE ANSWER TYPE

1. Let 'n' be the Cardinal number of a set A then the no. of subsets of A is

   1) $2^n - 1$ 2) 2^n 3) 2n 4) n

2. Which of the following is false ?

   1) Every set is a subset of itself. 2) Empty set is a subset of everyset.

   3) If $A = \{1, 2, 3,\}$ then $\{1\} \subset A$ 4) $\phi \in \{ \}$

3. If $A = \{a, e, i, o, u\}$ then the no. of proper subsets of A is

   1) 32 2) 31 3) 16 4) 4

4. A set Contains n elements. Then the no. of elements in power set

1) $2^n - 1$ 2) 2^n 3) 2n 4) n

5. If $A \subset B$, Then $A \cup B =$

1) A 2) B 3) ϕ 4) $A \cap B$

6. If $A = \phi$, then $A \cap B$

1) A 2) B 3) ϕ 4) $A \cup B$

7. If A, B are disjoint sets then

1) $A \cap B = \mu$ 2) $A \cap B = A \cup B$ 3) $A \cap B = \phi$ 4) $A \cup B = A$

8. If $A = \{1,2,3\}$, $B = \{1,2,3,4,5\}$ and $C = \{1,2,3,4,5,6\}$ then $(A \cup B) \cap C =$

1) A 2) B 3) C 4) $A \cap B$

9. A = set of all vowels then which of the following is correct ?

1) $a \subset A$ 2) $\{a,i,o,u\} > A$ 3) $a\{i,o,u\} \in A$ 4) $\{a,o\} \subset A$

10. If $A = \{1,2,5,7,11,\}$ and $B = \{3,5,11,13,7\}$ then $A \cup B$ =

1) $\{1,2,3,5,7,11,13\}$ 2) $\{5,7,11\}$ 3) $\{1,2,3,13\}$ 4) $\{3,5,7,11\}$

11. If $P = \{x : x = 2n, n \in N\}$, $Q = \{x : x = 3n, n \in N\}$, then $A \cap B =$

1) 5n 2) 6n 3) $6n^2$ 4) n

12. If $A = \{x: \ x = 4n+1, \ 2 \le n \le 5\}$, then the numbers of subset of A is

1) 16 2) 8 3) 4 4) 15

13. If $A = \{x: \ x = n^2 + 1, \ n < 4, \ n \in N\}$ then the number of proper subset of A

1) 3 2) 8 3) 7 4) 15

14. If $A = \{2,4,6,8\}$, $B = \{3,5,6,7\}$, $C = \{1,2,3,4,5,6,7,8\}$ then $(A \cap B) \cap C$

1) $A \cap B$ 2) A 3) B 4) C

15. $A = \{x/x = n+1, \ n \in N, \ 1 < n < 6\}$, $B = \{x: \ x = n^2 + 1, \ n \in N, \ n < 3\}$ then $A \cap B =$

1) $\{2,3,4,5,6\}$ 2) $\{2,5\}$ 3) $\{3,4,6\}$ 4) $\{5\}$

MULTI ANSWER TYPE

16. If A, B are two non-empty sets and $A \subset B$ then

1) $A \cup B = B$ 2) $A \cap B = A$ 3) $A \cup B = A$ 4) $A \cap B = B$

17. If $A=\{T,E,C,H,N,O\}$ then

1) The no. of subsets of A is 64
2) The no. of non-empty proper subsets of A is 62
3) $n(P(A))=64$
4) $n(A)=6$

18. Let $A=\{-1,-3,2,5,9\}$, $B=\{2,4,9,11\}$, $C=\{-3,5,7,11\}$ Then

1) $(A\cup B)\cap C=\{-3,5,11\}$
2) $A\cap(B\cup C)=\{-3,2,5,9\}$
3) $(A\cap B)\cap C=\phi$
4) $A\cap(B\cap C)=\phi$

