➤ **Choose the right answer from the given options. [1 Marks Each]** **[10]**

1. The product $(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$ is equal to:

(A) $a^6 + b^6$ (B) $a^6 - b^6$ (C) $a^3 - b^3$ (D) $a^3 + b^3$

Ans. :

b. $a^6 - b^6$

Solution:

$(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

$= (a^2 - b^2)(a^2 + b^2 - ab)(a^2 + b^2 - ab)$

$= (a^2 - b^2)\left\{(a^2 + b^2)^2 - (ab)^2\right\}$

$= (a^2 - b^2)\{a^4 + b^4 + 2a^2b^2 - a^2b^2\}$

$= (a^2 - b^2)\{a^4 + b^4 + a^2b^2\}$

$= \{a^6 + a^2b^4 + a^4b^2 - b^2a^4 - b^6 - b^4a^2\}$

$= a^6 - b^6$

Hence, correct option is (b).

2. The zeros of the polynomial $p(x) = x^2 + x - 6$ are:

(A) 2, 3 (B) -2, 3 (C) 2, -3 (D) -2, -3

Ans. :

c. 2, -3

Solution:

Let p(x) be a polynomial. If $\mathrm{p}(\alpha) = 0$, then we say that α is a zero of a polynomial.

$p(x) = x^2 + x - 6$

Now, p(x) = 0

$\Rightarrow x^2 + x - 6$

$\Rightarrow x^2 + 3x - 2x - 6 = 0$

$\Rightarrow x(x + 3) - 2(x + 3) = 0$

$\Rightarrow (x - 2)(x + 3) = 0$

$\Rightarrow (x - 2) = 0$ or $(x + 3) = 0$

$\Rightarrow x = 2$ or $x = -3$

$\therefore$ 2 and -3 are the zeroes of the polynomial p(x).

3. If x + y + z = 0 then $x^3 + y^3 + z^3$ is :

(A) 3xyz (B) xyz (C) 2xyz (D) 0

Ans. :

a. 3xyz

Solution:

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2\ xy - yz - zx)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$ if x + y + z = 0, then $x^3 + y^3 + z^3$ is 3xyz.

4. If (x + 5) is a factor of = $x^3 - 20x + 5k$ then k = ?

(A) -5 (B) 5 (C) 3 (D) -3

Ans. :

b. 5

Solution:

$p(x) = x^3 - 20x + 5k$

Now, $x + 5 = 0 \Rightarrow x = (-5)$

By factor theorem,

p(-5) = 0

$\Rightarrow (-5)^3 - 20(-5) + 5k = 0$

$\Rightarrow -125 + 100 + 5k = 0$

$\Rightarrow -25 + 5k = 0$

$\Rightarrow$ 5k = 25
$\Rightarrow$ k = 5

5. Write the correct answer in the following :
If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, the value of b is.
(A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Ans. :
c. $\frac{1}{4}$
Solution:
$49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$
$\Rightarrow 49x^2 - b = \left(7x\right)^2 - \left(\frac{1}{2}\right)^2$
$49^2 - \frac{1}{4}[\because (a+b)(a-b) = a^2 - b^2]$
So, we get $b = \frac{1}{4}$.

6. Write the correct answer in the following :
Which of the following is a factor of $(x + y)^3 - (x^3 + y^3)$?
(A) $x^2 + y^2 + 2xy$ (B) $x^2 + y^2 - xy$ (C) xy^2 (D) 3xy

Ans. :
d. 3xy
Solution:
$(x + y)^3 - (x^3 + y^3) = x^3 + y^3 + 3xy(x + y) - x^3 - y^3$
$[(a + b)^3 = a^3 + b^3 + 3ab(a + b)]$
= 3xy(x + y)
So, 3xy is a factor of $(x + y)^3 - (x^3 + y^3)$.

7. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is:
(A) 5x - 1 (B) 5 - x (C) 10x (D) 5 + x

Ans. :
c. 10x
Solution:
Now, $(25x^2 - 1) + (1 + 5x)^2$
$= 25x^2 - 1 + 1 + 25x^2 + 10x$ [using identity, $(a + b)^2 = a^2 + b^2 + 2ab$]
$= 50x^2 + 10x = 10x(5x + 1)$
Hence, one of the factor of given polynomial is 10x.

