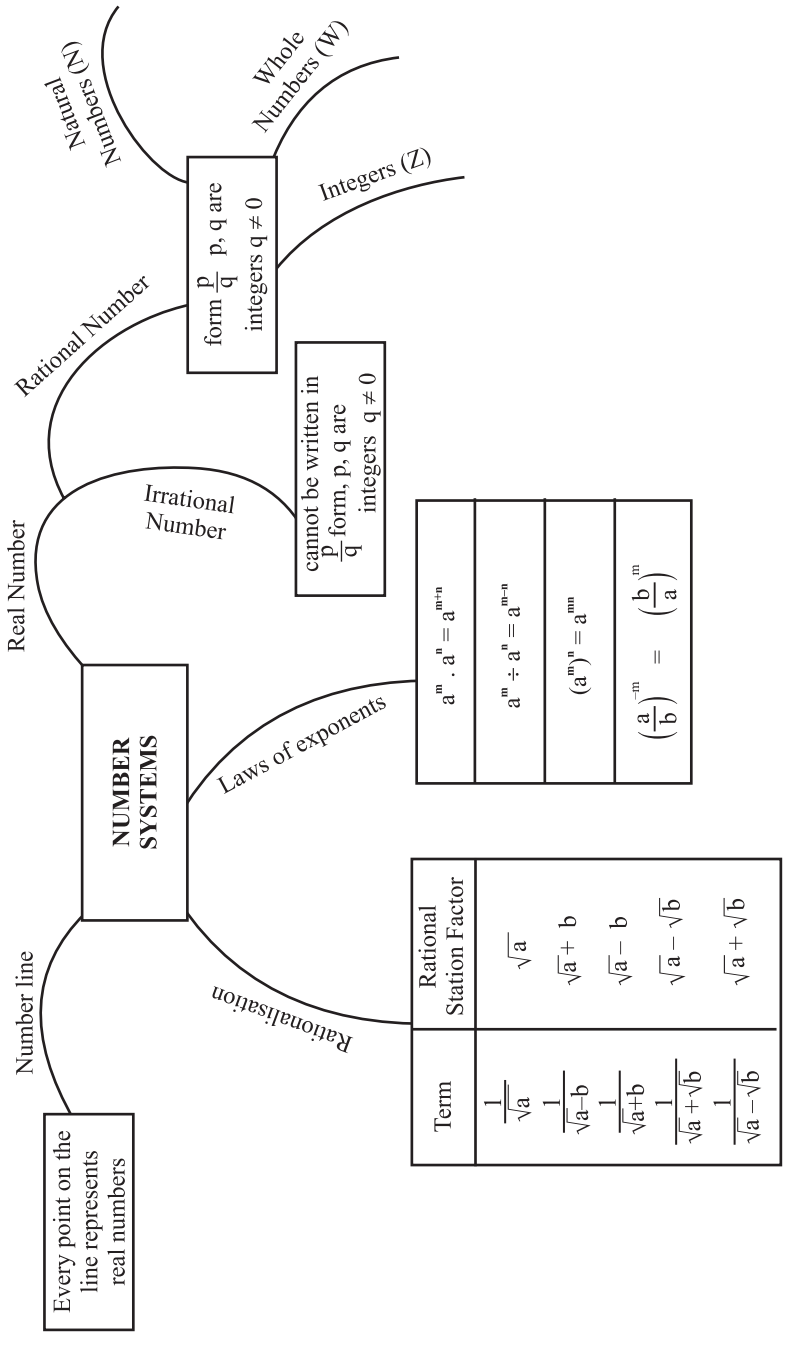
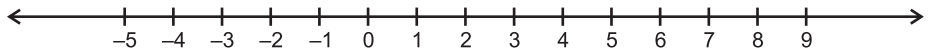
# CHAPTER-1
# NUMBER SYSTEMS
## MIND MAP



# CHAPTER-1
# NUMBER SYSTEMS
## KEY POINTS




- 1, 2, 3, ..... are natural numbers which are represented by N.
- 0, 1, 2, 3, ..... are whole numbers which are represented by W.
- ..... –3, –2, –1, 0, 1, 2, 3, ..... are Integers which are represented by Z or I.
- A number is a rational number if
  (a) it can be represented in the form of p/q, where p and q are integers and $q \neq 0$.