19. If $A=\{1,2\}$ then which of following is true ?

1) $\{1\}\subset P(A)$ 2) $A\in P(A)$ 3) $\{1\}\in P(A)$ 4) $\phi\subset P(A)$

20. If $A=\{x:x=2n,n\in N\}$, $B=\{x:x=2n-1,n\in N\}$ then

1) $A\cup B=N$ 2) $A\cap B=\phi$ 3) $A\cup B=R$ 4) $A\cup B=Q$

COMPREHENSION TYPE

Writeup-1

$A\cup B=\{x:x\in A \text{ or } x\in B\}$

21. If $A=\{x:\ x=2n-1,\ n\le 5,\ n\in N\}$ and

$n(A\cap B\cap C)=4$ then $A\cup B$ is

1) $\{1,3,5,7,9,11\}$ 2) $\{-1,0,3,5,7,9,11\}$ 3) $\{-1,1,3,5,7,9,11\}$ 4) $\{-1,3,7,11,13\}$

22. If $A=\{x:\ -3\le x\le 3,\ x\in z\}$, $B=\{x:\ 0\le x\le 3,\ x\in W\}$ and

$C=\{x:\ 1\le x\le 3,\ x\in N\}$ then which of the following is false

1) $n(A\cup C)=7$ 2) $n(B\cup C)=4$ 3) $n(A\cup(B\cup C))=7$ 4) $n(A\cap(B\cap C))=4$

23. The smallest set 'X' such that $X\cup\{1,3\}=\{1,3,5,7,9\}$ then X is

1) $\{1,3,5,7,9\}$ 2) $\{3,5,7,9\}$ 3) $\{1,5,7,9\}$ 4) $\{5,7,9\}$

Writeup-2

$A\cap B=\{x:\ x\in A\ \ and\ \ x\in B\}$

24. A= Set of natural numbers, B = Set of integers and C = Set of real numbers then $A\cap(B\cap C)$ is

1) N 2) W 3) z 4) R

25. If $A \cap \{1,3,5,7\} = \{1,3,5\}$ then the smallest possible set for A is

1) $\{3,5\}$ 2) $\{1,3,5\}$ 3) $\{3,5,7\}$ 4) $\{1,3,5,7\}$

26. If $n(A) = 6$ and $n(B) = 9$ then the maximum number of elements in $A \cap B$ is

1) 15 2) 9 3) 6 4) 3

MATRIX MATCHING TYPE

27. If A, B are any two non-empty sets and $n(A) = m, n(A) = n$ if $m > n$

Column - I	Column - II
1) Maximum value of $n(A \cup B)$	p) o
2) Minimum value of $n(A \cup B)$	q) m
3) Maximum value of $n(A \cap B)$	r) n
4) Minimum value of $n(A \cap B)$	s) $m+n$
	t) $m-n$

28. If $A = \{M, A, T, H, S\}$

Column - I	Column - II
1) No. of Subsets of A	p) 2^0
2) No. of proper subsets of A	q) 2^n
3) No. of elements in powerset of A	r) $\frac{2^n}{2}$
4) No. of improper subsets of A	s) 4.2^{n-2}
	t) 8.2^{n-4}

29. If $A = \{x: \ x = 2n+1, \ n \in W\}$ and $B = \{x: \ x = 2n, \ n \in N\}$ then $n(A \cap B) =$ ________

30. If A = letters from the word 'EAMCET' abd B = letters from the word "AIEEE" then $n(A \cup B) =$ ________

VERBAL REASONANING

Directions : In each of the following questions, a number series is given with one term missing. Choose the correct alternative that will continue the same pattern and replace

1) T, R, P, N, L, ?, ?