8. If $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^2$, then k =
(A) 1 (B) 2 (C) 8 (D) 4

Ans. :
c. 8
Solution:
We have,
$= (x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^3$
$= (x + y - x + y)^3 + 3(x + y)(x - y)(x + y - x + y) - 6y(x^2 - y^2) = ky^3$
$= 2y^3 + 6y(x^2 - y^2) - 6y(x^2 - y^2) = ky^3$
$= 8y^3 = ky^3$
= k = 8

9. The value of $\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)$ is:
(A) $x^4 - \frac{1}{x^4}$ (B) $x^2 + \frac{1}{x^2} - 2$ (C) $x^3 + \frac{1}{x^3} + 2$ (D) $x^4 + \frac{1}{x^4}$

Ans. :
a. $x^4 - \frac{1}{x^4}$
Solution:
$\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)$

$= \left(x^2 - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right)$ [Using identity (a + b) (a - b) = $a^2 - b^2$]

$= x^4 - \frac{1}{x^4}$ [Using identity (a + b) (a - b) = $a^2 - b^2$]

10. If $x^4 + \frac{1}{x^4} = 194$, than $x^3 + \frac{1}{x}^3 =$

(A) 64 (B) 52 (C) 76 (D) None of these.

Ans. :

b. 52

Solution:

$\left(x^4 + \frac{1}{x^4}\right) = 194$

$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right) + 2 \times x^2 \times \frac{1}{x^2} = 194 + 2 \times x^2 \times \frac{1}{x^2}$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 196$

$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{196} = 14$

Now,

$\Rightarrow (x^2) + \left(\frac{1}{x^2}\right) + 2 \times x \times \frac{1}{x} = 14 + 2 \times x \times \frac{1}{x}$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 16$

$\Rightarrow x + \frac{1}{x} = \sqrt{16} = 4$

Now, $\left(x + \frac{1}{x}\right)^3 = (4)^3$

$\Rightarrow (x)^3 + \left(\frac{1}{x}\right)^3 + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 64$

$\Rightarrow (x^3) + \left(\frac{1}{x^3}\right) + 3(4) = 64$

$\Rightarrow (x^3) + \left(\frac{1}{x^3}\right) = 64 - 12 = 52$

➤ **Answer the following short questions. [2 Marks Each]** **[8]**

11. Factorise:

$(5a - 7b)^3 + (7b - 9c)^3 + (9c - 5a)^3$

Ans. : Put (5a - 7b) = x, (7b - 9c) = y, (9c - 5a) = z. Here, x + y + z = 5a - 7b + 9c - 5a + 7b - 9c = 0 Thus, We have: $(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3 = x^3 + z^3 + y^3 = 3xyz$ [When x + y + z = 0, $x^3 + y^3 + z^3 = 3xyz$] = 3(5a - 7b)(9c - 5a)(7b - 9c)

12. Factorise:

$7(x - 2y)^2 - 25(x - 2y) + 12$

Ans. : Let x - 2y = z Then, $7(x - 2y)^2 - 25(x - 2y) + 12 = 7z^2 - 25z + 12 = 7z^2 - 21z - 4z + 12$ = 7z(z - 3) - 4(z - 3) = (z - 3)(7z - 4) Now replace z by (x - 2y), we get $7(x - 2y)^2 - 25(x - 2y) + 12$ = (x - 2y - 3)[7(x - 2y) - 4] = (x - 2y - 3)(7x - 14y - 4)

13. Factorise:

$2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$

Ans. : $2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$ $= \left(\sqrt{2}a\right)^3 + \left(\sqrt{3}b\right)^3 + c^3 - 3\left(\sqrt{2}a\right)\left(\sqrt{3}b\right)c$ We know $x^3 + y^3 + z^3 - 3xyz = (x + y + z)\left(x^2 + y^2 + z^2 - xy - yz - zx\right)$ $x = \sqrt{2}a,\ y = \sqrt{3}b,\ z = c$ $\left(\sqrt{2}a\right)^3 + \left(\sqrt{3}b\right)^3 + c^3 - 3\left(\sqrt{2}a\right)\left(\sqrt{3}b\right)c$

$= \left(\sqrt{2}a + \sqrt{3}b + c\right)\left(2a^2 + 3b^2 + c^2 - \sqrt{6ab} - \sqrt{3bc} - \sqrt{2}ac\right)$