  or

  (b) its decimal expansion is terminating (e.g. $\frac{2}{5} = 0.4$)

  or

  (c) its decimal expansion is non-terminating recurring (repeating) (e.g. $0.1\overline{234} = 0.1234234.....$)
- A number is irrational number if
  (a) it can not be represented in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

  or

  (b) its decimal expansion is non-terminating non-recurring (e.g. 0.1010010001.....)
- All rational and irrational numbers collectively form real numbers.
- There are infinite rational numbers between any two rational numbers.
- There is a unique real number corresponding to every point on the number line. Also, corresponding to each real number, there is a unique point on the number line.
- Rationalisation of a denominator means to change the Irrational denominator to rational form.
- To rationalise the denominator of $\frac{1}{\sqrt{a} + b}$, We multiply this by $\frac{\sqrt{a} - b}{\sqrt{a} - b}$, where a is a natural number and b is an integer.

- Laws of Exponents : Let a > 0 be a real number and m and n are rational numbers, then

1) $a^m \, a^n = a^{m+n}$ 2) $a^m \div a^n = a^{m-n}$

3) $(a^m)^n = a^{mn}$ 4) $a^m . b^m = (ab)^m$

5) $a^0 = 1$ 6) $a^{-m} = \frac{1}{a^m}$

- For positive real number a and b, the following Identities hold

1) $\sqrt{a} \, . \, \sqrt{b} = \sqrt{ab}$ 2) $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

3) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$ 4) $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$

5) $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

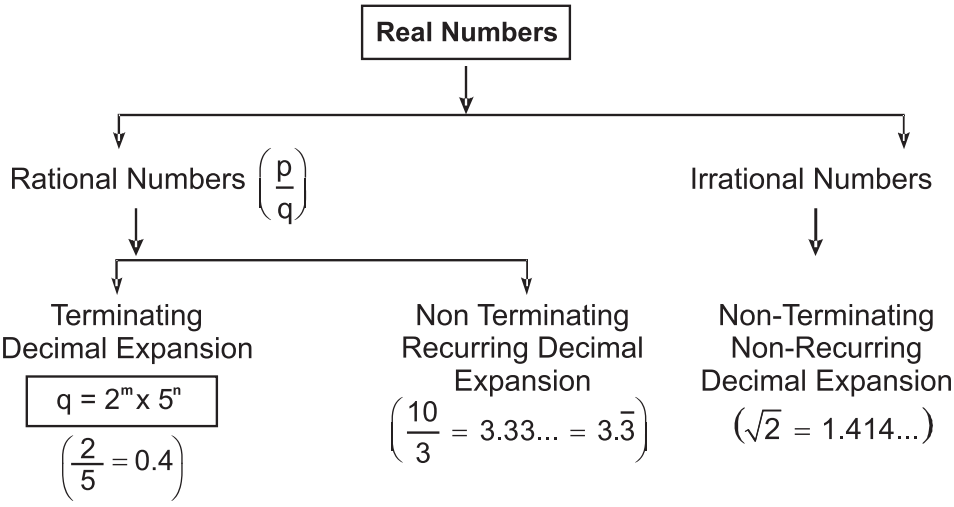
All natural numbers, whole numbers and integers are rational