(a) J, G (b) J, H (c) K, H (d) K, I

2) U, B, I, P, W, ?
(a) D (b) F (c) Q
(d) Z

3) Z, ?, T, ?, N, ?, H, ?, B
(a) W, Q, K, E (b) W,R,K,E (c) X,Q,K,E (d) X,R,K,E

4) a,d,c,f, ?, h,g,?,i
(a) e,j (b) e,k (c) f,j
(d) j,e

5) A, I, P, V, A, E, ?
(a) E (b) F (c) G
(d) H

6) Z, W, S, P, L I, E, ?
(a) B (b) D (c) F
(d) K

7) Y, W, T, P, K, E, X, ?, ?
(a) G, H (b) P,G (c) R,G
(d) S,R

8) A, B, N, C, D, O, E, F, P, ?, ?, ?
(a) G, H, I (b) G, H, J (c) G, H, Q (d) J, K, L

9) Y, B, T, G, O, ?
(a) N (b) M (c) L
(d) K

10) M, N, O, L, R, I, V, ?
(a) A (b) E (c) F
(d) H (e) Z

11) b e d f ? h j ? l
(a) I m (b) m I (c) i n
(d) j m

12) AI, BJ, CK, ?
(a) DL (b) DM (c) GH
(d) LM

13) GH, JL, NQ, SW, YD, ?
(a) EJ (b) FJ (c) EL
(d) FL

14) DF, GJ, KM, NQ, RT, ?
(a) UW (b) YZ (c) XZ
(d) UX (e) YA

15) PMT, OOS, NQR, MSQ, ?
(a) LUP (b) LVP (c) LVR (d) LWP

16) BZAM DYC, FXE, ?, JVI
(a) HUG (b) HWG (c) UHG (d) WHG

17) DHL, PTX, BFJ, ?
(a) CGK (b) KOS (c) NRV (d) RVZ

18) AZY, BUT, CXW, DWV, ?
(a) EVA (b) EVU (c) VEU (d) VUE

19) DEF, HIJ, MNO, ?
(a) STU (b) RST (c) SIJ
(d) SRQ

20) ejo tyd ins xch ?
(a) nrw (b) mrw (c) msx (d) nsx

21) AYBZC, DWEXF, GUHVI, JSKTL, ?
(a) MQORN (b) MQNRO (c) NQMOR (d) QMONR

22) ATTRIBUTION, TIRIBUTIO, RIBUTIO, IBUTI, ?
(a) IBU (b) UT (c) UTI (d) BUT

23) Consider the following series :
(a) ABCD.....XYZ|YX.....BA|BCD....YZ|YX....CBA|BC....YZ.... Which letter occupies the 1000th position in the above series ?
(a) B (b) C (c) X
(d) Y

KEY & HINTS

WORK SHEET – 1 (KEY)				
1) 4	2) 3	3) 4	4) 3	5) 1
6) 2	7) 2	8) 4	9) 4	10) 1
11) 2	12) 2	13) 3	14) 2	15) 2
16) 1,3,4	17) 1,2,3,4	18) 1,2,3	19) 2,4	20) 1,2,3,4
21) 2	22) 3	23) 4	24) 2	25) 4
26) 1	27) 1	28) 1	29) 1-Q 2-PR 3-PST 4-P	30) 1-QR 2-QS 3-T 4-P
31) 3	32) 4	33) 5	34) 1	

6. By using dens property (i.e. in between any two consecutine real numbers there are infinitly many real numbers.)

7. $x+1<12$

$x<11$

$\therefore A=\{1,2,3,....10\} \Rightarrow n(A)=10$ Ans : (2)

8. $\{M,A,T,H,E,M,A,T,I,C,S\}$

Ans : (4)

10. For any x, $x^2+1>0$ Ans : (1)

12. By Verification

Ans : (2)

14. $n(A) = n(B) = 4$

Both having same elements.