14. Factorise:

Prove that $\frac{0.85\times0.85\times0.85+0.15\times0.15\times0.15}{0.85\times0.85-0.85\times0.15+0.15\times0.15} = 1$

Ans. : Let 0.85 = a and 0.15 = b Then, we have $\text{L.H.S.} = \frac{0.85\times0.85\times0.85+0.15\times0.15\times0.15}{0.85\times0.85-0.85\times0.15+0.15\times0.15} = \frac{a\times a\times a+b\times b\times b}{a\times a-a\times b+b\times b}$

$= \frac{a^3+b^3}{a^2-ab+b^2} = \frac{(a+b)\left(a^2-ab+b^2\right)}{\left(a^2-ab+b^2\right)} = a + b = 0.85 + 0.15 = 1 = \text{R.H.S.}$

➤ **Answer the following questions. [3 Marks Each]** **[12]**

15. Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$.

Ans. : $(a + b + c)^3 = [a + (b + c)]^3 = a^3 + 3a^2(b + c) + 3a(b + c)^2 + (b + c)^3 = a^3 + 3a^2b + 3a^2c + 3a(b^2 + 2bc + c^2) + (b^3 + 3b^2c + 3bc^2 + c^3) = a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3$

$= a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2c + 3c^2a + 3c^2b + 6abc$

$= a^3 + b^3 + c^3 + 3a^2(b + c) + a^3 + b^3 + c^3 + 3a^2(b + c)$ Hence, above result can be put in the form $(a + b + c)^3 = (a + b + c)^3 + 3(a + b)(b + c)(c + a)$ $\therefore$ $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$

16. Find the value of m so that 2x - 1 be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$.

Ans. : Let $p(x) = 8x^4 + 4x^3 - 16x^2 + 10x + m$ Since, 2x - 1 is a factor of p(x), then put

$p\left(\frac{1}{2}\right) = 0$ $\quad \therefore 8\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^3 - 16\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) + m = 0$ $\quad \Rightarrow 8 \times \frac{1}{16} + 4 \times \frac{1}{8} - 16 \times \frac{1}{4} + 10\left(\frac{1}{2}\right) + m = 0$

$\Rightarrow \frac{1}{2} + \frac{1}{2} - 4 + 5 + m = 0$ $\Rightarrow 1 + 1 + m = 0$ $\therefore m = -2$ Hence, the value of m is -2.

17. For the polynomial $\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$, write.
    i. The degree of the polynomial.
    ii. The coefficient of x^3.
    iii. The coefficient of x^6.
    iv. The constant term.

Ans. : $\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$ $\frac{x^3}{5} + \frac{2x}{5} + \frac{1}{5} - \frac{7}{2}x^2 - x^6$

   i. We know that highest power of variable in a polynomial is the degree of the polynomial. In the given polynomial, the term with highest of x is - x, and the exponent of x in this term in 6.
   ii. The coefficient of x^3 is $\frac{1}{5}$
   iii. The coefficient of x^6 is -1.
   iv. The constant term is $\frac{1}{5}$

18. Show that (x - 2), (x + 3) and (x - 4) are factors of $x^3 - 3x^2 - 10x + 24$.

Ans. : $F(x) = x^3 - 3x^2 - 10x + 24$ Let, $x - 2 = 0 \Rightarrow x = 2$ $f(2) = 2^3 - 3(2)^2 - 10(2) + 24 = 8 - 12 - 20 + 24 = 32 - 32 = 0$ Let, $x + 3 = 0 \Rightarrow x = -3$ $f(-3) = (-3)^3 - 3(-3)^2 - 10(-3) + 24 = -27 - 3(9) + 30 + 24 = -27 - 27 + 30 + 24 = -54 + 54 = 0$ Let, $x - 4 = 0 \Rightarrow x = 4$ $f(4) = 4^3 - 3(4)^2 - 10(4) + 24 = 64 - 3(16) - 40 + 24 = 64 - 48 - 40 + 24 = 88 - 88 = 0$ $\because f(2) = 0, f(-3) = 0, f(4) = 0$ $\therefore$ By factore theoram (x - 2), (x + 3) and (x - 4) are factors of $x^3 - 3x^2 - 10x + 24$.

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