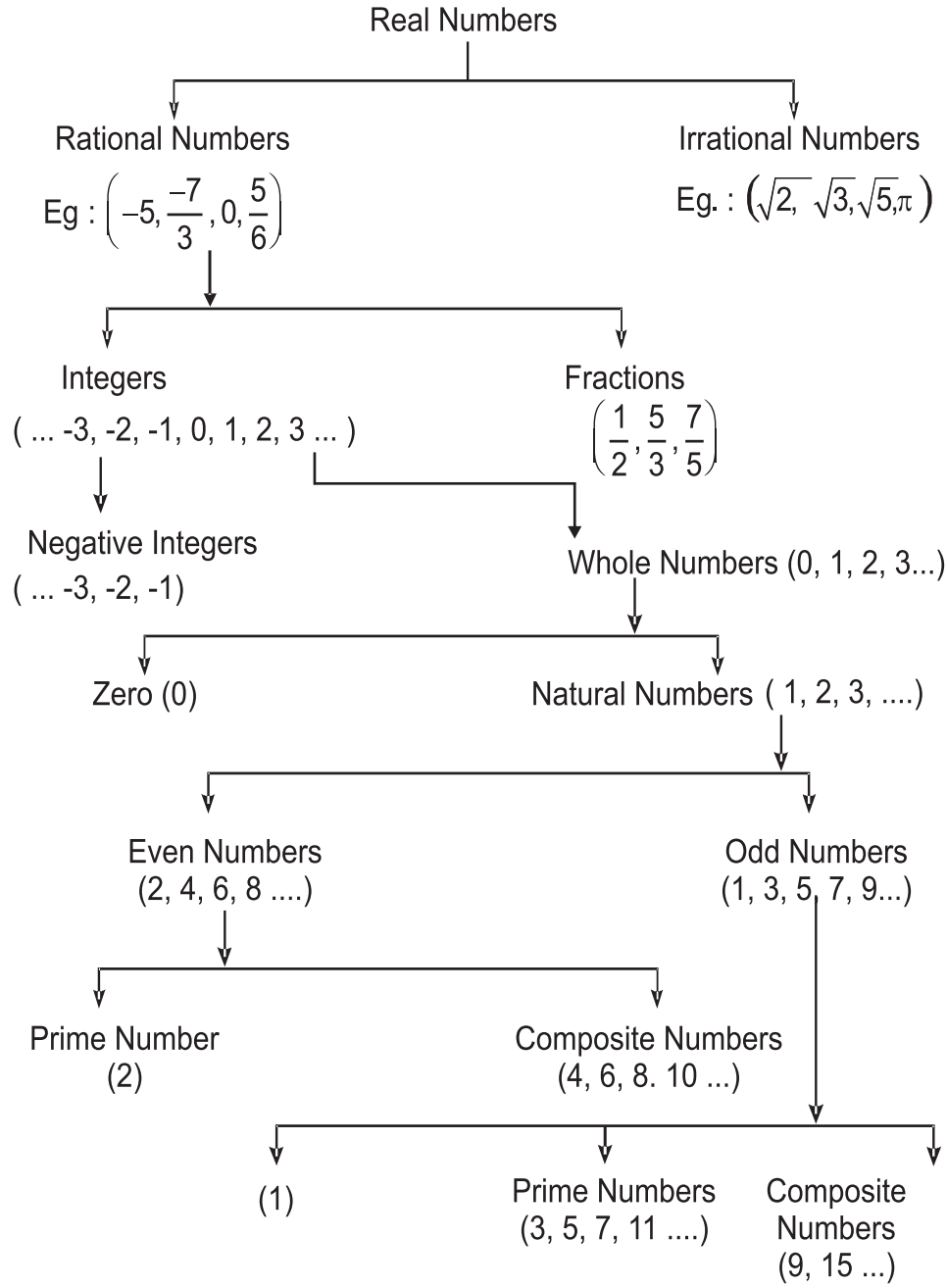
**Prime Numbers :** All natural numbers that have exactly two factors (i.e., 1 and itself) are called prime numbers. e.g., 2, 3, 5, 7, 11, 13, 17, 19, 23, … etc.

**Composite Numbers :** Those natural numbers which have more than two factors are known as composite numbers. e.g., 4, 6, 8, 10, 12, …

1 is neither prime nor composite.