Ans : (2)

15. $A=\{-1,0,1\}$

$n(A)=3$, $n(B)=3$

16. For $x \in R$, $x^2+1 \neq 0$ Ans : (1), (3), (4)

17. Ans : 1, 2, 3, 4

18. $A=\{1,2,3,4\}$ $B=\{x<5, x \in N\}$

$\therefore A=B$

Every equal set is equivalent also

Ans : 1, 2, 3

19. By Definition

20. 1, 2, 3, 4

21. $A=\{5,9,13,17,21\}$

$\therefore A=\{4n+1/n \leq 5, n \in N\}$

prime numbers of $A=\{5,13,17\}$

Composit numbers = $\{9,21\}$

22. $A=\{5,9,13,17,21\}$

$\therefore A=\{4n+1/n \leq 5, n \in N\}$

prime numbers of $A=\{5,13,17\}$

Composit numbers = $\{9,21\}$

23. $A=\{5,9,13,17,21\}$

$\therefore A=\{4n+1/n\le 5, n\in N\}$

prime numbers of $A=\{5,13,17\}$

Composit numbers = $\{9,21\}$

24. $A=\{1,3,5,7,9\}$

$A=\{x: \quad x=2n+1, \quad x\in W, \quad n<5\}$

$x:\{x=2n-1, \quad x\in N, \quad n\le 5\}$

25. $A=\{1,4,9,16,25\}$

$A=\{x: \quad x=n^2, \quad n\in N, \quad n\le 5\}$

or

$=\{x: \quad x=(n+1)^2, \quad n\in W, \quad n<5\}$

26. $A=\{2,9,28,65\}$

$A=\{x: \quad x=n^3+1, \quad n\in N, \quad n\le 4\}$

27. (1)
28. (1)

29. (1) $\to$ q (2) $\to$ p , r (3) $\to$ p, s, t (4) $\to$ p
30. (1) $\to$ q, r (2) $\to$ q, s (3) $\to$ t, (4) $\to$ p
31. 3
32. 4
33. 5
34. 1

W0RK SHEET – 2 (KEY)				
1) 2	2) 4	3) 2	4) 2	5) 2
6) 3	7) 3	8) 2	9) 4	10) 1
11) 2	12) 1	13) 3	14) 1	15) 4
16) 1,2	17) 1,2,3,4	18) 1,2,3,4	19) 2,3,4	20) 1,2
21) 3	22) 4	23) 4	24) 1	25) 2
26) 3	27) 1-S 2-Q 3-R 4-P	28) 1-QS 2-RT 3-QS 4-P	29) 0	30) 6

1. 2^n
   Ans : (2)
2. Since '$\in$' using relation between element and set.
   But $\phi, \{\ \}$ both are sets.
3. $2^n - 1 \ = \ 2^5 - 1 \ = 31$
   Ans : (2)
4. 2^n
   Ans : (2)
5. If $A \subset B \Rightarrow A \cup B = B$ (Greatest set)
   Ans : (2)
6. $A \cap B = \phi$ Since ϕ is subset of every set.
7. By definition
   Ans : (3)
8. $A \cup B = \{1, 2, 3, 4, 5\}$
   $\therefore (A \cup B) \cup C = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5, 6\} = \{1, 2, 3, 4, 5\} = B$
   Ans : (2)
9. Ans : (4)
10. Ans : (1)

11. $P=\{2,4,6,8,....\}$ $Q=\{3,6,9,12,....\}$

$\therefore P\cap Q=\{6,12,18,...\}$

$6n$, $n\in N$

Ans : (2)

12. $A=\{x: \quad x=4n+1, \quad 2\le n\le 5\}$

$\therefore A=\{9,13,17,21\}$

$\therefore$ No. of subsets of A is $2^4=16$

Ans : (1)

13. $A=\{x: \quad x=n^2+1, \quad n<4, \quad n\in N\}$

$\therefore A=\{2,5,10\}$

$\therefore$ No. of proper subsets of $A=2^3-1=7$

Ans : (3)

14. $A=\{2,4,6,8\}$ $B=\{3,5,6,7\}$

$\therefore A\cap B=\phi$

$(A\cap B)\cap C=\phi\cap C=\phi=A\cap B$

Ans : (1)