**Types of Numbers**




Real Numbers
Rational Numbers
Eg : $\left(-5, \frac{-7}{3}, 0, \frac{5}{6}\right)$
Irrational Numbers
Eg. : $\left(\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi\right)$
Integers
( ... -3, -2, -1, 0, 1, 2, 3 ... )
Fractions
$\left(\frac{1}{2}, \frac{5}{3}, \frac{7}{5}\right)$
Negative Integers
( ... -3, -2, -1)
Whole Numbers (0, 1, 2, 3...)
Zero (0)
Natural Numbers ( 1, 2, 3, ....)
Even Numbers
(2, 4, 6, 8 ....)
Odd Numbers
(1, 3, 5, 7, 9...)
Prime Number
(2)
Composite Numbers
(4, 6, 8. 10 ...)
(1)
Prime Numbers
(3, 5, 7, 11 ....)
Composite
Numbers
(9, 15 ...)

* $\sqrt[n]{a} = a^{1/n}$

where 'a' is a positive real number and n is a positive integer.

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

where 'a' is a positive real number, m and n are co prime integers, and $n > 0$.

## PART - A

1. If x = 2 and y = 4, then $\left(\frac{x}{y}\right)^{x-y} + \left(\frac{y}{x}\right)^{y-x}$ = _____

   a) 4 b) 8

   c 12 d) 2

2. Which of the following is the greatest ?

   a) $4^2$ b) $(16)^{3/2}$

   c) $\left(\frac{1}{64}\right)^{-1/3}$ d) $(256)^{-1/4}$

3. $\frac{(32)^{0.2} + (81)^{0.25}}{(256)^{0.5} - (121)^{0.5}}$ = _____

   a) 2 b) 5

   c) 1 d) 11

4. $\frac{3}{7}$ line between _____

   a) $\frac{4}{9}, \frac{5}{9}$ b) $\frac{43}{99}, \frac{4}{9}$

   c) $\frac{42}{99}, \frac{4}{9}$ d) $\frac{41}{99}, \frac{41}{9}$

5. The number 0.318564318564318564..... is

   a) a natural number b) an integer

   c) a rational number d) an irrational number

6. The number $0.\overline{7}$ in the form $\frac{p}{q}$ , where p and q are integers and q≠0, is

   a) $\frac{77}{100}$ b) $\frac{7}{10}$

   c) $\frac{7}{9}$ d) $\frac{7}{100}$

7. The value of $0.\overline{23} + 0.\overline{22}$ is

   a) 0.45 b) $0.\overline{45}$

   c) $\frac{45}{99}$ d) both (B) and (C)

8. The value of $[3-4(3-4)^4]^3$, is

a) 1 b) $-1$

c) 0 d) 7

9. The cube root of 125 divided by square root of 25, is

a) 5 b) 1

c) 1/5 d) None of these

10. If $y^2 = 625$ then y is

a) a rational number b) an irrational number

c) neither rational nor irrational d) a natural number

11. $\sqrt[2]{(81)^{2.5}}$ = _____

a) 1/81 b) 81

c) 243 d) 343

12. The value of x, if $5^{x-3} \cdot 3^{2x-8} = 225$, is

a) 2 b) 3

c) 5 d) 7

13. If $a = 2 + \sqrt{3}$, then the value of $\frac{1}{a}$ is

a) $2+\sqrt{3}$ b) $2-\sqrt{3}$

c) $\sqrt{3}-2$ d) 1

14. The smallest natural number is

a) $-1$ b) 0

c) 1 d) 2

15. Which of the following is not a rational number ?

a) $\sqrt{2}$ b) $\sqrt{4}$

c) $\sqrt{9}$ d) $\sqrt{25}$

16. Choose the wrong statement :

a) Every natural number is a whole number.

b) Every integer is a rational number.

c) Every rational number is an integer.

d) Every rational number is a real number.

17. The decimal expansion of the number $\sqrt{3}$ is

a) a finite decimal
b) 1.732
c) non-terminating recurring
d) non-terminating non-recurring

18. Between two rational numbers

a) there is no rational number.
b) there is exactly one rational number.
c) there are infinitely many rational numbers.
d) there are only rational numbers and no irrational number.