15. $A=\{x/x=n+1,n\in N,1<n<6\}$ $B=\{x:x=n^2+1,n\in N,n<3\}$

$A=\{3,4,5,6\}$ $B=\{2,5\}$

$\therefore A\cap B=\{5\}$

Ans :1,2

16. Properties

Ans : 1,2

17. $A=\{T,E,C,H,N,O\}$

$n(A)=6$

$\therefore$ no. of subsets $=2^6=64$

non-empty proper subsets $=2^n-2=64-2=62$

No. of elements in power ser $=2^n=64$

Ans : 1, 2, 3, 4

18. $A=\{-1,-3,2,5,9,\}$, $B=\{2,4,9,11\}$, $C=\{-3,5,7,11\}$

$A\cup B=\{-1,-3,2,4,5,9,11\}$

$(A\cup B)=\{-3,5,11\}$

$B\cup C=\{-3,2,4,5,7,9,11\}$

$\therefore A\cap(B\cup C)=\{-3,2,5,9\}$

$A\cap B=\{2,9\}$

$(A\cap B)\cap C=\phi$

Ans : 1,2 3, 4

19. $A=\{1,2\}$

$P(A)=\{\phi,\{1\},\{2\},\{1,2\}\}$

$\therefore \{1\}\in P(A)$; $A=\{1,2\}\in P(A)$

$\phi\subset P(A)$ Since ϕ is subset of every set.

Ans : 2,3,4

20. A= Set of even numbers $=\{2,4,6,8....\}$

B= Set of odd number $=\{1,3,5,7....\}$

$\therefore A\cup B=\{1,3,5,7...\}$

$A\cap B=\phi$

Ans : (1), (2)

21. $A=\{x: \quad x=2n-1, \quad n\le 5, \quad n\in N\}$

$A=\{1,3,5,7,9\}$

$B=\{x: \quad x=4n-1, \quad n\le 3, \quad n\in W\}$

$B=\{-1,3,7,11\}$

$\therefore A\cup B=\{-1,1,3,5,7,9,11\}$

Ans : (3)

22. $A=\{x:-3\le x\le 3, x\in z\}=\{-3,-2,-1,0,1,2,3\}$

$B=\{x:0\le x\le 3, x\in W\}=\{0,1,2,3\}$

$C=\{x:1\le x\le 3, x\in N\}=\{1,2,3\}$

$\therefore n(A\cup C)=7,\quad n(B\cup C)=4,\quad n(A\cup(B\cup C))=7,\quad n(A\cap(B\cap C))=3$

Ans : (4)

23. $X\cup\{1,3\}=\{1,3,5,7,9\}$

$\therefore$ Smallest possibility of $X=\{3,7,9\}$

Ans : (4)

24. $N\subset z\subset R$

$\therefore A\cap(B\cap C)=N$

Ans : (1)

25. $A\cap\{1,3,5,7\}=\{1,3,5\}$

$\therefore A=\{1,3,5\}$

Ans : (2)

26. $n(A)=6,\quad n(B)=9$

Maximum no. of elements in $A\cap B=6$

Ans : (3)

27. $(1)\to s,\quad (2)\to q\quad (3)\to r\quad (4)\to p$

28. $(1)\to q,s\quad (2)\to r,\quad (3)\to q,s\quad (4)\to p$

29. $A=\{1,3,5,7...\}$

$B=\{2,4,6,8,...\}$

$n(A\cap B)=0$

30. $A=\{E,A,M,C,T\}$

$B=\{A,I,E\}$

$A\cup B=\{A,E,C,I,M,T\}$

$\therefore n(A\cup B)=6$

VERBAL REASONING (KEY)				
1) B	2) A	3) A	4) A	5) D
6) A	7) B	8) C	9) C	10) B
11) A	12) A	13) D	14) D	15) A
16) B	17) C	18) B	19) A	20) B
21) B	22) C	23) A		