19. Which of the following is an irrational number ?

a) $\sqrt{\frac{4}{9}}$
b) $\frac{\sqrt{12}}{\sqrt{3}}$
c) $\sqrt{7}$
d) $\sqrt{81}$

20. Every rational number is

a) a natural number
b) an integer
c) a real number
d) a whole number

21. $\sqrt{6} \times \sqrt{8}$ is equal to

a) $3\sqrt{4}$
b) $4\sqrt{3}$
c) $\sqrt{14}$
d) $6\sqrt{8}$

22. After rationalising the denominator of $\frac{3\sqrt{2}}{3\sqrt{2} - 2\sqrt{2}}$, we get the denominator as

a) 13
b) 5
c) 19
d) 35

23. Which of the following is equal to 'a' ?

a) $a^{\frac{10}{6}} - a^{\frac{4}{6}}$
b) $\sqrt[12]{(a^4)^{1/3}}$
c) $(\sqrt{a^3})^{\frac{2}{3}}$
d) $a^{\frac{12}{7}} \times a^{\frac{7}{12}}$

24. The product of any two irrational numbers is

a) always an irrational number.

b) always a rational number.

c) always an integer.

d) sometimes rational, sometimes irrational.

25. a rational number between $\sqrt{2}$ and $\sqrt{3}$ is

a) $\frac{\sqrt{2}+\sqrt{3}}{2}$ b) $\frac{\sqrt{2}\times\sqrt{3}}{2}$

c) 1.5 d) 1.8

**Fill in the blanks**

26. The sum of a rational and an irrational numbers is always _____ number.

27. The difference of a rational and an irrational number is always _____ number.

28. The decimal expansion of every rational number is either _____ or non - terminating _____ .

29. The decimal expansion of every irrational number is always _____ .

30. Every number whose decimal expansion is non-terminating non-recurring is _____ number.

31. Between two distinct rational numbers there lie _____ rational numbers.

32. Between two distinct rational numbers there lie _____ irrational numbers.

33. Between two distinct irrational numbers there lie _____ rational numbers.

34. The reciprocal of every (non-zero) rational number is a _____ number.

**State whether the following statements are true or false.**

35. Every integer is a whole number.

36. Every integer can be written in the form $\frac{p}{q}$ , where p, q are integers, $q \neq 0$.

37. Every real number is an irrational number.

38. There are infinitely many integers between any two integers.

39. The square of an irrational number is always a rational number.

40. Reciprocal of every rational number is a rational number.

41. Write first five whole numbers in $\frac{p}{q}$ form, where p and q are integers and $q \neq 0$

42 Find decimal expansion of $\frac{17}{8}, \frac{3}{15}, \frac{2}{7}, \frac{50}{3}$.

43. Find four rational numbers between $\frac{2}{9}$ and $\frac{3}{7}$.

44. Find decimal form of $\sqrt{23}$ and $\sqrt{24}$ upto 3 decimal places.

45. Find two Irrational numbers between $\sqrt{23}$ and $\sqrt{24}$.

46. Find one Irrational and one rational number between 2 and $\sqrt{5}$.

47. Write two numbers whose decimal expansions are terminating.

48. What can be the maximum number of digits in the repeating block of digits in the decimal expansion of $\frac{5}{7}$ ?

49. Write two numbers whose decimal expansions are non-terminating non-repeating (non-recurring).

50. Find the value of $(256)^{0.16} \times (256)^{0.09}$

51. Find two Irrational numbers between 2016 and 2017.

52. Represent $\frac{-7}{5}$ on the number line.

53. Represent following on number line

i) $\sqrt{5}$ ii) $\sqrt{3}$ iii) $\sqrt{2}$

54. Insert two Irrational numbers between $\frac{2}{3}$ and $\frac{3}{2}$

55. Simplify : $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{80} + \sqrt{48} - \sqrt{45} - \sqrt{27}}$

56. Find the value of $[1^3 + 2^3 + 3^3 + 8^2]^{-5/2}$

57. Find the value of x if $x^{1/2} = (36)^{0.5}$

58. Find the value of x if $(\sqrt{3})^x = 3^7$

59. If $2^{5x} \div 2^{x} = \sqrt[5]{32}$. Then find the value of x.

60. Evaluate $a^{x-y}.a^{y-z}.a^{z-x}$.

61. Simplify $12^{\frac{2}{5}} . 5^{\frac{2}{5}}$.

62. Which of the following rational numbers will have a terminating decimal expansion or a non-terminating repeating (recurring) decimal expansion ?