1. (b): $a \xrightarrow{+3} b \xrightarrow{-1} c \xrightarrow{+3} f \xrightarrow{-1} (e) \xrightarrow{+3} h \xrightarrow{-1} g \xrightarrow{+3} (j) \xrightarrow{-1} i$
2. (a): $U \xrightarrow{+7} B \xrightarrow{+7} I \xrightarrow{+7} P \xrightarrow{+7} W \xrightarrow{+7}$
3. (a): $Z \xrightarrow{-6} T \xrightarrow{-6} N \xrightarrow{-6} H \xrightarrow{-6} B$ \

$$Z \xrightarrow{-3} (W) \xrightarrow{-3} T \xrightarrow{-3} (Q) \xrightarrow{-3} N \xrightarrow{-3} (K) \xrightarrow{-3} H \xrightarrow{-3} (E) \xrightarrow{-3} B$$

4. (a): $a \xrightarrow{+3} b \xrightarrow{-1} c \xrightarrow{+3} f \xrightarrow{-1} (e) \xrightarrow{+3} h \xrightarrow{-1} g \xrightarrow{+3} (j) \xrightarrow{-1} i$
5. (d): $A: \xrightarrow{+8} I \xrightarrow{+7} P \xrightarrow{+6} V \xrightarrow{+5} A \xrightarrow{+4} E \xrightarrow{+3} (H)$
6. (a): $Z \xrightarrow{-3} W \xrightarrow{-4} S \xrightarrow{-3} P \xrightarrow{-4} L \xrightarrow{-3} I \xrightarrow{-4} E \xrightarrow{-3} (B)$
7. (b): $Y \xrightarrow{-2} W \xrightarrow{-3} T \xrightarrow{-4} P \xrightarrow{-5} K \xrightarrow{-6} E \xrightarrow{-7} X \xrightarrow{-7} (P) \xrightarrow{-9} (G)$
8. (c): The given series may be divided into 2 groups :
I. A, B, C, D, E, F, ?, ? and II. N, O, P, ?
Clearly, the given series consists of two terms of I followed by one
term of II.
The missing terms in I are G and H while the missing term in II is Q.
9. (c): The given sequence is a combination of two series :
I. Y, T, O and II. B, G, ?
10. (b): The given sequence is a combination of two series :
I. M, O, R, V and II. N, L, I, ?
The pattern in I is : $M \xrightarrow{+2} O \xrightarrow{+3} R \xrightarrow{+4} V$
The pattern in II is : $N \xrightarrow{-2} L \xrightarrow{-3} I \xrightarrow{-4} (E)$
So, the missing letter is E.

11. (a): The series may be divided into groups as shown :
b e d / f ? h / j ? l
Clearly in the first group, the second and third letters are re-spectively
three and two steps ahead
of the first letter. A similar pattern would follow in the second and third
groups.

12. (a): 1st letter : $A \xrightarrow{+1} B \xrightarrow{+1} C \xrightarrow{+1} (D)$

2nd letter : $I \xrightarrow{+1} J \xrightarrow{+1} K \xrightarrow{+1} (L)$

13. (d): 1st letter : $G \xrightarrow{+3} J \xrightarrow{+4} N \xrightarrow{+5} S \xrightarrow{+6} Y \xrightarrow{+7} (F)$

2nd letter : $H \xrightarrow{+4} L \xrightarrow{+5} Q \xrightarrow{+6} W \xrightarrow{+7} D \xrightarrow{+8} (L)$

14. (d): 1st letter : $D \xrightarrow{+3} G \xrightarrow{+4} K \xrightarrow{+3} N \xrightarrow{+4} R \xrightarrow{+3} (U)$

2nd letter : $F \xrightarrow{+4} J \xrightarrow{+3} M \xrightarrow{+4} Q \xrightarrow{+3} T \xrightarrow{+4} (X)$