(i) $\frac{135}{50}$ (ii) $\frac{4}{11}$ (iii) $\frac{8}{7}$ (iv) $6\frac{3}{8}$

(v) $\frac{55}{9}$ (vi) $\frac{5^2 \times 3^3}{2 \times 5^3 \times 27}$ (vii) $\frac{51}{60}$.

63. Classify the following numbers as terminating decimal or non-terminating recurring decimal or non-terminating non-recurring decimal :

(i) 0.1666... (ii) 0.250 (iii) 1.01001000100001....

(iv) 0.27696 (v) 2.142857142857.... (vi) $0.\overline{3}$

(vii) 0.2359872785... (viii) 0.484848848.... (ix) 2.502500250002.....

(x) $4.\overline{123456789}$

Also classify these given numbers as Rational and Irrational numbers.

64. Classify the following numbers as rational or Irrational number :

(i) $\sqrt{27}$ (ii) $\sqrt{36}$ (iii) $\sqrt{5} \times \sqrt{125}$ (iv) $2\sqrt{3}$

(v) $\frac{7\sqrt{7}}{\sqrt{343}}$ (vi) $2 + \sqrt{21}$ (vii) $5 + 2\sqrt{23} - \left(\sqrt{25} + \sqrt{92}\right)$

(viii) $\frac{22}{7}$ (ix) $\pi$ (x) $\sqrt[3]{27}$

65. Express the following numbers in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) 0.0875 (ii) 2.123456789 (iii) 0.181818.....

(iv) $0.4\overline{37}$ (v) $3.65\overline{1}$

66. Do as directed :

(i) Add : $\sqrt{125} + 2\sqrt{27}$ and $-5\sqrt{5} - \sqrt{3}$

(ii) Add : $\sqrt{7} - \sqrt{11}$ and $\sqrt{5} - \sqrt{11} + \sqrt{13}$

(iii) Multiply : $2\sqrt{2}$ by $5\sqrt{2}$.

(iv) Multiply : $(-3 + \sqrt{5})$ by 3.

(v) Divide : $7\sqrt{5}$ by $-14\sqrt{125}$

(vi) Divide : $2\sqrt{216} - 3\sqrt{27}$ by 3.

**Part (C)**

67. Simplify :

(i) $(2\sqrt{2} + 3\sqrt{3})(2\sqrt{2} - 3\sqrt{3})$ (ii) $(2\sqrt{8} - 3\sqrt{2})^2$

(iii) $(\sqrt{7} + \sqrt{6})^2$ (iv) $(6 - \sqrt{2})(2 + \sqrt{3})$

68 Evaluate :

(i) $\dfrac{2^{38} + 2^{37} + 2^{36}}{2^{39} + 2^{38} + 2^{37}}$ (ii) $\left[\left(64^{\frac{1}{2}}\right)^{\frac{1}{6}}\right]^2$

69. Find the value of a if $\dfrac{6}{3\sqrt{2} - 2\sqrt{3}} = 3\sqrt{2} - a\sqrt{3}$.

70. Simplify : $\left[5\left(8^{1/3} + 27^{1/3}\right)^3\right]^{\frac{1}{4}}$

71. Simplify : $\dfrac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}}$

72. If $5^{2x-1} - (25)^{x-1} = 2500$, then find the value of *x*.

**Part (D)**

73. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$.

74. Rationalise the denominator of $\frac{1}{\sqrt{3}+\sqrt{5}+\sqrt{7}}$

75. Find *a* and *b* if $\frac{7+3\sqrt{5}}{2+\sqrt{5}} - \frac{7-3\sqrt{5}}{2-\sqrt{5}} = a + b\sqrt{5}$

76. If $x = (3-2\sqrt{2})$, show that $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = \pm 2$

77. If $xyz = 1$, then simplify

$$(1+x+y^{-1})^{-1} \times (1+y+z^{-1})^{-1} \times (1+z+x^{-1})^{-1}$$

78. Find the value of *x* if

(i) $25^{2x-3} = 5^{2x+3}$ (ii) $(4)^{2x-1} - (16)^{x-1} = 384$

79. Evaluate : $\frac{64^{\frac{a}{6}}}{4^a} \times \frac{2^{2a+1}}{2^{a-1}}$

80. Simplify : $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{a-c}+x^{b-c}}$

81. Simplify : $\left(\frac{x^a}{x^{-b}}\right)^{a-b} \times \left(\frac{x^b}{x^{-c}}\right)^{b-c} \times \left(\frac{x^c}{x^{-a}}\right)^{c-a}$

82. Show that :

$$\frac{1}{(3-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} - \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-2)} = 5$$

83. If $a = \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}}$ and $b = \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}}$, then find the value of $a^2 + b^2 + ab$.

84. Simplify : $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

85. If $x = 9 - 4\sqrt{5}$, then find

(i) $x + \frac{1}{x}$ (ii) $x - \frac{1}{x}$ (iii) $x^2 + \frac{1}{x^2}$ (iv) $x^2 - \frac{1}{x^2}$

(v) $x^3 + \frac{1}{x^3}$ (vi) $x^3 - \frac{1}{x^3}$ (vii) $\sqrt{x} + \frac{1}{\sqrt{x}}$ (viii) $\sqrt{x} - \frac{1}{\sqrt{x}}$

(ix) $x^4 + \frac{1}{x^4}$ (x) $x^6 + \frac{1}{x^6}$ (xi) $x + \frac{14}{x}$

86. If $a = 1 + \sqrt{7}$, find the value of $\frac{-6}{a}$

87. If $p = 5 - 2\sqrt{6}$, Find $p^2 + \frac{1}{p^2}$

88. Express $0.3\overline{178}$ in the form of p/q where p and q are integers and $q \neq 0$.

89. If $\sqrt{2} = 1.414$, then find the value of $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

90. Find the value of

$$\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{2}{(243)^{\frac{-1}{5}}}$$

# CHAPTER-1
# NUMBER SYSTEMS
# ANSWERS

1. b) 8

2. b) $(16)^{3/2}$

3. c) 1

4. c) $\frac{42}{99}, \frac{4}{9}$

5. c) a rational number

6. c) $\frac{7}{9}$

7. d) Both (B) and (C)

8. b) –1

9. b) 1

10. a) a rational number

11. c) 243

12. c) 5

13. b) $2 - \sqrt{3}$

14. c) 1

15. a) $\sqrt{2}$

16. c) Every rational number is an integer

17. d) Non-ternimating non-recurring

18. c) There are infinitely many rational numbers

19. c) $\sqrt{7}$

20 c) a real number

21. b) $4\sqrt{3}$

22. c) 19

23. c) $\left(\sqrt{9^3}\right)^{2/3}$

24. d) Sometimes rational, sometimes irrational

25. c) 1.5

26. an irrational

27. an irrational

28. Terminating, recurring

29. non-terminating non-recurring

30. an irrational

31. infinitely many

32. infinitely many

33. infinitely many

34. rational

35. False

36. True

37. False

38. False

39. False

40. False

41) $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}$

42) $\frac{17}{8} = 2.125, \frac{3}{15} = 0.2, \frac{2}{7} = 0.\overline{285714}, \frac{50}{3} = 16.\bar{6}$

43) $\frac{15}{63}, \frac{16}{63}, \frac{17}{63}, \frac{18}{63}$ (other answers are possible).

44) $\sqrt{23} = 4.795, \sqrt{24} = 4.898$

45) 4.8010010001 ....., 4.8020020002 ....., (other answers are possible)

46) 2.1, 2.010010001 ....., (other answers are possible).