15. (a): 1st letter : $P \xrightarrow{-1} O \xrightarrow{-1} N \xrightarrow{-1} M \xrightarrow{-1} (L)$

2nd letter : $M \xrightarrow{+2} O \xrightarrow{+2} Q \xrightarrow{+2} S \xrightarrow{+2} (U)$

3rd letter : $T \xrightarrow{-1} S \xrightarrow{-1} R \xrightarrow{-1} Q \xrightarrow{-1} (P)$

16. (b): 1st letter : $B \xrightarrow{+2} D \xrightarrow{+2} F \xrightarrow{+2} (H) \xrightarrow{+2} J$

2nd letter : $Z \xrightarrow{-1} Y \xrightarrow{-1} X \xrightarrow{-1} (W) \xrightarrow{-1} V$

3rd letter : $A \xrightarrow{+2} C \xrightarrow{+2} E \xrightarrow{+2} (G) \xrightarrow{+2} I$

17. (c): 1st letter : $D \xrightarrow{+12} P \xrightarrow{+12} B \xrightarrow{+12} (N)$

2nd letter : $H \xrightarrow{+12} T \xrightarrow{+12} F \xrightarrow{+12} (R)$

3rd letter : $L \xrightarrow{+12} X \xrightarrow{+12} J \xrightarrow{+12} (V)$

18. (b): 1st letter : $A \xrightarrow{+1} B \xrightarrow{+1} C \xrightarrow{+1} D \xrightarrow{+1} (E)$

2nd letter : $Z \xrightarrow{-5} U \xrightarrow{+(5-2)=+3} X \xrightarrow{-(-3-2)=-1} W \xrightarrow{+(1-2)=-1} (V)$

3rd letter : $Y \xrightarrow{-5} T \xrightarrow{+(5-2)=+3} W \xrightarrow{-(3-2)=-1} V \xrightarrow{+(1-2)=-1} (U)$

19. (a): 1st letter : $D \xrightarrow{+4} H \xrightarrow{+5} M \xrightarrow{+6} (S)$

2nd letter : $E \xrightarrow{+4} I \xrightarrow{+5} N \xrightarrow{+6} (T)$

3rd letter : $F \xrightarrow{+4} J \xrightarrow{+5} O \xrightarrow{+6} (U)$

20. (b): There is a gap of four letters between the first and second, the second and third letters of each term, and also between the last letter of a term and the first letter of the next term.

21. (b): 1st letter : $A \xrightarrow{+3} D \xrightarrow{+3} G \xrightarrow{+3} J \xrightarrow{+3} (M)$

2nd letter : $Y \xrightarrow{-2} W \xrightarrow{-2} U \xrightarrow{-2} S \xrightarrow{-2} (Q)$

3rd letter : $B \xrightarrow{+3} E \xrightarrow{+3} H \xrightarrow{+3} K \xrightarrow{+3} (N)$

4th letter : $Z \xrightarrow{-2} X \xrightarrow{-2} V \xrightarrow{-2} T \xrightarrow{-2} (R)$

5th letter : $C \xrightarrow{+3} F \xrightarrow{+3} I \xrightarrow{+3} L \xrightarrow{+3} (O)$

22. (c): In the first step, one letter from the beginning and one from the end of a term are removed to give the next term. In the second step, two letters from the beginning of a term are removed. These two steps are repeated alternately.

23. (a): We have 3 patterns :

I. ABCD ..... XYZ, which occurs only once.
II. YX ...... BA, which repeats alternately.
III. BC ..... YZ, which repeats alternately.

Now, I has 26 terms.

So, number of terms before the desired term $=(999-269)=973.$

Each of the patterns which occurs after I, has 25 letters.

Now, $973 \div 25$ gives quotient $=38$ and remainder $=23$.

Thus, the 1000th term of given series is the 24th term of the 39th pattern after I. Cleary, the 39th pattern is II and its 24th term is B.