48) 6

50) 4

51) 2016.1010010001 ..... ; 2016.2020020002 .....; (other answers are possible)

54) 0.909009000 .....; 1.10100100010000 ..... (other answers are possible)

55) 1 56) $\frac{1}{10^5}$ 57) 36 58) 14

59) $x = \frac{1}{4}$ 60) 1 61) $(60)^{2/5}$

62) (i) Terminating Decimal (ii) Non Terminating Repeating Decimal

(iii) Non-Terminating Repeating Decimal

(iv) Terminating Decimal (v) Non-Terminating Repeating Decimal

(vi) Terminating Decimal (vii) Terminating Decimal

63) (i) Non-Terminating Repeating Decimal (Rational).

(ii) Terminating Decimal (Rational).

(iii) Non-Terminating Non-Repeating Decimal (Irrational).

(iv) Terminating Decimal (Rational)

(v) Non-Terminating Repeating Decimal (Rational)

(vi) Non-Terminating Repeating Decimal (Rational)

(vii) Non-Terminating Non-Repeating Decimal (Irrational)

(viii) Non-Terminating Non-Repeating Decimal (Irrational)

(ix) Non-Terminating Non-Repeating Decimal (Irrational)

(x) Non-Terminating Repeating Decimal (Rational).

64. (i) Irrational (ii) Rational (iii) Rational (iv) Irrational

(v) Rational (vi) Irrational (vii)Rational (viii)Rational

(ix) Irrational (x) Rational

65. (i) $0.0875 = \frac{7}{80}$ (ii) $\frac{2123456789}{1000000000}$ (iii) $\frac{2}{11}$

(iv) $\frac{433}{990}$ (v) $\frac{1643}{450}$

66. (i) $5\sqrt{3}$ (ii) $\sqrt{5} - 2\sqrt{11} + \sqrt{7} + \sqrt{13}$ (iii) 20

(iv) $-9 + 3\sqrt{5}$ (v) $-\frac{1}{10}$ (vi) $4\sqrt{6} - 3\sqrt{3}$

67. (i) –19 (ii) 2 (iii) $13 + 2\sqrt{42}$

(iv) $12 + 6\sqrt{3} - 2\sqrt{2} - \sqrt{6}$

68. (i) $\frac{1}{2}$ (ii) 2

69. a = –2 70. 5 71. $\frac{3375}{512}$

72. x = 3 73. $\frac{167}{90}$

74. $\frac{1}{59}\left(9\sqrt{3}+5\sqrt{5}+\sqrt{7}-2\sqrt{105}\right)$

75. a = 0, b = 2

77. $\frac{1}{(1+y+xy)(1+z+yz)(1+x+zx)}$

78. (i) $\frac{9}{2}$ (ii) $\frac{11}{4}$

79. 4 80. 1 81. 1

83. $a^2+b^2+ab=675$

84. 0

85. (i) 18 (ii) $-8\sqrt{5}$ (iii) 322

(iv) $-144\sqrt{5}$ (v) 5778 (vi) $-2584\sqrt{5}$

(vii) $2\sqrt{5}$ (viii) 4 (ix) 103682

(x) 33385282 (xi) $8\sqrt{3}-14\sqrt{2}$

86. $1-\sqrt{7}$ 87. 98 88. $\frac{635}{1998}$

89. 28.28 90. 214

## Practice Test

## NUMBER SYSTEMS

Time : 50 Min. M.M. 20

1. If $\frac{4}{a} = \frac{a^2}{16}$, then check whether a is rational or irrational number. (1)

2. Find two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$. (1)

3. Simplify:

   $4\sqrt{3} + 3\sqrt{48} - \frac{5}{2}\sqrt{\frac{4}{3}}$ (2)

4. If $\sqrt{3} = 1.732$, find the value of $\frac{2}{\sqrt{3} - 1}$ (2)

5. Find the value of x and y (3)

   $$\frac{\sqrt{11} - \sqrt{7}}{\sqrt{11} + \sqrt{7}} = a - b\sqrt{77}$$

6. Represent $(2 + \sqrt{3})$ on the number line. (3)

7. Simplify : (4)

   $$\frac{16 \times 2^{a+1} - 4 \times 2^{a}}{16 \times 2^{a+2} - 2 \times 2^{a+2}}$$

8. Express the following in the form p/q where p and q are integers and $q \neq 0$ (4)

   $0.\overline{4} + 0.1\overline{8}$